Assignment - 2

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Are the following sets of vectors linearly independent ① or dependent?

1. $[1\,0\,0], [1\,1\,0], [1\,1\,1]$

$x_1 = [1,0,0]$. $x_2 = [1,1,0]$ $x_3 = [1,1,1]$

let $\lambda_1, \lambda_2, \lambda_3$ be three scalar

consider $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = 0$

$\lambda_1(1,0,0) + \lambda_2(1,1,0) + \lambda_3(1,1,1)$.

$\lambda_1 + \lambda_2 + \lambda_3 = 0$

$0\lambda_1 + \lambda_2 + \lambda_3 = 0$

$0\lambda_1 + 0\lambda_2 + 1\lambda_3 = 0$

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

$C = [A : B]$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - R_2$

$$\begin{bmatrix} \underbrace{\begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 0 \end{matrix}}_{A} & \underbrace{\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}}_{B} \end{bmatrix}$$

$A\lambda = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda_1 + \lambda_2 + \lambda_3 = 0$ —— (1)

$\lambda_2 + \lambda_3 = 0$ —— (2)

$-\lambda_2 = 0 \Rightarrow \boxed{\lambda_2 = 0}$

sub (2) in (1)

$\boxed{\lambda_1 = 0}$ $\boxed{\lambda_2 = 0}$ $\boxed{\lambda_3 = 0}$

3 scalars are 0, so the given vectors are linearly independent and there exists no solution.

2. $[7, -3, 11, -6]$ $[-56, 24, -88, 48]$

$x_1 = [7 \; -3 \; 11 \; -6]$ $x_2 = [-56 \; 24 \; -88 \; 48]$

let λ_1, λ_2 be scalars

consider $\lambda_1 x_1 + \lambda_2 x_2 = 0$

$\lambda_1(7, -3, 11, -6) + \lambda_2(-56, 24, -88, 48) = 0$

$7\lambda_1 - 56\lambda_2 = 0$

$-3\lambda_1 + 24\lambda_2 = 0$

$11\lambda_1 - 88\lambda_2 = 0$

$-6\lambda_1 + 48\lambda_2 = 0$

In Matrix form

$$\begin{bmatrix} 7 & -56 \\ -3 & -24 \\ 11 & -88 \\ -6 & 48 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$C = [A : B]$

$$\begin{bmatrix} 7 & -56 & 0 \\ -3 & -24 & 0 \\ 11 & -88 & 0 \\ -6 & 48 & 0 \end{bmatrix}$$

$R_2 \rightarrow 7R_2 + 3R_1$

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$$\begin{bmatrix} 7 & -56 & 0 \\ 0 & 0 & 0 \\ 11 & -88 & 0 \\ -6 & 48 & 0 \end{bmatrix}$$

$\boxed{\lambda_1 = 0}$ $\boxed{\lambda_2 = 0}$

$R_2 \leftrightarrow R_4$

$\therefore$ 2 scalars are 0 so, the given 2 vectors are linearly independent.

$$\begin{bmatrix} 7 & -56 & 0 \\ -6 & 48 & 0 \\ 11 & -88 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \longrightarrow 7R_2 + 6R_1$

$$\begin{bmatrix} 7 & -56 & 0 \\ 0 & 0 & 0 \\ 11 & -88 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 7 & -56 & 0 \\ 11 & -88 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \longrightarrow 7R_2 - 11R_1$

$$\begin{bmatrix} 7 & -56 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -56 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$7\lambda_1 - 56\lambda_2 = 0$

$0\lambda_1 + 0\lambda_2 = 0$

$0\lambda_1 + 0\lambda_2 = 0$

$0\lambda_1 + 0\lambda_2 = 0$

3. $[-1, 5, 0]$ $[16, 8, -3]$ $[-64, 56, 9]$

$\alpha_1 = [-1, 5, 0]$ $\alpha_2 = [16, 8, -3]$ $\alpha_3 = [-64, 56, 9]$

Let $\lambda_1, \lambda_2, \lambda_3$ be three scalar

consider $\lambda_1\alpha_1 + \lambda_2\alpha_2 + \lambda_3\alpha_3 = 0$

$\lambda_1(-1, 5, 0) + \lambda_2(16, 8, -3) + \lambda_3(-64, 56, 9) = 0$

$-\lambda_1 + 16\lambda_2 - 64\lambda_3 = 0$

$5\lambda_1 + 8\lambda_2 + 56\lambda_3 = 0$

$0\lambda_1 - 3\lambda_2 + 9\lambda_3 = 0$

$$\begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{A\lambda = B}$

$C = [A : B]$

$$\begin{bmatrix} -1 & 16 & -64 & 0 \\ 5 & 8 & 56 & 0 \\ 0 & -3 & 9 & 0 \end{bmatrix}$$

$R_2 \to R_2 + 5R_1$

$$\begin{bmatrix} -1 & 16 & -64 & 0 \\ 0 & 88 & -274 & 0 \\ 0 & -3 & 9 & 0 \end{bmatrix}$$

$A\lambda = B$

$$\begin{bmatrix} -1 & 16 & -64 \\ 0 & 88 & -274 \\ 0 & 0 & 190 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-\lambda_1 + 16\lambda_2 - 64\lambda_3 = 0$

$0\lambda_1 + 88\lambda_2 - 274\lambda_3 = 0$

$0\lambda_1 + 0\lambda_2 + 190\lambda_3 = 0$

$\boxed{\lambda_3 = 0}$

$88\lambda_2 - 0 = 0$

$\boxed{\lambda_2 = 0}$

$\boxed{\lambda_1 = 0}$

3 scalars are 0. So, the given vectors are linearly independent and there exists no solution.

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4. $[1,-1,1]$, $[1,1,-1]$, $[-1,1,1]$, $[0,1,0]$

Let $\lambda_1, \lambda_2, \lambda_3$ be three scalar

Consider $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = 0$

$\lambda_1(1,-1,1) + \lambda_2(1,1,-1) + \lambda_3(-1,1,1) + \lambda_4(0,1,0) = 0$

$\lambda_1 + \lambda_2 - \lambda_3 + 0\lambda_4 = 0$

$-1\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0$

$1\lambda_1 - 1\lambda_2 + 1\lambda_3 + 0\lambda_4 = 0$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$C = [A:B]$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 0 & -2 & 2 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda_1 + \lambda_2 - \lambda_3 + 0\lambda_4 = 0$

$0\lambda_1 + 2\lambda_2 + 0\lambda_3 + \lambda_4 = 0$

$0\lambda_1 + 0\lambda_2 + 3\lambda_3 + \lambda_4 = 0$

$2\lambda_2 + \lambda_4 = 0$

$\lambda_4 = -2\lambda_2$

Let $\boxed{\lambda_2 = K}$

$\boxed{\lambda_4 = -2K}$

$3\lambda_3 - 2K = 0$

$3\lambda_3 = 2K$

$\boxed{\lambda_3 = \frac{2}{3}K}$

$\lambda_1 + K - \frac{2K}{3} = 0$

$\lambda_1 = \frac{2K}{3} - K$

$\lambda_1 = \frac{2K - K}{3}$

$\boxed{\lambda_1 = -K/3}$

$\therefore$ since $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are non zero. The vectors are linear dependent.

5, $[2,-4], [1,9], [3,5]$

$x_1 = (2,-4) \quad x_2 = (1,9) \quad x_3 = (3,5)$

Let $\lambda_1, \lambda_2, \lambda_3$ be 3 scalars

Consider $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = 0$.

$\lambda_1(2,-4) + \lambda_2(1,9) + \lambda_3(3,5) = 0$

$2\lambda_1 + \lambda_2 + 3\lambda_3 = 0$

$-4\lambda_1 + 9\lambda_2 + 5\lambda_3 = 0$

$$\begin{bmatrix} 2 & 1 & 3 \\ -4 & 9 & 5 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$A\lambda = B$

$C = [A : B]$

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ -4 & 9 & 5 & 0 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & 7 & 11 & 0 \end{bmatrix}_{2\times 4}$$

$R_3 \longrightarrow R_2 - 7R_1$

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & 0 & -11 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & -11 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2\lambda_1 + \lambda_2 + 3\lambda_3 = 0$

$-11\lambda_3 = 0$

$\boxed{\lambda_3 = 0}$

$-\lambda_2 + \lambda_2 + 0 = 0$

$2\lambda_1 + \lambda_2 = 0$

$2\lambda_1 = -\lambda_2$

let $\boxed{\lambda_1 = K}$

E_1

$\boxed{\lambda_2 = 2K}$

since λ_1, λ_2, are non-zero

The given vectors are linearly dependent.

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6. $[3,-2,0,4]$, $[5,0,0,1]$, $[-6,1,0,1]$, $[2,0,0,3]$

$x_1=(3,-2,0,4)$ $x_2=(5,0,0,1)$ $x_3=(-6,1,0,1)$ $x_4=(2,0,0,3)$

Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ be scalars.

consider $x_1\lambda_1+x_2\lambda_2+x_3\lambda_3+x_4\lambda_4=0$

$\lambda_1(3,-2,0,4)+\lambda_2(5,0,0,1)+\lambda_3(-6,1,0,1)+\lambda_4(2,0,0,3)$

$3\lambda_1+5\lambda_2-6\lambda_3+2\lambda_4=0$

$-2\lambda_1+0\lambda_2+\lambda_3+0\lambda_4=0$

$0\lambda_1+0\lambda_2+0\lambda_3+0\lambda_4=0$

$4\lambda_1+1\lambda_2+1\lambda_3+3\lambda_4=0$

$$\begin{bmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 3 \end{bmatrix}\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$A\lambda=B$

$C=[A:B]$

$$\begin{bmatrix} 3 & 5 & -6 & 2 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 3 & 0 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\begin{bmatrix} 3 & 5 & -6 & 2 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 3 & 0 \end{bmatrix}$$

$R_2 \to 3R_2+2R_1$

$$\begin{pmatrix} 3 & 5 & -6 & 2 & 0 \\ 0 & 10 & 15 & 4 & 0 \\ 4 & 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$R_3 \to 3R_3 - 4R_1$

$$\begin{bmatrix} 3 & 5 & -6 & 2 & 0 \\ 0 & 10 & 15 & 4 & 0 \\ 0 & -17 & 27 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to 10R_3 + 17R_2$

$$\begin{bmatrix} 3 & 5 & -6 & 2 & 0 \\ 0 & 10 & 15 & 4 & 0 \\ 0 & 0 & 525 & 78 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$A\lambda = B$

$$\begin{bmatrix} 3 & 5 & -6 & 2 \\ 0 & 10 & 15 & 4 \\ 0 & 0 & 525 & 78 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$3\lambda_1 + 5\lambda_2 - 6\lambda_3 + 2\lambda_4 = 0$

$10\lambda_2 + 15\lambda_3 + 4\lambda_4 = 0$

$525\lambda_3 + 78\lambda_4 = 0$

$525\lambda_3 = -78\lambda_4$

$\lambda_3 = \frac{-78}{525}\lambda_4$

let $\boxed{\lambda_4 = k}$

$$\boxed{\lambda_3 = \frac{-78}{525}k}$$

$3\lambda_1 + 5\lambda_2 - 6\left(\frac{-78}{525}\right)k + 2k = 0$

$3\lambda_1 + 5\lambda_2 + \frac{468}{525}k + 2k = 0$

$3\lambda_1 + 5\lambda_2 = \frac{-468k}{525} - 2k$

$\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are non-zero vectors

so, the given vectors are linearly dependent.

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7. $[3,4,7]$, $[2,0,3]$, $[8,2,3]$, $[5,5,6]$

$x_1 = [3,4,7]$ $x_2 = [2,0,3]$ $x_3 = [8,2,3]$ $x_4 = [5,5,6]$

let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ be scalars

consider $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4 = 0$

$$\lambda_1(3,4,7) + \lambda_2(2,0,3) + \lambda_3(8,2,3) + \lambda_4(5,5,6)$$

$$3\lambda_1 + 2\lambda_2 + 8\lambda_3 + 5\lambda_4 = 0$$

$$4\lambda_1 + 0\lambda_2 + 2\lambda_3 + 5\lambda_4 = 0$$

$$7\lambda_1 + 3\lambda_2 + 3\lambda_3 + 6\lambda_4 = 0$$

$$\begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$A\lambda = B$

$C = [A:B]$

$$\begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 4 & 0 & 2 & 5 & 0 \\ 7 & 3 & 3 & 6 & 0 \end{bmatrix}$$

$R_2 \to 3R_2 - 4R_1$

$$\begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 0 & 0 & -26 & -5 & 0 \\ 7 & 3 & 3 & 6 & 0 \end{bmatrix}$$

$R_3 \to 3R_3 - 7R_1$

$$\begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 0 & 0 & -26 & -5 & 0 \\ 0 & -5 & -47 & -17 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 0 & -5 & -47 & -17 & 0 \\ 0 & 0 & -26 & -5 & 0 \end{bmatrix}$$

$R_2 \to 2R_2 + 5R_1$

$$\begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 0 & 0 & -54 & -9 & 0 \\ 0 & 0 & -26 & -5 & 0 \end{bmatrix}$$

$R_3 \to 27R_3 - 13R_2$

$$\begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 0 & 0 & -54 & -9 & 0 \\ 0 & 0 & 0 & -18 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 8 & 5 \\ 0 & 0 & -54 & -9 \\ 0 & 0 & 0 & -18 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$3\lambda_1 + 2\lambda_2 + 8\lambda_3 + 5\lambda_4 = 0$

$-54\lambda_3 - 9\lambda_4 = 0$

$-18\lambda_4 = 0$

$\boxed{\lambda_4 = 0}$

$-54\lambda_3 = 0$

$\boxed{\lambda_3 = 0}$

$3\lambda_1 + 2\lambda_2 = 0$

$3\lambda_1 = -2\lambda_2$

$\lambda_1 = \frac{-2}{3}\lambda_2$

$\boxed{\lambda_2 = K}$

$\boxed{\lambda_1 = -2/3\, k}$

λ_1, λ_2 are non-zero vectors so, the given vectors are linearly dependent.

8, $[6, 0, 3, 1, 4, 2]$ $[0, -1, 2, 7, 0, 5]$ $[12, 3, 0, -19, 8, -11]$

let $\lambda_1, \lambda_2, \lambda_3$, be scalars

$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = 0$

$\lambda_1(6, 0, 3, 1, 4, 2) + \lambda_2(0, -1, 2, 7, 0, 5) + \lambda_3(12, 3, 0, -19, 8, -11) = 0$

$6\lambda_1 + 0\lambda_2 + 12\lambda_3 = 0$

$0\lambda_1 - 1\lambda_2 + 3\lambda_3 = 0$

$3\lambda_1 + 2\lambda_2 + 0\lambda_3 = 0$

$1\lambda_1 + 7\lambda_2 - 19\lambda_3 = 0$

$4\lambda_1 + 0\lambda_2 + 8\lambda_3 = 0$

$2\lambda_1 + 5\lambda_2 - 11\lambda_3 = 0$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 3 & 2 & 0 \\ 1 & 7 & -19 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix}_{6\times 3} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$R_3 \to 2R_3 - R_1$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 4 & -12 \\ 1 & 7 & -19 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix}$$

$R_4 \to 6R_4 - R_1$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 4 & -12 \\ 0 & 42 & -102 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix}$$

$R_5 \to 6R_5 - 4R_5$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 4 & -12 \\ 0 & 42 & -102 \\ 0 & 0 & 0 \\ 2 & 5 & 11 \end{bmatrix}$$

$R_5 \leftrightarrow R_6$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 4 & -12 \\ 0 & 42 & -102 \\ 2 & 5 & -11 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_5 \to 3R_5 - R_1$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 4 & -12 \\ 0 & 42 & -102 \\ 0 & 15 & -45 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 + 4R_2$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 42 & -102 \\ 0 & 15 & -45 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_5 \leftrightarrow R_3$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 42 & -102 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 + 15R_1$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 42 & -102 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 42 & -102 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 = R_3 + 42R_2$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 0 & 24 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$6\lambda_1 + 12\lambda_3 = 0$

$-\lambda_2 + 3\lambda_3 = 0$

$24\lambda_3 = 0$

$\boxed{\lambda_3 = 0}$

$-\lambda_2 = 0$

$\boxed{\lambda_2 = 0}$

$6\lambda_1 + 0 = 0$

$\boxed{\lambda_1 = 0}$

$\therefore$ $\lambda_1, \lambda_2, \lambda_3$ are zeroes so, the given vectors are linearly independent.