

Ćwiczenia 5. Twierdzenie de l'Hospitala. Wyznaczanie asymptot funkcji.

Zad.1. Oblicz granice:

$$\frac{\frac{e^x - 1}{\sin(2x)}}{1 - \cos x}$$

$$\frac{\frac{x^2}{e^{3x}}}{\frac{x^2}{\ln x}}$$

$$\frac{x}{\arctg x}$$

$$\lim_{x \rightarrow 0} \frac{x - \arctg x}{x^3}$$

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 2\sin x}{\cos(3x)}$$

$$\lim_{x \rightarrow 0} (1 - e^{2x}) \operatorname{ctg} x$$

$$\lim_{x \rightarrow 1} \frac{\ln x}{1 - x^3}$$

$$\lim_{x \rightarrow 0} (e^{2x} + x)^{\frac{1}{x}}$$

$$\lim_{x \rightarrow 0} (1 - e^{2x}) \operatorname{ctg} x$$

$$\frac{\ln(\sin(3x))}{\ln(\sin(4x))}$$

$$\frac{\sqrt{x}}{\arcsin(2x)}$$

$$\frac{x^x}{(\sin x)^{\sqrt{x}}}$$

$$\frac{\frac{x - 1}{\ln x}}{x - \sin x}$$

$$\frac{\frac{x^3}{\operatorname{tg} x - \sin x}}{x - \sin x}$$

$$\frac{\operatorname{tg}(3x)}{1 - \operatorname{tg} x}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos(2x)}{e^{2x} - 1}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1 + 2x)}{\frac{1}{x \sin x} - \frac{1}{x^2}}$$

$$\lim_{x \rightarrow 1} \frac{e^x - e^{-x}}{\sin x}$$

$$\lim_{x \rightarrow \infty} \frac{\ln(\ln x)}{x}$$

$$x \ln x$$

$$\frac{\ln x}{\operatorname{ctg} x}$$

$$\frac{(\arcsin x)^{2x}}{(1 - x) \ln(1 - x)}$$

Zad.2. Wyznacz asymptoty funkcji f :

$$f(x) = \frac{x^2}{x - 1}$$

$$f(x) = \frac{x + 1}{x^2 - 4x + 3}$$

$$f(x) = \sqrt{x^2 - 4}$$

$$f(x) = x e^{\frac{1}{x}} - x$$

$$f(x) = \ln(1 - x^2)$$

$$f(x) = \frac{3x^2 + 1}{|x| + 2}$$

$$f(x) = \frac{x^3}{x^2 - 1}$$

$$f(x) = \frac{2x^2 - 4x}{x^2 - 4}$$

$$f(x) = x \arctg \frac{1}{x}$$

$$f(x) = \ln x - \arctg x$$

$$f(x) = \left(x + \frac{1}{x + 2}\right) \operatorname{arctg} x$$

$$f(x) = \frac{4x^2 - 1}{|x + 1| + 1}$$

$$f(x) = \frac{\sqrt{x^2 + 9}}{x + 3}$$

$$f(x) = \frac{x\sqrt{x^2 + 1}}{x + 2}$$

$$f(x) = \sqrt{4x^2 - x}$$

$$f(x) = \begin{cases} \frac{6x^2}{|x| - 1} & , |x| \\ \geq 2 \frac{1}{\sqrt{x^2 - 4}} & , |x| < 2 \end{cases}$$