PART 1

MATHEMATICS

God made natural numbers; all else is the work of man
– Leopold Kronecker

1. NUMBERS

TRANSLATION WORK:

number
numeral .....
numerical value of a .....
even number .....
odd number .....
natural number .....
rational number .....
irrational number .....
positive number .....
negative number .....
integer .....

READING

When reading numbers, students frequently find it difficult to read the combination of two letters: 'th'. Make sure you can read the following numbers accurately:
3 – three [θri:]
333 – three hundred (and) thirty three [θɜ:rti] [θri:]
3,333 – three thousand, three hundred (and) thirty three
3,003,333,333 – three billion, three million, three hundred thirty three thousand, three hundred (and) thirty three

PUZZLE:
If you took **th**ree **th**rushes* from under a **th**atched roof housing **th**irteen of **th**em, how many would you have?
*thrush – (PL) drozd

(See the bottom of page 13 for the answer)

MATCHING

Ex. 1 Match the terms (1, 2, 3…) with their definitions (a, b, c…):

1. a **number** [___]
2. a **numeral** [___]
3. an **odd** number [___]
4. an **even** number [___]
5. **natural** numbers [___]
6. a **positive** number [___]
7. a **negative** number [___]
8. **integers** [___]
9. an **irrational** number [___]
10. ordinal numbers [___]
11. a **numerical value** [___]
12. a **rational** number [___]

a. is a number that is less than 0 and with a – symbol in front of it
b. is an abstract entity that represents amount or measurement[1]
c. 1, 3, 5, 7, 9, 11, 13 …. – when divided by two, the result is a fraction
d. are the „whole" natural numbers, including negative ones
e. are the ordinary whole numbers that are used for counting (*There are 25 students in this class.*) or ordering (*This is the fifth tallest student...*)
f. i.e. nineteen, 19, XIX, represents a number
g. 2, 4, 6, 8, 10, 12, 14… – if it is a multiple of two
h. indicate the order in which objects appear in a well-ordered set, i.e. This is the fifth tallest student in [this particular set of students in] class
i. is a number that is greater than 0, it can be, but does not have to be written with a "+" symbol in front of it
j. is a real number regardless of its sign
k. any number that can be expressed as the quotient or fraction p/q of two integers, where $q \neq 0$
l. any real number that cannot be expressed as a ratio of integers, for instance $\sqrt{2}$, π, etc.

SPEAKING – PAIR WORK

Ex. 2 Student A: read the first 5 numbers, student B: listen and write them down. Then, students change roles for the remaining numbers 6 – 10.

[1] Krukiewicz-Gacek, Trzaska, 2010, p. 15.

1. 675
2. 37.89
3. 7,865
4. 908,076
5. 897.65
6. 1,209,398
7. 324,100,990
8. 77,543
9. 100,576,192,200
10. 509,489,099

TRUE/FALSE

Ex. 3 State whether the following sentences are true (T) or false (F):

1. A numeral represents a number. _____
2. When divided by two, an odd number gives a fraction. _____
3. Integers are the „whole" natural numbers, excluding negative ones. _____
4. An even number plus an even number gives an even number. _____
5. In English, we use a dot to refer to thousands and millions. _____
6. We use natural numbers for counting. _____
7. XIX represents a number and is a numeral. _____
8. *Nought* is usually used to refer to telephone numbers. _____

NOTE:

1. In Polish, the word *bilion* represents 10^{12} and the word *trylion* is equivalent to 10^{18}.

   English: 1,000,000,000 – one **billion (10^9)**
   1,000,000,000,000 – one **trillion** represents **10^{12}**

2. We say: three hundred (~~hundreds~~) soldiers, four thousand (~~thousands~~) children, thirty three million (~~millions~~) people, etc.
   But: hundreds of soldiers, thousands of children, tens of thousands of protesters, millions of people, etc.
3. Make sure you understand the use of a coma (i.e. 123,009) and a dot (i.e. 12.98 or 0.45), which is used to enumerate decimal fractions.
4. 0 – can be called ***zero***, or
   - ***nil*** (when it refers to numbers in sports games as in *Liverpool won the game three–nil.*),
   - ***nought / naught*** (esp US) in calculations and figures as in *GDP has decreased by nought point 25 per cent.*,
   - ***oh*** is used to refer to numbers such as telephone numbers: *7 oh 7 double 5 8 oh 7* (70755807).

2. ELEMENTS OF ALGEBRA, EQUATIONS AND SYMBOLS

TRANSLATION WORK:

the absolute value of a .....
the sum of .....
percent .....
per mil .....
equal to/not equal to .....
less/greater than or equal to .....
approximate (ly) .....
identical to .....
round, square brackets .....
parentheses .....
braces (also: curly brackets) .....
infinity .....
tends to .....
capital letter .....
subtraction .....
addition .....
division .....
multiplication .....
the sign of multiplication .....
quotient .....
product .....
rounding .....
ratio .....
directly proportional .....
variable .....
linear/quadratic/cubic equation .....
system of equations .....
solve an equation .....
solution/root of equation .....
unknown .....
substitute .....

READING

Read the text and do the exercise below (ex. 4).

Here are three useful reminders:

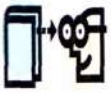
A. *Most mathematical operations: addition, subtraction, multiplication and division are normally performed in a particular order or sequence. Multiplication and division are done prior to addition and subtraction*[2].

B. Mathematical operations such as rounding a numerical value and solving an equation are very common. You ***round*** a numerical value when you replace the value with another that is ***approximately*** equal, i.e. $\sqrt{2} \approx 1.41$ (the square root of 2 is approximately equal to 1 point four one).

C. In order to **solve** this equation: $2\mathbf{x} - 4 = 10$

we can do the following:

1. **Transfer** the -4 from the left-hand side of the equation, to the right-hand side and change its sign:
   $2x = 10 + 4$
2. Since we can multiply or divide both sides of the equation, we divide it by 2 and **replace** our equation with an equivalent, simpler one:
   $2x : 2 = 14 : 2$
3. The **solution** (or **root**) of the equation is $x = 7$.



COMPREHENSION

Ex. 4 Choose the correct ending:

1. Multiplication and division are done
   a. before addition and subtraction
   b. after addition and subtraction
2. You ***round*** a numerical value when you replace the value with another that is
   a. identical to it.
   b. close to the exact value.
3. When you transfer a particular value from the left-hand side of the equation to the right-hand side
   a. you change its sign to the opposite.
   b. you always add the negative sign to it.

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[2] Krukiewicz-Gacek, Trzaska, 2010, p. 27.

The answer to the puzzle from the READING exercise in chapter 1.1 is three.

4. A solution of an equation can be called
   a. the unknown.
   b. the root of the equation.

MATCHING

Ex. 5 Match the terms (1, 2, 3…) with their definitions (a, b, c…):

1. **addition** [___]
2. **subtraction** [___]
3. **multiplication** [___]
4. **product** [___]
5. **quotient** [___]
6. a **linear** equation [___]
7. **quadratic** equation [___]
8. **cubic** equation [___]
9. **system** of equations [___]
10. **division** [___]

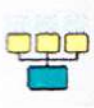
a. $f(x) = ax^3 + bx^2 + cx + d$
b. $a + b$
c. is the result of division
d. $a \times b$
e. $a : b$
f. is the result of multiplying
g. $y = 11 + x$
h. $a - b$
i. $ax^2 + bx + c = 0$
j. $\begin{cases} 3x + 2y = 19 \\ x - y = 3 \end{cases}$

GAP-FILL

Ex. 6 Complete the statements with the words given below:

less absolute capital in brackets equals variable braces greater directly proportional sum of a ratio much is not equal to square

1. Five minus four ..... 1.
2. The symbol $\propto$ (i.e. a $\propto$ b) means .....
3. Σ^{a}_{k} represents the ..... (sub) k
4. $\mathbf{X} \rightarrow \infty$ reads as follows: ..... x tends to infinity
5. 4:3 is the ..... of width to height in standard television
6. $2\mathbf{x} - 4 = 10$, in this equation, **x** is the ..... (or the unknown).
7. $|a| \geq 0$ is the ..... value of a is greater than or equal to 0
8. $X + 5 \neq X - 5$ reads as follows:
   capital X plus 5 ..... capital X minus 5
9. (a + b) reads as follows: a plus b .....
10. You can use {} – ....., where you cannot use either ( ) round or [ ] ..... brackets.
11. Other symbols are:
    < ..... than
    $\leq$ less than or equal to
    > ..... than
    $\geq$ greater than or equal to
    « ..... less than
    » much greater than



GAP-FILL

Ex. 7 Complete the statements below with the correct words:

1. Any number _____ by one is equal to the number itself.
2. A numerical value is _____ when the value is replaced by another that is approximately equal to it.
3. _____ by zero is impossible.
4. Multiplication and division are inverse _____.
5. We do not need any _____ to write the equation: $3 \times 5 - 2 \times 5 = 5$
6. _____ is the result of division.

MATCHING

Ex. 8 Match the symbols with their definitions

Symbols		Definitions
a. <	.....	1. is equal to / equals
b. =	.....	2. the absolute value of b
c. 0.	.....	3. infinity
d. {...}	.....	4. greater than or equal to
e. (...)	.....	5. not equal to
f. >	.....	6. less than or equal to
g. [...]	.....	7. divided by
h. +	.....	8. (in) brackets / parentheses
i. ÷	.....	9. approximately equal to
j. ≡	.....	10. (in) braces / curly brackets
k. –	.....	11. identical to
l. ≈	.....	12. plus
m. ≤	.....	13. the sum of (X values)
n. ∞	.....	14. (in) square brackets
o. →	.....	15. minus
p. ≥	.....	16. tends to
q. ≠	.....	17. decimal point
r. | b |	.....	18. greater than
s. ∑	.....	19. less than

NOTE:

% – **per cent** (also **percent** in US), **percentage – a percentage** in mathematics is defined as a **ratio** or **fraction** of 100, i.e. *20 per cent of 200 equals 40*. Other examples:

- 10 **percent** of school children are overweight.
- Tax is paid as **a percentage** of income.

‰ – **per mil** is defined as one part per thousand:

$1‰ = 10^{-3} = 0.001 = 0.1\%$

$1\% = 10‰$

Do not use the words: *smaller* or *bigger* to refer to < or >
Use *is equal **to** something* and *equal**s** something* accurately.
In English, we say: *open/close brackets*
In English *a **multiplied** by b* is equivalent to *a **times** b*,
× is the *sign of multiplication* or ***multiplication sign*** (**a × b**)
*a **divided** by b* is the same as *a **over** b*
: is the ***sign of division***

3. FRACTIONS

TRANSLATION WORK:

fraction .....
vulgar fraction .....
proper / improper fraction .....
numerator .....
denominator .....
common denominator .....
decimal fraction .....
repeating decimal .....
common factor .....
reduce to lowest terms .....
converting .....
add / subtract / multiply /divide fractions .....
reciprocal .....

READING

The cat's mystery

Here is a story of a tomcat. He was born on a sunny Sunday. He spent one third of his life enjoying the time on a pillow in a wealthy house. After another one sixth of his life, he hunted the first mouse. He escaped after the next two ninths of his life. He was found after the next two tenths of his life. It was then, when he brought a female cat with it back home. After another one sixth of a year, 4 kittens were born. They spent a subsequent year of the tomcat's life together. Unfortunately, the tomcat died in a car accident then.
How old was the cat when he died?
Clue:
In order to **add fractions**, you must first **convert** their **denominators** to a **common** one.

Should you need help, you will find the answer at the bottom of page 29.

MATCHING

Ex. 9 Match the terms (1, 2, 3…) with their definitions (a, b, c…):

1. **fraction** [___]
2. **vulgar (or a common) fraction** [___]
3. **proper fraction** [___]
4. **improper fraction** [___]
5. **decimal fraction** [___]
6. **repeating decimal (or recurring decimal)** [___]

a. consists of an integer numerator – 2/3 (2 is the numerator in this case) and a non-zero integer denominator 2/3 (3 is the denominator in this case)
b. (from Latin: *fractus* – broken) represents a part of a whole, i.e. ½, ¾, etc.
c. if the numerator is greater than the denominator, i.e. 8/5, 4/3, etc.
d. occurs when there is a finite sequence of digits that is repeated indefinitely, i.e. 2/3 = 0.666 – two thirds is equal to (nought) point six six six recurring
e. is a fraction written in the decimal numeral system and whose denominator is a power of ten
f. if the numerator is less than the denominator, i.e. ¼, ½, ¾, and if the absolute value of the fraction is less than 1

GAP-FILL

Ex. 10 Complete the statements below with the correct word(s):

1. In order to reduce a fraction to its lowest terms (to its more basic representation), you have to divide both the numerator and the denominator by the greatest common ……. .
2. In order to add fractions, you must first …….. their denominators to a common one.
3. Subtraction of fractions needs finding a …… …… .
4. In order to …….. a fraction by another fraction, you must multiply both the numerator and the denominator.
5. If you want to multiply a fraction by a whole number, you must convert the number to its equivalent …… fraction.
6. In order to divide a fraction by a fraction, you must multiply the fraction by the …… of the other.

MATCHING

Ex. 11 Match the terms (1, 2, 3…) with their definitions (a, b, c…):

1.	½	a.	is a reciprocal of 1/7
2.	0.45	b.	is an improper fraction
3.	8/7	c.	is a decimal fraction
4.	¼	d.	is the more basic representation of 4/8
5.	1/3	e.	is not a fraction
6.	7/1	f.	is a vulgar fraction
7.	2	g.	represents a recurring decimal 0.333

TRUE/FALSE

Ex. 12 State whether the following sentences are true (T) or false (F):

1. ¼ (a quarter) exemplifies an improper fraction. [___]
2. In order to divide a fraction by a fraction, one must multiply the fraction by their common denominator. [___]
3. A decimal fraction – is a fraction written in the decimal numeral system and whose denominator is a power of ten. [___]
4. In order to add fractions, one adds their numerators and denominators. [___]
5. Repeating decimal (or recurring decimal) occurs when there is a finite sequence of digits that is repeated indefinitely. [___]
6. A fraction represents a part of a whole. [___]
7. Subtraction of fractions is a reciprocal of addition of fractions. [___]
8. In order to reduce a fraction to its lowest terms, one has to divide both the numerator and the denominator by the smallest common factor. [___]
9. In ¾ , integer 3 represents the denominator. [___]

NOTE:

Here are some examples on how to write fractions in their word representations:

½ – **a half, one half**

⅓ – **a third / one third**

¼ – **a quarter / one quarter / one fourth**

⅛ – **an eighth / one eighth**

⅔ – **two thirds**

⅜ – **three eighths**

¾ – **three quarters / three fourths**

⅝ – **five eighths**

4¾ – four **and** three quarters/three fourths

11⅜ – eleven **and** three eighths

4. POWERS, LOGARITHMS AND ROOTS

TRANSLATION WORK:

power .....

raise a number to a power .....

square .....

squared .....

cubed .....

cubic .....

constant .....

logarithm .....

base .....

common logarithm .....

natural logarithm .....

superscript/subscript .....

root .....

to extract a root .....

READING

2 + 2 = 4, or else?

You might be wondering why there are so many mathematical operations and terms necessary for everyone to learn. Subtraction, addition, multiplication, division, raising numbers to powers, etc. make us wiser and civilized. In one case, we boastfully calculate a 15% discount off the price of the sweater we want to buy. In another case, we are proud when we explain to the less educated that a logarithm is not an abbreviation of low-ga-rhythm or that a square root has nothing to do with an ivy plant. We take mathematics and our skills for granted until we come across a mysterious discovery that 8 = 7.

Now, you have become intrigued. Read this[3]:
Let us assume that $x + y = z$,
Therefore $x = 8x - 7x$, $y = 8y - 7y$, and finally $z = 8z - 7z$
So, $8x - 7x + 8y - 7y = 8z - 7z$,
Next, $8x + 8y - 8z = 7x + 7y - 7z$
Then, $8(x + y - z) = 7(x + y - z)$
Finally, $8 = 7$ ☺

MATCHING

Ex. 13 Match the terms (1, 2, 3…) with their definitions (a, b, c…):

1. x^2 [___]
2. x^3 [___]
3. x^n [___]
4. x^{-n} [___]
5. **ln** [___]
6. $\log_b c$ [___]
7. $\sqrt{a} = x$ [___]
8. $\sqrt[n]{a} = x$ [___]
9. $\sqrt[3]{a} = x$ [___]

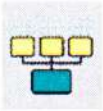
a. x to the power of n / x to the n-th power / x to the n-th
b. the natural logarithm, it has the constant *e* as its base, i.e. ln x (the logarithm of x to the base *e*, phonetically: [el en of eks])
c. the square root of a is/equals x
d. the cube/cubic root of a is /equals x
e. x cubed
f. x to the power of minus n / x to the minus n-th
g. the n^{th} root of a is /equals x
h. x squared
i. the logarithm of *c* to the base *b*

SPEAKING – PAIR WORK

Ex. 14 Student A: read out loudly the terms / equations 1-4;
Student B: write and confirm the correct versions. Next, Student B read out loudly the terms / equations 5 – 8 (p. 22) . Student A - write and confirm the correct versions.

1. 100^{-n}
2. k^3m^{4-c}
3. $\sqrt{x} = p : m$
4. $a^0 = 1$ (when $a \neq 0$)

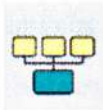
[3] Adapted from [www.ahajokes.com, 2013].



GAP-FILL

Ex. 15 Complete the following rules/definitions:

1. In x_q – q is called a _____ and is written slightly below the baseline.
2. Volume is given in _____ centimeters (cc), meters, etc.
3. Extracting a root is an inverse operation to _____ a number to a power, i.e. $\sqrt{\mathbf{a}} = \mathbf{x}$ and $\mathbf{x^2} = \mathbf{a}$.
4. In x^k – k can be called a _____ (or _____).
5. Any $a^0 = 1$, when a is not _____ to 0.
6. $x^m : x^n$ is equal to x^{m-n} (x to the _____ of m minus n).
7. You should add powers when you multiply numbers of the same _____.
8. The logarithm of a x to the base b ($\log_b x$) is the _____ to which the _____ must be raised to produce x.
9. If base b = 10, the logarithm is called _____ logarithm.
10. When we multiply numbers with the same base (i.e. $x^m \cdot x^n$), we _____ the powers (i.e. x^{m+n})



GAP-FILL

Ex. 16 Read this mathematical equation and fill in the blanks with the words given below. There are two extra words you do not need to use.

$$\{(x+y)^3 - \sqrt{a}\}^{-1} \; x^3 + \log_a x = \tfrac{2}{3}$$

a. **power**	b. **base**	c. **brackets**	d. **braces**	e. **root**
f. **cubed**	g. **over**	h. **multiplied**	i. **squared**	j. **thirds**
k. **times**				

X plus y in (1) _____ to the (2) _____ of three minus the square (3)_____ of a; all in (4) _____ and to the minus one. All this (5)_____ by x (6)_____ plus the logarithm of x to the (7) _____ a is equal to two (8) _____.

(Ex. 14) Student B:

5. $\log_b x$
6. $x^n - 9(n-1) = \sqrt[n]{b}$
7. $\sqrt[4]{x} = K$
8. $a^n = \frac{5}{8} + \sqrt{b} - b$

NOTE:

x^k – k can be called a superscript (or index)

x_q – q is called a subscript and is written slightly below the baseline

cc can mean the following:

- ***cubic centimetre(s) / centimeter(s)*** – for instance the cubic capacity of an engine
- or ***carbon copy*** – *used in a business letter or email to show that you are sending a copy to someone else*

$\sqrt[4]{a}$ is read the four**th,** $\sqrt[5]{a}$ – the fif**th**,$\sqrt[6]{a}$ – the six**th** etc. root of a

5. GEOMETRY

5.1. Two-dimensional geometry

A. Lines, angles, triangles

TRANSLATION WORK:

line .....

- solid .....
- broken .....
- dotted .....
- diagonal .....
- wavy .....
- straight .....
- curved .....
- parallel .....
- perpendicular .....
- intersecting .....

line segment .....

points/endpoints .....

vector .....

ray .....

angle .....

- acute .....

- obtuse .....
- right .....
- straight .....
- full .....

vertex .....

triangle .....

- acute .....
- obtuse .....
- equilateral .....
- isosceles .....
- scalene .....

MATCHING

Ex. 17 Match the names of lines (1, 2, 3…) with their equivalent representations (a, b, c…):

1.	dotted	[____]	a. 
2.	straight	[____]	b.
3.	curved	[____]	c. 
4.	perpendicular	[____]	d.
5.	intersecting	[____]	e. 
6.	diagonal	[____]	f. 
7.	broken	[____]	g. 
8.	parallel	[____]	h.
9.	vertical	[____]	i. 

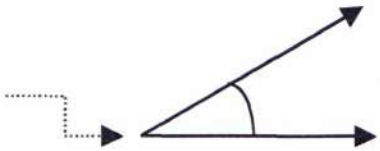
GAP-FILL

Ex. 18 Complete the following rules/definitions with the words given below:

full acute vertex segment obtuse extends right straight

1. a line _____ AB has two distinct endpoints: A and B
2. a ray/vector begins at its endpoint and _____ in one direction
3. an _____ angle measures between 0 and 90 degrees

4. a _____ angle is an angle measuring 90°
5. an _____ angle measures over 90° and less than 180°
6. a _____ angle = 180°
7. a _____ angle = 360°
8. the _____ of an angle *is the point where two rays that form the angle intersect*[4]



MATCHING

Ex. 19 Read the following definitions and match the triangles to their names (there is often more than one option possible).

a triangle is a three-sided polygon
an equilateral triangle has all three sides of equal length (all the angles measure 60°)
an isosceles triangle has at least two sides of equal length
a scalene triangle is a triangle having three sides of different length
an acute triangle has three acute angles
an obtuse triangle has one obtuse angle

1. ____
2. ____
3. ____
4. ____
5. ____
6. ____

a. equilateral ___
b. isosceles ___
c. obtuse ___
d. right-angled ___
e. scalene ___
f. acute ___

[4] Kucharska-Raczunas, Maciejewska, 2010, p. 56.

SPEAKING – PAIR WORK

Ex. 20 Read / dictate and draw:

Student A: read the first description given below
Student B: follow the description and do the drawing on a separate sheet of paper. Then, change roles for the other description.

1. *There is a horizontal base line. On the line, there are 2 equal circles supporting one rectangular shape. In the middle of the rectangle, there is a square whose bottom and top sides come within the sides of the rectangle. On the left side of the rectangle and at its lower corner, there is an isosceles triangle whose base comes within the side of the rectangle and is approximately equal to 2/3 of its side. The vertex of the triangle is the center of another circle whose diameter equals 1/3 of the side of the triangle.*
2. *There is a vertical line which intersects a horizontal line. On the right side of the vertical line, there is a square whose one side comes within the vertical line. On the other side of the vertical line, there is a circle. There is another horizontal line which joins the center of the circle to the center of the square.*

GAP-FILL

Ex. 21 Complete the following sentences:

1. An _____ angle measures between 0 and 90 degrees.
2. A _____ angle measures 90 degrees.
3. A _____ triangle has three sides of different lengths.
4. An _____ angle measures between 90 and 180 degrees.
5. An _____ triangle is a triangle in which all 3 sides are equal.
6. A triangle is a _____-_____ polygon.
7. The point where two rays that form an angle intersect is called the _____ of the angle.
8. A line segments has two distinct _____.

B. Polygons

TRANSLATION WORK:

polygons .....
quadrilateral .....

square .....
rectangle .....
parallelogram .....
rhombus .....
trapezoid .....
pentagon .....
hexagon .....
heptagon .....
octagon .....
nonagon .....
decagon .....
circle .....
chord .....
circumference .....
diameter .....
radius .....

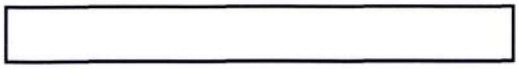
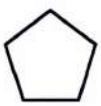
READING

Pentagon

The Pentagon, which is the Headquarters of the United States Department of Defense, takes its name after its design shape of a pentagon and is the biggest office building in the world. Hardly anybody knows that there is a central plaza inside it (also in the shape of a pentagon), which is informally known as *ground zero*[5].

How to make a pentagon?

In order to make a regular pentagon[6] (all edges of the same length), you need a rectangular strip of paper, yet it must be relatively long. For instance, it can be approximately 20 cm long and 3 cm wide.

Now, having a shape like this: 
make this: 

See the answer at the bottom of page 32.

[5] Wikipedia, The Pentagon, 2013.
[6] Adapted from [Steward, 2008, p. 34].

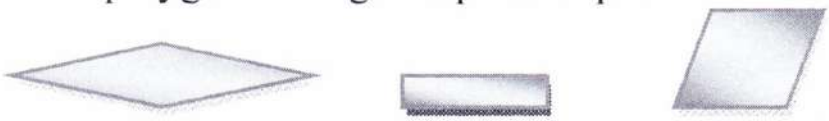
Polygons

Polygons can be **regular** (the sides are all of the same length and the angles are all the same) or **irregular/non-regular**

a square – a four-sided polygon having all sides of equal length

a rectangle – a four-sided polygon having all right angles

a parallelogram – a four-sided polygon having two pairs of parallel sides



a rhombus – a **quadrilateral** whose all sides are of the same length

a trapezoid – a four-sided polygon which has exactly one pair of parallel sides

a pentagon – a five-sided polygon

a hexagon – a six-sided polygon

a heptagon – a seven-sided polygon

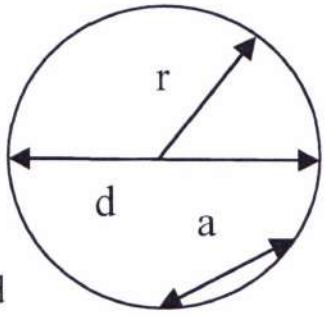
an octagon – an eight-sided polygon,

a nonagon – a nine-sided polygon

a decagon – a ten-sided polygon

Circle

r – radius c – ? (see TASK below)
d – diameter



a – chord

TASK: mark the circle's circumference in the picture of a circle,
circumference (c) $c = 2 \pi r$

TASK: True or false?
The mathematical constant **pi** (π) represents the ratio of a circle's circumference to its diameter. [____] (See the bottom of the page for the answer[7])

TRUE/FALSE

Ex. 22 State whether the following statements are true (T) or false (F)

1. The diagonals in a square intersect at a right angle. [____]
2. A hexagon is a seven-sided polygon. [____]
3. The sum of the angles of a rectangle is 360 degrees. [____]
4. The sides of a polygon intersect in exactly two places each. [____]
5. A trapezoid is a quadrilateral which has all sides of equal length. [____]
6. The point where two rays that form an angle intersect is called the bisector of the angle. [____]
7. A polygon can only be made of line segments. [____]

5.2. Three-dimensional geometric figures

TRANSLATION WORK:

cube .....
cuboid .....
cubical .....

---

[7] Yes.
Answer:
The cat lived 15 years, clue: x – cat's lifespan, so x/3 + x/6 + 2x/9 + 2x/10 + 1/6 + 1 = x

cone .....
cylinder .....
pyramid .....
sphere .....
hemisphere .....
tetrahedron .....
volume .....

READING

A spherical dilemma

The sphere is more important than the cube. This provocative thesis can be supported by countless arguments, and hands are raised when it comes to giving examples in favor of this statement. The arguments range from serious ones pointing to spherical planets, atoms, or lenses, to such touching ones showing the very first toy to be a spherical ball. Those in the minority then put forward quite a rational argument that the playing dice is cubical and so are the stairs. Finally, they claim that the brick, which is a basic building block most widely known to the human kind, is a cuboid. Of course, an intense scientific debate can be initiated emphasizing the fact that soap bubbles could never be cubical because their surface tension allows for spherical shapes exclusively. The discussion would continue. But does it really matter? ☺

MATCHING

Ex. 23 Match the following 3D figures to their names

a. 

b. 

c.

d. 

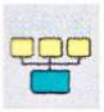
e. 

f. 

1. tetrahedron [____]
2. sphere [____]
3. cylinder [____]
4. pyramid [____]
5. cube [____]
6. cone [____]

TASK: Complete the table by writing the equivalent adjective, use the glossary at the end of the book:

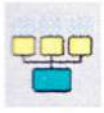
Noun	Adjective	Noun	Adjective
cone	.....	hemisphere	.....
cube	.....	cylinder	.....
sphere	.....	pyramid	.....



GAP-FILL

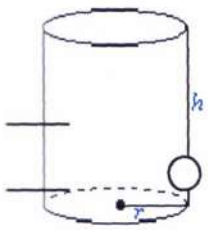
Ex. 24 Read the following descriptions and name the proper shape.

1. It is a three-dimensional figure which has six matching sides. [_____]
2. It is a three-dimensional shape having a circular base and a single vertex. [_____]
3. It is a three-dimensional shape having all of its points at the same distance from its center. [_____]
4. It is a four-sided three-dimensional shape, each face of which is a triangle. [_____]
5. It is a three-dimensional shape with a square base and 4 triangle sides. [_____]
6. It is a three-dimensional shape having two circular bases of the same shape and size that are parallel. [_____]



GAP-FILL

Ex. 25 Analyze this simple drawing of an electric boiler and complete the description below with the correct words.



A boiler is a (1) c _ _ _ _ _ _ _ with a (2) r _ _ _ _ _ ***r*** and the (3) h _ _ _ _ _ ***h***.

In order to calculate its surface area we need to add:

The area of the top and bottom (4) c _ _ _ _ _ _ $(2 \pi r^2)$ + the area of the side $(2\pi rh)$.

Therefore, the surface area (A) is (5) e _ _ _ _ to:

$A = 2 \pi r^2 + 2 \pi r h$

$A = 2 \pi r (r + h)$ A is equal to $2 \pi r$ times $r + h$ in (6) b_ _ _ _ _ _ _

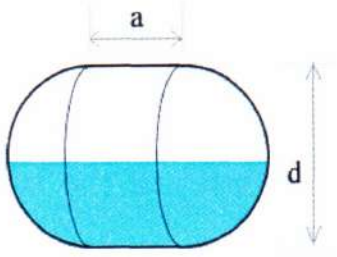
GAP-FILL

Ex. 26 Complete the following description of a pyramid and a sphere with the correct words.

The (1) v_ _ _ _ _ **V** of a **pyramid** is $V = 1/3\ Bh$, where ***B*** is the (2) a_ _ _ of the base and ***h*** is the (3) h_ _ _ _ _ (***h*** is (4) p_ _ _ _ _ _ _ _ _ _ _ _ _ to the plane of the base).
The base of a pyramid can be a regular (5) p_ _ _ _ _ _. If the base is circular, the pyramid becomes a (6) c _ _ _.
In classical geometry, the volume **V** of a **sphere** reads as follows:
$V = {}^4/_3 \pi r^3$ where $\mathbf{r^3}$ stands for (7) **r** _ _ _ _ _ to the (8) p_ _ _ _ of 3.

GAP-FILL

Ex. 27 How to calculate the volume V of a capsule tank[8]? Complete the description.



We treat a capsule as an object composed of a (1) s_ _ _ _ _ of diameter **d** split in half and separated by a (2) c_ _ _ _ _ _ _ of diameter **d** and (3) h_ _ _ _ _ **a**.

Therefore, the total volume $V = Vs + Vc$
$Vs = 4/3\pi\mathbf{r}^3$ where:

r is (4) r_ _ _ _ _ **r** = **d** : 2 **d** is (5) d_ _ _ _ _ _ by 2
$\mathbf{r}^3$ is **r** (6) c_ _ _ _
$Vc = \pi\, \mathbf{r}^2\mathbf{a}$, $\mathbf{r}^2$ is **r** (7) s _ _ _ _ _ _

NOTE:

a line in geometry **extends indefinitely** in both directions
a right angle = 90°, whereas a straight angle = 180°

---

[8] Adapted from: [www.calculatorsoup.com, 2012]
In order to make a pentagon, you must tie a knot (very carefully), and fold the ends:


[Steward, 2008, p. 269].

a 90-degree triangle is a right-angled triangle
an **equilateral** triangle – its sides are all of the same length
an **equiangular** triangle – its angles are all of the same measure
Remember the difference between solid vs straight lines.

Volume and capacity:
volume – volume is a measure of how much space a 3D shape takes up
capacity – a term in economics, management, engineering, etc., only similar to volume
half of a sphere is called a **hemisphere**