Predicting the car price based on its production year and mileage

1. Problem formulation:

The problem we focused on is predicting the car price based on its charakteristics. The dataset contains a large number of cars both new and used that were listed for sale in the otomoto.pl portal. We focused on one specific vehicle type - Audi A3 with the engine size of 2000cm^3.

We chose this problem, because we are interested in purchasing a car in near futer and the analysis of the data can help us rate if the car price of specific parameters is reasonable or not. Another use case is to apply this model to vehicles with different brands and characteristic and check how common the model is/

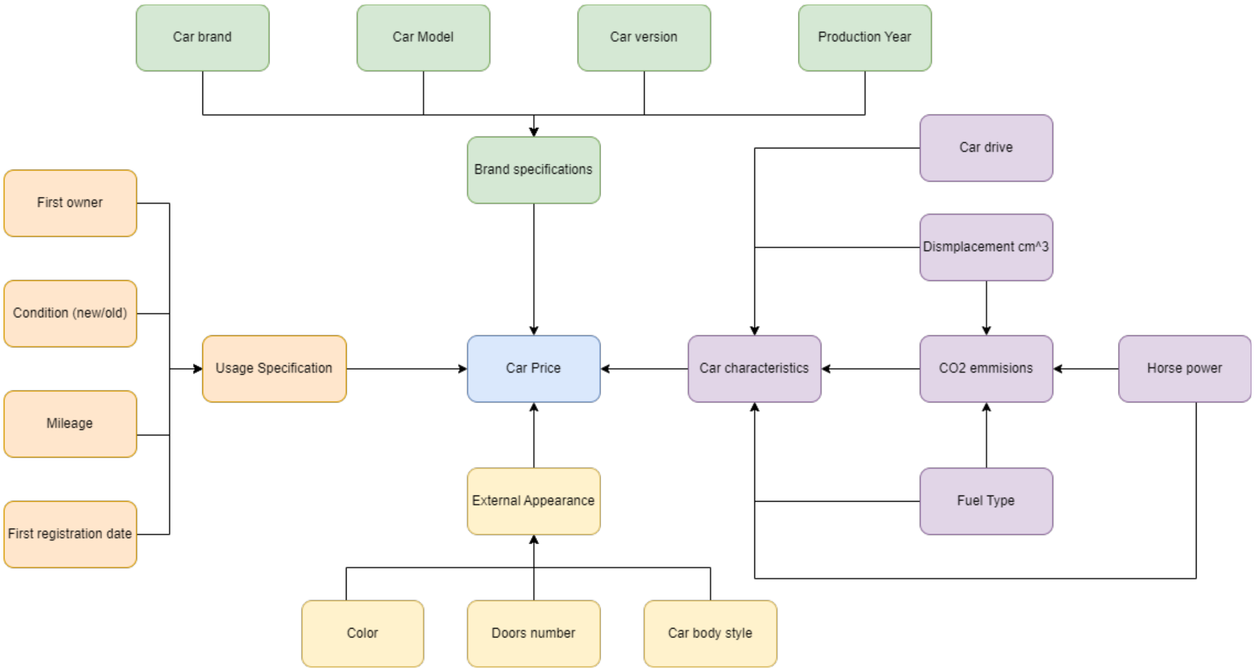
The chosen dataset is called "Poland cars for sale dataset (200k adverts)" and can be found under this link https://www.kaggle.com/datasets/bartoszpieniak/poland-cars-for-sale-dataset. This dataset was created by webscraping over 200,000 car offers from one of the largest car advertisement sites in Poland (otomoto). It contains 25 parameters listed below:

ID - unique ID of offer Price - value of the price Currency - currency of the price (mostly polish złoty, but also some euro) Condition - new or used Vehicle_brand - brand of vehicle in offer Vehicle_model - model of vehicle in offer Vehicle_generation - generation of vehicle in offer Vehicle_version - version of vehicle in offer Production_year - year of car production Mileage_km - total distance that the car has driven in kilometers Power_HP - car engine power in horsepower Displacement_cm3 - car engine size in cubic centimeters Fuel_type - car fuel type CO2_emissions - car CO2 emissions in g/km Drive - type of car drive Transmission - type of car transmission Type - car body style Doors_number - number of car doors Colour - car body color Origin_country - country of origin of the car First_owner - whether the owner is the first owner First_registration_date - date of first registration Offer_publication_date - date of publication of the offer Offer_location - address provided by the issuer Features - listed car features (ABS, airbag, parking sensors e.t.c)

DAG Diagram

Based on the data, we created a DAG diagram to describe what parameters affect the price and each other. We divided the data in categories - brand specification, useage specification, car characteristic and external appearance. We also draw the relation between CO2 emmision and parameters such as displacement, fuel type and horse type, which affect both the emmision and the price.

```python
from IPython.display import Image
image_path = "/home/DA/project/DAG_cars.png"
Image(filename=image_path)
```




```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import arviz as az
import seaborn as sns
import cmdstanpy
import pandas as pd
import numpy as np
from scipy import stats
from scipy.optimize import curve_fit
import matplotlib.pyplot as plt
import seaborn as sns
import os
from sklearn.preprocessing import MinMaxScaler
from fitter import Fitter, get_common_distributions, get_distributions


BINS = 20
```

Functions

```
def price_plot(df, column_name, plot_trend = False):
    price = df["Price"]
    data = df[column_name]
    plt.figure()
    plt.plot(data,price, 'o')
    plt.xlabel(column_name)
    plt.ylabel("Price_PLN")
    if plot_trend:
```

```
        z = np.polyfit(data, price, 1)
        p = np.poly1d(z)
        print(f"Polyfit equation: {p}")
        plt.plot(data, p(data))
        plt.axvline(data.mean(), color="red")
        plt.axhline(price.mean(), color="red")
    plt.show()
```

Loading the data:

```
df = pd.read_csv("data/Car_sale_ads.csv")
list(df.columns)
df.head()
```

```
   Index  Price Currency Condition Vehicle_brand Vehicle_model  \
0      0  86200      PLN       New        Abarth           595
1      1  43500      PLN      Used        Abarth         Other
2      2  44900      PLN      Used        Abarth           500
3      3  39900      PLN      Used        Abarth           500
4      4  97900      PLN       New        Abarth           595

  Vehicle_version Vehicle_generation  Production_year  Mileage_km  ...
\
0             NaN                NaN             2021         1.0  ...

1             NaN                NaN             1974     59000.0  ...

2             NaN                NaN             2018     52000.0  ...

3             NaN                NaN             2012     29000.0  ...

4             NaN                NaN             2021       600.0  ...


   Transmission        Type Doors_number  Colour Origin_country
First_owner  \
0        Manual  small_cars          3.0    gray            NaN
NaN
1        Manual       coupe          2.0  silver            NaN
NaN
2     Automatic  small_cars          3.0  silver            NaN
NaN
3        Manual  small_cars          3.0    gray            NaN
NaN
4        Manual  small_cars          3.0    blue            NaN
NaN

  First_registration_date  Offer_publication_date  \
0                     NaN              04/05/2021
1                     NaN              03/05/2021
```

```
2                     NaN              03/05/2021
3                     NaN              30/04/2021
4                     NaN              30/04/2021

                                      Offer_location  \
0  ul. Jubilerska 6 - 04-190 Warszawa, Mazowiecki...
1  kanonierska12 - 04-425 Warszawa, Rembertów (Po...
2                    Warszawa, Mazowieckie, Białołęka
3                                   Jaworzno, Śląskie
4  ul. Gorzysława 9 - 61-057 Poznań, Nowe Miasto ...

                                            Features
0                                                 []
1                                                 []
2  ['ABS', 'Electric front windows', 'Drivers air...
3  ['ABS', 'Electric front windows', 'Drivers air...
4  ['ABS', 'Electrically adjustable mirrors', 'Pa...

[5 rows x 25 columns]
```

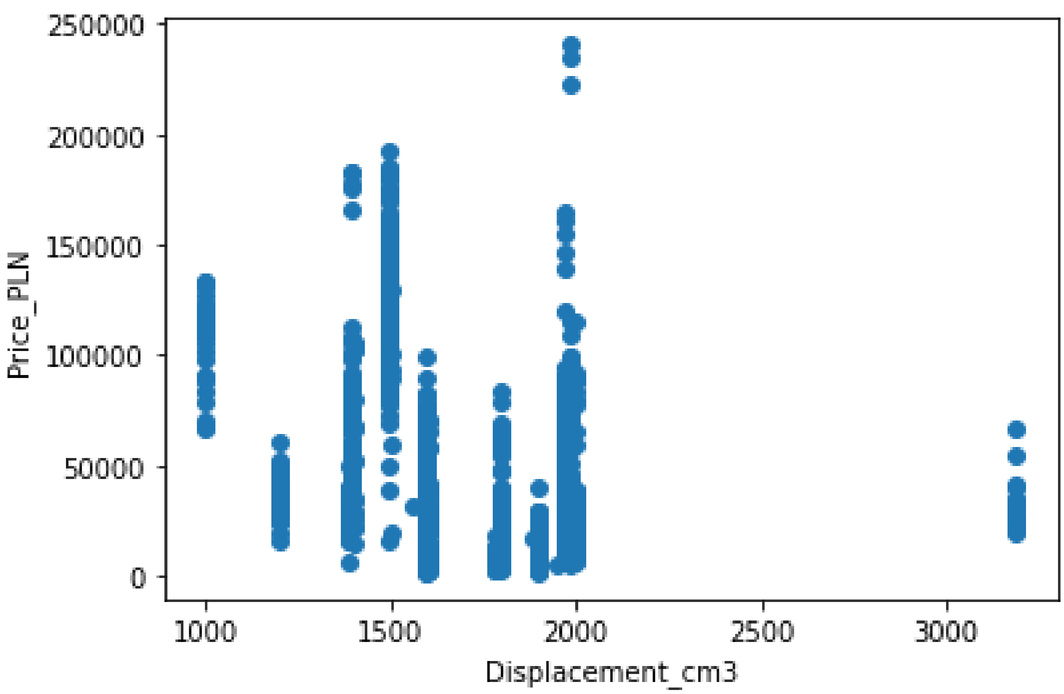
Unification of the price currency and selection of the desired columns

```
price = df["Price"].copy()
currency = df["Currency"].copy()

for idx, (p, c) in enumerate(zip(price, currency)):
    if c == "EUR":
        price_PLN = p * 4.6
        price[idx] = price_PLN
        currency[idx] = "PLN"


df["Currency"] = currency
df["Price"] = price

cols2add = ["Price", "Vehicle_brand", "Vehicle_model",
"Production_year", "Mileage_km", "Power_HP", "Displacement_cm3"]
test_df = df[cols2add]
test_df.head()

     Price Vehicle_brand Vehicle_model  Production_year  Mileage_km
Power_HP  \
0  86200.0        Abarth           595             2021         1.0
145.0
1  43500.0        Abarth         Other             1974     59000.0
75.0
2  44900.0        Abarth           500             2018     52000.0
180.0
3  39900.0        Abarth           500             2012     29000.0
160.0
```

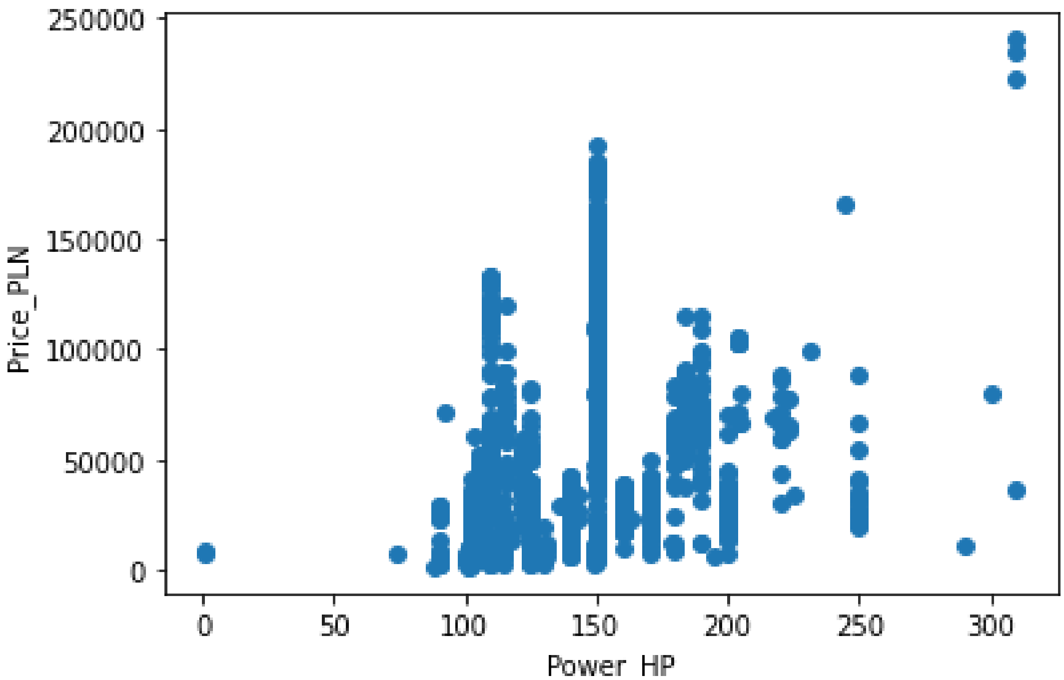
```
4  97900.0         Abarth              595               2021         600.0
165.0

   Displacement_cm3
0            1400.0
1            1100.0
2            1368.0
3            1368.0
4            1368.0
```

Due to the extensive size of the dataset and the wide range of car models included, we have made the decision to conduct our analysis solely on a single car model. ***Chosen car model:*** **Brand:** Audi **Model:** A3

```
audi_cars = test_df[test_df['Vehicle_brand'] == "Audi"]
audi_a3_cars = audi_cars[audi_cars["Vehicle_model"] == 'A3']

price_plot(audi_a3_cars,"Displacement_cm3")
```



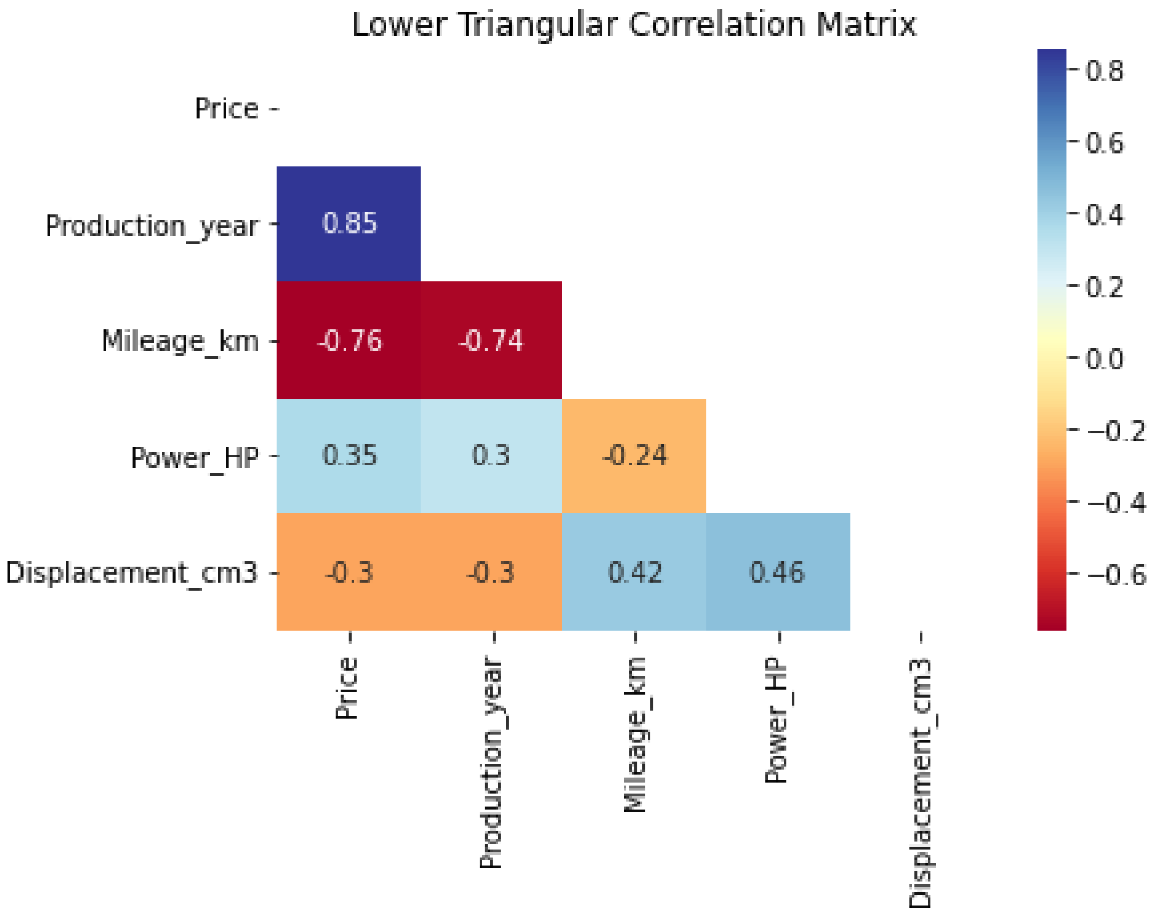

```
price_plot(audi_a3_cars,"Power_HP")
```




```
correlations = audi_a3_cars.iloc[:,
1:].corrwith(audi_a3_cars['Price'])
print(correlations)
```

```
Production_year     0.853472
Mileage_km         -0.764658
Power_HP            0.354174
Displacement_cm3   -0.301169
dtype: float64
```

```
correlation_matrix = audi_a3_cars.corr()
mask = np.triu(np.ones_like(correlation_matrix, dtype=bool))
sns.heatmap(data=correlation_matrix, mask=mask, annot=True,
cmap='RdYlBu')
plt.title('Lower Triangular Correlation Matrix')
plt.show()
```



Due to small effect of engine power and displacement on the price of a vehicle, it was decided that only cars with a displacement of 2000ccm would be analysed to simplify analizis.

```
audi_a3_2010 = audi_a3_cars[audi_a3_cars["Production_year"] == 2010]
audi_a3_2000ccm = audi_a3_cars[audi_a3_cars["Displacement_cm3"] >=
1950]
audi_a3_2000ccm = audi_a3_2000ccm[audi_a3_2000ccm["Displacement_cm3"]
<= 2050]
audi_a3_2000ccm = audi_a3_2000ccm.dropna()

if "audi_cars_data.csv" not in os.listdir("data"):
    audi_a3_2000ccm.to_csv('data/audi_cars_data.csv', index=False)
```

Summary

```
audi_a3_2000ccm.head()
```

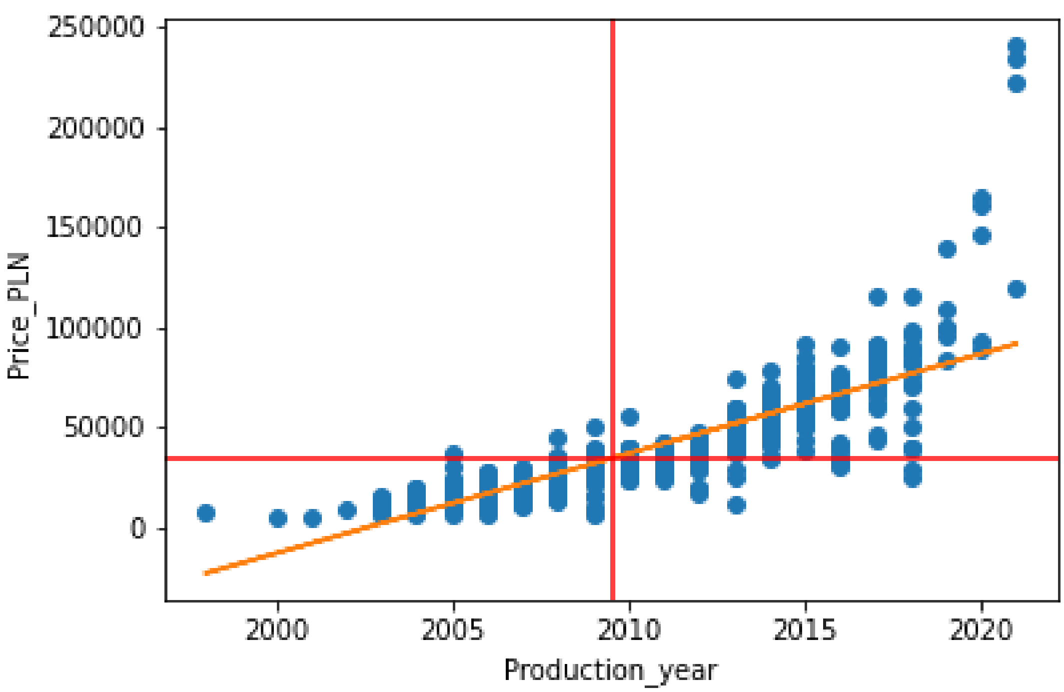
```
        Price Vehicle_brand Vehicle_model  Production_year  Mileage_km
\
1929  49900.0          Audi            A3             2015    208000.0

1932  13900.0          Audi            A3             2008    227000.0
```

1933	21900.0	Audi	A3	2008	313855.0
1934	19900.0	Audi	A3	2007	242000.0
1936	22900.0	Audi	A3	2006	240000.0

	Power_HP	Displacement_cm3
1929	150.0	1968.0
1932	140.0	1968.0
1933	140.0	1968.0
1934	170.0	1968.0
1936	200.0	1984.0

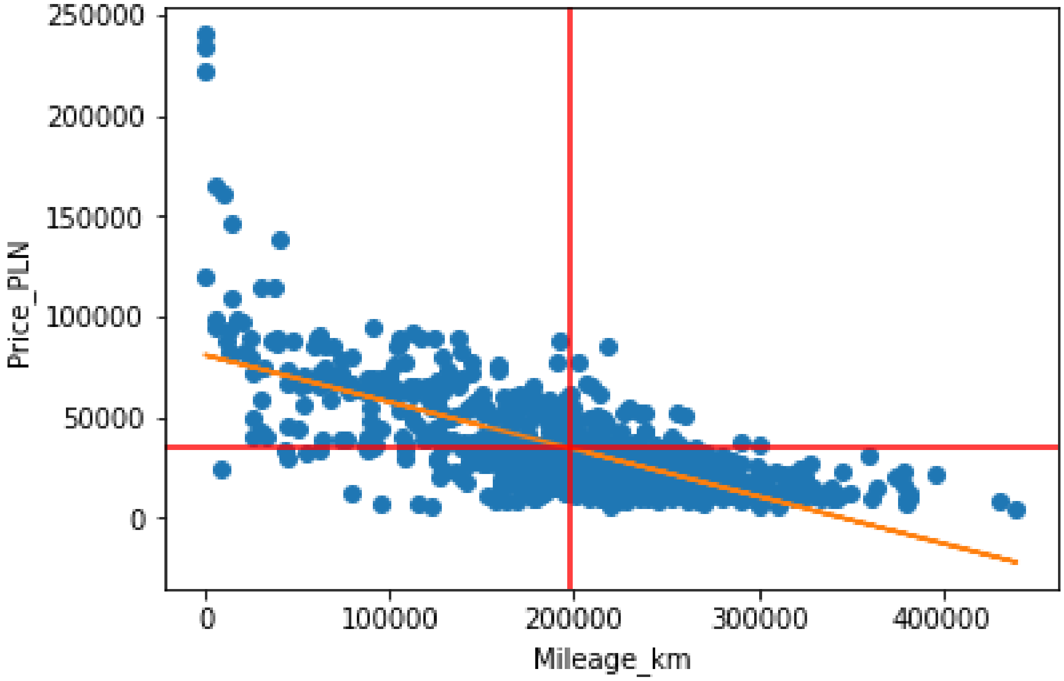
```
price_plot(audi_a3_2000ccm, "Production_year", True)
```

```
Polyfit equation:
4988 x - 9.99e+06
```

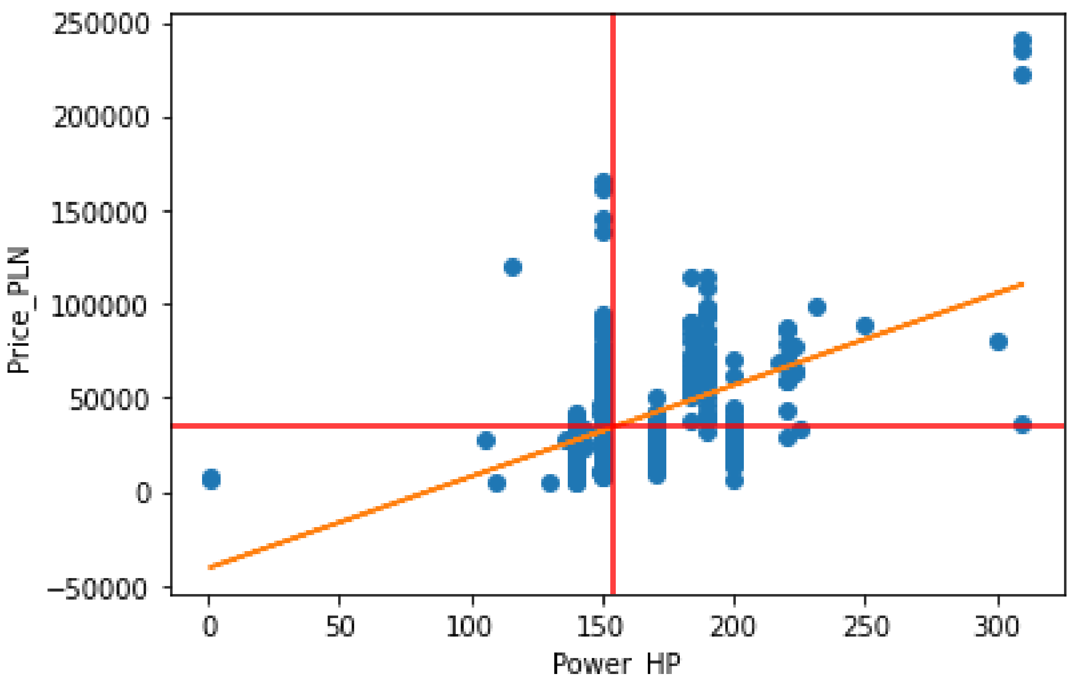



```
price_plot(audi_a3_2000ccm, "Mileage_km", True)
```

```
Polyfit equation:
-0.2344 x + 8.109e+04
```

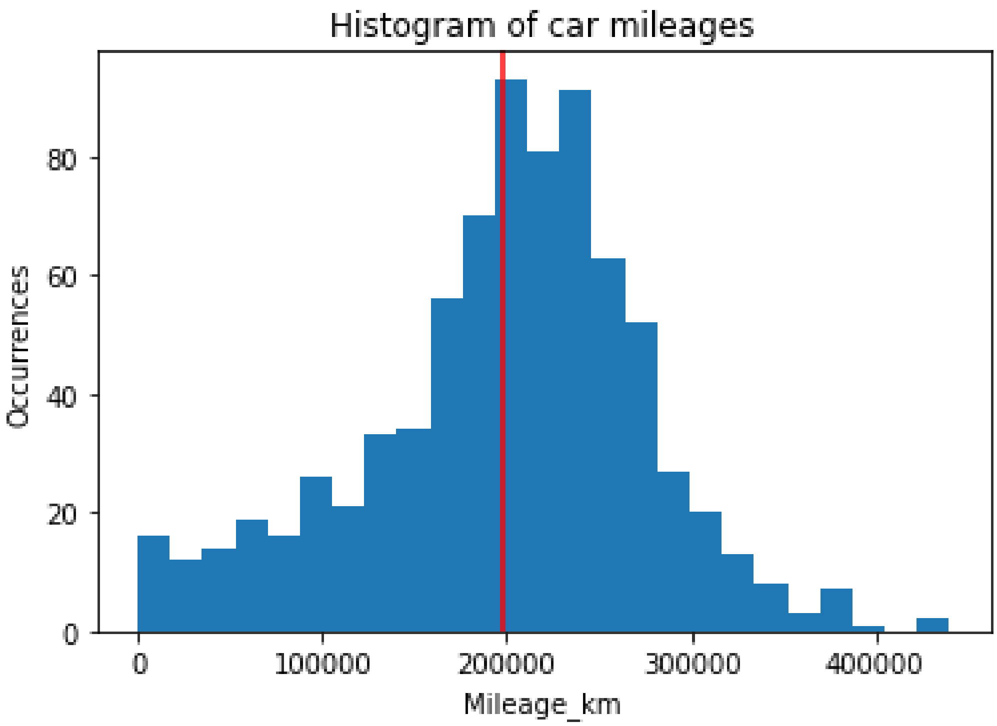



```
price_plot(audi_a3_2000ccm, "Power_HP", True)

Polyfit equation:
487.6 x - 4.072e+04
```



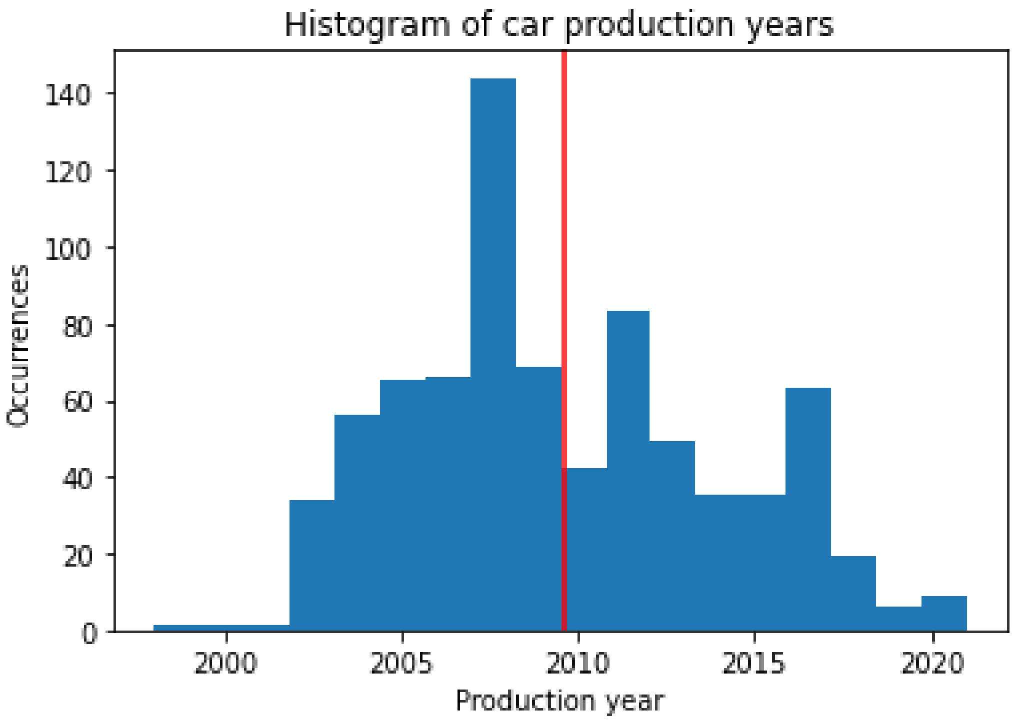
```
mileage_mean = np.mean(audi_a3_2000ccm["Mileage_km"])
print(f"Mean: {mileage_mean}")
plt.figure()
plt.hist(audi_a3_2000ccm["Mileage_km"], bins = 25)
plt.axvline(mileage_mean, color="red")
plt.xlabel("Mileage_km")
plt.ylabel("Occurrences")
plt.title("Histogram of car mileages")
plt.show()
```

```
Mean: 198361.37403598972
```



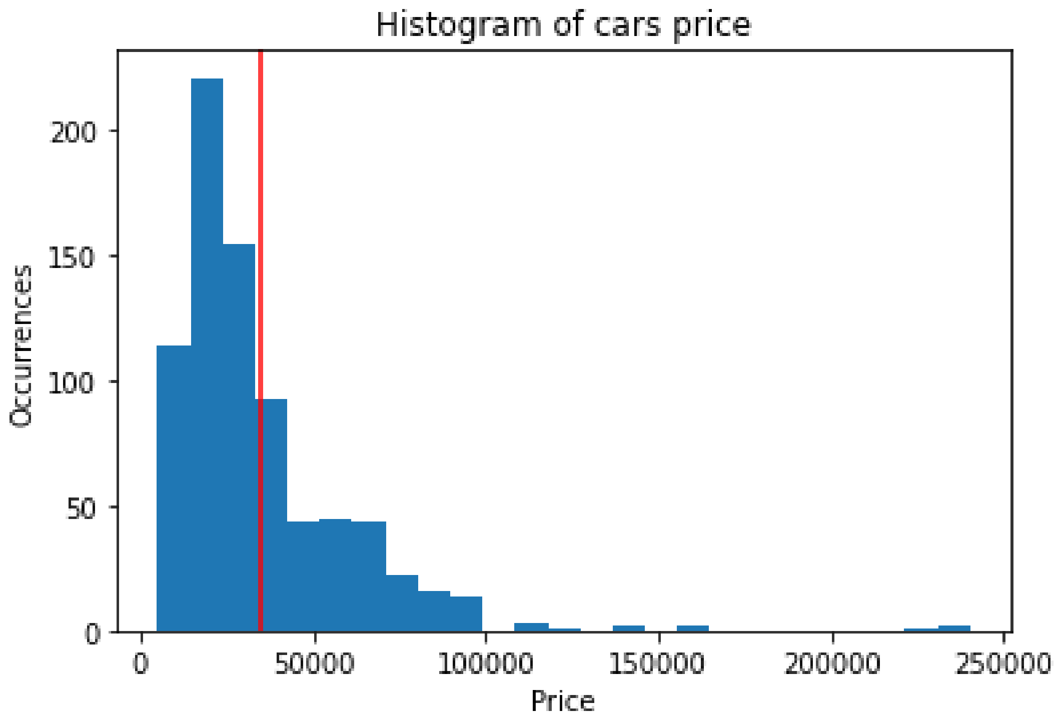

```
prod_mean = np.mean(audi_a3_2000ccm["Production_year"])
print(f"Mean: {prod_mean}")
plt.figure()
plt.hist(audi_a3_2000ccm["Production_year"], bins = 18)
plt.axvline(prod_mean, color="red")
plt.xlabel("Production year")
plt.ylabel("Occurrences")
plt.title("Histogram of car production years")
plt.show()
```

```
Mean: 2009.5719794344473
```




```
price_mean = np.mean(audi_a3_2000ccm["Price"])
print(f"Mean: {price_mean}")
plt.figure()
plt.hist(audi_a3_2000ccm["Price"], bins = 25)
plt.axvline(price_mean, color="red")
plt.xlabel("Price")
plt.ylabel("Occurrences")
plt.title("Histogram of cars price")
plt.show()
```

```
Mean: 34600.235218509
```



Data standarization

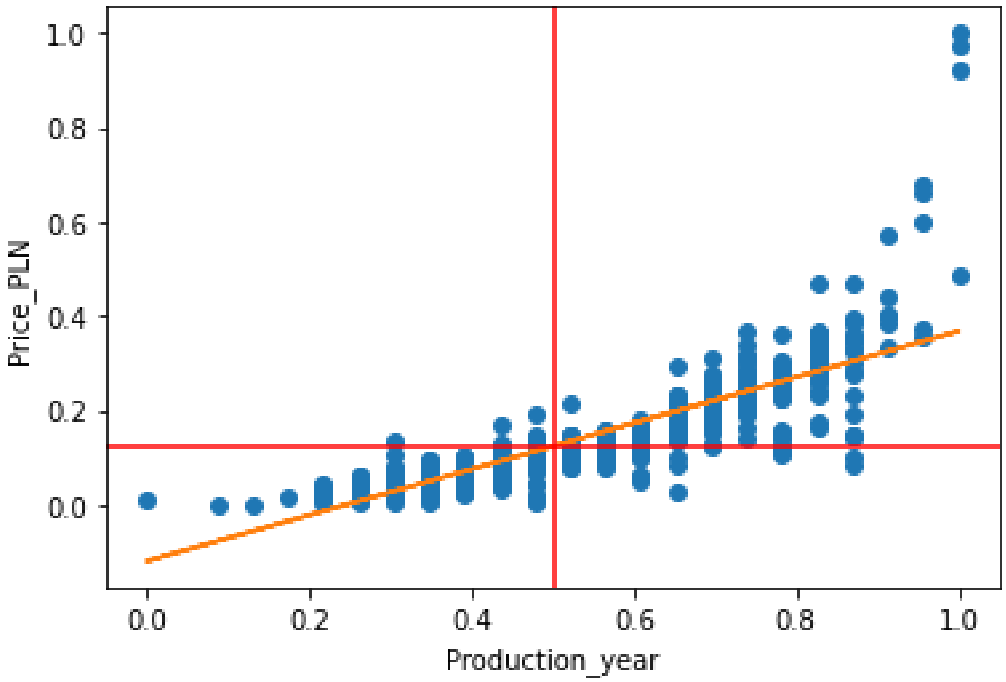
Due to the diversity of the data (production year (values form 2003 to 2021), mileage (values from 0 to 400000), price (values from 0 to 160000)), we decided to standardise the data using the MinMax scalar. This way we got all the data in the range from 0 to 1, without loosing information about data and making it easier to analyze it.

```
scaler = MinMaxScaler()
audi_a3_2000ccm_standarized_data =
scaler.fit_transform(audi_a3_2000ccm.loc[:,["Price",
"Production_year", "Mileage_km"]])
audi_a3_2000ccm_standarized =
pd.DataFrame(audi_a3_2000ccm_standarized_data,columns=["Price",
"Production_year", "Mileage_km"])
audi_a3_2000ccm_standarized.describe()
```

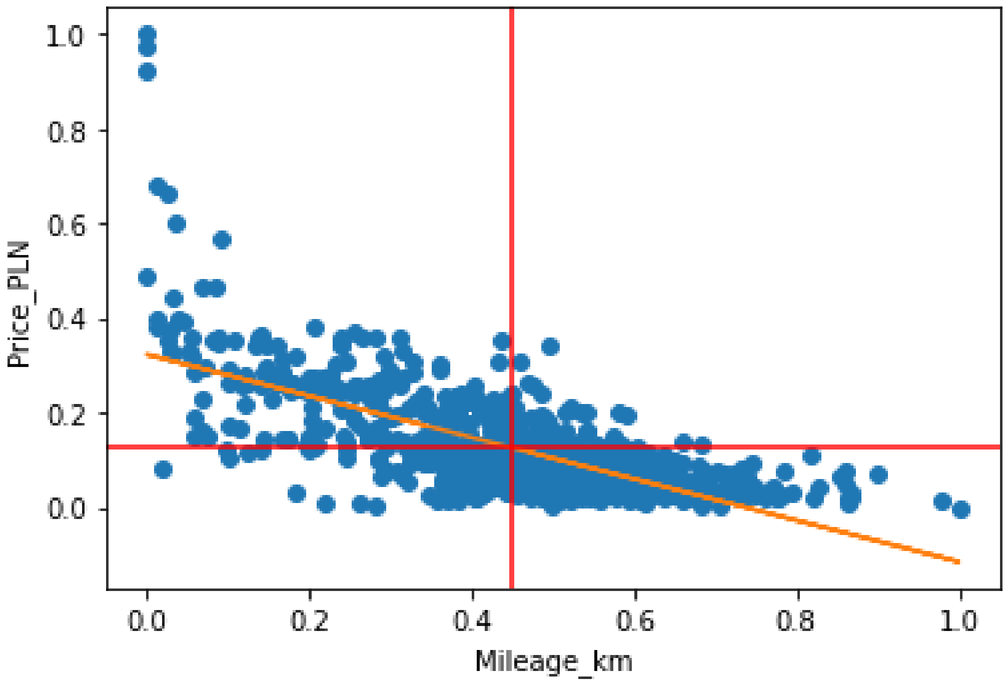
	Price	Production_year	Mileage_km
count	778.000000	778.000000	778.000000
mean	0.125764	0.503130	0.450820
std	0.110705	0.190895	0.173469
min	0.000000	0.000000	0.000000
25%	0.053109	0.347826	0.363635
50%	0.093048	0.478261	0.470453
75%	0.160497	0.652174	0.561363
max	1.000000	1.000000	1.000000

```
price_plot(audi_a3_2000ccm_standarized,"Production_year",True)
price_plot(audi_a3_2000ccm_standarized,"Mileage_km",True)
```

```
Polyfit equation:
0.4875 x - 0.1195
```

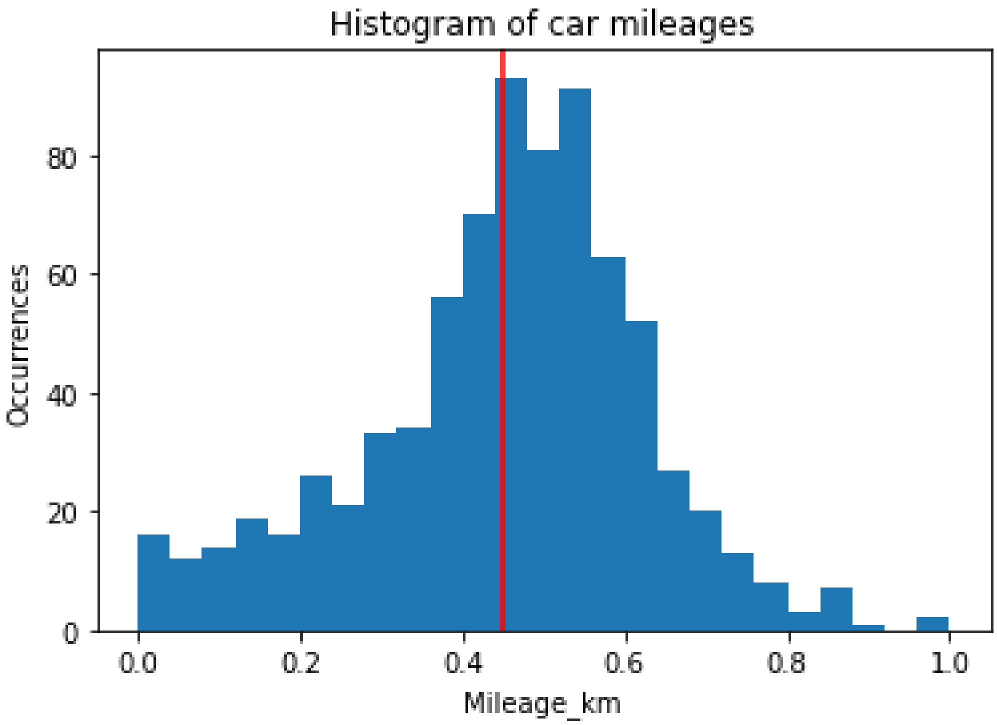



```
Polyfit equation:
-0.4382 x + 0.3233
```

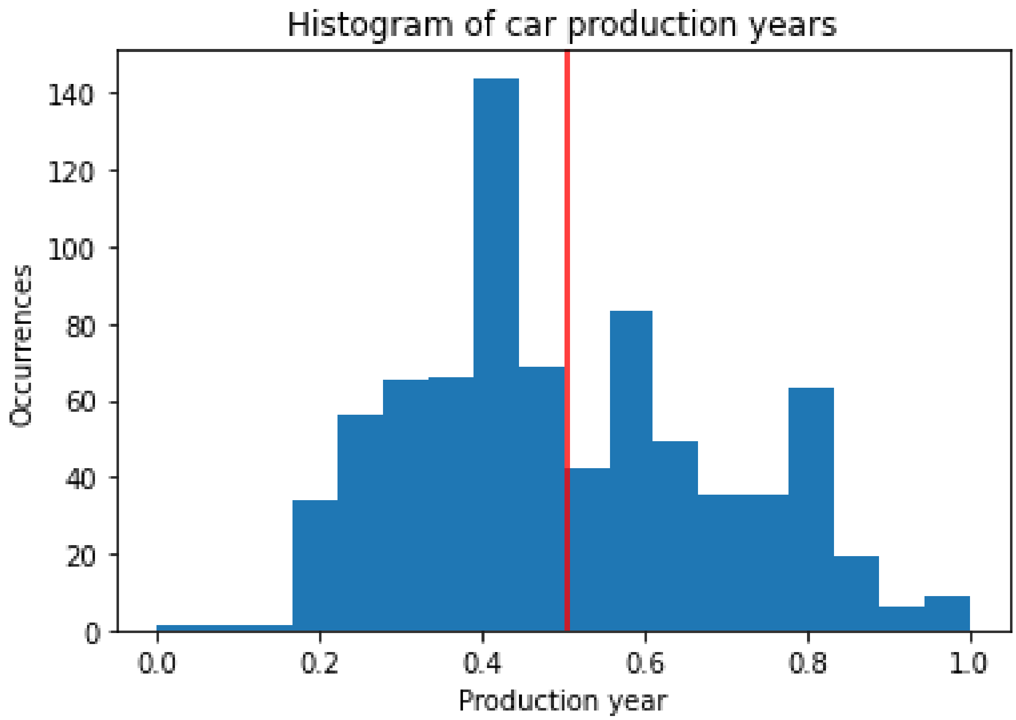



```python
mileage_mean = np.mean(audi_a3_2000ccm_standarized["Mileage_km"])
print(f"Mean: {mileage_mean}")
plt.figure()
plt.hist(audi_a3_2000ccm_standarized["Mileage_km"], bins = 25)
plt.axvline(mileage_mean, color="red")
plt.xlabel("Mileage_km")
plt.ylabel("Occurrences")
plt.title("Histogram of car mileages")
plt.show()

prod_mean = np.mean(audi_a3_2000ccm_standarized["Production_year"])
print(f"Mean: {prod_mean}")
plt.figure()
plt.hist(audi_a3_2000ccm_standarized["Production_year"], bins = 18)
plt.axvline(prod_mean, color="red")
plt.xlabel("Production year")
plt.ylabel("Occurrences")
plt.title("Histogram of car production years")
plt.show()

price_mean = np.mean(audi_a3_2000ccm_standarized["Price"])
price_var = np.var(audi_a3_2000ccm_standarized["Price"])
print(f"Mean: {price_mean}")
print(f"Var: {price_var}")
plt.figure()
plt.hist(audi_a3_2000ccm_standarized["Price"], bins = 25)
plt.axvline(price_mean, color="red")
plt.xlabel("Price")
```

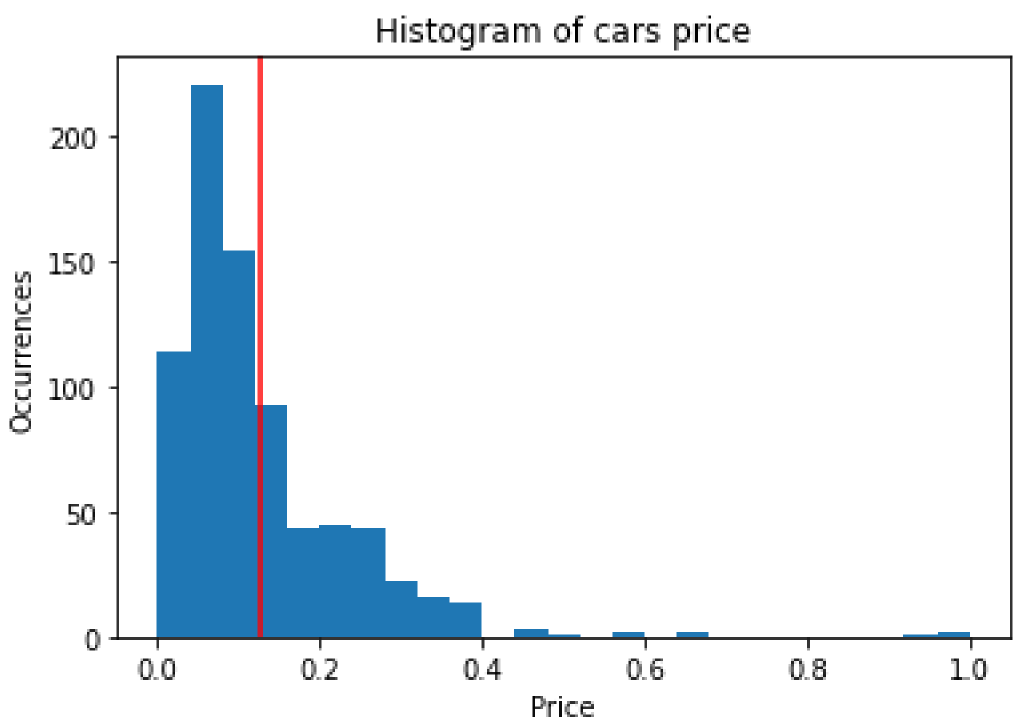
```
plt.ylabel("Occurrences")
plt.title("Histogram of cars price")
plt.show()

Mean: 0.45082005649101414
```




```
Mean: 0.5031295406281482
```




```
Mean: 0.12576418221431998
Var: 0.012239734156792626
```



```python
if 'audi_data_standarized.csv' not in os.listdir("data"):

audi_a3_2000ccm_standarized.to_csv('data/audi_data_standarized.csv',
index=False)

audi_a3_2000ccm_standarized =
pd.read_csv("data/audi_data_standarized.csv")
audi_a3_2000ccm_standarized.head()
```

	Price	Production_year	Mileage_km
0	0.190769	0.739130	0.472726
1	0.037814	0.434783	0.515908
2	0.071804	0.434783	0.713306
3	0.063306	0.391304	0.549999
4	0.076053	0.347826	0.545454

3. Model

For this project we specified two prior models of exponential range distribution. We wanted to check how adding highly corelated parameter to the model will affect price estimation. In the first model we used linear regression model with exponnenital distribution. We estimate price only based on the production year. In the second model we add mileage as well. Model 1 formula:

$$price = exponential((\alpha + \beta * production_year) * \lambda)$$

Model 2 formula:

$$price = exponential((\alpha - \beta_1 * mileage + \beta_2 * production_year) * \lambda)$$

3.1 Model 1- prior

Priors selection The choice of an exponential distribution for modeling used car prices is justified by the observation that newer and less used cars tend to have higher prices. The exponential distribution captures this pattern with its right-skewed shape, accommodating a higher concentration of lower-priced cars and a smaller number of higher-priced cars. The min-max scaling ensures that the production year variable is on a comparable scale for accurate analysis and modeling of the relationship between production year and used car prices.

The choice to use a normal distribution for α, β and λ allows for capturing the natural variability of these parameters and is a common approach in statistical modeling for estimation and significance assessment.

```python
model_exp1_ppc =
cmdstanpy.CmdStanModel(stan_file='stan_files/exp_model1_ppc.stan')
```

```
INFO:cmdstanpy:found newer exe file, not recompiling
```

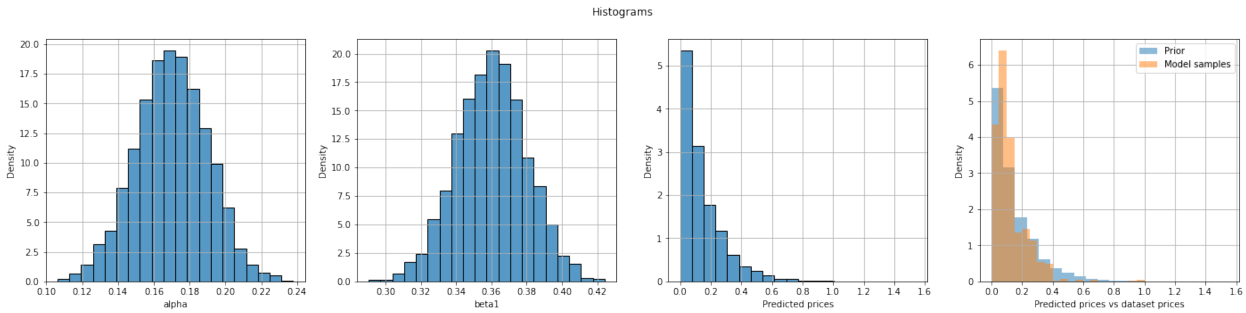
```python
N = len(audi_a3_2000ccm_standarized)

data = {"N": N,
        "mileage" : np.linspace(0.01,1,N),
        "production_year" : np.linspace(0.01,1,N)
}
sim_exp_fit1=model_exp1_ppc.sample(data=data)
sim_exp_fit1_pd = sim_exp_fit1.draws_pd()
sim_exp_fit1_pd.head()
```

```
INFO:cmdstanpy:CmdStan start processing
chain 1 |          | 00:00 Status
▍         | 00:00 Status

█         | 00:00 Iteration: 100 / 1000 [ 10%]  (Sampling)

chain 1 |██        | 00:00 Iteration: 300 / 1000 [ 30%]  (Sampling)
███       | 00:00 Iteration: 500 / 1000 [ 50%]  (Sampling)
████      | 00:00 Iteration: 700 / 1000 [ 70%]  (Sampling)
██████████| 00:00 Sampling completed
chain 2 |██████████| 00:00 Sampling completed

chain 3 |██████████| 00:00 Sampling completed
chain 4 |██████████| 00:00 Sampling completed




INFO:cmdstanpy:CmdStan done processing.



   lp__  accept_stat__  price[1]  price[2]  price[3]  price[4]
price[5]  \
0   0.0            0.0  0.105195  0.167174  0.711009  0.006327
0.023779
1   0.0            0.0  0.032936  0.001719  0.102338  0.353666
0.249956
2   0.0            0.0  0.016656  0.043789  0.036351  0.616655
0.228353
3   0.0            0.0  0.285560  0.081365  0.241231  0.167318
0.126170
4   0.0            0.0  0.201843  0.072561  0.105279  0.081733
0.194447

   price[6]  price[7]  price[8]  ...  price[773]  price[774]
price[775]  \
0  0.014575  0.093657  0.034181  ...    0.067077    0.000468
0.044011
1  0.017459  0.101930  0.106334  ...    0.033959    0.137571
0.005238
```

```
2  0.077383  0.064425  0.008720  ...    0.061551    0.011364
0.246932
3  0.216949  0.131051  0.096908  ...    0.049293    0.033840
0.007374
4  0.047238  0.087672  0.080498  ...    0.020130    0.095196
0.112169

   price[776]  price[777]  price[778]     alpha      beta     sigma
lambda
0    0.021837    0.064800    0.078109  0.158116  0.352988  0.116625
39.7923
1    0.040567    0.002834    0.070587  0.184970  0.385498  0.176368
40.0152
2    0.007619    0.038981    0.027535  0.150371  0.357884  0.181312
39.9036
3    0.022735    0.001567    0.061044  0.199287  0.343763  0.154765
40.1491
4    0.049858    0.057298    0.001569  0.158854  0.332186  0.171331
39.9415

[5 rows x 784 columns]
```

```
_, ax = plt.subplots(1, 4, figsize=(24, 5))
ax = ax.flatten()
sns.histplot(data=sim_exp_fit1_pd, x="alpha", stat="density",
ax=ax[0], bins=BINS)
sns.histplot(data=sim_exp_fit1_pd, x="beta", stat="density", ax=ax[1],
bins=BINS)
sns.histplot(data=sim_exp_fit1_pd, x="price[1]", stat="density",
ax=ax[2], bins=BINS)

ax[3].hist(sim_exp_fit1_pd["price[1]"], bins=BINS, alpha=0.5,
density=True, label="Prior")
ax[3].hist(audi_a3_2000ccm_standarized["Price"], bins=BINS, alpha=0.5,
density=True, label="Model samples")


ax[0].grid()
ax[1].grid()
ax[2].grid()
ax[3].grid()

ax[0].set_xlabel("alpha"),
ax[1].set_xlabel("beta1"),
ax[2].set_xlabel("Predicted prices"),
ax[3].set_xlabel("Predicted prices vs dataset prices")

ax[3].set_ylabel("Density")
ax[3].legend()
```

```
plt.suptitle("Histograms")
plt.show()
```



The prior parameters were chosen through a semi-empirical process. Initially, a standard parameter from the literature was used, but further modifications were made to align the simulated data with the observed data. This adjustment ensured a closer match between the chosen priors and the actual data.

3.2 Model 1- posterior

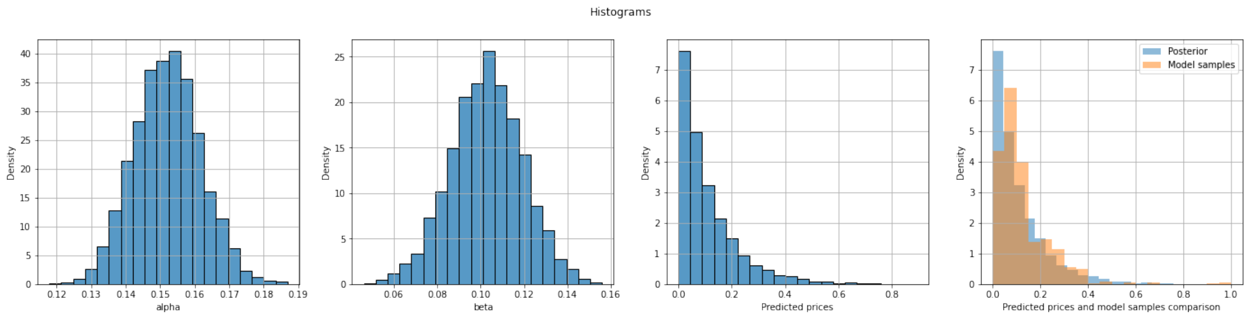
```
model_exp_fit =
cmdstanpy.CmdStanModel(stan_file='stan_files/exp_model1_fit.stan')
N = len(audi_a3_2000ccm_standarized)
#Parameters

data = {"N": N,
        "mileage" : audi_a3_2000ccm_standarized['Mileage_km'],
        "production_year" :
audi_a3_2000ccm_standarized['Production_year'],
        "price_observed": audi_a3_2000ccm_standarized['Price']
        }

sim_exp_pos1_fit=model_exp_fit.sample(data=data)
sim_exp_pos1_fit_pd = sim_exp_pos1_fit.draws_pd()
sim_exp_pos1_fit_pd.head()

INFO:cmdstanpy:found newer exe file, not recompiling
INFO:cmdstanpy:CmdStan start processing
chain 1 |          | 00:00 Status
▏         | 00:00 Status
▊         | 00:00 Iteration:    1 / 2000 [  0%]  (Warmup)
chain 1 |█         | 00:00 Iteration:  100 / 2000 [  5%]  (Warmup)

█         | 00:00 Iteration:  200 / 2000 [ 10%]  (Warmup)

█▏        | 00:00 Iteration:  300 / 2000 [ 15%]  (Warmup)

██▎       | 00:01 Iteration:  400 / 2000 [ 20%]  (Warmup)
███▎      | 00:01 Iteration:  600 / 2000 [ 30%]  (Warmup)
████      | 00:01 Iteration:  800 / 2000 [ 40%]  (Warmup)
```

```
chain 1 |█████     | 00:01 Iteration:  900 / 2000 [ 45%]  (Warmup)

██████    | 00:01 Iteration: 1001 / 2000 [ 50%]  (Sampling)

chain 1 |██████    | 00:02 Iteration: 1100 / 2000 [ 55%]  (Sampling)
██████▉   | 00:02 Iteration: 1200 / 2000 [ 60%]  (Sampling)
███████▎  | 00:02 Iteration: 1300 / 2000 [ 65%]  (Sampling)
███████▋  | 00:03 Iteration: 1400 / 2000 [ 70%]  (Sampling)
████████▏ | 00:03 Iteration: 1500 / 2000 [ 75%]  (Sampling)
████████▋ | 00:03 Iteration: 1600 / 2000 [ 80%]  (Sampling)
█████████ | 00:04 Iteration: 1700 / 2000 [ 85%]  (Sampling)
█████████▌| 00:04 Iteration: 1800 / 2000 [ 90%]  (Sampling)
██████████| 00:04 Iteration: 1900 / 2000 [ 95%]  (Sampling)
██████████| 00:04 Sampling completed
chain 2 |██████████| 00:04 Sampling completed
chain 3 |██████████| 00:04 Sampling completed
chain 4 |██████████| 00:04 Sampling completed



INFO:cmdstanpy:CmdStan done processing.


      lp__  accept_stat__  stepsize__  treedepth__  n_leapfrog__
divergent__  \
0  688.774       0.992027    0.522092          3.0           7.0
0.0
1  692.248       0.976427    0.522092          2.0           7.0
0.0
2  691.642       0.898055    0.522092          3.0           7.0
0.0
3  691.417       0.928710    0.522092          3.0           7.0
0.0
4  689.678       0.938565    0.522092          3.0           7.0
0.0

   energy__     alpha      beta     sigma  ...  log_lik[769]
log_lik[770]  \
0  -688.483  0.163078  0.093421  0.098915  ...       1.12857
1.50514
1  -687.583  0.151551  0.095218  0.132216  ...       1.12853
1.47620
2  -689.180  0.144877  0.101951  0.180190  ...       1.12830
1.46794
3  -690.580  0.153526  0.085891  0.125413  ...       1.12802
1.47365
4  -688.286  0.168723  0.082123  0.169052  ...       1.12845
1.51232
```

```
   log_lik[771]  log_lik[772]  log_lik[773]  log_lik[774]
log_lik[775]  \
0       1.13224       1.68721      0.880598       1.26481
0.118243
1       1.13200       1.64954      0.893021       1.26439
0.170924
2       1.13175       1.64009      0.894674       1.26449
0.171860
3       1.13144       1.64493      0.895328       1.26331
0.191424
4       1.13214       1.69543      0.878974       1.26490
0.122992

   log_lik[776]  log_lik[777]  log_lik[778]
0       1.40232       1.45739      0.875509
1       1.38141       1.44823      0.888959
2       1.37591       1.44893      0.890476
3       1.37874       1.44262      0.891911
4       1.40666       1.45560      0.874382

[5 rows x 2345 columns]
```

```
_, ax = plt.subplots(1, 4, figsize=(24, 5))
ax = ax.flatten()
sns.histplot(data=sim_exp_pos1_fit_pd, x="alpha", stat="density",
ax=ax[0], bins=BINS)
sns.histplot(data=sim_exp_pos1_fit_pd, x="beta", stat="density",
ax=ax[1], bins=BINS)
sns.histplot(data=sim_exp_pos1_fit_pd, x="price_estimated[1]",
stat="density", ax=ax[2], bins=BINS)

ax[3].hist(sim_exp_pos1_fit_pd["price_estimated[1]"], bins=BINS,
alpha=0.5, density=True, label="Posterior")
ax[3].hist(audi_a3_2000ccm_standarized["Price"], bins=BINS, alpha=0.5,
density=True, label="Model samples")


ax[0].grid()
ax[1].grid()
ax[2].grid()
ax[3].grid()

ax[0].set_xlabel("alpha"),
ax[1].set_xlabel("beta"),
ax[2].set_xlabel("Predicted prices"),
ax[3].set_xlabel("Predicted prices and model samples comparison")

ax[3].set_ylabel("Density")
ax[3].legend()
```

```
plt.suptitle("Histograms")
plt.show()
```



The posterior data analysis, which includes comparing the histogram of the prior distribution with the posterior distribution and real data, indicates a good fit. This comparison demonstrates that the chosen priors align well with the observed data, suggesting that the model captures the underlying patterns and provides reliable estimates.

```
summary = sim_exp_pos1_fit.summary()
summary.head()
```

```
          Mean     MCSE  StdDev      5%    50%    95%   N_Eff
N_Eff/s  \
name

lp__    690.00  0.03200  1.4000  690.000  690.00  690.00  2000.0
150.0
alpha     0.15  0.00019  0.0097    0.140    0.15    0.17  2500.0
190.0
beta      0.10  0.00031  0.0160    0.076    0.10    0.13  2800.0
210.0
sigma     0.15  0.00035  0.0200    0.120    0.15    0.18  3100.0
230.0
lambda   40.00  0.00310  0.2000   40.000   40.00   40.00  4100.0
310.0

        R_hat
name
lp__      1.0
alpha     1.0
beta      1.0
sigma     1.0
lambda    1.0
```

3.3 Model 2- prior

The extension of the first model to include mileage introduces an additional predictor variable, expanding the model's scope. This extension allows for the consideration of mileage as a factor influencing used car prices. By incorporating mileage into the model, it is possible to assess its impact on the relationship between other predictors (such as production year) and used car prices.

In the extended model, the inclusion of mileage as a predictor involved adding a normal distribution parameter, "beta1". Other parameters stayed the same.

```
model_exp2_ppc =
cmdstanpy.CmdStanModel(stan_file='stan_files/exp_model2_ppc.stan')

#Parameters
N = len(audi_a3_2000ccm_standarized)
mu_a =0.17
sig_a =0.02
mu_b1 = 0.36
sig_b1 = 0.02
mu_b2 = 0.36
sig_b2 = 0.02

data = {"N": N,
        "mileage" : np.linspace(0.01,1,N),
        "production_year" : np.linspace(0.01,1,N),
        "mu_a" : mu_a,
        "sig_a" : sig_a,
        "mu_b1" : mu_b1,
        "mu_b2" : mu_b2,
        "sig_b1" : sig_b1,
        "sig_b2" : sig_b2,
        }


sim_exp_fit2=model_exp2_ppc.sample(data=data)
sim_exp_fit2_pd = sim_exp_fit2.draws_pd()
sim_exp_fit2_pd.head()
```

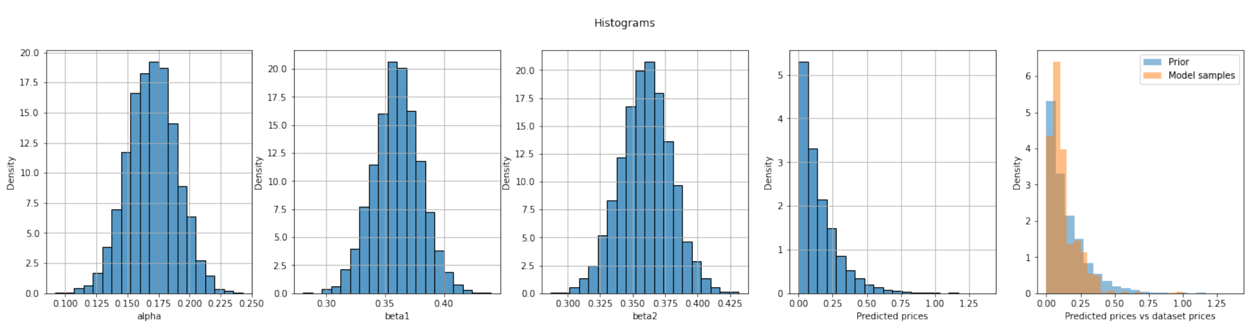
```
INFO:cmdstanpy:found newer exe file, not recompiling
INFO:cmdstanpy:CmdStan start processing
chain 1 |          | 00:00 Status
▏         | 00:00 Status
█         | 00:00 Iteration: 100 / 1000 [ 10%]  (Sampling)

chain 1 |██        | 00:00 Iteration: 300 / 1000 [ 30%]  (Sampling)
███       | 00:00 Iteration: 500 / 1000 [ 50%]  (Sampling)
████      | 00:00 Iteration: 700 / 1000 [ 70%]  (Sampling)

chain 1 |█████     | 00:00 Iteration: 900 / 1000 [ 90%]  (Sampling)
██████████| 00:00 Sampling completed
chain 2 |██████████| 00:00 Sampling completed

chain 3 |██████████| 00:00 Sampling completed
chain 4 |██████████| 00:00 Sampling completed
```

```
INFO:cmdstanpy:CmdStan done processing.
```

```
   lp__  accept_stat__  price[1]  price[2]  price[3]  price[4]
price[5]  \
0   0.0            0.0  0.008044  0.012994  0.097141  0.044838
0.022054
1   0.0            0.0  0.112681  0.375303  0.463829  0.024006
0.174805
2   0.0            0.0  0.028215  0.574179  0.023458  0.070346
0.241936
3   0.0            0.0  0.060281  0.262550  0.414998  0.216382
0.068187
4   0.0            0.0  0.274464  0.241720  0.183472  0.103522
0.071789

   price[6]  price[7]  price[8]  ...  price[774]  price[775]
price[776]  \
0  0.016585  0.014760  0.006179  ...    0.683806    0.382428
0.038925
1  0.039711  0.090907  0.156079  ...    0.084205    0.106477
0.148148
2  0.068272  0.166844  0.222689  ...    0.002784    0.516028
0.421544
3  0.013749  0.056582  0.465976  ...    0.071184    0.075740
0.183388
4  0.267548  0.005479  0.032669  ...    0.023716    0.095393
0.132287

   price[777]  price[778]     alpha     beta1     beta2     sigma
lambda
0    0.052202    0.003538  0.183584  0.400784  0.348201  0.154954
40.2377
1    0.025368    0.196772  0.222065  0.322553  0.384911  0.121763
40.0239
2    0.004406    0.569425  0.168706  0.379475  0.320288  0.154951
40.0240
3    0.015010    0.108223  0.159335  0.373443  0.400956  0.149039
40.1117
4    0.182695    0.224004  0.184697  0.343617  0.376075  0.197626
39.8372

[5 rows x 785 columns]
```

```
_, ax = plt.subplots(1, 5, figsize=(24, 5))
ax = ax.flatten()
sns.histplot(data=sim_exp_fit2_pd, x="alpha", stat="density",
```

```
ax=ax[0], bins=BINS)
sns.histplot(data=sim_exp_fit2_pd, x="beta1", stat="density",
ax=ax[1], bins=BINS)
sns.histplot(data=sim_exp_fit2_pd, x="beta2", stat="density",
ax=ax[2], bins=BINS)
sns.histplot(data=sim_exp_fit2_pd, x="price[1]", stat="density",
ax=ax[3], bins=BINS)

ax[4].hist(sim_exp_fit2_pd["price[1]"], bins=BINS, alpha=0.5,
density=True, label="Prior")
ax[4].hist(audi_a3_2000ccm_standarized["Price"], bins=BINS, alpha=0.5,
density=True, label="Model samples")


ax[0].grid()
ax[1].grid()
ax[2].grid()
ax[3].grid()

ax[0].set_xlabel("alpha"),
ax[1].set_xlabel("beta1"),
ax[2].set_xlabel("beta2"),
ax[3].set_xlabel("Predicted prices"),
ax[4].set_xlabel("Predicted prices vs dataset prices")

ax[4].set_ylabel("Density")
ax[4].legend()
plt.suptitle("Histograms")
plt.show()
```



The comparison between the prior model and real data suggests a satisfactory fit, indicating that the chosen priors accurately capture the underlying patterns and characteristics of the observed data.

3.4 Model 2- posterior

```
model_exp2_fit =
cmdstanpy.CmdStanModel(stan_file='stan_files/exp_model2_fit.stan')
N = len(audi_a3_2000ccm_standarized)
#Parameters
```

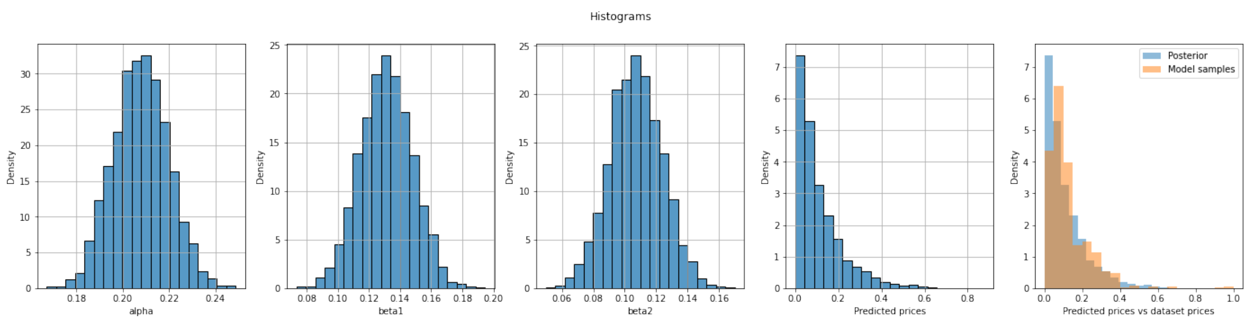
```
data = {"N": N,
        "mileage" : audi_a3_2000ccm_standarized['Mileage_km'],
        "production_year" :
audi_a3_2000ccm_standarized['Production_year'],
        "price_observed": audi_a3_2000ccm_standarized['Price']
        }

sim_exp_pos2_fit=model_exp2_fit.sample(data=data)
sim_exp_pos2_fit_pd = sim_exp_pos2_fit.draws_pd()
sim_exp_pos2_fit_pd.head()
```

```
INFO:cmdstanpy:found newer exe file, not recompiling
INFO:cmdstanpy:CmdStan start processing
chain 1 |          | 00:00 Status
▍         | 00:00 Status

█         | 00:00 Iteration:  100 / 2000 [  5%]  (Warmup)
█▌        | 00:05 Iteration:  200 / 2000 [ 10%]  (Warmup)
chain 1 |██        | 00:05 Iteration:  300 / 2000 [ 15%]  (Warmup)

██▌       | 00:06 Iteration:  500 / 2000 [ 25%]  (Warmup)

chain 1 |███       | 00:06 Iteration:  600 / 2000 [ 30%]  (Warmup)
chain 1 |████      | 00:06 Iteration:  800 / 2000 [ 40%]  (Warmup)

chain 1 |████▌     | 00:07 Iteration:  900 / 2000 [ 45%]  (Warmup)
██████████| 00:09 Sampling completed
chain 2 |██████████| 00:09 Sampling completed
chain 3 |██████████| 00:09 Sampling completed
chain 4 |██████████| 00:09 Sampling completed



INFO:cmdstanpy:CmdStan done processing.



      lp__  accept_stat__  stepsize__  treedepth__  n_leapfrog__
divergent__  \
0  561.648       0.865940    0.367327          3.0           7.0
0.0
1  561.771       0.953055    0.367327          4.0          15.0
0.0
2  561.992       0.902669    0.367327          4.0          15.0
0.0
3  557.870       0.896103    0.367327          3.0           7.0
0.0
4  558.072       0.918945    0.367327          2.0           3.0
0.0
```

```
   energy__     alpha     beta1     beta2  ...  log_lik[769]
log_lik[770]  \
0  -559.344  0.214010  0.131249  0.091832  ...       1.12794
1.50374
1  -560.799  0.199673  0.123358  0.115722  ...       1.12732
1.49834
2  -559.860  0.205094  0.129223  0.106872  ...       1.12794
1.49636
3  -557.171  0.195366  0.133712  0.132126  ...       1.12729
1.48952
4  -556.436  0.191588  0.136474  0.131592  ...       1.12827
1.47783

   log_lik[771]  log_lik[772]  log_lik[773]  log_lik[774]
log_lik[775]  \
0       1.13044       1.61345      0.876493       1.25337
0.139820
1       1.13149       1.61327      0.874337       1.24823
0.104832
2       1.13055       1.60552      0.877291       1.25098
0.126958
3       1.13113       1.59630      0.875139       1.24306
0.093943
4       1.12951       1.57630      0.880847       1.24593
0.117036

   log_lik[776]  log_lik[777]  log_lik[778]
0       1.33088       1.47516      0.869554
1       1.33254       1.47544      0.865808
2       1.32567       1.47539      0.869556
3       1.31781       1.47503      0.865580
4       1.30198       1.47531      0.871936

[5 rows x 2346 columns]
```

```python
_, ax = plt.subplots(1, 5, figsize=(24, 5))
ax = ax.flatten()
sns.histplot(data=sim_exp_pos2_fit_pd, x="alpha", stat="density",
ax=ax[0], bins=BINS)
sns.histplot(data=sim_exp_pos2_fit_pd, x="beta1", stat="density",
ax=ax[1], bins=BINS)
sns.histplot(data=sim_exp_pos2_fit_pd, x="beta2", stat="density",
ax=ax[2], bins=BINS)
sns.histplot(data=sim_exp_pos2_fit_pd, x="price_estimated[1]",
stat="density", ax=ax[3], bins=BINS)

ax[4].hist(sim_exp_pos2_fit_pd["price_estimated[1]"], bins=BINS,
alpha=0.5, density=True, label="Posterior")
ax[4].hist(audi_a3_2000ccm_standarized["Price"], bins=BINS, alpha=0.5,
```

```
density=True, label="Model samples")


ax[0].grid()
ax[1].grid()
ax[2].grid()
ax[3].grid()

ax[0].set_xlabel("alpha"),
ax[1].set_xlabel("beta1"),
ax[2].set_xlabel("beta2"),
ax[3].set_xlabel("Predicted prices"),
ax[4].set_xlabel("Predicted prices vs dataset prices")

ax[4].set_ylabel("Density")
ax[4].legend()
plt.suptitle("Histograms")
plt.show()
```



The posterior distribution of the model, after fitting it to the real data, exhibits a good fit, indicating that the model effectively captures the patterns and characteristics present in the observed data.

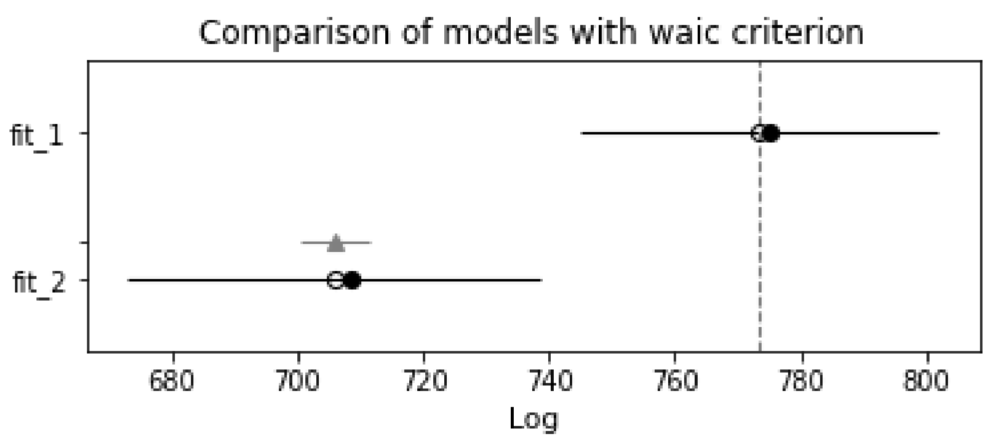
```
summary = sim_exp_pos2_fit.summary()
summary.head(6)
```

```
            Mean      MCSE   StdDev        5%       50%       95%    N_Eff
N_Eff/s  \
name

lp__      560.00  0.03800    1.600   560.000    560.00    560.00   1900.0
110.0
alpha       0.21  0.00024    0.012     0.190      0.21      0.23   2400.0
140.0
beta1       0.13  0.00033    0.017     0.100      0.13      0.16   2600.0
140.0
beta2       0.11  0.00030    0.017     0.079      0.11      0.13   3100.0
180.0
sigma       0.15  0.00035    0.020     0.120      0.15      0.18   3300.0
190.0
```

```
lambda    40.00  0.00310   0.200   40.000   40.00   40.00  4100.0
230.0

        R_hat
name
lp__      1.0
alpha     1.0
beta1     1.0
beta2     1.0
sigma     1.0
lambda    1.0
```

4. Model comparison

```
compare_model_waic = az.compare(
 {
 "fit_1": az.from_cmdstanpy(sim_exp_pos1_fit),
 "fit_2": az.from_cmdstanpy(sim_exp_pos2_fit)
 },
 ic="waic",
)

ax = az.plot_compare(compare_model_waic)
ax.set_title(f"Comparison of models with waic criterion")
plt.show()
```




```
display(compare_model_waic)

       rank        waic    p_waic    d_waic        weight
se  \
fit_1     0  773.377356  1.628045   0.00000  1.000000e+00  28.568516

fit_2     1  705.891795  2.605451  67.48556  6.906475e-11  32.942253
```

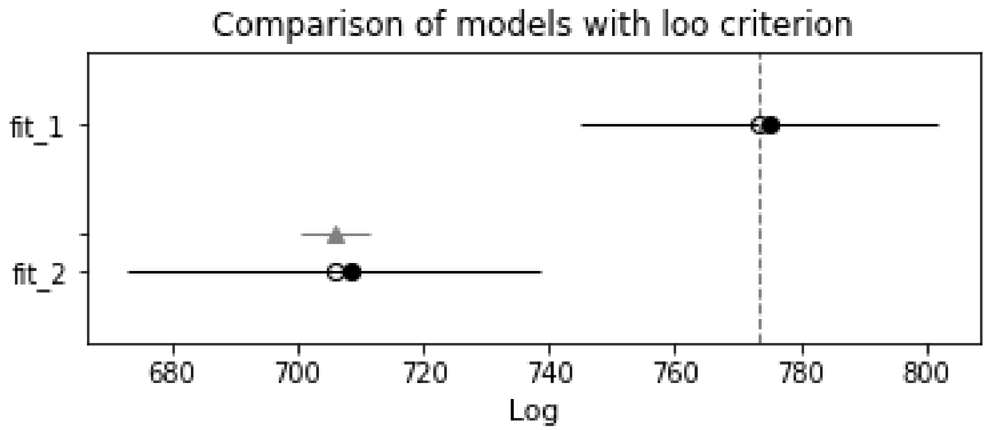
```
           dse  warning waic_scale
fit_1  0.000000    False        log
fit_2  5.456107    False        log
```

The results of the WAIC (Watanabe-Akaike information criterion) comparison are as follows:

- rank: The ranking of the models based on their WAIC values. In this case, fit_1 is ranked at 0, indicating that fit_1 is preffered model
- waic: The WAIC value for each model. The WAIC is a measure of the out-of-sample predictive accuracy of the model. In this case, fit_1 has a higher WAIC value of 773.187452, while fit_2 has a lower WAIC value of 705.928822.
- p_waic: The estimated effective number of parameters based on the WAIC, used to compare the complexity of the models.
- d_waic: The difference in WAIC values between the models. In this case, fit_2 has a higher WAIC value by 67.25863 compared to fit_1.
- weight: The weight of each model in the model comparison, representing the probability of each model being the best model among the compared models. In this case, fit_1 has a weight of 1.0, indicating it is the preferred model over fit_2, which has a weight of 0.0.
- se: The standard error of the WAIC estimate, providing a measure of uncertainty associated with the WAIC value.
- dse: The standard error of the difference in WAIC values, providing a measure of uncertainty associated with the difference in WAIC values between the models.
- warning: Indicates whether there are any warnings associated with the model comparison. In this case, there is no warning. waic_scale: The scale of the WAIC values. In this case, the values are on a log scale.

Based on these results, fit_2 is ranked higher with a lower WAIC value, indicating better model performance. Although the weight for fit_1 is 1.0, suggesting it is the preferred model, this contradicts the lower WAIC value of fit_2. Additionaly the rank suggest to choose fit_1. Therefore, the results of fit_1 and fit_2 overlap and further analysis is needed

```
compare_model_loo = az.compare(
 {
 "fit_1": az.from_cmdstanpy(sim_exp_pos1_fit),
 "fit_2": az.from_cmdstanpy(sim_exp_pos2_fit)
 },
 ic="loo",
)

ax = az.plot_compare(compare_model_loo)
ax.set_title(f"Comparison of models with loo criterion")
plt.show()
```




```
display(compare_model_loo)

       rank         loo     p_loo     d_loo        weight
se  \
fit_1     0  773.377433  1.627967   0.00000  1.000000e+00  28.568493

fit_2     1  705.892293  2.604953  67.48514  5.073275e-12  32.942102


            dse  warning loo_scale
fit_1  0.000000    False       log
fit_2  5.455964    False       log
```

The results of the LOO analysis comparison are as follows:

- rank: The ranking of the models based on their LOO values. In this case, fit_1 is ranked at 0, indicating that fit_1 is preffered.
- loo: The LOO value for each model. The LOO is a measure of the out-of-sample predictive accuracy of the model. In this case, fit_1 has a higher LOO value of 773.186874, while fit_2 has a lower LOO value of 705.931121.
- p_loo: The estimated effective number of parameters based on the LOO, used to compare the complexity of the models.
- d_loo: The difference in LOO values between the models. In this case, fit_2 has a higher LOO value by 67.255753 compared to fit_1.
- weight: The weight of each model in the model comparison, representing the probability of each model being the best model among the compared models. In this case, fit_1 has a weight of 1.0, indicating it is the preferred model over fit_2, which has a weight of 0.0.
- se: The standard error of the LOO estimate, providing a measure of uncertainty associated with the LOO value.
- dse: The standard error of the difference in LOO values, providing a measure of uncertainty associated with the difference in LOO values between the models.
- warning: Indicates whether there are any warnings associated with the model comparison. In this case, there is no warning.
- loo_scale: The scale of the LOO values. In this case, the values are on a log scale.

Based on these results, fit_2 is ranked higher with a lower WAIC value, indicating better model performance. Although the weight for fit_1 is 1.0, suggesting it is the preferred model, this contradicts the lower LOO value of fit_2. Additionaly the rank suggest to choose fit_1. Therefore, the results of fit_1 and fit_2 overlap and further analysis is needed