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Assignment - 4

1, Find the rank of the matrix A by reducing in row echelon form.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}_{4\times4}$$

$R_2 \to R_2 - 2R_1$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \to R_4 + R_3$,

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \to NZ \\ \to NZ \\ \to NZ \\ \to Z \end{matrix} \Bigg\} \text{Rank} = 3$$

3, Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ find the eigen values and eigen vectors of A^{-1}

and $A + 4I$.

$$A^{-1} = \frac{1}{4-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

characteristic equation.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} \frac{2}{3} - \lambda & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} - \lambda \end{vmatrix} = 0$$

$$\left(\frac{2}{3} - \lambda\right)^2 - \left(\frac{1}{3}\right)^2 = 0$$

$$\left(\frac{2}{3} - \lambda - \frac{1}{3}\right)\left(\frac{2}{3} - \lambda + \frac{1}{3}\right) = 0$$

$$\left(\frac{1}{3} - \lambda\right)(1 - \lambda) = 0$$

$\boxed{\lambda = 1, 1/3}$ → These are eigen values.

sub $\lambda = 1$

Let $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be eigen vectors

$$\begin{bmatrix} \frac{2}{3} - 1 & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} - 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-\frac{1}{3}x_1 + \frac{1}{3}x_2 = 0$ $\quad$ $x_1 = x_2 = k \Rightarrow -\frac{1}{3}k + \frac{1}{3}k = 0$

$\frac{1}{3}x_1 - \frac{1}{3}x_2 = 0$ $\quad$ $\frac{1}{3}k - \frac{1}{3}k = 0$

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eigen vectors $= K\begin{bmatrix}-1\\1\end{bmatrix}$

sub $\lambda = 1/3$

$$\begin{bmatrix}\frac{2}{3}-\frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{2}{3}-\frac{1}{3}\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}\frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{3}\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$\frac{1}{3}x_1 + \frac{1}{3}x_2 = 0$

$\frac{1}{3}x_1 + \frac{1}{3}x_2 = 0$

$\frac{1}{3}(x_1+x_2) = 0$

$x_1 + x_2 = 0$

$x_1 = -x_2$

let $x_1 = K$

$x_2 = -K$

eigen vectors $= \begin{bmatrix}K\\-K\end{bmatrix}$

$= 1\begin{bmatrix}1\\-1\end{bmatrix}$

$A + 4I$

$$\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix} + 4\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix} + \begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}$$

$$\begin{bmatrix}6 & -1\\-1 & 6\end{bmatrix}$$

$|A-\lambda I| = 0$

$$= \begin{bmatrix}6-\lambda & -1\\-1 & 6-\lambda\end{bmatrix}$$

$(6-\lambda)^2 - (-1)^2$

$= (6-\lambda)(7-\lambda) = 0$

eigen values $\lambda = 5, 7$

sub $\lambda = 5$

$$= \begin{bmatrix}1 & -1\\-1 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$x_1 - x_2 = 0$

$-x_1 + x_2 = 0$

$x_1 = x_2$

let $x_1, x_2 = K$

eigen vectors $= \begin{bmatrix}K\\K\end{bmatrix} = K\begin{bmatrix}1\\1\end{bmatrix}$

sub $\lambda = 7$

$$= \begin{bmatrix}-1 & -1\\-1 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$-x_1 - x_2 = 0$

$-x_1 - x_2 = 0$

$-x_1 = x_2$

let $x_1, x_2 = K$

eigenvectors $= \begin{bmatrix}-K\\K\end{bmatrix} = K\begin{bmatrix}-1\\1\end{bmatrix}$

4, solve by gauss-seidel Method (Take three iteration)

$$8x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

with initial values $x(0) = 0$, $y(0) = 0$, $z(0) = 0$

$$x = \frac{1}{3}[7.85 + 0.1y - 0.2z]$$
$$y = \frac{1}{7}[-19.3 - 0.1x + 0.3z]$$
$$z = \frac{1}{10}[7.14 - 0.3x + 0.2y]$$

iteration - 1 :-

$$z = y = 0$$
$$x = \frac{1}{3}[7.85]$$
$$x = 2.6167$$

Now $z = 0$

$$y_1 = \frac{1}{7}[-19.3 - 0.1(2.616) + 0.3(0))] = -2.794$$
$$z_1 = \frac{1}{10}[7.14 - 0.3(2.616) + 0.2(-2.794)] = \frac{70.056}{10} = 7.0056$$

iteration - 2 :-

$$x(2) = \frac{7.85 - 0.2(7.00) + 0.1(-2.79)}{3} = \frac{6.175}{3} = 2.057$$

$$y(2) = \frac{-19.3 + 0.3(7.00) - 0.1(2.05)}{3} = -2.487.$$

$$z(2) = \frac{7.14 + 0.2(-2.487) - 0.3(2.23)}{10} = 7.023$$

iteration - 3 :-

$$x(3) = \frac{7.85 - 0.2(7.023) + 0.1(-2.487)}{3} = 2.064$$

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$$y(3) = \frac{-19.3 + 0.3(7.023) - 0.3(2.064)}{7} = -2.544$$

$$z(3) = \frac{71.4 + 0.2(-2.54) - 0.3(2.064)}{10} = 7.027$$

5. Define consistent and inconsistent system of equations. Hence, solve the following system of equations if consistent
$x+3y+2z=0$, $2x-y+3z=0$, $3x-5y+4z=0$, $x+17y+4z=0$.

Consistent :- A system of equations is consistent if it has at least one solution, meaning the equations have a common solution.

Inconsistent :- A system of equations is inconsistent if it has no solutions meaning the equations do not intersect at any point and are contradictory.

$$x + 3y + 2z = 0$$
$$2x - y + 3z = 0$$
$$3x - 5y + 4z = 0$$
$$x + 17y + 4z = 0$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [A : B]$$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{array}{c} \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \to NZ \\ \to NZ \\ \to Z \\ \to Z \end{array} \quad \text{Rank} = 2.$$

Rank(C) = Rank(A) ≠ n.o of unknowns.

so, we get infinite solutions.

$AX = B$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + 3y + 2z = 0$

$0 - 7y - 1z = 0$

$-7y = 1z$

$\boxed{y = -1/7\, z}$

$x - \frac{3}{7}z + \frac{2}{1}z = 0$

$x + \frac{3}{7}z - \frac{2}{1}z = 0$

$x = \frac{3z - 14z}{7}$

$\boxed{x = \frac{-11z}{7}}$

let $\boxed{z = k}$

$\boxed{y = \frac{-1}{7}k}$

$\boxed{x = \frac{-11}{7}k}$

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9. Explain one application of matrix operations in image processing with example.

Image transformation:

Application: Transformation matrices are used for operations like scalling, rotation, translation, etc.

Ex:- A 2D transformation matrix can be applied to each pixel in the image to perform operations like rotation or scalling.

Give a brief description of linear transformation for 2D image.

10. In a computer vision, a linear transformation is widely used for tasks like rotating, scaling, shearing and translating images. A common linear transformation for rotating a 2D images is the rotation matrix.

The rotation of a point (x, y) in a 2D space by an angle θ

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

To apply this transformation to an entire image, each pixel's coordinates can be transformed using the rotation matrix. If the original image is represented as a matrix of pixel values, each pixel's position in the new image after rotation can be calculated using this transformation.

2, $T: W \longrightarrow P_2$

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b)+(b-c)x+(c-a)x^2$$

find the rank and nullity of T.

$$\Rightarrow T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a-b)+(b-c)x+(c-a)x^2$$

let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ then

$$T(A) = (a-b)+(b-c)x+(c-a)x^2$$
$$= a - bx + c(x^2 - x + 1)$$

$\therefore$ The image of T is the set of all polynomials of degree atmost 2, denoted as P_2.

Rank of T:-

The rank of T is the dimension of its image since P_2 has a dimension of 3 (coeffiencts for x^0, x^1 and x^2) the rank of T is 3.

The Null space of symmetric matrix

$T(A) = 0$ this leads to the system of equations

$a - b = 0 \qquad b - c = 0 \qquad c - a = 0$

$\therefore a = b = c$

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6, $T: P_2 \longrightarrow P_2$ is linear transformation.

$$T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$$

$\Rightarrow$ $T(a+bx+c) = (a+1)+(b+1)x+(c+1)x^2$ is a linear transformation, we need to check two properties.

1, Additivity $\cdot\, T(U+V) = T(U)+T(V)$

2, Homogeneity of degree 1:

$T(ku) = kT(U)$ for all u in the domain of T

and all scalars k.

1, $T(U+V) = T((a_1+b_1x+c_1)+(a_2+b_2x+c_2))$

$= T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2))$

$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$

$= (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x+(c_2+1)x^2$

$T(a_1+b_1x+c_1)+T(a_2+b_2x+c_2)$

so function is additive.

$\Rightarrow$ 2, Homogenity of Degree 1:-

$T(kU) = T(k(a+bx+c))$

$= T(ka+kbx+kc) = (ka+1)+(kb+1)x+(kc+1)$

$= k(a+1)+k(b+1)x+k(c+1)x^2$

$= kT(a+bx+c)$

so, the function is homogeneous of degree 1.

$\therefore$ It included linear transformation.

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7. $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In ⑥ cases S is not a basis determine sub space spanned by S.

$\Rightarrow$ $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ can be arranged as a matrix.

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

Now, lets perform row reduction to obtain the echelon form.

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} \quad R_2 \leftarrow R_2 - 2R_1, \quad R_3 \leftarrow R_3 - 3R_1$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \quad R_3 \rightarrow R_3 + \frac{9}{5}R_2 \quad \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Third row of zeros indicates that the vectors in S are linearly dependent. for basics of the subspace spanned by S.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \end{bmatrix}$$

$(1,3,2)$ and $(0,-5,5)$ these vectors form a basis for the subspace spanned by S.

$\therefore$ Dimension of subspace spanned by $S = 2$.

$\therefore$ set S is not a basis of R^3 because the row reduced form has a row of zeros.

$\therefore$ The basis for the subspace spanned by S is $\{(1,3,-2), (0,-5,5)\}$

$\therefore$ The dimension of the sub space is 2.

8, Using Jacobs method (perform 3 iterations) solve.

$3x-6y+2z=23$, $-4x+y-z=-15$, $x-3y+7z=16$, with

initial values, $x_0=1$, $y_0=1$, $z_0=1$.

$$3x-6y+2z=23$$
$$-4x+y-z=-15 \qquad \text{with } x_0=1$$
$$x-3y+7z=16 \qquad y_0=1$$
$$z_0=1$$

Iteration-1:-

$$x(1)=\frac{23+6y(0)-2(z(0))}{3}$$

$$=\frac{23+6(1)-2(1)}{3}=9.0$$

$$y(1)=\frac{-15+4(x(0))-z(0)}{7}=-9.0$$

$$z(1)=\frac{16-x(1)-3(y(1))}{7}$$

$$=\frac{16-9-3(-9)}{7}=2.0$$

Iteration-2:-

$$x(2)=\frac{23+6y(1)-2(z(1))}{3}$$

$$=\frac{23+6(-9)-2(2)}{3}=5.0$$

$$y(2)=\frac{-15+4(x(1))+z(1)}{7}$$

$$=\frac{-15+4(9)+z(2)}{7}=-5.$$

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$$z(2) = \frac{16 - x(1) + 3y(1)}{7} = 3.0$$

Iteration-3

$$x(3) = \frac{23 + 6y(2) - 2(z(2))}{3}$$

$$= \frac{23 + 6(-5) - 2(3)}{3} = 6.0$$

$$y(3) = \frac{-15 + 4x(2) - z(2)}{7}$$

$$= \frac{-15 + 4(5) - 3}{7} = -6.0$$

$$z(3) = \frac{16 - x(2) + 3y(2)}{7} = 2.0.$$