Primjena elektromagnetskih valova u inženjerstvu

Formule - MI

Teorija linija:

- $c = \lambda f$
- $\Delta\varphi = \frac{\Delta l}{\lambda} \cdot 360^{\circ}$
- $\gamma = \alpha + j\beta$

  Linija bez gubitaka:

  - $R = G = 0$
  - $\alpha = 0 \, , \beta = \omega\sqrt{LC}$
- $\Gamma = \frac{U^{-}}{U^{+}} = \frac{Z_L - Z_0}{Z_L + Z_0}$
- $SWR = \frac{U_{max}}{U_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$
- $P_{avg} = \frac{1}{2} \cdot \left|\frac{U_0^{+}}{Z_o}\right|^2 (1 - |\Gamma|^2)$

  $; \; Z_G = Z_o$

- $Z_0 = \sqrt{\frac{Z}{Y}}$

  Linija bez gubitaka:

  - $Z_0 = \sqrt{\frac{L}{C}}$
- $U(x) = U_R[\frac{Z_0}{Z_L} sh(\gamma x) + ch(\gamma x)]$
- $I(x) = I_R[\frac{Z_L}{Z_0} sh(\gamma x) + ch(\gamma x)]$
- $z_{IN}(x) = \frac{U(x)}{I(x)} = Z_0 \cdot \frac{Z_L + Z_0\, th(\gamma x)}{Z_0 + Z_L\, th(\gamma x)}$
- $\lambda = \frac{v_P}{f}$
- $\beta = \frac{2\pi}{\lambda}$
- $v_p = \frac{c}{\sqrt{\varepsilon_r}}$

Dodatak:

- *$\lambda/4$ linija (transformator)*
  - $Z_{in} = \frac{Z_0^2}{Z_R}$

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Formule – ZI

Valna jednadžba:

- $\nabla^2\vec{E} + k^2\vec{E} = 0$; $k = \omega\sqrt{\mu\varepsilon}$

  Analogija s linijama:

  - $k = \beta - j\alpha$
  - $\gamma = \alpha + j\beta$
  - $k = -j\gamma$

Općeniti oblici električnog i magnetskog polja:

- $\vec{E}(z,t) = \hat{x}E_0\cos(\omega t - kz + \varphi)$
- $\vec{H}(z,t) = \hat{y}E_0\frac{1}{\eta}\cos(\omega t - kz + \varphi)$

- $\frac{E_{x0}}{H_{y0}} = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$
  - $\eta_0 \approx 377\Omega$
  - $\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}}$
  - $k = \frac{\omega}{c}\sqrt{\varepsilon_r}$
  - $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \approx 3\cdot 10^8\,\frac{m}{s}$

- $v_p = \frac{\omega}{k} = \lambda f$

- $p = EH$; $(p - \text{protok snage})$

Medij s gubicima:

- $\hat{\varepsilon} = \varepsilon' - j\varepsilon'' = \varepsilon - j\frac{\sigma}{\omega}$
- $\frac{\varepsilon''}{\varepsilon'} = \frac{v}{\omega\varepsilon} = \frac{\sigma}{\omega\varepsilon_r\varepsilon_0}$

Okomiti upad:

- $\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$
- $T = 1 + \Gamma = \frac{2\eta_2}{\eta_2+\eta_1}$
- $E'' = E'\cdot|\Gamma|$
- $E''' = E'\cdot|T|$
- $H' = \frac{E'}{\eta_1}$
- $H'' = \frac{E''}{\eta_1}$
- $H''' = \frac{E'''}{\eta_2}$

  Dubina $|E|$:

  - $\tilde{E}(z) = E'''\cdot e^{-\alpha z}$

Snellov zakon:

- $\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{n_2}{n_1} = \frac{\sqrt{\varepsilon_{r2}\mu_{r2}}}{\sqrt{\varepsilon_{r1}\mu_{r1}}} = \frac{\lambda_1}{\lambda_2} = \frac{\eta_1}{\eta_2} = \frac{k_2}{k_1}$

  Kritični kut $(n_1 > n_2)$:

  - $\sin(\theta_c) = \frac{n_2}{n_1}$
  - $\sin(\theta_c) = \frac{1}{\sqrt{\varepsilon_r}}$

Polje na granici između dvaju dielektrika:

- $\hat{n} \cdot \overrightarrow{D_1} = \hat{n} \cdot \overrightarrow{D_2}$
- $\hat{n} \cdot \overrightarrow{B_1} = \hat{n} \cdot \overrightarrow{B_2}$
- $\hat{n} \times \overrightarrow{E_1} = \hat{n} \times \overrightarrow{E_2}$
- $\hat{n} \times \overrightarrow{H_1} = \hat{n} \times \overrightarrow{H_2}$

Kosi upad:

TE val:

- $E' + E'' = E'''$
- $H'cos(\theta_i) + H''cos(\theta_i) = H'''cos(\theta_t)$
- $\frac{cos(\theta_i)}{\eta_1}(E' + E'') = \frac{cos(\theta_t)}{\eta_2}E'''$
- $\Gamma_\perp = \frac{E''}{E'} = \frac{\eta_2 cos(\theta_i) - \eta_1 cos(\theta_t)}{\eta_2 cos(\theta_i) + \eta_1 cos(\theta_t)}$
- $T_\perp = 1 + \Gamma_\perp = \frac{2\eta_2 cos(\theta_i)}{\eta_2 cos(\theta_i) + \eta_1 cos(\theta_t)}$

TM val:

- $H' + H'' = H'''$
- $E'cos(\theta_i) + E''cos(\theta_i) = E'''cos(\theta_t)$
- $\frac{1}{\eta_1}(E' + E'') = \frac{1}{\eta_2}E'''$
- $\Gamma_{||} = \frac{E''}{E'} = \frac{\eta_2 cos(\theta_t) - \eta_1 cos(\theta_i)}{\eta_2 cos(\theta_t) + \eta_1 cos(\theta_i)}$
- $T_{||} = (1 + \Gamma_{||})\frac{cos(\theta_i)}{cos(\theta_t)} = \frac{2\eta_2 cos(\theta_i)}{\eta_2 cos(\theta_t) + \eta_1 cos(\theta_t)}$

Pravokutni valovod:

TE mod:

- $\tilde{H}_z = H_{z0} cos\left(\frac{m\pi x}{a}\right) cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$
  - Dominantni mod: λ_{10}, f_{10}

TM mod:

- $\tilde{E}_z = E_{z0} sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$
  - Dominantni mod: λ_{11}, f_{11}

- $\beta = \sqrt{k^2 - k_c^2} = \sqrt{\omega^2 \mu \varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$

$k > k_c \rightarrow \beta \in \mathbb{R}$ - Propagacijski mod
$k < k_c \rightarrow \beta \in \mathbb{C} \setminus \mathbb{R}$ - Evanescentno polje
Zaporna frekvencija $k = k_c$

- $f_{mn} = \frac{v_{po}}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$
- $\lambda_{mn} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$
- $\beta_{mn} = \frac{\omega}{v_{po}} \sqrt{1 - \left(\frac{f_{mn}}{f}\right)^2}$
- $Z_{TM} = \eta \sqrt{1 - \left(\frac{f_{mn}}{f}\right)^2}$
- $Z_{TE} = \frac{\eta}{\sqrt{1 - \left(\frac{f_{mn}}{f}\right)^2}}$

Brewsterov kut (TM):

- $\theta_B = tan^{-1}\left(\frac{n_2}{n_1}\right)$; $n_2 > n_1$
- $\theta_B = sin^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}}\right)$; $\mu_1 = \mu_2$

Fazna i grupna brzina u valovodu:

- $v_p \cdot v_g = v_{po}^2$
- $v_p = \frac{\omega}{\beta} = \frac{v_{po}}{\sqrt{1 - \left(\frac{f_{11}}{f}\right)^2}}$

Antene:

- $\Omega_P = \iint_\Omega F(\theta, \varphi)\, d\Omega = \int_0^{\theta'} \sin(\theta)\, d\theta \int_0^{2\pi} d\varphi = 2\pi \int_0^{\theta'} \sin(\theta)\, d\theta$
- $D = \frac{4\pi}{\Omega_P} = \frac{U_{max}}{U_{avg}} = \frac{4\pi U_{max}}{P_z}$; $D[dB] = 10\, log(D)$; $dBi = dBd + 2.15$
- $k = \frac{P_z}{P_{ul}} = 1 - \frac{P_{dis}}{P_{ul}}$
- $G = kD$; (k – faktor učinkovitosti, G – dobitak) ; $G[dB] = 10\, log(G)$
- $S_{max} = \frac{W_0}{4\pi R^2} \cdot G = \frac{E_{ef}^2}{\eta}$

Table 1: Formulation in terms of free charge and current

Name	Differential form	Integral form
Gauss' law:	$\nabla \cdot \mathbf{D} = \rho_f$	$\oint_S \mathbf{D} \cdot d\mathbf{A} = Q_{f,S}$
Gauss' law for magnetism:	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$
Maxwell-Faraday equation (Faraday's law of induction):	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial \Phi_{B,S}}{\partial t}$
Ampère's circuital law (with Maxwell's correction):	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_{\partial S} \mathbf{H} \cdot d\mathbf{l} = I_{f,S} + \frac{\partial \Phi_{D,S}}{\partial t}$

Table 2: Formulation in terms of *total* charge and current

Name	Differential form	Integral form
Gauss' law:	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$	$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_S}{\epsilon_0}$
Gauss' law for magnetism:	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$
Maxwell-Faraday equation (Faraday's law of induction):	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial \Phi_{B,S}}{\partial t}$
Ampère's circuital law (with Maxwell's correction):	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\oint_{\partial S} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_S + \mu_0 \epsilon_0 \frac{\partial \Phi_{E,S}}{\partial t}$

Dodatak:

- $\vec{D} = \varepsilon \vec{E}$
- $\vec{B} = \mu \vec{H}$
- $\Phi_{B,S} = \int_S \vec{B}\, dS$
- $\nabla \cdot (\nabla \times \vec{F}) = 0$
- $\nabla \times (\nabla f) = 0$
- $\nabla \times \nabla \times \vec{F} = \nabla(\nabla \cdot \vec{F}) - \nabla^2 \vec{F}$

Gaussov teorem (divergencije):

- $\int_V (\nabla \cdot \vec{F})\, dV = \oint_S \vec{F} \cdot \hat{n}\, dS$

Stokesov teorem:

- $\int_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS = \oint_C \vec{F}\, d\vec{l}$