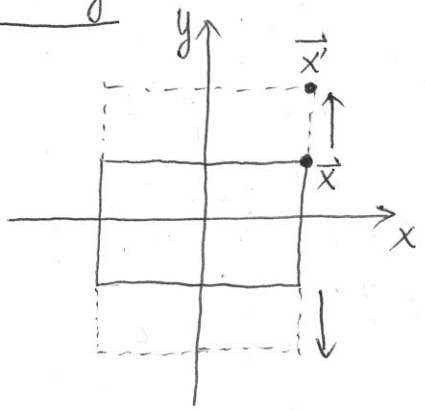
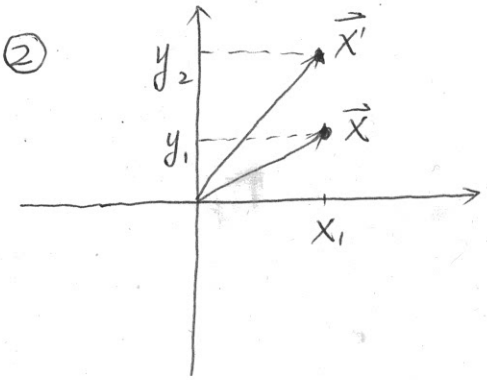
Scaling

scale ↑ 2 times in y direction

① 

② 

③. $\vec{x} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ $\vec{x}' = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

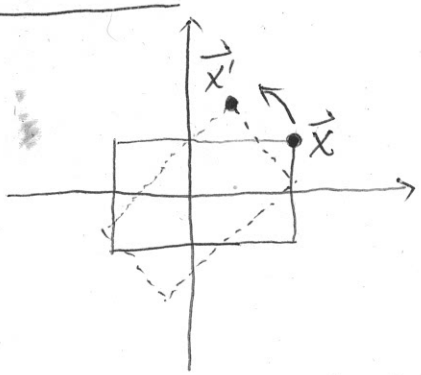
$\left.\begin{aligned} x_2 &= x_1 \\ y_2 &= 2y_1 \end{aligned}\right\} \Rightarrow \begin{aligned} x_2 &= 1 \cdot x_1 + 0 \cdot y_1 \\ y_2 &= 0 \cdot x_1 + 2 y_1 \end{aligned}$

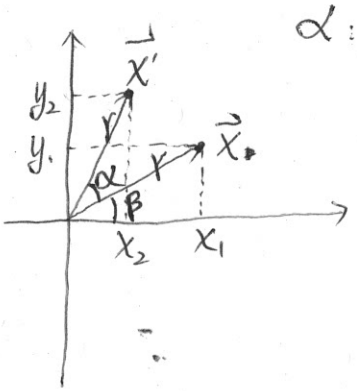
④. $\vec{x}' = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \cdot x_1 + 0 \cdot y_1 \\ 0 \cdot x_1 + 2 \cdot y_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \vec{x}$

⑤. $f(\vec{x}) = \vec{x}'$ what is $f(\cdot)$? $f(\cdot)$

Rotation

α : angle of rotation

① 

② 

③. $\vec{x} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ $\vec{x}' = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$x_1 = r\cos\beta$ $\quad x_2 = r\cos(\alpha+\beta)$

$y_1 = r\sin\beta$ $\quad y_2 = r\sin(\alpha+\beta)$

④ $\vec{x}' = f(\vec{x})$ $\quad f(\cdot) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad \vec{x}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \vec{x}$

⑤. $x_2 = r\cos(\alpha+\beta) = \underline{r}\cos\alpha\,\underline{\cos\beta} - \underline{r}\sin\alpha\,\underline{\sin\beta}$

$= x_1\cos\alpha - y_1\sin\alpha$

$y_2 = r\sin(\alpha+\beta) = \underline{r}\sin\alpha\,\underline{\cos\beta} + \underline{r}\cos\alpha\,\underline{\sin\beta}$

$= x_1\sin\alpha + y_1\cos\alpha$

⑥

$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha \cdot x_1 - \sin\alpha \cdot y_1 \\ \sin\alpha \cdot x_1 + \cos\alpha \cdot y_1 \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$