Computer Assignment 1
European options and structured products

Begränsad delning

Introduction

Here you will price European options where S(t) denotes the value of the underlying stock, which is assumed to follow a geometric Brownian motion. Use 20 000 simulations for each price calculation and present your results in a table, one for each of the assignments (see table formats at the end). Give your answers with 3 decimals.

You shall also, for problems 1 and 2, compare the standard MC method to the variance reduction method for an increasing number of simulations. For each problem calculate the price using 10 000, 20 000, ... and up to 100 000 simulations and then plot the prices, for both standard MC and the variance reduction method, using a line plot with the number of simulations on the x-axis.

1. Call and put option

Let S(0) = 100, r = 0.03, T = 1 (year), $\sigma = 20\%$ and $K \in \{70, 100, 130\}$:

a) Calculate the option price using the Black-Scholes formula
b) Study how the price for the call and put option (for K = 100) depends on the volatility, time to maturity and the interest rate by plotting the option values against these variables in three separate plots. Vary
   - the volatility in the range [1%,50%] with steps of size 1%,
   - the time to maturity in the range [1/12,5] with steps of size 1/12, and
   - the interest rate in the range [0, 0.1] with steps of size 0.01.
c) Calculate the price using standard MC, determine a 95% confidence interval
d) Calculate the price using antithetic variables and determine a 95% confidence interval. Here you should only use 10000 simulations to make the comparison fair.
e) Produce a comparison plot for the call option with K = 100 (including: exact price, SMC-price, AV-price)

Discussion: Provide intuitive (not based on formulas) explanations for the plots in b) for volatility and time to maturity.

2. Asian call

Choose parameters as in problem 1 with $0 = t_0 < t_1 < \cdots < t_m = T$, where $t_i - t_{i-1} = 1/m$ for $m \in \{12, 52\}$, for the following two types of Asian options.

i. Asian call based on the geometric mean: $Payoff = \left(\sqrt[m]{S(t_1)\cdot \ldots \cdot S(t_m)} - K\right)^+$
   a. Calculate the price using the exact formula

ii. Asian call based on the arithmetic mean: $Payoff = \left(\frac{S(t_1)+\cdots+S(t_m)}{m} - K\right)^+$
   a. Calculate the price using standard MC for all strikes
   b. Calculate the price using a control variate (the geometric asian call) for all strikes
   c. Produce a comparison plot for K = 100 and m = 52 (including SMC-price and CV-price)

Discussion:

- *Why is the price of the geometric option lower than the arithmetic option?*
- *Why does the price decrease as m increases?*

3. Barrier option, down-and-in-call

Payoff $= 1\{\tau(b) \leq T\} * (S(T) - K)^+$ where $\tau(b) = min\,\{t_i : S(t_i) \leq b\}$

Let *S(0) = 100, r = 0.03, T = 1 (year), K = 100,* $\sigma \in \{0.2, 0.4\}$, $b = 80$ and $m \in \{12, 52\}$.

a) Calculate the price using standard MC
b) Calculate analytical approximations using the Broadie, Glasserman and Kou continuity-correction-technique[1]

Discussion: Why is the price higher when the volatility is higher?

4. Basket call and put option on two underlying stocks

$Call\,payoff = (\frac{1}{2}(S_1(T) + S_2(T)) - K)^+$. $Put\,payoff = (K - \frac{1}{2}(S_1(T) + S_2(T)))^+$. Let $S_i(0) = 100, \sigma_i =$ 20%. For $K \in \{70, 100, 130\}$, calculate prices using standard MC for the following two correlations (using the same r and T as before):

a) $\rho = 0$
b) $\rho = 0.5$.

Discussion: Why are the prices lower than in problem 1?

[1] *See the section on Barrier options in the chapter on Exotic options in Hull's book "Options, futures and other derivatives" (section 25.8 in the 8th edition). A copy is available in Canvas, under References.*

5. Autocall on four underlying stocks

Consider an autocall on four underlying stocks $S_1, \ldots, S_4$ with nominal 100, autocall barrier at 100, coupon barrier at 95 and risk barrier at 70. The (accumulated) coupon is 10% and the capital protection is 90% (see the accompanying sales material on Canvas for more information and examples). Let $S_i(0) = 100, r = 0.03, \sigma_i = 20\%$ and price the autocall using the following sets of correlations: (i) $\rho_{i,j} = 0$ and (ii) $\rho_{i,j} = 0.5$ for all $i \neq j$.

a) Calculate prices using standard MC
b) Find the coupon rates that makes the price 100 for both sets of correlations

Discussion: Why does higher correlation imply a higher price?

Grading

These are the requirements for the grades 3, 4 and 5:

Problem/Grade	3	4	5
1	All	All	All
2	All	All	All
3	-	All	All
4	All	All	All
5	-	-	All

Presentation of results

1. Call and put option. One table for each option. In the table, lower (upper) means the lower (upper) confidence interval bound,

Method	K=70	K=100	K=130
Exact	Price (no c.i.)	Price (no c.i.)	Price (no c.i.)
Standard MC	[lower,**price**,upper]	[lower,**price**,upper]	[lower,**price**,upper]
Antithetic	[lower,**price**,upper]	[lower,**price**,upper]	[lower,**price**,upper]

2. Asian call.

	K = 70		K = 100		K = 130	
Option - method	m=12	m=52	m=12	m=52	m=12	m=52
GAC – exact						
AAC – standard MC						
AAC – CV						

3. Barrier option. One table for each of the two values of the time step.

Method	$\sigma = 20\%$	$\sigma = 40\%$
Standard MC		
BGK		

4. Basket option. One table for each option.

Method/K	K = 70	K = 100	K = 130
$\rho = 0$			
$\rho = 0.5$			

5. Autocall.

	$\rho = 0$	$\rho = 0.5$
Standard MC		
Coupon rate		