

Particle Swarm Optimization (PSO)



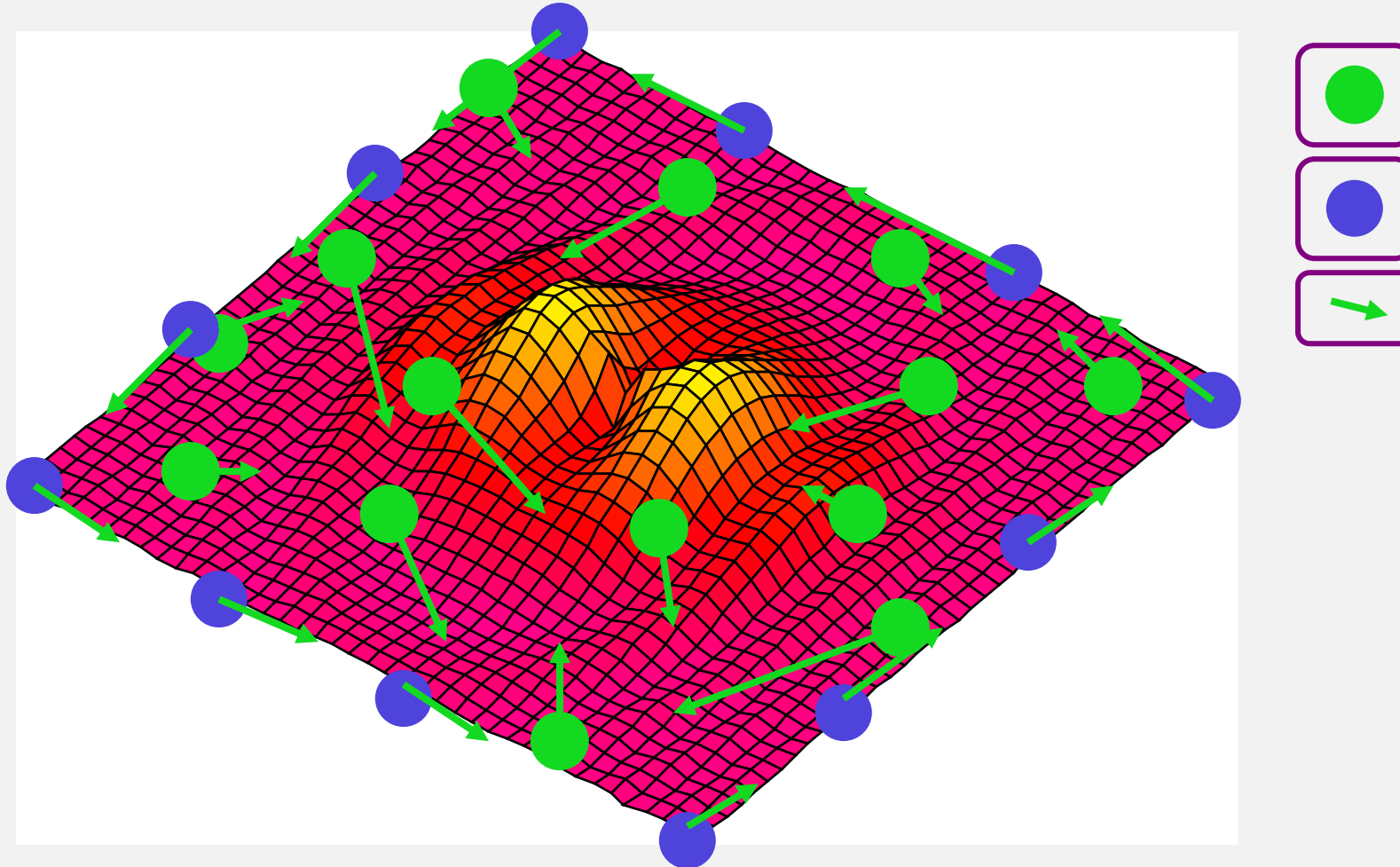
The basic idea

- Each particle is searching for the optimum.
- Each particle is moving and therefore has a velocity.
- Each particle remembers the position it had when it got its best result so far (personal best).
- But this is not good enough by itself; the particles need help deciding where to look.

The basic idea II

- **The particles in the swarm cooperate.** They exchange information about what they have discovered in the places they have visited.
- **Cooperation is very simple.** In the basic PSO it is like this:
 - One particle has a neighborhood associated with it.
 - A particle knows the fitnesses of its neighbors and uses the position of the one with the greatest fitness.
 - This position is simply used to adjust the velocity of the particle.

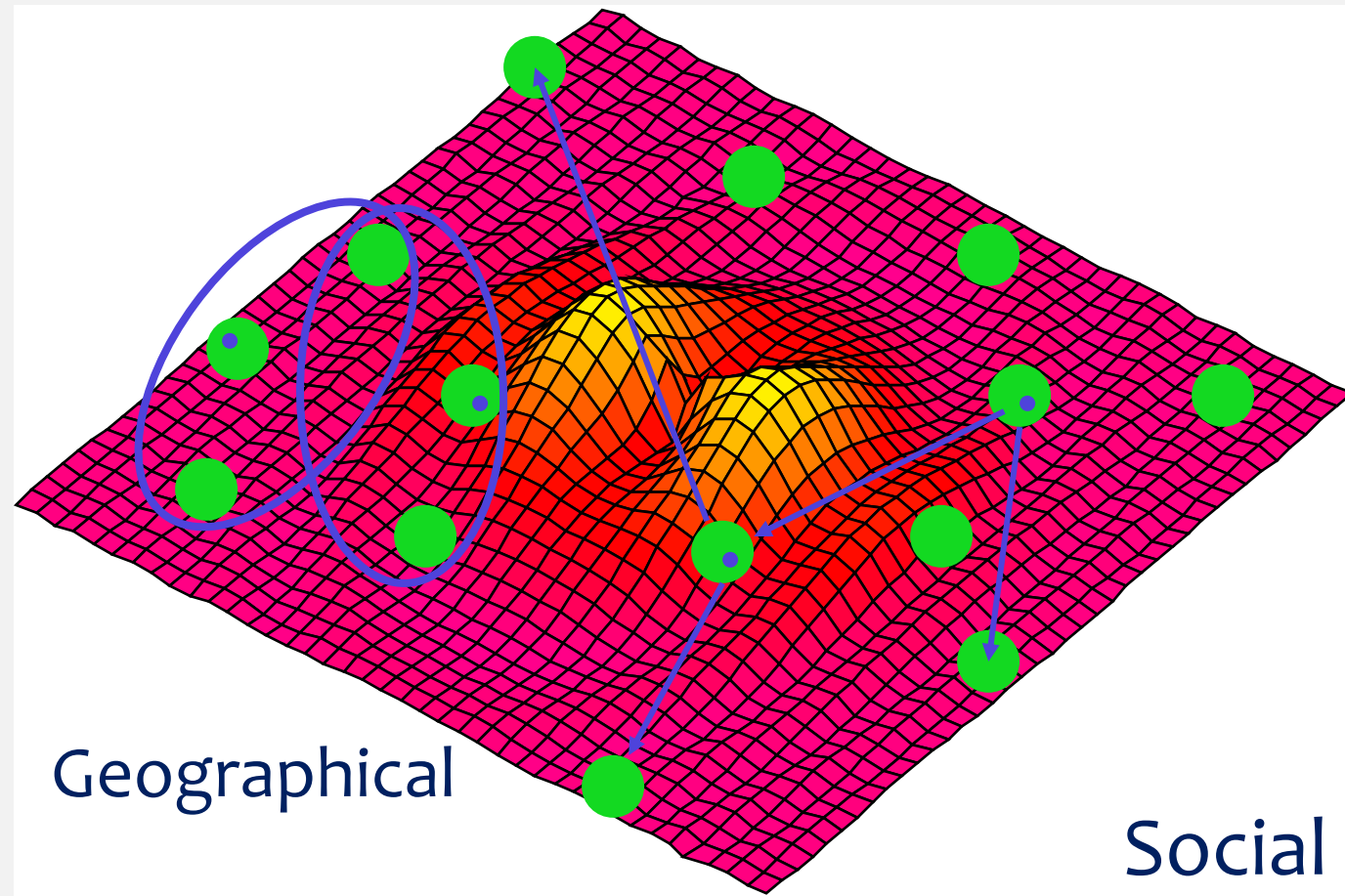
Initialization: Positions and velocities



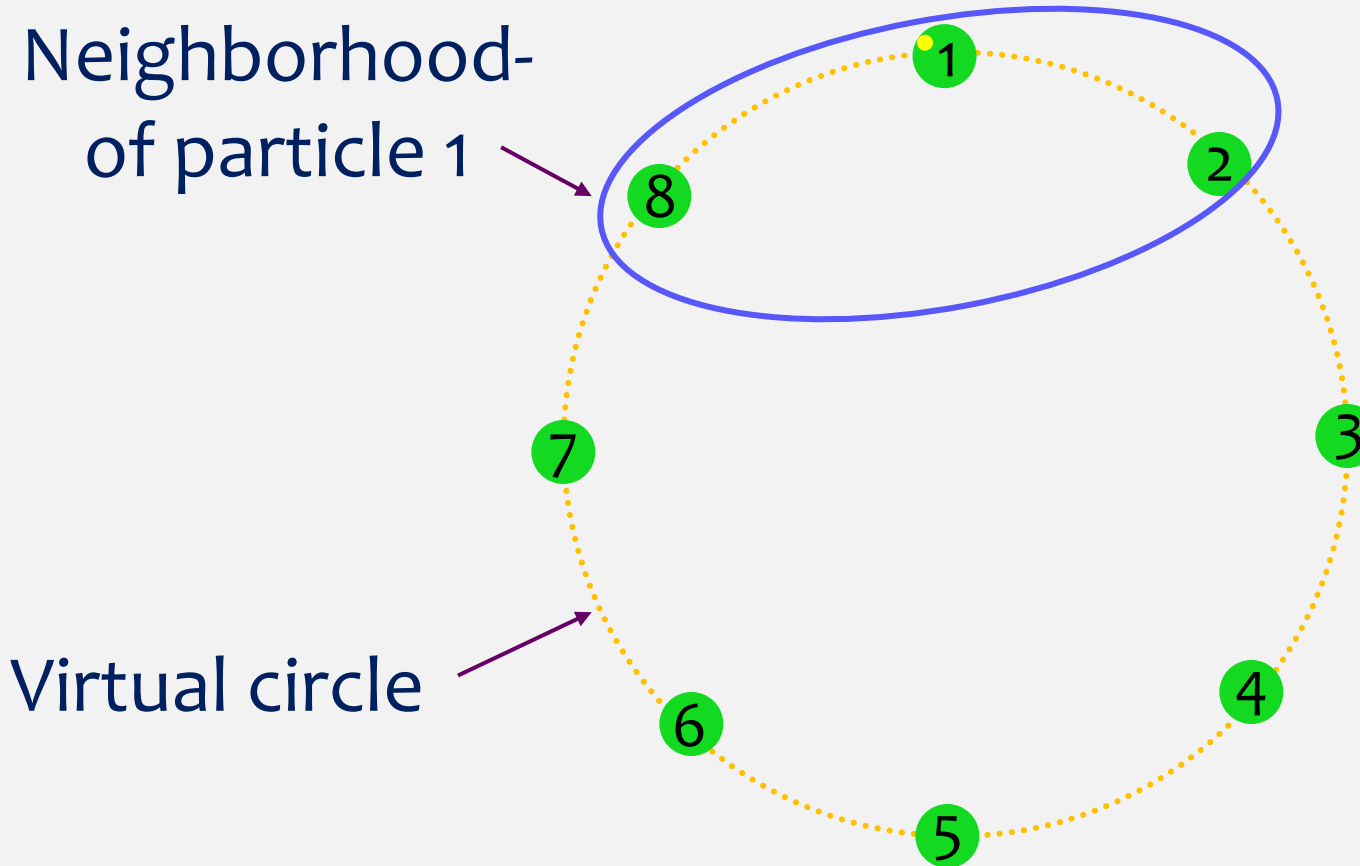
How a particle moves?

- At each time step, a particle must move to a new position. It does this by adjusting its velocity.
 - The adjustment is essentially this:
 - The current velocity PLUS
 - A portion randomly weighted in the direction of its personal best PLUS
 - A portion randomly weighted in the direction of its best neighbor.
- Having calculated the new velocity, its position is simply its previous position plus its new velocity.

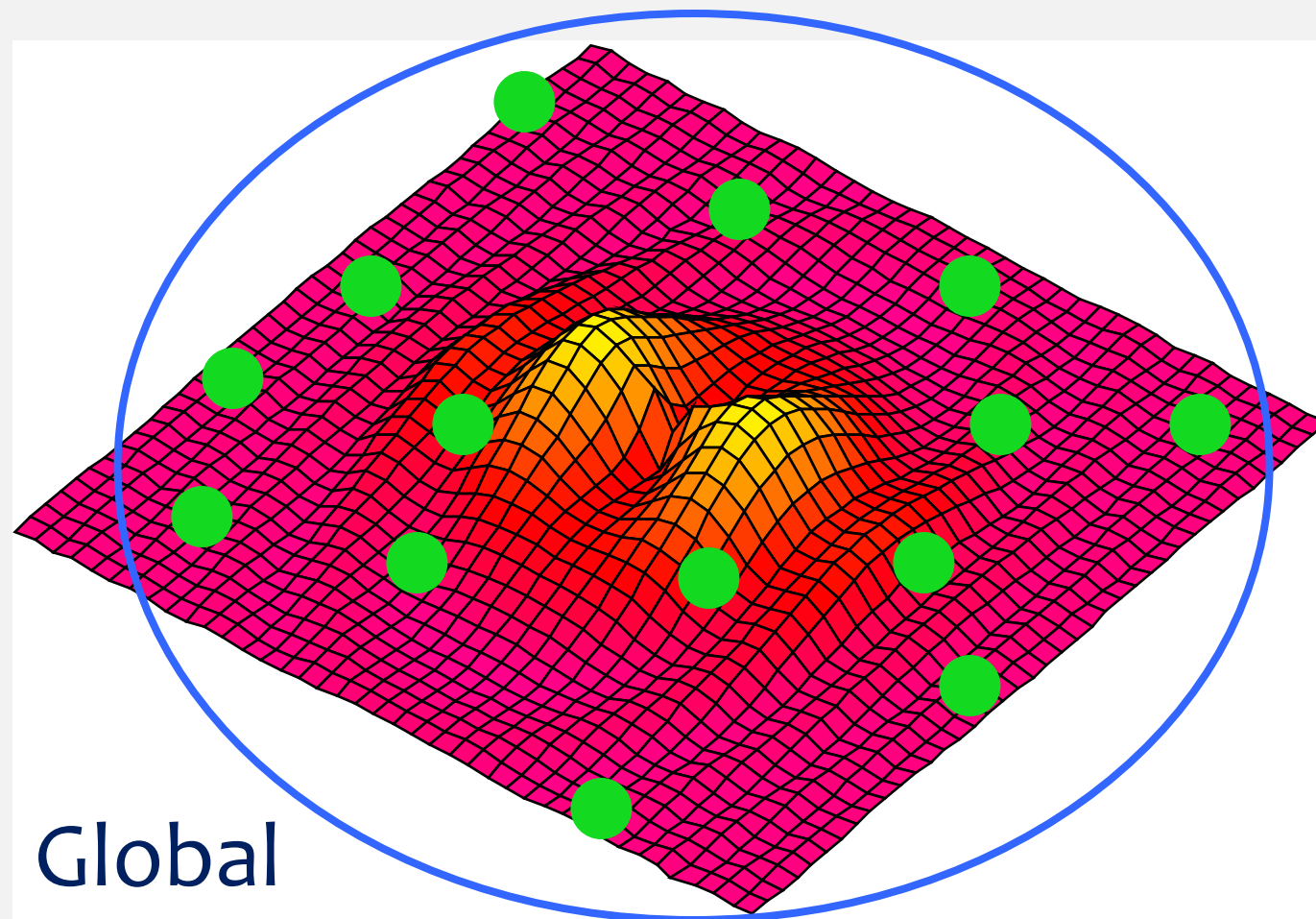
Neighborhoods



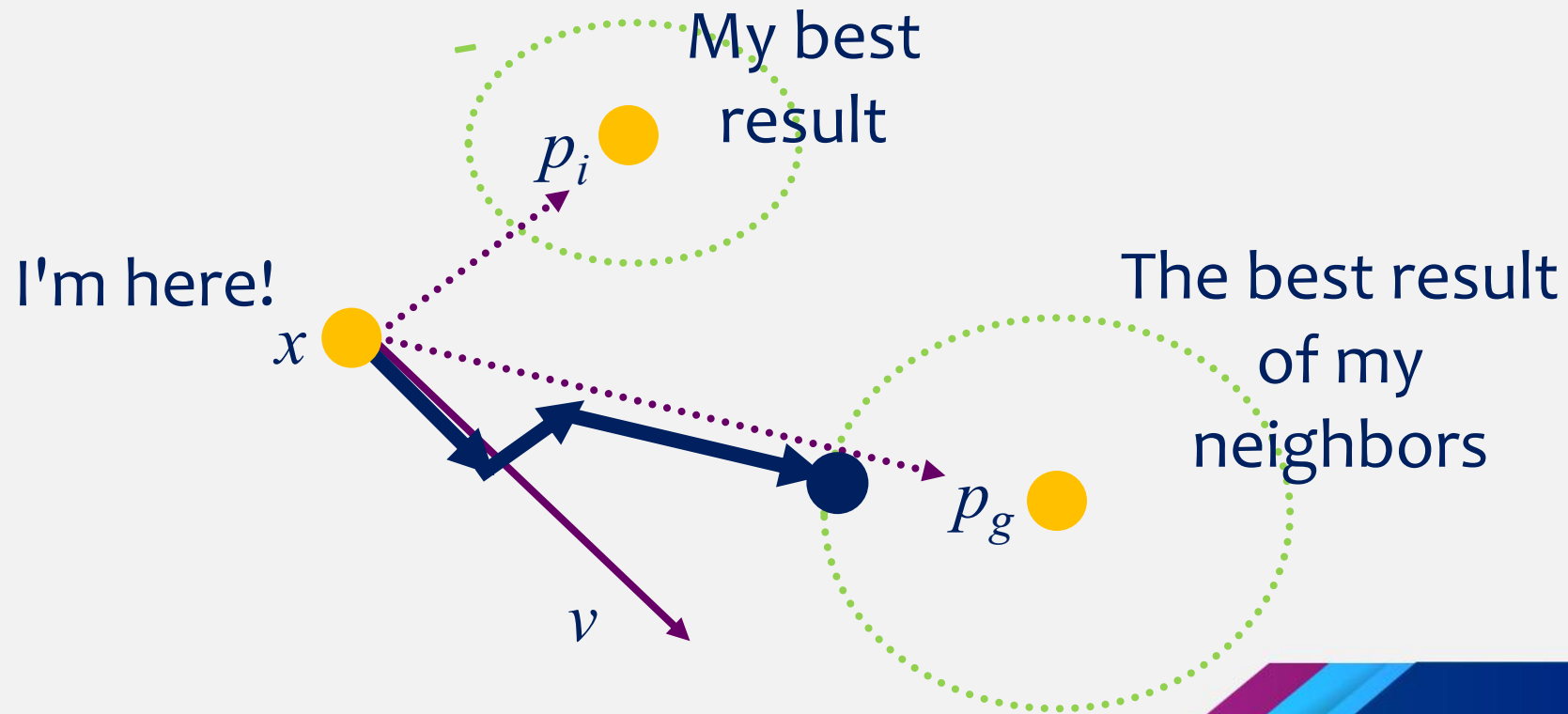
Circular neighborhood



Neighborhoods



- Particles adjust their positions according to a "psychosocial compromise" between what an individual considers comfortable and what society says.



Equations

Equation (a) to adjust the velocity

$$\begin{aligned} \text{vel}[] = & C0 * \text{vel}[] \\ & + C1 * \text{rand}() * (\text{pbest}[] - \text{pos}[]) \\ & + C2 * \text{rand}() * (\text{gbest}[] - \text{pos}[]) \end{aligned}$$

(In the original method, $C0 = 1$, but now many researchers play with this parameter)

Equation (b) to adjust the position

$$\text{pos}[] = \text{pos}[] + \text{vel}[]$$

Pseudocode for PSO

```
Repeat for each particle  
    Initializes the particle  
End
```

```
Repeat  
    Repeat for each particle  
        Calculate its fitness value  
        If the fitness value is better than its personal best  
            It sets its current value as the new pbest  
    End
```

```
Choose the particle with the overall best fitness as gbest
```

```
Repeat for each particle  
    Calculate the new velocity of the particle  
        with equation (a)  
    Calculate the new position of the particle  
        with equation (b)  
End
```

```
While maximum iterations or minimum error criterion is not reached
```

Pseudocode for PSO

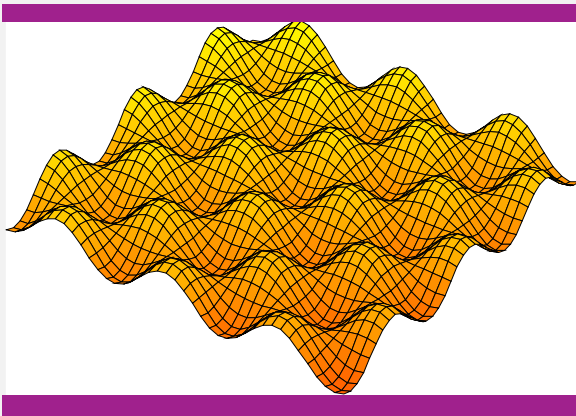
- The particle velocities in each dimension are subject to a maximum speed **V_{max}** which is a parameter specified by the user.
- If the sum of the accelerations will cause that V_{max} is exceeded in one dimension, then the velocity in this dimension would be established as V_{max}.

How to choose the parameters?

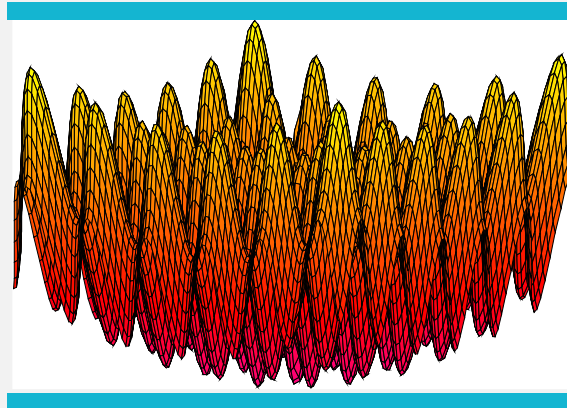
- **Number of particles**
(10-50) are reported as usually sufficient.
- **C1** (Importance of personal best)
- **C2** (Importance of best neighbor)
- Usually $C1 + C2 = 4$. Without better reason than as product of empirical results.
- **Vmax** - too low → too slow;
 too high → too unstable.

Some functions that are frequently used to test optimization algorithms with real values.

Griewank



Rastrigin



Rosenbrock

