Diversity Techniques: An Examination of MRRC and Alamouti Codes

Karol Wadolowski

***Abstract*—This document serves to explain to diversity techniques for wireless communications systems and how Figure 4 of Alamouti's paper [1] was recreated. The first transmit diversity technique is maximal-ratio receive combining (MRRC) which employs one transmit and M receive antennas. The second technique is Alamouti codes which employ two transmit antennas and M receive antennas. Both techniques are used to increase the diversity gain of the system which results in a lower required SNR to acheive the same bit error rate (BER) performance.**

***Index Terms*—Diversity, MRRC, Alamouti Codes, Rayleigh Fading, Maximum-Likelihood Estimation**

I. Introduction

THIS report covers the diversity techniques that are MRRC and Alamouti codes. Both techniques will first be explained in depth. Next a brief introduction into Rayleigh fading channels and their simulation will be provided. Afterwards both techniques will be tested in a Rayleigh fading channel and their performance will be evaluated individually and against one another.

II. Maximal-Ratio Receive Combining

MRRC is covered for the case of two receiver antennas in [1]. The following derivation is an extension of Alamouti's two receiver case.

Fig. 1 provides a baseband representation of MRRC with M receive antennas. This provides a diversity order of M.

At time t symbol s_0 is transmitted. It travels to each of the M receiver antennas. The channel between the transmitter and receive antenna k for $k \in \{0, 1, ..., M-1\}$ is given by h_k. The received signal at receive antenna k is given by Eq. 1 where n_k is zero mean additive white Gaussian noise (AWGN) that also encapsulates interference.

$$r_k = h_k s_0 + n_k \tag{1}$$

After being received, channel estimation is done to retrieve the h_k channel values. These are then conjugated and multiplied by the received signal for its respective receiver. The outputs from each receiver are then summed to get the combined signal. The value of the combined signal is given by $\tilde{s}_0$ in Eq. 2.

$$\tilde{s}_0 = \sum_{k=0}^{M-1} h_k^* r_k \tag{2}$$

This combined signal is then sent to the maximum likelihood (ML) detector to make a final decision on what the recovered

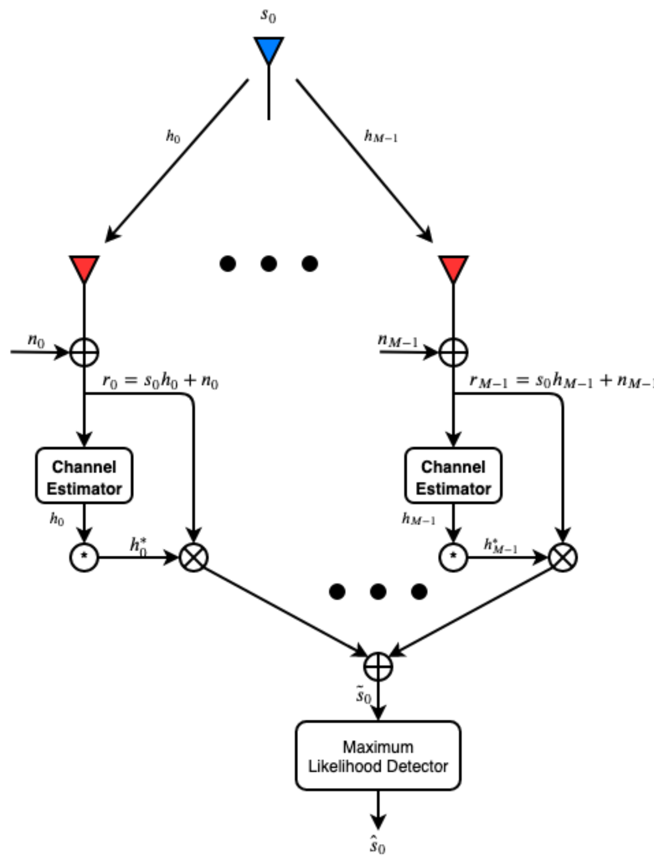

Figure 1. M-branch MRRC structure.

symbol is. The ML decision rule is to choose the signal s_i if and only if

$$\sum_{k=0}^{M-1} d^2(r_k, h_k s_i) \leq \sum_{k=0}^{M-1} d^2(r_k, h_k s_p), \quad \forall i \neq p \tag{3}$$

where $d^2(x, y)$ is the standard squared Euclidean distance measure between x and y given by:

$$d^2(x, y) = (x - y)(x^* - y^*) \tag{4}$$

The intuition behind Eq. 3 is to choose the symbol s_i which when sent through the channels would result in received signals that are closest to those actually measured. Expanding Eq. 3 using Eq. 4 gives the equivalent ML decision rule choose

s_i if and only if

$$\begin{aligned} \rho|s_i|^2 + d^2(\tilde{s}_0, s_i) \leq \rho|s_p|^2 + d^2(\tilde{s}_0, s_p), \quad \forall i \neq p \\ \rho = \sum_{k=0}^{M-1} \|h_k\|_2^2 - 1 \end{aligned} \tag{5}$$

Here $\|h_k\|_2$ represents the l_2-norm of the channel. For equal energy constellations (Eq. 6) such as phase shift keying (PSK) the decision can be simplified even further as the ρ terms in Eq. 5 are equal giving the decision rule choose s_i if it satisfies Eq. 7.

$$|s_i|^2 = |s_p|^2, \quad \forall i, p \tag{6}$$

$$d^2(\tilde{s}_0, s_i) \leq d^2(\tilde{s}_0, s_p), \quad \forall i \neq p \tag{7}$$

The constellation used to make Fig. 4 in [1] was binary phase shift keying (BPSK). The possible symbols are $\{-1, +1\}$. BPSK is a special case of PSK. The MRRC decision rule can be simplified even further for BPSK. The symbol decision $\hat{s}_0 \in \{-1, +1\}$ is given by

$$\hat{s}_0 = \begin{cases} -1, & \Re(\tilde{s}_0) < 0 \\ +1, & \Re(\tilde{s}_0) > 0 \end{cases} \tag{8}$$

where $\Re(z)$ returns the real part of complex number z. Note the case of $\Re(\tilde{s}_0) = 0$ is ignored as it occurs with probability 0.

III. Alamouti Codes

Alamouti codes where first introduced by Alamouti in [1]. He introduced and derived the ML detector for the case of one and two receiver antennas. The following is the derivation of the general case of M antennas.

Fig. 2 provides a baseband representation of Alamouti codes with M receive antennas. Alamouti codes provide a diversity order of 2M due to the extra transmit antenna.

At time t transmit antenna 0 sends s_0 to the M receive antennas and transmit antenna 1 send s_1 to those M antennas as well. The channels between transmit antenna 0 and receive antenna k for $k \in \{0, 1, ..., M-1\}$ are given by h_{2k} and the channels between transmit antenna 1 and receive antenna k are given by h_{2k+1}. The received signal at the k^{th} receive antenna corresponding to the symbols sent at time t is given by Eq. 9. Now assume the channel stays constant across two adjacent symbol times. Then at time $t+T$ transmitter 0 sends $-s_1^*$ and transmitter 1 sends s_0^*. Under our assumption both these symbols experience the same channels as the previous symbol sent out from their respective transmit antennas. The received signal by the k^{th} receive antenna at time $t+T$ is given by Eq. 10.

$$r_{2k} = h_{2k}s_0 + h_{2k+1}s_1 + n_{2k} \tag{9}$$

$$r_{2k+1} = h_{2k}(-s_1^*) + h_{2k+1}(s_0^*) + n_{2k+1} \tag{10}$$

Note that at each of the two times each of the M receive antennas experiences different AWGN. Also since two symbols are being sent for every two transmission times Alamouti codes are a rate 1 transmission scheme.

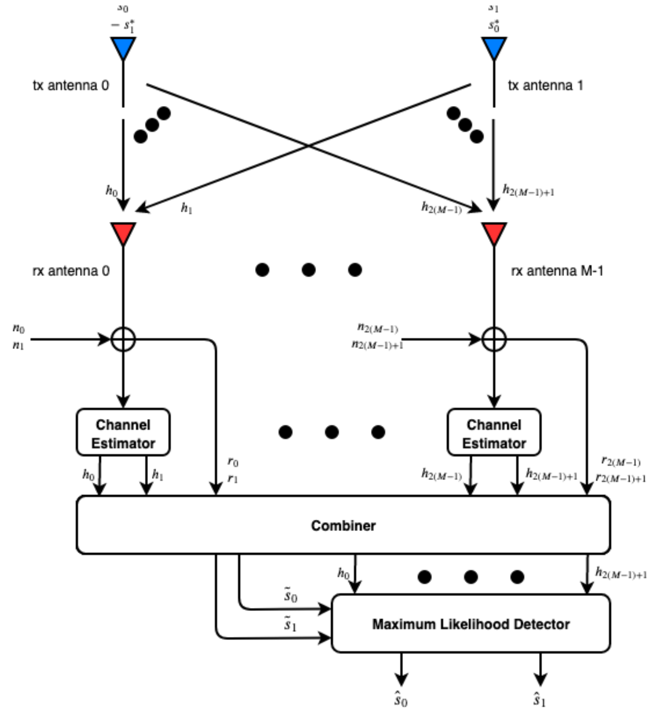

Figure 2. M-branch Alamouti code structure.

Once $r_0, ..., r_{2(M-1)+1}$ are obtained they are sent to the combiner along with the channel estimates of $h_0, ..., h_{2(M-1)+1}$. The combiner uses this information to get the two combined signal $\tilde{s}_0$ and $\tilde{s}_1$ are given by:

$$\begin{aligned} \tilde{s}_0 &= \sum_{k=0}^{M-1} \left(h_{2k}^* r_{2k} + h_{2k+1} r_{2k+1}^* \right) \\ \tilde{s}_1 &= \sum_{k=0}^{M-1} \left(h_{2k+1}^* r_{2k} - h_{2k} r_{2k+1}^* \right) \end{aligned} \tag{11}$$

Both of these combined signals as well as the channel estimate are then sent to the ML decoder to get the decoded values $\hat{s}_0$ and $\hat{s}_1$. The ML decoder is the same as the one in Eq. 5 except that the $\tilde{}$ term can be either $\tilde{s}_0$ or $\tilde{s}_1$. It also follows that Eq. 7 can be used for PSK constellations for each of the two combined signals and that the BPSK again is just given by Eq. 8.

IV. Rayleigh Fading Channels

In [1], Alamouti tested both MMRC and his technique, Alamouti codes, using Rayleigh fading channels. Here is some background on how to simulate a Rayleigh fading channel based off of the presentation of Clarke's Model in [2]

Fig. 3 provides the flow chart of how a Rayleigh fading channel $r(t)$ can be simulated.

The first step in producing a Rayleigh fading channel is to choose N (the number of points) and f_D the maximum doppler frequency. These two values define the frequency spacing as $\Delta f = \frac{2f_D}{N-1}$ and the duration of the waveform as $T = \frac{1}{\Delta f}$. (Note T here is separate from the T in Sec. III). Then generate

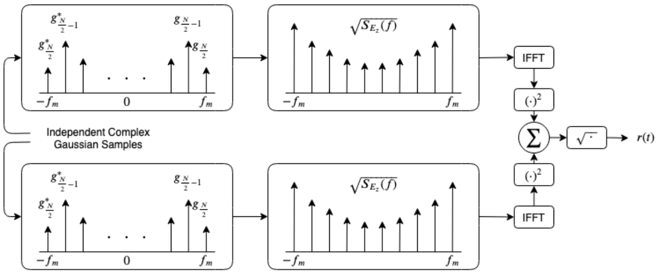

Figure 3. Baseband Rayleigh fading channel simulator. Figure based off of Fig. 5.25 in [2].

two sets of $\lceil \frac{N}{2} \rceil$ complex Gaussian random variables. These two sets correspond to the upper and lower branches seen in Fig. 3. If N is even then negative frequency portion of the spectrum in the left most blocks is just obtained by taking the negative conjugate of these $\frac{N}{2}$ samples. If N is odd however one of the generated samples is assigned to $f = 0$ and the rest are negated and conjugated to form the rest of the spectrum.

The next step is to acquire a sampled version of the fading spectrum $\sqrt{S_{E_z}(f)}$. The $S_{E_z}(f)$ is the doppler power spectrum and is given at baseband by Eq. 12.

$$S_{E_z}(f) = \frac{1.5}{\pi f_D \sqrt{1 - \left(\frac{f}{f_D}\right)^2}} \tag{12}$$

The sampling points are the frequencies $-f_D + k\Delta f$ where $k \in \{0, 1, ..., N-1\}$. Note for $k = 0, N-1$ the value of the fading spectrum is ∞ so the values used for those two points are taken from the linear interpolations about the $k = 1, N-2$ points. After sampling from the fading spectrum, the Gaussian spectrums are pointwise multiplied with the fading spectrum. The results from the upper and lower branches are then put through IFFTs and are then squared. The two branches are then added together and the square root of the result is taken to give the simulated Rayleigh fading spectrum $r(t)$ to a scaling factor. Following [3] $r(t)$ is then divided by the mean value of all the samples of $r(t)$. This is done so that the Rayleigh fading channel has a mean 0 dB power.

Two Rayleigh fading channels simulated using this method can be seen in Fig. 4 and 5. Both simulated channels have a duration of 1 s. As can be seen the channel with a max doppler frequency of 10 Hz is less sporadic than the 100 Hz channel.

V. Performance of MRRC and Alamouti Codes

The goal of this section is to show that the simulated MRRC and Alamouti codes diversity techniques match the performance shown in Fig. 4 of [1] which is reproduced here as Fig. 6. In addition to this, the performance of the diversity techniques are also shown for other diversity orders.

Fig. 6 was obtained by Alamouti in the following way. Alamouti used only coherent BPSK in a Rayleigh fading channel. This information was sent in 5 ways. The first, labeled no diversity (1 Tx, 1 Rx), is equivalent to MRRC with M = 1. For this curve and all the others he assumed that his channel estimator was perfect and he knew the channel

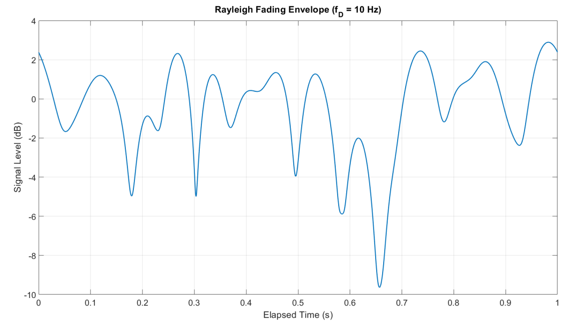

Figure 4. Rayleigh fading channel with $T = 1s$ and $f_D = 10Hz$.

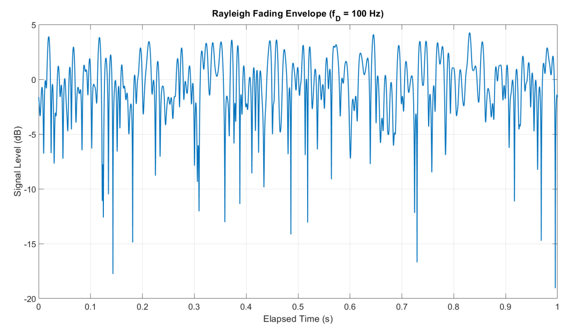

Figure 5. Rayleigh fading channel with $T = 1s$ and $f_D = 100Hz$.

exactly. The next two curves listed in the legend correspond to MRRC with M = 2 and M = 4 respectively. The final two curves where Alamouti codes with M = 1 and M = 2 receive antennas respectively. These were chosen as to maintain the same diversity order as the respective MRRC system. The transmitters for the Alamouti codes were only emitting half power compared to the transmitter in MRRC so as to make both diversity techniques use the same amount of total power. Also note that the x axis is not labeled as either E_b/N_o or SNR on a dB scale. This is because the two are equivalent for uncoded BPSK.

Fig. 7 shows my attempt at implementing these two techniques. They are spot on! In his paper Alamouti also claimed that if each transmitter radiated the same amount of power as MRRC, then the curves of equal diversity order would overlap (double power for his method). The results of doubling the power can be seen in Fig. 8. Bam! They overlap just as predicted.

To also show off the extension of the two techniques to M receive antennas both the MRRC and Alamouti codes techniques were simulated for antenna amounts not specified in [1]. Fig. 9 shows this off for MRRC and Fig. 10 shows this off for Alamouti codes using the same amount of transmit power.

VI. Conclusion

Two diversity techniques presented in [1] were formally extended to the case of an arbitrary amount of receive antennas. A method for simulating Rayleigh fading channels from [2]

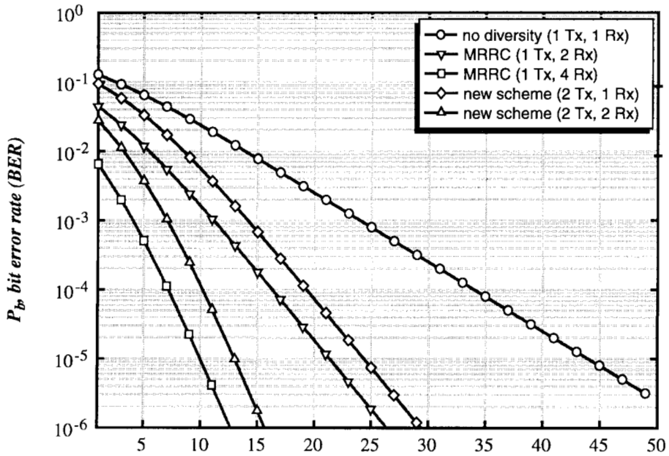

Figure 6. The results presented by Alamouti [1]. Here new scheme refers to Alamouti codes.

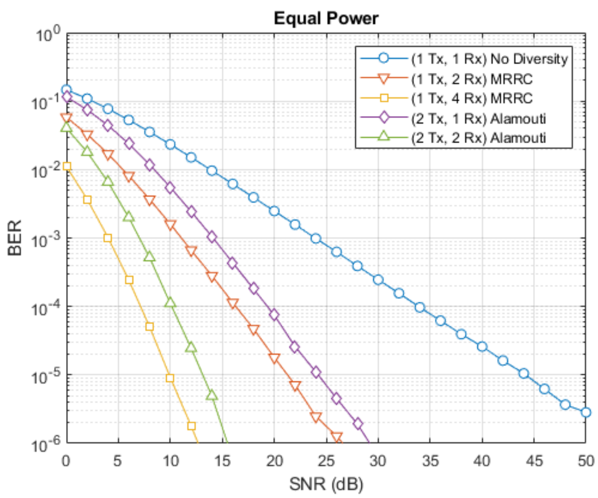

Figure 7. Recreation of Alamouti's results. Each scheme radiates the same amount of power.

was also explained. After obtaining the background for these three necessary components the results Alamouti had in [1] were successfully recreated.

References

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Select Areas in Communications*, vol. 16, no. 8, pp. 1451–1458, October 1998.

[2] T. S. Rappaport, *Wireless Communications - Principles and Practice*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 2001.

[3] J. I. Smith, "A computer generated multipath fading simulation for mobile radio," *IEEE Transactions on Vehicular Technology*, vol. 24, no. 3, pp. 39–40, August 1975.

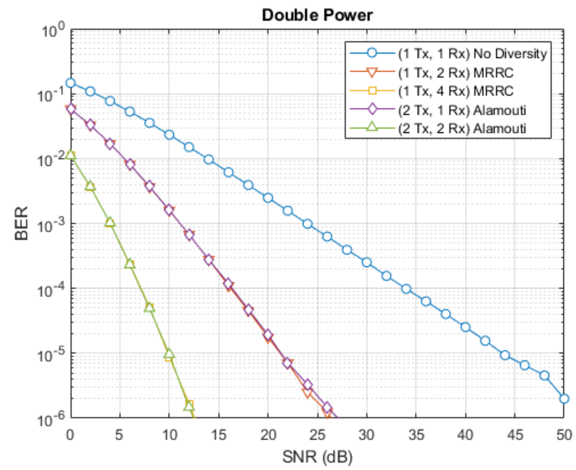

Figure 8. Recreation of Alamouti's results. The Alamouti code scheme uses double the power of MRRC.

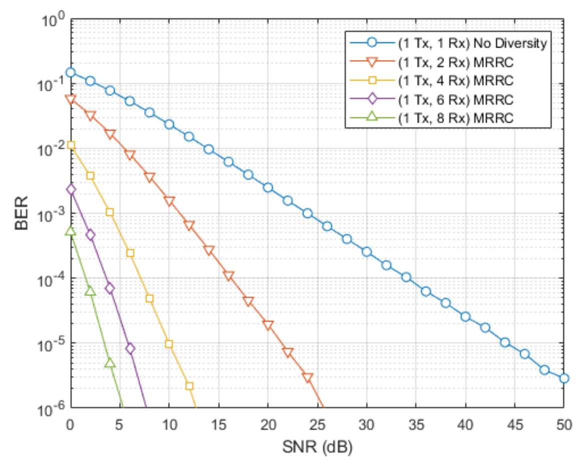

Figure 9. MRRC simulated with different amounts of receive antennas.

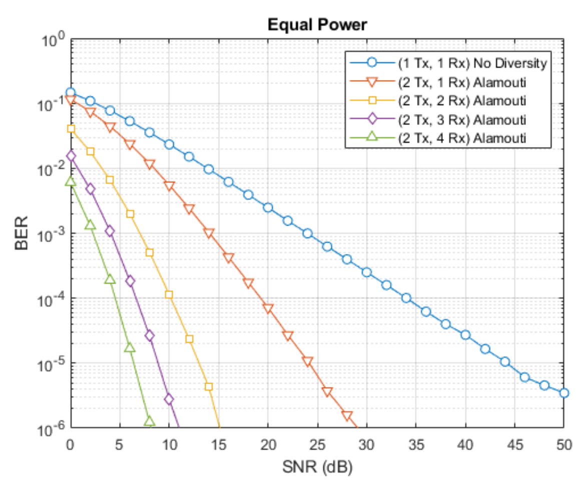

Figure 10. Alamouti codes simulated with different amounts of receive antennas with equal transmit power when compared to MRRC.