P2 - 27/05/19

01

a) $\iint_R \frac{x^2 y}{y^2+4}\, dA$, onde $R = [-2,2] \times [0,1]$.

$$\iint_R \frac{x^2 y}{y^2+4}\, dA = \int_{-2}^{2} \int_{0}^{1} \frac{x^2 y}{y^2+4}\, dy\, dx$$

$\rightarrow u = y^2+4$
$du = 2y\,dy$

$$= \int_{-2}^{2} \frac{x^2}{2} \ln(y^2+4) \Big|_0^1 dx$$

$$= \int_{-2}^{2} \frac{x^2}{2}\left[\ln 5 - \ln 4\right] dx = \frac{\ln 5 - \ln 4}{2} \cdot \frac{x^3}{3} \Big|_{-2}^{2}$$

$$= \frac{\ln 5 - \ln 4}{2}\left(\frac{8}{3} - \left(-\frac{8}{3}\right)\right) = \frac{16}{6}(\ln 5 - \ln 4) = \frac{8}{3}\ln\left(\frac{5}{4}\right).$$

b) $\iiint_B x\, \text{sen}(xy)\, dV$, onde $B = [0,1/2] \times [0,\pi] \times [0,\pi/3]$

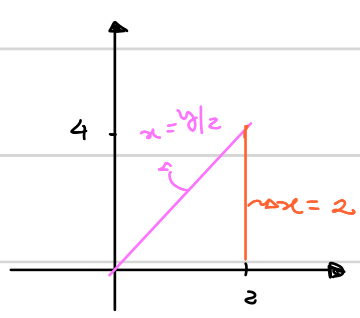
$$\iiint_B x\,\text{sen}(xy)\,dV = \int_0^{\pi/3}\int_0^{1/2}\int_0^{\pi} x\,\text{sen}(xy)\,dy\,dx\,dz$$

$$= \int_0^{\pi/3}\int_0^{1/2} -\cos(xy)\Big|_{y=0}^{y=\pi} dx\,dz = \int_0^{\pi/3}\int_0^{1/2} (1 - \cos(\pi x))\,dx\,dz$$

$$= \int_0^{\pi/3} x - \frac{\text{sen}(\pi x)}{\pi}\Big|_0^{1/2} dz = \int_0^{\pi/3}\left(\frac{1}{2} - \frac{\text{sen}(\pi/2)}{\pi}\right) dz$$

$$= \int_0^{\pi/3}\left(\frac{1}{2} - \frac{1}{\pi}\right) dz = \frac{\pi}{3}\left(\frac{1}{2} - \frac{1}{\pi}\right) = \frac{\pi}{6} - \frac{1}{3}$$

02 Calcule a integral $\int_0^4 \int_{y/2}^{2} e^{x^2}\, dx\, dy$.



Como não conseguimos calcular $\int_0^{y/2} e^{x^2} dx$ diretamente, mudamos a ordem de integração pois $\int e^{x^2} dy$ é possível de ser resolvida facilmente.

Assim, trocamos $0 \leq y \leq 4$ e $y/2 \leq x \leq 2$ por

$0 \leq x \leq 2$ e $0 \leq y \leq 2x$

Portanto,

$$\int_0^4 \int_{y/2}^2 e^{x^2}\, dx\, dy = \int_0^2 \int_0^{2x} e^{x^2}\, dy\, dx = \int_0^2 e^{x^2}\, y \Big|_0^{2x} dx$$

$$= \int_0^2 e^{x^2} \cdot 2x\, dx = e^{x^2} \Big|_0^2 = e^4 - 1.$$

$\hookrightarrow u = x^2$

$du = 2x\,dx$

(03) Para calcular o volume de um sólido E, basta calcular a integral $\iiint_E 1\, dV$. Usando esta informação, calcule o volume do sólido E delimitado pelo parabolóide $z = 4 - (x^2 + y^2)$ e pelo plano $z = 0$.

Temos que z está limitado pelas funções $z = 0$ e $z = 4 - (x^2 + y^2)$. Assim, $0 \leq z \leq 4 - (x^2 + y^2)$. Além disso, x e y estão limitados no disco $x^2 + y^2 \leq 4$; em coordenadas polares: $0 \leq r \leq 2$ e $0 \leq \theta \leq 2\pi$.

Calculando a integral utilizando coordenadas cilíndricas:

$$\iiint_E 1 \cdot dV = \int_0^{2\pi} \int_0^2 \int_0^{4 - r^2} 1 \cdot r\, dz\, dr\, d\theta = \int_0^{2\pi} \int_0^2 r z \Big|_0^{4 - r^2} dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^2 (4r - r^3)\, dr\, d\theta = \int_0^{2\pi} 2r^2 - \frac{r^4}{4} \Big|_0^2 d\theta = \int_0^{2\pi} 4\, d\theta$$

$$= 4\theta \Big|_0^{2\pi} = 8\pi.$$

(04) Calcule $\iiint_E z^2\, dV$, onde E é o sólido delimitado pelas semi-esferas $x^2 + y^2 + z^2 = 1$ e $x^2 + y^2 + z^2 = 9$, com $z \geq 0$.

Vamos utilizar coordenadas esféricas:

$x = \rho \operatorname{sen}\phi \cos\theta$, $y = \rho \operatorname{sen}\phi \operatorname{sen}\theta$ e $z = \rho \cos\phi$.

Assim, temos

* $z^2 = \rho^2 \cos^2\phi$

* $0 \leq \theta \leq 2\pi$, $0 \leq \phi \leq \pi/2$ e $1 \leq \rho \leq 3$

de onde segue que

$$\iiint_E x^2+y^2\,dV = \int_0^{2\pi}\int_0^{\pi/2}\int_1^3 \rho^2\cos^2\phi\,\rho^2 \operatorname{sen}\phi\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/2}\int_1^3 \cos^2\phi\operatorname{sen}\phi\,\rho^4\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/2} \cos^2\phi\operatorname{sen}\phi\,\frac{\rho^5}{5}\Big|_1^3\,d\phi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/2} \frac{242}{5}\cos^2\phi\operatorname{sen}\phi\,d\phi\,d\theta$$

$$= \int_0^{2\pi}\frac{242}{5}\left(-\frac{\cos^3\phi}{3}\Big|_0^{\pi/2}\right)d\theta = \int_0^{2\pi}\frac{242}{5}\cdot\frac{1}{3}\,d\theta = \frac{242}{15}\,\theta\Big|_0^{2\pi} = \frac{484\pi}{15}$$