

Hands-on training session 2

Hui-Walter models for diagnostic test evaluation

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Introduction

Date/time:

- 19th February 2020
- 16.00 - 17.00

Teachers:

- Matt Denwood (presenter)
- Giles Innocent

Recap

Important points from session 1

TODO

Session 2a: Hui-Walter models for 2 tests and 1 population

Hui-Walter Model

Background (not necessarily Bayesian)

Rabbits and hats

Model Specification

```
1  model{
2    Tally ~ dmulti(prob, TotalTests)
3
4    # Test1- Test2-
5    prob[1] <- (prev * ((1-se[1])*(1-se[2]))) + ((1-prev) *
6      ↪ ((sp[1])*(sp[2])))
7
8    # Test1+ Test2-
9    prob[2] <- (prev * ((se[1])*(1-se[2]))) + ((1-prev) *
10     ↪ ((1-sp[1])*(sp[2])))
11
12    # Test1- Test2+
13    prob[3] <- (prev * ((1-se[1])*(se[2]))) + ((1-prev) *
14     ↪ ((sp[1])*(1-sp[2])))
15
16    # Test1+ Test2+
17    prob[4] <- (prev * ((se[1])*(se[2]))) + ((1-prev) *
18     ↪ ((1-sp[1])*(1-sp[2])))
19
20    prev ~ dbeta(1, 1)
21    se[1] ~ dbeta(1, 1)
```

- And run it:

```
1  twoXtwo <- matrix(c(48, 12, 4, 36), ncol=2, nrow=2)
2  twoXtwo

1  ##      [,1] [,2]
2  ## [1,]   48   4
3  ## [2,]   12  36

1  library('runjags')
2
3  Tally <- as.numeric(twoXtwo)
4  TotalTests <- sum(Tally)
5
6  prev <- list(chain1=0.05, chain2=0.95)
7  se <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
8  sp <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
9
10 results <- run.jags('basic_hw.bug', n.chains=2)

1  ## Loading required namespace: rjags
1  ## Finished running the simulation
```



```

1  results

1  ##
2  ## JAGS model summary statistics from 20000 samples (chains = 2;
   ↪ adapt+burnin = 5000):
3  ##
4  ##           Lower95   Median Upper95      Mean      SD
5  ## prev          0.30463  0.43905  0.5673  0.43957  0.068498
6  ## prob[1]       0.36893  0.46065  0.55862  0.46158  0.049011
7  ## prob[2]       0.072141  0.13274  0.20048  0.13503  0.033383
8  ## prob[3]       0.01705  0.054976  0.10335  0.058038  0.023171
9  ## prob[4]       0.25229  0.34432  0.43441  0.34535  0.04675
10 ## se[1]         0.82443  0.9326  0.99999  0.92407  0.05346
11 ## se[2]         0.69339  0.85114  0.99992  0.84739  0.090355
12 ## sp[1]         0.7445  0.87349  0.99997  0.87109  0.074509
13 ## sp[2]         0.85954  0.94725  0.99999  0.94012  0.042404
14 ##
15 ##           Mode      MCerr MC%ofSD SSeff      AC.10
16 ## prev          0.43487  0.0010748      1.6  4062  0.031192
17 ## prob[1]       0.45985  0.00041111      0.8 14213 -0.0059391
18 ## prob[2]       0.12712  0.00027833      0.8 14386  0.0063839
19 ## prob[3]       0.052209  0.00024004      1   9318  -0.011454
20 ## prob[4]       0.344  0.00040718      0.9 13182  -0.007306

```

	Lower95	Median	Upper95	SSeff	psrf
prev	0.305	0.439	0.567	4062	1.000
prob[1]	0.369	0.461	0.559	14213	1.000
prob[2]	0.072	0.133	0.200	14386	1.000
prob[3]	0.017	0.055	0.103	9318	1.000
prob[4]	0.252	0.344	0.434	13182	1.000
se[1]	0.824	0.933	1.000	5875	1.001
se[2]	0.693	0.851	1.000	3455	1.001
sp[1]	0.744	0.873	1.000	3419	1.001
sp[2]	0.860	0.947	1.000	5478	1.000

- Note wide confidence intervals

Care with order of combinations in multinom

Lots of data needed

- And/or strong priors for one of the tests

Convergence can be tricky

Label Switching

How to interpret a test with $Se=0\%$ and $Sp=0\%$?

...

The test is perfect - we are just holding it upside down...

...

We can force $se+sp \geq 1$:

```
1 se[1] ~ dbeta(1, 1)
2 sp[1] ~ dbeta(1, 1)T(1-se[1], )
```

...

Or:

```
1 se[1] ~ dbeta(1, 1)T(1-sp[1], )
2 sp[1] ~ dbeta(1, 1)
```

Simulating data

Analysing simulated data is useful to check that we can recover parameter values.

Some simulation code:

```
1  se1 <- 0.9
2  sp1 <- 0.95
3  sp2 <- 0.99
4  se2 <- 0.8
5  prevalence <- 0.5
6  N <- 100
7
8  truestatus <- rbinom(N, 1, prevalence)
9  Test1 <- rbinom(N, 1, (truestatus * se1) + ((1-truestatus) *
   ↪ (1-sp1)))
10 Test2 <- rbinom(N, 1, (truestatus * se2) + ((1-truestatus) *
   ↪ (1-sp2)))
11
12 twoXtwo <- table(Test1, Test2)
13 twoXtwo
```

Exercise

Modify JAGS code to force tests to be better than useless

Simulate data and recover parameters for:

- $N=10$, $N=100$, $N=1000$

Optional Exercise

Use priors for test1 taken from session 1 and compare the results

Solution

Model definition:

```
1  model{
2    Tally ~ dmulti(prob, TotalTests)
3
4    # Test1- Test2-
5    prob[1] <- (prev * ((1-se[1])*(1-se[2]))) + ((1-prev) *
6      ↪ ((sp[1])*(sp[2])))
7
8    # Test1+ Test2-
9    prob[2] <- (prev * ((se[1])*(1-se[2]))) + ((1-prev) *
10     ↪ ((1-sp[1])*(sp[2])))
11
12    # Test1- Test2+
13    prob[3] <- (prev * ((1-se[1])*(se[2]))) + ((1-prev) *
14     ↪ ((sp[1])*(1-sp[2])))
15
16    # Test1+ Test2+
17    prob[4] <- (prev * ((se[1])*(se[2]))) + ((1-prev) *
18     ↪ ((1-sp[1])*(1-sp[2])))
```


Optional Solution

```
1  HPSe[1,] <- c(148.43, 16.49)
2  HPSp[1,] <- c(240.03, 12.63)
```

```
3
```

```
4  HPSe
```

```
1  ##           [,1]  [,2]
2  ## [1,] 148.43 16.49
3  ## [2,]   1.00   1.00
```

```
1  HPSp
```

```
1  ##           [,1]  [,2]
2  ## [1,] 240.03 12.63
3  ## [2,]   1.00   1.00
```

```
1  results <- run.jags('basic_hw.bug', n.chains=2)
```

```
1  ## Finished running the simulation
```

Session 2b: Hui-Walter models for 2 tests and N populations

Independent intercepts for populations

```
1  model{
2    for(p in 1:Populations){
3      Tally[1:4, p] ~ dmulti(prob[1:4, p], TotalTests[p])
4
5      # Test1- Test2- Pop1
6      prob[1, p] <- (prev[p] * ((1-se[1])*(1-se[2]))) +
↪    ((1-prev[p]) * ((sp[1])*(sp[2])))
7
8      ## etc ##
9
10     prev[p] ~ dbeta(1, 1)
11   }
12
13   se[1] ~ dbeta(HPSse[1,1], HPSse[1,2])T(1-sp[1], )
14   sp[1] ~ dbeta(HPSp[1,1], HPSp[1,2])
15   se[2] ~ dbeta(HPSse[2,1], HPSse[2,2])T(1-sp[2], )
16   sp[2] ~ dbeta(HPSp[2,1], HPSp[2,2])
17
18   #data# Tally, TotalTests, Populations, HPSse, HPSp
19   #monitor# prev, prob, se, sp
```

We would usually start with individual-level data in a dataframe
e.g.:

```
1  se1 <- 0.9
2  sp1 <- 0.95
3  sp2 <- 0.99
4  se2 <- 0.8
5  prevalences <- c(0.1, 0.5, 0.9)
6  N <- 100
7
8  simdata <- data.frame(Population = sample(seq_along(prevalences),
  ↪   N, replace=TRUE))
9  simdata$probability <- prevalences[simdata$Population]
10 simdata$truestatus <- rbinom(N, 1, simdata$probability)
11 simdata$Test1 <- rbinom(N, 1, (simdata$truestatus * se1) +
  ↪   ((1-simdata$truestatus) * (1-sp1)))
12 simdata$Test2 <- rbinom(N, 1, (simdata$truestatus * se2) +
  ↪   ((1-simdata$truestatus) * (1-sp2)))
13
14 head(simdata)
```

The model code and data format for an arbitrary number of populations (and tests) can be determined automatically

There is a function (soon to be included in the runjags package, but for now provided in the GitHub repo) that can do this for us:

```
1  simdata$Population <- factor(simdata$Population,  
  ↪  levels=seq_along(prevalences), labels=paste0('Pop_',  
  ↪  seq_along(prevalences)))  
2  
3  source("autohuiwalter.R")  
4  auto_huiwalter(simdata[,c('Population', 'Test1', 'Test2')],  
  ↪  outfile='autohw.bug')  
  
1  ## The model and data have been written to autohw.bug in the  
  ↪  current working directory  
2  ## You should check and alter priors before running the model
```

This generates self-contained model/data/initial values etc (ignore covse and covsp for now):

```
1  ## ## Auto-generated Hui-Walter model created by script version
   ↪ 0.1 on 2020-02-12
2  ##
3  ## model{
4  ##
5  ##   ## Observation layer:
6  ##
7  ##   # Complete observations (N=100):
8  ##   for(p in 1:Populations){
9  ##       Tally_RR[1:4,p] ~ dmulti(prob_RR[1:4,p], N_RR[p])
10 ##
11 ##       prob_RR[1:4,p] <- se_prob[1:4,p] + sp_prob[1:4,p]
12 ##   }
13 ##
14 ##
15 ##   ## Observation probabilities:
16 ##
17 ##   for(p in 1:Populations){
18 ##
19 ##       # Probability of observing Test1- Test2- from a true
```

And can be run directly from R:

```
1 results <- run.jags('autohw.bug')  
  
1 ## Note: The monitored variables 'covse12' and 'covsp12'  
2 ## appear to be non-stochastic; they will not be  
3 ## included in the convergence diagnostic  
4 ## Finished running the simulation
```

```

1  results

1  ##
2  ## JAGS model summary statistics from 20000 samples (chains = 2;
   ↪ adapt+burnin = 5000):
3  ##
4  ##           Lower95  Median Upper95      Mean      SD      Mode
5  ## se[1]      0.71517 0.84356 0.96496 0.83979 0.064616 0.85345
6  ## se[2]      0.65745 0.79572 0.9177 0.79285 0.066441 0.7994
7  ## sp[1]      0.87605 0.96673      1 0.95596 0.039059 0.98493
8  ## sp[2]      0.86925 0.95868 0.99998 0.94921 0.040251 0.98013
9  ## prev[1] 0.072539 0.18993 0.3269 0.19543 0.066634 0.18175
10 ## prev[2] 0.26969 0.46757 0.66678 0.4676 0.10224 0.47244
11 ## prev[3] 0.68481 0.84386 0.98271 0.83654 0.078561 0.85803
12 ## covse12      0      0      0      0      0      0
13 ## covsp12      0      0      0      0      0      0
14 ##
15 ##           MCerr MC%ofSD SSeff      AC.10      psrf
16 ## se[1] 0.00075224      1.2 7378 0.0014165 1.0003
17 ## se[2] 0.0007197      1.1 8522 0.014384 1.0001
18 ## sp[1] 0.00058798      1.5 4413 0.025856 1.0001
19 ## sp[2] 0.00054376      1.4 5479 -0.0065604 1.0002
20 ## prev[1] 0.0006472      1 10600 0.0049727 1.0003

```


Observation-level model specification

```
1  model{
2
3    for(i in 1:N){
4      Status[i] ~ dcat(prob[i, ])
5
6      prob[i,1] <- (prev[i] * ((1-se[1])*(1-se[2]))) +
7                  ((1-prev[i]) * ((sp[1])*(sp[2])))
8      prob[i,2] <- (prev[i] * ((se[1])*(1-se[2]))) +
9                  ((1-prev[i]) * ((1-sp[1])*(sp[2])))
10     prob[i,3] <- (prev[i] * ((1-se[1])*(se[2]))) +
11                 ((1-prev[i]) * ((sp[1])*(1-sp[2])))
12     prob[i,4] <- (prev[i] * ((se[1])*(se[2]))) +
13                 ((1-prev[i]) * ((1-sp[1])*(1-sp[2])))
14
15     logit(prev[i]) <- intercept +
16       ↪ population_effect[Population[i]]
17   }
18
19   intercept ~ dnorm(0, 0.33)
20   population_effect[1] <- 0
21   for(p in 2:Pops){
```

Just like in session 1, the main difference is the prior for prevalence (this time in each population)

We also need to give initial values for intercept and population_effect rather than prev, and tell run.jags the data frame from which to extract the data (except N and Pops):

```
1 intercept <- list(chain1=-1, chain2=1)
2 population_effect <- list(chain1=c(NA, 1, -1), chain2=c(NA, -1,
  ↪ 1))
3 se <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
4 sp <- list(chain1=c(0.5,0.99), chain2=c(0.99,0.5))
5
6 simdata$Status <- with(simdata, factor(interaction(Test1, Test2),
  ↪ levels=c('0.0', '1.0', '0.1', '1.1'))))
7 N <- nrow(simdata)
8 Pops <- length(levels(simdata$Population))
9 glm_results <- run.jags('glm_hw.bug', n.chains=2, data=simdata)

1 ## Note: The monitored variable 'population_effect[1]'
2 ## appears to be non-stochastic; it will not be included
3 ## in the convergence diagnostic
```

Also like in session 1, the estimates for se/sp should be similar, although this model runs more slowly.

Note: this model could be used as the basis for adding covariates

For a handy way to generate a GLM model see
`runjags::template.jags`

- Look out for integration with `autohuiwalter` in the near (ish) future. . .

Need to be very careful with tabulating the data, or use automatically generated code

Works best when populations have very different prevalences

Exercise

Play around with the `autohwiwalter` function

Notice the model and data and initial values are in a self contained file

Ignore the `covse` and `covsp` for now