

# Hands-on training session 2

Hui-Walter models for diagnostic test evaluation

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# Introduction

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Date/time:

- 19th February 2020
- 16.00 - 17.00

Teachers:

- Matt Denwood (presenter)
- Giles Innocent

# Recap

- Fitting models using MCMC is easy with JAGS / runjags
- But we must **never forget** to check convergence and effective sample size!
- More complex models become easy to implement
  - For example imperfect diagnostic tests
  - But remember to be realistic about what is possible with your data

# Recap

- Fitting models using MCMC is easy with JAGS / runjags
- But we must **never forget** to check convergence and effective sample size!
- More complex models become easy to implement
  - For example imperfect diagnostic tests
  - But remember to be realistic about what is possible with your data
- So how do we extend these models to multiple diagnostic tests?

## **Session 2a: Hui-Walter models for 2 tests and 1 population**

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# Hui-Walter Model

- A particular model formulation that was originally designed for evaluating diagnostic tests in the absence of a gold standard
- Not necessarily (or originally) Bayesian but now usually implemented using Bayesian MCMC
- But evaluating an imperfect test against another imperfect test is a bit like pulling a rabbit out of a hat
  - If we don't know the true disease status, how can we estimate sensitivity or specificity for either test?

## Model Specification

```
model{  
  Tally ~ dmulti(prob, TotalTests)  
  
  # Test1- Test2-  
  prob[1] <- (prev * ((1-se[1])*(1-se[2]))) + ((1-prev) *  
  
  # Test1+ Test2-  
  prob[2] <- (prev * ((se[1])*(1-se[2]))) + ((1-prev) *  
  
  # Test1- Test2+  
  prob[3] <- (prev * ((1-se[1])*(se[2]))) + ((1-prev) *
```



```

# Test1+ Test2+
  prob[4] <- (prev * ((se[1])*(se[2]))) + ((1-prev) * ((1-

prev ~ dbeta(1, 1)
se[1] ~ dbeta(1, 1)
sp[1] ~ dbeta(1, 1)
se[2] ~ dbeta(1, 1)
sp[2] ~ dbeta(1, 1)

#data# Tally, TotalTests
#monitor# prev, prob, se, sp
#inits# prev, se, sp
}

```

```
1 twoXtwo <- matrix(c(48, 12, 4, 36), ncol=2, nrow=2)
2 twoXtwo
```

```
##      [,1] [,2]
## [1,]  48   4
## [2,]  12  36
```

```
1 library('runjags')
2
3 Tally <- as.numeric(twoXtwo)
4 TotalTests <- sum(Tally)
5
6 prev <- list(chain1=0.05, chain2=0.95)
7 se <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
8 sp <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
9
10 results <- run.jags('basic_hw.bug', n.chains=2)
```

```
## Warning: You should update the rjags package to version
## (This warning is given once per R session)
```

## 1 results

|         | Lower95 | Median | Upper95 | SSeff | psrf |
|---------|---------|--------|---------|-------|------|
| prev    | 0.311   | 0.441  | 0.576   | 4269  | 1    |
| prob[1] | 0.362   | 0.462  | 0.554   | 14011 | 1    |
| prob[2] | 0.074   | 0.132  | 0.203   | 13515 | 1    |
| prob[3] | 0.017   | 0.055  | 0.103   | 9019  | 1    |
| prob[4] | 0.253   | 0.343  | 0.437   | 12884 | 1    |
| se[1]   | 0.821   | 0.932  | 1.000   | 5613  | 1    |
| se[2]   | 0.689   | 0.846  | 1.000   | 3493  | 1    |
| sp[1]   | 0.747   | 0.879  | 1.000   | 3492  | 1    |
| sp[2]   | 0.862   | 0.948  | 1.000   | 5691  | 1    |

- Note the wide confidence intervals!

- These models need A LOT of data
  - And/or strong priors for one of the tests
- Convergence is more problematic than usual
- Be **very** careful with the order of combinations in `dmultinom`!
- Check your results carefully to ensure they make sense!

## Label Switching

How to interpret a test with  $Se=0\%$  and  $Sp=0\%$ ?

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- The test is perfect - we are just holding it upside down...

We can force  $se+sp \geq 1$ :

```
1 se[1] ~ dbeta(1, 1)
2 sp[1] ~ dbeta(1, 1)T(1-se[1], )
```

Or:

```
1 se[1] ~ dbeta(1, 1)T(1-sp[1], )
2 sp[1] ~ dbeta(1, 1)
```

But not both!

This allows the test to be useless, but not worse than useless

# Simulating data

Analysing simulated data is useful to check that we can recover parameter values.

```
1  se1 <- 0.9; sp1 <- 0.95;
2  se2 <- 0.8; sp2 <- 0.99
3  prevalence <- 0.5; N <- 100
4
5  truestatus <- rbinom(N, 1, prevalence)
6  Test1 <- rbinom(N, 1, (truestatus * se1) + ((1-truestatus) *
   ↪  (1-sp1)))
7  Test2 <- rbinom(N, 1, (truestatus * se2) + ((1-truestatus) *
   ↪  (1-sp2)))
8
9  twoXtwo <- table(Test1, Test2)
10 Tally <- as.numeric(twoXtwo)
```

Can we recover these parameter values?



## Exercise

Modify the code in the Hui Walter model to force tests to be no worse than useless

Simulate data and recover parameters for:

- $N=10$ ,  $N=100$ ,  $N=1000$

## Optional Exercise

Use priors for test1 taken from session 1 and compare the results

## **Session 2b: Hui-Walter models for 2 tests and N populations**

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# Hui-Walter models with multiple populations

- Basically an extension of the single-population model
- Works best with multiple populations each with differing prevalences
  - These could even be subgroups of individuals within the same population if there are e.g. known risk factors for disease status
  - Remember that the focus is usually to estimate the diagnostic test parameters and not the prevalence in the different populations/subgroups!

# Independent intercepts for populations

```
1  model{
2    for(p in 1:Populations){
3      Tally[1:4, p] ~ dmulti(prob[1:4, p], TotalTests[p])
4      # Test1- Test2- Pop1
5      prob[1, p] <- (prev[p] * ((1-se[1])*(1-se[2]))) +
↪    ((1-prev[p]) * ((sp[1])*(sp[2])))
6      ## etc ##
7      prev[p] ~ dbeta(1, 1)
8    }
9
10   se[1] ~ dbeta(HPSe[1,1], HPSe[1,2])T(1-sp[1], )
11   sp[1] ~ dbeta(HPSp[1,1], HPSp[1,2])
12   se[2] ~ dbeta(HPSe[2,1], HPSe[2,2])T(1-sp[2], )
13   sp[2] ~ dbeta(HPSp[2,1], HPSp[2,2])
14
15   #data# Tally, TotalTests, Populations, HPSe, HPSp
16   #monitor# prev, prob, se, sp
17   #inits# prev, se, sp
18 }
```

We would usually start with individual-level data in a dataframe:

```
1  se1 <- 0.9; sp1 <- 0.95; sp2 <- 0.99; se2 <- 0.8
2  prevalences <- c(0.1, 0.5, 0.9)
3  N <- 100
4
5  simdata <- data.frame(Population = sample(seq_along(prevalences),
   ↪   N, replace=TRUE))
6  simdata$probability <- prevalences[simdata$Population]
7  simdata$truestatus <- rbinom(N, 1, simdata$probability)
8  simdata$Test1 <- rbinom(N, 1, (simdata$truestatus * se1) +
   ↪   ((1-simdata$truestatus) * (1-sp1)))
9  simdata$Test2 <- rbinom(N, 1, (simdata$truestatus * se2) +
   ↪   ((1-simdata$truestatus) * (1-sp2)))
10
11 head(simdata)
```

| ##   | Population | probability | truestatus | Test1 | Test2 |
|------|------------|-------------|------------|-------|-------|
| ## 1 | 2          | 0.5         | 0          | 0     | 0     |
| ## 2 | 3          | 0.9         | 1          | 1     | 1     |
| ## 3 | 1          | 0.1         | 0          | 0     | 0     |
| ## 4 | 2          | 0.5         | 1          | 1     | 1     |
| ## 5 | 3          | 0.9         | 1          | 0     | 1     |
| ## 6 | 2          | 0.5         | 0          | 0     | 0     |

[Except that probability and truestatus would not normally be known!]

The model code and data format for an arbitrary number of populations (and tests) can be determined automatically

There is a function (provided in the GitHub repo) that can do this for us:

```
1  simdata$Population <- factor(simdata$Population,  
  ↪  levels=seq_along(prevalences), labels=paste0('Pop_',  
  ↪  seq_along(prevalences)))  
2  
3  source("autohuiwalter.R")  
4  auto_huiwalter(simdata[,c('Population', 'Test1', 'Test2')],  
  ↪  outfile='autohw.bug')
```



This generates self-contained model/data/initial values etc (ignore covse and covsp for now):

```
model{  
  
  ## Observation layer:  
  
  # Complete observations (N=100):  
  for(p in 1:Populations){  
    Tally_RR[1:4,p] ~ dmulti(prob_RR[1:4,p], N_RR[p])  
  
    prob_RR[1:4,p] <- se_prob[1:4,p] + sp_prob[1:4,p]  
  }  
  
  ## Observation probabilities:
```

And can be run directly from R:

```
1 results <- run.jags('autohw.bug')
2 results
```

|         | Lower95 | Median | Upper95 | SSeff | psrf |
|---------|---------|--------|---------|-------|------|
| se[1]   | 0.752   | 0.865  | 0.960   | 9289  | 1    |
| se[2]   | 0.774   | 0.894  | 1.000   | 7079  | 1    |
| sp[1]   | 0.761   | 0.869  | 0.963   | 8255  | 1    |
| sp[2]   | 0.927   | 0.982  | 1.000   | 4086  | 1    |
| prev[1] | 0.000   | 0.056  | 0.160   | 6423  | 1    |
| prev[2] | 0.267   | 0.428  | 0.598   | 10994 | 1    |
| prev[3] | 0.696   | 0.852  | 0.976   | 8634  | 1    |
| covse12 | 0.000   | 0.000  | 0.000   | NA    | NA   |
| covsp12 | 0.000   | 0.000  | 0.000   | NA    | NA   |

- Modifying priors must still be done directly in the model file
- The model needs to be re-generated if the data changes
  - But remember that your modified priors will be reset
- There must be a single column for the population (as a factor), and all of the other columns (either factor, logical or numeric) are interpreted as being test results
- The function will soon be included in the runjags package
  - Feedback welcome!

## Observation-level model specification

```
model{  
  
  for(i in 1:N){  
    Status[i] ~ dcat(prob[i, ])  
  
    prob[i,1] <- (prev[i] * ((1-se[1])*(1-se[2]))) +  
                 ((1-prev[i]) * ((sp[1])*(sp[2])))  
    prob[i,2] <- (prev[i] * ((se[1])*(1-se[2]))) +  
                 ((1-prev[i]) * ((1-sp[1])*(sp[2])))  
    prob[i,3] <- (prev[i] * ((1-se[1])*(se[2]))) +  
                 ((1-prev[i]) * ((sp[1])*(1-sp[2])))  
    prob[i,4] <- (prev[i] * ((se[1])*(se[2]))) +  
                 ((1-prev[i]) * ((1-sp[1])*(1-sp[2])))
```

```

intercept ~ dnorm(0, 0.33)
population_effect[1] <- 0
for(p in 2:Pops){
  population_effect[p] ~ dnorm(0, 0.1)
}
se[1] ~ dbeta(1, 1)T(1-sp[1], )
sp[1] ~ dbeta(1, 1)
se[2] ~ dbeta(1, 1)T(1-sp[2], )
sp[2] ~ dbeta(1, 1)

#data# Status, N, Population, Pops
#monitor# intercept, population_effect, se, sp
#inits# intercept, population_effect, se, sp
}

```

- The main difference is the prior for prevalence in each population
- We also need to give initial values for intercept and population\_effect rather than prev, and tell run.jags the data frame from which to extract the data (except N and Pops):

```
1 intercept <- list(chain1=-1, chain2=1)
2 population_effect <- list(chain1=c(NA, 1, -1), chain2=c(NA, -1,
  ↪ 1))
3 se <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
4 sp <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
5
6 simdata$Status <- with(simdata, factor(interaction(Test1, Test2),
  ↪ levels=c('0.0','1.0','0.1','1.1')))
7 N <- nrow(simdata)
8 Pops <- length(levels(simdata$Population))
9 glm_results <- run.jags('glm_hw.bug', n.chains=2, data=simdata)
```

Also like in session 1, the estimates for  $se/sp$  should be similar, although this model runs more slowly.

Note: this model could be used as the basis for adding covariates

For a handy way to generate a GLM model see  
`runjags::template.jags`

- Look out for integration with `autohuiwalter` in the near (ish) future. . .

## Exercise

Play around with the `autohuiwalter` function

Notice the model and data and initial values are in a self contained file

Ignore the `covse` and `covsp` for now

[There is no particular solution to this exercise!]



# Summary

- Estimating sensitivity and specificity is like pulling a rabbit out of a hat
- Multiple populations helps **a lot**
- Strong priors for one of the tests helps even more!
- Make sure you tabulate the data correctly . . . or use the automated model generator!