Computer Assignment 3
Greeks and American options

Introduction

In the first problem you will examine how dynamic delta hedging works for different rebalancing frequencies.

In the second and third problem you will calculate greeks.

In the last two problems you will apply binomial trees and the least squares Monte Carlo method to American options.

1. Dynamic hedging of a European call option

Let S(0) = 100, K = 90, r = 0.03, $\sigma = 30\%$ and T = 1.

Here you will investigate how well delta hedging performs (during a year) using three different rebalancing strategies: daily (assume 252 trading days), weekly and monthly. Perform 10 000 simulations for each strategy and in each simulation calculate the present value of the stream of profits and losses arising from delta hedging the option (you make a profit in a time period if the hedge portfolio outperforms the option, otherwise a loss). Use the BS-formula for calculating the option price and use the exact expression when calculating the delta of the option.

a) Calculate the mean and the standard deviation of these 10 000 present values and present these values in a table
b) Plot the distribution of the present values, for each strategy, using a histogram (all in the same plot)

Discussion: What effect does the rebalancing frequency have?

2. Delta and vega for an Asian call

Let S(0) = 100, K $\in$ {80, 100, 120}, r = 0.03 and $\sigma = 20\%$ and $0 = t_0 < t_1 < \cdots < t_m = 1$ where $t_i - t_{i-1} = 1/m$ for $m \in \{12, 52\}$. Using 20 000 simulations,

a) Estimate delta and vega using finite differences (both f.d. and c.d.) with h = 1 for delta and h = 0.01 for vega
b) Estimate delta and vega using pathwise derivatives

Discussion: Are these vegas higher/lower than regular call vegas? Why is that?

3. Estimation of gamma using LRM and QMC

With S0, K, r, T and σ as above, estimate gamma for a regular call option using (in a and b only for K=100):

a) LRM and plot the estimated gammas when using 10 000, …, 100 000 simulations (include the exact value).
b) by combining LRM with QMC and plot the estimated gammas when using 10 000, …, 100 000 simulations.
c) Produce a table of gamma values for the two methods and all strikes using 20 000 simulations.

Discussion: How do these methods compare?

4. Implied volatilities and greeks for American options

The files "TSLA_calls.csv" and "TSLA_puts.csv" contain prices of options[1], on Tesla, maturing in 2024-09-20. These option prices were extracted on 2024-03-15 when the stock price, S(0), was 165.8. With r = 5%:

a) What would it cost to buy a long strangle with strikes 135 and 195?
b) Based on the put option prices, calculate the implied volatilities for strikes 100 - 230 and present them in a line plot and a table. Use the average of Bid and Ask as the market price and calculate the implied volatilities by "inverting" the binomial tree, with a weekly time step.
c) For K = 165, estimate theta (daily), delta and gamma from the binomial tree at t = 0.

Discussion: What is the shape of the plot in b (a smile or a skew)?

5. American basket put option on two underlying stocks

We assume that the option is a Bermudan option that can only be exercised at $t_i = i \cdot T/6$ for $i \in \{1, \dots, 6\}$, where T = 1.

Let the underlying stock prices be 100, the volatilities 30% and r = 3%. Use the least squares MC-method with 40 000 simulated paths to

a) Price the option for all combinations of correlations: $\rho \in \{0, 0.5\}$, and strikes: $K \in \{80, 100, 120\}$
b) Calculate the early exercise premiums (use your implementation from assignment 1 of the European basket put option but now with 40 000 simulations)

[1] *Read csv-files in Python (pandas) using: call_data = pd.read_csv('…\TSLA_calls.csv', sep=';', index_col='Contract Name')*

Grading

These are the requirements for the grades 3, 4 and 5:

Problem/Grade	3	4	5
1	All	All	All
2	a	All	All
3	-	3 or 5	All
4	All	All	All
5	-	3 or 5	All

Presentation of results

1. Hedging of a European call option

Strategy	Mean	Standard deviation
Daily		
Weekly		
Monthly		

2. Asian call.

Greek & method	K = 80 (m=12)	K = 80 (m=52)	K = 100 (m=12)	K = 100 (m=52)	K = 120 (m=12)	K = 120 (m=52)
Delta (f.d.)						
Delta (c.d.)						
Delta (p.d.)						
Vega (f.d.)						
Vega (c.d.)						
Vega (p.d.)						

3. LRM

Two plots and a table

4. Put option.

Table and plot of the implied volatility. Table of greeks.

5. Basket option. One table for the price and one for the early exercise premium.

Correlation / K	80	100	120
$\rho = 0$			
$\rho = 0.5$			