Probability & Statistics

Problem set №5. Week starting March 25th

1. Continuous random variable (X, Y) has density function $f(x, y)$. Prove that
$$\mathrm{E}(X+Y) = \mathrm{E}(X) + \mathrm{E}(Y).$$

2. X is a discrete random variable. Seuences $\{x_i\}, \{p_i\}$ – values and probabilities of the r.v. Prove that if $Y = aX + b$ then $\mathrm{V}(Y) = a^2\mathrm{V}(X)$, $(a, b \in \mathbb{R})$.

3. Random variable X has standard normal distribution, i.e. $f(x) = \dfrac{1}{\sqrt{2\pi}}\exp\left(-\dfrac{x^2}{2}\right), \quad x \in \mathbb{R}$. (In short: $X \sim N(0, 1)$). Find distribution (density $f_Y(y) \equiv g(y)$) of variable $Y = X^2$.

4. Check that $\Gamma(1/2) = \sqrt{\pi}$. (Hint: In exercise 1.3 substitution $t = x^2/2$ and compare with exercise 1.6)

5. R.v. X has Gamma distribution with parameters $b, p > 0$ (Abb.: $X \sim \mathrm{Gamma}(b, p)$) if and only if $f(x) = \dfrac{b^p}{\Gamma(p)}\, x^{p-1}\exp(-bx)$, where $x \in (0, \infty)$. Does Y from exercise 3. has Gamma distribution? If answer is positive give the values of parameters b, p.

6. Random variable X has standard normal distribution $X \sim N(0, 1)$. Let $\sigma > 0, \mu \in \mathbb{R}$. Find distribution (density) of variable $Y = \sigma X + \mu$.

7. **2p.** Variable (X, Y) has density fnction of the form $f(x, y) = xy$, in area $[0, 2] \times [0, 1]$. Find cummulative distribution function of this variable. In other words find formula(e) of the integral $F_{XY}(s, t) = \displaystyle\int_{-\infty}^{s}\int_{-\infty}^{t} xy\, dy\, dx$.

8. **2p.** (X, Y) like in exercise 7. Find distribution of $Z = X + Y$.

9. Variable (X, Y) is of continuous type, variables X, Y are independent. Prove that $\mathrm{Cov}(X, Y) = 0$.

Witold Karczewski