Computer Assignment 2
IS, QMC and interest rate derivatives

Introduction

In the first four problems you will price European stock options using importance sampling and Quasi-MC and 20 000 simulations and you shall also, for problems 1 and 2, compare the standard MC method to all the variance reduction methods (even those in computer assignment 1) for an increasing number of simulations. For each problem calculate the price using 10 000, 20 000, ... and up to 100 000 simulations and then plot the prices, for both standard MC and the variance reduction methods (including Quasi-MC), using a line plot with the number of simulations on the x-axis.

In the last two problems you will price interest rate derivatives using Vasicek's short rate model.

1. Call and put option

Let S(0) = 100, r = 0.03, T = 1, $\sigma = 20\%$ and $K \in \{70, 100, 130\}$:

a) Calculate the price using quasi-MC for all strikes
b) Calculate the price using importance sampling, but only for K = 130 for the call and K = 70 for the put. Use μ = 1.84 for the call and μ = -2.26 for the put.
c) Produce a comparison plot including the methods (and the exact price) from assignment 1 for the call with K = 130.

Discussion: Based on the plot in c) which pricing method seems to be the most accurate?

2. Asian call

Choose parameters as in problem 1 with $0 = t_0 < t_1 < \cdots < t_m = T$, $t_i - t_{i-1} = 1/m$ and consider an Asian call on arithmetic mean.

a) Calculate the price using quasi-MC for all strikes $K \in \{70, 100, 130\}$ and $m \in \{12, 52\}$
b) Calculate the price using importance sampling when K = 130. Use $\mu = 5/m$ in each time step, for $m \in \{12, 52\}$.
c) Produce a comparison plot including the methods from assignment 1 for K = 130 and m = 52

3. Basket call and put option on two underlying stocks

Let $S_i(0) = 100$, $\sigma_i = 20\%$, r = 0.03, T = 1 and $K \in \{70, 100, 130\}$. Calculate prices of the basket options using quasi-MC for:

a) $\rho_{\square} = 0$
b) $\rho_{\square} = 0.5$

4. Barrier option

$Payoff = 1\{\tau(B) \leq T\} * (S(T) - K)^+ \; where \; \tau(B) = \inf\{t_i : S(t_i) \leq B\}$.

Let *S(0) = 100, r= 0.03, T = 1, K = 100,* $\sigma \in \{0.2, 0.4\}$, $B = 80$ and $m \in \{12, 52\}$.

a) Calculate prices assuming the stock follows the Heston model with $a = 0.1, b = \sigma^2, \sigma_v = 0.1$ and for $\rho = -0.7$ and $\rho = 0.7$. Use the squared volatilities above, $\sigma \in \{0.2, 0.4\}$, as initial values in the variance process.
b) Calculate prices using importance sampling, as in example 4.6.4 in the book, now assuming a GBM.

Discussion: For a), which correlation results in higher prices and why?

5. Call option on a zero-coupon bond

Payoff function: $\phi(p(T_1, T_2)) = (p(T_1, T_2) - K)^+$. Here $p(T_1, T_2)$ denotes the price of a zcb at time T_1 with face value 1 that is paid out at time T_2. Let *K = 0.8,* $T_1 = 5$ and $T_2 = 10$. Assume that the short rate evolves according to Vasiceks model (as defined in Glasserman) with parameters r(0) = 0.0137, $a = 0.2, b = 0.02, \sigma = 0.01$.

a) Plot the zero-coupon interest rate curve (YTM-curve), at *t = 0,* given by the parameters in Vasicek's model. Also include the values in a table. Note: You are <u>not</u> asked to plot a path of the short rate.
b) Calculate the price at *t = 0* of the option using the exact formula
c) Calculate the price at *t = 0* of the option using standard MC with a weekly time-step. Use 20 000 simulations.

6. Adjustable rate mortgage with interest rate ceiling (ARMWIRC)

Using Vasicek's model for the short rate, with the same parameters as in the previous problem, calculate the fair premium (spread) for a five year ARMWIRC, without amortization, based on the one-month rate and with a ceiling of 2.3% (excluding the spread). Use the short rate as the one-month rate.

The fair premium is the spread that makes the market price of the bank's and the customer's cash flows equal.

Use 100 000 simulated paths of the short rate to determine the premium.

Grading

These are the requirements for the grades 3, 4 and 5:

Problem/Grade	3	4	5
1	All	All	All
2	All	All	All
3	-	All	All
4	-	4a or 4b	4a and (4b or 6)
5	All	All	All
6	-	-	4b or 6

Presentation of results

1. Call and put option (one table for each option)

Method	K=70	K=100	K=130
Quasi-MC			
Importance Sampling			

2. Asian call

	K = 70		K = 100		K = 130	
Option - method	m=12	m=52	m=12	m=52	m=12	m=52
Quasi-MC						
IS						

3. Basket option call and put option (one table for each option)

Method/K	K = 70	K = 100	K = 130
$\rho = 0$			
$\rho = 0.5$			

4. Barrier option. For a) it's one table for each correlation

	$\sigma = 20\%$	$\sigma = 40\%$
12		
52		

5. Bond option.

a) Plot and the following table

Year	1	2	3	4	5	6	7	8	9	10
Zcb-rate										

b and c)

Method	Price
Exact	
Standard MC	