CSE 380 **Discrete Mathematics II**

Quote by Paul Davies

January 5, 2021 *Rick Neff*

I was once asked to write an article about the most profound discovery ever made. I unhesitatingly chose Kurt Gödel's incompleteness theorem. In the normal use of the term, the word "discovery" implies that we learn something that was not known before. Gödel's theorem, by contrast, tells us what we don't know and can't know. It sets a fundamental and inescapable limit on knowledge of what is. It pinpoints the boundaries of ignorance — not just human ignorance, but that of any sentient being.

This remarkable book addresses the question of what can and cannot be known. It is about the nature of existence and reality and truth. Before Gödel, it was widely supposed that mathematics offered the most secure form of knowledge. Mathematics is a vast labyrinth of definitions and relationships open to independent scrutiny and supported by the iron scaffolding of unassailable logic.

Human beings may legitimately quarrel about the facts of history or religion or politics, or even about the content of scientific theories, but properly-formulated mathematics leaves no scope for disagreement. The statement "eleven is a prime number" is not a matter of learned opinion, it is simply true, as may be demonstrated by systematic proof. That is to say, the statement is true because it can be proved to be true, step by step, starting with the assumed axioms of arithmetic and applying the standard rules of logic at each point in the argument.

The end result is thus beyond any doubt.

The utterly shocking import of Gödel's theorem, and the work of Emil Post and Alan Turing that flowed from it, is that the mighty edifice of mathematics is ultimately built on

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because the nexus between proof and truth is demonstrably shaky. The problem that Gödel uncovered is that in mathematics, and in fact in almost all formal systems of reasoning, statements can be true yet unprovable — not just unproved, but unprovable, even in principle. Mathematical propositions can no longer be [seen] as a colossal list of statements to which yes-no answers may always be appended by exhaustive systematic investigation; rather, some of the propositions may be intrinsically undecidable, thus demolishing the concept of a closed, consistent and complete body of rules and objects.

Incompleteness is unavoidable.

The concept of absolute truth, even in the orderly world of mathematics, [...] was dealt a shattering blow by Gödel's work.