

WeGotYouCovered

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Abstract

We present the winning solver of the PACE 2019 Implementation Challenge Vertex Cover Track. The vertex cover problem is one of a handful of problems for which *kernelization*—the repeated reducing of the input size via *data reduction rules*—is known to be highly effective in practice. Our algorithm uses a portfolio of techniques, including an aggressive kernelization strategy with all known reduction rules, local search, branch-and-reduce, and a state-of-the-art branch-and-bound solver. Of particular interest is that several of our techniques were *not* from the literature on the vertex cover problem: they were originally published to solve the (complementary) maximum independent set and maximum clique problems. Lastly, we perform extensive experiments to show the impact of the different solver techniques on the number of instances solved during the challenge.

1 Introduction

A *vertex cover* of a graph $G = (V, E)$ is a set of vertices $S \subseteq V$ of G such that every edge of G has at least one of member of S as an endpoint (i.e., $\forall(u, v) \in E [u \in S \text{ or } v \in S]$). A minimum vertex cover is a vertex cover of minimum cardinality. Complementary to vertex covers are independent sets and cliques. An independent set is a set of vertices $I \subseteq V$, all pairs of which are not adjacent, and an clique is a set of vertices $K \subseteq V$ all pairs of which are adjacent. A maximum independent set (maximum clique) is an independent set (clique) of maximum cardinality. The goal of the maximum independent set problem (maximum clique problem) is to compute a maximum independent set (maximum clique).

Many techniques have been proposed for solving these problems, and papers in the literature usually focus on one of these problems in particular. However, all of these problems are equivalent: a minimum vertex cover C in G is the complement of a maximum independent set $V \setminus C$ in G , which is a maximum clique $V \setminus C$

in \overline{G} . Thus, an algorithm that solves one of these problems can be used to solve the others. For our approach, we use a portfolio of solvers, using techniques from the literature on all three problems. These include data reduction rules and branch-and-reduce for the minimum vertex cover problem [?], iterated local search for the maximum independent set problem [?], and a state-of-the-art branch-and-bound maximum clique solver [?].

We first briefly describe related work. Then we outline each of the techniques that we use, and finally describe how we combine all of the techniques in our final solver that scored most of the points during the PACE 2019 Implementation Challenge. Lastly, we perform an experimental evaluation to show the impact of the components used on the final number of instances solved during the challenge.

2 Related Work

We now present important related work. This includes exact branch-and-bound algorithms as well as reduction based approaches. Much research has been devoted to improve exact branch-and-bound algorithms for the independent set and its complementary problems. These improvements include different pruning methods and sophisticated branching schemes [?, ?, ?, ?]. Warren and Hicks [?] proposed three combinatorial branch-and-bound algorithms that are able to quickly solve DIMACS and weighted random graphs. These algorithms use weighted clique covers to generate upper bounds that reduce the search space via pruning. Furthermore, they all use a branching scheme proposed by Balas and Yu [?]. In particular, their first algorithm is an extension and improvement of a method by Babel [?]. Their second one uses a modified version of the algorithm by Balas and Yu that uses clique covers that borrow structural features from the ones by Babel [?]. Finally, their third approach is a hybrid of both previous algorithms. Overall, their algorithms are able to quickly solve instances with hundreds of vertices.

An important technique to reduce the base of the exponent for exact branch-and-bound algorithms are so-called *reduction rules*. Reduction rules are able to reduce the input graph to an irreducible *kernel* by removing well-defined subgraphs. This is done by

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selecting certain vertices that are provably part of some maximum independent set, thus maintaining optimality. We can then extend a solution on the kernel to a solution on the input graph by undoing the previously applied reductions. There exist several well-known reduction rules for the unweighted vertex cover problem (and in turn for the unweighted MIS problem) [?].

As noted by Larson [?], it is possible that in the unweighted case the initial critical set found by Butenko and Trukhanov might be empty. To prevent this case, Larson [?] proposed an algorithm that finds a *maximum* (unweighted) critical independent set. His algorithm accumulates vertices that are in some critical set and removes their neighborhood. Additionally, he provides a method to quickly check if a given vertex is part of some critical set. Later Iwata [?] has shown how to remove a large collection of vertices from a maximum matching all at once; however, it is not known if these reductions are equivalent.

For the maximum weight clique problem, Cai and Lin [?] give an exact branch-and-bound algorithm that interleaves between clique construction and reductions. In particular, their algorithm picks different starting vertices to form a clique and then maintains a candidate set to iteratively extend this clique. In each iteration, the vertex to be added is selected using a benefit estimation function and a dynamic best from multiple selection heuristic [?]. Once the candidate set is empty, the new solution is compared to the best solution found so far. If an improvement is found, their algorithm then tries to apply reductions and reduce the graph size. To be more specific, they use two reduction rules that are able to remove a vertex v by computing upper bounds related to the weight of different neighborhoods of v . We briefly note that their algorithm and reductions are targeted at sparse graphs, and therefore their reductions would likely work well for the maximum weighted independent set problem on *dense* graphs.

3 Solver Techniques

We now describe the main techniques that we use within our solver.

Kernelization. The most efficient algorithms for computing a minimum vertex cover in both theory and practice use *data reduction rules* to obtain a much smaller problem instance. If this smaller instance has size bounded by a function of some parameter, it's called a *kernel*.

We use an extensive (though not exhaustive) collection of data reduction rules whose efficacy was studied by Akiba and Iwata [?]. To compute a kernel, Akiba and Iwata [?] apply their reductions r_1, \dots, r_j by iterating over all reductions and trying to apply the cur-

rent reduction r_i to all vertices. If r_i reduces at least one vertex, they restart with reduction r_1 . When reduction r_j is executed, but does not reduce any vertex, all reductions have been applied exhaustively, and a kernel is found. Following their study we order the reductions as follows: degree-one vertex (i.e., pendant) removal, unconfined vertex removal [?], a well-known linear-programming relaxation [?, ?] related to crown removal [?], vertex folding [?], and twin, funnel, and desk reductions [?].

Branch-and-Reduce. Branch-and-reduce is a paradigm that intermixes data reduction rules and branching. We use the algorithm of Akiba and Iwata, which exhaustively applies their full suite of reduction rules before branching, and includes a number of advanced branching rules. When branching, a vertex is chosen at random for inclusion into the vertex cover.

Branch-and-Bound. Experiments by Strash [?] show that the full power of branch-and-reduce is only needed *very rarely* in real-world instances; kernelization followed by standard branch-and-bound solver is sufficient for many real-world instances. Furthermore, branch-and-reduce does not work well on many synthetic benchmark instances, where data reduction rules are ineffective [?], and instead add significant overhead to branch-and-bound. We use a state-of-the-art branch-and-bound maximum clique solver (MoMC) by Li et al. [?], which uses incremental MaxSAT reasoning to prune search, and a combination of static and dynamic vertex ordering to select the vertex for branching. We run the clique solver on the complement graph, giving a maximum independent set from which we derive a minimum vertex cover. In preliminary experiments, we found that a kernel can sometimes be harder for the solver than the original input; therefore, we run the algorithm on both the kernel and on the original graph.

Iterated Local Search. Batsyn et al. [?] showed that if branch-and-bound search is primed with a high-quality solution from local search, then instances can be solved up to thousands of times faster. We use iterated local search algorithm by Andrade et al. [?] to prime the branch-and-reduce solver with a high-quality initial solution. Iterated local search was originally implemented for the maximum independent set problem, and is based on the notion of (j, k) -swaps. A (j, k) -swap removes j nodes from the current solution and inserts k nodes. The authors present a fast linear-time implementation that, given a maximal independent set, can find a $(1, 2)$ -swap or prove that none exists. Their algorithm applies $(1, 2)$ -swaps until reaching a local maximum, then perturbs the solution and repeats. We implemented the algorithm to find a high-quality solution on *the kernel*. Calling local search on the kernel has been shown to pro-

duce a high-quality solution much faster than without kernelization [?, ?].

4 Putting it all Together

Our algorithm first runs a preprocessing phase, followed by 4 phases of solvers.

Phase 1. (Preprocessing) Our algorithm starts by computing a kernel of the graph using the reductions by Akiba and Iwata [?]. From there we use iterated local search to produce a high-quality solution S_{init} on the (hopefully smaller) kernel.

Phase 2. (Branch-and-Reduce, short) We prime a branch-and-reduce solver with the initial solution S_{init} and run it with a short time limit.

Phase 3. (Branch-and-Bound, short) If Phase 2 is unsuccessful, we run the MoMC [?] clique solver on the complement of the kernel, also using a short time limit. Sometimes kernelization can make the problem harder for MoMC. Therefore, if the first call was unsuccessful we also run MoMC on the complement of the original (unkernelized) input with the same short time limit.

Phase 4. (Branch-and-Reduce, long) If we have still not found a solution, we run branch-and-reduce on the kernel using initial solution S_{init} and a longer time limit. We opt for this second phase because, while most graphs amenable to reductions are solved very quickly with branch-and-reduce (less than a second), experiments by Akiba and Iwata [?] showed that other slower instances either finish in at most a few minutes, or take significantly longer—more than the time limit allotted for the challenge. This second phase of branch-and-reduce is meant to catch any instances that still benefit from reductions.

Phase 5. (Branch-and-Bound, remaining time)

If all previous phases were unsuccessful, we run MoMC on the original (unkernelized) input graph until the end of the time given to the program by the challenge. This is meant to capture only the most hard-to-compute instances.

The ordering and time limits were carefully chosen so that the overall algorithm outputs solutions of the “easy” instances *quickly*, while still being able to solve hard instances.

5 Experimental Results

We now look at the impact of the algorithmic components on the number of instances solved. Here, we use the public instances – obtained from <https://pacechallenge.org/files/pace2019-vc-exact-public-v2.tar.bz2> – of the PACE 2019 Track A implementation challenge. This set contains 100 instances overall. Afterwards, we present the results comparing against the second and third best competing algorithms during the challenge.

5.1 Methodology and Setup. All of our experiments were run on a machine with four Sixteen-Core Intel Xeon Haswell-EX E7-8867 processors running at 2.5 GHz, 1 TB of main memory, and 32768 KB L2-Cache. The machine runs Debian GNU/Linux 9 and Linux kernel version 4.9.0-9. All algorithms were implemented in C++11 and compiled with gcc version 6.3.0 with optimization flag `-O3`. Each algorithm was run sequentially with a time limit of 30 minutes. Our evaluations focus on the total number of instances solved.

5.2 Algorithm Configurations. Variants:

- MoMC solves 30 / 100 instances
- Kernel + MoMC solves 68 / 100 instances
- Kernel + BnR solves 55 / 100 instances
- Kernel + BnR - Local Search solves 42 / 100 instances
- Full solver solves 82 / 100 instances

6 TODOs

- TODO add more exact stuff, heuristics?
- TODO insert final three solvers of pace challenge
- create a table with instances solved

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inst #	n	m	n'	m'	MoMC	RMoMC	BnR	BnR-LS	FullA	$ VC $
001	6160	40207	0	0	-	X	X	X	X	2586
003	60541	74220	0	0	-	X	X	X	X	12190
005	200	819	192	800	X	X	X	X	X	129
007	8794	10130	0	0	-	X	X	X	X	4397
009	38452	174645	0	0	-	X	X	X	X	21348
011	9877	25973	0	0	-	X	X	X	X	4981
013	45307	55440	0	0	-	X	X	X	X	8610
015	53610	65952	0	0	-	X	X	X	X	10670
017	23541	51747	0	0	-	X	X	X	X	12082
019	200	884	194	862	X	X	X	X	X	130
021	24765	30242	0	0	-	X	X	X	X	5110
023	27717	133665	0	0	-	X	X	X	X	16013
025	23194	28221	0	0	-	X	X	X	X	4899
027	65866	81245	0	0	-	X	X	X	X	13431
029	13431	21999	0	0	-	X	X	X	X	6622
031	200	813	198	818	X	X	X	X	X	136
033	4410	6885	138	471	-	X	X	X	X	2725
035	200	884	189	859	X	X	X	X	X	133
037	198	824	194	810	X	X	X	X	X	131
039	6795	10620	219	753	-	X	X	X	X	4200
041	200	1040	200	1023	X	X	X	X	X	139
043	200	841	198	844	X	X	X	X	X	139
045	200	1044	200	1020	X	X	X	X	X	137
047	200	1120	198	1080	X	X	X	X	X	140
049	200	957	198	930	X	X	X	X	X	136
051	200	1135	200	1098	X	X	X	X	X	140
053	200	1062	200	1026	X	X	X	X	X	139
055	200	958	194	938	X	X	X	X	X	134
057	200	1200	197	1139	X	X	X	X	X	142
059	200	988	193	954	X	X	X	X	X	137
061	200	952	198	914	X	X	X	X	X	135
063	200	1040	200	1011	X	X	X	X	X	138
065	200	1037	200	1011	X	X	X	X	X	138
067	200	1201	200	1174	X	X	X	X	X	143
069	200	1120	196	1077	X	X	X	X	X	140
071	200	984	200	952	X	X	X	X	X	136
073	200	1107	200	1078	X	X	X	X	X	139
075	26300	41500	500	3000	-	-	X	-	X	16300
077	200	988	193	954	X	X	X	X	X	137
079	26300	41500	500	3000	-	-	X	-	X	16300
081	199	1124	197	1087	X	X	X	X	X	141
083	200	1215	198	1182	X	X	X	X	X	144
085	11470	17408	3539	25955	-	-	-	-	-	
087	13590	21240	441	1512	-	X	-	-	X	8400
089	57316	77978	16834	54847	-	-	-	-	-	
091	200	1196	200	1163	X	X	X	X	X	145
093	200	1207	200	1162	X	X	X	X	X	143
095	15783	24663	510	1746	-	X	-	-	X	9755
097	18096	28281	579	1995	-	X	-	-	X	11185
099	26300	41500	500	3000	-	-	X	-	X	16300

inst#	n	m	n'	m'	MoMC	RMoMC	BnR	BnR-LS	FullA	
101	26300	41500	500	3000	-	-	X	-	X	16300
103	15783	24663	513	1752	-	X	-	-	X	9755
105	26300	41500	500	3000	-	-	X	-	X	16300
107	13590	21240	435	1500	-	X	-	-	X	8400
109	66992	90970	20336	66350	-	-	-	-	-	
111	450	17831	450	17831	X	X	-	-	X	420
113	26300	41500	500	3000	-	-	X	-	X	16300
115	18096	28281	573	1986	-	X	-	-	X	11185
117	18096	28281	582	2007	-	X	-	-	X	11185
119	18096	28281	588	2016	-	X	-	-	X	11185
121	18096	28281	579	1998	-	X	-	-	X	11185
123	26300	41500	500	3000	-	-	X	-	X	16300
125	26300	41500	500	3000	-	-	X	-	X	16300
127	18096	28281	582	2001	-	X	-	-	X	11185
129	15783	24663	507	1752	-	X	-	-	X	9755
131	2980	5360	2179	6951	X	-	-	-	-	
133	15783	24663	507	1746	-	X	-	-	X	9755
135	26300	41500	500	3000	-	-	X	-	X	16300
137	26300	41500	500	3000	-	-	X	-	X	16300
139	18096	28281	579	1995	-	X	-	-	X	11185
141	18096	28281	576	1995	-	X	-	-	X	11185
143	18096	28281	582	2001	-	X	-	-	X	11185
145	18096	28281	576	1989	-	X	-	-	X	11185
147	18096	28281	567	1974	-	X	-	-	X	11185
149	26300	41500	500	3000	-	-	X	-	X	16300
151	15783	24663	501	1728	-	X	-	-	X	9755
153	29076	45570	2124	16266	-	-	-	-	-	
155	26300	41500	500	3000	-	-	X	-	X	16300
157	2980	5360	2169	6898	X	-	-	-	-	
159	18096	28281	582	2004	-	X	-	-	X	11185
161	138141	227241	41926	202869	-	-	-	-	-	
163	18096	28281	582	2004	-	X	-	-	X	11185
165	18096	28281	576	1995	-	X	-	-	X	11185
167	15783	24663	510	1746	-	X	-	-	X	9755
169	4768	8576	3458	11014	-	-	-	-	-	
171	18096	28281	576	1989	-	X	-	-	X	11185
173	56860	77264	17090	55568	-	-	-	-	-	
175	3523	6446	2723	8570	-	-	-	-	-	
177	5066	9112	3704	11797	-	-	-	-	-	
179	15783	24663	504	1740	-	X	-	-	X	9755
181	18096	28281	573	1989	-	X	X	-	X	11185
183	72420	118362	30340	133872	-	-	-	-	-	
185	3523	6446	2723	8568	-	-	-	-	-	
187	4227	7734	3264	10286	-	-	-	-	-	
189	7400	13600	5802	18212	-	-	-	-	-	
191	4579	8378	3539	11137	-	-	-	-	-	
193	7030	12920	5510	17294	-	-	-	-	-	
195	1150	81068	1150	81068	-	-	-	-	-	
197	1534	127011	1534	127011	-	-	-	-	-	
199	1534	126163	1534	126163	-	-	-	-	-	

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