Pattern Recognition HW 1

171240510 Yuheng Ma

Math Major, Kuang Yaming Honors School

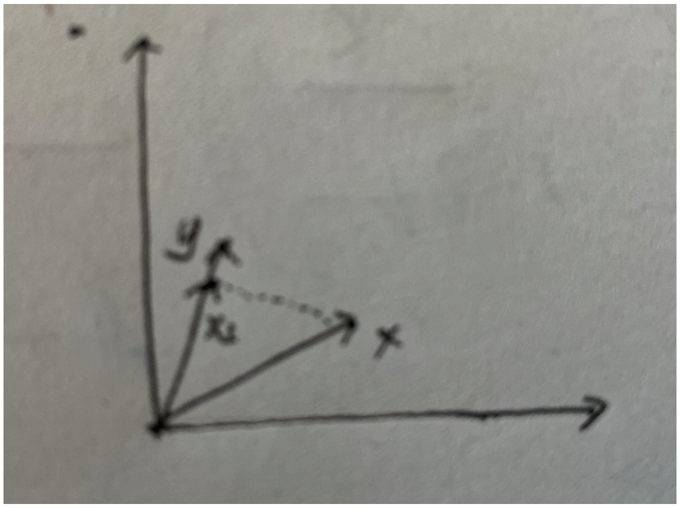
March 3, 2020

1 01 Introduction

1. We need $\sqrt{\frac{8a-1}{3}} > 0$ and thus $a > \frac{1}{8}$.
2. 1
3. a=$\frac{1}{2}$, where output value is 1. It might be constantly 1.
4. 1.3956439237389602+0.2284251258739286j
5. use
$$\sqrt[3]{a+\frac{a+1}{3}\sqrt{\frac{8a-1}{3}}}+sign(a-\frac{a+1}{3}\sqrt{\frac{8a-1}{3}})\sqrt[3]{|a-\frac{a+1}{3}\sqrt{\frac{8a-1}{3}}|}$$
6. For any a$\in\mathbb{R}$, $\sqrt[3]{a+\frac{a+1}{3}\sqrt{\frac{8a-1}{3}}}+\sqrt[3]{a-\frac{a+1}{3}\sqrt{\frac{8a-1}{3}}}=1$.
   Proof: After some algebra, we have
$$\sqrt[3]{a+\frac{a+1}{3}\sqrt{\frac{8a-1}{3}}}+\sqrt[3]{a-\frac{a+1}{3}\sqrt{\frac{8a-1}{3}}}=\frac{\sqrt[3]{(1+\sqrt{\frac{8a-1}{3}})^3}+\sqrt[3]{(1+\sqrt{\frac{8a-1}{3}})^3}}{2}$$
   and thus proved.
7. Take a=2, we have
$$\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}=1$$
8. Cardano stated that
$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$
   is a root of x^3+px+q while $4p^3+27q^2>0$. Thus when 1 is a root of x^3+px+q, we have p+q=-1. Then
$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$
$$=\sqrt[3]{-\frac{q}{2}+\sqrt{-\frac{q^3}{27}+\frac{5}{36}q^2-\frac{q}{9}-\frac{1}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{-\frac{q^3}{27}+\frac{5}{36}q^2-\frac{q}{9}-\frac{1}{27}}}$$
   By replacing q by -2a, we have Equation 1. Also, by
$$-\frac{q^3}{-27}+\frac{5}{36}q^2-\frac{q}{9}-\frac{1}{27}=-\frac{(q/2-1)^2}{9}\frac{4q+1}{3}$$
   , we can see that constraint of q is q$<-\frac{1}{4}$ which is just a$>\frac{1}{8}$.

2 02 Math 1

1. $(\frac{\sqrt{3}}{2}, \frac{3}{2})$
2. $(\boldsymbol{x} - \boldsymbol{x}_\perp) \cdot \boldsymbol{y}$=$(\frac{\sqrt{3}}{2}, -\frac{1}{2}) \cdot (1, \sqrt{3})$=0
3. As below



4. By hint

$$\|\boldsymbol{x} - \boldsymbol{x}_\perp\|^2 + \|\boldsymbol{x}_\perp - \lambda \boldsymbol{y}\|^2 = \|\boldsymbol{x} - \lambda \boldsymbol{y}\|^2$$

   the inequality holds natually and equals when $x_\perp = \lambda y$

3 02 Math 2

1. x>0
2. 6

4 02 Math 6

1. $E(x) = \beta^{-1}, Var(x) = \beta^{-1}$.
2. c(x)=$1 - e^{-\beta x}$
3.

$$\begin{aligned} P(X \geq a+b | X \geq a) =& \frac{P(X \geq a+b, X \geq a)}{X \geq a} \\ =& \frac{P(X \geq a+b)}{X \geq a} \\ =& \frac{1 - c(a+b)}{1 - c(a)} \\ =& e^{-\beta b} \\ =& P(X \geq b) \end{aligned}$$

4. 1000, 1000.

5 02 Math 10

1. f is smooth and f"(x)=a^2e^{ax}>0, thus convex.
2. g is smooth and g"(x)=$-\frac{1}{x^2}$<0, thus concave.
3. h is convex on (0,+∞). When x_1=0,

   $$\frac{x_1+x_2}{2}\log(\frac{x_1+x_2}{2}) = \frac{x_2}{2}\log(\frac{x_2}{2}) > \frac{x_2}{2}\log x_2 = \frac{x_1\log x_1+x_2\log x_2}{2}$$

   Thus the inequality hold for $x_1,x_2 \in [0,+\infty)$
4. Since p_is are probabilities,

   $$L(p) = H - \lambda(\sum p_i - 1)$$

   $$\frac{\partial L}{\partial p_i} = -log_2 p_i - \lambda - \frac{1}{\log 2}, i = 1,\cdots,n$$

   Thus if partial derivatives all equal 0, p_is are equal, which is $p_i = \frac{1}{n}$.