

## UNIVERSIDADE FEDERAL DA GRANDE DOURADOS Prof<sup>a</sup>. Karla Lima

Cálculo III

27 de Abril de 2019

(1) Calcule as integrais iteradas:

(a) 
$$\int_{1}^{4} \int_{0}^{2} (6x^{2}y - 2x) dy dx$$

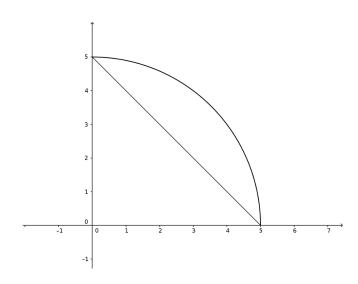
(b) 
$$\int \int_R \frac{xy^2}{x^2+1} dA$$
, onde  $R = [0,1] \times [-3,3]$ .

(c) 
$$\int \int_R x e^{xy} dA$$
, onde  $R = [1, 3] \times [0, 1]$ .

(d) 
$$\int_0^1 \int_x^{2x} (2x+4y)dydx$$
.

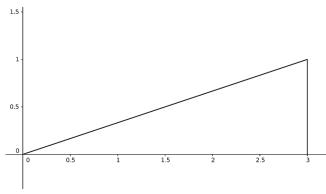
(e) 
$$\int_1^e \int_{\ln x}^1 x dy dx$$
.

(2) Calcule  $\int \int_R y dA$ , onde R é a região do primeiro quadrante compreendida pelo círculo  $x^2+y^2=25$  e a reta x+y=5.

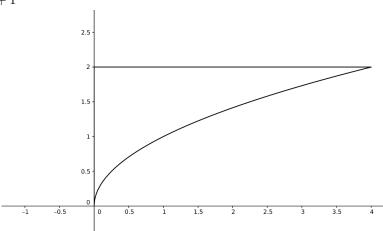


(3) Mude a ordem de integração e calcule as integrais abaixo:

(a) 
$$\int_0^1 \int_{3y}^3 e^{x^2} dx dy$$

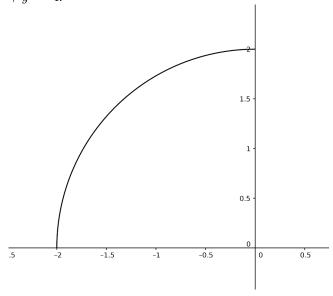


(b) 
$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3 + 1} dy dx$$

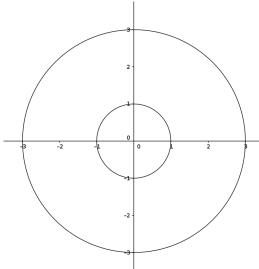


(4) Usando coordenadas polares, calcular:

(a)  $\int \int_R \frac{dA}{1+x^2+y^2}$ , onde R é a região do segundo quadrante delimitada pela circunferência  $x^2+y^2=4$ .



(b) 
$$\int \int_R \sqrt{x^2 + y^2} dA$$
, onde  $R$  é a região delimitada por  $x^2 + y^2 = 1$  e  $x^2 + y^2 = 9$ .



- (5) Calcule as integrais triplas abaixo:
  - (a)  $\iiint_B 2y \mathrm{sen}(yz) dV \text{ onde } B = [0,\pi] \times [0,\tfrac{\pi}{2}] \times [0,\tfrac{\pi}{3}].$
  - (b)  $\int_1^3 \int_x^{x^2} \int_0^{\ln z} x e^y dy dz dx.$
  - (c)  $\int_{1/3}^{1/2} \int_0^{\pi} \int_0^1 zx \operatorname{sen}(xy) dz dy dx$ .
- (6) Calcule  $\int \int \int_E x^2 + y^2 dV$ , onde E é o cilindro  $x^2 + y^2 \le 1, \ 1 \le z \le 4.$

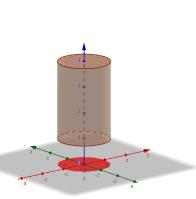


Figure 1. Região E

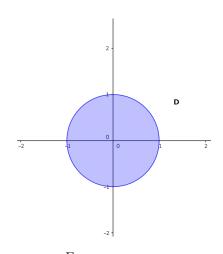


FIGURE
2. Projeção
no plano XY

(7) Calcular  $\int \int \int_E xy dV$ , onde E é a região delimitada pelos planos  $y=0,\ y=4,\ z=0$  e por  $z=4-x^2.$ 

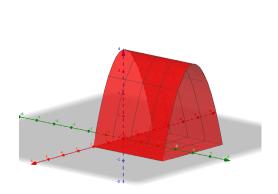


FIGURE 3. Região E

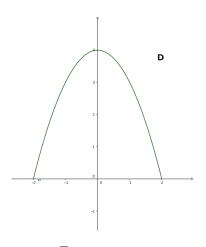
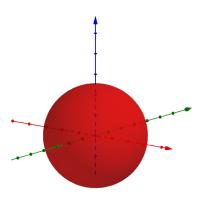
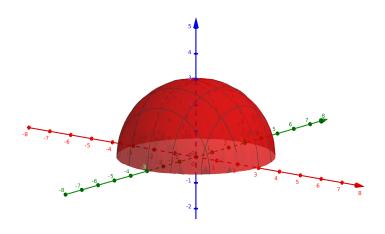


FIGURE 4. Projeção no plano ZX

(8) Calcular  $\int \int \int_E x^2 + y^2 + z^2 dV$ , onde E é a esfera  $x^2 + y^2 + z^2 = 25$ .



(9) Calcular  $\int \int \int_E (9-x^2-y^2) dV$ , onde E é a semi-esfera  $x^2+y^2+z^2=9,\,z\geq 0.$ 



## Gabarito

- (1) (a) 222 (b)  $9 \ln 2$ (c)  $e^3 e 2$ (d)  $\frac{8}{3}$ (e)  $\frac{e^2 3}{4}$
- (2)  $\frac{125}{6}$
- (3) a)  $\frac{1}{6}(e^9 1)$ 
  - b)  $\frac{\ln 9}{3}$
- (4) a)  $\frac{\pi}{4} \ln 5$ 
  - b)  $\frac{52\pi}{3}$
- (5) b)  $\pi^2 6 \operatorname{sen}(\frac{\pi^2}{6})$ 
  - c)  $\frac{118}{3}$
- $\begin{array}{c}
  3 \\
  d) \frac{\pi 6 + 3\sqrt{3}}{12\pi} \\
  (6) \frac{3\pi}{2}
  \end{array}$
- (7) 0
- (8)  $2500\pi$