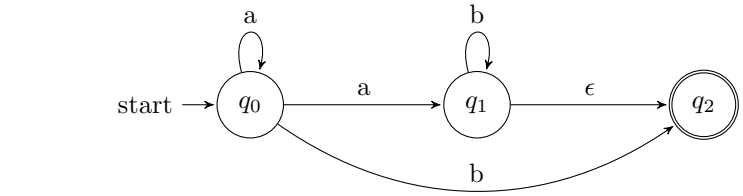
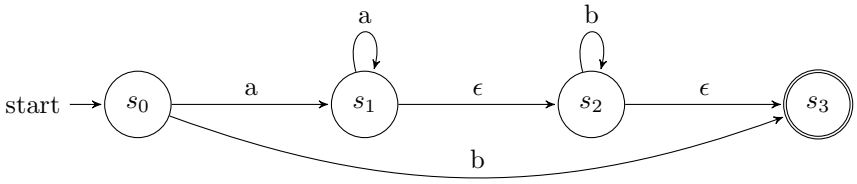
Assignment 4 by Lucas Karlsson

1. Prove that $a^*(b + ab^*) = b + aa^*b^*$.

Solution 1:
I read online about an algorithm to determine whether two RE are equal, one could construct NFA's from both and then using these convert them into DFA's using subset construction and then minimize these using a standard dfa minimization algorithm. And then compare these to see if they accpets the same language.

Step 1. Turning them into NFA's where $\mathbf{Q} = a^*(b + ab^*)$ and $\mathbf{S} = b + aa^*b^*$





Step 2. Now we can turn these NFA's into DFA's by fist removing the epsilon transitions, this can be done by everytime we have a state with a epsilon transition to another state, for every occurence of that first state we write both the first state and the state it had a epsilon transition to. I will keep on using **Q** and **S** to represent my regular expression. Here are their tables first with epsilon transitions and then after I've removed them.

		ϵ	**a**	**b**
$\rightarrow$	q_0	$\emptyset$	$\{q_0, q_1\}$	q_2
	q_1	q_2	$\emptyset$	q_1
$*$	q_2	$\emptyset$	$\emptyset$	$\emptyset$

		ϵ	**a**	**b**
$\rightarrow$	s_0	$\emptyset$	s_1	s_3
	s_1	s_2	s_1	$\emptyset$
	s_2	s_3	$\emptyset$	s_2
$*$	s_3	$\emptyset$	$\emptyset$	$\emptyset$

		a	**b**
$\rightarrow$	q_0	$\{q_0, q_1, q_2\}$	q_2
	q_1	$\emptyset$	$\{q_1, q_2\}$
$*$	q_2	$\emptyset$	$\emptyset$

		a	**b**
$\rightarrow$	s_0	$\{s_1, s_2, s_3\}$	s_3
	s_1	$\{s_1, s_2, s_3\}$	$\emptyset$
	s_2	$\emptyset$	$\{s_2, s_3\}$
$*$	s_3	$\emptyset$	$\emptyset$

Step 3. Now we can use the above table, the one without epsilon transitions to construct our DFA. The method for this is quite simple but involves a lot of calculations. How this works is I'm going to start at **Q** and **S** initial state and from there expand by calculating $\delta(state, aorb) = \delta_n(state1, aorb) \cup \delta_n(state2, aorb) = \{states\}$ Where δ_n is the transition function for our NFA and δ is the transition function for our new DFA.

The calculations for our new **Q** DFA will be:
$\delta(q_0, a) = \delta_n(q_0, a) = \{q_0, q_1, q_2\}$
$\delta(q_0, b) = \delta_n(q_0, b) = \{q_2\}$
$\delta(q_0q_1q_2, a) = \delta_n(q_0, a) \cup \delta_n(q_1, a) \cup \delta_n(q_2, a) = \{q_0, q_1, q_2\}$
$\delta(q_0q_1q_2, b) = \delta_n(q_0, b) \cup \delta_n(q_1, b) \cup \delta_n(q_2, b) = \{q_1, q_2\}$
$\delta(q_2, a) = \delta_n(q_2, a) = \{\emptyset\}$
$\delta(q_2, b) = \delta_n(q_2, b) = \{\emptyset\}$
$\delta(q_1q_2, a) = \delta_n(q_1, a) \cup \delta_n(q_2, a) = \{\emptyset\}$
$\delta(q_1q_2, b) = \delta_n(q_1, b) \cup \delta_n(q_2, b) = \{q_1, q_2\}$

The calculations for our new **S** DFA will be:
$\delta(s_0, a) = \delta_n(s_0, a) = \{s_1, s_2, s_3\}$
$\delta(s_0, b) = \delta_n(s_0, b) = \{s_3\}$
$\delta(s_1s_2s_3, a) = \delta_n(s_1, a) \cup \delta_n(s_2, a) \cup \delta_n(s_3, a) = \{s_1, s_2, s_3\}$
$\delta(s_1s_2s_3, b) = \delta_n(s_1, b) \cup \delta_n(s_2, b) \cup \delta_n(s_3, b) = \{s_2, s_3\}$
$\delta(s_3, a) = \delta_n(s_3, a) = \{\emptyset\}$
$\delta(s_3, b) = \delta_n(s_3, b) = \{\emptyset\}$
$\delta(s_2s_3, a) = \delta_n(s_2, a) \cup \delta_n(s_3, a) = \{\emptyset\}$
$\delta(s_2s_3, b) = \delta_n(s_2, b) \cup \delta_n(s_3, b) = \{s_2, s_3\}$

This gives the new tables:

		a	**b**
$\rightarrow$ (q_0)	q_0	$q_0q_1q_2$	q_2
* (q_1)	$q_0q_1q_2$	$q_0q_1q_2$	q_1q_2
* (q_2)	q_2	q_4	q_4
* (q_3)	q_1q_2	q_4	q_1q_2
(q_4)	q_4	q_4	q_4

		a	**b**
$\rightarrow$ (s_0)	s_0	$s_1s_2s_3$	s_3
* (s_1)	$s_1s_2s_3$	$s_1s_2s_3$	s_2s_3
* (s_2)	s_3	s_4	s_4
* (s_3)	s_2s_3	s_4	s_2s_3
(s_4)	s_4	s_4	s_4

Step 4. we now have two identical tables that represent the same automaton and we have equality between **Q** and **S**. We can also see this automaton in the jflap file **ass4question1.jff**.

2. Prove that every non regular language over an alphabet Σ has an infinite complement (with respect to Σ^*)

Solution 2:
To start this off I will cite one of the regular languages closures that says "The complement of a regular language is also regular" this implies that if we have the language L which is regular, $\overline{L}$ which is the complement of L will also by definition of the closure be regular. Another thing that is important that we also need to prove is that all finite languages are regular, this because regular languages are built recursively from our alphabet, this results in that every regular finite language can be represented by a automaton or a RE because the language will consist of a finite amount of string with a finite length. Clearly the finite language will always be regular. We can also see this with a easy proof by contradiction:

If we assume the language L is non-regular and defined as $L = w_1, w_2, w_3, w_4...w_x$ where $x \in \mathbb{N}$ we can contruct this language with the regular expression $w_1 + w_2 + w_3 + w_4 + .. + w_x$ and thus the language has to be regular, we have our contradiction.

Now we know that the non-regular language we are looking at will be infinite, now we need to prove that the complement to this language also will be infinite. This can be done by proving that the complement of a non-regular language also will be non-regular. We can do this with the following calculation:

We assume that our language L is non-regular, and that $\overline{L}$ is regular, that by the definition before would result in that $\overline{\overline{L}}$ would also be regular but $\overline{\overline{L}} = L$ and L is non-regular. This translates into that if L is non-regular then $\overline{L}$ is also non- regular and every non-regular language is infinite with respect to Σ^*

□

3. Show that the language $\{w \in \{0,1,2\}^* \mid \#_0(w) + \#_1(w) = \#_2(w)\}$ over the alphabet $\{0,1,2\}$ is not regular by using the pumping lemma for regular languages. Here $\#_a(w)$ is the number of occurences of a in w.

Solution 3:
We start by assuming that the language defined above, lets call it L, is regular. Then the pumping lemma for regular languages will also apply for L.

Step 1. Let n be the constant given by the pumping lemma and $w = 0^n1^n2^{2n}$, obviously $w \in L$ and $|w| \geqslant n$

Step 2. By the definition of the pumping lemma we know that $w = xyz$ with $|xy| \leqslant n$ and $y \neq \epsilon$

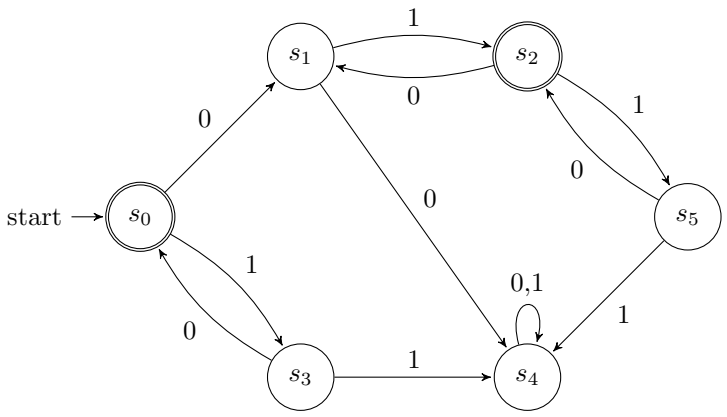
Because we chose w the way we did we know that our xy is located in the beginning of the word and that it is no longer than n, then xy can only contain 0's. Because $y \neq \epsilon$ then y should contain at least one 0 and because xy only contain 0's then y will also contain only 0's.

Step 3. Let $x = 0^j$ for $j \geqslant 0$, $y = 0^i$ for $i \geqslant 1$ because $y \neq \epsilon$. and $z = 0^{n-j-i}1^n2^{2n}$ our word will then be $w = 0^j0^i0^{n-i-j}1^n2^{2n}$ then for $k = 2$, the word $xy^2z = xyyz = 0^j0^{2i}0^{n-j-i}1^n2^2 = 0^{n+i}1^n2^{2n}$ will contain more 0's and 1's than $2's$ we can see that $n + i + n \geqslant 2n$ and then xy^2z will not belong to L which contradicts the pumping u lemma therefore L cannot be regular.

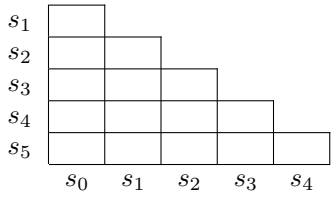
4. Minimize the following DFA: (Do not just give the answer do a step by step)

		0	**1**
$\rightarrow$ *	s_0	s_1	s_3
	s_1	s_4	s_2
*	s_2	s_1	s_5
	s_3	s_0	s_4
	s_4	s_4	s_4
	s_5	s_2	s_4

Solution 4:
When minimizing a DFA you need to do two things, remove all states that cannot be reached. After that we partition the rest of the states so that every partition only contains states that are equal. We can start by creating a automata from the given DFA, this because its easier to find equal states when we have visualized the transitions.



Step 1. We create a table representing the DFA and the all the state tuples. The table will look like this:



Step 2. Now we can start filling our table, firstly every pair where exactly one of the states is accepting. This because they cannot be equal.

	s_0	s_1	s_2	s_3	s_4
s_1	x				
s_2		x			
s_3	x		x		
s_4	x		x		
s_5	x		x		

Step 3. Now we have fewer states that we manually need to check. Lets start

s_0, s_2, $\delta(s_0, 0)$ and $\delta(s_2, 0)$ both transition into the same state s_1 doing the other transition, $\delta(s_0, 1)$ and $\delta(s_2, 1)$ we see that they both go $(01)^*$ on s_3 and s_5 respectively and thus is equal again.

s_1, s_3 not equal because $\delta(s_1, 0) = s_4$ and $\delta(s_3, 0) = s_0$ and s_4 and s_0 is not equal.

s_1, s_4 not equal because $\delta(s_1, 1) = s_2$ and $\delta(s_4, 1) = s_4$ and s_2 and s_4 is not equal.

s_1, s_5 not equal because $\delta(s_1, 1) = s_2$ and $\delta(s_5, 1) = s_4$ and s_2 and s_4 is not equal.

s_3, s_4 not equal because $\delta(s_3, 0) = s_0$ and $\delta(s_4, 0) = s_4$ and s_0 and s_4 is not equal.

s_3, s_5 not equal because $\delta(s_3, 0) = s_0$ and $\delta(s_5, 0) = s_2$ and s_0 and s_2 is equal and $\delta(s_3, 1) = s_4 = \delta(s_5, 1)$ and these are also equal. s_3, s_5 is then equal!

s_4, s_5 not equal because $\delta(s_4, 0) = s_4$ and $\delta(s_5, 0) = s_2$ and s_4 and s_2 is not equal.

Our table will then look like:

	s_0	s_1	s_2	s_3	s_4
s_1	x				
s_2		x			
s_3	x	x	x		
s_4	x	x	x	x	
s_5	x	x	x		x

Step 4. We can now construct our minimized dfa, which can also be found in the jflap file **ass4question4.jff**