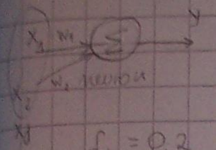
Neuronske mreže

## 1. LMS ALGORITAM

uvježbavanje jednog neurona

$$W^{(k+1)} = W^{(k)} - \alpha_k \cdot x_{d,k} \cdot e_k$$




$f(x) = 2x_1 - x_2$

| | $x_1$ | $x_2$ | $x_d$ | |
|---|---|---|---|---|
| [0,5 1,5] | 0,5 | 1,5 | -0,5 | 0 |
| | 1 | 0,5 | 1,5 | 1 |
| | 1,5 | 1 | 2 | 2 |
| | 2 | 2 | 2 | 3 |

$w_1^0 = 0$

$w_2^0 = 0$

$W = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\alpha_k = 0{,}2$

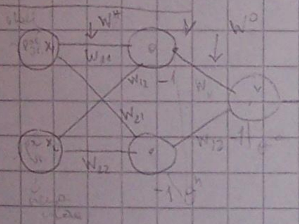
$e_k = x_{d,k}^T \cdot W^{(k)} - y_{d,k}$

$$W^{(1)} = W^0 - \alpha_0 \cdot x_{d,0} \cdot (x_{d,0}^T \cdot W^0 - y_{d,0}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - 0{,}2 \cdot \begin{bmatrix} 0{,}5 \\ 1{,}5 \end{bmatrix} \cdot \left( \begin{bmatrix} 0{,}5 & 1{,}5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} - (-0{,}5) \right) = W^{(1)} = \begin{bmatrix} -0{,}05 \\ -0{,}15 \end{bmatrix}$$

$$W^{(2)} = \begin{bmatrix} -0.05 \\ -0.15 \end{bmatrix} - 0.2 \cdot \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \cdot (0.125 - 1.5) = \begin{bmatrix} -0{,}05 \\ -0{,}15 \end{bmatrix} + 1{,}375 \begin{bmatrix} 0{,}2 \\ 0{,}1 \end{bmatrix}$$

$W^5 \to$ uzimamo $x_{d,0}$

## uvježbavanje mreže




$f(x) = \frac{1}{1+e^{-x}}$

$f'(x) = f(x)(1 - f(x))$

$$W^h = \begin{bmatrix} w_{11}^h & w_{12}^h \\ w_{21}^h & w_{22}^h \end{bmatrix}$$

$W^0 = \begin{bmatrix} w_{1} & w_{2} \end{bmatrix}$

1. unaprijed izračunavanje

2. back propagation

$V = W^H \cdot x - \theta^h$ $\quad$ $z = \text{sigmoid}(V)$

$+(-1) \cdot \theta^h$

$u = W^0 \cdot z + (-1) \cdot \theta^0$ $\quad$ $y = \text{sigmoid}(u)$

$$\boxed{\delta^0 = (y - y_d) \cdot y \cdot (1 - y)}$$

$\delta^h = ((W^0)^T \cdot \delta^0) .* z .* (1 - z)$

$W^0 = W^0 - \eta \cdot \delta^0 \cdot z^T$

$\theta^0 = \theta^0 - \eta \cdot \delta^0$

$W^h = W^h - \eta \cdot \delta^h \cdot x^T$

$\theta^h = \theta^h - \eta \cdot \delta^h$

| $x_1$ | $x_2$ | $y_d$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$\eta = 0{,}1$

$W^h = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$W^0 = \begin{bmatrix} 0 & 0 \end{bmatrix}$

$\theta^h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\theta^0 = [0]$

$$v = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v^h \cdot x$

$$z = sig(v) = \frac{1}{1-e^{v}} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$u = W^o z + (-1) \cdot \theta = \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} - 0 = 0 \qquad y = sig(u) = \frac{1}{2}$$

<u>unatrag (back propagation)</u>

$$\delta^o = (y - y_d) .* y .* (1-y) = \left(\frac{1}{2} - 0\right) \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

$$\delta^h = ((W^o)^T \cdot \delta^o) .* z .* (1-z) = \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot \frac{1}{8}\right) \cdot \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$W_o^{(2)} = W_o - \eta \cdot \delta^o \cdot z^T = \begin{bmatrix} 0 & 0 \end{bmatrix} - 0{,}1 \cdot \frac{1}{8} \cdot \begin{bmatrix} 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} \frac{-1}{160} & \frac{-1}{160} \end{bmatrix}$$

$$\theta^{o(2)} = \theta^o - \eta \delta^o \cdot (-1) = [0] + 0{,}1 \cdot \frac{1}{8} = \frac{1}{80}$$

$$W^{h(2)} = W^h - \eta \cdot \delta^h \cdot x_d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\theta^{h(2)} = \theta^h - \eta \cdot \delta^h \cdot (-1) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - 1 \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

<u>korak 2</u>

| $x_1$ | $x_2$ | $y$ |
|---|---|---|
| 0 | 0 | |
| 0 | 1 | 1 |

unaprijed

$$v = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{matrix} 0 \\ 0 \end{matrix} \qquad z = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$u = \begin{bmatrix} \frac{-1}{160} & \frac{-1}{160} \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} - \begin{bmatrix} \frac{1}{80} \end{bmatrix} = A$$

$$y = sig(A) = \frac{1}{1+e^{-A}}$$

<u>back propagation</u>

$$\delta^o (y - y_d) .* y .* (1-y) = (sig(A) - 1) \cdot sig^A \cdot (1 - sig A) = B$$

$$\delta^h = \begin{bmatrix} \frac{-1}{160} \\ \frac{-1}{160} \end{bmatrix} \cdot \delta^o \cdot \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} D \\ E \end{bmatrix}$$

$$W^{o(3)} = W^{o(2)} - \eta \cdot \delta^o \cdot z^T = \frac{-1}{160} \cdot \frac{-1}{160} - \delta^o \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$\theta^{o(3)}$

$W^{h(3)}$

$\theta^{h(3)}$

| $x_1$ | $x_2$ | $y$ |
|---|---|---|
| 1 | 0 | 1 |

# DELTA PRAVILO

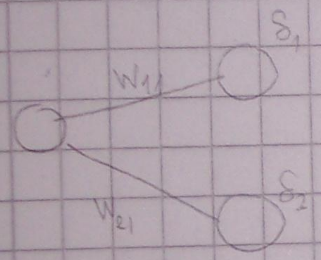
$$(W_{ji}) = (W_{ji})_{\text{TRENUTNA}} - \eta \cdot \delta_j \cdot z_i$$

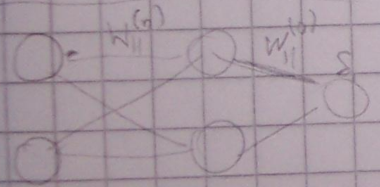
$z_i \rightarrow$ izlaz i-tog prethodnika

$\delta_j \xrightarrow{f(x)}$

$$\delta_j = y(1-y) \cdot (y - y_d) \quad \text{za izlazni sloj}$$

$$\delta_j = z \cdot (1 - z) \cdot \sum_k \delta_k \cdot W_{kj} \quad \text{za skriveni sloj}$$




$$W_{11}^{*} = W_{11}^{*} - \eta \cdot \delta_1 \cdot x_1 = W_{11} - 0.1 \cdot x_1 \cdot \delta_1$$

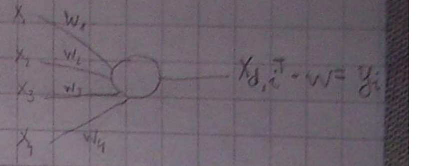
$$\delta_1 = z \cdot (1 - z) \cdot \sum_k \delta_k \cdot w_{k1} = \delta_1 \cdot w_{11}^{o}$$

→ Linearni neuron s 4 ulaza treba uvježbati sa sljedećim nizom podataka

| ulaz 1 | 2 | 3 | 4 | izlaz |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 |

← $y_d$

Je li rješenje jedinstveno ili ne?




$$X_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

3 para

$$X_d^T \cdot w = y$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$w_1 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + w_2 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + w_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + w_4 \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$w_1 = 1, \quad w_2 = \times, \quad w_3 = 0, \quad w_4 = 1 \quad \longrightarrow \quad w = \begin{bmatrix} 1 \\ \times \\ 0 \\ 1 \end{bmatrix}$$

Ima manje parova od parametara, rješenje nije jedinstveno. Rješenja ima beskonačno.

Biramo rješenje najmanje norme. !!

$$A^T (AA^T)^{-1}$$

$$A = X_d^T : \; A \cdot A^T = I_3 \implies (AA^T)^{-1} = I_3$$

optimalno rješenje $w = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$$w = A^T (AA^T)^{-1} \cdot \underset{y_d}{b} = A^T \cdot I_3 y_d = A^T \cdot y_d = X_d \cdot y_d = \dots = \text{rješenje}$$

← optimalni parametri

2. način preko Kaczmarz algoritma (minimalne norme)

$$w_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mu = 1 \qquad w_{k+1} = w_k - \mu \cdot \frac{e_k \cdot X_{d,k}}{\| X_{d,k} \|^2}$$

korak 1: $w_1 = w_0 - 1 \cdot \frac{e_0 \cdot X_{d,0}}{\| X_{d,0} \|^2}$ $\quad X_{d,0} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad e_0 = X_{d,0}^T \cdot w_0 - y_{d,0} = -1$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - 1 \frac{(-1) \cdot X_{d,0}}{1} = X_{d,0} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

korak 2: $w_2 = w_1 - 1 \frac{e_1 \cdot X_{d,1}}{\| X_{d,1} \|^2} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - 1 \frac{\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T}{1}$ $\quad e_1 = X_{d,1}^T \cdot w_1 - y_{d,1} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - 1 = -1$

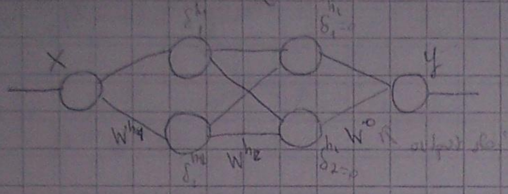
$$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Prvi način uvježbavanja iterativnim postupkom

• 1x1 algoritam

(3.d) Zadana je potpuno povezana unaprijedna neuronska mreža 4 sloja strukture $1 \times 2 \times 2 \times 1$. Aktivacijska funkcija svih u mreži je opći sigmoid.

a) Skicirati tu mrežu.




b) provesti prvi korak uvježbavanja te mreže (dajte račun potreban za određivanje popravke) ako ju treba uvježbati za logičku funkciju NE

| ulaz | izlaz |
| --- | --- |
| 0 | 1 |
| 1 | 0 |

Početne vrijednosti svih param. mreže postavite na nulu.

$$W_0^{h_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad W_0^{h_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad W_0^{o} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

(prvi skriveni sloj; izlazni sloj)

$$\theta^{h_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \theta^{h_2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \theta^{o} = 0$$

$$z_1 = \text{sigmoid}(v_1 - \theta_1) = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \qquad v_1 = W^{h_1} \cdot x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_2 = W^{h_2} \cdot z_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow z_2 = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$v^o = w^o \cdot z_2 = 0 \Rightarrow y_0 = 1/2$$

backprop.

$$EA_0^o = (y_0 - y_{d_0}) = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\delta \Rightarrow EI_0 = EA_0^o .* y_0 .* (1 - y_0) = \left(-\frac{1}{2}\right) \cdot \frac{1}{2} \cdot \left(1 - \frac{1}{2}\right) = -\frac{1}{8} \qquad E\theta^o = -EI^o = \frac{1}{8}$$

$$EW_0^o = EI_0 \cdot z_2^T = \begin{bmatrix} -1/16 \\ -1/16 \end{bmatrix} \qquad EA_0^{h_2} = (W^o)^T \cdot EI_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot \left(-\frac{1}{8}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

delta pravilo

$$\delta^o = -\frac{1}{8}$$

$$EI_0^{h_2}$$

$$\delta_1^{h_1} = f'(v_{11}) \cdot \left(w_{11}^{h_2} \cdot \delta_1^{h_2} + w_{21}^{h_2} \cdot \delta_2^{h_2}\right) = z_{11} \cdot (1 - z_{11}) \cdot (\dots)$$

## Simplex algoritam

min $5x_1 + x_2$

s.t. $\begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 3 & 7 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq \begin{bmatrix} 50 \\ 30 \\ 40 \end{bmatrix}$

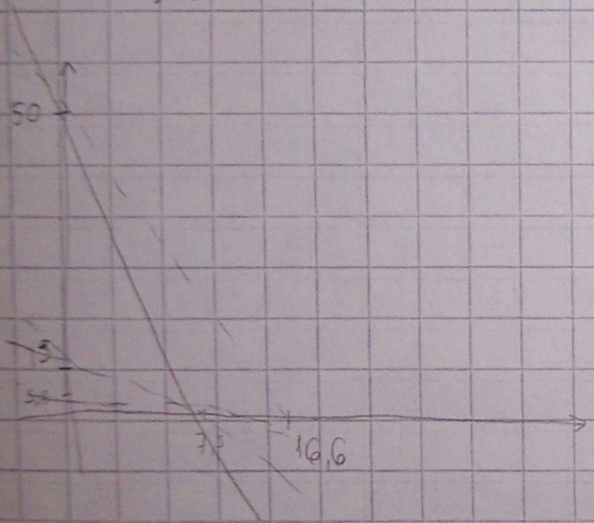
$x \geq 0$

$3x_1 + x_2 = 50$
$x_2 = 50 - 3x_1$

$4x_1 + 2x_2 = 30$
$x_2 = 15 - 2x_1$

nije dobro zadan zadatak

$3x_1 + 7x_2 = 40$
$x_2 = \frac{40}{7} - \frac{3}{7}x_1$




(0,50)