

# Hierarchical Projection Model of Quantum Measurement: Effective Decoherence in Multi-Qubit Systems

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## Abstract

We present an effective model of decoherence in multi-qubit quantum systems using hierarchical projection formalism from computational theory. The framework models measurement as projection  $P_Q : \mathcal{H}_j \rightarrow \mathcal{H}_i$  between computational levels, with reconstruction operators  $C_Q : \mathcal{H}_i \rightarrow \mathcal{H}_j$  forming an approximate adjunction  $(C_Q \dashv P_Q)$  in the category-theoretic sense. All evolution is described through completely positive trace-preserving (CPTP) maps consistent with standard quantum mechanics.

**Important disclaimer:** This model does not modify the postulates of quantum mechanics. Rather, it provides an effective description of decoherence timescales and transient coherence effects in hierarchically structured quantum systems. The "collapse" terminology refers to effective diagonalization via environmental decoherence, not a fundamental process.

The model predicts: (1) effective decoherence time scaling as  $\tau(m) \approx \tau_0 + \gamma m \ln m$  for  $m$ -qubit systems, arising from cumulative environmental coupling in complex registers; and (2) transient oscillatory deviations from Born-rule probabilities during the decoherence process, with amplitude  $A(m) \lesssim c/(m \ln m)$  where  $c < 0.5$  is platform-dependent. Importantly, final measurement outcomes remain Born-rule compliant after complete decoherence.

These effects should be observable in mid-scale systems (10–20 qubits) using current technology. The framework offers testable predictions about decoherence dynamics while remaining fully consistent with standard quantum theory.

**Keywords:** Quantum measurement, decoherence, CPTP maps, computational hierarchy, information theory, entropic scaling

**Note:** Adjunction refers to category-theoretic functorial duality  $(C_Q \dashv P_Q)$ , not Hermitian conjugation ( $A^\dagger$ ). Throughout, log and ln denote the natural logarithm unless stated otherwise.

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# 1 Glossary of Symbols

For ease of reference, we collect the main notation:

Symbol	Meaning
$\mathcal{H}_n$	Hilbert space at level $n$ with effective capacity $I(n) = \kappa n \ln n$
$I(n)$	Information capacity at level $n$ : $\kappa n \ln n$ bits
$m$	Number of qubits in quantum system
$\Delta n$	Level separation: $\Delta n = j - i$ for levels $j > i$
$\rho$	Density operator (density matrix)
$U_n$	Unitary operator on $\mathcal{H}_n$
$P_Q$	Quantum projection (CPTP map modeling effective measurement)
$C_Q$	Reconstruction operator (embedding)
$\eta_Q$	Unit of adjunction $\text{id} \Rightarrow C_Q \circ P_Q$
$\varepsilon_Q$	Counit of adjunction $P_Q \circ C_Q \Rightarrow \text{id}$
$\text{Tr}_E$	Partial trace over environment $E$
$S(\rho)$	Von Neumann entropy: $-\text{Tr}(\rho \ln \rho)$
$D(\rho, \sigma)$	Trace distance: $\frac{1}{2}\text{Tr} \rho - \sigma $
$\tau(m)$	Effective decoherence time: $\tau_0 + \gamma m \ln m$
$\kappa$	Entropic scaling constant (units: bit/level)
$\gamma$	Temporal scaling factor (units: sec/bit)
$\varepsilon$	Approximation error in adjunction (measured by trace distance)
$\mathbf{Hilb}_{\text{fsm}}$	Category of finite-dimensional Hilbert spaces

## 2 Introduction

### 2.1 Decoherence and information-theoretic approaches

Standard quantum mechanics describes measurement outcomes via the Born rule, treating wavefunction reduction as instantaneous. However, realistic measurements involve finite-time decoherence processes wherein environmental interactions drive the system toward diagonal (classical) states. Understanding the timescales and dynamics of this decoherence is crucial for quantum computing and foundational physics.

Recent information-theoretic approaches (QBism, relational QM, constructor theory) reframe measurement as knowledge update or agent-relative state assignment. We adopt a complementary perspective: modeling decoherence as information processing within an explicitly structured computational hierarchy with entropic scaling.

### 2.2 Hierarchical computation and quantum systems

The *Adjoint Projections on Computational Hierarchies* framework [6] formalizes nested computational levels  $\{M_n\}$  with effective information capacity  $I(n) = \kappa n \ln n$  bits. Projection operators  $P_{j \rightarrow i}$  compress information from level  $j$  to  $i < j$ , while reconstruction operators  $C_{i \rightarrow j}$  recover higher-level structure. The pair  $(C, P)$  forms a category-theoretic adjunction.

**Key modeling assumption:** We map this structure to quantum systems by identifying level index with qubit number. Decoherence (effective measurement) is modeled as projection between levels, requiring finite time proportional to entropic information capacity  $I(m) = \kappa m \ln m$  bits for  $m$  qubits.

### 2.3 Scope and relationship to standard QM

This framework provides an *effective model* of decoherence dynamics within standard quantum mechanics. We explicitly do **not** claim:

- To modify the postulates or axioms of quantum mechanics
- That computational levels are fundamental constituents of reality
- That measurement involves non-standard physics beyond CPTP evolution

Rather, we model decoherence in multi-qubit systems using hierarchical information-processing concepts. The resulting timescales and transient effects are predictions about *how standard decoherence manifests* in complex quantum registers, not about new physics.

## 2.4 Main predictions

Two observable consequences follow from entropic scaling in decoherence:

1. **Effective decoherence time:** The characteristic timescale for environmental decoherence in  $m$ -qubit systems follows  $\tau(m) \approx \tau_0 + \gamma m \ln m$  due to cumulative environmental coupling. This contrasts with simple models where  $\tau$  is constant or linear in  $m$ .
2. **Transient coherence oscillations:** Before complete decoherence, probability distributions may exhibit transient oscillations with amplitude  $A(m) \lesssim c/(m \ln m)$ , where  $c < 0.5$  is platform-dependent. **Crucially, final probabilities after decoherence remain Born-rule compliant.**

Both predictions are testable with current technology in 10–20 qubit systems.

## 2.5 Critical concerns and limitations

Before proceeding, we acknowledge fundamental limitations:

*Critical Concern 2.1* (Physical justification). The entropic scaling  $I(m) = \kappa m \ln m$  is phenomenological, motivated by computational complexity arguments rather than derived from microscopic Hamiltonians. While plausible for certain coupling topologies, it requires empirical validation.

*Critical Concern 2.2* (Relationship to established decoherence theory). Standard decoherence theory (Zurek, Joos, Schlosshauer) successfully explains measurement without invoking hierarchical levels. Our model must be consistent with established results or clearly identify where it makes distinct predictions. The  $m \ln m$  scaling may emerge from specific environmental coupling structures but needs microscopic justification.

*Critical Concern 2.3* (Testability vs. parameter freedom). The model contains phenomenological parameters ( $\gamma, c$ ) that could be fitted post-hoc. True falsification requires predicting functional forms (especially the  $m \ln m$  dependence) independent of experimental data.

These concerns motivate our emphasis on *falsifiable predictions* of scaling behavior rather than philosophical claims about measurement.

## 3 Mathematical Framework

### 3.1 CPTP maps and effective measurement

All quantum evolution in our framework is described by completely positive trace-preserving (CPTP) maps, the standard formalism for open quantum systems. A CPTP map  $\mathcal{E} : \mathcal{B}(\mathcal{H}_A) \rightarrow \mathcal{B}(\mathcal{H}_B)$  can be expressed in Kraus form:

$$\mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger, \quad \sum_k E_k^\dagger E_k = I$$

We model effective measurement as a CPTP map  $P_Q : \mathcal{H}_j \rightarrow \mathcal{H}_i$  that produces decoherence through environmental interaction. The "projection" terminology reflects the map's effect of reducing off-diagonal coherences, not a claim about instantaneous collapse.

**Definition 3.1** (Effective measurement via partial trace). Given system-environment initial state  $\rho_{SE}$ , effective measurement is:

$$P_Q(\rho_S) = \text{Tr}_E[U_{SE}(\rho_S \otimes \rho_E)U_{SE}^\dagger]$$

where  $U_{SE}$  is unitary system-environment evolution and  $\text{Tr}_E$  is partial trace over the environment. This is manifestly a CPTP map within standard QM.

### 3.2 Collapse as effective diagonalization

What we term "collapse" refers to the process by which off-diagonal elements of  $\rho$  in the measurement basis decay to zero through environmental decoherence. Formally, for measurement in basis  $\{|i\rangle\}$ :

$$\rho(t=0) = \sum_{i,j} \rho_{ij} |i\rangle\langle j| \xrightarrow{\text{decoherence}} \rho(t \gg \tau) \approx \sum_i \rho_{ii} |i\rangle\langle i|$$

This is *not* a fundamental process but an effective description of standard decoherence dynamics described by Lindblad or Kraus formalisms.

### 3.3 Adjunction as formal reconstruction

Given projection  $P_Q$ , we define a reconstruction map  $C_Q : \mathcal{H}_i \rightarrow \mathcal{H}_j$  that formally inverts the projection in a computational sense. The pair  $(C_Q \dashv P_Q)$  forms an approximate adjunction with error  $\varepsilon$ :

**Definition 3.2** ( $\varepsilon$ -adjunction). Maps  $C_Q$  and  $P_Q$  form an  $\varepsilon$ -adjunction if:

$$D(P_Q \circ C_Q, \text{id}_i) \leq \varepsilon \tag{1}$$

$$D(C_Q \circ P_Q, \text{id}_j) \leq \varepsilon \tag{2}$$

where  $D(\rho, \sigma) = \frac{1}{2}\text{Tr}|\rho - \sigma|$  is the trace distance.

**Important clarification:** The reconstruction map  $C_Q$  is *not* a physical reverse measurement. It is a formal computational construct useful for analyzing information flow in the hierarchy. Physical irreversibility is captured by  $\varepsilon > 0$ .

## 4 Effective Decoherence Time

### 4.1 Entropic scaling from environmental coupling

Consider an  $m$ -qubit system with information capacity  $I(m) = \kappa m \ln m$  bits. In a hierarchically structured quantum register, qubits at different "levels" experience different environmental coupling strengths. The effective decoherence time arises from summing contributions across these coupling channels.

**Proposition 4.1** (Effective decoherence timescale). *For an  $m$ -qubit system with hierarchical environmental coupling, the characteristic decoherence time is:*

$$\tau(m) \approx \tau_0 + \gamma m \ln m$$

where  $\tau_0$  is a baseline hardware-dependent timescale and  $\gamma$  (units: sec/bit) parametrizes the coupling strength per information unit.

**Physical interpretation:** This is *not* a fundamental law but an effective description. In systems where environmental coupling follows a logarithmic hierarchy (e.g., nearest-neighbor interactions in a tree topology with depth  $\ln m$ ), the total decoherence time accumulates as  $m \ln m$ . This is a phenomenological model requiring microscopic validation.

## 4.2 Contrast with standard models

Standard decoherence models typically predict:

- **Constant  $\tau$ :** If environment couples uniformly to all qubits independently
- **Linear  $\tau \propto m$ :** If decoherence is dominated by sequential processes
- **Sublinear  $\tau \propto m^\alpha$  with  $\alpha < 1$ :** For certain collective decoherence modes

The  $m \ln m$  scaling emerges from a specific ansatz about hierarchical coupling topology. Experimental observation of this scaling would validate the hierarchical structure; deviation would falsify it.

# 5 Transient Coherence Effects

## 5.1 Oscillations during decoherence

During the decoherence process (for  $t \lesssim \tau(m)$ ), the system has not yet reached the fully diagonal state. In this transient regime, interference between coherent and decohered components can produce small oscillations in measurement probabilities.

**Proposition 5.1** (Transient probability oscillations). *For times  $t \lesssim \tau(m)$ , measurement probabilities may exhibit:*

$$P(\text{outcome}) \approx |\alpha|^2 + A(m) \cdot \sin(2\pi t/T) + O(A^2)$$

where  $A(m) \lesssim c/(m \ln m)$  with  $c < 0.5$  platform-dependent, and  $T$  is determined by system-environment coupling parameters.

**Critical point:** These oscillations are *transient effects during decoherence*. After full decoherence ( $t \gg \tau(m)$ ), probabilities converge to Born-rule values:

$$P(\text{outcome}) \rightarrow |\alpha|^2 \quad (\text{Born rule})$$

This prediction does **not modify** the Born rule. It predicts observable transient dynamics during the decoherence process itself.

## 5.2 Physical origin

The oscillations arise from coherent evolution during partial decoherence. As off-diagonal elements decay according to:

$$\rho_{ij}(t) = \rho_{ij}(0)e^{-\Gamma_{ij}t}e^{i\omega_{ij}t}$$

the interference between decaying and surviving coherences produces oscillatory corrections to instantaneous measurement statistics. The amplitude  $A(m)$  reflects the fraction of coherence surviving at the measurement timescale.

## 5.3 Relationship to standard decoherence theory

This prediction is consistent with standard theory. Decoherence is not instantaneous; during the transition, transient effects are expected. What our model adds is a specific scaling prediction:  $A(m) \sim 1/(m \ln m)$  based on information-theoretic arguments about hierarchical systems.

## 6 Thermodynamic Considerations

### 6.1 Landauer's principle and measurement cost

Landauer's principle states that erasing one bit of information requires dissipating at least  $k_B T \ln 2$  of energy. Quantum measurement involves effective information erasure (loss of coherence), suggesting a thermodynamic cost.

We estimate the energy cost of decoherence as:

$$E_{\text{decohere}} \sim \kappa k_B T \ln 2 \cdot \Delta S$$

where  $\Delta S = j - i$  is the information difference (in bits) between levels and  $\kappa$  is an order-unity efficiency factor.

**Important caveat:** This is a heuristic estimate based on information-theoretic analogy, not a fundamental derivation. The actual energy dissipation depends on microscopic details of the system-environment coupling. We do **not** claim this as a modification of Landauer's principle, merely an application to estimate measurement energetics.

### 6.2 Consistency with thermodynamic bounds

Standard thermodynamic bounds require:

$$\Delta F \geq k_B T \ln 2 \quad (\text{per bit erased})$$

Our estimate is consistent with this bound, providing an upper estimate rather than a tight constraint. Precise validation requires detailed modeling of the measurement apparatus and environment.

## 7 Experimental Protocols

### 7.1 Protocol 1: Measuring decoherence timescales

**Setup:** Multi-qubit quantum processor (ion trap or superconducting qubits)

**Procedure:**

1. Prepare  $m$ -qubit superposition state (e.g., GHZ:  $|0\rangle^{\otimes m} + |1\rangle^{\otimes m}$ )
2. Initiate environmental coupling (begin measurement process)
3. Monitor coherence decay via off-diagonal density matrix elements
4. Extract characteristic decoherence time  $\tau_{\text{decoh}}(m)$
5. Repeat for  $m = 5, 10, 15, 20$  qubits
6. Fit to  $\tau(m) = \tau_0 + \gamma m \ln m$  and compare with linear/quadratic alternatives

**Expected result:** If hierarchical coupling dominates,  $\gamma \approx 10^{-9}$  sec/bit (order of magnitude, platform-dependent). Linear or constant scaling would falsify the model.

### 7.2 Protocol 2: Detecting transient oscillations

**Setup:** High-precision time-resolved quantum state tomography

**Procedure:**

1. Prepare superposition  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

2. Initiate measurement at  $t = 0$
3. Perform rapid repeated measurements at intervals  $\Delta t \ll \tau(m)$
4. Record time-dependent probabilities  $P_0(t), P_1(t)$
5. Fourier analyze for oscillatory components with frequency  $1/T$
6. Verify amplitude scaling:  $A(m) \lesssim c/(m \ln m)$  with  $c < 0.5$

**Expected result:** Small ( $\sim 1\%$ ) oscillations for 10-qubit systems during decoherence, decaying as  $1/(m \ln m)$ . **Final probabilities after decoherence must match Born rule.** Absence of oscillations or wrong amplitude scaling falsifies the prediction.

**Critical control:** Verify that oscillations vanish for  $t \gg \tau(m)$  and that time-averaged probabilities match Born rule predictions.

## 8 Interpretation and Physical Meaning

### 8.1 Effective vs. fundamental description

This model is an *effective field theory* of decoherence, not a fundamental modification of quantum mechanics. It provides useful heuristics for:

- Estimating decoherence timescales in multi-qubit systems
- Understanding information flow in hierarchical quantum architectures
- Connecting computational complexity to physical timescales

It does **not** claim to reveal fundamental ontology of measurement or consciousness-related phenomena.

### 8.2 Relationship to other interpretations

The framework is *interpretation-neutral*. It is compatible with:

- **Copenhagen:** Models the collapse process as effective decoherence
- **Many-Worlds:** Describes branch decoherence timescales
- **Objective collapse:** Could be reinterpreted as effective description of collapse dynamics
- **QBism:** Models agent's information update timescales

The predictions are orthogonal to interpretational disputes about the "reality" of wavefunction collapse.

## 9 Falsification Criteria and Limitations

### 9.1 Clear falsification conditions

The framework is falsified if:

1. Decoherence time shows no systematic dependence on qubit number  $m$
2. Scaling follows  $\tau \propto m$  (linear) or  $\tau \propto m^2$  (quadratic), not  $m \ln m$
3. No transient oscillations detected within sensitivity limits ( $< 0.1\%$  for  $m \approx 10$ )

4. Oscillation amplitude doesn't follow  $\sim 1/(m \ln m)$  decay, or persists after decoherence
5. Final measurement probabilities deviate from Born rule by more than  $10^{-6}$  (experimental precision)

Current quantum computing technology can test these predictions.

## 9.2 Acknowledged limitations

*Limitation 9.1* (Phenomenological scaling). The  $m \ln m$  scaling is not derived from first principles but assumed based on computational complexity heuristics. Microscopic derivation from specific Hamiltonians is needed.

*Limitation 9.2* (Parameter freedom). The model contains adjustable parameters ( $\gamma, c, \tau_0$ ) that could be fitted to data. True validation requires predicting the functional form (specifically  $m \ln m$ ) before measurement, then testing parameter values.

*Limitation 9.3* (Scope). The model applies to decoherence in multi-qubit registers with hierarchical coupling structure. It may not apply to systems with uniform environmental coupling or fundamentally different topologies.

## 10 Conclusion

We have presented an effective model of decoherence in multi-qubit quantum systems using hierarchical projection formalism. The model makes two testable predictions within standard quantum mechanics:

1. **Decoherence timescale:**  $\tau(m) \approx \tau_0 + \gamma m \ln m$  arising from cumulative environmental coupling in hierarchical quantum registers
2. **Transient oscillations:** Probability oscillations with amplitude  $A(m) \lesssim c/(m \ln m)$  during decoherence, vanishing asymptotically with Born-rule-compliant final probabilities

### Critical assessment:

- **Strengths:** Falsifiable predictions, consistent with QM postulates, rigorous CPTP formalism, testable with current technology
- **Weaknesses:** Phenomenological scaling assumptions, limited microscopic justification, parameter freedom requires careful experimental design

This work should be viewed as a **testable effective model**, not a fundamental theory. The predictions constrain how decoherence manifests in complex quantum systems. Whether the hierarchical information-processing picture provides accurate effective description remains an empirical question.

The framework reframes decoherence timescales using computational complexity concepts, but whether nature actually implements this structure—or whether it merely provides useful computational heuristics—requires experimental validation.

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