(1D001) Sjecište dvije ravnine u implicitnom obliku

$R_1 = [-5 \quad -6 \quad 6 \quad -10]^T$

$R_2 = [-10 \quad -5 \quad -10 \quad -10]^T$

Skalarni produkt normala je pravac koji leži u dvije ravnine

$\vec{n_1} = [-5 \quad -6 \quad 6]^T$

$n_2 = [-10 \quad -5 \quad -10]^T$

$$\Rightarrow p = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -6 & 6 \\ -10 & -5 & -10 \end{vmatrix} = \begin{bmatrix} 90 \\ -110 \\ -35 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$-5x + (-6)y + 6z - 10 = 0$

$-10x - 5y - 10z - 10 = 0$

$\underline{z = 0}$

$-5x - 6y - 10 = 0$

$-10x - 5y - 10 = 0 \Rightarrow 5y = -10x - 10 \;/ \cdot 5$

$y = -2x - 2$

$-5x - 6(-2x - 2) - 10 = 0$

$-5x + 12x + 12 - 10 = 0$

$+7x + 2 = 0$

$x = -\frac{2}{7}$ $\quad y = \frac{4}{7} - 2 \Rightarrow y = -\frac{10}{7}$

$X = -0.2857142857$ $\quad Y = -1.428571429$ $\quad Z = 0.0$

(1D002) Određivanje površine trokuta - 2D slučaj

$T_1(11,2)$

$T_2(17,0)$ $\Rightarrow$ Proširimo u 3D

$T_3(3,7)$

$T_1(11,2,0)$

$T_2(17,0,0)$

$T_3(3,7,0)$

$$\vec{v}_{12} = \begin{bmatrix} 6 \\ -2 \\ 0 \end{bmatrix} \qquad \vec{v}_{13} = \begin{bmatrix} -8 \\ 5 \\ 0 \end{bmatrix}$$

$$\vec{v}_{12} \times \vec{v}_{13} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -2 & 0 \\ -8 & 5 & 0 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ 14 \end{bmatrix}$$

$$\left\| \begin{bmatrix} 0 \\ 0 \\ 14 \end{bmatrix} \right\| = \sqrt{14^2} = \underline{14}$$

$$P = \frac{14}{2} = 7$$

(1D002) Određivanje površine trokuta (3D slučaj)

$T_1 (17, 0, 12)$

$T_2 (17, 0, 16)$

$T_3 (2, 13, 17)$

$$\vec{v}_{12} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \qquad \vec{v}_{23} = \begin{bmatrix} -15 \\ 13 \\ 5 \end{bmatrix}$$

$$P = \frac{\| \vec{v}_{12} \times \vec{v}_{23} \|}{2}$$

$$\vec{v}_{12} \times \vec{v}_{23} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 4 \\ -15 & 13 & 5 \end{vmatrix} = \begin{bmatrix} -52 \\ -60 \\ 0 \end{bmatrix}$$

$$\| \vec{v}_{12} \times \vec{v}_{23} \| = \sqrt{(-52)^2 + (-60)^2} = 79.3977$$

$$P = \frac{79.3977}{2} = \underline{39.699}$$

(1D 003) (2D) Ispitivanje je li točka u trokutu

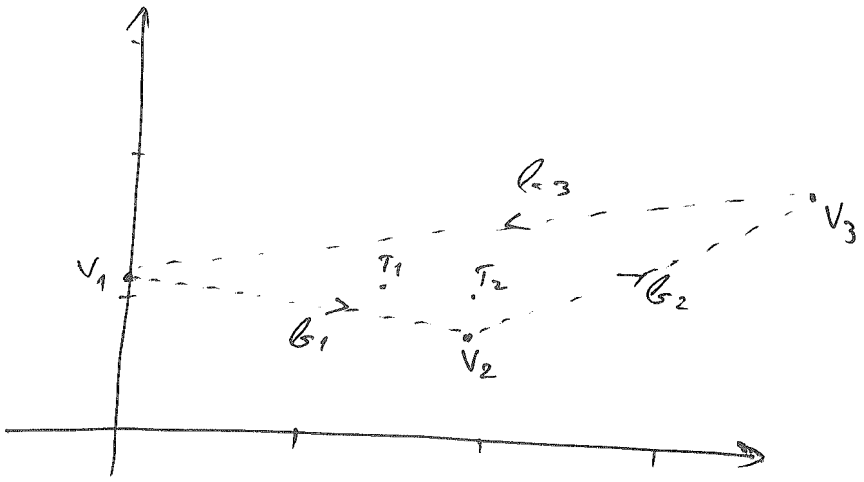
T_1 (7.31, 5.9)

T_2 (9.94 5.21)

V_1 (0, 6)

V_2 (9, 4)

V_3 (20, 9)



$$l_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 6 & 1 \\ 9 & 4 & 1 \end{vmatrix} = \begin{bmatrix} 2 \\ 9 \\ -54 \end{bmatrix} \qquad l_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9 & 4 & 1 \\ 20 & 9 & 1 \end{vmatrix} = \begin{bmatrix} -5 \\ 11 \\ 1 \end{bmatrix}$$

$$l_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 20 & 9 & 1 \\ 0 & 6 & 1 \end{vmatrix} = \begin{bmatrix} 3 \\ -20 \\ 120 \end{bmatrix}$$

Zbog toga što su vrhovi zadani CCW, točke moraju biti lijevo/~~izvan~~ svih bridova da li bile u trokutu

T_1

$$T_1 \cdot l_1 = \begin{bmatrix} 7.31 & 5.9 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \\ -54 \end{bmatrix} = 13.72 \qquad T_2 l_1 = \begin{bmatrix} 9.94 & 5.21 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \\ -54 \end{bmatrix} = 12.7$$

$$T_1 \cdot l_2 = \begin{bmatrix} 7.31 & 5.9 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 11 \\ 1 \end{bmatrix} = 29.35 \qquad T_2 l_2 = \begin{bmatrix} 9.94 & 5.21 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 11 \\ 1 \end{bmatrix} = 8.6$$

$$T_1 \cdot l_3 = \begin{bmatrix} 7.31 & 5.9 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -20 \\ 120 \end{bmatrix} = 23.93 \qquad T_2 l_3 = \begin{bmatrix} 9.94 & 5.21 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -20 \\ 120 \end{bmatrix} = 45.6$$

T_2

Sve vrijednosti su veće od nula $\Rightarrow$ T_1 i T_2 su u trokutu

(ID 004) (3D) Sjecište dva pravca u parametarskom obliku

$$G_1 = [t \;\; 1]\begin{bmatrix} -1 & 1 & 1 & 0 \\ -2 & 1 & 1 & -1 \end{bmatrix} \qquad G_2 = [t \;\; 1]\begin{bmatrix} 1 & 2 & 2 & 0 \\ -1 & -2 & -2 & -2 \end{bmatrix}$$

"Ulijemo" homogenu koordinatu $\Rightarrow$ 2. red podjelino s njom

$$G_1 = [t \;\; 1]\begin{bmatrix} -1 & 1 & 1 & 0 \\ 2 & -1 & -1 & 1 \end{bmatrix} \qquad G_2 = [t \;\; 1]\begin{bmatrix} 1 & 2 & 2 & 0 \\ 0.5 & 1 & 1 & 1 \end{bmatrix}$$

"Izračunam točku" $\Rightarrow$ za t iod G_1 stavimo λ, a iod G_2 μ

$$\begin{matrix} & x & y & z & h \end{matrix}$$
$$T_1 = [-\lambda+2 \quad \lambda-1 \quad \lambda-1 \quad 1] \qquad T_2 = [\mu+0.5 \quad 2\mu+1 \quad 2\mu+1 \quad 1]$$

Izjednačimo vrijednosti točaka

$$-\lambda+2 = \mu+0.5 \Rightarrow \mu = -\lambda+1.5$$

$$\lambda-1 = 2\mu+1 \Rightarrow \lambda-1 = -2\lambda+3+1 \Rightarrow 3\lambda = 5 \Rightarrow \lambda = \frac{5}{3}$$

λ uvrstimo u T_1

$$T_1 = \left[-\frac{5}{3}+2 \quad \frac{5}{3}-1 \quad \frac{5}{3}-1 \quad 1\right]$$

$$T_1 = [0.333 \quad 0.666 \quad 0.666 \quad 1.0]$$

(ID005) Sjecište dva pravca implicitni i matrični oblik

$G_1 \dots \overset{a}{3}x + \overset{b}{2}y + \overset{c}{3} = 0$

$G_2 \dots \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

$$T = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 3 \\ 0 & 0 & 3 \end{vmatrix} = \begin{bmatrix} 6 \\ -9 \\ 0 \end{bmatrix}^T \Rightarrow$$

Homogena je $\neq 0$

Točka je u beskonačnosti.

$G_1 \dots 3x - y + 1 = 0$

$G_2 \dots \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

$$T = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 1 \\ 2 & 2 & 2 \end{vmatrix} = \begin{bmatrix} -4 \\ -4 \\ 8 \end{bmatrix}^T$$

(ID 005) Sjecište dva pravca - parametarski oblik

$$G_1 = [t \quad 1] \begin{bmatrix} 1 & 4 & 0 \\ 76 & 40 & 96 \end{bmatrix} \qquad G_2 = [t \quad 1] \begin{bmatrix} 0 & 1 & 0 \\ 4 & 30 & 34 \end{bmatrix}$$

T_S - "početna" točka

v_p - vektor pravca

za $t = 1$ dobivamo T_E - "završna" točka

$$T_{S1} = [76 \quad 40 \quad 96]$$

$$T_{E1} = [77 \quad 44 \quad 96]$$

$$T_{S2} = [4 \quad 30 \quad 34]$$

$$T_{E2} = [4 \quad 31 \quad 34]$$

$$G_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 76 & 40 & 96 \\ 77 & 44 & 96 \end{vmatrix} = \begin{bmatrix} -384 \\ 96 \\ 264 \end{bmatrix}$$

$$G_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 30 & 34 \\ 4 & 31 & 34 \end{vmatrix} = \begin{bmatrix} -34 \\ 0 \\ 4 \end{bmatrix}$$

$$X = G_1 \times G_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -384 & 96 & 264 \\ -34 & 0 & 4 \end{vmatrix} = \begin{bmatrix} 384 \\ -7440 \\ 3264 \end{bmatrix}^T$$

$$X = \begin{bmatrix} 0.117647 \\ -2.279412 \\ 1.0 \end{bmatrix}^T$$

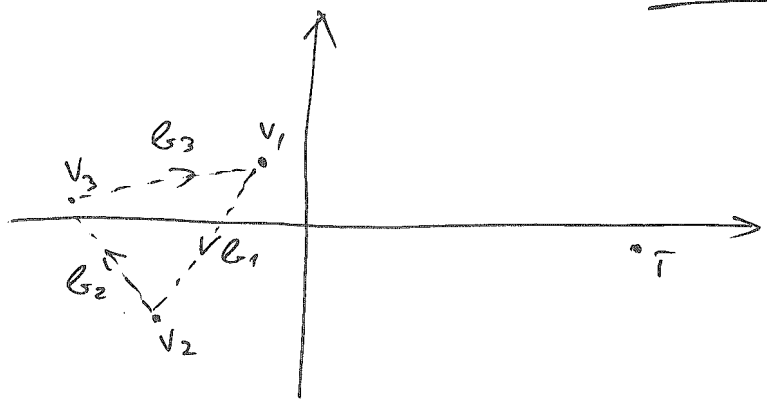
(ID006) Određivanje odnosa točke i poligona – trokut –

$V_1(-1, 3)$ $\quad T(10, -1)$

$V_2(-6, -8)$

$V_3(-7, 1)$

$L(V_1, V_2, V_3) \Rightarrow$ CW



$$p_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 3 & 1 \\ -6 & -8 & 1 \end{vmatrix} = \begin{bmatrix} 11 \\ -5 \\ 26 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$$p_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -6 & -8 & 1 \\ -7 & 1 & 1 \end{vmatrix} = \begin{bmatrix} -9 \\ -1 \\ -62 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$$p_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -7 & 1 & 1 \\ -1 & 3 & 1 \end{vmatrix} = \begin{bmatrix} -2 \\ 6 \\ -20 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$$T_1 \cdot p_1 = \begin{bmatrix} 10 & -1 & 1 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ 26 \end{bmatrix} = 141 \Rightarrow \text{iznad}$$

$$T_1 \cdot p_2 = \begin{bmatrix} 10 & -1 & 1 \end{bmatrix} \begin{bmatrix} -9 \\ -1 \\ -62 \end{bmatrix} = -151 \Rightarrow \text{ispod}$$

$$T_1 \cdot p_3 = \begin{bmatrix} 10 & -1 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 6 \\ -20 \end{bmatrix} = -46 \Rightarrow \text{ispod}$$

(1D006) Određivanje odnosa točke i poligona – konveksan četverokut –

$V_1(6,-2)$

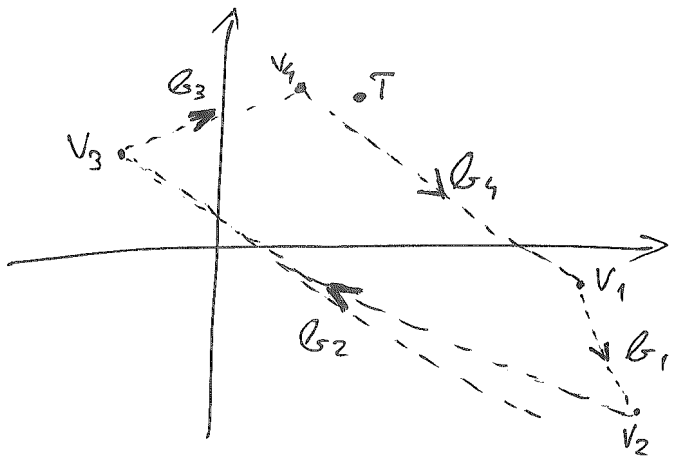
$V_2(7,-8)$

$V_3(-2,4)$

$V_4(1,7)$

$T(2,6)$

$L(V_1, V_2, V_3, V_4) \Rightarrow$ CW



$$p_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -2 & 1 \\ 7 & -8 & 1 \end{vmatrix} = \begin{bmatrix} 6 \\ 1 \\ -34 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$$T_1 p_1 = [2\ 6\ 1] \begin{bmatrix} 6 \\ 1 \\ -34 \end{bmatrix} = -16 \Rightarrow \text{ISPOD}$$

$$p_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & -8 & 1 \\ -2 & 4 & 1 \end{vmatrix} = \begin{bmatrix} -12 \\ -9 \\ 12 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$$T_1 p_2 = [2\ 6\ 1] \begin{bmatrix} -12 \\ -9 \\ 12 \end{bmatrix} = -66 \Rightarrow \text{ISPOD}$$

$$p_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 4 & 1 \\ 1 & 7 & 1 \end{vmatrix} = \begin{bmatrix} -3 \\ 3 \\ -18 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$$T_1 p_3 = [2\ 6\ 1] \begin{bmatrix} -3 \\ 3 \\ -18 \end{bmatrix} = -6 \Rightarrow \text{ISPOD}$$

$$p_4 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 7 & 1 \\ 6 & -2 & 1 \end{vmatrix} = \begin{bmatrix} 9 \\ 5 \\ -44 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$$T_1 p_4 = [2\ 6\ 1] \begin{bmatrix} 9 \\ 5 \\ -44 \end{bmatrix} = 4 \Rightarrow \text{IZNAD}$$

(1D006) Odredivanje odnosa točke i poligona

- Konkavan četverokut -

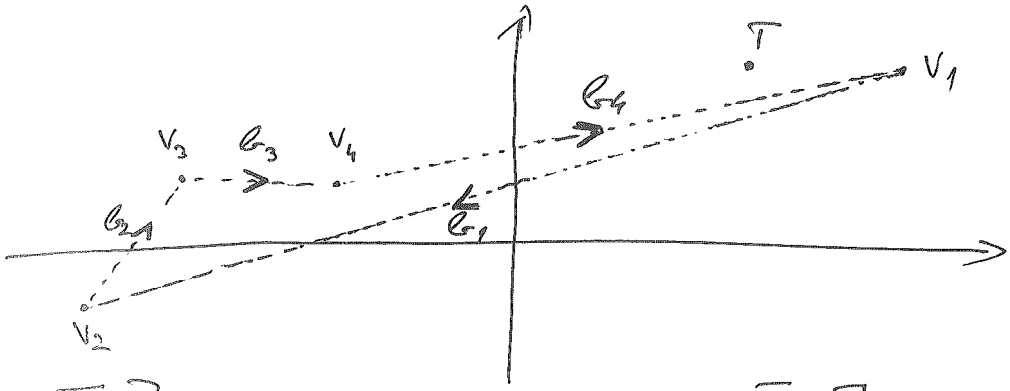
$V_1(6,7)$ $L(V_1, V_2, V_3, V_4)$

$V_2(-5,-1)$

$V_3(-4,2)$

$V_4(-2,2)$

$T(4,7)$



$$p_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 7 & 1 \\ -5 & -1 & 1 \end{vmatrix} = \begin{bmatrix} 8 \\ -11 \\ 29 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$$T_1 p_1 = \begin{bmatrix} 4 & 7 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ -11 \\ 29 \end{bmatrix} = -16$$

ISPOD

$$p_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -1 & 1 \\ -4 & 2 & 1 \end{vmatrix} = \begin{bmatrix} -3 \\ 1 \\ -14 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$$T_1 p_2 = \begin{bmatrix} 4 & 7 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ -14 \end{bmatrix} = -19$$

ISPOD

$$p_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 2 & 1 \\ -2 & 2 & 1 \end{vmatrix} = \begin{bmatrix} 0 \\ 2 \\ -4 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$$T_1 p_3 = \begin{bmatrix} 4 & 7 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} = 18$$

IZNAD

$$p_4 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 2 & 1 \\ 6 & 7 & 1 \end{vmatrix} = \begin{bmatrix} -5 \\ 8 \\ -26 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

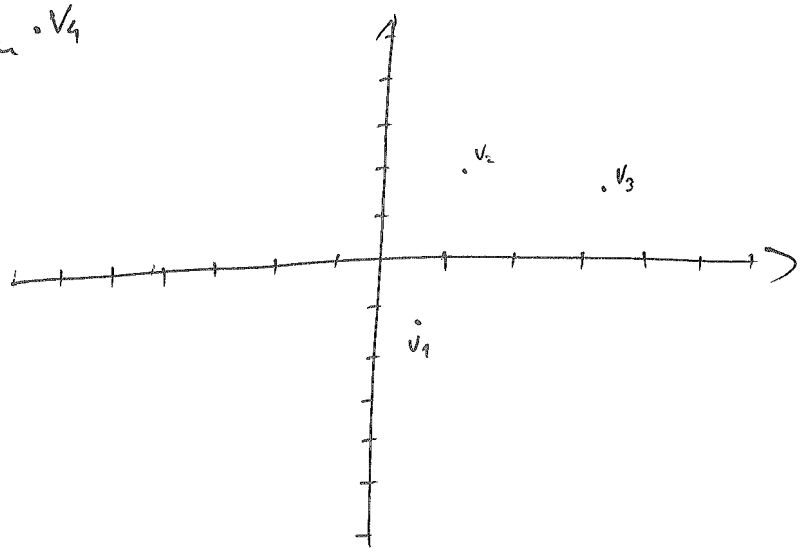
$$T_1 p_4 = \begin{bmatrix} 4 & 7 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 8 \\ -26 \end{bmatrix} = 10$$

IZNAD

(ID008) Afina transformacija koja preslikava jednu dužinu u drugu.

P_1 { V_1 (3.61, −6.71)
V_2 (6.06, 9.08)

P_2 { V_3 (16.97, 8.1)
V_4 (−36.82, 25.57)



1. Pomak V_1 u ishodište

$$T_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -3.61 & +6.71 & 1 \end{bmatrix}$$

$$V_1' = [0 \quad 0 \quad 1]$$

$$V_2' = [2.45 \quad 15.79 \quad 1]$$

2. Skaliramo da V_2' dođe na (1, 1)

$$T_2 = \begin{bmatrix} \frac{1}{2.45} & 0 & 0 \\ 0 & \frac{1}{15.79} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$V_1'' = [0 \quad 0 \quad 1]$$

$$V_2'' = [1 \quad 1 \quad 1]$$

3. Skaliramo na razliku V_3 i V_4

$$\vec{v}_{34} = [-36.82 - 16.97 \quad 25.57 - 8.1] = [-53.79 \quad 17.47]$$

$$T_3 = \begin{bmatrix} -53.79 & 0 & 0 \\ 0 & 17.47 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$V_1^{III} = [0 \quad 0 \quad 1]$$

$$V_2^{III} = [-53.79 \quad 17.47 \quad 1]$$

4. Pomaknemo iz ishodišta u V_3

$$T_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 16.97 & 8.1 & 1 \end{bmatrix}$$

$$V_1^{IV} = [16.97 \quad 8.1 \quad 1]$$

$$V_2^{IV} = [-36.82 \quad 25.57 \quad 1]$$

$$T = T_1 \cdot T_2 \cdot T_3 \cdot T_4 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -3.61 & 6.71 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2.45} & 0 & 0 \\ 0 & \frac{1}{15.79} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -53.79 & 0 & 0 \\ 0 & 17.47 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 16.97 & 8.1 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 21.95 & 0 & 0 \\ 0 & 1.1063 & 0 \\ 211.15 & 125.32 & 1 \end{bmatrix} \begin{bmatrix} -21.9551 & 0 & 0 \\ 0 & 1.1063965 & 0 \\ 96.2279 & 15.5239 & 1 \end{bmatrix}$$

(1D009) Sjecište pravca i ravnine

Dvanima zadana implicitnim oblikom

$$G_1 = [t \quad 1] \begin{bmatrix} 2 & 2 & -1 & 0 \\ -1 & 1 & 1 & -2 \end{bmatrix}$$

$$\vec{t}(\lambda) = T_S + \lambda \vec{d}$$

NORMIRAMO

$$R = [-1 \quad -1 \quad 1 \quad -1]^T$$

$$\vec{n} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$d = \frac{\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}}{\left\| \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \right\|} = \frac{\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}}{3} = \begin{bmatrix} 2/3 \\ 2/3 \\ -1/3 \end{bmatrix}$$

$$T_S = [-1 \quad 1 \quad 1 \quad -2]$$

$$T_S = [0.5 \quad -0.5 \quad -0.5 \quad 1]$$

$$\vec{n} \cdot \vec{t} + D = 0$$

$$\vec{n} \cdot (T_S + \lambda \vec{d}) + D = 0$$

$$\vec{n} \cdot T_S + \lambda \vec{d} \cdot \vec{n} + D = 0$$

$$\lambda \vec{d} \vec{n} = -(\vec{n} T_S + D)$$

$$\lambda = -\frac{\vec{n} T_S + D}{\vec{d} \cdot \vec{n}} = \frac{\cancel{(-1)\cdot(-1) + (1)(-1)}}{} = -\frac{0.5(-1) + (-0.5)(-1) + 1(-0.5) + (-}{-\frac{2}{3} - \frac{2}{3} - \frac{1}{3}}$$

$$\lambda = +\frac{\cancel{-0.5 + 0.5 - 0.5} + 1}{+\frac{5}{3}} = \frac{-\frac{3}{2}}{\frac{5}{3}} = -\frac{9}{10}$$

~~$I = [$~~ $T = T_S + \left(-\frac{9}{10}\right)\vec{d}$

$$T = [0.5 \quad -0.5 \quad -0.5 \quad 1] + \left(-\frac{9}{10}\right) \begin{bmatrix} 2/3 \\ 2/3 \\ -1/3 \end{bmatrix} =$$

$$T = [-0.1 \quad -1.1 \quad -0.2 \quad 1]$$

(1D009) Specify prava i ravnina

Ravnina zadana parametarskim oblikom

$$G = [t \quad 1] \begin{bmatrix} 1 & -1 & 2 & 0 \\ -2 & -2 & -2 & -1 \end{bmatrix} \Rightarrow d = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\Rightarrow T_S = [-2 \quad -2 \quad -2 \quad -1]$$

$$\Downarrow$$

$$T_S = [2 \quad 2 \quad 2 \quad 1]$$

$$R = [u \quad v \quad 1] \begin{bmatrix} 1 & -2 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$t(n) = T_S + n\vec{d}$$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 1 \\ -1 & 1 & -1 \end{vmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$Ax + By + Cz + D = 0$$

$$(-1)(1) + (-1)(0) + 1(-1) + D = 0$$

$$-1 - 1 + D = 0$$

$$\boxed{D = 2}$$

$$\vec{n} \cdot \vec{t} + D = 0$$

$$\vec{n}(T_S + n\vec{d}) + D = 0$$

$$\vec{n} T_S + n\vec{n}\vec{d} + D = 0$$

$$n = -\frac{\vec{n} T_S + D}{\vec{n}\vec{d}} = -\frac{(1)(2) + (0)(2) + (-1)(2) + 2}{(1)(1) + (-1)(0) + (-1)(2)} = \frac{-2}{-1} = 2$$

$$T = [2 \quad 2 \quad 2 \quad 1] + 2\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = [4 \quad 0 \quad 6 \quad 1]$$

(1D019) Udaljenost točke do pravca - 2D slučaj

$p: 9x - 8y + 4 = 0 \Rightarrow 8y = 9x + 4$

$T(-14, 12)$

$y = \frac{9}{8}x + \frac{4}{8}$

$y = \frac{9}{8}x + \frac{1}{2}$

$\downarrow$

$k_1 = \frac{9}{8}$

$y - y_1 = l(x - x_1)$

$l_2 = -\frac{1}{l_1} = -\frac{8}{9}$

$y - 12 = -\frac{8}{9}(x + 14)$

$y = -\frac{8}{9}x - \frac{112}{9} + 12$

$y = -\frac{8}{9}x - \frac{4}{9}$

$\frac{9}{8}x + \frac{4}{8} = -\frac{8}{9}x - \frac{4}{9} \quad / \cdot 72$

$9^2x + 4 \cdot 9 = -8^2x - 4 \cdot 8$

$81x + 36 = -64x - 32$

$145x = -68$

$x = -0.4689655$

$y = -0.027586$

$\Delta = \sqrt{(-14 + 0.4689655)^2 + (12 + 0.027586)^2}$

$\Delta = 18.1039$

(1D019) Udaljenost točke do pravca - 38) sljedeći

$$G_1 = [t \quad 1] \begin{bmatrix} -10 & 8 & 11 & 0 \\ -10 & 8 & 3 & 1 \end{bmatrix} \Rightarrow T_1 = [-10t-10 \quad 8t+8 \quad 11t+3 \quad 1]$$

$$T = (-8, -3, -3)$$

Računamo razmak u kojoj leži točka T, a okomita je na pravac $p \Rightarrow \vec{n} = \vec{p}$

$$R = [-10 \quad 8 \quad 11 \quad D] \qquad \vec{n} = \begin{bmatrix} -10 \\ 8 \\ 11 \end{bmatrix}$$

Umnožak normale i točke T daje D

$$-8(-10) - 3 \cdot 8 - 3 \cdot 11 + D = 0$$

$$80 - 24 - 33 = -D$$

$$\boxed{D = -23}$$

$$R = [-10 \quad 8 \quad 11 \quad -23]$$

Ubacimo točku T_1 u R, da dobijemo sjecište pravca p i R

$$-10(-10t-10) + 8(8t+8) + 11(11t+3) - 23 = 0$$

$$100t + 100 + 64t + 64 + 121t + 33 - 23 = 0$$

$$285t = -174 \Rightarrow t = -0.6105 \Rightarrow T_1 = [-3.895 \quad 3.116 \quad -3.7155$$

$$\Delta = \sqrt{(-8+3.895)^2 + (-3-3.116)^2 + (-3+3.7153)^2}$$

$$\boxed{\Delta = 7.4}$$

(ID 020) Udaljenost dva pravca - 3D slučaj

$$G_1 = [t \;\; 1] \begin{bmatrix} -9 & 14 & -5 & 0 \\ -14 & -7 & -2 & 1 \end{bmatrix}$$

$$G_2 = [t \;\; 1] \begin{bmatrix} 13 & -1 & -2 & 0 \\ 10 & 7 & -14 & 1 \end{bmatrix}$$

v spaja 2 točke na dva pravca

$$v = \begin{bmatrix} 24 \\ 14 \\ -12 \end{bmatrix}$$

Najkraća udaljenost između 2 mimosmjerna pravca:

$$d(p_1, p_2) = \frac{|v \cdot (p_1 \times p_2)|}{|p_1 \times p_2|}$$

$$p_1 \times p_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -9 & 14 & -5 \\ 13 & -1 & -2 \end{vmatrix} = \begin{bmatrix} -33 \\ -83 \\ -173 \end{bmatrix}$$

$$|v \cdot (p_1 \times p_2)| = |24 \cdot (-33) + 14(-83) - 12(-173)| = \underline{122}$$

$$|p_1 \times p_2| = \sqrt{(-33)^2 + (-83)^2 + (-173)^2} = 194.697$$

$$d(p_1, p_2) = \frac{122}{194.697} = \boxed{0.6266}$$

(1D021) Bilinearna interpolacija - složeniji slučaj

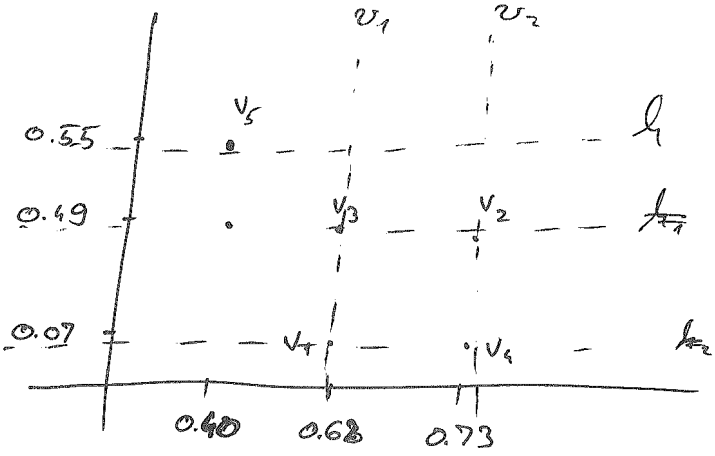
$V_1(0.62, 0.07) = 3.00$

$V_2(0.73, 0.49) = 13.00$

$V_3(0.62, 0.49) = 7.00$

$V_4(0.73, 0.07) = 14.00$

$V_5(0.40, 0.55) = ?$



$$v_1 = 0 + \lambda \frac{7-0}{0.49-0.07}$$

$$v_1 = \lambda * 16.666$$

$\lambda = y - y_0$

$$v_2 = 14 + \lambda \frac{13-14}{0.49-0.07}$$

$$v_2 = 14 - 2.38095\,\lambda$$

$v_1(0.55) = (0.55-0.07) \cdot 16.666$

$v_1(0.55) = 7.99968$

$v_2(0.55) = 14 - 2.38095(0.55-0.07)$

$v_2(0.55) = 12.857144$

$\mu = x - x_0$

$$h = 7.99968 + \mu \frac{12.857144 - 7.99968}{0.73 - 0.62}$$

$h = 7.99968 + \mu\ 44.15876364$

$h(0.40) = 7.99968 + (0.40-0.62)\ 44.15876364$

$$h(0.40) = -1.715248$$

(ID024) Interpolacija boje baricentričnim koordinatama

$A(2,2)$ $I_A(255, 29, 23)$ (I, G, B) $T_1(11,13)$

$B(14,2)$ $I_B(13, 253, 31)$ $T_2(10,6)$

$C(12,18)$ $I_C(35, 31, 233)$ $T_3(10,12)$

$I_{T_1} = (t_1 I_{A,R} + t_2 I_{B,R} + t_3 I_{B,R}\ ,\ t_1 I_{A,G} + t_2 I_{B,G} + t_3 I_{C,G}\ ,\ t_1 I_{A,B} + t_2 I_{B,B} + t_3 I_{C,B})$

T_1

$$t_1\begin{bmatrix}2\\2\\1\end{bmatrix} + t_2\begin{bmatrix}14\\2\\1\end{bmatrix} + t_3\begin{bmatrix}12\\18\\1\end{bmatrix} = \begin{bmatrix}11\\13\\1\end{bmatrix} \Rightarrow t_1 = \frac{13}{96},\ t_2 = \frac{17}{96},\ t_3 = \frac{11}{16}$$

$I_{T_1}(61, 70, 169)$

T_2

$$t_1\begin{bmatrix}2\\2\\1\end{bmatrix} + t_2\begin{bmatrix}14\\2\\1\end{bmatrix} + t_3\begin{bmatrix}12\\18\\1\end{bmatrix} = \begin{bmatrix}10\\6\\1\end{bmatrix} \Rightarrow t_1 = \frac{7}{24},\ t_2 = \frac{11}{24},\ t_3 = \frac{1}{4}$$

$I_{T_2}(89, 132, 79)$

T_3

$$t_1\begin{bmatrix}2\\2\\1\end{bmatrix} + t_2\begin{bmatrix}14\\2\\1\end{bmatrix} + t_3\begin{bmatrix}12\\18\\1\end{bmatrix} = \begin{bmatrix}10\\12\\1\end{bmatrix} \Rightarrow t_1 = \frac{11}{48},\ t_2 = \frac{7}{48},\ t_3 = \frac{3}{8}$$

$I_{T_3}(82, 63, 155)$

(ID 025) Probodište pravca i trokuta

A (4.93, 1.30, -4.77)
B (-6.24, -5.95, 6.03)
C (2.20, -3.42, -2.13)
} trokut

P (3.04, -2.64, 6.13)

K (-1.90, 2.49, -27.26)

1. Normala ravnine

$$\upsilon_{AB} = [-11.17 \quad -7.25 \quad +10.80]^T$$

$$\upsilon_{AC} = [-2.73 \quad -4.72 \quad 2.64]^T$$

$$n = \upsilon_{AB} \times \upsilon_{AC} = [31.836 \quad 0.005 \quad 32.93]^T$$

2. Implicitni oblik ravnine

$$Ax + By + Cz + D = 0 \Rightarrow D = -Ax - By - Cz = -\vec{n} \cdot \vec{t}$$

$$n = \begin{bmatrix} A \\ B \\ C \end{bmatrix} \qquad D = -\begin{bmatrix} 31.836 \\ 0.005 \\ 32.93 \end{bmatrix} \cdot [4.93 \quad 1.30 \quad -4.77] = \underline{0.11812}$$

3. Jednadžba pravca

$$\upsilon_p = [-4.94 \quad 5.13 \quad -33.39]^T$$

$$\vec{t}(\lambda) = \vec{T}_S + \lambda \vec{d}$$

$$\vec{d} = \frac{\vec{T}_E - \vec{T}_S}{\| \vec{T}_E - \vec{T}_S \|} \Rightarrow d = \frac{\upsilon_p}{\cancel{171.36}\ 34.11} = \begin{matrix} \cancel{-0.0290} \\ \cancel{0.02999} \\ \cancel{-0.194} \end{matrix} \begin{bmatrix} -0.145 \\ 0.150 \\ -0.978 \end{bmatrix}$$

4. Probodište

$$\vec{n} \cdot \vec{t} + D = 0 \Rightarrow \vec{n}(\vec{T}_S + \lambda \vec{d}) + D = 0 \rightarrow \vec{n}\vec{T}_S + \vec{n}\lambda\vec{d} + D = 0$$

$$\Rightarrow \lambda = -\frac{(\vec{n}\vec{T}_S + D)}{\vec{n}\vec{d}} = -\frac{298.63 + 0.118}{-36.82} = \underline{+8.114} \qquad \lambda \neq 0 \Rightarrow \text{ima presjeka}$$

$$\vec{r}(\lambda) = \vec{T_S} + \lambda \vec{d}$$

$$\vec{T_S} = P = (3.04, -2.64, 6.13)$$

$$\lambda = +8.114$$

$$\vec{d} = \begin{bmatrix} -0.145 \\ 0.150 \\ -0.978 \end{bmatrix}^T$$

$$\vec{r} = [3.04 \quad -2.64 \quad 6.13] + [-1.177 \quad +1.217 \quad -7.935]$$

~~$t = [4.217 \quad 3.857 \quad 14.065]$~~

$$t = [1.863 \quad -1.423 \quad -1.805]$$

5. Provjera je li točka u trokutu - baricentrične koordinate

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = t_1 \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} + t_2 \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} + t_3 \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix}$$

$$\begin{bmatrix} 1.863 \\ -1.423 \\ -1.805 \end{bmatrix} = t_1 \begin{bmatrix} 4.93 \\ 1.30 \\ -4.77 \end{bmatrix} + t_2 \begin{bmatrix} -6.24 \\ -5.95 \\ 6.03 \end{bmatrix} + t_3 \begin{bmatrix} 2.20 \\ -3.42 \\ -2.13 \end{bmatrix}$$

$t_1 = 0.687$
$t_2 = 0.295$
$t_3 = 0.155$

Sve baricentrične koordinate su < 1, stoga je točka unutar trokuta

(1D027) Ispitivanje točke Mandelbrotovog skupa

$(0.0, 0.0) - (800.0, 600.0)$ x, y $\quad \varepsilon = 234$

$(-2.0, -1.0) - (0.25, 1.0)$ u, v

$(x', y') = (304, 274)$ iteracija max 8

$z_0 = 0$

$$u = \frac{x' - x_{min}}{x_{max} - x_{min}} (u_{max} - u_{min}) + u_{min} = \frac{304}{800} (2.25) - 2.0 = -1.145$$

$$v = \frac{y' - y_{min}}{y_{max} - y_{min}} (v_{max} - v_{min}) + v_{min} = \frac{274}{600} (2.0) - 1.0 = -0.087$$

$z' = -1.145 - 0.087i$

$z_0 = 0$

$z_1 = -1.145 - 0.087i \qquad \delta_1^2 = 1.318594$

$z_2 = 0.158456 + 0.11223i$

$z_3 = -1.132487 - 0.05143297i$

$z_4 = 0.13488 + 0.02949i$

$z_5 = -1.12767 - 0.079i$

$z_6 = 0.1204 + 0.09127i$

$z_7 = -1.1388 - 0.065i$

$z_8 = 0.1497 + 0.06109i$

točka nije probila granicu => PRIPADA SKUPU

(1D040) Specifiche implicite i parametrele racumine

$$D_1 = [-5 \quad 1 \quad 5 \quad -10]^T$$

$$V_a = [10 \quad -10 \quad -6]$$

$$T_r = V_a \cdot t + V_b \cdot u + T_s$$

$$V_b = [-7 \quad -10 \quad 2]$$

$$T_s = [1 \quad 3 \quad 7]$$

$$\vec{m_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & -10 & -6 \\ -7 & -10 & 2 \end{vmatrix} = \begin{bmatrix} -80 \\ 22 \\ -170 \end{bmatrix}$$

Unde $\vec{m_2} \cdot T_s + D = 0 \Rightarrow D = -\vec{m_2} T_s$

$$D = -(-80 \cdot 1 + 22 \cdot 3 - 170 \cdot 7) = 1204$$

$$\vec{m_1} = [-5 \quad 1 \quad 5]^T$$

$$P = \vec{m_1} \times \vec{m_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & 1 & 5 \\ -80 & 22 & -170 \end{vmatrix} = \begin{bmatrix} -280 \\ -1250 \\ -30 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$z = 0$

$$-5x + y + 5z - 10 = 0 \Rightarrow y = 5x + 10$$

$$-80x + 22y - 170z + 1204 = 0$$

$$-80x + 22(5x + 10) + 1204 = 0$$

$$-80x + 110x + 220 + 1204 = 0$$

$$30x = -1424$$

$x = -47.4666$ $\quad y = -227.333$ $\quad z = 0$

(1D044) Interpolacija baricentrnih koordinata

- Vrijednosti na slari -

A $(-3, 9)$ $t_1 = 0.35$ $I_A = 239$

B $(-9, 6)$ $t_2 = 0.47$ $I_B = 119$

C $(9, -10)$ $t_3 = 0.18$ $I_C = 205$

$$I_T = t_1 \cdot I_A + t_2 \cdot I_B + t_3 \cdot I_C = \boxed{176.48}$$

$$0.35\begin{bmatrix} -3 \\ 9 \end{bmatrix} + 0.47\begin{bmatrix} -9 \\ 6 \end{bmatrix} + 0.18\begin{bmatrix} 9 \\ -10 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x = -3 \cdot 0.35 - 9 \cdot 0.47 + 9 \cdot 0.18 = \boxed{-3.66}$$

$$y = 9 \cdot 0.35 + 6 \cdot 0.47 + 10 \cdot 0.18 = \boxed{4.17}$$

(1D044) Interpolacija baricentričnih koordinata

$A(3,2)$ $\quad t_1 = 0.06$ $\quad V_A = [0.12 \quad -0.99]$

$B(0,-6)$ $\quad t_2 = 0.5$ $\quad V_B = [0.68 \quad -0.72]$

$C(5,-7)$ $\quad t_3 = 0.43$ $\quad V_C = [0.67 \quad -0.73]$

$$V_T = [0.06 \cdot 0.12 + 0.5 \cdot 0.68 + 0.43 \cdot 0.67 \quad 0.06(-0.99) + 0.5(-0.72) + 0.43(-0.7$$

$$\boxed{V_T = [0.6353 \quad -0.7333]}$$

$$0.06 \begin{bmatrix} 3 \\ 2 \end{bmatrix} + 0.5 \begin{bmatrix} 0 \\ -6 \end{bmatrix} + 0.43 \begin{bmatrix} 5 \\ -7 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x = 0.06 \cdot 3 + 0.5 \cdot 0 + 0.43 \cdot 5 = 2.33$$

$$y = 0.06 \cdot 2 + 0.5(-6) + 0.43(-7) = -5.89$$

$$\boxed{\begin{bmatrix} 2.33 \\ -5.89 \end{bmatrix}}$$

(1D054) Određivanje baricentričnih koordinata

$V_1 = (-4, -3, -5)$

$V_2 = (0, -3, 3)$

$V_3 = (0, 0, -3)$

$P = (3.35, -0.25, 4.2)$

$$t_1 \begin{bmatrix} -4 \\ -3 \\ -5 \end{bmatrix} + t_2 \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix} + t_3 \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 3.35 \\ -0.25 \\ 4.2 \end{bmatrix}$$

Sustav jednadžbi → kalkulator da riješi

$$t_1 = -\frac{67}{80} = -0.8375$$

$$t_2 = \frac{221}{240} = 0.9208$$

$$t_3 = \frac{11}{12} = 0.9167$$

(ID059) Transformacija iz globalnog u lokalni 2oo sustav

$O = (0, 0)$

$\vec{x} = [1, 0]$

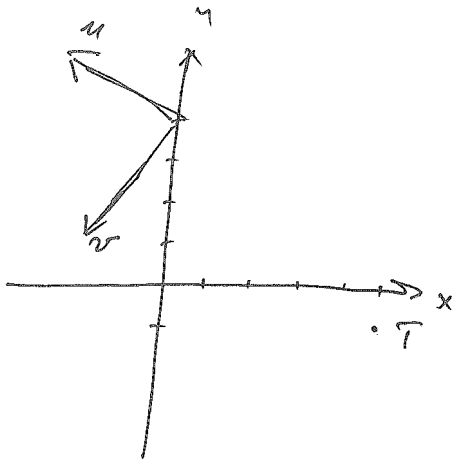
$\vec{y} = [0, 1]$

$T = (5, -1)$

$O_1 = (0, 4)$

$\varphi = \frac{5}{6}\pi$ rad

$\vec{n}_1 = 2\vec{n}_2$



1. Translacija

$\Delta x = 0$

$\Delta y = -4$

$T' = (5, -5)$

"Isti redoslijed" operacija, ali vrijednosti *(-1) od zadanih

2. Rotacija

$\varphi = -\frac{5}{6}\pi$

$T' = [5 \quad -5 \quad 1]$

$$R = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$T'' = [-6.83 \quad 1.83 \quad 1]$

3. Skaliranje zbog različitih normi

$$S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$T''' = [-13.66 \quad 3.66 \quad 1]$

(ID 059) Transformacija iz lokalnog u globalni koo

$O = (0,0)$

$\vec{x} = [1\ 0]$

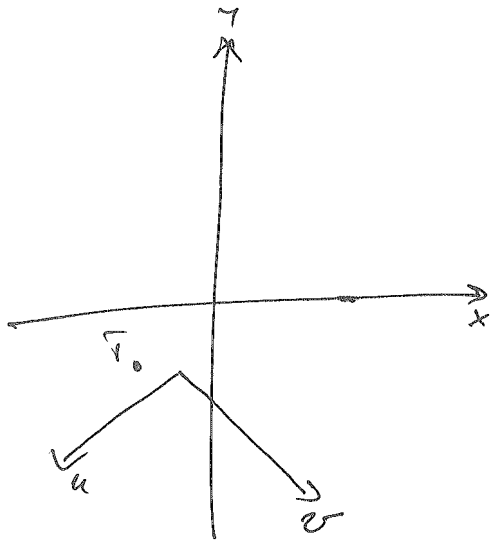
$\vec{y} = [0\ 1]$

$O_1 = (-2,-3)$

$\varphi = \frac{7}{6}\pi$ rad

$\vec{m_1} = 2\vec{m_2}$

$T = (1,-2)$



Obrnuti redoslijed transformacija od globalni → lokalni

Vrijednosti transformacija jednake zadanim podacima

1. Skaliranje

$$S = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$T' = [0.5 \quad -1 \quad 1]$

2. Rotacija

$\varphi = \frac{7}{6}\pi$

$$R = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$T'' = [-0.933 \quad 0.616 \quad 1]$

3. Translacija

$$T_r = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -3 & 1 \end{bmatrix}$$

$T''' = [-2.933 \quad -2.383 \quad 1]$

(ID 0608) Ljudsko oko i percepcija intenziteta

Broj razina → 128

$I_0 = 0.018$

$I_{127} = 1$

$I_{30} = ?$

$$I_j = (I_0)^{1-\frac{j}{127}} = 0.018^{1-\frac{30}{127}} = 0.0469857$$

BROJ RAZINA − 1

(ID 062) Projekcija vektora na vektor - 2D slučaj

$\vec{a}\ [-6 \quad -25]$

$\vec{b}\ [89 \quad 63]$

$a \Rightarrow b$

$$\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\cdot\|\vec{b}\|}$$

$$\vec{v} = \frac{\vec{b}}{\|\vec{b}\|}\cdot\|\vec{a}\|\cdot\cos\alpha = \frac{\vec{b}}{\|\vec{b}\|}\cdot\|\vec{a}\|\cdot\frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\|\vec{b}\|}$$

$$\vec{v} = \vec{b}\cdot\frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|^2} = \vec{b}\cdot(-0.177375946 2)$$

$$\vec{v} = [-15.78645921 \quad -11.17468461]$$

(1D062) Projekcija vektora na vektor - 3D slučaj

$a = [41 \quad 87 \quad 32]$

$b = [55 \quad 82 \quad -47]$

$a \Rightarrow b$

$$\cos\alpha = \frac{\vec{a}\vec{b}}{\|\vec{a}\|\|\vec{b}\|}$$

$$\vec{v} = \vec{b} \cdot \frac{\|\vec{a}\|}{\|\vec{b}\|} \cdot \cos\alpha = \vec{b} \frac{\|\vec{a}\|}{\|\vec{b}\|} \cdot \frac{\vec{a}\vec{b}}{\|\vec{a}\|\|\vec{b}\|} = \vec{b} \frac{\vec{a}\vec{b}}{\|\vec{b}\|^2}$$

$$\vec{v} = \vec{b} \cdot 0.659391202 5$$

$$\vec{v} \quad [36.26651614 \quad 54.07007861 \quad -30.99138652]$$