A Study of a 2x2 MIMO-OFDM System

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***Abstract*—This document serves to explain how MIMO and OFDM function and how they perform. The performance of both techniques was observed using different receiver techniques such as MMSE and ZF equalizers and additionally channel precoding and receiver shaping for MIMO.**

***Index Terms*—OFDM, MIMO, MMSE Equalizer, ZF Equalizer, SVD**

I. Introduction

THIS report covers the principles of how MIMO and OFDM systems function. It also serves to show how different transmitter and receiver techniques can be implemented. The principles of MIMO using precoding and receiver shaping, an minimum-mean square error (MMSE) equalizer, and a zero forcing (ZF) equalizer will be covered. Afterwards OFDM will be covered along with how an MMSE and ZF equalizer are used in this case. Following this the results for MIMO, OFDM, and MIMO-OFDM systems will be shown functioning in different channels.

II. MIMO

Multiple-Input Multiple-Output systems are the present and the future. They allow wireless communications systems to increase performance in various ways such as increasing SNR and data rate. In the simulations performed in this paper the main benefit that was being sought was an increase in data rate.

The general layout of a MIMO system is as follows: given M_t transmit antennas and M_r receive antennas, the data vector being sent over the channel $x \in \mathbb{C}^{M_t \times 1}$ is received as the vector $y \in \mathbb{C}^{M_r \times 1}$. The relation is given by:

$$y = Hx + n \tag{1}$$

where $H \in \mathbb{C}^{M_r \times M_t}$ is the channel matrix and $n \in \mathbb{C}^{M_r \times 1}$ is AWGN noise. There are several ways to recover the sent data x given y. The following sections explain some of these methods.

A. Precoding and Receiver Shaping

Precoding and receiver shaping is a technique that can be used given channel state information at the transmitter (CSIT) and channel state information at the receiver (CSIR), *i.e.* H is known at both ends. H can be decomposed using the singular value decomposition (SVD) as:

$$H = U\Sigma V^H \tag{2}$$

where $U \in \mathbb{C}^{M_r \times M_r}$, $\Sigma \in \mathbb{C}^{M_r \times M_t}$, $V \in \mathbb{C}^{M_t \times M_t}$, and $()^H$ denoted the Hermitian transpose. U and V are both unitary matrices and Σ is a diagonal matrix with real non-negative entries that are the singular values, or, in context of signal processing, the channel gains. With this decomposition, we can precode data $\tilde{x}$ as:

$$x = V\tilde{x} \tag{3}$$

After precoding, x is sent through the channel and is recieved. At the receiver, receiver shaping is performed and the sent data is approximated as $\tilde{y}$, given by:

$$\begin{aligned}\tilde{y} &= U^H(Hx + n)\\ &= U^H(U\Sigma V^H V\tilde{x} + n)\\ &= \Sigma\tilde{x} + U^H n\\ &= \Sigma\tilde{x} + \tilde{n}\end{aligned} \tag{4}$$

Applying U^H to the noise n does not change its power as U is unitary. This means that by applying precoding and receiver shaping leads to an SNR increase due to the channel gains encoded in Σ. It is useful to think of this not as the signal power increasing but rather the noise power decreasing.

B. MMSE and ZF Equalizers

For MMSE and ZF equalizers CSIR is necessary. The main idea behind both of the them is to, in effect, apply the inverse of the channel. The ZF equalizer is given by:

$$W_{ZF} = (H^H H)^{-1} H^H = H^\dagger \tag{5}$$

where $()^\dagger$ is the Moore-Penrose psuedoinverse of a matrix. This equalizer is applied as so:

$$\tilde{y} = W_{ZF}(Hx + n) \tag{6}$$

The problem with the ZF equalizer is that at some frequencies the noise present in the channel is stronger than the attenuated signal. This means that ZF equalizer will in effect boost the noise which is not desirable. This issue is addressed in the MMSE equalizer which is given by:

$$W_{MMSE} = (H^H H + \sigma_n^2 I)^{-1} H^H \tag{7}$$

where σ_n^2 is the noise variance. It is applied in the same way as the ZF equalizer. Having the extra $\sigma_n^2 I$ term prevents noise from being amplified by a large amount at frequencies where the signal is attenuated a lot by the channel.

III. OFDM

Orthogonal Frequency Division Multiplexing is a technique that uses multiple carriers, the IFFT, and a cyclic prefix to counteract the affects of frequency selective channels that arise from inter symbol interference (ISI). Here we will explore how the 802.11a WiFi standard generates an OFDM symbol and then a different form of the MMSE and ZF equalizers.

A. OFDM in 802.11a

The 802.11a standard supports four different types of modulation, namely 2, 4, 16, and 64 QAM. It also supports three different coding rates $\frac{1}{2}$. $\frac{2}{3}$, and $\frac{3}{4}$. It doesn't support all 12 combinations of modulation and rate, but all are possible. To generate one OFDM symbol using M-QAM and code rate R, $M \in \{2, 4, 16, 64\}$ and $R \in \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}\}$, start with $48R\log_2(M)$ data bits. These data bits are first encoded using a rate $\frac{1}{2}$ convolutional encoder given by the generator polynomials:

$$\begin{aligned} g_0(x) &= 1 + x^2 + x^3 + x^5 + x^6 &= 133_8 \\ g_1(x) &= 1 + x + x^2 + x^3 + x^6 &= 171_8 \end{aligned} \tag{8}$$

The encoder can be seen in the figure below.

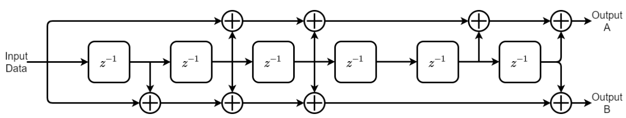


Figure 1. 802.11a convolutional encoder.

To get the desired rate do the following. For $R = \frac{1}{2}$ do nothing. For $R = \frac{2}{3}$ and $R = \frac{3}{4}$ use the appropriate puncturing pattern shown in the figure below.

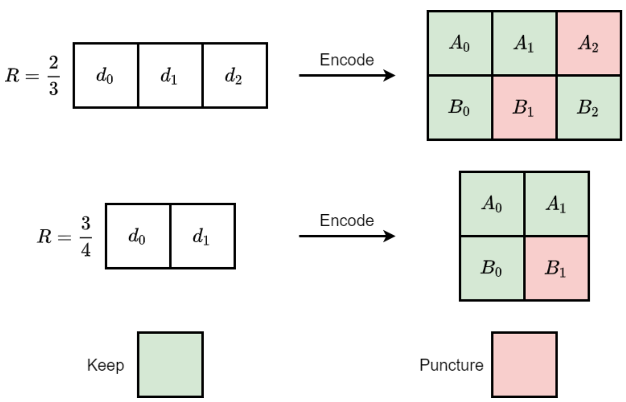


Figure 2. 802.11a convolutional encoder puncturing patterns.

After encoding the data there are $N_{CBPS} = 48\log_2(M)$ bits. N_{CBPS} stands for the number of coded bits per symbol. The encoded bits are then interleaved according to the following rule:

$$\begin{aligned} i &= \frac{N_{CBPS}}{16}(k \bmod 16) + \left\lfloor \frac{k}{16} \right\rfloor \\ j &= s\left\lfloor \frac{i}{s} \right\rfloor + \left(\left(i + N_{CBPS} - \left\lfloor \frac{16i}{N_{CBPS}} \right\rfloor \right) \bmod s \right) \\ s &= \frac{\log_2(M)}{2} \end{aligned} \tag{9}$$

where $k = 0, 1, ..., N_{CBPS} - 1$ represents the original position of the bit and j represents the interleaved position of the bit. After this, groups of $\log_2(M)$ adjacent interleaved bits are mapped to the M-QAM constellation. For every OFDM symbol there are 48 symbols coming from the coded bits.

In 802.11a each user is assigned a 20 MHz bandwidth with 64 subcarriers that split the bandwidth into equal portions. The 48 symbols are mapped to 48 of these subcarriers. Of the remaining 16 subcarriers, 12 are used for the guard band and 4 for pilot channels. All of these are assigned in the frequency domain. In order to have a time domain signal take a 64-point IFFT to get 64 time domain samples. The inputs to the IFFT are shown in the following figure. Here $s_1, ...s_{48}$ represent the symbols and $p_1, ..., p_4$ represent the pilots. For the purposes of

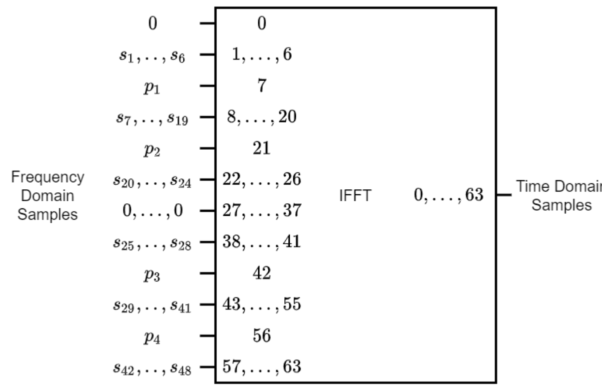


Figure 3. Assignment of the symbols to the 64 point IFFT

this study all the pilot values were all considered to be 1 since they are not used. In the standard their values come from PN sequences.

After obtaining the 64 time domain samples $(x[0], ..., x[63])$ a cyclic prefix is applied. The cyclic prefix is applied by taking the last 16 time domain samples $(x[48], ..., [63])$ and copying them to the front. This gives a time domain sequence that is 80 symbol periods long and has the 80 symbols $(x[48], ...x[63], x[0], ..., x[63])$. This 80 symbol sequence is 1 OFDM symbol. The cyclic prefix helps with preventing ISI interefence between OFDM symbols since there is a buffer period of 16 samples that are repeated and contain no new information.

Since the bandwidth for a single user is 20 MHz, 20 Msps can be sent. Since there are 80 time domain symbols per second this means that the 250,000 OFDM symbols are sent every second. The lowest data rate corresponds to $M = 2$ and $R = \frac{1}{2}$. The highest occurs for $M = 64$ and $R = \frac{3}{4}$. In general the data rate D is given by:

$$\begin{aligned} D = &\frac{2.5 \times 10^5 \,\text{OFDM syms}}{1\,\text{sec}} * \frac{48\,\text{syms}}{1\,\text{OFDM sym}} * \\ &\frac{\log_2(M)\,\text{coded bits}}{1\,\text{sym}} * \frac{R\,\text{bits}}{1\,\text{coded bit}} \end{aligned} \tag{10}$$

Therefore the minimum data rate is $D_{min} = 6$ Mbps and the maximum data rate is $D_{max} = 54$ Mbps.

B. MMSE and ZF Equalizers

In this take of MMSE and ZF equalizers we consider a single input single output frequency selective channel with frequency response $H(z)$. Again the goal of the ZF equalizer is to simply invert the channel. Therefore the ZF equalizer is given by:

$$W_{ZF}(z) = \frac{1}{H(z)} \tag{11}$$

This version has the same problem as before. Noise at certain frequencies is amplified too much and corrupts the signal. Again the MMSE equalizer solves this issue. The MMSE equalizer is given by:

$$W_{MMSE}(z) = \frac{1}{H(z) + \sigma_n^2} \tag{12}$$

IV. Testing MIMO

The performance of a 2x2 MIMO system was simulated. The system had a symbol rate of 20 Msps so that it would be comparable to the 802.11a standard. The data was also encoded using the same convolutional encoder. The interleaver for this test was a 10x10 block interleaver. The encoded bits were modulated using M-QAM and were sent (no IFFT and cyclic prefix) through Rayleigh fading channels with various max Doppler frequencies. Using the same modulation sizes and encoding rates as described in the previous section, the minimum data rate for this system is $D_{min} = 20$ Mbps and the maximum is $D_{max} = 180$ Mbps. This increase in data rate comes from the fact that there are less protections, *i.e.* no cyclic prefix, guard bands, or pilots. It also comes from the fact that twice the amount of data is being sent since we are using a 2x2 MIMO channel.

The 2x2 MIMO set up was tested at an SNR of 35 dB with 3×10^5 symbols sent. There were three different Rayleigh fading channels tested. These corresponded to a max Doppler frequency of 100 Hz, 1 kHz, and 10 kHz. A sample Rayleigh fading envelope over 0.1 sec for each of these three frequencies can be seen in Figures 4 - 6.

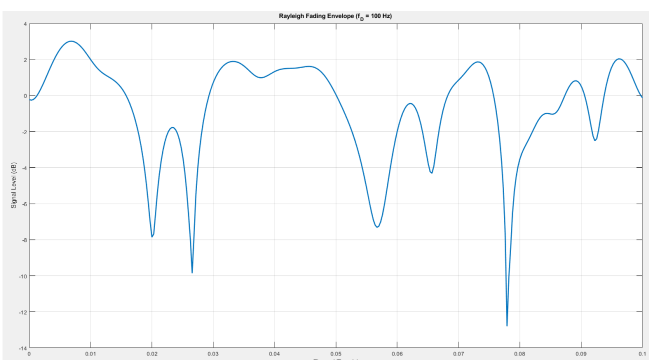


Figure 4. Rayleigh Fading Envelope. Max Doppler frequency 100 Hz.

For each Doppler frequency all the modulation and encoding rate combinations were simulated using precoding and receiver shaping, the MMSE equalizer, and the ZF equalizer. The BER for each was recorded and the best configuration was reported. The best configuration, for a given target BER, is the configuration corresponding to the highest data rate with a BER lower than the target system BER. If no configuration

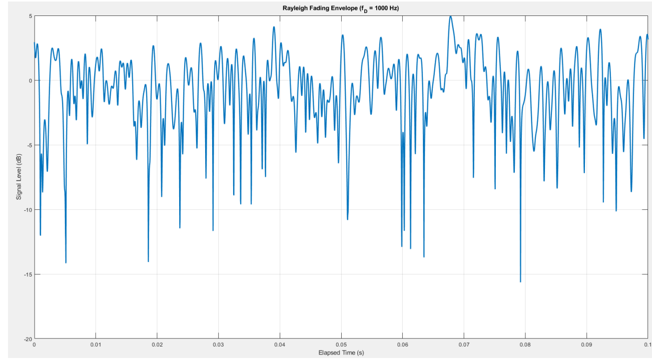


Figure 5. Rayleigh Fading Envelope. Max Doppler frequency 1 kHz.

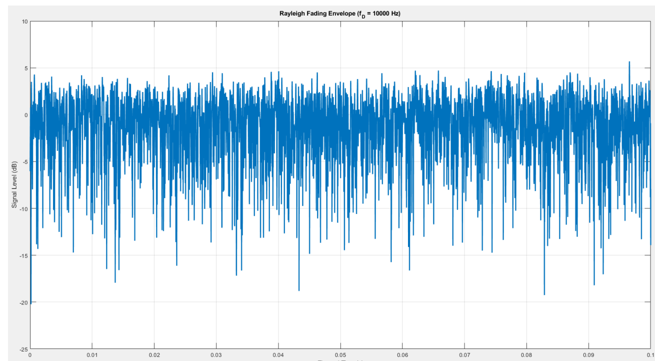
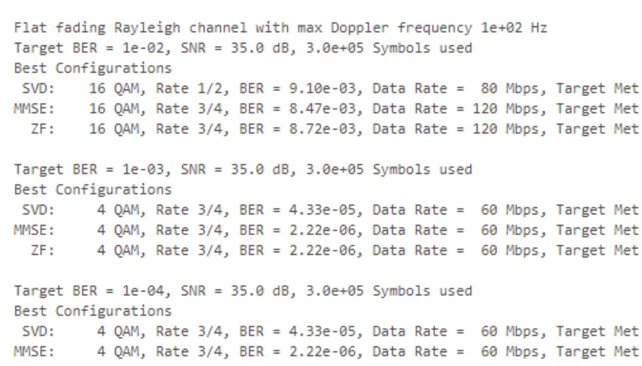


Figure 6. Rayleigh Fading Envelope. Max Doppler frequency 10 kHz.

achieves the required BER than the system achieving the lowest BER is considered best. Figures 7 - 9 show the best configurations.

These results should not be taken as a full indication as to which system is best. This is because for each Doppler frequency only one instance of the Rayleigh fading values was generated. Ideally this experiment would be run over multiple iterations and the BERs would be averaged before picking a best configuration for each scenario. This is however very computationally expensive and would take too much time. These results only serve as a possible indication as to which



```
Flat fading Rayleigh channel with max Doppler frequency 1e+02 Hz
Target BER = 1e-02, SNR = 35.0 dB, 3.0e+05 Symbols used
Best Configurations
 SVD:    16 QAM, Rate 1/2, BER = 9.10e-03, Data Rate =  80 Mbps, Target Met
MMSE:    16 QAM, Rate 3/4, BER = 8.47e-03, Data Rate = 120 Mbps, Target Met
  ZF:    16 QAM, Rate 3/4, BER = 8.72e-03, Data Rate = 120 Mbps, Target Met

Target BER = 1e-03, SNR = 35.0 dB, 3.0e+05 Symbols used
Best Configurations
 SVD:     4 QAM, Rate 3/4, BER = 4.33e-05, Data Rate =  60 Mbps, Target Met
MMSE:     4 QAM, Rate 3/4, BER = 2.22e-06, Data Rate =  60 Mbps, Target Met
  ZF:     4 QAM, Rate 3/4, BER = 2.22e-06, Data Rate =  60 Mbps, Target Met

Target BER = 1e-04, SNR = 35.0 dB, 3.0e+05 Symbols used
Best Configurations
 SVD:     4 QAM, Rate 3/4, BER = 4.33e-05, Data Rate =  60 Mbps, Target Met
MMSE:     4 QAM, Rate 3/4, BER = 2.22e-06, Data Rate =  60 Mbps, Target Met
  ZF:     4 QAM, Rate 3/4, BER = 2.22e-06, Data Rate =  60 Mbps, Target Met
```

Figure 7. MIMO: Best configurations for a max Doppler frequency of 100 Hz.

```
Flat fading Rayleigh channel with max Doppler frequency 1e+03 Hz
Target BER = 1e-02, SNR = 35.0 dB, 3.0e+05 Symbols used
Best Configurations
 SVD:     4 QAM, Rate 3/4, BER = 7.34e-03, Data Rate =  60 Mbps, Target Met
MMSE:     4 QAM, Rate 3/4, BER = 6.92e-03, Data Rate =  60 Mbps, Target Met
  ZF:     4 QAM, Rate 3/4, BER = 7.57e-03, Data Rate =  60 Mbps, Target Met

Target BER = 1e-03, SNR = 35.0 dB, 3.0e+05 Symbols used
Best Configurations
 SVD:     2 QAM, Rate 3/4, BER = 1.56e-03, Data Rate =  30 Mbps, Target Not Met
MMSE:     2 QAM, Rate 1/2, BER = 8.50e-04, Data Rate =  20 Mbps, Target Met
  ZF:     2 QAM, Rate 1/2, BER = 1.25e-03, Data Rate =  20 Mbps, Target Not Met

Target BER = 1e-04, SNR = 35.0 dB, 3.0e+05 Symbols used
Best Configurations
 SVD:     2 QAM, Rate 3/4, BER = 1.56e-03, Data Rate =  30 Mbps, Target Not Met
MMSE:     2 QAM, Rate 1/2, BER = 8.50e-04, Data Rate =  20 Mbps, Target Not Met
  ZF:     2 QAM, Rate 1/2, BER = 1.25e-03, Data Rate =  20 Mbps, Target Not Met
```

Figure 8. MIMO: Best configurations for a max Doppler frequency of 1 kHz.

```
Flat fading Rayleigh channel with max Doppler frequency 1e+04 Hz
Target BER = 1e-02, SNR = 35.0 dB, 3.0e+05 Symbols used
Best Configurations
 SVD:     2 QAM, Rate 2/3, BER = 5.67e-03, Data Rate =  27 Mbps, Target Met
MMSE:     4 QAM, Rate 2/3, BER = 8.28e-03, Data Rate =  53 Mbps, Target Met
  ZF:     2 QAM, Rate 3/4, BER = 9.99e-03, Data Rate =  30 Mbps, Target Met

Target BER = 1e-03, SNR = 35.0 dB, 3.0e+05 Symbols used
Best Configurations
 SVD:     2 QAM, Rate 1/2, BER = 1.73e-03, Data Rate =  20 Mbps, Target Not Met
MMSE:     2 QAM, Rate 1/2, BER = 7.23e-04, Data Rate =  20 Mbps, Target Met
  ZF:     2 QAM, Rate 1/2, BER = 1.34e-03, Data Rate =  20 Mbps, Target Not Met

Target BER = 1e-04, SNR = 35.0 dB, 3.0e+05 Symbols used
Best Configurations
 SVD:     2 QAM, Rate 1/2, BER = 1.73e-03, Data Rate =  20 Mbps, Target Not Met
MMSE:     2 QAM, Rate 1/2, BER = 7.23e-04, Data Rate =  20 Mbps, Target Not Met
  ZF:     2 QAM, Rate 1/2, BER = 1.34e-03, Data Rate =  20 Mbps, Target Not Met
```

Figure 9. MIMO: Best configurations for a max Doppler frequency of 10 kHz.

system is best and what kind of data rates are achievable.

Looking at just these results it can be seen that the MMSE equalizer performs better than ZF equalizer.

V. Testing OFDM

The performance of the OFDM system was tested at an SNR of 10 dB using 4×10^4 OFDM symbols. It was tested using four different frequency selective channels. The transfer functions of these channels were given by:

$$H_1(z) = 1 \tag{13}$$

$$H_2(z) = 1 - \frac{3}{10}z^{-1} \tag{14}$$

$$H_3(z) = 1 + \frac{1}{5}z^{-1} + \frac{2}{5}z^{-2} \tag{15}$$

$$H_4(z) = 1 + \frac{1}{5}z^{-1} + \frac{2}{5}z^{-2} - \frac{3}{10}z^{-3} + \frac{1}{10}z^{-4} - \frac{7}{10}z^{-5} \tag{16}$$

These represent no ISI to severe ISI. The magnitude responses of$H_2(z)$ through $H_4(z)$ can be seen in Figures 10 - 12. As can be seen they are clearly frequency selective. It should be noted that these channels cause ISI within an OFDM symbol as well as among two adjacent OFDM symbols.

All the modulation and rate combinations were tested for each of these channels. The best configuration was defined in the same way as for the MIMO tests. The results can be seen

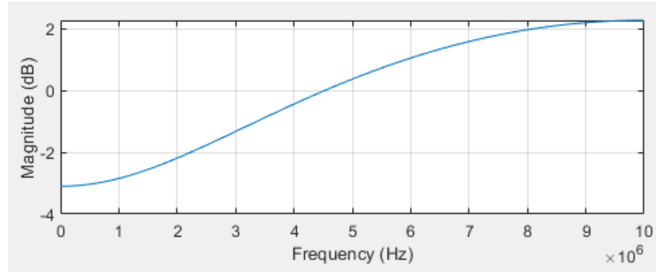


Figure 10. Magnitude response of $H_2(z)$.

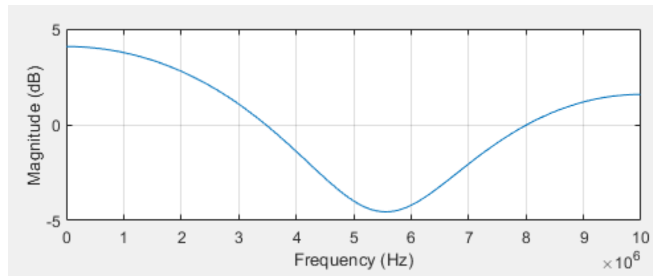


Figure 11. Magnitude response of $H_3(z)$.

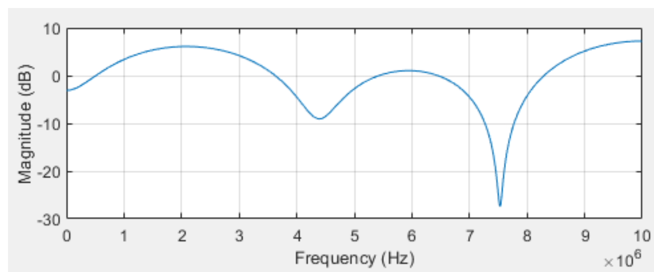


Figure 12. Magnitude response of $H_4(z)$.

in Figures 13 - 16. Again these results should not be taken to be fully accurate. To be more accurate more OFDM symbols should be sent during each test.

```
Target BER = 1e-02, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:    16 QAM, Rate 1/2, BER = 1.10e-03, Data Rate = 24 Mbps, Target Met
  ZF:    16 QAM, Rate 1/2, BER = 1.11e-03, Data Rate = 24 Mbps, Target Met

Target BER = 1e-03, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:     4 QAM, Rate 3/4, BER = 0.00e+00, Data Rate = 18 Mbps, Target Met
  ZF:     4 QAM, Rate 3/4, BER = 0.00e+00, Data Rate = 18 Mbps, Target Met

Target BER = 1e-04, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:     4 QAM, Rate 3/4, BER = 0.00e+00, Data Rate = 18 Mbps, Target Met
  ZF:     4 QAM, Rate 3/4, BER = 0.00e+00, Data Rate = 18 Mbps, Target Met
```

Figure 13. OFDM: Best configuration for channel $H_1(z)$.

From these results it seems that the MMSE and ZF equalizers perform about the same at the given SNR.

```
Target BER = 1e-02, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:    16 QAM, Rate 1/2, BER = 3.10e-03, Data Rate = 24 Mbps, Target Met
  ZF:    16 QAM, Rate 1/2, BER = 3.09e-03, Data Rate = 24 Mbps, Target Met

Target BER = 1e-03, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:     4 QAM, Rate 3/4, BER = 1.49e-05, Data Rate = 18 Mbps, Target Met
  ZF:     4 QAM, Rate 3/4, BER = 1.49e-05, Data Rate = 18 Mbps, Target Met

Target BER = 1e-04, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:     4 QAM, Rate 3/4, BER = 1.49e-05, Data Rate = 18 Mbps, Target Met
  ZF:     4 QAM, Rate 3/4, BER = 1.49e-05, Data Rate = 18 Mbps, Target Met
```

Figure 14. OFDM: Best configuration for channel $H_2(z)$.

```
Target BER = 1e-02, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:     4 QAM, Rate 3/4, BER = 5.33e-04, Data Rate = 18 Mbps, Target Met
  ZF:     4 QAM, Rate 3/4, BER = 5.31e-04, Data Rate = 18 Mbps, Target Met

Target BER = 1e-03, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:     4 QAM, Rate 3/4, BER = 5.33e-04, Data Rate = 18 Mbps, Target Met
  ZF:     4 QAM, Rate 3/4, BER = 5.31e-04, Data Rate = 18 Mbps, Target Met

Target BER = 1e-04, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:     4 QAM, Rate 2/3, BER = 2.19e-05, Data Rate = 16 Mbps, Target Met
  ZF:     4 QAM, Rate 2/3, BER = 2.19e-05, Data Rate = 16 Mbps, Target Met
```

Figure 15. OFDM: Best configuration for channel $H_3(z)$.

```
Target BER = 1e-02, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:     4 QAM, Rate 1/2, BER = 3.16e-03, Data Rate = 12 Mbps, Target Met
  ZF:     4 QAM, Rate 1/2, BER = 3.19e-03, Data Rate = 12 Mbps, Target Met

Target BER = 1e-03, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:     2 QAM, Rate 1/2, BER = 1.03e-04, Data Rate =  6 Mbps, Target Met
  ZF:     2 QAM, Rate 1/2, BER = 1.03e-04, Data Rate =  6 Mbps, Target Met

Target BER = 1e-04, SNR = 10.0 dB, 4.0e+04 Symbols used
Best Configurations
MMSE:     2 QAM, Rate 1/2, BER = 1.03e-04, Data Rate =  6 Mbps, Target Not Met
  ZF:     2 QAM, Rate 1/2, BER = 1.03e-04, Data Rate =  6 Mbps, Target Not Met
```

Figure 16. OFDM: Best configuration for channel $H_4(z)$.

VI. MIMO-OFDM Testing

The 2x2 MIMO OFDM system takes 2 OFDM streams and sends them in the typical MIMO way. Precoding and receiver shaping, and the MMSE and ZF equalizers were all tested. The same SNR and max Doppler frequencies were used for this test as in the MIMO test. There were 3×10^3 OFDM symbols generated for each transmit antenna. With this set the minimum data rate is $D_{min} = 12$ Mbps and the max is $D_{max} = 108$ Mbps, twice the regular OFDM amounts. The best configurations can be seen in Figures 17 - 19.

From these limited tests, it seems that the best overall MIMO method to use is the MMSE equalizer. It consistently gave the lowest BER. This cannot be directly compared to the OFDM test since different channels were used at a different SNR.

```
Flat fading Rayleigh channel with max Doppler frequency 1e+02 Hz
Target BER = 1e-02, SNR = 35.0 dB, 3.0e+03 OFDM Symbols used
Best Configurations
 SVD:    16 QAM, Rate 3/4, BER = 9.85e-03, Data Rate =  72 Mbps, Target Met
MMSE:    16 QAM, Rate 3/4, BER = 8.76e-03, Data Rate =  72 Mbps, Target Met
  ZF:    16 QAM, Rate 3/4, BER = 8.76e-03, Data Rate =  72 Mbps, Target Met

Target BER = 1e-03, SNR = 35.0 dB, 3.0e+03 OFDM Symbols used
Best Configurations
 SVD:     4 QAM, Rate 2/3, BER = 7.81e-06, Data Rate =  32 Mbps, Target Met
MMSE:     4 QAM, Rate 2/3, BER = 0.00e+00, Data Rate =  32 Mbps, Target Met
  ZF:     4 QAM, Rate 2/3, BER = 0.00e+00, Data Rate =  32 Mbps, Target Met

Target BER = 1e-04, SNR = 35.0 dB, 3.0e+03 OFDM Symbols used
Best Configurations
 SVD:     4 QAM, Rate 2/3, BER = 7.81e-06, Data Rate =  32 Mbps, Target Met
MMSE:     4 QAM, Rate 2/3, BER = 0.00e+00, Data Rate =  32 Mbps, Target Met
  ZF:     4 QAM, Rate 2/3, BER = 0.00e+00, Data Rate =  32 Mbps, Target Met
```

Figure 17. MIMO-OFDM: Best configurations for a max Doppler frequency of 100 Hz.

```
Flat fading Rayleigh channel with max Doppler frequency 1e+03 Hz
Target BER = 1e-02, SNR = 35.0 dB, 3.0e+03 OFDM Symbols used
Best Configurations
 SVD:     4 QAM, Rate 3/4, BER = 3.54e-03, Data Rate =  36 Mbps, Target Met
MMSE:     4 QAM, Rate 3/4, BER = 3.42e-03, Data Rate =  36 Mbps, Target Met
  ZF:     4 QAM, Rate 3/4, BER = 4.16e-03, Data Rate =  36 Mbps, Target Met

Target BER = 1e-03, SNR = 35.0 dB, 3.0e+03 OFDM Symbols used
Best Configurations
 SVD:     2 QAM, Rate 3/4, BER = 5.05e-04, Data Rate =  18 Mbps, Target Met
MMSE:     2 QAM, Rate 3/4, BER = 3.15e-04, Data Rate =  18 Mbps, Target Met
  ZF:     2 QAM, Rate 3/4, BER = 4.72e-04, Data Rate =  18 Mbps, Target Met

Target BER = 1e-04, SNR = 35.0 dB, 3.0e+03 OFDM Symbols used
Best Configurations
 SVD:     2 QAM, Rate 1/2, BER = 1.18e-04, Data Rate =  12 Mbps, Target Not Met
MMSE:     2 QAM, Rate 1/2, BER = 0.00e+00, Data Rate =  12 Mbps, Target Met
  ZF:     2 QAM, Rate 1/2, BER = 6.94e-05, Data Rate =  12 Mbps, Target Met
```

Figure 18. MIMO-OFDM: Best configurations for a max Doppler frequency of 1 kHz.

```
Flat fading Rayleigh channel with max Doppler frequency 1e+04 Hz
Target BER = 1e-02, SNR = 35.0 dB, 3.0e+03 OFDM Symbols used
Best Configurations
 SVD:     4 QAM, Rate 1/2, BER = 7.80e-03, Data Rate =  24 Mbps, Target Met
MMSE:     4 QAM, Rate 2/3, BER = 6.83e-03, Data Rate =  32 Mbps, Target Met
  ZF:     4 QAM, Rate 1/2, BER = 8.19e-03, Data Rate =  24 Mbps, Target Met

Target BER = 1e-03, SNR = 35.0 dB, 3.0e+03 OFDM Symbols used
Best Configurations
 SVD:     2 QAM, Rate 1/2, BER = 2.42e-03, Data Rate =  12 Mbps, Target Not Met
MMSE:     2 QAM, Rate 1/2, BER = 5.14e-04, Data Rate =  12 Mbps, Target Met
  ZF:     2 QAM, Rate 1/2, BER = 2.98e-03, Data Rate =  12 Mbps, Target Not Met

Target BER = 1e-04, SNR = 35.0 dB, 3.0e+03 OFDM Symbols used
Best Configurations
 SVD:     2 QAM, Rate 1/2, BER = 2.42e-03, Data Rate =  12 Mbps, Target Not Met
MMSE:     2 QAM, Rate 1/2, BER = 5.14e-04, Data Rate =  12 Mbps, Target Not Met
  ZF:     2 QAM, Rate 1/2, BER = 2.98e-03, Data Rate =  12 Mbps, Target Not Met
```

Figure 19. MIMO-OFDM: Best configurations for a max Doppler frequency of 10 kHz.

VII. Conclusion

The performance of MIMO, OFDM, and MIMO-OFDM systems was found using limited simulations. The simulations found the best configurations of modulation and encoding rate to meet a specific BER and maximize the data rate. These configurations were found for various equalizers. Using MIMO and OFDM in conjuction with one another allows for higher data rate systems at reasonable BERs and SNRs.