

Convolutional Neural Networks (8DC00)

Friso G. Heslinga

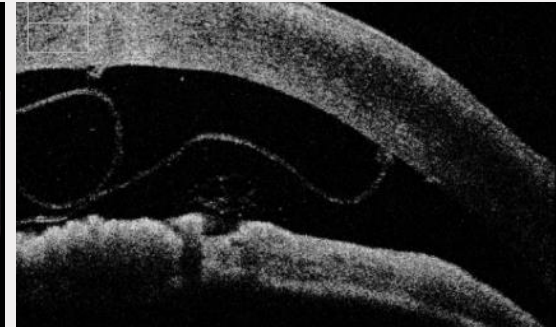
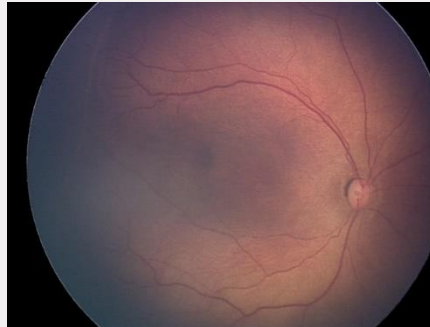
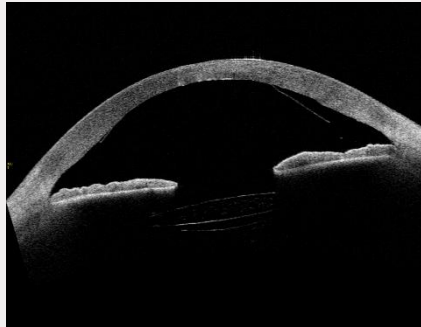
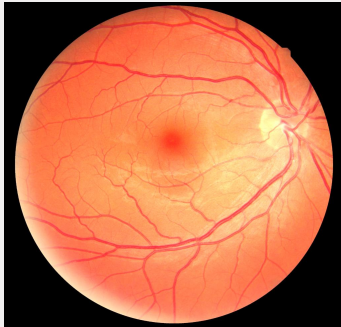
Friso G. Heslinga MSc.



Background: BSc and MSc in Biomedical Engineering, MSc in Health Sciences (University of Twente).

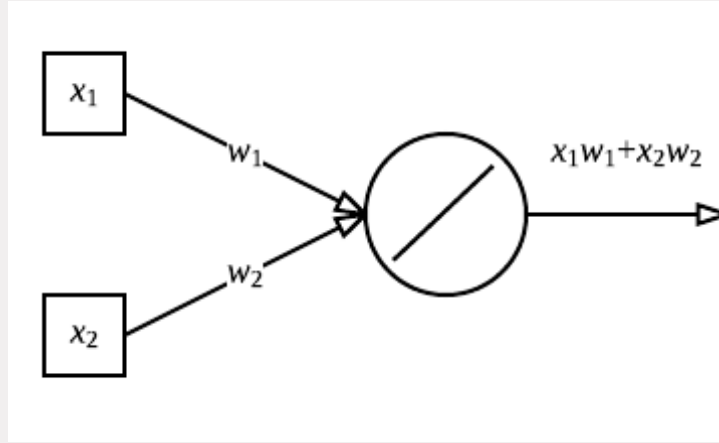
Work/internship experience: University of Western Australia, University of California - Berkeley, Harvard Medical School

PhD Research: Deep learning, Medical image analysis, Ophthalmology

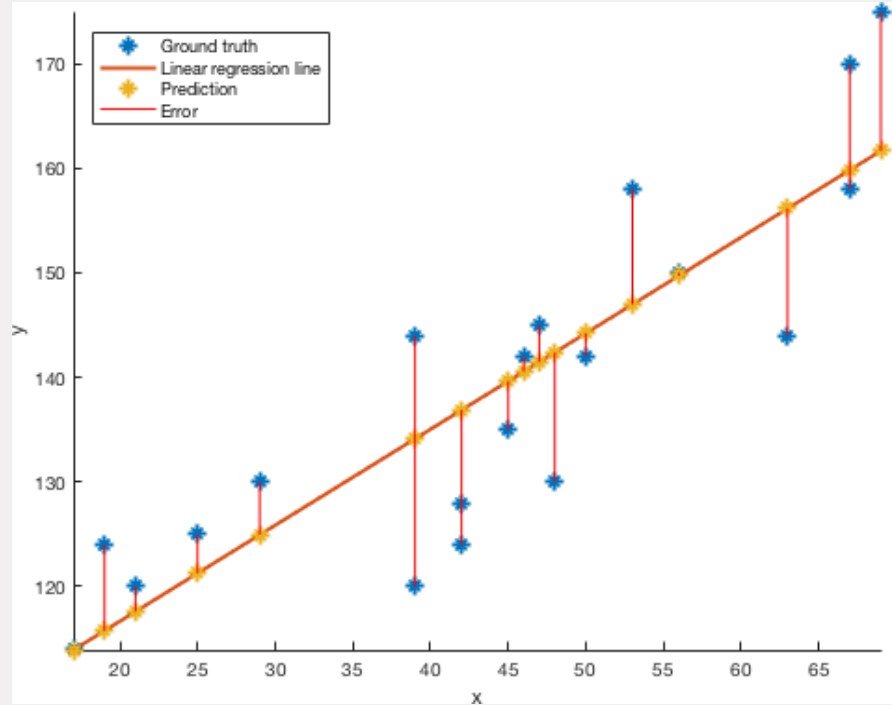


Week	Date	Lecturer	Topics
1	1 Sept.	Maureen	Course introduction; Software demo; Image registration (1)
	3 Sept.	Maureen	Image registration (2); Geometrical transformations
2	8 Sept.	Maureen	Point-based registration
	10 Sept.	Maureen	Intensity-based registration; Evaluation metrics
3	15 Sept.	<i>Catch-up day (no lecture)</i>	
	17 Sept.	Cornel Zachiu (UMCU)	Guest lecture 1: Image analysis for adaptive radiotherapy
4	22 Sept.	Mitko	Introduction to CAD; k-NN; Decision trees
	24 Sept.	Mitko	Generalization and overfitting
5	29 Sept.	Mitko	Logistic regression; Neural networks
	1 Oct.	Friso	Convolutional neural networks
6	6 Oct.	Friso	Deep learning frameworks and applications
	8 Oct.	Friso	Unsupervised machine learning
7	13 Oct.	Maureen	Deep learning for deformable image registration
	15 Oct.	Geert-Jan Rutten (ETZ)	Guest lecture 2: Image analysis in neurosurgery applications
8	20 Oct	<i>Self-study (no lecture)</i>	Active shape models
	22 Oct	<i>Self-study (no lecture)</i>	Active shape models

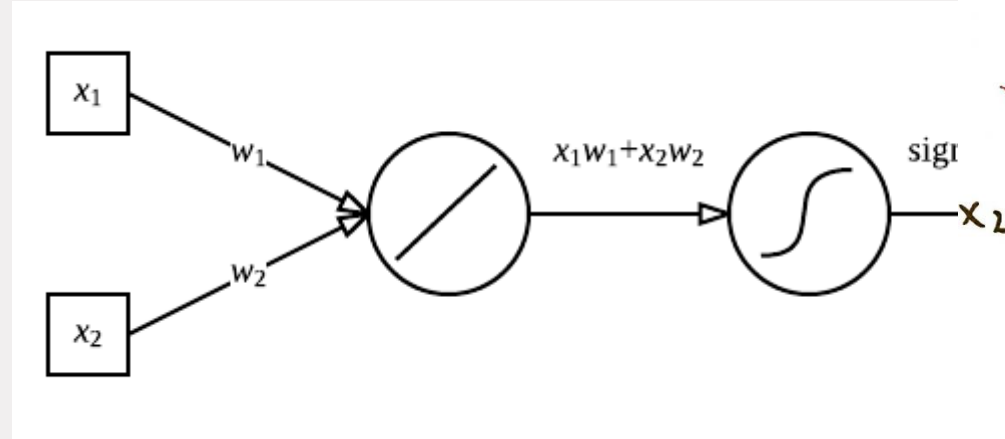
Previously – Linear regression



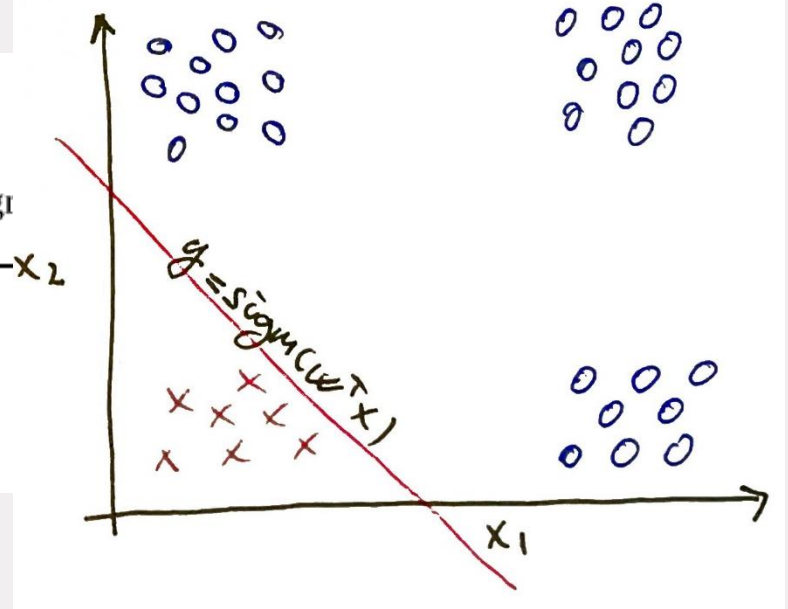
$$\hat{y} = \theta^T \mathbf{x}$$



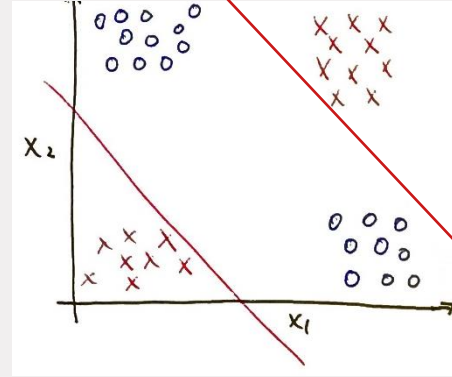
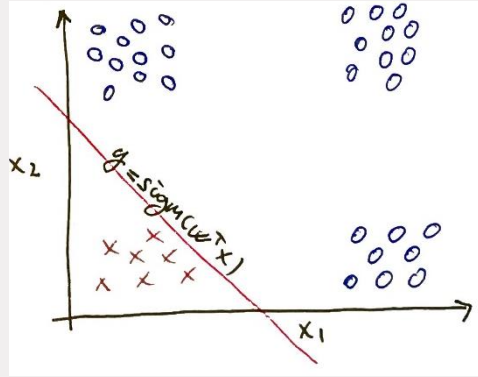
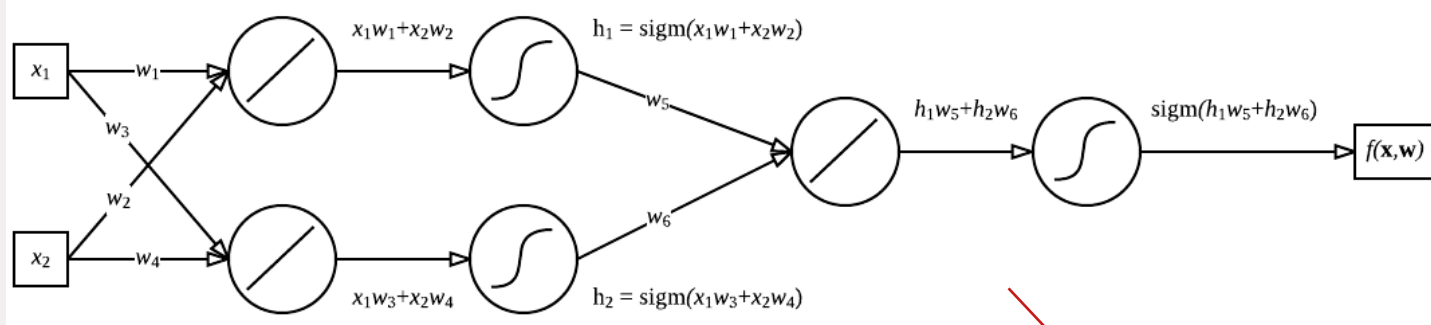
Previously – Logistic regression



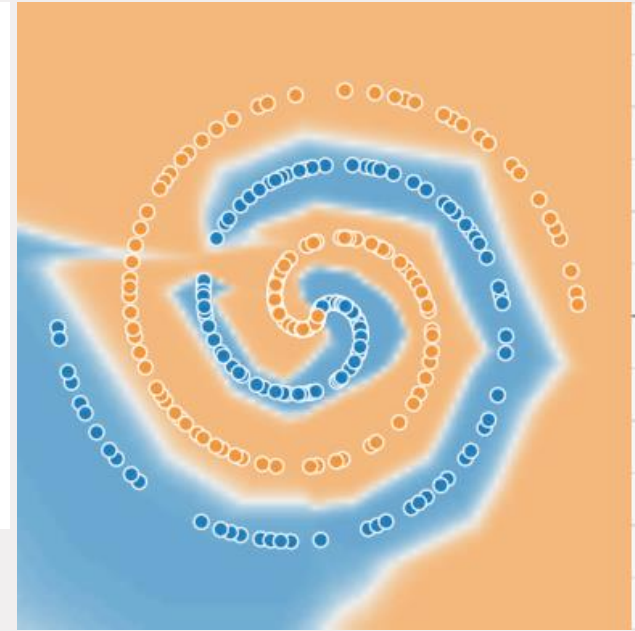
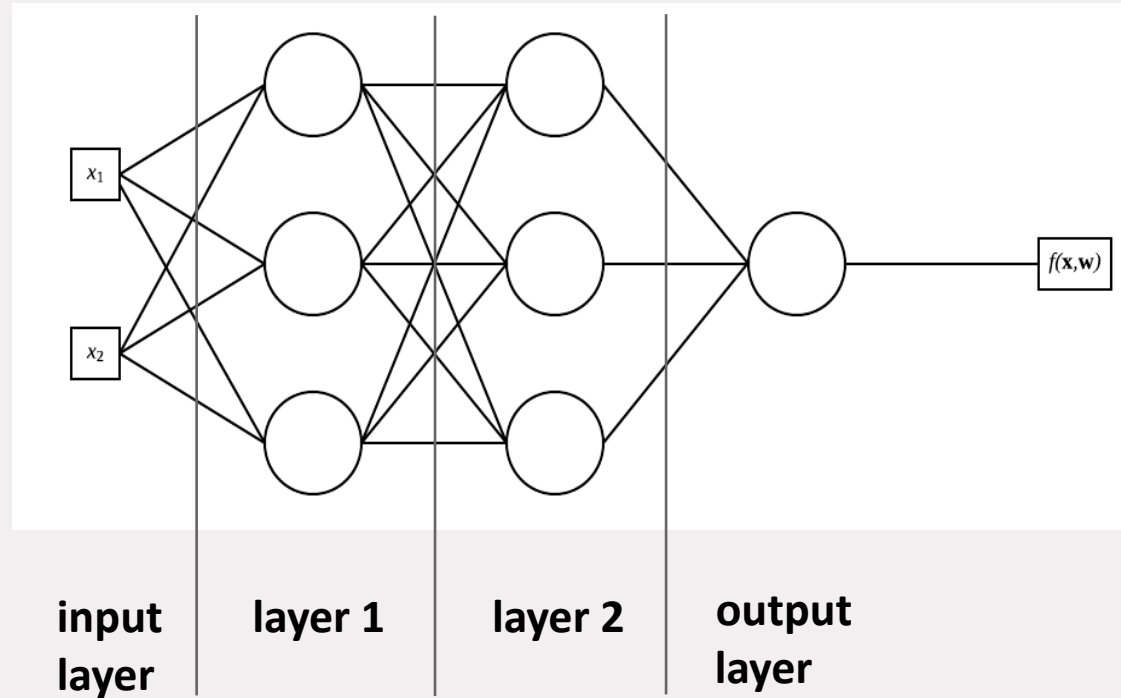
$$p(y = 1 \mid \mathbf{x}) = \sigma(\boldsymbol{\theta}^T \mathbf{x})$$



Previously – Neural networks

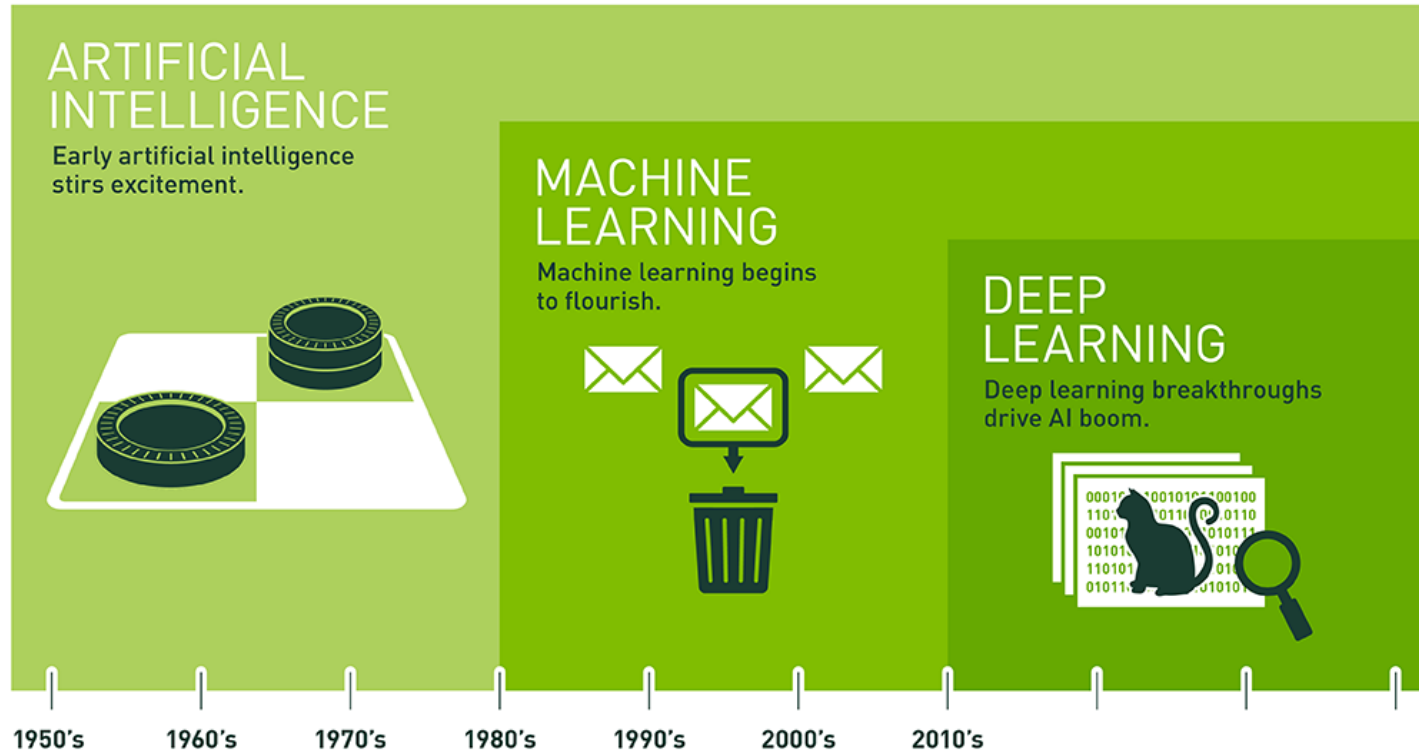


Previously – Neural networks



Previously – Machine learning

Figure source: nvidia.com



Today: Convolutional neural networks (CNN)

- Neural networks → Convolutional neural networks

Building blocks for deep learning models for image analysis:

- Convolutional layer
- Max-pooling layer
- Not needed for the project, but will be on the exam

Learning outcomes

- Student can explain the concept of **convolutions** in a neural network
- Student can describe why we can use a **convolutional approach** for (medical) **images**
- Student can explain why convolutions enable development of **deep** (and large) **neural networks**
- Students can explain and apply the **max-pooling layer** in a convolutional neural network
- Students can motivate the choice for a **kernel size**

Lecture outline

- Images as input to neural network
- Reducing # of weights
- 1D convolutions
- 2D convolutions
- *Break (15 mins)*
- Kernels
- Max-pooling
- Interactive example

Images as input to neural networks

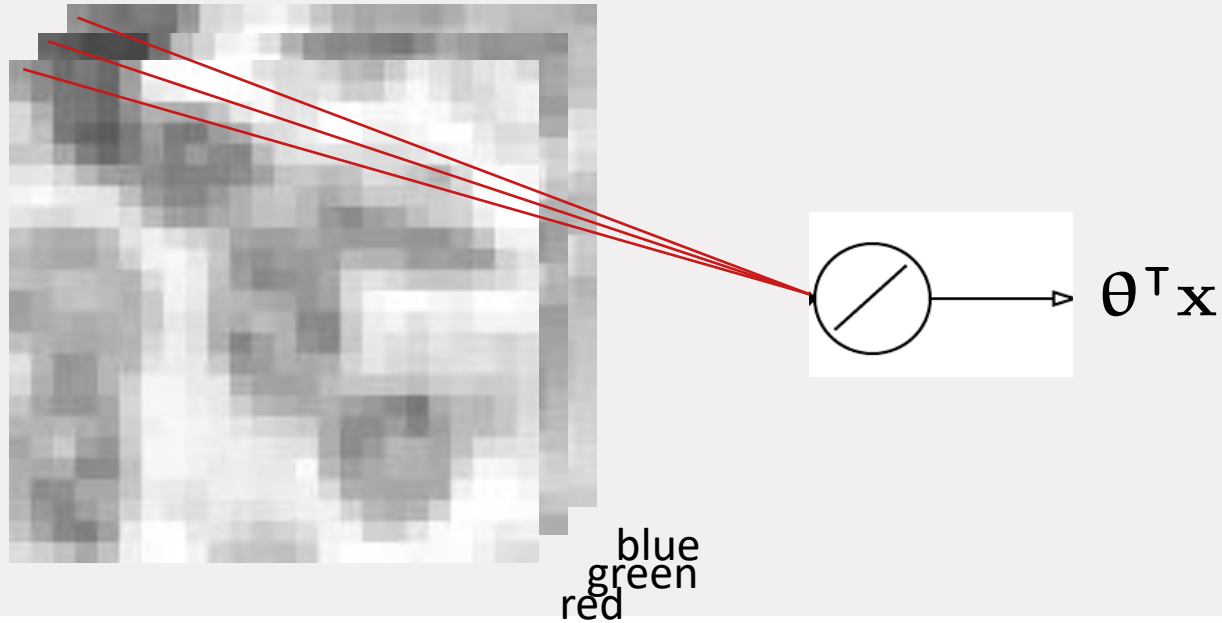


= 1728 features (\mathbf{x}_i are 1728 dimensional vectors)

If we train linear regression with these inputs (such as in the first practical), we will have 1728 weights w and a bias b .

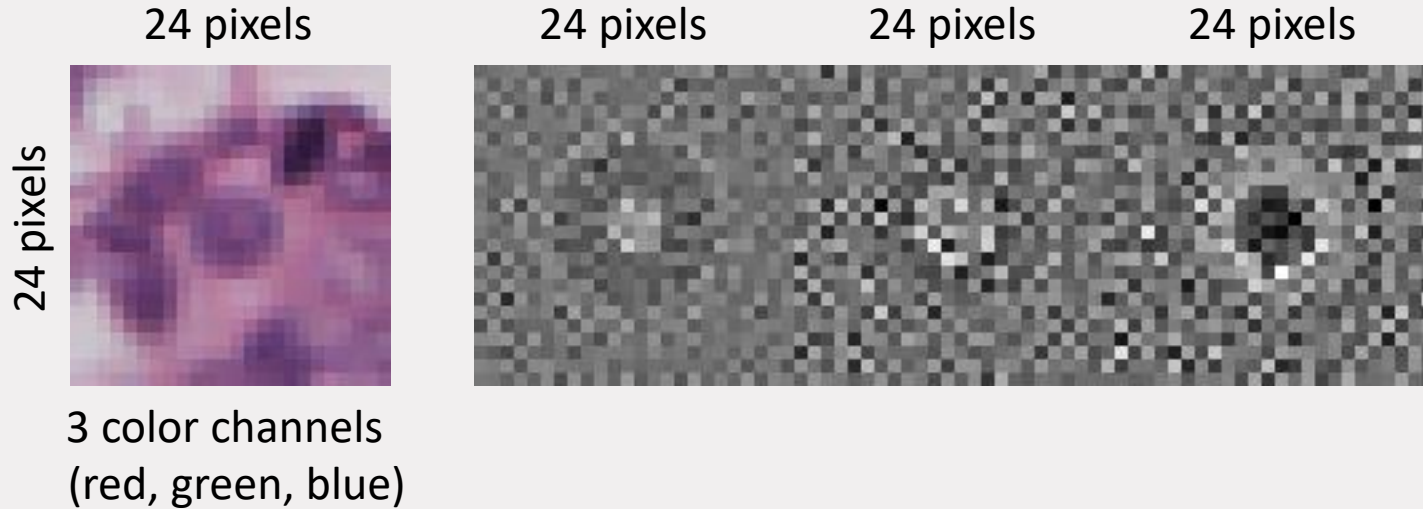
Every pixel is an input

Every pixel from every color channel is multiplied by a weight



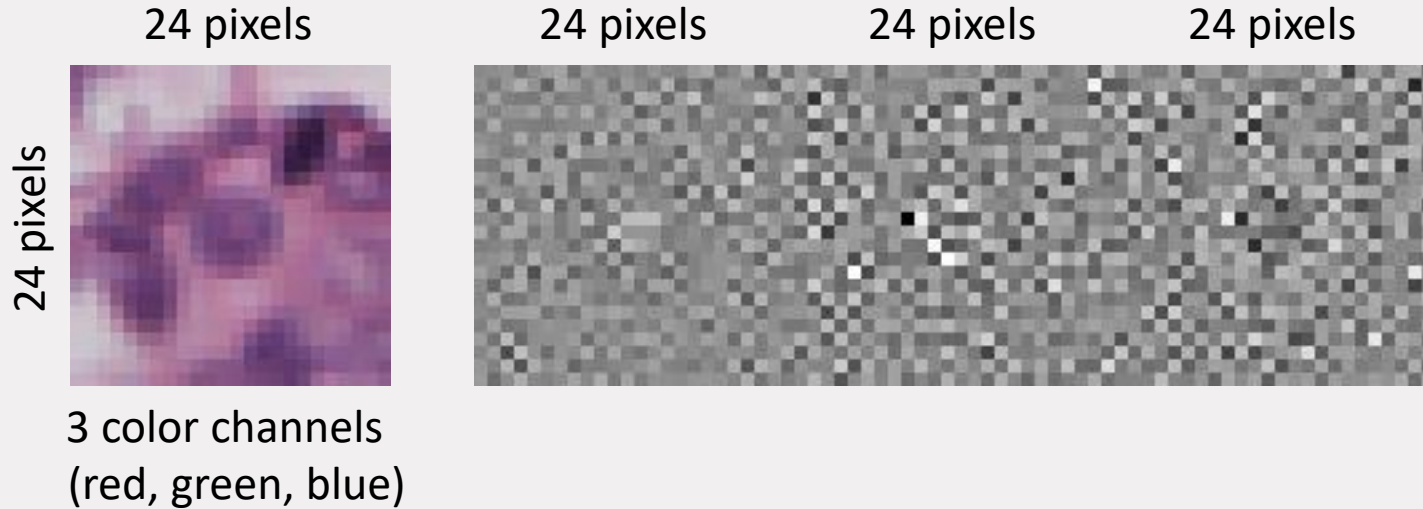
Post-training: visualizing what model has learned

Reshape vector of parameters into 24x24x3 image



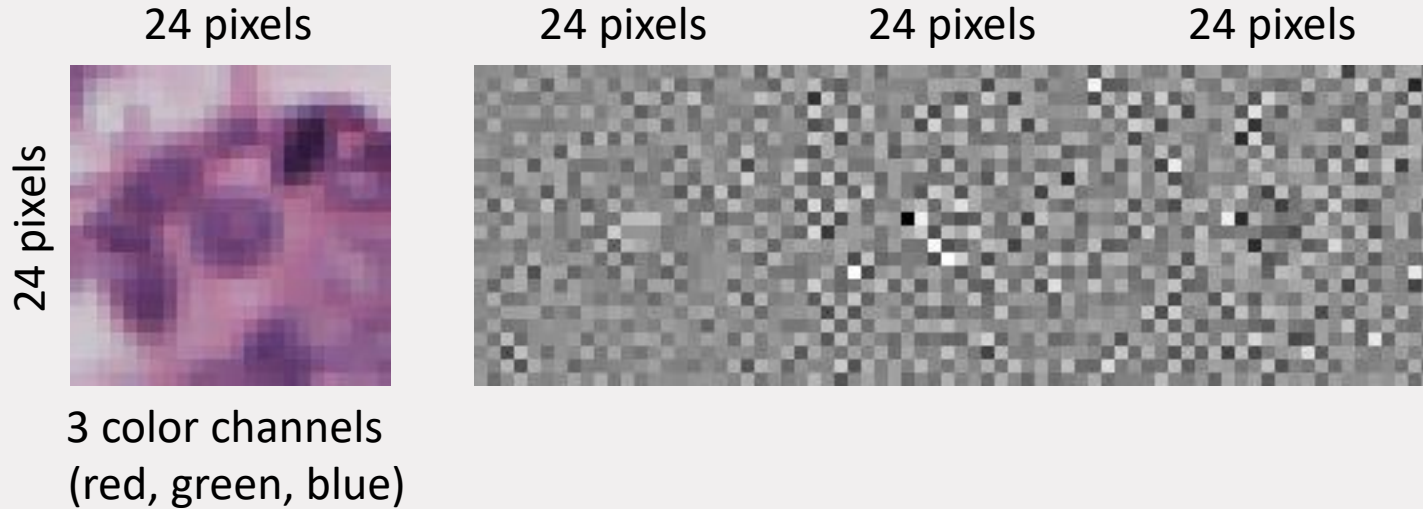
Post-training: visualizing what model has learned

Only 25% of training samples.. Looks noisy!

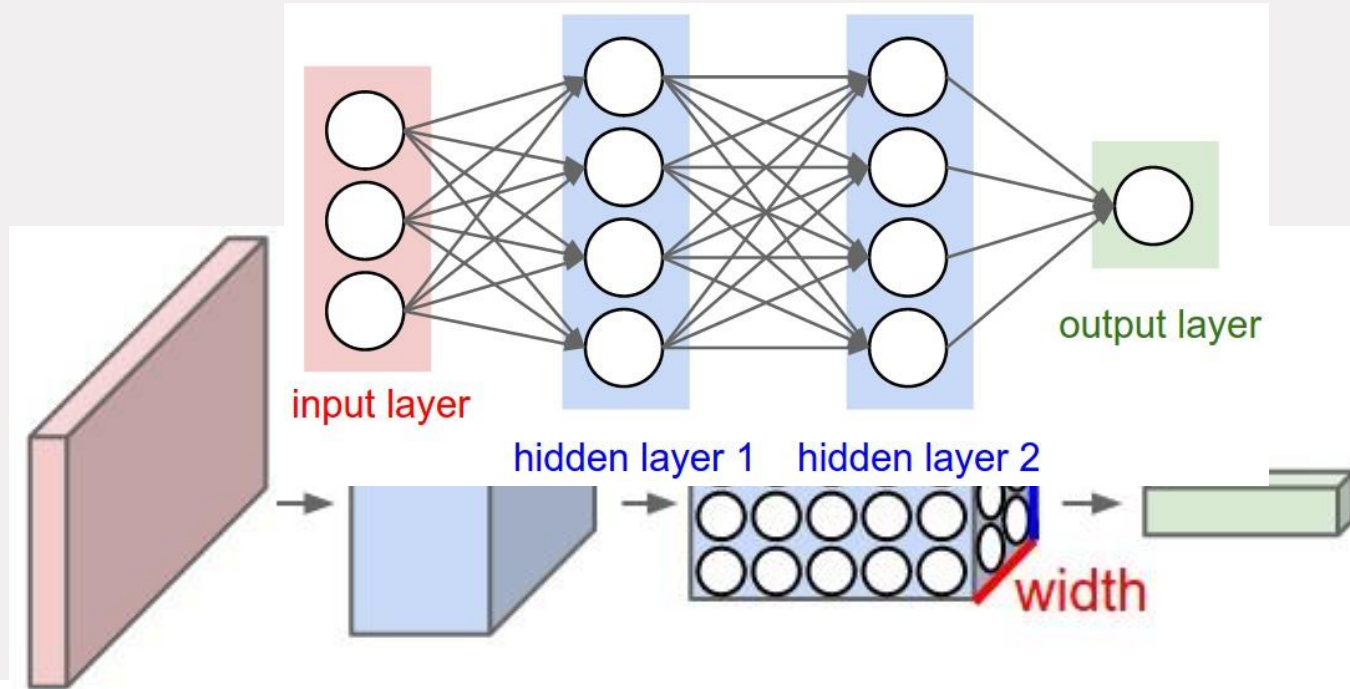


Post-training: visualizing what model has learned

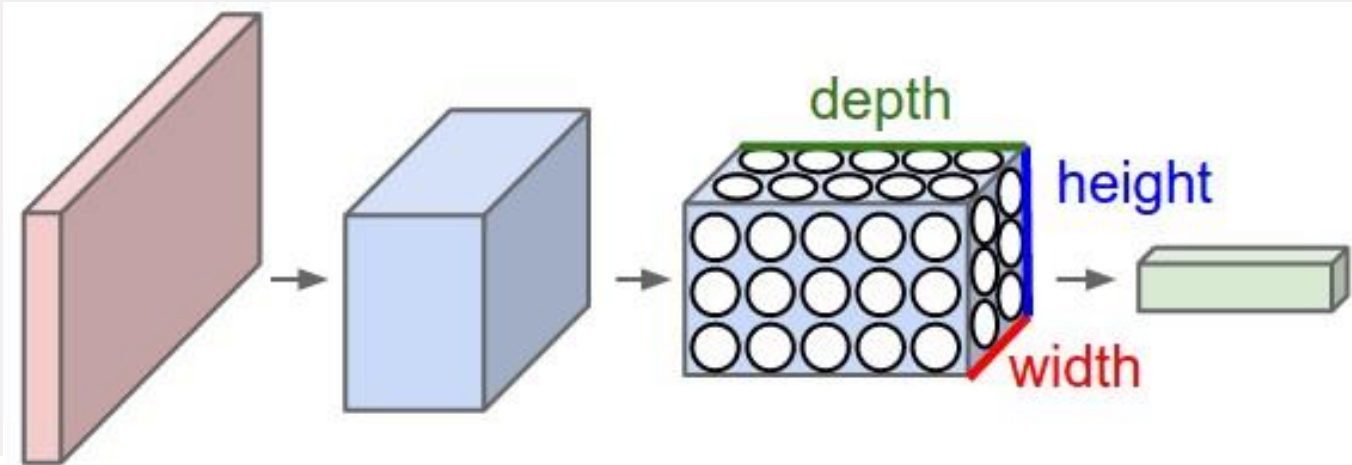
You can think of it as “there is not enough training data to reliably estimate all model weights”.



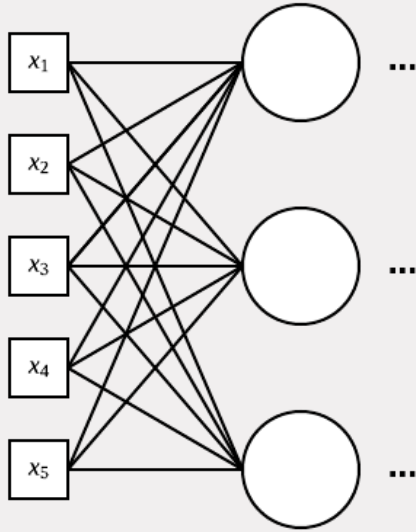
Many weights



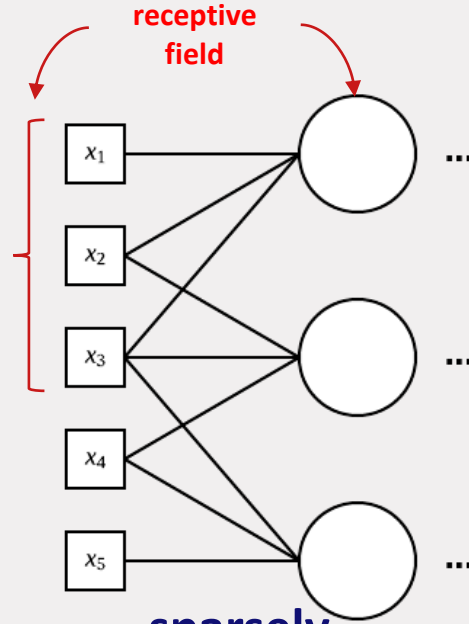
With large inputs such as images and deep networks, the number of weights "explodes". We need a way to reduce the number of weights, without sacrificing performance.



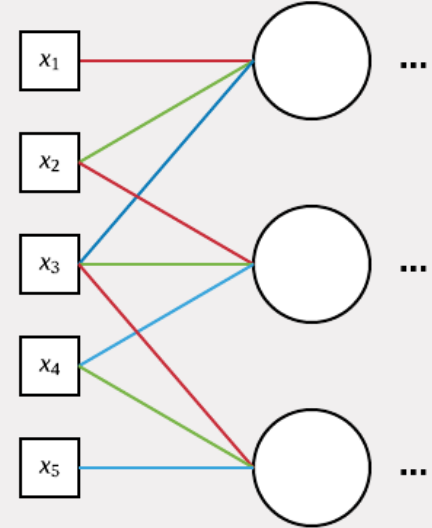
How many weights?



“regular” NN
15 weights

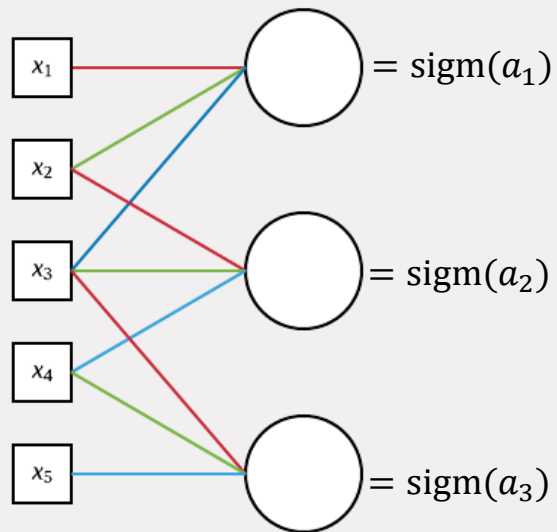


**sparsely
connected NN**
9 weights



shared weights
3 weights

Shared weights



shared weights
3 weights

$$a_1 = x_1 w_1 + x_2 w_2 + x_3 w_3$$

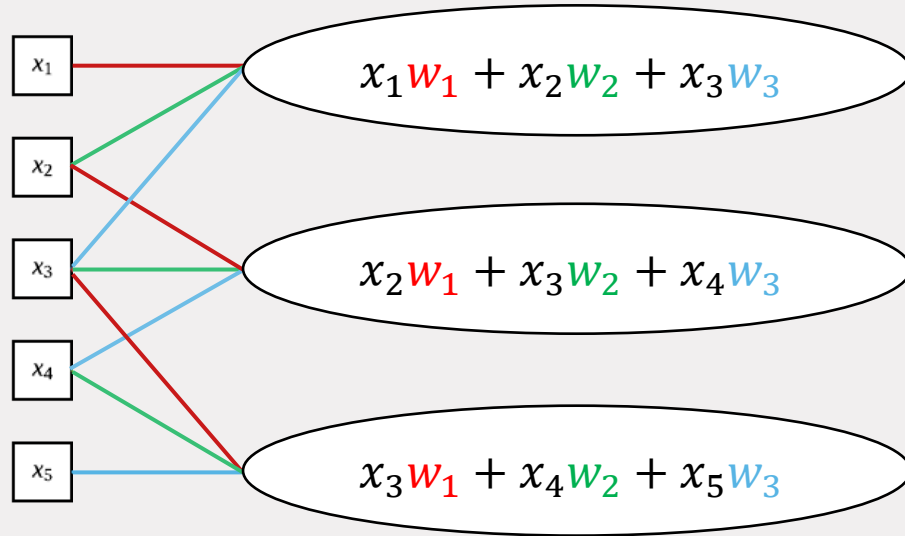
$$a_2 = x_2 w_1 + x_3 w_2 + x_4 w_3$$

$$a_3 = x_3 w_1 + x_4 w_2 + x_5 w_3$$

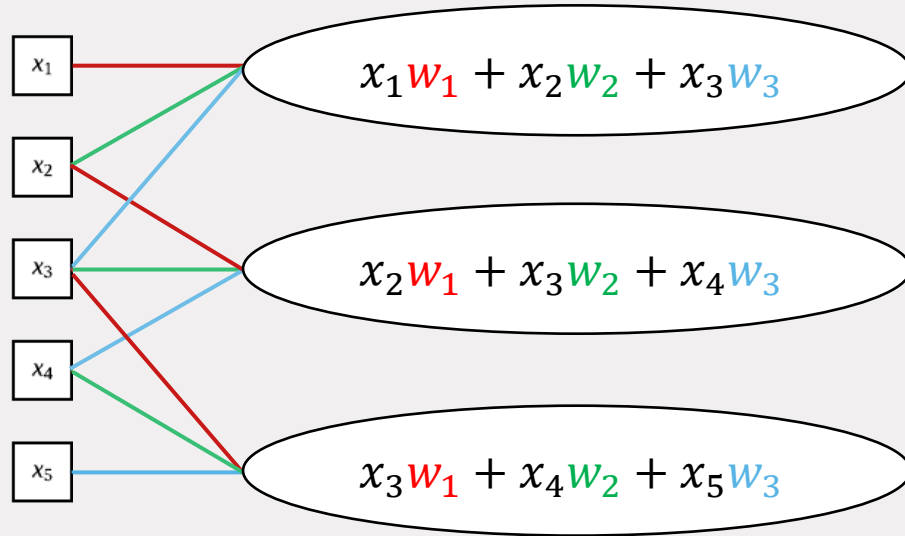
$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix} * \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix}$$

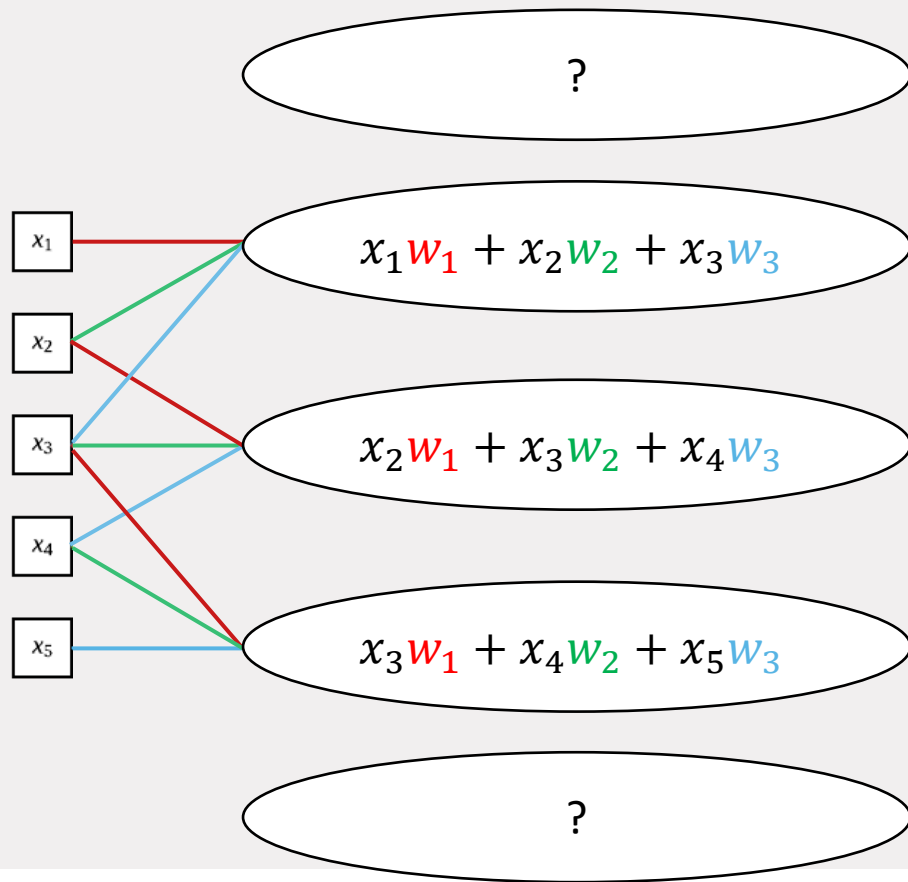
convolution, thus convolutional NN

1-D Convolution



1-D Convolution



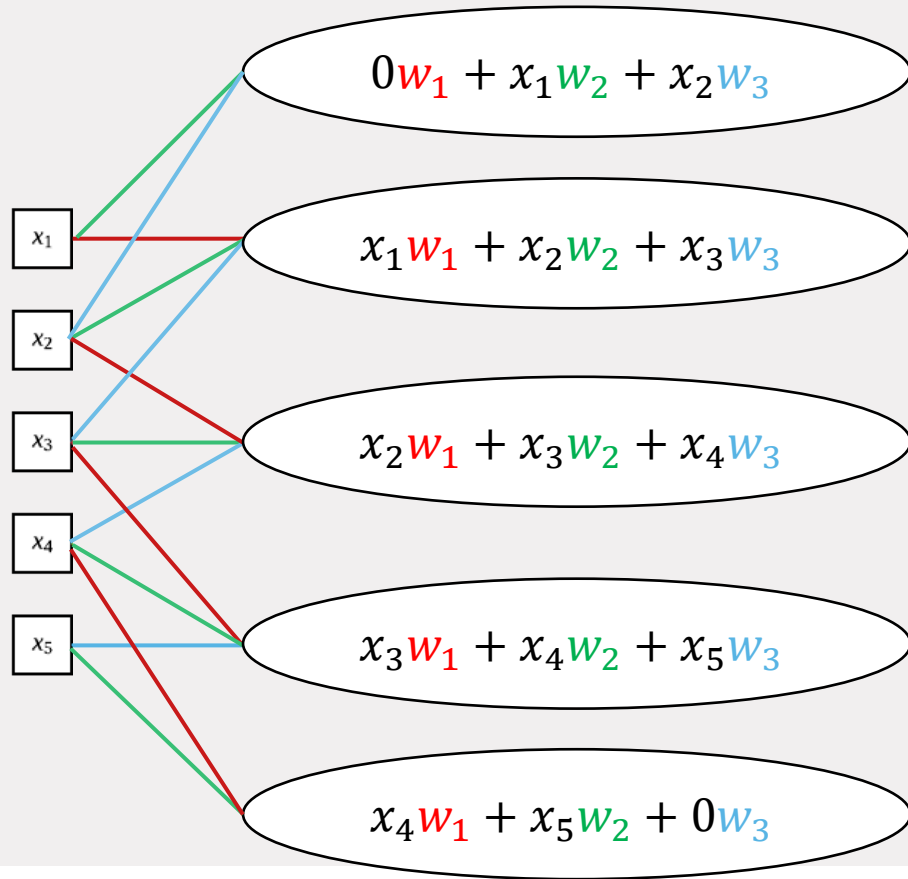


1-D Convolution

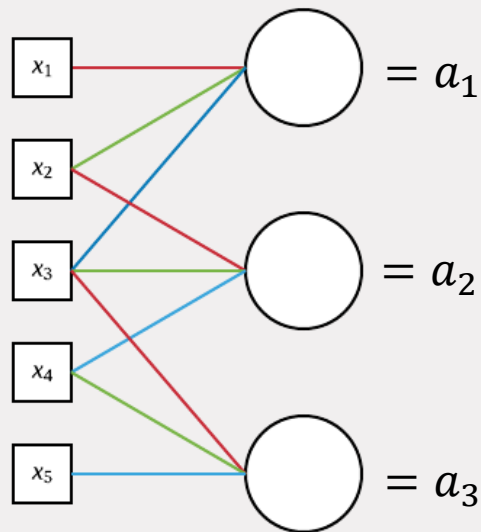
How can we keep the same number of features in hidden layer 1?

1-D Convolution

Zero-padding!



1-D Convolution



$$a_1 = x_1 w_1 + x_2 w_2 + x_3 w_3$$

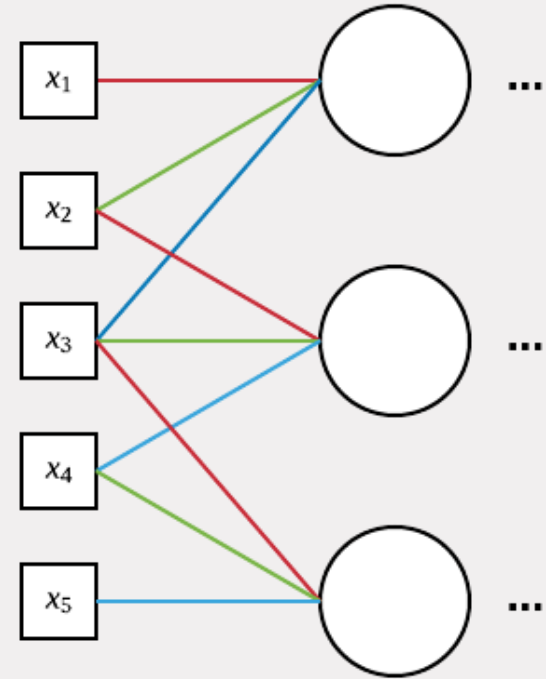
$$a_2 = x_2 w_1 + x_3 w_2 + x_4 w_3$$

$$a_3 = x_3 w_1 + x_4 w_2 + x_5 w_3$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix} * \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix}$$

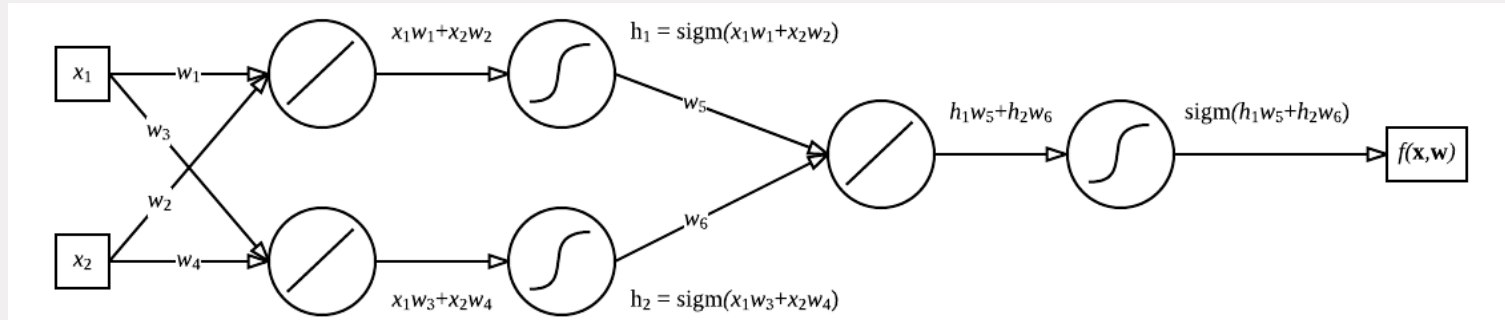
Properties of convolutional neural networks

- Sparse connectivity
- Weight sharing
- Parallel computations



Why does this work?

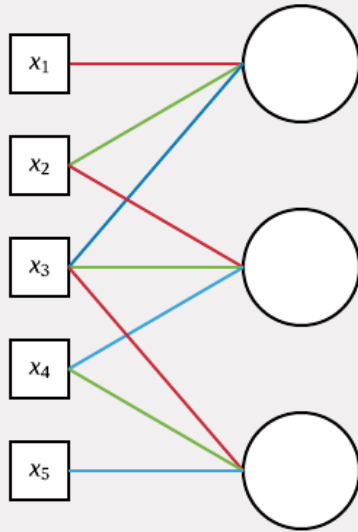
- Multiple layers
- Hidden layers contain features calculated from previous layers



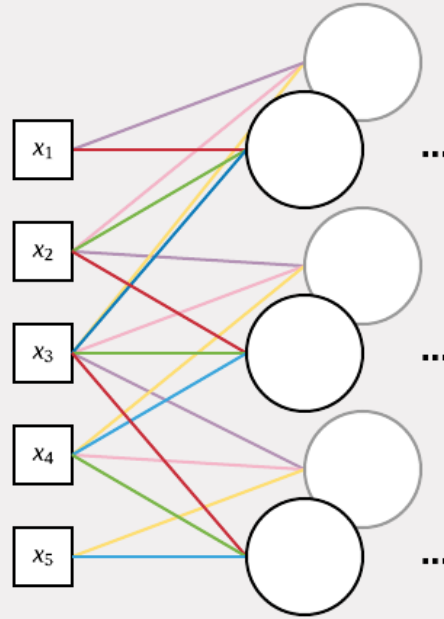
Why does this work?

- Different layers contain different transformation
 - Simple (e.g. edges, colors)
 - Complex (final layers)

One added benefit is that the learned transformations will be equivariant with translation (if the features/image is shifted up/down the features will still be detected).



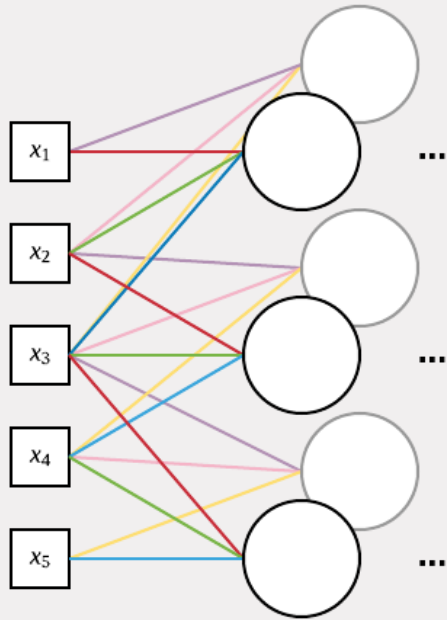
shared weights
3 weights



two sets shared weights
6 weights

We can add additional sets of weights that can learn additional interesting transformation of the input.

Note that the added neurons are not a new layer. They are part of layer 1.



two sets shared weights
6 weights

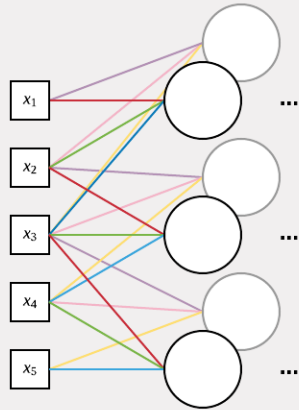
$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix} * \begin{bmatrix} w_{1,1} & w_{1,2} & w_{1,3} \end{bmatrix}$$

$$\begin{bmatrix} a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix} * \begin{bmatrix} w_{2,1} & w_{2,2} & w_{2,3} \end{bmatrix}$$

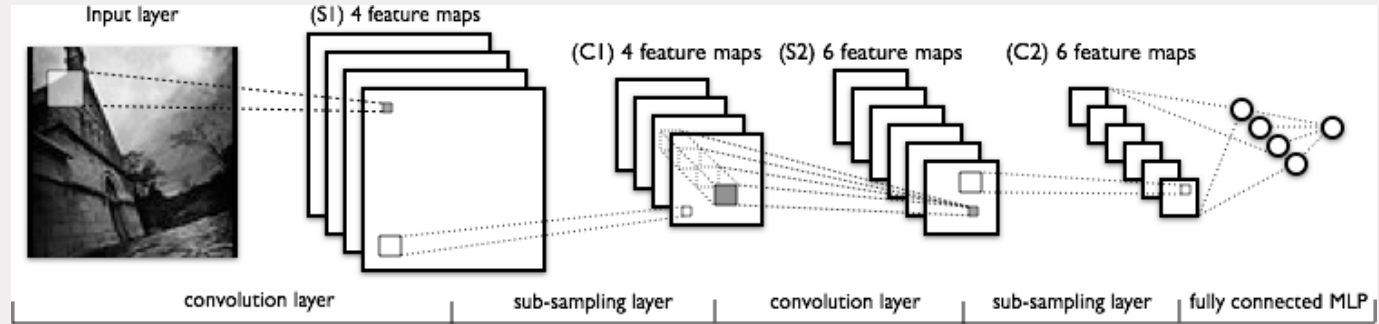
$\begin{bmatrix} w_{1,1} & w_{1,2} & w_{1,3} \end{bmatrix}$, and $\begin{bmatrix} w_{2,1} & w_{2,2} & w_{2,3} \end{bmatrix}$

are **convolution kernels**. They extract features. However, they are not hand-designed features – they were learned by the neural network.

Convolutional neural networks are ideal for images



1D NN



2D NN (images)

Because of the weight sharing, convolutional neural networks only work with structured data (such as images) as inputs.

$$\begin{array}{|c|c|c|c|c|} \hline I_1 & I_2 & I_3 & I_4 & I_5 \\ \hline I_6 & I_7 & I_8 & I_9 & I_{10} \\ \hline I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\ \hline I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\ \hline I_{21} & I_{22} & I_{23} & I_{24} & I_{25} \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline w_1 & w_2 & w_3 \\ \hline w_4 & w_5 & w_6 \\ \hline w_7 & w_8 & w_9 \\ \hline \end{array} =$$

$$\begin{array}{|c|c|c|} \hline I_1 w_1 + I_2 w_2 + I_3 w_3 & & \\ \hline + & & \\ I_6 w_4 + I_7 w_5 + I_8 w_6 & & \\ \hline + & & \\ I_{11} w_7 + I_{12} w_8 + I_{13} w_9 & & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|} \hline I_1 & I_2 & I_3 & I_4 & I_5 \\ \hline I_6 & I_7 & I_8 & I_9 & I_{10} \\ \hline I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\ \hline I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\ \hline I_{21} & I_{22} & I_{23} & I_{24} & I_{25} \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline w_1 & w_2 & w_3 \\ \hline w_4 & w_5 & w_6 \\ \hline w_7 & w_8 & w_9 \\ \hline \end{array} =$$

$$\begin{array}{|c|c|c|} \hline \begin{array}{c} I_1 w_1 + I_2 w_2 + I_3 w_3 \\ + \\ I_6 w_4 + I_7 w_5 + I_8 w_6 \\ + \\ I_{11} w_7 + I_{12} w_8 + I_{13} w_9 \end{array} & \begin{array}{c} I_2 w_1 + I_3 w_2 + I_4 w_3 \\ + \\ I_7 w_4 + I_8 w_5 + I_9 w_6 \\ + \\ I_{12} w_7 + I_{13} w_8 + I_{14} w_9 \end{array} & \\ \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|} \hline I_1 & I_2 & I_3 & I_4 & I_5 \\ \hline I_6 & I_7 & I_8 & I_9 & I_{10} \\ \hline I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\ \hline I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\ \hline I_{21} & I_{22} & I_{23} & I_{24} & I_{25} \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline w_1 & w_2 & w_3 \\ \hline w_4 & w_5 & w_6 \\ \hline w_7 & w_8 & w_9 \\ \hline \end{array} =$$

$$\begin{array}{|c|c|c|} \hline I_1 w_1 + I_2 w_2 + I_3 w_3 + I_6 w_4 + I_7 w_5 + I_8 w_6 + I_{11} w_7 + I_{12} w_8 + I_{13} w_9 & I_2 w_1 + I_3 w_2 + I_4 w_3 + I_7 w_4 + I_8 w_5 + I_9 w_6 + I_{12} w_7 + I_{13} w_8 + I_{14} w_9 & I_3 w_1 + I_4 w_2 + I_5 w_3 + I_8 w_4 + I_9 w_5 + I_{10} w_6 + I_{13} w_7 + I_{14} w_8 + I_{15} w_9 \\ \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|} \hline I_1 & I_2 & I_3 & I_4 & I_5 \\ \hline I_6 & I_7 & I_8 & I_9 & I_{10} \\ \hline I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\ \hline I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\ \hline I_{21} & I_{22} & I_{23} & I_{24} & I_{25} \\ \hline \end{array}
 * \begin{array}{|c|c|c|} \hline w_1 & w_2 & w_3 \\ \hline w_4 & w_5 & w_6 \\ \hline w_7 & w_8 & w_9 \\ \hline \end{array} =$$

$$\begin{array}{|c|c|c|} \hline \begin{array}{c} I_1 w_1 + I_2 w_2 + I_3 w_3 \\ + \\ I_6 w_4 + I_7 w_5 + I_8 w_6 \\ + \\ I_{11} w_7 + I_{12} w_8 + I_{13} w_9 \end{array} & \begin{array}{c} I_2 w_1 + I_3 w_2 + I_4 w_3 \\ + \\ I_7 w_4 + I_8 w_5 + I_9 w_6 \\ + \\ I_{12} w_7 + I_{13} w_8 + I_{14} w_9 \end{array} & \begin{array}{c} I_3 w_1 + I_4 w_2 + I_5 w_3 \\ + \\ I_8 w_4 + I_9 w_5 + I_{10} w_6 \\ + \\ I_{13} w_7 + I_{14} w_8 + I_{15} w_9 \end{array} \\ \hline \begin{array}{c} I_6 w_1 + I_7 w_2 + I_8 w_3 \\ + \\ I_{11} w_4 + I_{12} w_5 + I_{13} w_6 \\ + \\ I_{16} w_7 + I_{17} w_8 + I_{18} w_9 \end{array} & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

$$\begin{array}{ccccc}
 I_1 & I_2 & I_3 & I_4 & I_5 \\
 I_6 & I_7 & I_8 & I_9 & I_{10} \\
 I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\
 I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\
 I_{21} & I_{22} & I_{23} & I_{24} & I_{25}
 \end{array}
 *
 \begin{array}{ccc}
 w_1 & w_2 & w_3 \\
 w_4 & w_5 & w_6 \\
 w_7 & w_8 & w_9
 \end{array}
 =$$

$$\begin{array}{ccc}
 \begin{array}{c} I_1 w_1 + I_2 w_2 + I_3 w_3 \\ + \\ I_6 w_4 + I_7 w_5 + I_8 w_6 \\ + \\ I_{11} w_7 + I_{12} w_8 + I_{13} w_9 \end{array} &
 \begin{array}{c} I_2 w_1 + I_3 w_2 + I_4 w_3 \\ + \\ I_7 w_4 + I_8 w_5 + I_9 w_6 \\ + \\ I_{12} w_7 + I_{13} w_8 + I_{14} w_9 \end{array} &
 \begin{array}{c} I_3 w_1 + I_4 w_2 + I_5 w_3 \\ + \\ I_8 w_4 + I_9 w_5 + I_{10} w_6 \\ + \\ I_{13} w_7 + I_{14} w_8 + I_{15} w_9 \end{array} \\
 \begin{array}{c} I_6 w_1 + I_7 w_2 + I_8 w_3 \\ + \\ I_{11} w_4 + I_{12} w_5 + I_{13} w_6 \\ + \\ I_{16} w_7 + I_{17} w_8 + I_{18} w_9 \end{array} &
 \begin{array}{c} I_7 w_1 + I_8 w_2 + I_9 w_3 \\ + \\ I_{12} w_4 + I_{13} w_5 + I_{14} w_6 \\ + \\ I_{17} w_7 + I_{18} w_8 + I_{19} w_9 \end{array} &
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 \begin{array}{c} \\ \\ \\ \end{array} &
 \begin{array}{c} \\ \\ \\ \end{array} &
 \begin{array}{c} \\ \\ \\ \end{array}
 \end{array}$$

$$\begin{array}{ccccc}
 I_1 & I_2 & I_3 & I_4 & I_5 \\
 I_6 & I_7 & I_8 & I_9 & I_{10} \\
 I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\
 I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\
 I_{21} & I_{22} & I_{23} & I_{24} & I_{25}
 \end{array}
 *
 \begin{array}{ccc}
 w_1 & w_2 & w_3 \\
 w_4 & w_5 & w_6 \\
 w_7 & w_8 & w_9
 \end{array}
 =$$

$$\begin{array}{ccc}
 \begin{array}{c} I_1 w_1 + I_2 w_2 + I_3 w_3 \\ + \\ I_6 w_4 + I_7 w_5 + I_8 w_6 \\ + \\ I_{11} w_7 + I_{12} w_8 + I_{13} w_9 \end{array} &
 \begin{array}{c} I_2 w_1 + I_3 w_2 + I_4 w_3 \\ + \\ I_7 w_4 + I_8 w_5 + I_9 w_6 \\ + \\ I_{12} w_7 + I_{13} w_8 + I_{14} w_9 \end{array} &
 \begin{array}{c} I_3 w_1 + I_4 w_2 + I_5 w_3 \\ + \\ I_8 w_4 + I_9 w_5 + I_{10} w_6 \\ + \\ I_{13} w_7 + I_{14} w_8 + I_{15} w_9 \end{array} \\
 \begin{array}{c} I_6 w_1 + I_7 w_2 + I_8 w_3 \\ + \\ I_{11} w_4 + I_{12} w_5 + I_{13} w_6 \\ + \\ I_{16} w_7 + I_{17} w_8 + I_{18} w_9 \end{array} &
 \begin{array}{c} I_7 w_1 + I_8 w_2 + I_9 w_3 \\ + \\ I_{12} w_4 + I_{13} w_5 + I_{14} w_6 \\ + \\ I_{17} w_7 + I_{18} w_8 + I_{19} w_9 \end{array} &
 \begin{array}{c} I_8 w_1 + I_9 w_2 + I_{10} w_3 \\ + \\ I_{13} w_4 + I_{14} w_5 + I_{15} w_6 \\ + \\ I_{18} w_7 + I_{19} w_8 + I_{20} w_9 \end{array} \\
 \square & \square & \square
 \end{array}$$

$$\begin{array}{ccccc}
 I_1 & I_2 & I_3 & I_4 & I_5 \\
 I_6 & I_7 & I_8 & I_9 & I_{10} \\
 I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\
 I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\
 I_{21} & I_{22} & I_{23} & I_{24} & I_{25}
 \end{array}
 *
 \begin{array}{ccc}
 w_1 & w_2 & w_3 \\
 w_4 & w_5 & w_6 \\
 w_7 & w_8 & w_9
 \end{array}
 =$$

$$\begin{array}{ccc}
 \begin{array}{c} I_1 w_1 + I_2 w_2 + I_3 w_3 \\ + \\ I_6 w_4 + I_7 w_5 + I_8 w_6 \\ + \\ I_{11} w_7 + I_{12} w_8 + I_{13} w_9 \end{array} &
 \begin{array}{c} I_2 w_1 + I_3 w_2 + I_4 w_3 \\ + \\ I_7 w_4 + I_8 w_5 + I_9 w_6 \\ + \\ I_{12} w_7 + I_{13} w_8 + I_{14} w_9 \end{array} &
 \begin{array}{c} I_3 w_1 + I_4 w_2 + I_5 w_3 \\ + \\ I_8 w_4 + I_9 w_5 + I_{10} w_6 \\ + \\ I_{13} w_7 + I_{14} w_8 + I_{15} w_9 \end{array} \\
 \begin{array}{c} I_6 w_1 + I_7 w_2 + I_8 w_3 \\ + \\ I_{11} w_4 + I_{12} w_5 + I_{13} w_6 \\ + \\ I_{16} w_7 + I_{17} w_8 + I_{18} w_9 \end{array} &
 \begin{array}{c} I_7 w_1 + I_8 w_2 + I_9 w_3 \\ + \\ I_{12} w_4 + I_{13} w_5 + I_{14} w_6 \\ + \\ I_{17} w_7 + I_{18} w_8 + I_{19} w_9 \end{array} &
 \begin{array}{c} I_8 w_1 + I_9 w_2 + I_{10} w_3 \\ + \\ I_{13} w_4 + I_{14} w_5 + I_{15} w_6 \\ + \\ I_{18} w_7 + I_{19} w_8 + I_{20} w_9 \end{array} \\
 \begin{array}{c} I_{11} w_1 + I_{12} w_2 + I_{13} w_3 \\ + \\ I_{16} w_4 + I_{17} w_5 + I_{18} w_6 \\ + \\ I_{21} w_7 + I_{22} w_8 + I_{23} w_9 \end{array} &
 \begin{array}{c} \\ \\ \\ \end{array} &
 \begin{array}{c} \\ \\ \\ \end{array}
 \end{array}$$

I_1	I_2	I_3	I_4	I_5
I_6	I_7	I_8	I_9	I_{10}
I_{11}	I_{12}	I_{13}	I_{14}	I_{15}
I_{16}	I_{17}	I_{18}	I_{19}	I_{20}
I_{21}	I_{22}	I_{23}	I_{24}	I_{25}

 \ast

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

 $=$

$I_1 w_1 + I_2 w_2 + I_3 w_3$ $+$ $I_6 w_4 + I_7 w_5 + I_8 w_6$ $+$ $I_{11} w_7 + I_{12} w_8 + I_{13} w_9$	$I_2 w_1 + I_3 w_2 + I_4 w_3$ $+$ $I_7 w_4 + I_8 w_5 + I_9 w_6$ $+$ $I_{12} w_7 + I_{13} w_8 + I_{14} w_9$	$I_3 w_1 + I_4 w_2 + I_5 w_3$ $+$ $I_8 w_4 + I_9 w_5 + I_{10} w_6$ $+$ $I_{13} w_7 + I_{14} w_8 + I_{15} w_9$
$I_6 w_1 + I_7 w_2 + I_8 w_3$ $+$ $I_{11} w_4 + I_{12} w_5 + I_{13} w_6$ $+$ $I_{16} w_7 + I_{17} w_8 + I_{18} w_9$	$I_7 w_1 + I_8 w_2 + I_9 w_3$ $+$ $I_{12} w_4 + I_{13} w_5 + I_{14} w_6$ $+$ $I_{17} w_7 + I_{18} w_8 + I_{19} w_9$	$I_8 w_1 + I_9 w_2 + I_{10} w_3$ $+$ $I_{13} w_4 + I_{14} w_5 + I_{15} w_6$ $+$ $I_{18} w_7 + I_{19} w_8 + I_{20} w_9$
$I_{11} w_1 + I_{12} w_2 + I_{13} w_3$ $+$ $I_{16} w_4 + I_{17} w_5 + I_{18} w_6$ $+$ $I_{21} w_7 + I_{22} w_8 + I_{23} w_9$	$I_{12} w_1 + I_{13} w_2 + I_{14} w_3$ $+$ $I_{17} w_4 + I_{18} w_5 + I_{19} w_6$ $+$ $I_{22} w_7 + I_{23} w_8 + I_{24} w_9$	

I_1	I_2	I_3	I_4	I_5
I_6	I_7	I_8	I_9	I_{10}
I_{11}	I_{12}	I_{13}	I_{14}	I_{15}
I_{16}	I_{17}	I_{18}	I_{19}	I_{20}
I_{21}	I_{22}	I_{23}	I_{24}	I_{25}

 \ast

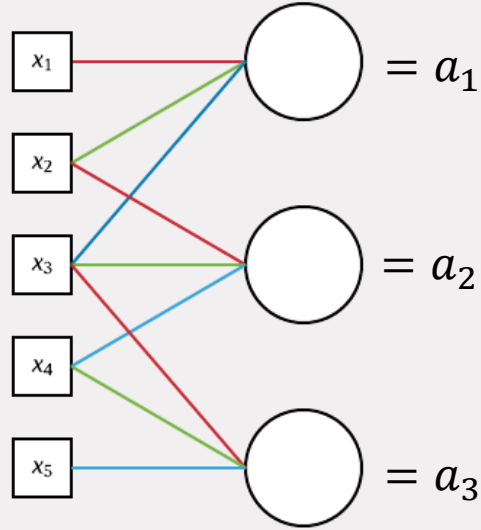
w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

 $+ \mathbf{b} =$

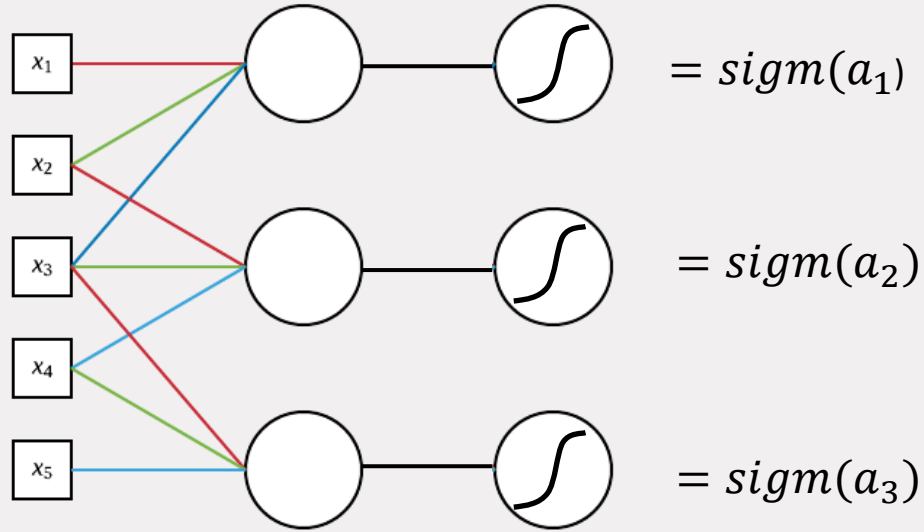
$I_1 w_1 + I_2 w_2 + I_3 w_3$ + $I_6 w_4 + I_7 w_5 + I_8 w_6$ + $I_{11} w_7 + I_{12} w_8 + I_{13} w_9$	$I_2 w_1 + I_3 w_2 + I_4 w_3$ + $I_7 w_4 + I_8 w_5 + I_9 w_6$ + $I_{12} w_7 + I_{13} w_8 + I_{14} w_9$	$I_3 w_1 + I_4 w_2 + I_5 w_3$ + $I_8 w_4 + I_9 w_5 + I_{10} w_6$ + $I_{13} w_7 + I_{14} w_8 + I_{15} w_9$
$I_6 w_1 + I_7 w_2 + I_8 w_3$ + $I_{11} w_4 + I_{12} w_5 + I_{13} w_6$ + $I_{16} w_7 + I_{17} w_8 + I_{18} w_9$	$I_7 w_1 + I_8 w_2 + I_9 w_3$ + $I_{12} w_4 + I_{13} w_5 + I_{14} w_6$ + $I_{17} w_7 + I_{18} w_8 + I_{19} w_9$	$I_8 w_1 + I_9 w_2 + I_{10} w_3$ + $I_{13} w_4 + I_{14} w_5 + I_{15} w_6$ + $I_{18} w_7 + I_{19} w_8 + I_{20} w_9$
$I_{11} w_1 + I_{12} w_2 + I_{13} w_3$ + $I_{16} w_4 + I_{17} w_5 + I_{18} w_6$ + $I_{21} w_7 + I_{22} w_8 + I_{23} w_9$	$I_{12} w_1 + I_{13} w_2 + I_{14} w_3$ + $I_{17} w_4 + I_{18} w_5 + I_{19} w_6$ + $I_{22} w_7 + I_{23} w_8 + I_{24} w_9$	$I_{13} w_1 + I_{14} w_2 + I_{15} w_3$ + $I_{18} w_4 + I_{19} w_5 + I_{20} w_6$ + $I_{23} w_7 + I_{24} w_8 + I_{25} w_9$

 $+ \mathbf{b}$

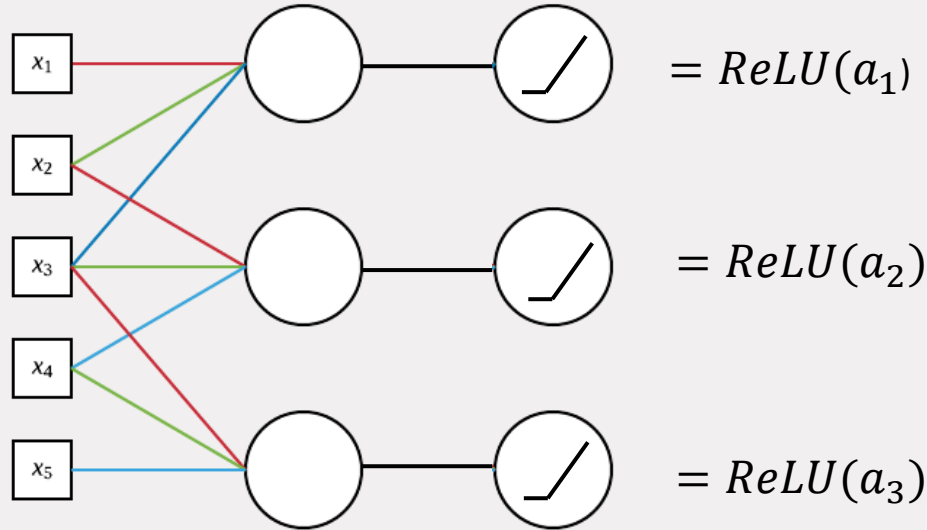
Convolutions + non-linearity



Convolutions + non-linearity (sigmoid)



Convolutions + non-linearity (ReLU)



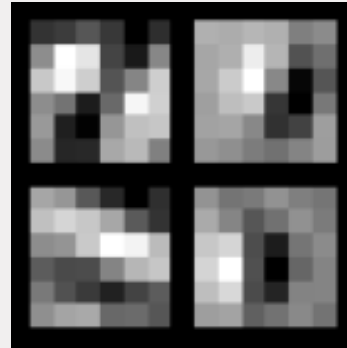
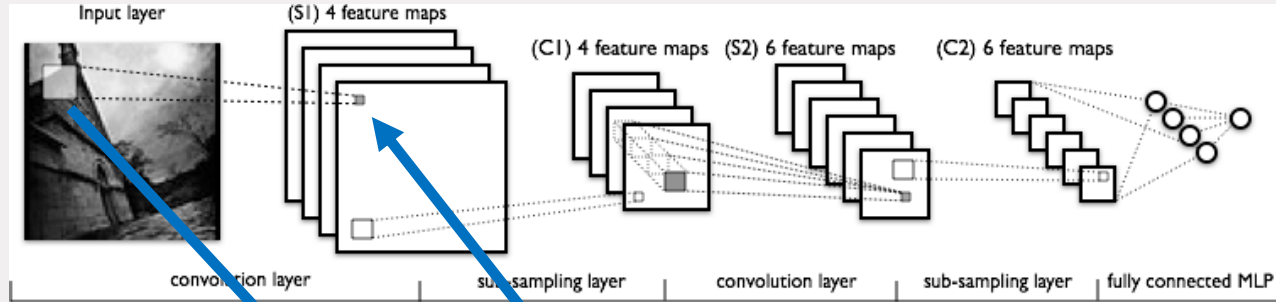
Break!

Quokka (Australia)

<https://imgur.com/r/aww/GqJi0il>



Convolutional kernels



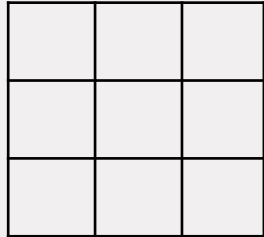
these are the weights
(convolutional kernels)
that produce the four
feature maps

Kernel size

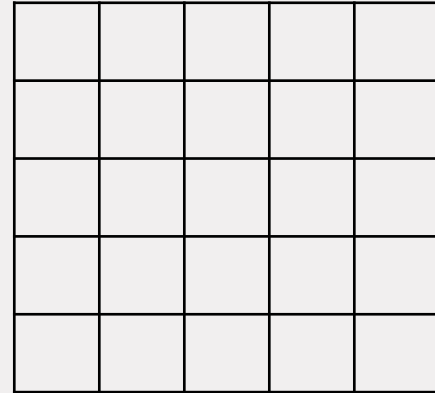
1 x 1



3 x 3

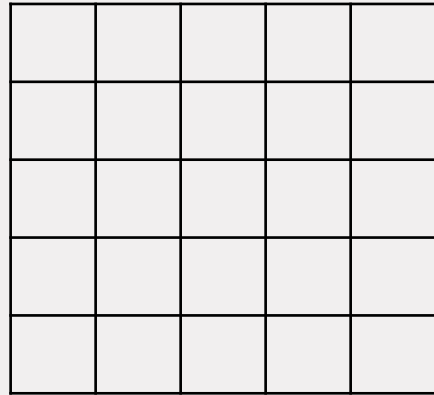


5 x 5



- More weights → more information
- More computations / memory
- Receptive field

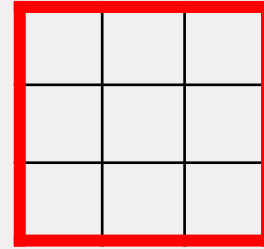
Receptive field



5 x 5

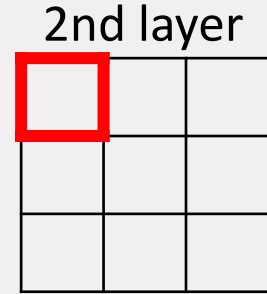
25 weights

Same receptive
field



3 x 3

9 weights

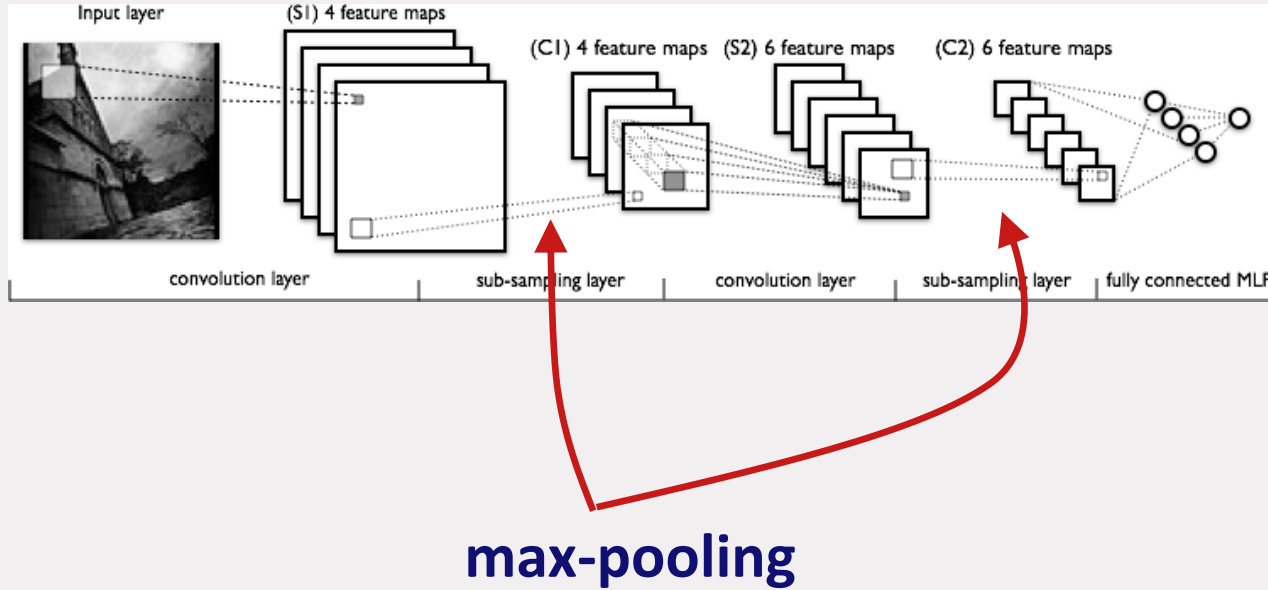


3 x 3

9 weights

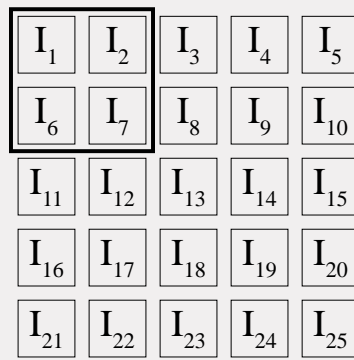
= 18 weights

Max-pooling

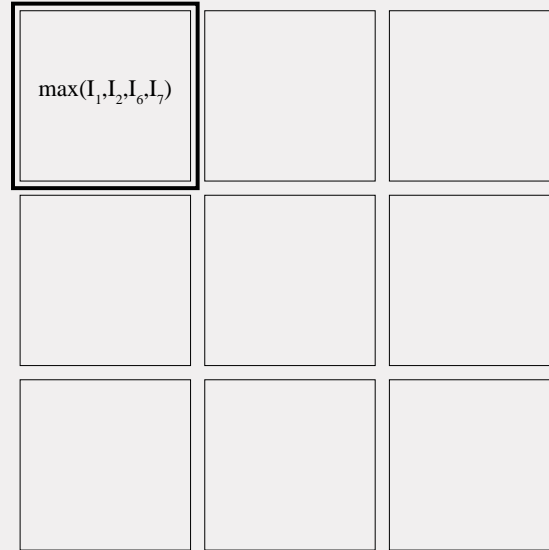


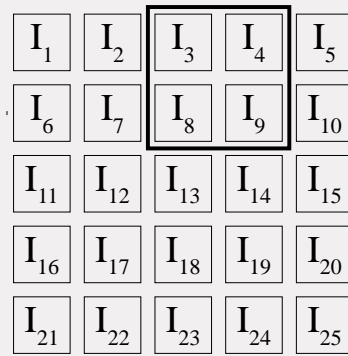
Max-pooling

- Reduce size of feature space
- Maximum of features
- Typical kernel size = 2×2

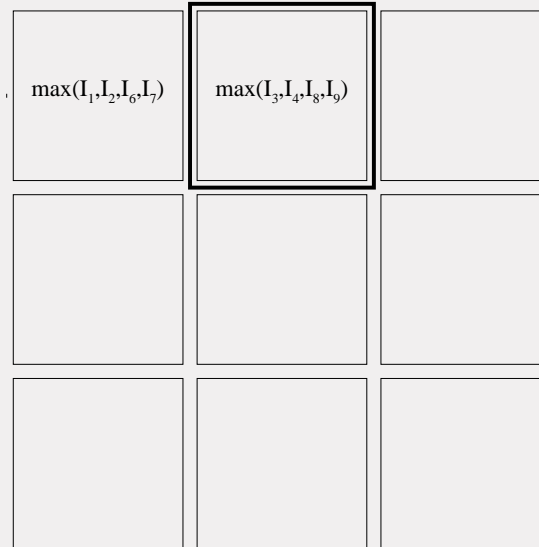


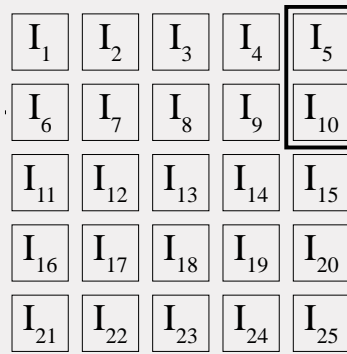
2×2 max pooling



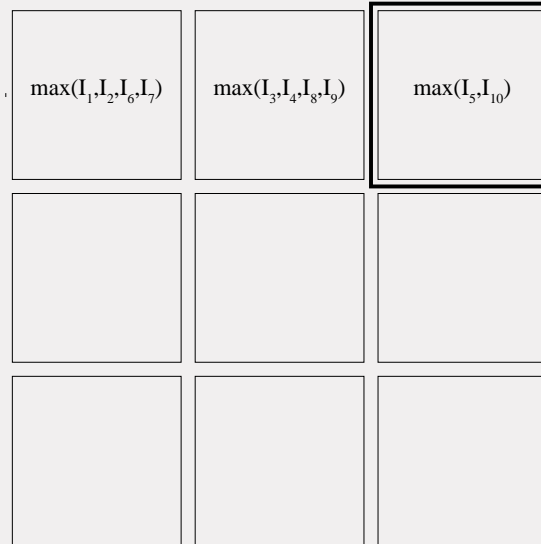


2×2 max pooling

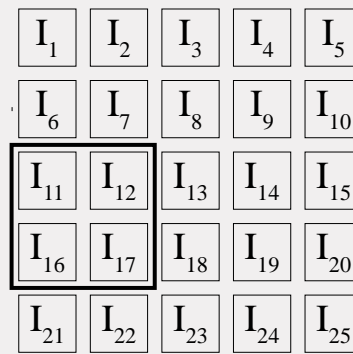




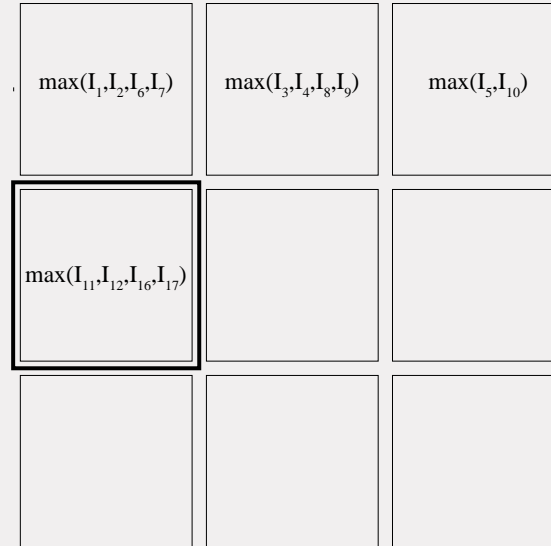
2×2 max pooling

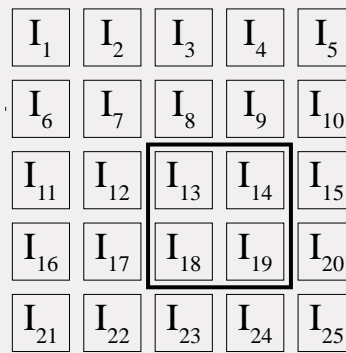


In this example:
no zero padding

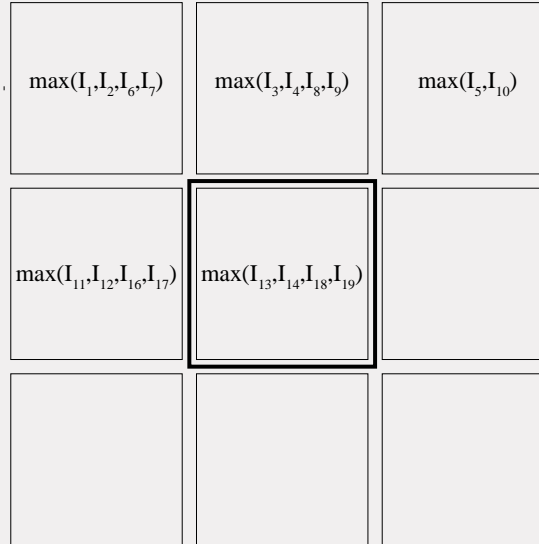


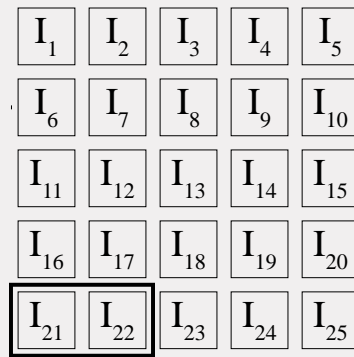
2×2 max pooling



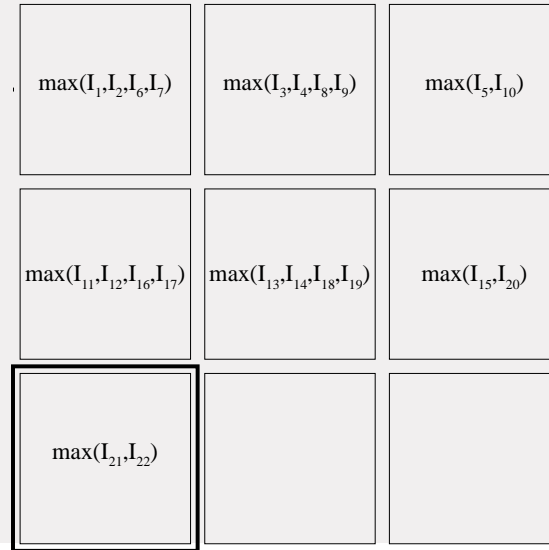


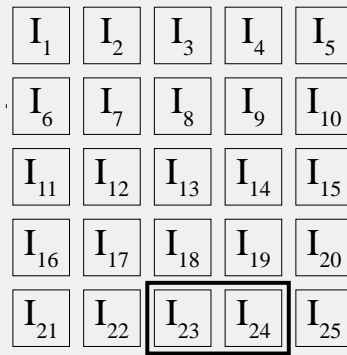
2×2 max pooling



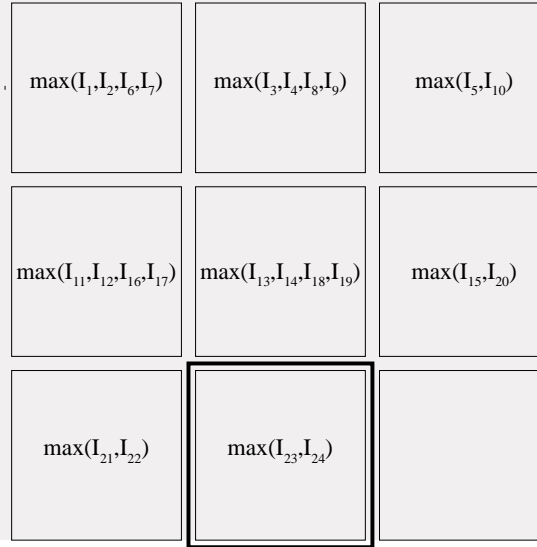


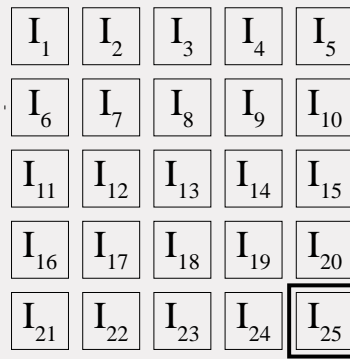
2×2 max pooling



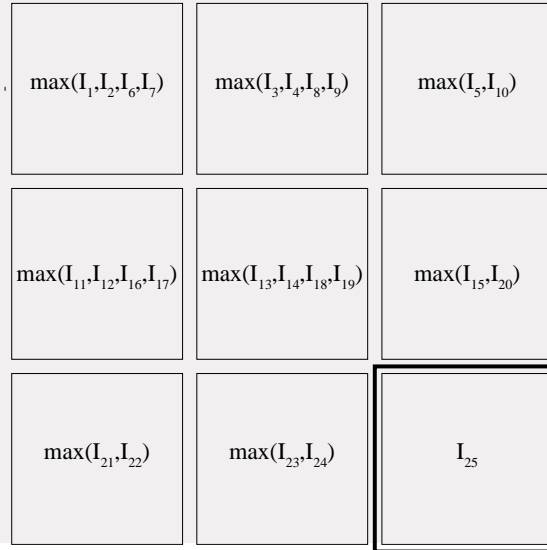


2×2 max pooling





2×2 max pooling



Benefits of max-pooling

- “Quickly” reduces the number size of the feature maps
- Introduces translation invariance (slightly translated version of the input image will result in the same output)

Alternative: Average Pooling

Let try it out!

<http://scs.ryerson.ca/~aharley/vis/conv/flat.html>

An Interactive Node-Link Visualization of Convolutional Neural Networks

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Toronto, ON M5B 2K3, Canada
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Training of Convolutional Neural Networks

- Similar to training of 'fully connected' neural networks
- Optimize networks weights with respect to a loss function that describes difference between network output and label/annotation
- Update networks weights iteratively
- Keep track of model performance on train & validation set

Summary

- Concept of **convolutions** in a neural network
- Why can we use a **convolutional approach** for (medical) **images**
- Convolutions enable development of **deep** (and large) **neural networks**
- **Max-pooling layer** in a convolutional neural network
- **Kernel size**
- **Receptive field**

