K. Ujwala
054

Assignment - 1

Test for consistency and solve.

i, $2x-3y+7z=5$

$3x+y-3z=13$

$2x+19y-47z=32$

$$\begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$AX = B$

C matrix is combination of A and B matrix.

$C = [A:B]$

$$= \begin{bmatrix} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix}$$

$R_2 \to 2R_2 - 3R_1$

$$= \begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 2 & 19 & -47 & 32 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 0 & 22 & -54 & 27 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{array}{c} \begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \to NZ \\ \to NZ \\ \to NZ \end{array}$$

Rank = 3.

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 0 & 11 & -27 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \rightarrow NZ \\ \rightarrow NZ \\ \rightarrow Z \end{matrix}$$

$$\text{Rank}(C) \neq \text{Rank}(A)$$

The given system is not consistent and there exists no solution.

ii, $2x - y + 3z = 8$

$-x + 2y + z = 4$

$3x + y - 4z = 0$

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

$AX = B$

C matrix is combination A & B matrix

$C = [A : B]$

$$= \begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix}$$

$R_2 \rightarrow 2R_2 - R_1$

$$= \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 5 & -1 & 0 \\ 3 & 1 & -4 & 0 \end{bmatrix}$$

$R_3 \rightarrow 2R_3 - 3R_1$

$$= \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 5 & -1 & 0 \\ 0 & 5 & -17 & -24 \end{bmatrix}$$

K. Vijwala

054 (2)

$R_3 \to R_3 - R_2$

$$= \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 5 & -1 & 0 \\ 0 & 0 & -16 & -24 \end{bmatrix} \begin{array}{l} \to NZ \\ \to NZ \\ \to NZ \end{array}$$

Rank $[A:B] = 3$

$$\text{Rank} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 5 & -1 \\ 0 & 0 & -16 \end{bmatrix} \begin{array}{l} \to NZ \\ \to NZ \\ \to NZ \end{array}$$

Rank $(A) = 3$.

Rank $[A:B]$ = Rank $[B]$ and it is equal to no. of unknown then we get unique solution.

iii) $4x - y = 12$

$-x + 5y - 2z = 0$

$-2x + 4z = -8$

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$AX = B$

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$C = [A:B]$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & 8 \end{bmatrix}$$

$R_2 \rightarrow 4R_2 - R_1$

$$= \begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 21 & -8 & -12 \\ -2 & 0 & 4 & 8 \end{bmatrix}$$

$R_3 \rightarrow 2R_3 - R_1$

$$= \begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 21 & -8 & -12 \\ 0 & 1 & 8 & -28 \end{bmatrix}$$

$R_3 \rightarrow 21R_3 - R_2$

$$= \begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 21 & -8 & -12 \\ 0 & 0 & 172 & -576 \end{bmatrix} \begin{matrix} \rightarrow NZ \\ \rightarrow NZ \\ \rightarrow NZ \end{matrix}$$ Rank $[A:B] = 3$

$$= \begin{bmatrix} 4 & -1 & 0 \\ 0 & 21 & -8 \\ 0 & 0 & 172 \end{bmatrix}$$ Rank $[A] = 3$

Rank $[A:B]$ = Rank $[A]$ and it is equal to no of unknown then we get unique solution.

c, Find for what values of λ, the given equations $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ have a solution and solve them completely in each case.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$C = [A:B]$

K. Ujwala
054 (3)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - R_1 \quad R_3 \longrightarrow R_3 - R_1$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 3R_1$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1-3(\lambda-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1-3\lambda+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix} \rightarrow \text{Rank}(A) \neq \text{Rank}(B) \neq n$$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - \lambda - 2\lambda + 2 = 0$

$\lambda(\lambda-2) - 1(\lambda-2) = 0$

$(\lambda-1)(\lambda-2) = 0$

$\lambda = 1, \lambda = 2$

case-1 :- $\boxed{\lambda = 1}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x + y + z = 1$

$y + 3z = 1$

let $z = k$

$x + y + k = 0$

$x + y = -k$

$x = -k - y + 1$

$y = -3k$

$x = -k + 3k + 1$

$\boxed{x = 2k + 1}$

case-2 :- $\boxed{\lambda = 2}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x + y + z = 1$

$y + 3z = 1$

$y + 3k = 1$

$y = 1 - 3k$

$x + 1 - 3k + k = 1$

$x - 2k = 0$

$\boxed{x = 2k}$

$\boxed{y = 1 - 3k}$

$\boxed{z = k}$

d. Find the solution of the system of equations, $x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [A:B]$$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 1 & -11 & 14 & 0 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of $[A:B] = 2$

Rank of $[A] = 2$ and not equal to n.o of unknowns

so, the system of equation is consistent. it will have infinite solutions.

K. Ujwala (4)
054

6) $x+y+z=6$

$x+2y+3z=10$

$x+2y+\lambda z=\mu$

$$\begin{bmatrix}1&1&1\\1&2&3\\1&2&\lambda\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\10\\\mu\end{bmatrix}$$

$C=[A:B]$

$$\begin{bmatrix}1&1&1&6\\1&2&3&10\\1&2&\lambda&\mu\end{bmatrix}$$

$R_2 \to R_3 - R_1$

$$\begin{bmatrix}1&1&1&6\\0&1&2&4\\1&2&\lambda&\mu\end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix}1&1&1&6\\0&1&2&4\\0&0&\lambda-3&\mu-10\end{bmatrix} \Rightarrow \begin{bmatrix}1&1&1&6\\0&1&2&4\\0&0&3-3&8-10\end{bmatrix}$$

Here $A=\begin{bmatrix}1&1&1\\0&1&2\\0&0&\lambda-3\end{bmatrix}$

If $\lambda=3$ then $\begin{bmatrix}1&1&1\\0&1&2\\0&0&0\end{bmatrix}\begin{matrix}\to NZ\\\to NZ\\\to Z\end{matrix}$

rank = 2.

Case-1:- If $\lambda=3$, $\mu\neq10$ then $R(A)=2$

$R(C)=3$ $R(A)\neq R(C)$

$\therefore$ The system is inconsistent and there exists no solution.

Case-2:- If $\lambda\neq3$, $\mu\neq10$ then $R(A)=3$

$R(C)=3$

$R(A)=R(C)$ n is also = 3

The system is consistent and there exists unique solution.

Case-3:- If $\lambda=3$, $\mu=10$

then $R(A)=2$, $R(C)=2$.

Many solutions.

e, Find for what values of λ the given equations $3x+y-\lambda z=0$, $4x-2y-3z=0$, $2\lambda x+4y+\lambda z=0$, may possess non-trivial solution and solve them completely in each case.

$$A=\begin{bmatrix}3 & 1 & -\lambda\\ 4 & -2 & -3\\ 2\lambda & 4 & \lambda\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & -\lambda\\ 4 & -2 & -3\\ 2\lambda+3 & 4+1 & 0\end{bmatrix}$$

$R_3 \rightarrow R_3+R_1$

$$\begin{bmatrix}12 & 4 & -4\lambda\\ 12 & -6 & -9\\ 2\lambda+3 & 5 & 0\end{bmatrix}$$

$$=\begin{bmatrix}12 & 4 & -4\lambda\\ 0 & -10 & -9+4\lambda\\ 2\lambda+3 & 5 & 0\end{bmatrix}$$

$R_1=4R_1 / R_2=3R_2$

$$=\begin{bmatrix}12 & 4 & -4\lambda & 0\\ 0 & -10 & -9+4\lambda & 0\\ 2\lambda+3 & 5 & 0 & 0\end{bmatrix}$$

$R_2 \rightarrow R_2-R_1$

$$=\begin{bmatrix}12 & -6 & -9 & 0\\ 0 & -10 & -9+4\lambda & 0\\ 2\lambda+3 & 5 & 0 & 0\end{bmatrix}$$

$\Rightarrow 12x+6y-9z=0$

$-10y+(-9+4\lambda)z=0$

$(2\lambda+3)x+5y=0$

$= \left(\frac{-2\lambda+3}{5}\right)x = y.$

$12x-6y-9z=0$

$= 12x+\frac{6(2\lambda+3)}{5}x-9\left(\frac{10y}{(-9+4\lambda)}\right)=0$

$= 12x+\frac{6(2\lambda+3)}{5}x+9\left(\frac{\cancel{10}\,2}{(9+4\lambda)}\left(\frac{2\lambda+3}{\cancel{5}}\right)x\right)=0$

$= 12+\frac{6(2\lambda+3)}{5}+\frac{18(2\lambda+3)}{((4\lambda-9))}=0$

$= 12(4\lambda-9)5+6(2\lambda+3)(4\lambda-9)(12\lambda+18)$
$+18(5)(2\lambda+3)=0$

$= 240\lambda-540+180\lambda+270+48\lambda^2$
$-108\lambda+72\lambda-162=0$

$= 48\lambda^2+348\lambda-432=0$

$= \lambda^2+8\lambda-9=0$

$= \lambda^2+9\lambda-\lambda-9=0$

$= \lambda(\lambda-1)+9(\lambda-1)=0$

$(\lambda+9)(\lambda-1)=0$

$\Rightarrow \lambda=1, \lambda=-9.$

K. Ujwala
054 5

$\lambda = 1$

$-x = y, \; z = -2y$

$12x - 6y - 9z = 0$

$12(-y) - 6y - 9(-2y) = 0$

$18y - 18y = 0$

$y = 0$ trivial solution.

$\lambda = -9$

$y = \left(\dfrac{-(2\lambda + 3)x}{5} \right)$

$= 2 - \left(\dfrac{-18 + 3}{5} \right) x = 3x.$

$z = \dfrac{10y}{(-9 + 4\lambda)}$

$= \dfrac{10y}{-45} = \dfrac{2y}{-9}$

$\boxed{z = 2y/-9}$

$12x - 6y - 9z = 0$

$\cancel{12}\left(\dfrac{4}{\cancel{3}}\right) - 6y - \cancel{9}\left(\dfrac{2y}{-\cancel{9}}\right) = 0$

$4y - 6y + 2y = 0$

$y = 0$

trivial solution.

$\therefore$ It has trivial solutions.