Corporate Finance

Lecture 8.2: Capital Structure:

M&M Theorem, Cost of Capital (with tax)

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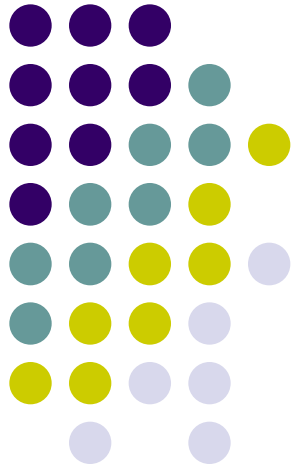

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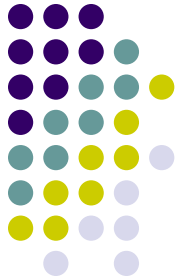
Irrelevant?

M&M Theorem: *Without friction*, capital structure is irrelevant.

What does *"No Frictions"* mean?

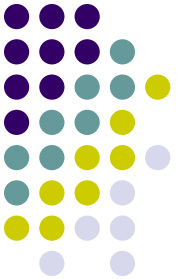
- **No tax shield (no taxes, or interest is paid after tax)**
- No consequences of *financial distress* for assets
- No *information asymmetry* and *agency frictions*
- Individuals can borrow and lend at the same rate as corporations

Taxes



- By replacing some equity with debt, a firm would pay more in interest (less in dividend), but **less in tax payments.**
- The *total cash flows to equity and debt holders* are larger by the amount of tax saving.
- So, the value of a levered company is **greater** than that of a unlevered firm.
- Notice that we are assuming taxes only affect the distribution of cash flows, but not the FCF *per se* generated from the firm's continuing operations.

The M&M Theorem with Taxes



Proposition I:

Firm value increases with leverages.

$$V_L = V_U + V_{ts}$$

- where V_L is the firm value with debt (levered firm value),
- V_U is the firm value without debt (unlevered firm value),
- V_{ts} is the present value of all future tax shields of interest payments.

JELLYBEANS, INC.

Assume that there was a tax of 40%

	Current	Proposed
EBIT	$1,083,000	$1,083,000
Interest	$0	$250,000
EBT	$1,083,000	$833,000
Tax (40%)	$433,000	$333,000
Net income	$650,000	$500,000
Total cash flow to shareholders and debtholders	$650,000	$750,000

Tax Shield

Tax shield (tax savings *every year*):

- $\$100{,}000 = 0.4 \times \$250{,}000 = \tau \cdot Interest = \tau \cdot (R_D \cdot D)$.
- τ is the corporate tax rate.

PV of all future tax shields is: $V_{ts} = \tau \cdot D$

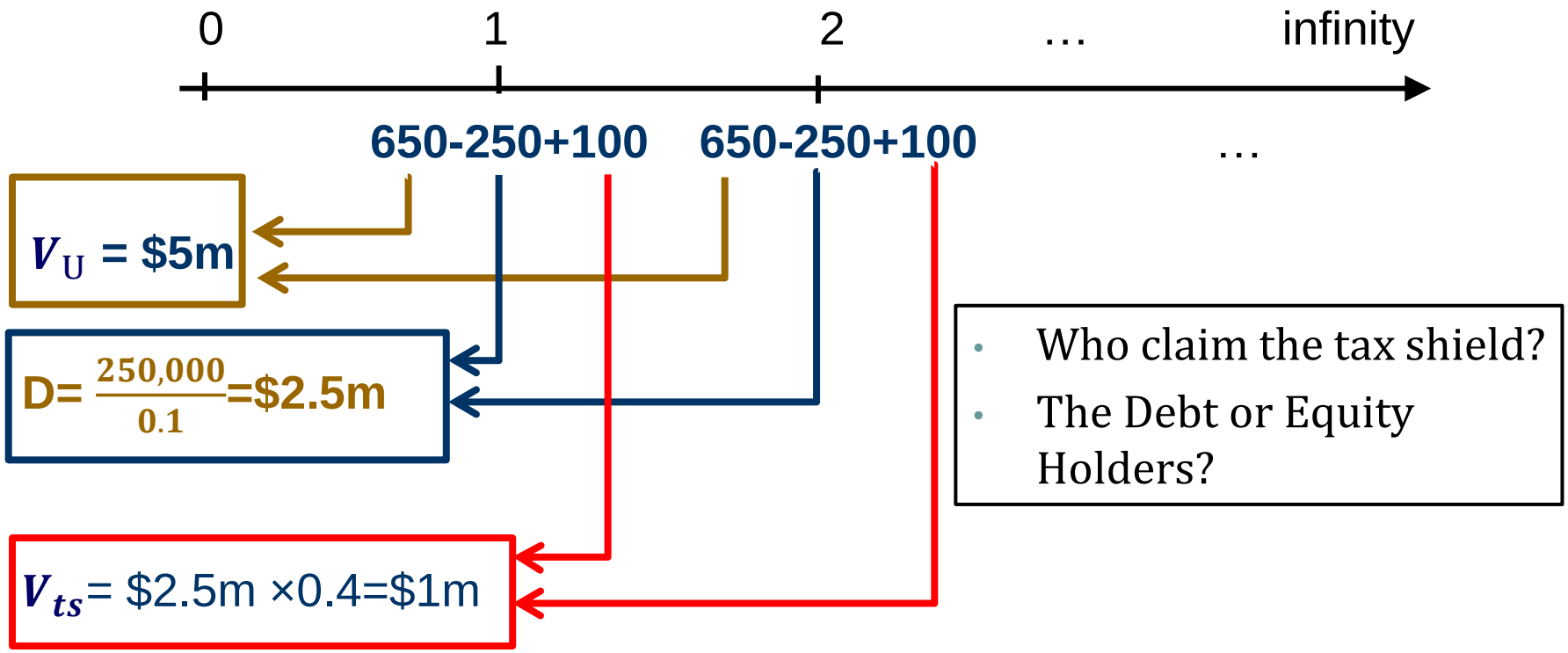
- Suppose the interest payment is perpetuity.
- Present value of tax shield is: $\frac{\tau \cdot (R_D \cdot D)}{R_D} = \tau \cdot D$.

Discount rate for tax shield is the *expected return on debt*. Why?

- The **risk** of the *tax saving on interest* has the same risk as the *interest on debt* itself.

Cash Flows to Equity Holders

With taxes



Adjusted Present Value

Considering Taxes (but not the other frictions):

$$V_L - V_U = \frac{\tau \times r_D \times D}{r_D} = \tau \times D \equiv V_{ts}$$

- V_L is the value of the levered firm,
- V_U is the value of the unlevered firm,
- $r_D \times D$ is the interest payment,
- τ is the tax rate.

Thus, $V_L = V_U + V_{ts} = V_U + \tau \times D$.

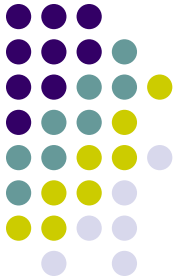
Value of tax shield is increasing in debt level, so is the (levered) firm value.

Adjusted Present Value

$$V_L = V_U + \tau \times D$$

- This gives us a method to value a company with debt.
- However, we need to find the unlevered firm value, $\boldsymbol{V_U}$, first.
  - Recall: Free cash flow is the after-tax cash flow a firm would generate *if had no debt*. So, FCF is the unlevered *cash flow* for V_U.
  - What is the proper *discount rate* for it?
  - *Return on asset* of *full-equity firm*, i.e., $\boldsymbol{r_A}$.
  - r_A is **independent** to leverage, i.e., $\frac{D}{E}$ (by definition).

The M&M Theorem with Taxes



Proposition II:

The *expected return on equity* of a levered firm increases in proportion to leverage (D/E):

$$r_E = r_A + (1 - \tau)(r_A - r_D) \cdot \frac{D}{E}$$

- r_A the *return on assets if the firm had no debt*.

M&M #2 With Taxes (Optional)

- Given M&M #1, we have a levered firm's market value balance sheet as below.

Tax Shield=$\boldsymbol{\tau \cdot D}$	Equity Value
Unlevered Firm Value = V_U	Debt Value

- The expected cash flow from the left side is: $V_U \cdot R_A + \tau \cdot D \cdot R_D$
- The expected cash to debt holders and stockholders is: $D \cdot R_D + E \cdot R_E$
- Assume all cash flows are paid out as dividends and the firm generate perpetuity cash flows. (The result holds without such assumptions.)

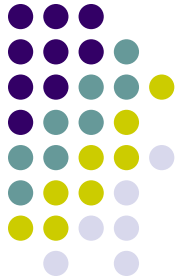
$$V_U \cdot R_A + \tau \cdot D \cdot R_D = D \cdot R_D + E \cdot R_E \ldots (1)$$

- From M&M#1, we know:

$$V_U = D + E - \tau \cdot D \ldots (2)$$

- (1) & (2) gives M&M#2.

The Effect of Financial Leverage



- M&M #2 with taxes:

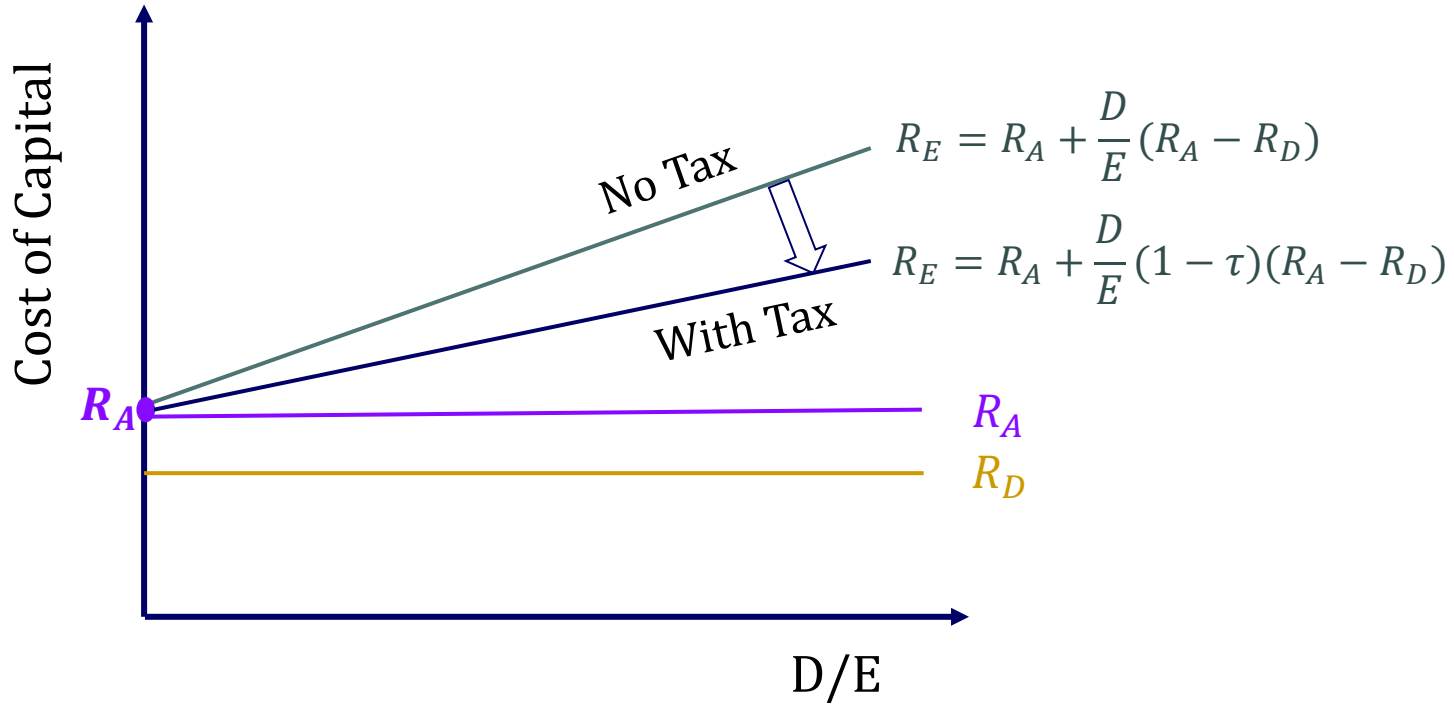
$$r_E = r_A + (r_A - r_D) \cdot \frac{D}{E} - \tau \cdot (r_A - r_D) \cdot \frac{D}{E}$$

- As debt increases, the risk of cash flows remains the same (R_A is a constant), but the risk of equity (R_E):
  - *Increases* because a higher fraction of *fixed* cash flows to the debt holder;
  - *Decreases* because tax shield goes to the *equity holders* and tax shield is less risky than the general cash flows.
  - The net effect of *leverage* on *equity risk* is $(r_A - r_D)(1 - \tau)$.
- Same effect for beta: $\beta_E = \beta_A + (1 - \tau)(\beta_A - \beta_D) \cdot \frac{D}{E}$

The Effect of Financial Leverage

Other than tax shield, there are no other frictions

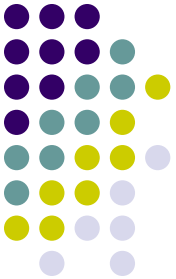
- No consequences of *financial distress* for assets; No *information asymmetry* and *agency frictions;* Well functioning financial markets.



JELLYBEANS, INC.

- Assume that corporate tax rate is 40%.
- Currently, JellyBeans, Inc. has no debt. The expected after-tax cash flow (assuming the same as net income) is \$650,000 forever, and cost of equity is 13%. $\longrightarrow R_A = ?\ V_U = ?$
- $R_A = R_E^U = 13\%$
- $V_U = \frac{650{,}000}{13\%} = \$5{,}000{,}000.$
- After recapitalization (borrowing \$2.5m debt with 10% interest rate, and repurchasing half of the equity): $\longrightarrow V_L = V_U + V_{TS}$=?
- $V_{TS} = D \cdot \tau = \$2.5m \cdot 40\% = \$1{,}000{,}000.$
- $V_L = V_U + V_{TS} = \$5m + \$1m = \$6m.$

WACC Revisited



WACC captures the weighted average cost of capital (debt and equity)

Is the WACC of a levered firm the same as the return on asset of unlevered equity, r_A?

No!

Which one is larger?

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Assume that there was a tax of 40%

What is the **WACC with Tax**?

	Current	Proposed
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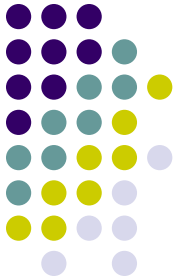
Review: WACC with Taxes



Tax shield reduces ***the effective cost of debt*** by a fraction of **the marginal tax rate**:

$$WACC = \frac{E}{V_L} \cdot R_E + \frac{D}{V_L} \cdot R_D \cdot (1 - \tau)$$

Why?

- Interest payment is tax-deductible
- The **after-tax cost of debt** is $r_D \times (1 - T_c)$

As before, D, E, and V are **market values**.

Try it!

Effective After-Tax Cost of Debt

Cash Flow generated by Firm Assets

Our World		A Parallel World	
EBIT	**$1,083,000**	EBIT	**$1,083,000**
Interest	$250,000	Tax (40%) ★	$433,000
EBT	$833,000	EBI	$650,000
Tax (40%)	$333,000	Interest	$150,000
Net income	**$500,000**	Net income	**$500,000**
Interest Rate	10%	Interest Rate	6%

Cash Flow entitled to Equity Holders

6%=150,000/2.5m
=10%*(1-0.4)

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- Before recapitalization:
  - WACC = r_E =13%
  - $r_A = r_E = 13\%$
- After recapitalization:
  - $V_L = 6m; D = 2.5m; E = V_L - D = 3.5m.$
  - r_A does not change with leverage.
  - Using M&M#2, we find: r_E =?

$$r_E = 13\% + (1 - 0.4) \times (13\% - 10\%) \times \frac{2.5m}{3.5m} = 14.28\%$$

  - Thus: $WACC$ =?

$$WACC = 10\% \times (1 - 0.4) \times \frac{2.5m}{6m} + 14.28\% \times \frac{3.5m}{6m} = 10.83\%$$

WACC and R_A

What's the relation between r_A and WACC?

- M&M#2: $r_E = r_A + (1-\tau)(r_A - r_D)\frac{D}{E}$
- $\leftrightarrow r_E + \frac{D}{E}(1-\tau)r_D = r_A \cdot \frac{E+D(1-\tau)}{E}$
- $\leftrightarrow \frac{E}{V_L} \cdot r_E + \frac{D}{V_L} \cdot (1-\tau) \cdot r_D = r_A \cdot \frac{V_L - \tau D}{V_L}$
- $\leftrightarrow$ $\boldsymbol{WACC = r_A \cdot (1 - \frac{\tau \cdot D}{V_L})}$

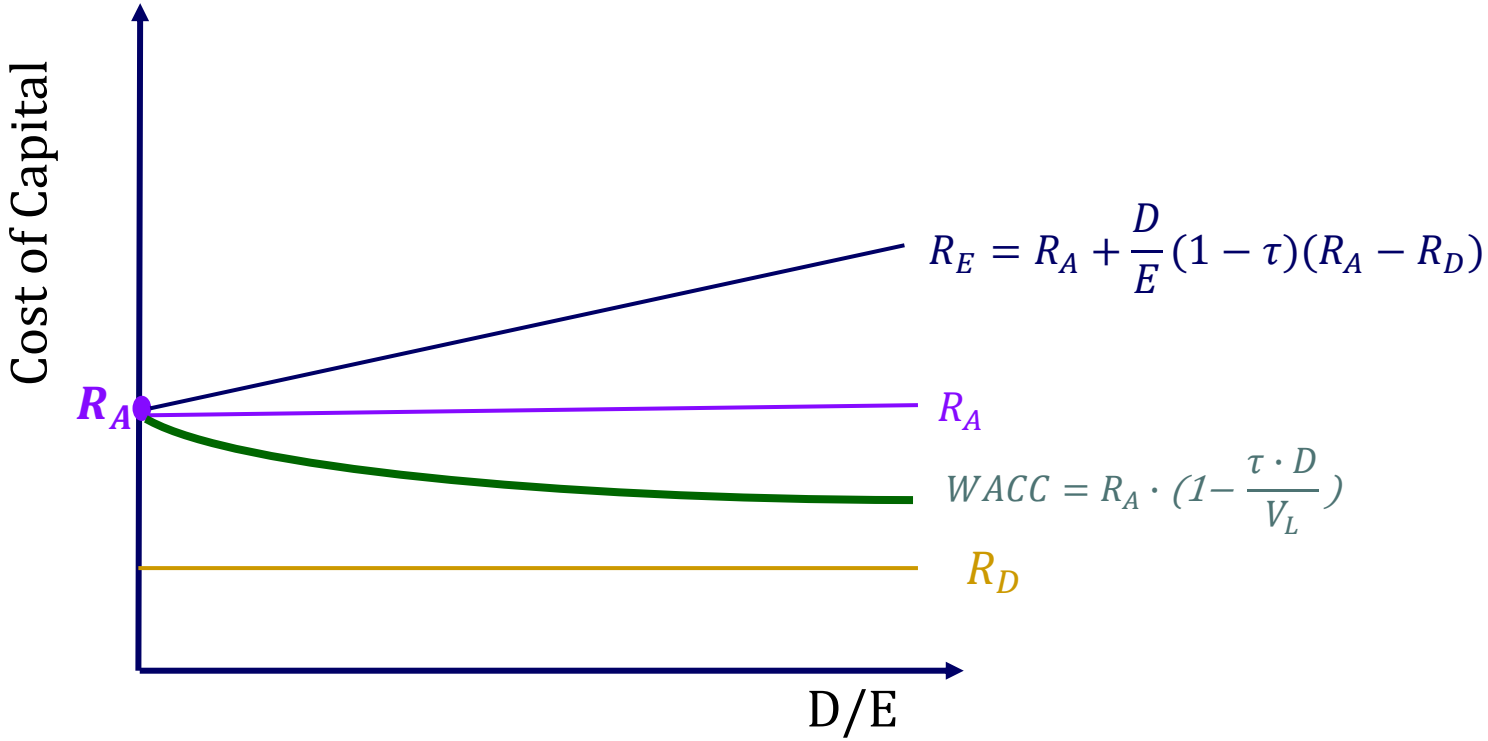
$WACC < r_A$ (since $\tau D < V_L$).

- The gap between r_A and WACC is driven by the tax shield of debt.

WACC and R_A

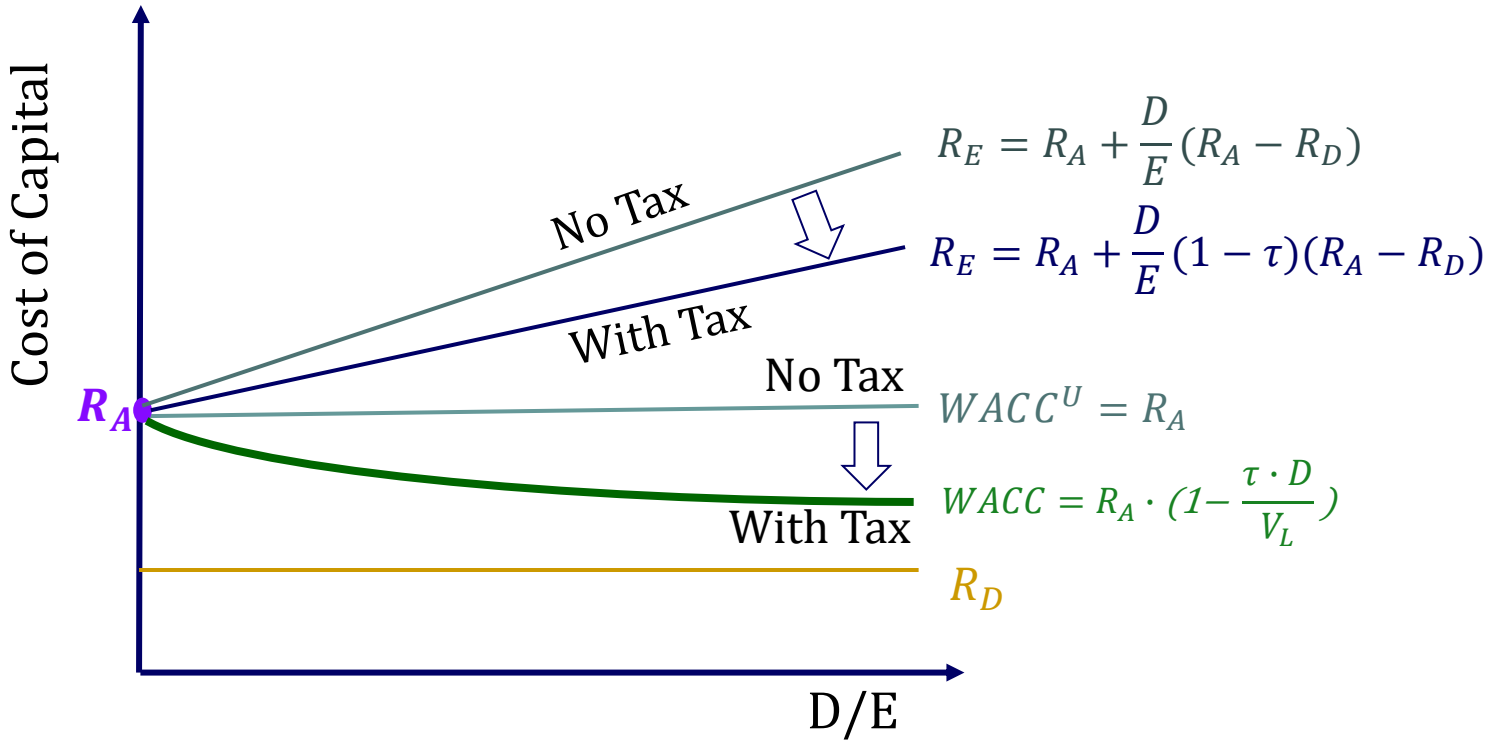
Other than tax shield, there are no other frictions

- $WACC = \frac{D}{V_L}(1-\tau)R_D + \frac{E}{V_L}R_E = R_A \cdot (1 - \frac{\tau \cdot D}{V_L})$



WACC and R_A

From "No Tax Shield" to "With Tax Shield":



WACC and Firm Value

- *From* $WACC = r_A \cdot (1 - \frac{\tau \cdot D}{V_L})$ *:*
- $V_L \cdot WACC = r_A \cdot (V_L - \tau \cdot D)$
- $V_L \cdot WACC = r_A \cdot V_U = FCF$
- Thus,

Recall: $V_U = \frac{FCF}{r_A}$

$$\boldsymbol{V_L = \frac{FCF}{WACC}}$$

- Firm value is the total *unlevered cash flows* discounted at the *WACC of a levered firm*.

WACC and Firm Value

- More generally (FCF might not be constant over time),

$$V_L = \sum_t \frac{FCF_t}{(1 + WACC)^t}$$

- Recall: FCF is generated from *operations* and is *free to be distributed* to capital suppliers. *FCF doesn't change with leverage*.

$$FCF = (1 - \tau) \cdot EBIT + Dep - \Delta NWC - CPX$$

- WACC reflects both **risk** and **financing**.

$$WACC = \frac{\boldsymbol{D}}{\boldsymbol{V_L}} \cdot \boldsymbol{(1 - \tau)} \cdot \boldsymbol{R_D} + \frac{\boldsymbol{E}}{\boldsymbol{V_L}} \cdot \boldsymbol{R_E}$$

JELLYBEANS, INC.

What's the firm value after recapitalization? (Use WACC)

- FCF = \$650,000.
- WACC = 10.83%.
- $V_L = \frac{FCF}{WACC} = \frac{\$650K}{10.83\%} = \$6m.$

Verify: $V_L = V_U + V_{TS}$

- $V_U = \frac{FCF}{r_A} = \frac{\$650K}{13\%} = \$5m.$
- $V_{TS} = \tau D = \$2.5m \times 0.4 = \$1m.$

Takeaways

I. Without any frictions

- Capital structure is irrelevant as long as it does not affect the total cash flows generated by the assets
- Return on equity increases with leverage, since debt increases the risk of equity.
- WACC is the discount rate of firm's cash flow (FCF) for finding firm value

Takeaways

II. With tax shields

- M&M#1: $V_L = V_U + V_{ts}$
- M&M#2: $r_E = r_A + (1-\tau)(r_A - r_D) \cdot \frac{D}{E}$
- $WACC = \frac{E}{V_L} \cdot R_E + \frac{D}{V_L} \cdot R_D \cdot (1-\tau)$
- Connection: $WACC = \mathrm{r_A} \cdot (1 - \frac{\tau \cdot \mathrm{D}}{\mathrm{V_L}})$
- $V_L = \sum_t \frac{FCF_t}{(1+WACC)^t}$; $V_U = \sum_t \frac{FCF_t}{{r_A}^t}$

Exercise

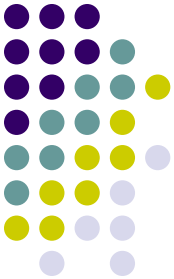
Ch16-25 MM with Taxes Dickson, Inc., has a debt-equity ratio of 2.3. The firm's weighted average cost of capital is 10 percent and its pretax cost of debt is 6 percent. The tax rate is 24 percent.

a. What is the company's cost of equity capital?

b. What is the company's unlevered cost of equity capital?

c. What would the company's weighted average cost of capital be if the firm's debt-equity ratio were .75? What if it were 1.3?

Exercise



- $WACC = \frac{E}{V_L} \cdot R_E + \frac{D}{V_L} \cdot R_D \cdot (1 - \tau)$
- M&M#2: $r_E = r_A + (1 - \tau)(r_A - r_D) \cdot \frac{D}{E}$
- A. $10\% = \frac{1}{3.3} R_E + \frac{2.3}{3.3} \times 6\% \times (1 - 24\%) \rightarrow R_E = 22.51\%$
- B. $22.51\% = R_A + (1 - 24\%) \times (R_A - 6\%) \times 2.3 \rightarrow R_A = 12.01\%$
- C.1 $R_E = 12.01\% + 0.75 \times (1 - 24\%) \times (12.01\% - 6\%) = 15.43\%$
- $WACC = \frac{0.75}{1.75} \times (1 - 24\%) \times 6\% + \frac{1}{1.75} \times 15.43\% = 10.77\%$
- C.2 $R_E = 12.01\% + 1.3 \times (1 - 24\%) \times (12.01\% - 6\%) = 17.95\%$
- $WACC = \frac{1.3}{2.3} \times (1 - 24\%) \times 6\% + \frac{1}{2.3} \times 17.95\% = 10.38\%$