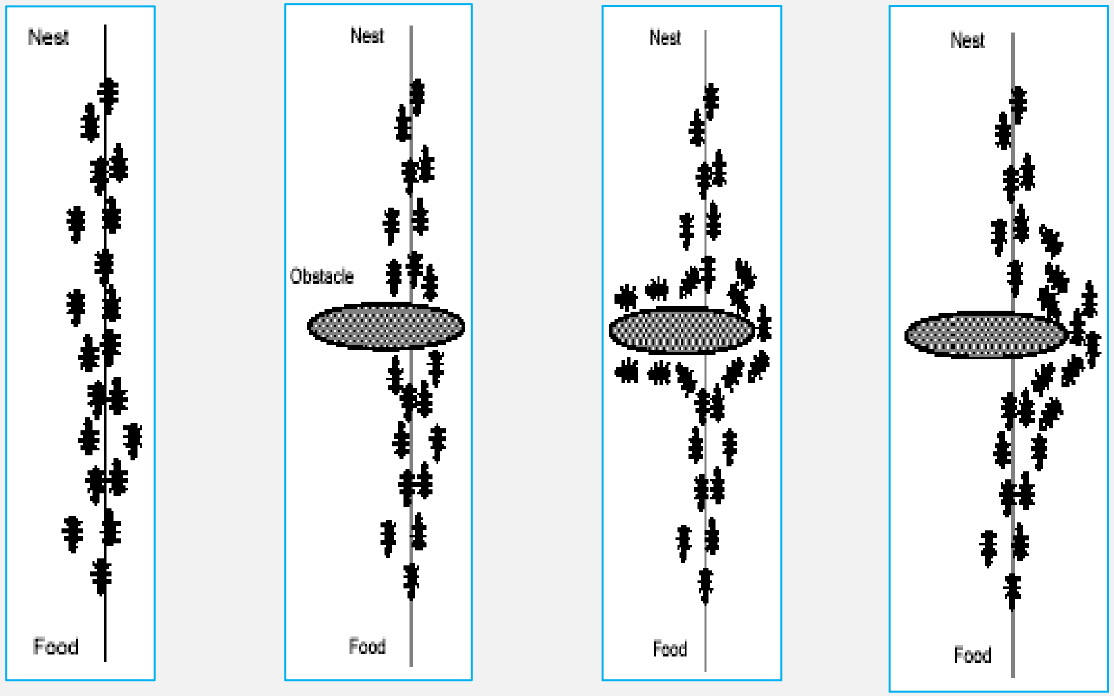
Ant System (AS)

Natural behavior of an ant

Foraging modes

- Wander mode
- Search mode
- Way back mode
- Attraction mode
- Tracking mode
- Transport mode



Ant Algorithms - (P.Koumoutsakos - based on notes L. Gamberdella (Www.idsia.ch)

How to implement an AS?

- **Ants**: Simple agents.

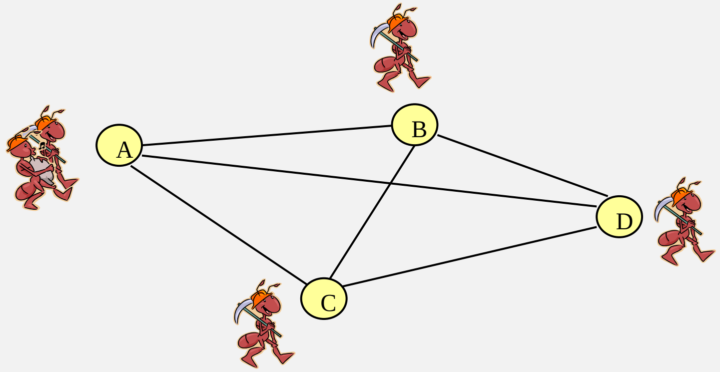
- **Move an ant**: Choose the next component in the construction of the solution.
- **Pheromone**: $\Delta\tau_{i,j}^{k}$

- **Memory**: M_K or $Tabu_K$

- ***Next move***: Use probability to move an ant.

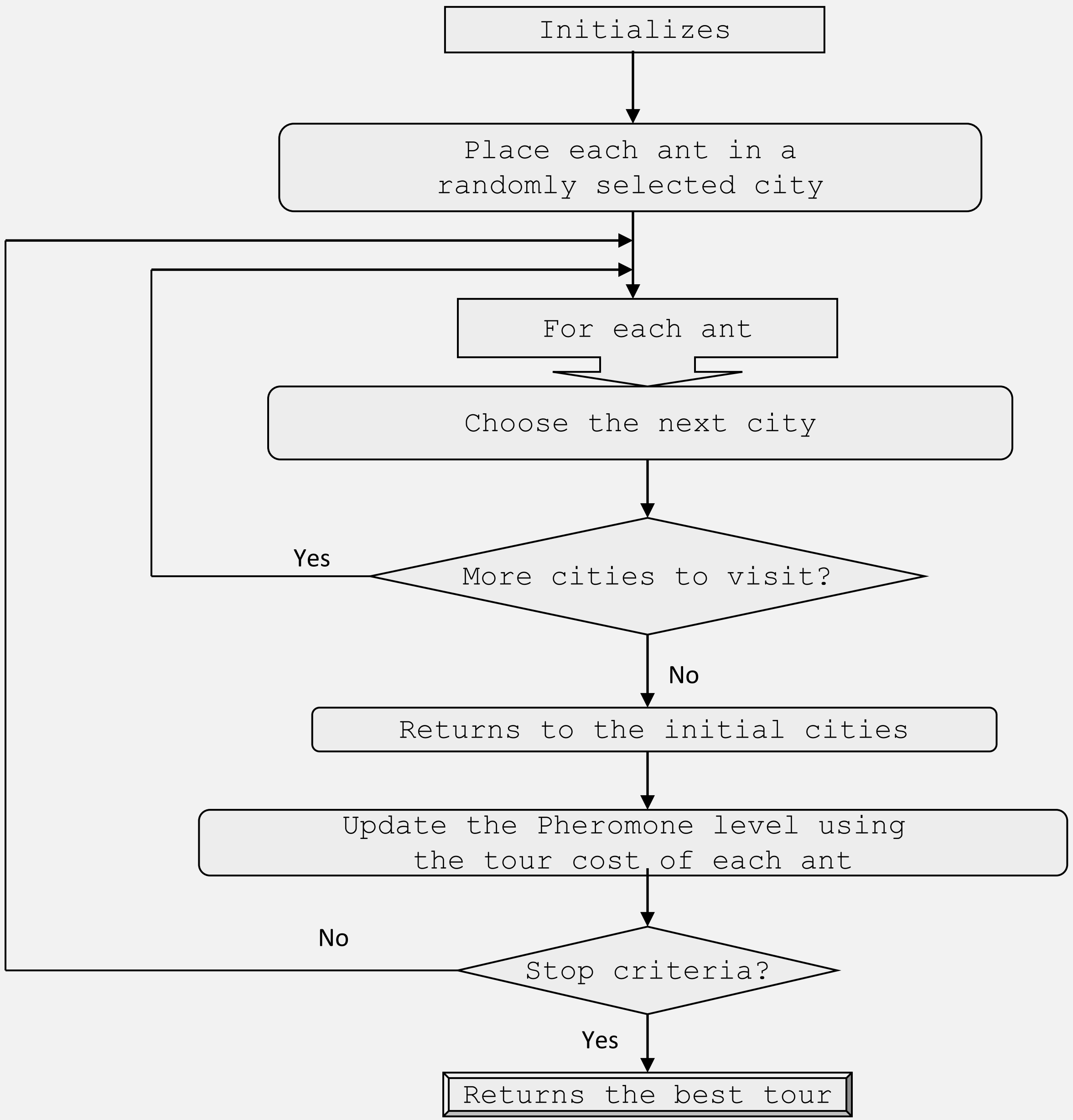
Example: Ant System for the Traveling Salesman Problem (TSP)

- Given a network of N cities, find a minimum total distance tour that visits each city only once.
- Graph (N, E), where:
  - *N* = cities / nodes,
  - E = arcs
  - d_{ij}= the cost of going to the city i to city j (edge weight)



- An ant moves from one city to the next with some transition probability.
- Each arc is assigned a static value returned by a heuristic function η_{ij} based on the cost of the arc.
- Each arc of the graph is augmented with a pheromone trace τ_{ij} deposited by the ants.
- Pheromone is dynamic and is learned at runtime.

AS algorithm for TSP



Computation of transition probability

Transition probability so that ant k goes from city i to j while building its tour.

$$p_{ij}^{k}(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum\limits_{k \in permitido_k} [\tau_{ik}(t)]^{\alpha}[\eta_{ik}]^{\beta}} & \text{if } \ j \in \text{permitted}_{\text{k}} \\ 0 & \text{on the contrary} \end{cases}$$

where:

$\tau_{ij}(t)$ = **pheromone trail** : a type of global information

$\eta_{ij} = \dfrac{1}{d_{ij}}$ = **visibility** : heuristic desirability of choosing the city j when in city i.

$\text{permitted}_{\text{k}}$ = **memory** : Set of cities that have not been visited by ant k

α y β = **search parameters**

Pheromone trail in AS

After completing a tour, each ant leaves some pheromones for each arc it has used, depending on how well the ant has performed.

$$\tau_{ij}(t+n) = \rho\tau_{ij}(t) + \Delta\tau_{ij}$$

$where:$

ρ is a parameter of pheromone consumption

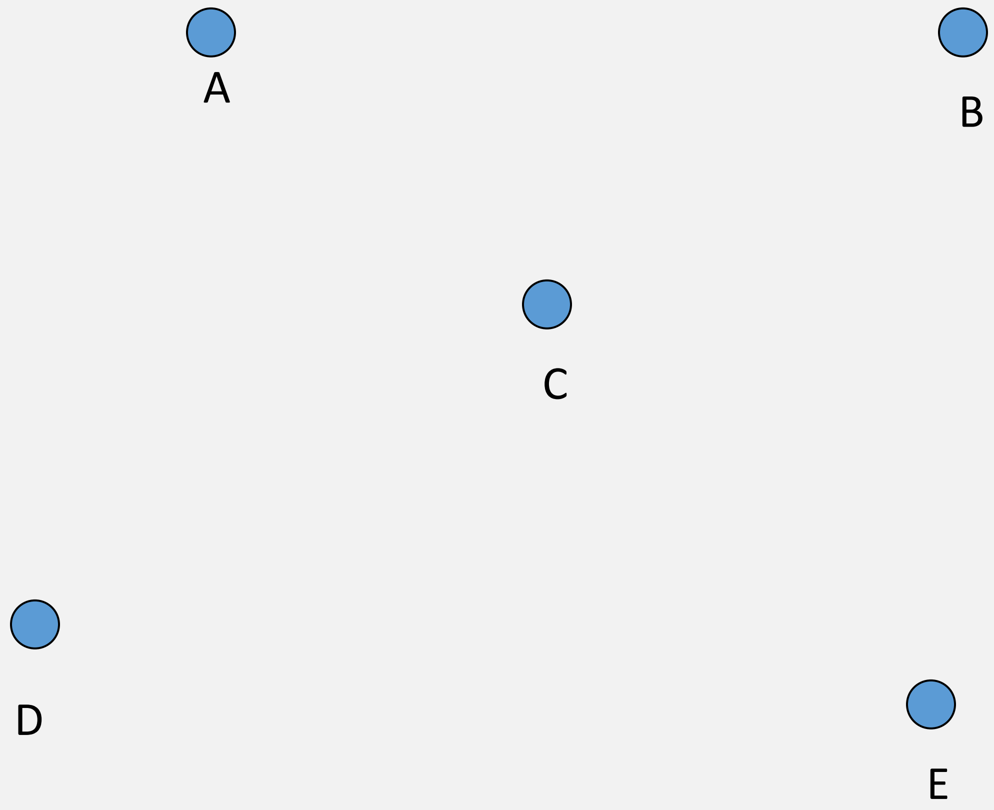
$\Delta\tau_{ij} = \Delta\tau_{ij}^{1} + \Delta\tau_{ij}^{2} + \ldots + \Delta\tau_{ij}^{m}$, for the m ants

$$\Delta\tau_{i,j}^{k} = \begin{cases} \dfrac{Q}{L_k} & si\,(i,\,j) \in \text{tour of ant k} \\ 0 & \text{on the contrary} \end{cases}$$

Q is a parameter of contribution of pheromone, and

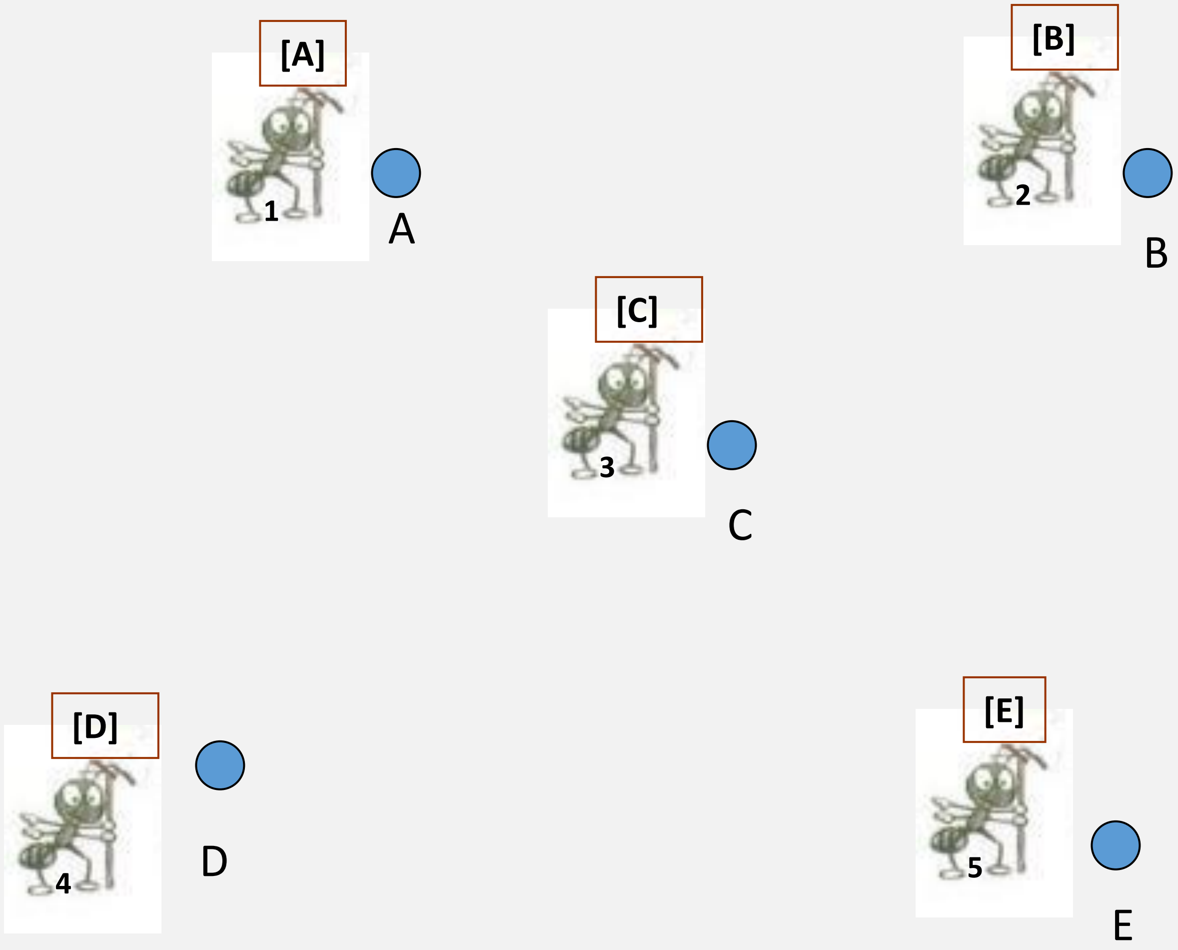
L_k is the length of the tour of ant k

An example of simple TSP

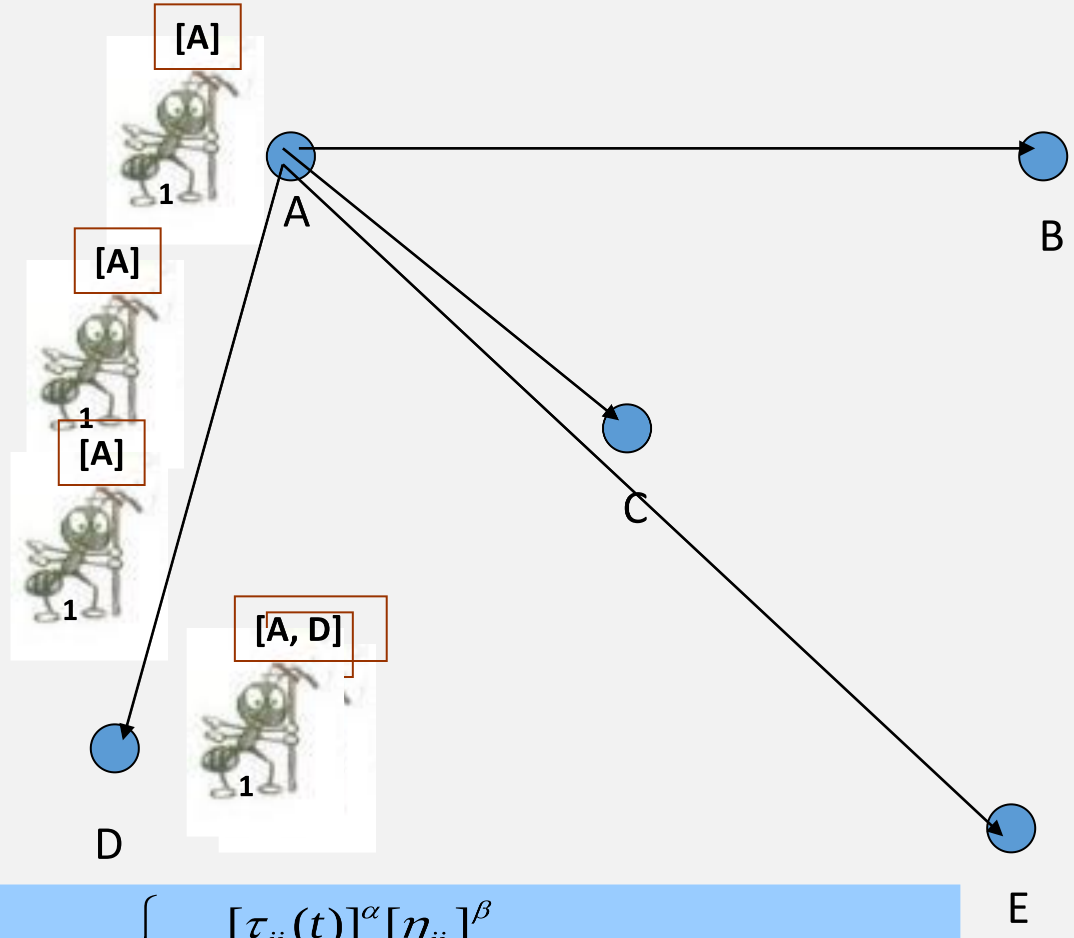


$d_{AB} = 100$ $d_{BC} = 60$ ... $d_{DE} = 150$

Choose a random city for each ant

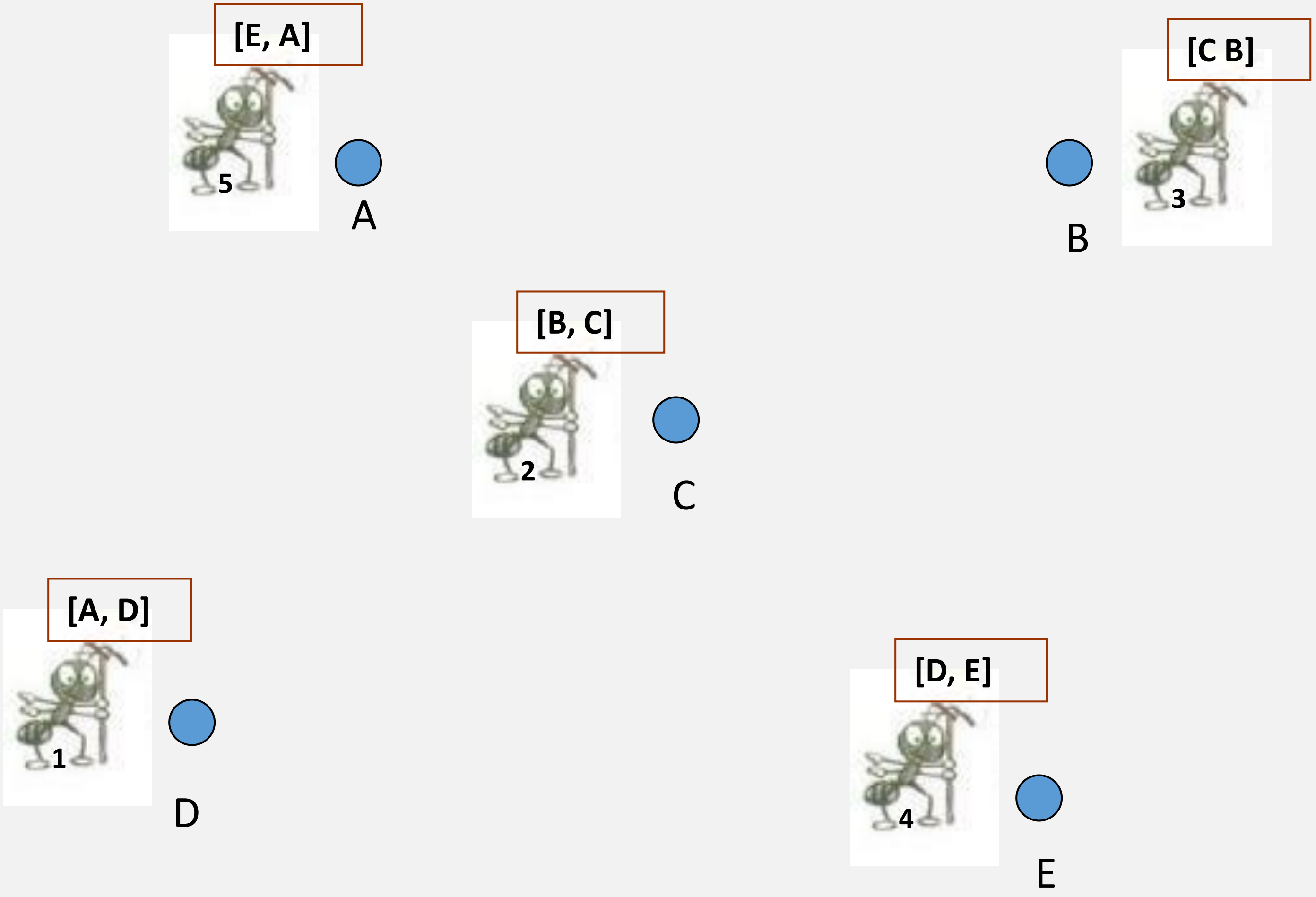


Selects the next city for each ant = **roulette wheel**

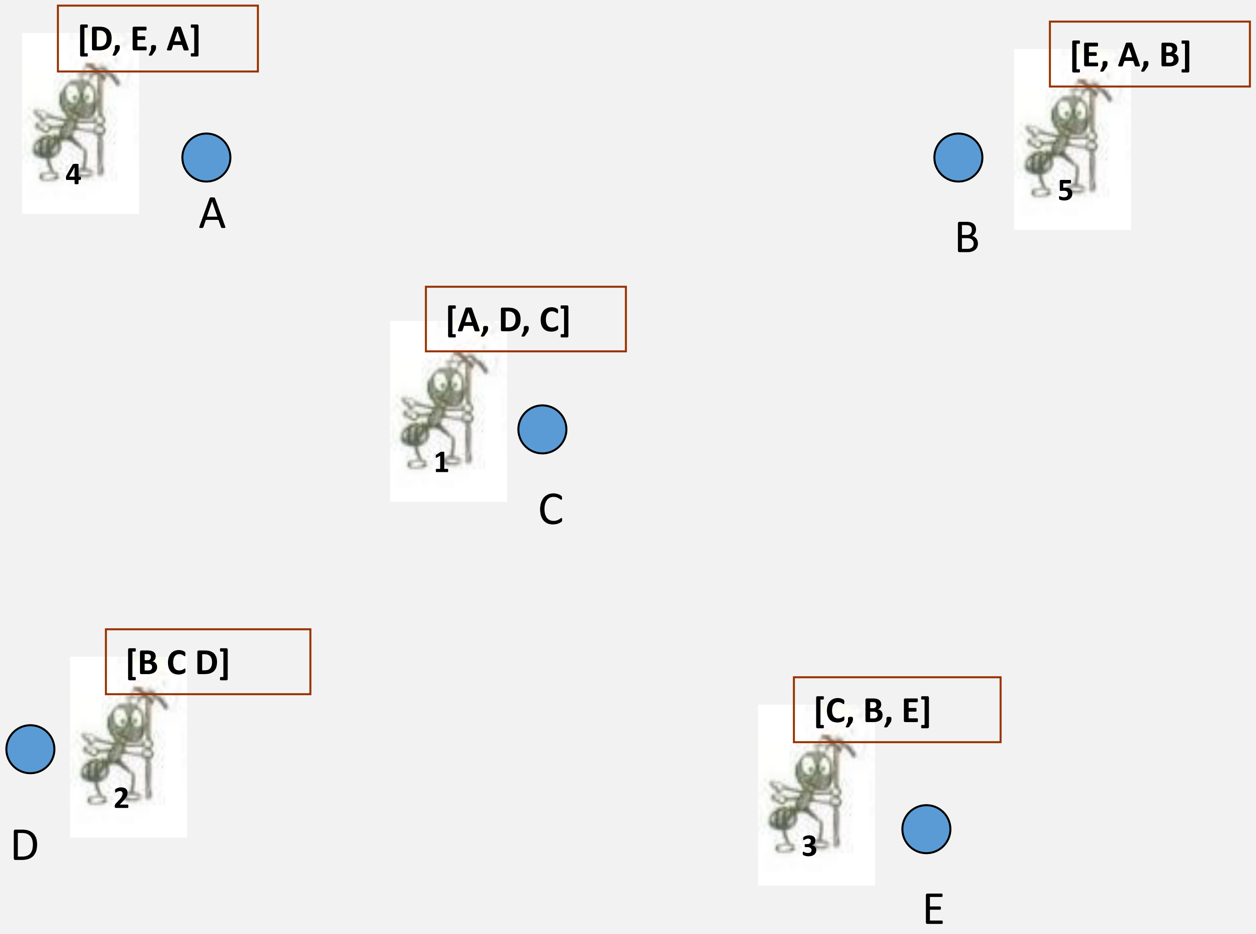


$$p_{ij}^{k}(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum\limits_{k \in permitted_k} [\tau_{ik}(t)]^{\alpha}[\eta_{ik}]^{\beta}} & \text{si } j \in \text{permitted}_k \\ 0 & \text{on the contrary} \end{cases}$$

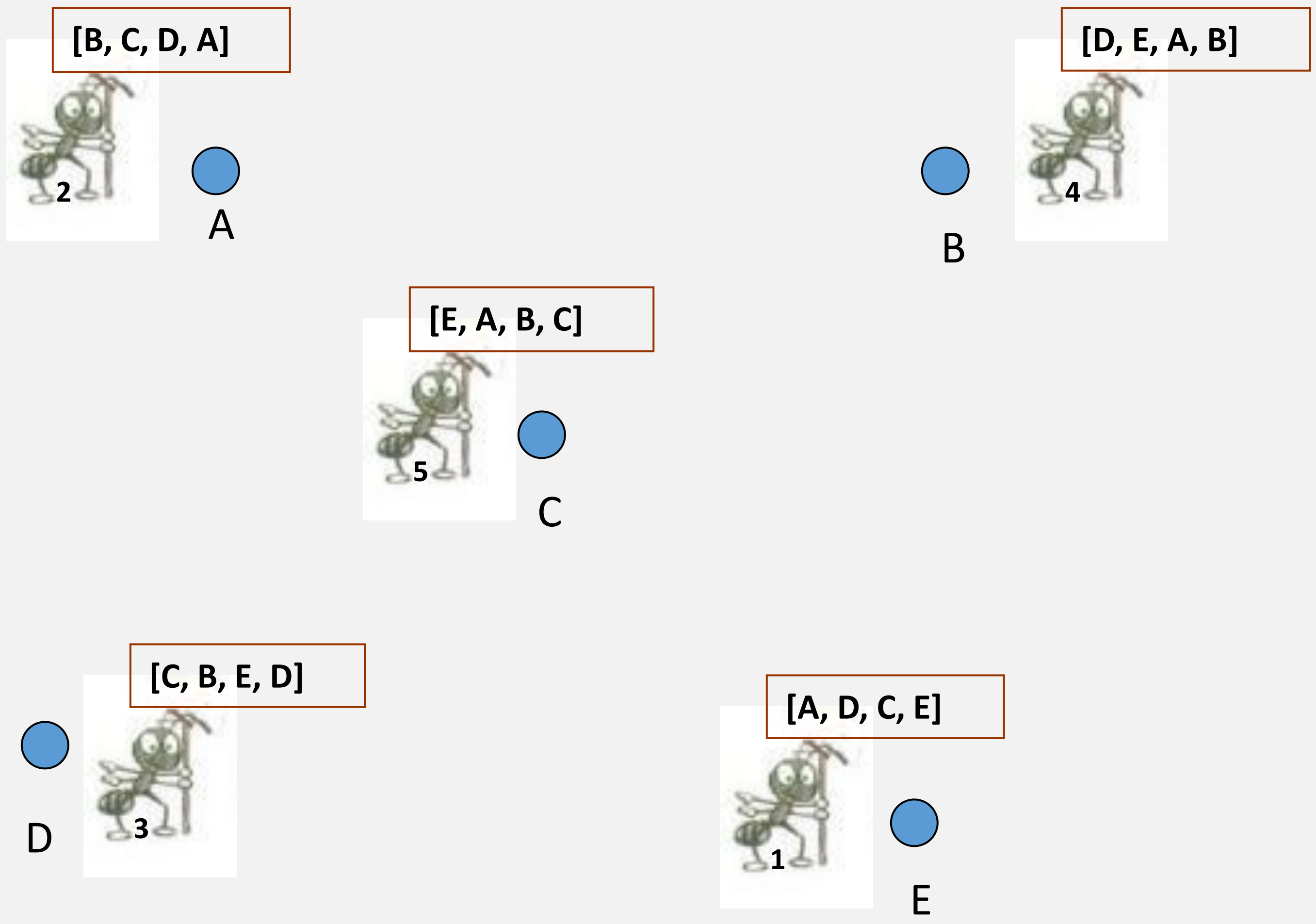
Iteration 2



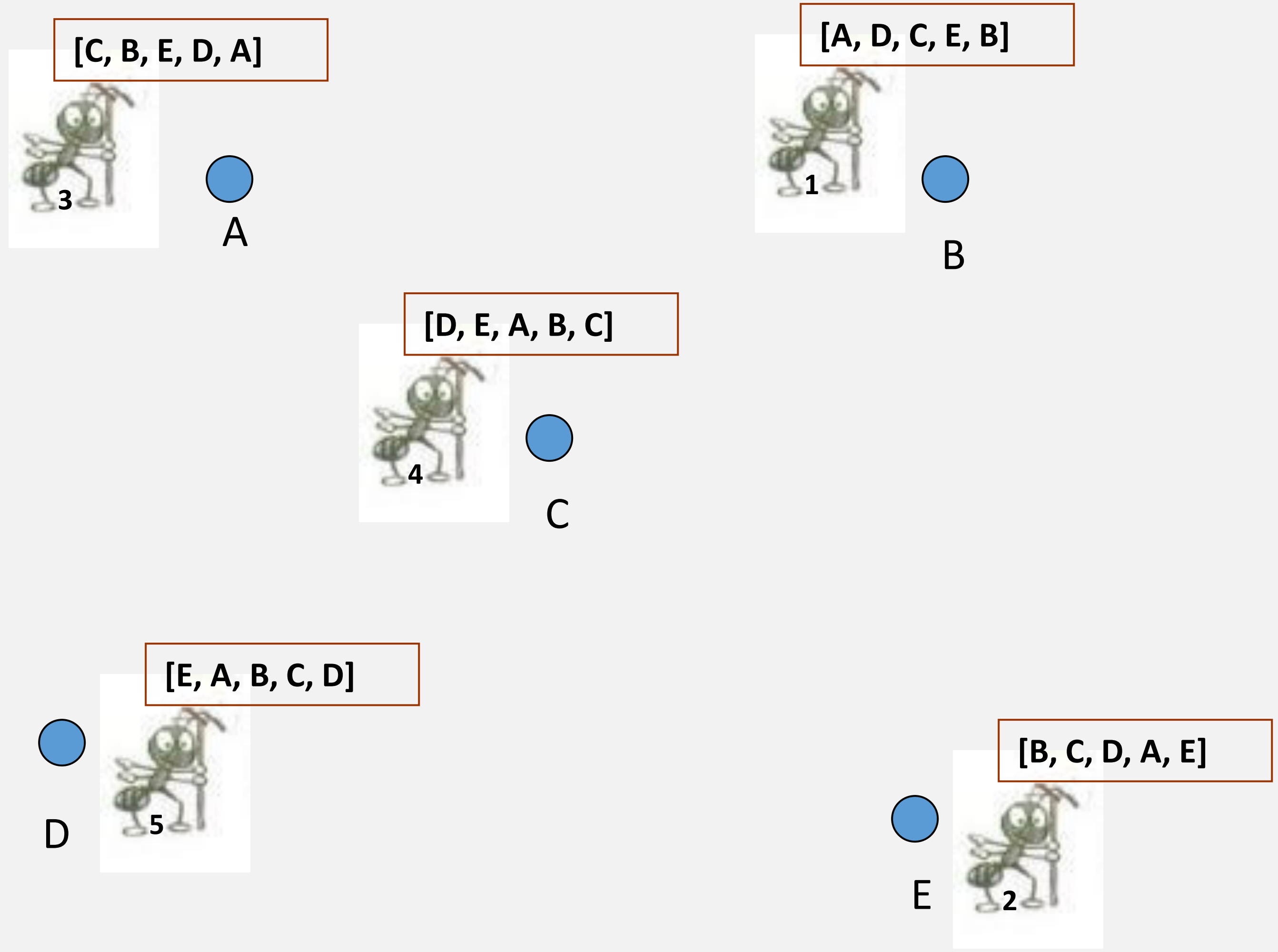
Iteration 3



Iteration 4



Iteration 5



Evaluates the route and updates the pheromone trail

[A, D, C, E, B]

1

$L_1 = 300$

[B, C, D, A, E]

2

$L_2 = 450$

[C, B, E, D, A]

3

$L_3 = 260$

[D, E, A, B, C]

4

$L_4 = 280$

[E, A, B, C, D]

5

$L_5 = 420$

$$\Delta \tau_{i,j}^{k} = \begin{cases} \dfrac{Q}{L_k} & \text{si } (i, j) \in \text{tour} \\ 0 & \text{on the contrary} \end{cases}$$

$$\Delta \tau_{A,B}^{total} = \Delta \tau_{A,B}^{1} + \Delta \tau_{A,B}^{2} + \Delta \tau_{A,B}^{3} + \Delta \tau_{A,B}^{4} + \Delta \tau_{A,B}^{5}$$

Closing of a cycle

- Stop criteria
  - Stagnation
  - Maximum iterations
- Preparation for the next run

End of the first run

Saving the best tour (Sequence and length)

All ants die

Newborn ants

Problems resolved with AS

- Traveling salesman
- Quadratic assignment
- Scheduling tasks
- Vehicle routing
- Graph coloring
- Routing networks
- Constraint Satisfaction
- Multiple backpack
- ...

Conclusions about Ant Systems

- Stochastic search methods.
- Build a solution probabilistically.
- Iteratively add components to partial solutions:
  - Heuristic information
  - Pheromone trail
- A type of reinforcement learning.
- Modify the representation of the problem in each iteration.
- Ants work concurrently and independently.
- Collective interaction via indirect communication leads to good solutions.

Some inherent advantages

- Positive feedback helps in the rapid discovery of good solutions.
- Distributed computing prevents premature convergence.
- Greedy heuristics helps to find an acceptable solution in the early stages of the search process.
- Collective interaction of a population of agents.

Disadvantages of ant systems

- Slower convergence than other heuristics.
- Perform poorly for TSP with more than 75 cities.
- Do not have a centralized processor to guide the AS towards good solutions.