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## Practical Sheet 2

Solutions due **June 7**, 23:59, in the Moodle system.

**Submission Instructions** For each exercise, you must submit three different files:

- 1. A SAT file in the format specified below.
- 2. The log output resulting of running the SAT solver with this input file.
- 3. A PDF file explaining your model and the connection between the model and the SAT file submitted.

Put all files into an archive called "name1-name2-name3.zip", where "name1", "name2", "name3" are the family names of all authors. There are six files in total, that must be called as follows:

- "name1-name2-name3-ex2.{pdf, sat, log}"
- "name1-name2-name3-ex3.{pdf, sat, log}"

Additionally, add a file called "authors.txt" to the archive that contains one line per author, detailing the full name and matriculation number. To upload the archive to the Moodle system, go to the corresponding assignment, click "Add submission", and upload it. (obviously, you are not restricted to "\*.zip", you can use any popular format, that you like). Note that only one author per group needs to do the submission.

**SAT file** In this practical sheet you have to model problems as satisfiability problems. For that, we will use the standard CNF format used in the SAT competition<sup>1</sup>. Lines starting with "c" are comments. The first line specifies that the formula is in CNF and sets the number of variables and clauses. Then, each line corresponds to a clause. Since the formula is in CNF, it suffices to specify the literals (possibly negated with a "-") as a list of integers.

<sup>1</sup>http://www.satcompetition.org/2004/format-solvers2004.html

## SAT file example:

```
c start with comments (comments are only allowed at the beginning of the file)
c This file represents the formula:
c x1 or not x5 or x4 AND
c not x1 or x5 or x2 or x4 AND
c not x3 or not x4
c
c p cnf <# variables> <# clauses>
p cnf 5 3
1 -5 4 0
-1 5 2 4 0
-3 -4 0
```

**LOG file** The SAT file specified above can be used with many different SAT solvers. We chose *Yices* (http://yices.csl.sri.com/) because it is already precompiled for Linux, Windows, and Mac. You'll find the executable in bin/yices-sat. Run the solver with the command ./yices-sat -m -v filename.SAT

**PDF file** To describe your model you must specify the set of variables and the formula. The **formula must be specified in CNF**, i.e., as a set of clauses/constraints. You can follow the example on slide 19 of chapter 9, and then transform everything to CNF. You do not need to specify the whole formula, but rather you can use quantifiers. For example:  $\bigwedge_{x \in set1} (A_x \vee \neg B_x)$ , or  $\bigwedge_{x \in set1} (\bigvee_{y \in set2} A_{x,y})$ .

It is really important that the correspondance between your model description in the PDF file and the model in the SAT file is clear. To do so, specify the mapping between your model and the SAT file. In particular, specify the mapping between the variables in your model and their corresponding ID on the SAT file, e.g.,  $A_1, \ldots, A_5 = 1, \ldots 5$ . Also, for each type of constraint, put some examples of constraints in the SAT file that correspond to this type of constraint.

**Note** You can write the SAT file by hand, but since the format is quite unreadable this may be difficult and prone to error. We recommend you to write a script in which you do the mapping of variables to IDs and use that mapping to automatically write the SAT file. Alternative encodings are possible. To get an idea of the size of the models, our master solution for each exercise does not exceed 50 variables, nor 100 clauses.

Exercise 2. (8 Points)

5 students enter an overcrowded tutorial<sup>2</sup>, where there are only 4 available seats left. Each student has to get its own seat. No seat can be shared between different students. Encode this problem as a satisfiability problem and prove that it is impossible for all students to be seated.

Argue how the performance of the DPLL algorithm from the lecture slides scales with the number of students N.

Exercise 3. (12 Points)

There are 3 ships in a port (positions 1, 2, and 3). Each of the ships:

- Is from a different nationality: German, French, or Spanish.
- Goes to a different destination: Marseille, Hamburg, or Barcelona.
- Carries a different good: coffee, rice, or tea.

Given that we know the following facts:

- 1. The German ship goes to Barcelona and carries coffee.
- 2. The French ship is to the left of a ship that carries coffee.
- 3. The ship that carries tea is in the border (position 1 or 3).
- 4. The ship carrying tea is anchored next to the ship carrying rice.
- 5. The ship carrying rice is either going to Marseille or Barcelona.

Encode the problem into SAT and fill the following table:

| Position    | 1 | 2 | 3 |
|-------------|---|---|---|
| Nationality |   |   |   |
| Destination |   |   |   |
| Good        |   |   |   |

How can you modify your model to verify that the solution provided by the solver is unique?

<sup>&</sup>lt;sup>2</sup>This is a fictional exercise. Any resemblance to the names, character, or history of any person is coincidental and unintentional. You should keep coming to the tutorials.