Assignment 5 by Lucas Karlsson

Consider the following language over the alphabet $\Sigma = \{a, b\}$:
$M = \{a^i b^{i+j} | i, j \in \mathbf{N}\}$

1. Give an ambiguous context-free grammar $G = (\{S\}, \Sigma, P, S)$ for M. Note that the grammar should have exactly one nonterminal, S.

2. Prove that G is ambiguous.

3. Define a recursive function (in haskell ofc) that takes two natural numbers i and j and returns a proof showing that the string $a^i b^{i+j}$ can be derived from S. The proof should be represented by a list of derivation steps:

- The derivation step $\alpha A \beta \implies \alpha\gamma\beta$ should be represented by the four tuple $(\alpha, A, \gamma, \beta)$
- If the list contains n elements, then there should be a derivation $S \implies^* a^i b^{i+j}$ of the length n, where k-th step of the derivation matches the k-th element of the list.

4. Prove $\forall w \in L(G, S). w \in M$, where $L(\_, \_)$ is the inductive construction introduced in the eleventh lecture under the heading of **Recursive inference**.

5. Give an unambiguous context-free grammar G' for M.

6. (**BONUS**) Is there an algorithm for checking whether an arbitrary context-free grammar implements the language M?