Probability & Statistics

Problem set №3. Week starting March 11th

1. A and B are events such that $P(A \cap B) = 1/4$, $P(A^C) = 1/3$, $P(B) = 1/2$. Find $P(A \cup B)$.
2. Is it true that 13. day of the month is connected with Friday? (January 1, 1601 – December 31, 2000)
   EXPLANATION: Year n is a leap year if $n \equiv_4 0$, with the exception of years divisible by 100 ($n \not\equiv_{100} 0$); unless $n \equiv_{400} 0$ (i.e. year 2000). How many times in 400-year cycle 13. day of the month was Monday, Tuesday, ..., Sunday?

   Random variables X, Y are independent, iff, in discrete case, condition $P(X = x_i, Y = y_k) = P(X = x_i) \cdot P(Y = y_k)$ holds.
3. R.v. X has binomial distribution $B(n_1, p)$ and r.v. Y $B(n_2, p)$ distribution. X, Y are independent. Prove that $Z = X + Y$ has $B(n_1 + n_2, p)$ distribution.
4. Independent r.vs. X, Y have Poisson distribution with parametersλ_1 i λ_2. Prove that r.v. $Z = X + Y$ has Poisson distribution with parameter $\lambda_1 + \lambda_2$.
5. Given $\mathrm{E}(X) = 1$ i $\mathrm{V}(X) = 5$. Find the values $\mathrm{E}((2 + X)^2)$ i $\mathrm{V}(3X + 2)$
6. Probability of success in trial equals p. We perform trials until 2 successes occur. R.v. X is equal to number of performed trials. Find distribution of X, i.e. find density function (probabilities) and expected value X.
7. Readable and thoroughly - without using the notes - write upper and lower Greek letters: alpha α, beta β, zeta ζ, eta η, lambda λ, chi χ, xi ξ.

8. Card is randomly chosen form the deck of 24 playing cards (an old game "thousand"). X is a random variable with values

X	Club	Diamond	Heart	Spade
	0	1	2	4

   ,

   and Y is r.v. with values

Y	Ace	King	Queen	nes
	5	4	2	0

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   nes ≡ not elsewhere specified
   (a) What is the distribution of r.v. (X, Y)? What are the marginal distribution?
   (b) Check if X and Y are independent.
   (c) Find distribution of $Z = X + Y$.

9. Let X be r.v. with geometric distribution ($X \sim \mathrm{Geom}(p)$). Check that $\mathrm{V}(X) = \dfrac{1 - p}{p^2}$.
10. Cardinality of sets $A_1, \ldots, A_4$ is equal – respectively – 40, 32, 20, 50. An element (from set of 142 elements) is randomly chosen. Cardinality of the set from which chosen element was taken is the value of random variable X. Next a set is randomly chosen. Cardinality of the chosen set is the value of random variable Y. Find $\mathrm{E}(X)$ i $\mathrm{E}(Y)$.

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