# Notes

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### Notation

- N number of time series
- T length of time series
- $y_t$  N×1 vector of observation at  $\mathbf{t}$
- $f_t \neq 1$  factor at  $\mathbf{t}$
- B $N\times q$  loading matrix

我们考虑加权主成分方法(Weighted PCA)。对 p 维观察数据  $\mathbf{y}_t = (y_{1t}, \dots, y_{pt})' \in \mathbb{R}^p$ ,有

$$\mathbf{y}_t = \mathbf{B}\mathbf{f}_t + \mathbf{u}_t, \quad t = 1, \dots, T$$

其中  $\mathbf{f}_t \in \mathbb{R}^K$  是 K 维公共因子,B 为因子载荷矩阵。根据 Bai and Liao 2013,

$$\widehat{\mathbf{F}} = \left(\widehat{\mathbf{f}}_1, \dots, \widehat{\mathbf{f}}_T\right)'$$

$$\widehat{\mathbf{B}} = T^{-1}\mathbf{Y}\widehat{\mathbf{F}}$$

即为F与B的相合估计。

### Method

PCA:

$$\operatorname{argmin}_{\mathbf{f}_{\mathbf{f}} \in \mathbb{R}^K} \sum_{t=1}^{T} (\mathbf{y}_t - \mathbf{B}\mathbf{f}_t)' (\mathbf{y}_t - \mathbf{B}\mathbf{f}_t)$$
 (1)

WPCA:

$$\operatorname{argmin}_{\mathbf{f}_{t} \in \mathbb{R}^{K}} \sum_{t=1}^{T} (\mathbf{y}_{t} - \mathbf{B}\mathbf{f}_{t})' \Sigma_{u}^{-1} (\mathbf{y}_{t} - \mathbf{B}\mathbf{f}_{t})$$
(2)

Karlman Filter

## Experiment

Cholesky decomposition is not positive definite.

# Error in chol.default(denom) : the leading minor of order 71 is not positive definite

### Reference

- Quefeng Li, Guang Cheng, Jianqing Fan & Yuyan Wang (2018) Embracing the Blessing of Dimensionality in Factor Models, Journal of the American Statistical Association, 113:521, 380-389, DOI: 10.1080/01621459.2016.1256815
- Bai, J., and Liao, Y. (2013), "Statistical Inferences Using Large Estimated Covariances for Panel Data and Factor Models," arXiv:1307.2662. [380,381,382,383]