CSE 380	**Discrete Mathematics II**

Wrapping Up Week Four

28 January 2021 — *As We End*

At the end (of class meetings) of this fourth week, let's recap:

GCD

Revisiting this important concept one more time:

Important?

Important enough to be a library-import away:

```
from math import gcd

print(gcd(8993287598798794850007, 5677777777777777777777))
```

Trace and Test

```
from math import floor, log10

def trace_gcd(a, b):
    r = -1
    num_digits_in_a = str(floor(log10(a)) + 1)
    num_digits_in_b = str(floor(log10(b)) + 1)
    num_digits_in_r = str(floor(log10(a % b)) + 1)
    format_specifier = '{0:' + num_digits_in_a + \
                       '} = {1:' + num_digits_in_b + \
                       '}({2}) + {3:' + num_digits_in_r + '}'
    while r:
        q = a // b
        r = a % b
        print(format_specifier.format(a, b, q, r))
        a, b = b, r
```

```
trace_gcd(87,55)
```

```
trace_gcd(89, 56)
```

```
trace_gcd(56, 89)
```

```
from contfrac import *

print(frac2contfrac(frac(89, 56)))
```

```
from math import sqrt
from contfrac import *

phi = (sqrt(5) + 1) / 2
print(phi)

phi_as_frac = frac(phi).limit_denominator(10000000)

cfr_phi_as_frac = frac2contfrac(phi_as_frac)
print(cfr_phi_as_frac)
print(all(map(lambda x: x == 1, cfr_phi_as_frac[:-1])))
```

```
trace_gcd(377, 233)
```

```
trace_gcd(899, 493)
```

Now About that Address

5/2 is at address 110, equals 2.5000000

```
trace_gcd(5, 2)
```

8/3 is at address 1101, equals 2.666666…

```
trace_gcd(8, 3)
```

11/4 is at address 11011, equals 2.7500000…

```
trace_gcd(11, 4)
```

19/7 is at address 110110, equals 2.7142857…

```
trace_gcd(19, 7)
```

30/11 is at address 1101101, equals 2.7272727…

```
trace_gcd(30, 11)
```

49/18 is at address 11011010, equals 2.7222222…

```
trace_gcd(49, 18)
```

```
from contfrac import *

print(frac2contfrac(frac(49, 18)))
```

87/32 is at address 1101101000, equals 2.7187500…

```
trace_gcd(87, 32)
```

106/39 is at address 11011010000, equals 2.7179487…

```
trace_gcd(106, 39)
```

```
print('{0:.50f}'.format(1038929163353808 / 382200680031313))
```

```
from mpmath import mp
mp.dps = 50
print(mp.fdiv(1038929163353808, 382200680031313))
```

```
from contfrac import *
from math import e

e_as_frac = frac(e).limit_denominator(10000000000000000)
print(e_as_frac, frac2contfrac(e_as_frac))
print(e_cont_frac(20))
```

```
from contfrac import *
from math import pi

pi_as_frac = frac(pi).limit_denominator(10)
print(pi_as_frac, frac2contfrac(pi_as_frac))
```

```
from contfrac import *

print(contfrac2frac([2,1,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,12,1,1,14,1,1,16,1,1,18,1,1,20,1,1,22
```

```
from mpmath import mp
mp.dps = 50
print(mp.fdiv(52061284670617417, 19152276311294112))
```

Answer Selected Questions

1

How can relations be encoded as sets of pairs?

Answer

Try it!

```
from pyprimesieve import primes
# ftmpl = first ten million primes list
ftmpl = list(primes(1, 179424674))
print(len(ftmpl))
```

```
print(ftmpl[0])
```

```
def encode_set(nset):
  return 1 if not nset else encode_set(nset[1:]) * ftmpl[nset[0]]
```

```
print(encode_set([1, 2, 3]))
```

```
R = [(1, 2), (2, 3), (1, 3)]
print(R)
```

```
pair_encodings = list(map(encode_set, R))
print(pair_encodings)
encoding_of_R = encode_set(pair_encodings)
print(encoding_of_R)
```

2

Why is an ordered set which is reflexive, antisymmetric, and transitive called a partial order? If all the elements preceding a given element are less than in comparable value and a set can have no duplicates (meaning if x and y as elements of set S are equal by x = y, then they are duplicates), why is it called a partial order? It sounds to me more like a total order - a complete order. Incomplete is a synonym to partial, the opposite of complete.

Answer

What we have here is a category error.

A Partial Order is a **relation**, not a set.

A poster-child poset is the pair (P(S), $\subseteq$). For example

Given S = {1, 2, 3, 4}, P(S) = Power Set of S = {∅, {1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}}

The nodes of the Hasse Diagram of this poset (you supply the links):

```
             {1, 2, 3, 4}



{1, 2, 3}  {1, 2, 4}  {1, 3, 4}  {2, 3, 4}



 {1, 2} {1, 3} {1, 4} {2, 3} {2, 4} {3, 4}



           {1} {2} {3} {4}



                   ∅
```

3

From the reading we learn that a Gödel hash can uniquely identify the original object. Does this mean that most hashes are larger than their original objects?

Answer

It depends on the prime map and the elements being hashed.

4

As defined in the book "An equivalence relation is a binary relation that is reflexive, symmetric, and transitive." This being the case, if a relation is symmetric and transitive, would it not also **have** to be reflexive? Why or why not?

Answer

The is-sibling-of relation is symmetric and transitive, but not reflexive.

5

In the book an example is given of a relation $R_1 = \{(1, 1)\}$, which is characterized as both symmetric and antisymmetric.

With the standard meaning of the word symmetry it's hard to envision how something could be both symmetric and antisymmetric. Are the only relations that possess both properties those in which $x = y$?

Answer

Yes. In fact, any subset of the identity relation possesses both.

(Note: The empty relation vacuously does too.)

6

What, exactly, is the prime basis of a poset?

According to the reading, the prime basis is a subset of the poset. Also, "A poset has a prime basis if every (non-bottom) element has a unique decomposition as the least upper bound of a finite number of basis elements." Is this saying that each element in the poset has a unique decomposition, or each element in the prime basis has a unique decomposition?

Answer

A good example, related to the poster-child poset mentioned earlier, is given in V.B.d) *factorable power sets*: A partially ordered power set (P(S),) where the order is inclusion is easily factorable: its prime basis, $B_{P(S)}$, consists of the singleton sets over S: $\{\{s\} : s \in S\}$

7

When considering binary relations, does a partial order always imply divisibility? If so, is this the reason Gödel hashes are considered partial order-preserving?

Answer

"Divides" (e.g. $3 \mid 12$) is a partial order. It can profitably be thought of when pondering the abstract nature of these objects.

For Your Consideration

Feature Column of the American Mathematical Society

Monthly essays on mathematical topics:

Trees, Teeth, and Time: The mathematics of clock making

This link to the original AMS site is broken sometimes, so here's a PDF version someone captured:

`https://gaurish4math.files.wordpress.com/2016/10/feature-column-from-the-ams.pdf`