

# Hands-on training session 2

Hui-Walter models for diagnostic test evaluation

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Matt Denwood    Giles Innocent

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# Introduction

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Date/time:

- 19th February 2020
- 16.00 - 17.00

Teachers:

- Matt Denwood (presenter)
- Giles Innocent

# Recap

- Fitting models using MCMC is easy with JAGS / runjags
- But we must *never forget* to check convergence and effective sample size!
- More complex models become easy to implement
  - For example imperfect diagnostic tests
  - But remember to be realistic about what is possible with your data

# Recap

- Fitting models using MCMC is easy with JAGS / runjags
- But we must *never forget* to check convergence and effective sample size!
- More complex models become easy to implement
  - For example imperfect diagnostic tests
  - But remember to be realistic about what is possible with your data
- So how do we extend these models to multiple diagnostic tests?

## **Session 2a: Hui-Walter models for 2 tests and 1 population**

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TODO

Background (not necessarily Bayesian)

Rabbits and hats

# Model Specification

```
1  model{
2    Tally ~ dmulti(prob, TotalTests)
3
4    # Test1- Test2-
5    prob[1] <- (prev * ((1-se[1])*(1-se[2]))) + ((1-prev) *
6      ↪ ((sp[1])*(sp[2])))
7
8    # Test1+ Test2-
9    prob[2] <- (prev * ((se[1])*(1-se[2]))) + ((1-prev) *
10     ↪ ((1-sp[1])*(sp[2])))
11
12    # Test1- Test2+
13    prob[3] <- (prev * ((1-se[1])*(se[2]))) + ((1-prev) *
14     ↪ ((sp[1])*(1-sp[2])))
15
16    # Test1+ Test2+
17    prob[4] <- (prev * ((se[1])*(se[2]))) + ((1-prev) *
18     ↪ ((1-sp[1])*(1-sp[2])))
```



```
1
2   prev ~ dbeta(1, 1)
3   se[1] ~ dbeta(1, 1)
4   sp[1] ~ dbeta(1, 1)
5   se[2] ~ dbeta(1, 1)
6   sp[2] ~ dbeta(1, 1)
7
8   #data# Tally, TotalTests
9   #monitor# prev, prob, se, sp
10  #inits# prev, se, sp
11 }
```

```

1  twoXtwo <- matrix(c(48, 12, 4, 36), ncol=2, nrow=2)
2  twoXtwo

1  ##      [,1] [,2]
2  ## [1,]  48   4
3  ## [2,]  12  36

1  library('runjags')
2
3  Tally <- as.numeric(twoXtwo)
4  TotalTests <- sum(Tally)
5
6  prev <- list(chain1=0.05, chain2=0.95)
7  se <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
8  sp <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
9
10 results <- run.jags('basic_hw.bug', n.chains=2)

1  ## Warning: You should update the rjags package to version 5.x -
   ↪ if the version on CRAN is not currently up to date, try
   ↪ downloading from
   ↪ https://sourceforge.net/projects/mcmc-jags/files/rjags/
   ↪ instead

```

## 1 results

	Lower95	Median	Upper95	SSeff	psrf
prev	0.306	0.442	0.574	3860	1.000
prob[1]	0.366	0.461	0.558	13086	1.000
prob[2]	0.072	0.133	0.201	14066	1.000
prob[3]	0.019	0.055	0.104	9199	1.000
prob[4]	0.253	0.344	0.438	12555	1.000
se[1]	0.823	0.931	1.000	5625	1.000
se[2]	0.689	0.847	1.000	3293	1.000
sp[1]	0.745	0.877	1.000	3204	1.000
sp[2]	0.862	0.948	1.000	5311	1.001

- Note the wide confidence intervals!

TODO

Care with order of combinations in `dmultinom`

Lots of data needed

- And/or strong priors for one of the tests

Convergence can be tricky

## Label Switching

How to interpret a test with  $Se=0\%$  and  $Sp=0\%$ ?

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How to interpret a test with  $Se=0\%$  and  $Sp=0\%$ ?

- The test is perfect - we are just holding it upside down...

We can force  $se+sp \geq 1$ :

```
1 se[1] ~ dbeta(1, 1)
2 sp[1] ~ dbeta(1, 1)T(1-se[1], )
```

...

Or:

```
1 se[1] ~ dbeta(1, 1)T(1-sp[1], )
2 sp[1] ~ dbeta(1, 1)
```

But not both!

This allows the test to be useless, but not worse than useless

# Simulating data

Analysing simulated data is useful to check that we can recover parameter values.

Some simulation code:

```
1  se1 <- 0.9
2  sp1 <- 0.95
3  sp2 <- 0.99
4  se2 <- 0.8
5  prevalence <- 0.5
6  N <- 100
7
8  truestatus <- rbinom(N, 1, prevalence)
9  Test1 <- rbinom(N, 1, (truestatus * se1) + ((1-truestatus) *
   ↪ (1-sp1)))
10 Test2 <- rbinom(N, 1, (truestatus * se2) + ((1-truestatus) *
   ↪ (1-sp2)))
11
12 twoXtwo <- table(Test1, Test2)
13 twoXtwo
```



## Exercise

Modify JAGS code to force tests to be better than useless

Simulate data and recover parameters for:

- $N=10$ ,  $N=100$ ,  $N=1000$

## Optional Exercise

Use priors for test1 taken from session 1 and compare the results

# Solution

Model definition:

```
1  model{
2    Tally ~ dmulti(prob, TotalTests)
3
4    # Test1- Test2-
5    prob[1] <- (prev * ((1-se[1])*(1-se[2]))) + ((1-prev) *
6      ↪ ((sp[1])*(sp[2])))
7
8    # Test1+ Test2-
9    prob[2] <- (prev * ((se[1])*(1-se[2]))) + ((1-prev) *
10     ↪ ((1-sp[1])*(sp[2])))
11
12    # Test1- Test2+
13    prob[3] <- (prev * ((1-se[1])*(se[2]))) + ((1-prev) *
14     ↪ ((sp[1])*(1-sp[2])))
15
16    # Test1+ Test2+
17    prob[4] <- (prev * ((se[1])*(se[2]))) + ((1-prev) *
18     ↪ ((1-sp[1])*(1-sp[2])))
```

## Optional Solution

```
1  HPSe[1,] <- c(148.43, 16.49)
2  HPSp[1,] <- c(240.03, 12.63)
```

```
3
```

```
4  HPSe
```

```
1  ##           [,1]  [,2]
2  ## [1,] 148.43 16.49
3  ## [2,]   1.00   1.00
```

```
1  HPSp
```

```
1  ##           [,1]  [,2]
2  ## [1,] 240.03 12.63
3  ## [2,]   1.00   1.00
```

```
1  results <- run.jags('basic_hw.bug', n.chains=2)
```

```
1  ## Finished running the simulation
```

## **Session 2b: Hui-Walter models for 2 tests and N populations**

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# Independent intercepts for populations

```
1  model{
2    for(p in 1:Populations){
3      Tally[1:4, p] ~ dmulti(prob[1:4, p], TotalTests[p])
4
5      # Test1- Test2- Pop1
6      prob[1, p] <- (prev[p] * ((1-se[1])*(1-se[2]))) +
↪    ((1-prev[p]) * ((sp[1])*(sp[2])))
7
8      ## etc ##
9
10     prev[p] ~ dbeta(1, 1)
11   }
12
13   se[1] ~ dbeta(HPSe[1,1], HPSe[1,2])T(1-sp[1], )
14   sp[1] ~ dbeta(HPSp[1,1], HPSp[1,2])
15   se[2] ~ dbeta(HPSe[2,1], HPSe[2,2])T(1-sp[2], )
16   sp[2] ~ dbeta(HPSp[2,1], HPSp[2,2])
17
18   #data# Tally, TotalTests, Populations, HPSe, HPSp
19   #monitor# prev, prob, se, sp
```

We would usually start with individual-level data in a dataframe  
e.g.:

```
1  se1 <- 0.9
2  sp1 <- 0.95
3  sp2 <- 0.99
4  se2 <- 0.8
5  prevalences <- c(0.1, 0.5, 0.9)
6  N <- 100
7
8  simdata <- data.frame(Population = sample(seq_along(prevalences),
  ↪   N, replace=TRUE))
9  simdata$probability <- prevalences[simdata$Population]
10 simdata$truestatus <- rbinom(N, 1, simdata$probability)
11 simdata$Test1 <- rbinom(N, 1, (simdata$truestatus * se1) +
  ↪   ((1-simdata$truestatus) * (1-sp1)))
12 simdata$Test2 <- rbinom(N, 1, (simdata$truestatus * se2) +
  ↪   ((1-simdata$truestatus) * (1-sp2)))
13
14 head(simdata)
```

The model code and data format for an arbitrary number of populations (and tests) can be determined automatically

There is a function (soon to be included in the runjags package, but for now provided in the GitHub repo) that can do this for us:

```
1  simdata$Population <- factor(simdata$Population,  
  ↪  levels=seq_along(prevalences), labels=paste0('Pop_',  
  ↪  seq_along(prevalences)))  
2  
3  source("autohuiwalter.R")  
4  auto_huiwalter(simdata[,c('Population', 'Test1', 'Test2')],  
  ↪  outfile='autohw.bug')  
  
1  ## The model and data have been written to autohw.bug in the  
  ↪  current working directory  
2  ## You should check and alter priors before running the model
```



This generates self-contained model/data/initial values etc (ignore covse and covsp for now):

```
1  ## ## Auto-generated Hui-Walter model created by script version
   ↪ 0.1 on 2020-02-18
2  ##
3  ## model{
4  ##
5  ##   ## Observation layer:
6  ##
7  ##   # Complete observations (N=100):
8  ##   for(p in 1:Populations){
9  ##       Tally_RR[1:4,p] ~ dmulti(prob_RR[1:4,p], N_RR[p])
10 ##
11 ##       prob_RR[1:4,p] <- se_prob[1:4,p] + sp_prob[1:4,p]
12 ##   }
13 ##
14 ##
15 ##   ## Observation probabilities:
16 ##
17 ##   for(p in 1:Populations){
18 ##
19 ##       # Probability of observing Test1- Test2- from a true
```

And can be run directly from R:

```
1 results <- run.jags('autohw.bug')  
  
1 ## Note: The monitored variables 'covse12' and 'covsp12'  
2 ## appear to be non-stochastic; they will not be  
3 ## included in the convergence diagnostic  
4 ## Finished running the simulation
```

```

1  results

1  ##
2  ## JAGS model summary statistics from 20000 samples (chains = 2;
   ↪ adapt+burnin = 5000):
3  ##
4  ##           Lower95      Median Upper95      Mean      SD
5  ## se[1]      0.87189    0.96816      1    0.95577  0.042197
6  ## se[2]      0.60525    0.76164  0.89516    0.75667  0.07517
7  ## sp[1]      0.92611    0.9804      1    0.97362  0.02416
8  ## sp[2]      0.88409    0.94591  0.99308    0.9417  0.029668
9  ## prev[1]    0.0069498  0.069215  0.16152  0.076369  0.043371
10 ## prev[2]    0.18076    0.32083  0.46915    0.32322  0.075138
11 ## prev[3]    0.7327     0.88449  0.99128    0.87414  0.072603
12 ## covse12      0          0          0          0          0
13 ## covsp12      0          0          0          0          0
14 ##
15 ##           Mode      MCerr MC%ofSD SSeff      AC.10
16 ## se[1]      0.98579  0.00066739      1.6   3998    0.015552
17 ## se[2]      0.76962  0.00071055      0.9  11192    0.014574
18 ## sp[1]      0.99125  0.00035858      1.5   4540    0.0054374
19 ## sp[2]      0.9504    0.0003137      1.1   8944   -0.0064593
20 ## prev[1]    0.057135  0.00049806      1.1   7583   -0.00044204

```

## Observation-level model specification

```
1  model{
2
3    for(i in 1:N){
4      Status[i] ~ dcat(prob[i, ])
5
6      prob[i,1] <- (prev[i] * ((1-se[1])*(1-se[2]))) +
7                  ((1-prev[i]) * ((sp[1])*(sp[2])))
8      prob[i,2] <- (prev[i] * ((se[1])*(1-se[2]))) +
9                  ((1-prev[i]) * ((1-sp[1])*(sp[2])))
10     prob[i,3] <- (prev[i] * ((1-se[1])*(se[2]))) +
11                 ((1-prev[i]) * ((sp[1])*(1-sp[2])))
12     prob[i,4] <- (prev[i] * ((se[1])*(se[2]))) +
13                 ((1-prev[i]) * ((1-sp[1])*(1-sp[2])))
14
15     logit(prev[i]) <- intercept +
16       ↪ population_effect[Population[i]]
17   }
18
19   intercept ~ dnorm(0, 0.33)
20   population_effect[1] <- 0
21   for(p in 2:Pops){
```

Just like in session 1, the main difference is the prior for prevalence (this time in each population)

We also need to give initial values for intercept and population\_effect rather than prev, and tell run.jags the data frame from which to extract the data (except N and Pops):

```
1 intercept <- list(chain1=-1, chain2=1)
2 population_effect <- list(chain1=c(NA, 1, -1), chain2=c(NA, -1,
  ↪ 1))
3 se <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
4 sp <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
5
6 simdata$Status <- with(simdata, factor(interaction(Test1, Test2),
  ↪ levels=c('0.0', '1.0', '0.1', '1.1'))))
7 N <- nrow(simdata)
8 Pops <- length(levels(simdata$Population))
9 glm_results <- run.jags('glm_hw.bug', n.chains=2, data=simdata)

1 ## Note: The monitored variable 'population_effect[1]'
2 ## appears to be non-stochastic; it will not be included
3 ## in the convergence diagnostic
```

Also like in session 1, the estimates for  $se/sp$  should be similar, although this model runs more slowly.

Note: this model could be used as the basis for adding covariates

For a handy way to generate a GLM model see  
`runjags::template.jags`

- Look out for integration with `autohuiwalter` in the near (ish) future. . .

Need to be very careful with tabulating the data, or use automatically generated code

Works best when populations have very different prevalences

## Exercise

Play around with the `autohwiwalter` function

Notice the model and data and initial values are in a self contained file

Ignore the `covse` and `covsp` for now