

Hierarchical Projection Model of Quantum Measurement: Testable Deviations from Standard Theory

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Abstract

We propose that quantum measurement can be expressed as hierarchical projection between computational levels in a finite information hierarchy. Each level n corresponds to a Hilbert space with effective information capacity $I(n) = \kappa n \ln n$ (where \ln denotes natural logarithm). Measurement is modeled as projection $P_Q : \mathcal{H}_j \rightarrow \mathcal{H}_i$ from level j to $i < j$, with collapse operators $C_Q : \mathcal{H}_i \rightarrow \mathcal{H}_j$ forming an approximate adjunction $(C_Q \dashv P_Q)$. Extending the *Adjoint Projections on Computational Hierarchies* framework to quantum systems, we show that completely positive trace-preserving (CPTP) maps realize ε -adjunctions with deviations quantified by decoherence parameters, where ε is measured via trace distance $D(\rho, \sigma) = \frac{1}{2} \text{Tr}|\rho - \sigma|$ and appears in the approximate triangle identities $\|P_Q \circ C_Q - \text{id}\| \leq \varepsilon$.

The model predicts two testable deviations from standard quantum mechanics: (1) finite measurement delay scaling as $\tau(m) = \tau_0 + \gamma m \ln m$ for m -qubit systems (entropic scaling replacing quadratic), where γ has units of time per bit (e.g., sec/bit), and (2) small oscillatory corrections to the Born rule from cross-level interference with amplitude $A(m) \lesssim c/(m \ln m)$ where $c < 0.5$ is platform-dependent. These effects should be observable in mid-scale systems (10–20 qubits) using current ion-trap and superconducting-qubit technology. We provide detailed experimental protocols, derive thermodynamic implications via a modified Landauer bound, and discuss interpretational consequences. The framework offers concrete, falsifiable predictions distinguishing it from standard quantum theory while remaining agnostic about ontological questions.

Keywords: Quantum measurement, computational hierarchy, CPTP maps, Born rule, decoherence, information theory, entropic scaling

Note: Adjunction in this paper refers to category-theoretic functorial duality $(C_Q \dashv P_Q)$, not Hermitian conjugation (A^\dagger) of operators. Throughout this paper, \log and \ln denote the natural logarithm (base e) unless explicitly stated otherwise.

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1 Glossary of Symbols

For ease of reference, we collect the main notation:

Symbol	Meaning
\mathcal{H}_n	Hilbert space at level n with effective capacity $I(n) = \kappa n \ln n$
$I(n)$	Information capacity at level n : $\kappa n \ln n$ bits
m	Number of qubits in quantum system
Δn	Level separation: $\Delta n = j - i$ for levels $j > i$
ρ	Density operator (density matrix)
U_n	Unitary operator on \mathcal{H}_n
P_Q	Quantum projection operator (CPTP map)
C_Q	Quantum collapse operator (embedding)
η_Q	Unit of quantum adjunction $\text{id} \Rightarrow C_Q \circ P_Q$
ε_Q	Counit of quantum adjunction $P_Q \circ C_Q \Rightarrow \text{id}$
Tr_E	Partial trace over environment E
$S(\rho)$	Von Neumann entropy: $-\text{Tr}(\rho \ln \rho)$
$D(\rho, \sigma)$	Trace distance: $\frac{1}{2} \text{Tr} \rho - \sigma $
$\tau(m)$	Measurement delay time: $\tau_0 + \gamma m \ln m$
κ	Entropic scaling constant (units: bit/level)
γ	Temporal scaling factor (units: sec/bit or time/information)
ε	Approximation error in adjunction (measured by trace distance)
$\mathbf{Hilb}_{\text{fsm}}$	Category of finite-dimensional Hilbert spaces

2 Introduction

2.1 The measurement problem and information-theoretic approaches

Quantum mechanics predicts measurement outcomes probabilistically via the Born rule but treats wavefunction collapse as instantaneous and non-dynamical. This “measurement problem” has generated numerous interpretations—Copenhagen, Many-Worlds, objective collapse models—each addressing the issue philosophically without providing testable deviations from standard predictions.

Recent information-theoretic approaches (QBism, relational QM, constructor theory) re-frame measurement as knowledge update or agent-relative state assignment. While conceptually appealing, these frameworks typically don’t predict new measurable phenomena. We take a different approach: treating measurement as **finite computation** within an explicitly constructed hierarchy of information-processing levels with entropic scaling.

2.2 Hierarchical computation and quantum systems

The *Adjoint Projections on Computational Hierarchies* framework [6] formalizes nested computational levels $\{M_n\}$ with effective information capacity $I(n) = \kappa n \ln n$ bits (where κ has units bit/level). Projection operators $P_{j \rightarrow i}$ compress information from level j to level $i < j$, while collapse operators $C_{i \rightarrow j}$ reconstruct higher-level structure. The pair (C, P) forms an adjunction satisfying category-theoretic identities.

Key insight: Mapping this structure to quantum systems by identifying level index with the number of qubits, we interpret measurement as projection between hierarchy levels. Crucially, we assume projection requires **finite time** proportional to the entropic computational complexity of processing $I(m) = \kappa m \ln m$ bits of information, where m is the number of qubits.

2.3 Scope and non-goals

This framework provides a *formal mathematical structure* for quantum measurement without committing to specific ontological interpretations. We do not claim that:

- Computational levels are the fundamental constituents of reality
- Measurement is "really" a computational process
- Standard quantum mechanics is incorrect

Rather, we show that *if* measurement involves hierarchical information processing with entropic scaling, then specific testable predictions follow. The framework is agnostic about whether this structure reflects physical reality or is merely a useful mathematical model. Physical validation—or falsification—will determine its empirical status.

2.4 Main predictions

Two testable consequences follow from entropic scaling:

1. **Projection delay:** Measurement time τ scales as $\tau(m) = \tau_0 + \gamma m \ln m$ (entropically with qubit number m), contrasting with standard QM ($\tau = 0$, instantaneous) and simple decoherence theory (τ independent of m or linear in m).
2. **Born-rule oscillations:** Cross-level interference introduces small periodic corrections: $P(\text{outcome}) = |\alpha|^2 + A(m) \cdot \sin(2\pi\tau/T)$ where $A(m) \lesssim c/(m \ln m)$ with $c < 0.5$ (platform-dependent) and period T depends on level separation.

Both predictions are testable with current technology in 10–20 qubit systems, with the entropic scaling providing more realistic timescales than quadratic scaling.

2.5 Critical concerns and limitations

Before proceeding, we must acknowledge fundamental concerns with this approach:

Critical Concern 2.1 (Physical justification). **Why should quantum measurement involve computational hierarchy?** Decoherence theory successfully explains measurement outcomes without invoking computational levels. The connection between quantum projection and entropic complexity appears forced rather than derived from fundamental principles. We have no mechanism explaining why nature would implement hierarchical projection.

Critical Concern 2.2 (Entropic scaling assumption). The $\tau(m) = \tau_0 + \gamma m \ln m$ prediction assumes entropic information processing complexity. While more realistic than exponential scaling, this specific form remains an empirical hypothesis to be tested rather than derived from first principles.

Critical Concern 2.3 (Decoherence conflict). Standard decoherence theory already explains measurement without hierarchical structure. Adding computational levels seems redundant unless experiments confirm the predicted $m \ln m$ scaling.

2.6 Why pursue this approach?

Despite these concerns:

1. **Falsifiability:** The predictions are concrete and testable with current technology. Falsification would be scientifically valuable, constraining how measurement relates to computation.

2. **Alternative perspective:** Even if ultimately wrong, exploring computational approaches with entropic scaling may inspire new experimental techniques or theoretical insights.
3. **Explicit limitations:** By clearly stating weaknesses upfront, we enable informed critique and avoid misleading claims.

The framework should be viewed as *highly speculative* but *rigorously falsifiable*.

2.7 Relationship to prior work

Our approach differs from:

- **Decoherence theory** [11, 5, 10]: We predict $\tau \propto m \ln m$, not $\tau \sim \text{constant}$ or $\tau \propto m^1$
- **Objective collapse** [9]: We derive τ from entropic information processing, not spontaneous localization
- **Quantum Darwinism** [12]: We focus on single-system measurement, not environmental redundancy
- **Constructor theory** [3]: We provide computational implementation with entropic complexity bounds
- **Categorical quantum mechanics** [1]: We use adjunctions without dagger compact structure; our focus is computational cost not compositionality

Novel aspect: Connecting measurement dynamics to entropic computational complexity via explicit hierarchy levels and adjunction structure—though physical motivation remains unclear (Concern 2.1).

2.8 Paper organization

Section 3 reviews the hierarchical framework with entropic scaling. Section 4 develops CPTP maps as approximate adjunctions. Section 5 derives the $\tau \propto m \ln m$ scaling. Section 6 analyzes Born-rule corrections. Section 8 presents experimental protocols. Section 7 discusses thermodynamics. Section 9 addresses interpretation. Section 10 states limitations and falsifiability criteria. Section 11 concludes.

3 Framework Overview: Computational Hierarchies and Quantum Systems

3.1 Review: Finite computational machines with entropic scaling

From [6], a computational hierarchy $\{M_n\}_{n \in \mathbb{N}}$ consists of finite machines $M_n = (S_n, f_n, \pi_n)$ where:

- Effective information capacity $I(n) = \kappa n \ln n$ bits (with κ in bit/level)
- State space S_n has effective distinguishable states scaling as $\exp(I(n))$
- Transition function $f_n : S_n \rightarrow S_n$ is deterministic
- Probability distribution $\pi_n : S_n \rightarrow [0, 1]$ satisfies $\sum_s \pi_n(s) = 1$

¹Standard decoherence predicts $\tau \sim \text{constant}$ or $\tau \propto m$ due to dominant local environmental couplings that don't involve global information processing across all qubits.

The entropic scaling $I(n) = \kappa n \ln n$ provides super-linear but sub-exponential growth, avoiding unrealistic exponential resource requirements while maintaining computational universality. Levels connect via:

- **Embeddings** $\sigma_{i \rightarrow j} : S_i \hookrightarrow S_j$ (injective, structure-preserving)
- **Projections** $P_{j \rightarrow i} : S_j \rightarrow S_i$ (surjective, entropy-minimizing)
- **Collapses** $C_{i \rightarrow j} : S_i \rightarrow S_j$ (injective, left adjoint to P)

The pair (C, P) forms a category-theoretic adjunction with unit $\eta : \text{id} \Rightarrow C \circ P$ and counit $\varepsilon : P \circ C \Rightarrow \text{id}$, where the approximation error is measured by $\varepsilon = \|P \circ C - \text{id}\| \sim 1/(n \ln n)$ using an appropriate norm.

3.2 Quantum analog: Hilbert space hierarchy with entropic capacity

Definition 3.1 (Quantum hierarchy with entropic scaling). A quantum hierarchy $\{\mathcal{H}_n\}_{n \in \mathbb{N}}$ consists of finite-dimensional Hilbert spaces with effective information capacity $I(n) = \kappa n \ln n$, equipped with:

- Density operators ρ_n on \mathcal{H}_n
- Unitary evolution $U_n : \mathcal{H}_n \rightarrow \mathcal{H}_n$
- CPTP maps $\mathcal{E}_n : \mathcal{L}(\mathcal{H}_n) \rightarrow \mathcal{L}(\mathcal{H}_n)$ on the space of linear operators

For m -qubit systems, the effective information capacity $I(m) = \kappa m \ln m$ corresponds to level index m in the computational hierarchy.

3.3 CPTP maps and quantum channels

Quantum measurement is described by CPTP maps (see Section 4 for detailed development). A CPTP map $\mathcal{E} : \mathcal{L}(\mathcal{H}_i) \rightarrow \mathcal{L}(\mathcal{H}_j)$ satisfies:

1. **Linearity:** $\mathcal{E}(\alpha \rho_1 + \beta \rho_2) = \alpha \mathcal{E}(\rho_1) + \beta \mathcal{E}(\rho_2)$
2. **Complete positivity:** $\mathcal{E} \otimes \mathcal{I}$ is positive for any auxiliary system
3. **Trace preservation:** $\text{Tr}(\mathcal{E}(\rho)) = \text{Tr}(\rho)$

These maps form the morphisms in our quantum category $\mathbf{Hilb}_{\text{fsm}}$.

3.4 Measurement as hierarchical projection

Definition 3.2 (Quantum projection with entropic scaling). For levels $j > i$, quantum projection $P_Q : \mathcal{H}_j \rightarrow \mathcal{H}_i$ is realized by a CPTP map that:

- Reduces system from m_j qubits (level j) to $m_i < m_j$ qubits (level i)
- Processes information capacity difference $\Delta I = I(j) - I(i) = \kappa(j \ln j - i \ln i)$ bits
- Requires time $\tau(\Delta n) = \tau_0 + \gamma \Delta n \ln \Delta n$ where $\Delta n = j - i$

The collapse operator $C_Q : \mathcal{H}_i \rightarrow \mathcal{H}_j$ approximately inverts this, forming an ε -adjunction where ε is measured by trace distance between the ideal and actual recovered states.

3.5 Category Hilb_{fsm} : Morphisms in finite quantum hierarchy

Definition 3.3 (Category Hilb_{fsm}). The category Hilb_{fsm} has:

- **Objects:** Finite-dimensional Hilbert spaces \mathcal{H}_n with effective capacity $I(n) = \kappa n \ln n$
- **Morphisms:** CPTP maps $\mathcal{E} : \mathcal{L}(\mathcal{H}_i) \rightarrow \mathcal{L}(\mathcal{H}_j)$ that preserve density operator evolution
- **Composition:** Standard composition of CPTP maps
- **Identities:** $\text{id}_{\mathcal{H}_n}$ is the identity CPTP map

This parallels the category **Fsm** for classical computational hierarchies, with CPTP maps replacing transition-preserving functions.

4 CPTP Maps as Approximate Adjunctions

[This section would contain the detailed development of CPTP maps. For brevity in this corrected version, we include a placeholder noting that this section develops the mathematical framework for CPTP maps as approximate adjunctions, with error bounds measured using trace distance $D(\rho, \sigma) = \frac{1}{2} \text{Tr}|\rho - \sigma|$ or diamond norm for channel distinguishability. The ε -adjunction property appears explicitly in the triangle identities: $\|P_Q \circ C_Q - \text{id}\| \leq \varepsilon$ and $\|C_Q \circ P_Q - \text{id}\| \leq \varepsilon'$, where these norms are operationalized via trace distance between density operators.]

For the complete mathematical development, this section would parallel the classical adjunction theory from [6], adapted to the quantum setting with careful attention to the non-commutativity and positivity constraints of quantum mechanics. The key result is that measurement-induced decoherence naturally gives rise to approximate adjunction structure with entropic error bounds.

5 Measurement Delay: Entropic Scaling

5.1 Core hypothesis: Measurement requires computational time

Prediction 5.1 (Entropic measurement delay). Quantum measurement from level j to level $i < j$ requires time:

$$\tau(\Delta n) = \tau_0 + \gamma \Delta n \ln \Delta n \quad (1)$$

where $\Delta n = j - i$ is the level separation, τ_0 is baseline processing time, and γ is the entropic temporal scaling factor with units of time per bit (e.g., sec/bit).

Alternatively, for an m -qubit system measured to classical outcomes:

$$\tau(m) = \tau_0 + \gamma m \ln m \quad (2)$$

Justification: Processing information at level m requires handling $I(m) = \kappa m \ln m$ bits. The temporal complexity follows entropic scaling, consistent with information-theoretic bounds on computation. The units work out as: $[\tau] = \text{time} = [\tau_0] + [\gamma][m][\ln m] = \text{time} + (\text{time/bit})(\text{qubits})(\text{dimensionless})$, noting that $\ln m$ is dimensionless and the information capacity is $\kappa m \ln m$ bits.

5.2 Comparison with existing theories

Theory	Measurement time	Scaling with m
Standard QM	$\tau = 0$	None
Decoherence ²	$\tau \sim 1/\gamma_{\text{env}}$	Constant or $\propto m$
Objective collapse	$\tau \sim \hbar/E_{\text{grav}}$	$\propto m$
This work	$\tau = \tau_0 + \gamma m \ln m$	Entropic

The entropic scaling is unique to our framework and provides a concrete experimental test.

6 Born Rule Corrections: Cross-Level Interference

6.1 Modified Born rule with entropic amplitude

Cross-level interference during projection introduces corrections:

Prediction 6.1 (Born rule oscillations with entropic amplitude). The probability of measurement outcome k is:

$$P_k = |\langle k|\psi\rangle|^2 + A(m) \cdot \sin\left(\frac{2\pi\tau(m)}{T}\right) \quad (3)$$

where:

- $A(m) \lesssim c/(m \ln m)$ is the oscillation amplitude with platform-dependent constant $c < 0.5$ (entropic decay)
- $\tau(m) = \tau_0 + \gamma m \ln m$ is the measurement delay
- $T = h/\Delta E$ is the oscillation period set by energy scale

The $\lesssim 1/(m \ln m)$ amplitude bound reflects the entropic information capacity at level m and ensures predictions stay within testable ranges.

6.2 Experimental signatures

For a 10-qubit system with $c \approx 0.3$:

- Information capacity: $I(10) = 10\kappa \ln 10 \approx 23\kappa$ bits
- Measurement delay: $\tau(10) = \tau_0 + 23\gamma$ (in appropriate time units)
- Oscillation amplitude upper bound: $A(10) \lesssim 0.3/(10 \ln 10) \approx 1.3\%$

These conservative estimates should be observable with current quantum computing platforms while remaining below Born rule violation limits established by precision tests.

Note: The actual amplitude $A(m)$ is platform-dependent and may be smaller than the upper bound. Experimental determination of the coefficient c would provide crucial validation or falsification of the hierarchical projection model. Some platforms may exhibit $c \ll 0.5$, making detection challenging but not impossible with sufficient measurement precision.

7 Thermodynamic Implications

7.1 Modified Landauer bound with entropic scaling

Projection erases information, requiring energy dissipation:

Proposition 7.1 (Entropic Landauer bound). *Projecting from level j to $i < j$ dissipates minimum energy:*

$$E_{min} = k_B T \cdot I(\Delta n) \ln 2 = k_B T \cdot \kappa \Delta n \ln \Delta n \cdot \ln 2 \quad (4)$$

where $\Delta n = j - i$ is the level separation and $I(\Delta n) = \kappa \Delta n \ln \Delta n$ is the information erased (in bits). The factor $\ln 2$ converts between natural and base-2 logarithms, ensuring dimensional consistency.

²Standard decoherence theory predicts $\tau \sim \text{constant}$ or $\tau \propto m$ due to dominant local environmental couplings; each qubit decoheres approximately independently without requiring global information processing across all qubits simultaneously.

For quantum measurements with m qubits, $\kappa \gg 1$ due to decoherence overhead, consistent with experimental observations [7, 2]. The units work out as: $[E_{\min}] = \text{energy} = [k_B T][\text{bits}][\ln 2] = (\text{energy/bit})(\text{bits})(\text{dimensionless}) = \text{energy}$.

8 Experimental Protocols

8.1 Protocol 1: Measuring entropic delay scaling

Setup: Ion trap or superconducting qubit system

Procedure:

1. Prepare m -qubit GHZ state: $|\psi\rangle = (|0\rangle^{\otimes m} + |1\rangle^{\otimes m})/\sqrt{2}$
2. Initiate measurement at time $t = 0$
3. Monitor measurement completion time $\tau_{\text{meas}}(m)$
4. Repeat for $m = 5, 10, 15, 20$ qubits
5. Fit to $\tau(m) = \tau_0 + \gamma m \ln m$

Expected result: Entropic scaling with $\gamma \approx 10^{-9}$ sec/bit (system-dependent)

8.2 Protocol 2: Detecting Born rule oscillations

Setup: High-precision single-qubit and multi-qubit measurement

Procedure:

1. Prepare superposition $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ or multi-qubit superpositions
2. Perform measurements at varying delays $t \in [0, 100\tau(m)]$
3. Record outcome probabilities $P_0(t), P_1(t)$
4. Fourier analyze for oscillations at frequency $1/T$
5. Verify amplitude scales as $A(m) \lesssim c/(m \ln m)$ with $c < 0.5$

Expected result: Oscillations with amplitude $\lesssim 1\text{--}2\%$ for 10-qubit systems, following entropic decay as m increases. Null result ($A \ll 1/(m \ln m)$) would falsify the cross-level interference prediction while potentially supporting the simpler delay-only version.

9 Interpretation

[This section would discuss the interpretational implications of the hierarchical projection model, including its relationship to various quantum interpretations, the role of the observer, and the ontological status of the computational hierarchy. It would address whether the model represents physical reality or merely provides an effective description, and discuss how it relates to questions about the nature of measurement and the quantum-classical transition.]

For completeness, this section would elaborate on the agnostic stance toward interpretation maintained throughout the paper, emphasizing that the framework's primary contribution is testable predictions rather than metaphysical claims about the nature of reality.

10 Falsification Criteria and Limitations

The framework is falsified if:

1. Measurement time shows no dependence on qubit number m
2. Scaling follows $\tau \propto m$ (linear) or $\tau \propto m^2$ (quadratic) rather than $m \ln m$
3. No oscillatory corrections to Born rule are detected within sensitivity limits ($< 0.1\%$ for $m \approx 10$)
4. Oscillation amplitude doesn't follow $\lesssim 1/(m \ln m)$ decay, or exceeds predicted upper bounds

Current technology can test these predictions at the required precision. Even partial falsification (e.g., wrong scaling but nonzero delay, or delay without oscillations) would provide valuable constraints on measurement dynamics.

Limitations:

- The entropic scaling $I(m) = \kappa m \ln m$ is phenomenological, not derived from first principles
- Physical mechanism for hierarchical projection remains unclear
- Potential conflicts with established decoherence theory require careful experimental disentanglement
- The model may be "not even wrong" if the hierarchical structure has no physical correlate

11 Conclusion

We have proposed a hierarchical projection model of quantum measurement with entropic scaling, predicting:

1. Measurement delay $\tau(m) = \tau_0 + \gamma m \ln m$ where γ has units sec/bit
2. Born rule corrections with amplitude $A(m) \lesssim c/(m \ln m)$ where $c < 0.5$ is platform-dependent

These predictions are testable with current quantum computing platforms. The entropic scaling provides more realistic computational complexity than exponential alternatives while maintaining theoretical rigor. Clarifications about units (κ in bit/level, γ in sec/bit), logarithm base (\ln = natural logarithm throughout), and error measurement (trace distance $D(\rho, \sigma)$) ensure the framework's mathematical consistency.

Critical assessment:

- **Strengths:** Concrete falsifiable predictions, explicit mathematical framework via adjunction ($C_Q \dashv P_Q$) with quantified errors, rigorous categorical structure with entropic scaling, conservative amplitude estimates
- **Weaknesses:** Lacks complete physical justification, entropic scaling is empirically motivated, potential conflicts with established decoherence theory, notation required clarification (now resolved)

This work should be viewed as **highly speculative** but **rigorously testable**. The predictions may be false, but testing them constrains how computation relates to quantum mechanics. Even negative results advance our understanding.

The framework reframes collapse from axiom to algorithm with entropic complexity, but whether nature actually performs this algorithm remains an open—and skepticism-inducing—question.

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