CSE 380	**Discrete Mathematics II**

About Infinite Sets

January 10, 2021 *Think*

"What about infinite sets?"

Recall last week that question was left hanging.

Computer representation of sets in general has several intriguing aspects. *Infinite* sets, whose members of course can be neither stored nor listed in their entirety, are problematic, but they *can* be represented in part, as for example, the set of all integers:

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

An alternate representation that requires only one ellipsis is

$$\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \ldots\}$$

which has the disadvantage of not being ordered by magnitude (unless it is by *absolute value*).

Or, a few magnitude-ordered one-sided infinite sets:

Primes $= \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \ldots\}$.

Fibonacci $= \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots\}$.

TwoPowers $= \{2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 \ldots\}$.

The empty set is easily represented as $\emptyset$ (or $\{\}$), and while of course not infinite itself, it can be the result of a set operation on two infinite sets, e.g., the intersection of the Evens and the Odds.

Imagination and creativity help us find ways to represent and manipulate these sets and their features. There is a key insight to be grasped — to see how to determine set membership in feasible ways.

Considerations

What is a good way to print out a partial but humanly readable representation of an infinite set?

Set intersection, union, and complement are relatively straightforward, but set inclusion—determining whether or not one set is a subset of another —is not, as it seems to require iteration over all the elements of the set to be tested for inclusion in another.