Decline Curves
Part 2

Hyperbolic Equations $\quad 0 < b < 1$

Calculate the time interval spanning the initial and final rate for the hyperbolic case.

$$t = \frac{\left[\frac{q_i}{q_2}\right]^b - 1}{bD_i}$$

Rearrange the rate-decline relationship

$$\frac{D_i}{D_2} = \left[\frac{q_i}{q_2}\right]^b$$

For cumulative production

$$Q_p = \frac{q_i}{D_i(1-b)}\left[1 - \frac{1}{(1+bD_it)^{(1-b)/b}}\right]$$

Sub in the flow equation for $b \neq 0$

$$Q_p = \frac{q_i}{D_i(1-b)}\left[1 - \left[\frac{q_i}{q_2}\right]^{1-b}\right]$$

Assume $q_2 = 0$, maximum cumulative production

$$Q_{p\,max} = \frac{q_i}{D_i(1-b)}$$

$$q = q_i (1 + bD_i t)^{-\frac{1}{b}}$$

q = production rate at time t
q_i = initial rate of production
D_i = initial nominal decline rate
b = hyperbolic exponent (~ 0 and 1) (Special case b = 1 is harmonic decline)

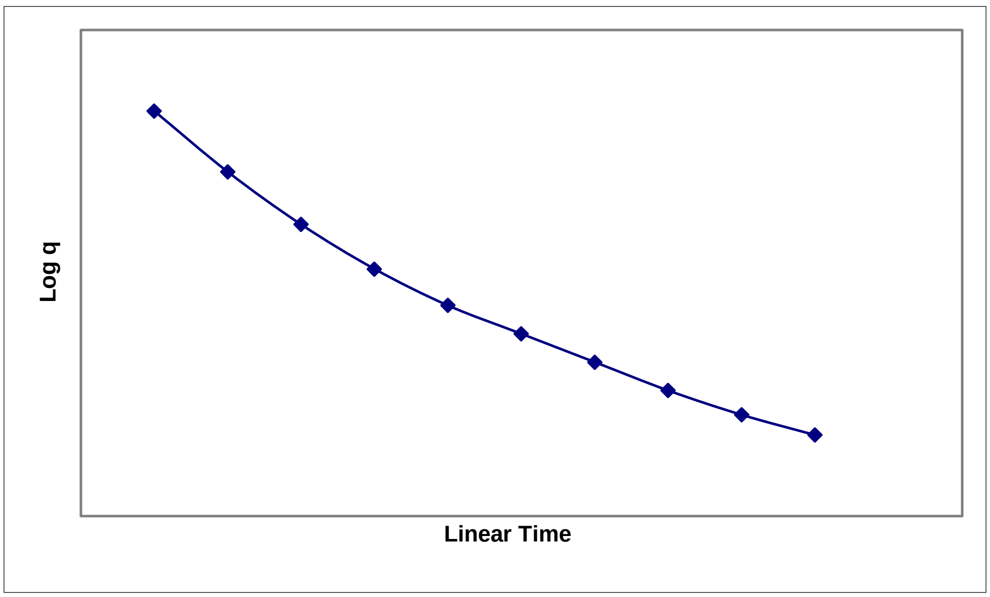
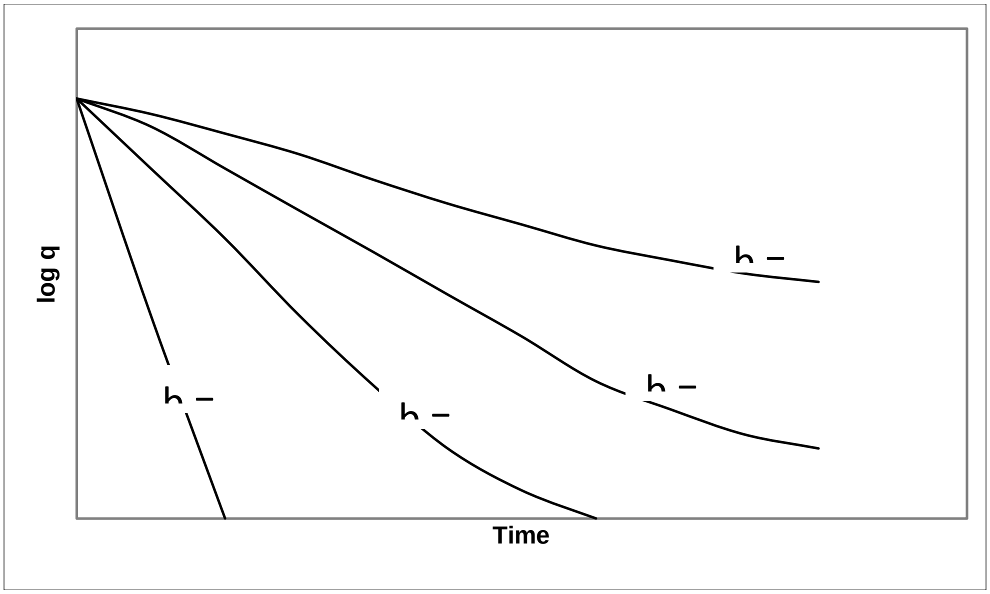


Figure 8-2: Hyperbolic Decline Plot.

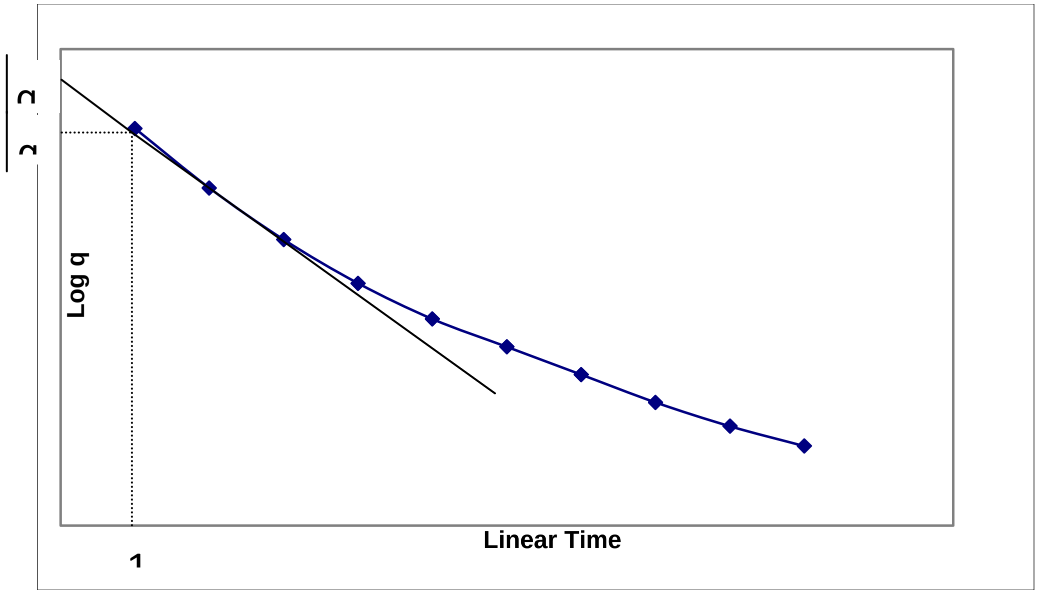


Hyperbolic Decline b values.

Effective decline rate is

$$D_{ei} = \frac{q_i - q}{q_i}$$

where a tangent is drawn at t = 0



Hyperbolic Decline Plot.

Initial nominal decline rate is

$$D_i = \frac{(1 - D_{ei})^{-b} - 1}{b}$$

Cumulative production

$$N_p = \int_0^t q\,dt$$

$$N_p = \int_0^t q_i (1 + bD_i t)^{-\frac{1}{b}} dt$$

Integrating (b $\neq$ 1)

$$N_p = \frac{1}{-\frac{1}{b}+1}\frac{q_i}{bD_i}\left|(1+bD_i t)^{-\frac{1}{b}+1}\right|_0^t$$

$$N_p = \frac{q_i}{(b-1)D_i}\left[(1+bD_i t)^{\frac{1-b}{-b}} - 1\right]$$

$$N_p = \frac{q_i b}{(1-b)D_i}\left[q_i^{1-b} - q^{1-b}\right]$$

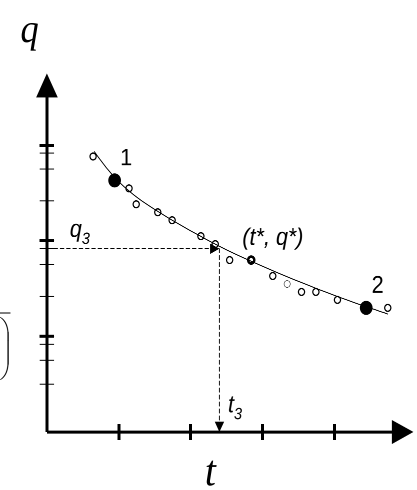
NOTE: When dealing with hyperbolic decline, we will use the cumulative equation for each individual year. Then to get year production we will subtract from the total to get each year.

To find *b* for hyperbolic decline

1. Select points (t_1, q_1) and (t_2, q_2)
2. Read t_3 at $q_3 = \sqrt{q_1 q_2}$
3. Calculate $\left(\frac{b}{a}\right) = \frac{t_1 + t_2 - 2t_3}{t_3^2 - t_1 t_2}$
4. Find q_0 at $t = 0$
5. Pick up any point (t_*, q_*)
6. Use

$$q_* = \frac{q_0}{\left(1+\left(\frac{b}{a}\right)t_*\right)^a} \Rightarrow a = \frac{\log\left(\frac{q_0}{q_*}\right)}{\log\left(1+\left(\frac{b}{a}\right)t_*\right)}$$

7. Finally $b = \left(\frac{b}{a}\right)a$



The *b* calculated here is *D* in the above equations, and 1/a is the *b* in the above equations

Example 4: The following data for a hyperbolic decline is known

$$q_i = 100 \text{ BOPD}$$

$$D_i = 0.5 / yr$$

$$b = 0.9$$

Find production for the first year

$$q = q_i (1 + bD_i t)^{-\frac{1}{b}}$$

$$q = 100(1 + 0.9(0.5)(1))^{-\frac{1}{0.9}}$$

$$q = 66.176 \text{ BOPD}$$

$$N_p = \frac{q_i^b}{(1-b)D_i}\left[q_i^{1-b} - q^{1-b}\right]$$

$$N_p = \frac{(100)^{0.9}}{(0-0.9)0.5}\left[(100)^{1-0.9} - (66.176)^{1-0.9}\right]$$

$$N_p = 126.915(1.585 - 1.521)$$

$$N_p = 80.889 \ \frac{\text{STB / day}}{1 / \text{yr}}\left(365 \frac{\text{days}}{\text{year}}\right)$$

$$N_p = 29{,}524 \text{ STB}$$

Example 5: For the previous example find the yearly production for 5 years.

Step 1: Tabulate the producing rate at the end of each year using

$$q = q_i\left(1 + bD_i t\right)^{-\frac{1}{b}}$$

Step 2: Calculate the cumulative production from time 0 to the end of each year using

$$N_p = \frac{q_i^b}{(1-b)D_i}\left[q_i^{1-b} - q^{1-b}\right]$$

Step 3: Subtract the previous year's cumulative from this year's cumulative to fine the production during the year.

Table 8-2: Examples 4 and 5 Data

Year	Rate of End of Year BOPD	Cumulative Production Thru Year STB	Yearly Production STB
0	100	0	
1	66.176	29,524	29,524
2	49.009	50,249	20,724
3	38.699	66,115	15,867
4	31.854	78,914	12,799
5	26.992	89,606	10,692