

$P(S|T) = \frac{P(T|S)P(S)}{P(T)}$

(  $P(S)$  ) — proba to be sick = 0,01%  
 (  $P(T)$  ) — proba of a positive test = 0,06%  
 probability that the test  
 is positive given that you are sick!

$$P(S|T) = \frac{(1-0,001) \times 0,01\%}{0,06\%} = \boxed{0,1665} \text{ so it is not much}$$

In order to have more chance to be sure to be sick  
 when you come back with a positive test, you  
 take a second one.

We want to compute the following:  $P(S|TT)$  and  
 $P(S|T\bar{T})$ , that account for the probability of being sick  
 given 2 positive tests and the probability of being sick after  
 the first test positive and the second negative respectively.  
 To achieve that we will need to compute particular  
 terms such as the following:  $P(TT)$ ,  $P(T\bar{T})$ .

We know that:

- .)  $P(S) = 0,000$
- .)  $P(T|\bar{S}) = 0,0005$
- .)  $P(S|T) = 0,1665$
- .)  $P(\bar{T}|S) = 0,001$
- .)  $P(T) = 0,0006$

$$P(S|TT) = \frac{P(TT|S) \times P(S)}{P(TT)} \quad \text{from Bayes' rule and respectively:}$$

$$P(S|T\bar{T}) = \frac{P(T\bar{T}|S) P(S)}{P(T\bar{T})}$$

So we need to compute  $P(TT)$ ,  $P(T\bar{T})$ ,  $P(TT|S)$  and  $P(T\bar{T}|S)$ .

$$\begin{aligned} *) P(TT) &= P(TT|S)P(S) + P(TT|\bar{S})P(\bar{S}) \\ &= P(T|S)^2 P(S) + P(T|\bar{S})^2 P(\bar{S}) \\ &= 0,999^2 \times 0,0001 + 0,0005^2 \times 0,9999 = \underline{110^{-9}} \end{aligned}$$

$$\begin{aligned} *) P(T\bar{T}) &= P(T\bar{T}|S)P(S) + P(T\bar{T}|\bar{S})P(\bar{S}) \\ &= P(T|S)P(\bar{T}|S)P(S) + P(T|\bar{S})P(\bar{T}|\bar{S})P(\bar{S}) \\ &= 0,999 \times 0,001 \times 0,0001 + 0,0005 \times 0,9995 \times 0,9999 \\ &\approx \underline{510^{-9}} \end{aligned}$$

Therefore, from these results, we can now compute  $P(TT|S)$  and  $P(T\bar{T}|S)$ .

$$*) P(S|TT) = \frac{P(TT|S) \times P(S)}{P(TT)}$$

$$= \frac{P(T|S)^2 P(S)}{2 \cdot 10^{-4}} = 0,998$$

$$\begin{aligned} *) P(S|T\bar{T}) &= \frac{P(T\bar{T}|S) P(S)}{P(T\bar{T})} \\ &= \frac{P(T|S) P(\bar{T}|S) P(S)}{5 \cdot 10^{-4}} = 1,998 \cdot 10^{-4} \end{aligned}$$

We can then conclude that by taking a second test we are way more sure of being sick or not.

If the second test is positive, then there is almost 100% chance (99,8%) that you are sick and if

the second test is negative, then there is a ridiculous  $\approx 0,2\%$  chance that it is a false negative.