Assignment - 8

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Exercise: Find the Eigen values and eigen vectors of following matrices

①

1. $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

The characteristic equation is

$|A-\lambda I| = 0$

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{vmatrix} = 0$$

$= (-2-\lambda)\{(1-\lambda)(-\lambda) - 12\} - 2(2(-\lambda)-6) - 3(-4+1(1-\lambda)) = 0$

$= (-2-\lambda)(-\lambda+\lambda^2-12) - 2(-2\lambda-6) - 3(-3-\lambda) = 0$

$= -\lambda(-2-\lambda) + \lambda^2(-2-\lambda) - 12(-2-\lambda) + 4\lambda + 12 + 9 + 3\lambda = 0$

$= 2\lambda + \lambda^2 - 2\lambda^2 - \lambda^3 + 24 + 12\lambda + 4\lambda + 12 + 9 + 3\lambda = 0$

$= -\lambda^3 - \lambda^2 + 21\lambda + 45 = 0$

$\Rightarrow \lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

This is characteristic polynomial

Now, we have to find roots of this polynomial

The eq^n

$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

Trail & error method

sub -1 in the place of λ

$(-1)^3 + (-1)^2 - 21(-1) - 45 = 0$

$-1 + 1 + 21 - 45 = 0$

$21 - 45 = 0$

$-24 \neq 0$ ✗

sub -2

$(-2)^3 + (-2)^2 - 21(-2) - 45 = 0$

$-8 - 4 + 42 - 45 = 0$

$57 + 42 = 0$

$99 \neq 0$

sub -3 in the place of λ

$$(-3)^3+(-3)^2-21(-3)-45=0$$

$$-27+9+63-45=0$$

$$-72+72=0$$

$$0=0$$

so '-3' is one of the roots of the eqn

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -21 & -45 \\ -1 & 1 & -3 & +6 & +45 \\ \hline & 1 & -2 & -15 & \boxed{0} \end{array}$$

$$x^2-2x-15=0$$

$$=\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{2\pm\sqrt{4-4(1)(-15)}}{2} = \frac{2\pm\sqrt{4+60}}{2}$$

$$= \frac{2\pm\sqrt{64}}{2} = \frac{2\pm 8}{2} = \frac{2+8}{2} \text{ \& } \frac{2-8}{2}$$

$$= 5 \text{ \& } -3$$

Therefore the roots are $-3, -3, 5$

So, they are the eigen values of the given matrix

Let $x=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the eigen vectors corresponding to eigen value λ.

$$[A-\lambda I]x=0$$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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Now consider $\lambda = 5$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-7x_1 + 2x_2 - 3x_3 = 0 \rightarrow (1)$

$2x_1 - 4x_2 - 6x_3 = 0 \rightarrow (2)$

$-x_1 - 2x_2 - 5x_3 = 0 \rightarrow (3)$

considering (1) & (2)

$$\frac{x_1}{\begin{vmatrix} 2 & -3 \\ -4 & -6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -7 & -3 \\ 2 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -7 & 2 \\ 2 & -4 \end{vmatrix}}$$

$$\frac{x_1}{-12-12} = \frac{-x_2}{42+6} = \frac{x_3}{28-4}$$

$$\frac{x_1}{-24} = \frac{-x_2}{48} = \frac{x_3}{24}$$

$$\frac{x_1}{-1} = \frac{x_2}{-2} = \frac{x_3}{1}$$

$[-1, -2, 1] \rightarrow$ These are the eigen vectors for $\lambda = 5$

Now $\lambda = -3$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 2x_2 - 3x_3 = 0 \rightarrow (4)$

$2x_1 + 4x_2 + 6x_3 = 0 \rightarrow (5)$

$-x_1 - 2x_2 + 3x_3 = 0 \rightarrow (6)$

consider (4) & (5)

$$\frac{x_1}{\begin{vmatrix} 2 & -3 \\ 4 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & -3 \\ 2 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}}$$

$$\frac{x_1}{24} = \frac{-x_2}{12} = \frac{x_3}{0}$$

$$\frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{0}$$

$[2, -1, 0] \rightarrow$ These are the eigen vectors for $\lambda = -3$.

2. $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

The characteristic equation is

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = 0$$

$$= (4-\lambda)((1-\lambda)(1-\lambda) - 0) - 0(-2(1-\lambda)+0) + 1(0+2(1-\lambda))$$

$$= (4-\lambda)((1-\lambda-\lambda+\lambda^2) + 2 - 2\lambda = 0$$

$$= (4-\lambda)(1-2\lambda+\lambda^2) + 2 - 2\lambda = 0$$

$$= 1(4-\lambda) - 2\lambda(4-\lambda) + \lambda^2(4-\lambda) + 2 - 2\lambda = 0$$

$$= 4 - \lambda - 8\lambda + 2\lambda^2 + 4\lambda^2 - \lambda^3 + 2 - 2\lambda = 0$$

$$= -\lambda^3 + 6\lambda^2 - 11\lambda + 6 = 0$$

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$= \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

This is characterstic polynomial.

Now, we have to find roots

sub 1.

$(1)^3 - 6(1)^2 + 11(1) - 6 = 0$

$1 - 6 + 11 - 6 = 0$

$12 - 12 = 0$

$0 = 0$

$\therefore$ Therefore one is the root

sub -1

$(-1)^3 - 6(-1)^2 + 11(-1) - 6 = 0$

$-1 - 6 - 11 - 6 = 0$ ✗

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & \downarrow & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$x^2 - 5x + 6 = 0$

$x^2 - 2x - 3x + 6 = 0$

$x(x-2) - 3(x-2) = 0$

$(x-3)(x-2) = 0$

$x - 3 = 0 \quad x - 2 = 0$

$x = 3 \quad x = 2$

$\therefore$ The roots of the characteristic polynomial are 1, 2, 3.

so, they are the eigen values of the given matrix.

let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the eigen vectors corresponding to eigen values.

$|A - \lambda I|x = 0$

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now Consider 1

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3x_1 + 0x_2 + 1x_3 = 0$$
$$-2x_1 + 0x_2 + 0x_3 = 0$$
$$-2x_1 + 0x_2 + 0x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 3 & 1 \\ -2 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 3 & 0 \\ -2 & 0 \end{vmatrix}}$$

$$\frac{x_1}{0} = \frac{-x_2}{0+2} = \frac{x_3}{0}$$

$x_1 = 0 \quad x_2 = -2 \quad x_3 = 0$

$[0, -2, 0] \rightarrow$ These are the eigen vectors of $\lambda = 1$

Now $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 + 0x_2 + 1x_3 = 0$$
$$-2x_1 - 1x_2 + 0x_3 = 0$$
$$-2x_1 + 0x_2 - 1x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 2 & 1 \\ -2 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 2 & 0 \\ -2 & -1 \end{vmatrix}}$$

$$\frac{x_1}{1} = \frac{-x_2}{2} = \frac{x_3}{-2}$$

$x_1 = 1 \quad x_2 = -2 \quad x_3 = -2$

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$[1, -2, -2] \rightarrow$ These are eigen vectors for $\lambda = 2$

Now $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$1x_1 + 0x_2 + 1x_3 = 0$$
$$-2x_1 - 2x_2 + 0x_3 = 0$$
$$-2x_1 + 0x_2 - 2x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 1 \\ -2 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 0 \\ -2 & -2 \end{vmatrix}}$$

$x_1 = 2$, $x_2 = -2$ $x_3 = -2$

$[2, -2, -2] \rightarrow$ These are eigen vectors for $\lambda = 3$

3. $$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

The characterstic equation is

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$= \begin{vmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{vmatrix} = 0$$

$(5-\lambda)((-\lambda)(3-\lambda) - 0(0-0) + 0(0-\lambda)) = 0$

$(5-\lambda)(-3\lambda + \lambda^2) = 0$

$(-3\lambda)(5-\lambda) + \lambda^2(5-\lambda) = 0$

$-15\lambda + 3\lambda^2 + 5\lambda^2 - \lambda^3 = 0$

$-\lambda^3 + 8\lambda^2 - 15\lambda + 0 = 0$

$\lambda^3 - 8\lambda^2 + 15\lambda + 0 = 0$

This is characteristic polynomial

Now, we have to find roots of this polynomial

The eq^n

$\lambda^3 - 8\lambda^2 + 15\lambda + 0 = 0$

sub -1 in the place of λ

$(-1)^3 - 8(-1)^2 + 15(-1) + 0 = 0$

$-1 - 8 - 15 = 0$ ✗

sub 1 in the place of λ

$(1)^3 - 8(1)^2 + 15(1) = 0$

$1 - 8 + 15 = 0$ ✗

sub 2 in the place of λ

$(2)^3 - 8(2)^2 + 15(2) = 0$

$8 - 32 + 30 = 0$ ✗

sub 3 in the place of λ

$(3)^3 - 8(3)^2 + 15(3) = 0$

$27 - 72 + 45 = 0$

$-72 + 72 = 0$ ✓

so 3 is one of the root

$$\begin{array}{c|cccc} 3 & 1 & -8 & 15 & 0 \\ & 1 & 3 & -15 & 0 \\ \hline & 1 & -5 & 0 & 0 \end{array}$$

$x^3 - 5x^2 = 0$

$x^2(x-5) = 0$

$x = 0 \quad x = 5.$

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$\therefore$ The roots are $0, -3, 5$

so, these are the eigen values of the matrix

let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the eigen vectors corresponding to eigen value λ.

$$|A - \lambda I| x = 0$$

$$\begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

consider $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 + 0x_2 + 0x_3 = 0$$

$$0 - 3x_2 + 0x_3 = 0$$

$$-1x_1 + 0x_2 + 0x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 0 & 0 \\ -3 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 2 & 0 \\ 0 & -3 \end{vmatrix}}$$

$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0$

$\therefore$ $[0, 0, 0]$ $\rightarrow$ eigen vectors for $\lambda = 3$.

consider $\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x_1 + 0x_2 + 0x_3 = 0$$
$$0x_1 + 0x_2 + 0x_3 = 0$$
$$-x_1 + 0x_2 + 3x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & 0 \\ 0 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & 0 \\ 0 & 0 \end{vmatrix}}$$

$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0$

$[0, 0, 0] \rightarrow$ eigen vectors for $\lambda = 0$

consider $\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0x_1 + 0x_2 + 0x_3 = 0$$
$$0x_1 - 5x_2 + 0x_3 = 0$$
$$-1x_1 + 0x_2 - 2x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 0 & 0 \\ -5 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 0 \\ 0 & -5 \end{vmatrix}}$$

$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0$

$[0, 0, 0] \rightarrow$ eigen vectors for $\lambda = 5$

4. $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$

The characteristic equation is

$$|A - \lambda I| = 0$$

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$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix} = 0$$

$$= -\lambda((3-\lambda)(-2-\lambda) - 0)$$

$$= -\lambda((-6 - 3\lambda + 2\lambda + \lambda^2))$$

$$= -\lambda((\lambda^2 - \lambda - 6))$$

$$= -\lambda^3 + \lambda^2 + 6\lambda + 0 = 0$$

$$= \lambda^3 - \lambda^2 - 6\lambda + 0 = 0$$ → This is characteristic polynomial.

The roots of the polynomial

sub -2 in the place of λ.

$$(-2)^3 - (-2)^2 - 6(-2) + 0 = 0$$

$$-8 - 4 + 12 = 0$$

$$-12 + 12 = 0$$

$$0 = 0$$

-2 is one of the roots of the equation

-2	1	-1	-6	0
		-2	6	
	1	-3	0	

$$x^3 - 3x^2 = 0$$

$$x^2(x-3) = 0$$

$$x = 0 \quad x = 3$$

∴ The roots are $0, 3, -2$ and these are eigen values.

Let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the eigen vectors corresponding to eigen value λ.

$|A-\lambda I| = 0$

$$\begin{bmatrix} -\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Now Consider 0

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0x_1 + 0x_2 + 0x_3 = 0$

$0x_1 + 3x_2 + 4x_3 = 0$

$0x_1 + 0x_2 - 2x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 0 & 0 \\ 3 & 4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 0 \\ 0 & 4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix}}$$

$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0$

$[0,0,0] \to$ eigen vectors of $\lambda = 0$

Now Consider -2

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 + 0x_2 + 0x_3 = 0$

$0x_1 + 5x_2 + 4x_3 = 0$

$0x_1 + 0x_2 + 0x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 0 & 0 \\ 5 & 4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -2 & 0 \\ 0 & 4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 2 & 0 \\ 0 & 5 \end{vmatrix}}$$

$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0$

$[0,0,0] \to$ eigen vectors of $\lambda = -2$.

Now Consider 3

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_1 + 0x_2 + 0x_3 = 0$

$0x_1 + 0x_2 + 4x_3 = 0$

$0x_1 + 0x_2 - 5x_3 = 0$

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$$\frac{x_1}{\begin{vmatrix} 0 & 0 \\ 0 & 4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -3 & 0 \\ 0 & 4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -3 & 0 \\ 0 & 0 \end{vmatrix}}$$

$x_1 = 0$ $x_2 = 0$ $x_3 = 0$

$[0, 0, 0] \rightarrow$ eigen vectors of $\lambda = 3$.

5. For following matrix find one eigen value without calculation and justify your answer $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

since all the rows are same so, the matrix is singular, meaning its determinant is 0.

so we can conclude that the determinant is 0 without calculation. Therefore, one eigen value of this matrix is 0,