Homework 2

Question 1. Derive two competing platforms' optimal prices when users are single-homing. Interpret the results.

Answer.

<u>Basic Assumptions:</u> There are two horizontally differentiated platforms, 1 and 2, and a Hotelling Line with a length of 1. Platform 1 and 2 are located at the two ends. There is a unit mass of buyers and a unit mass of sellers. Each buyer or seller can at most join one platform (both sides are single-homing). And we assume full coverage:

let n_B^i denote the number of buyers joining platform i, then

$$n_B^1 + n_B^2 = 1,$$

let n_S^i denote the number of sellers joining platform i, then

$$n_S^1 + n_S^2 = 1.$$

Both sellers and buyers are **uniformly** distributed on the Hotelling Line. Hotelling transportation cost parameter: τ_S for sellers and τ_B for buyers. Platforms compete by designing membership fees $(A_B^1, A_S^1), (A_B^2, A_S^2)$, so:

buyers' utility from joining platform i (net of transportation cost):

$$u_B^i = n_S^i b_B^i - A_B^i,$$

sellers' utility from joining platform i (net of transportation cost):

$$u_S^i = n_B^i b_S^i - A_S^i.$$

<u>Calculate:</u> We first look at the subgame induced by **any** $(A_B^1, A_S^1), (A_B^2, A_S^2)$. First, we need to identify the buyer (seller) who is indifferent between joining platform 1 and joining 2:

For buyers, equating $u_B^1 - x\tau_B = u_B^2 - (1-x)\tau_B$ yields the indifferent buyer's location

$$\overline{x_B} = \frac{1}{2} + \frac{u_B^1 - u_B^2}{2\tau_B}.$$

Buyers with $x < \overline{x_B}$ prefers platform 1 and buyers with $x > \overline{x_B}$ prefer platform 2.

For sellers, equating $u_S^1 - x\tau_S = u_S^2 - (1-x)\tau_S$ yields the indifferent seller's location

$$\overline{x_S} = \frac{1}{2} + \frac{u_S^1 - u_S^2}{2\tau_S}.$$

Sellers with $x < \overline{x_S}$ prefer platform 1 and sellers with $x > \overline{x_S}$ prefer platform 2.

So, we can get the Platform i's demand:

$$n_S^i = \frac{1}{2} + \frac{u_S^i - u_S^j}{2\tau_S} \text{ and } n_B^i = \frac{1}{2} + \frac{u_B^i - u_B^j}{2\tau_B}.$$

Using $u_B^i = n_S^i b_B^i - A_B^i$ and $u_S^i = n_B^i b_S^i - A_S^i$ and the fact that $n_B^j = 1 - n_B^i$ and $n_S^j = 1 - n_S^i$, we can obtain:

$$n_S^i(n_B^i) = \frac{1}{2} + \frac{\left[(2n_B^i - 1)b_S - \left(A_S^i - A_S^j\right)\right]}{2\tau_S} \text{ and } n_B^i(n_S^i) = \frac{1}{2} + \frac{\left[(2n_S^i - 1)b_B - \left(A_B^i - A_B^j\right)\right]}{2\tau_B}.$$

Combining the two equations, we can solve:

$$n_S^i = \frac{1}{2} + \frac{b_S\left(A_B^j - A_B^i\right) + \tau_B\left(A_S^j - A_S^i\right)}{2(\tau_B\tau_S - b_S b_B)} \text{ and } n_B^i = \frac{1}{2} + \frac{b_B\left(A_S^j - A_S^i\right) + \tau_S\left(A_B^j - A_B^i\right)}{2(\tau_B\tau_S - b_S b_B)}.$$

The profit of platform i is:

$$\Pi^i = \left(A_S^i - c_S\right) * n_S^i + \left(A_B^i - c_B\right) * n_B^i,$$

which equals to:

$$\begin{aligned} \Pi^i = \left(A_S^i - c_S\right)\left[\frac{1}{2} + \frac{b_S\left(A_B^j - A_B^i\right) + \tau_B\left(A_S^j - A_S^i\right)}{2(\tau_B\tau_S - b_S b_B)}\right] \\ + \left(A_B^i - c_B\right)\left[\frac{1}{2} + \frac{b_B\left(A_S^j - A_S^i\right) + \tau_S\left(A_B^j - A_B^i\right)}{2(\tau_B\tau_S - b_S b_B)}\right] \end{aligned}.$$

To maximize Π^i for $i = 1,2$, we solve the derivative of them, imposing symmetry $A_B^1 = A_B^2 = A_B$ and $A_S^1 = A_S^2 = A_S$ yields:

$$A_S = c_S + \tau_S - \frac{b_B}{\tau_B}(b_S + A_B - c_B) \text{ and } A_B = c_B + \tau_B - \frac{b_S}{\tau_S}(b_B + A_S - c_S)$$

Again, by solving the two equations, we can get the final result of this question:

$$A_S = c_S + \tau_S - b_B$$

$$A_B = c_B + \tau_B - b_S$$

From this result we can get:

- The side of the market which exerts larger externality tends to be subsidized.
- The side of the market with little horizontal differentiation tends to pay low membership fee.

Question 2. Read the attached article about review website TripAdvisor and address the following questions:

a) Why is there a concern regarding the biased or fake review on TripAdvisor? What are your suggestions to address it (for the platform and for the hotels)?

Answer. First of all, biased or fake reviews do exist. It was estimated that between 2% and 6% of website ratings and reviews were fake or deceptive. Secondly, creating reviews has become an industry to get consumers to fictionalize positive reviews by giving them cash or gift cards, etc. What's more, platform consumers can easily recognize those fake reviews, which can greatly affect the credibility of TripAdvisor. Also, as a competitive industry, rivals of hotel operations will likely compete maliciously by writing a large number of fake reviews. **Without a way to control the biased or fake reviews, TripAdvisor will lose its user appeal**. So, this is a very important concern. My suggestions are as follows:

- For the platform. 1) Strictly set rules so that only those who have actually consumed are qualified to write reviews; 2) Develop recognition models to accurately identify those fake reviews and delete them in time.
- For the hotels. 1) Be a good hotel yourself and don't break the rules or maliciously create comments that are favorable to you and unfavorable to others; 2) Respond to some of the negative comments on time.

b) How would you measure the impact of online reviews on reservations? What are the potential problems?

Answer. We learned three methods in class: natural experiments, A/B tests as instruments, and regression discontinuity. Here, I'll choose to use regression discontinuity, considering that the TripAdvisor also builds in a mechanism for scoring reviews. Because the rounding generates a discontinuous jump in the perceived rating. Those products near the rounding threshold are likely to be similar except for their rounded star ratings. Hence, the causal impact of the star ratings can be obtained by comparing demand for products marginally above and marginally below the rounding threshold.

The specific measurements are as follows: 1) Collect samples of hotels that do not have a large difference in scores but have a difference in star ratings; 2) Regress them above and below the threshold; see if there is a significant difference in performance between the two groups.
The potential problems are: 1) Sample Limits: only applicable to platforms that round the star ratings; 2) Scope Limits: Only suitable for analysis of valence, not number or content of reviews.

c) Will those actions by hotels to encourage guests to write reviews affect the informativeness of the online reviews?
Answer. Yes! The informativeness of online reviews is based on the fact that the number and content of reviews must be highly correlated to the quality of hotel services. If there are hotels that encourage consumers to review in special ways, this can affect the content and number of reviews, making the information from the reviews not a true reflection of the quality of the hotel's services.

d) Comment on Four Seasons Hotels' and Homewood Suites' strategies of managing the online reviews.
Answer. Both hotels have a good strategy of relying on consumer reviews to build word of mouth, which in turn attracts more consumers.

- Four Seasons Hotels displays users' reviews on TripAdvisor on each Four Seasons property's Website. This strategy shows guests' reviews instead of the hotel's recommendation, which increases the reliability. It creates a stronger sense of trust among consumers.
- Homewood Suites replies to every review to show they attach importance to customer satisfaction, which will have positive effects on review number and review rate. This creates a positive interaction and a strong relational bond with the consumer and will make the hotel more attractive.

Question 3. Prove the following claim: In the standard second-price auction, it is a weakly dominant strategy for bidder i to bid exactly his true value v_i (Truth-telling).

Answer.

<u>Basic Settings:</u>

N (ex-ante) symmetric players with private information v_i;

Action: b_i;

Bidder i's payoff: $v_i - p_i$ if bidder i wins where p_i is the winning price, 0 if bidder i loses

<u>Prove:</u> We prove this argument using two steps:

1) $\boldsymbol{b_i = v_i}$ weakly dominates bidding any $\boldsymbol{b_i < v_i}$.

Consider player i bids $b_i = x < v_i$.

- Suppose $\max b_{-i} > v_i$ Both bids, v_i and x, will lose. **The payoffs are the same.**
- Suppose $x < \max b_{-i} < v_i$. Bidding v_i will win while bidding x will lose. Bidding v_i yields **positive payoff** ($v_i - \max b_{-i}$) while bidding x yields payoff 0.
- Suppose $\max b_{-i} < x$. Both bids, v_i and x will win. **Both payoffs are the same**: $v_i - \max b_{-i}$.

Then, in all the 3 situations, bidding the truth value is weakly better than bidding $\boldsymbol{b_i < v_i}$.

2) $\boldsymbol{b_i = v_i}$ weakly dominates bidding any $\boldsymbol{b_i > v_i}$.

Consider player i bids $b_i = x > v_i$.

- Suppose $\max b_{-i} > x$. Both bids, v_i and x, will lose. **The payoffs are the same.**
- Suppose. $v_i < \max b_{-i} < x$. Bidding v_i will lose while bidding x will win. Bidding v_i yields payoff 0**,** while bidding x yields **negative payoff** ($v_i - \max b_{-i}$).
- Suppose $\max b_{-i} < v_i$. Both bids, v_i and x will win. **Both payoffs are the same**: $v_i - \max b_{-i}$.

Then, in all the 3 situations, bidding the truth value is weakly better than bidding $\boldsymbol{b_i > v_i}$.

According to the two kinds pf stating above, we have that it is a weakly dominant strategy for bidder i to bid exactly his true value v_i (Truth-telling).

Question 4. If a platform is using generalized second price auction to sell advertising slots. There are two slots and three advertisers. The first slot receives 200 clicks while the second receives 100. Advertiser 1, 2, and 3 have value per click of $10, 4 and 2, respectively. Could you find an equilibrium and prove it?

Answer.

As mentioned in the class, there is an equilibrium that 3 advertisers bid $10, 4 and 2.

- **For advertiser 1**, in this situation, he will make profits $1200. If the other 2 do not change, he changes to $a, while a > 4, the profit is also $1200. If he changes to $b, while 4 > b > 2, the profit will be $100*8=$800. If he changes to $c, while c<2, the profit will be 0. **So he has no motivation to change the bid.**
- **For advertiser 2,** in this situation, he will make profits $200. If the other 2 do not change, he changes to $a, while a > 10, the profit is $-1200. If he changes to $b, while 10 > b > 2, the profit will be also $200. If he changes to $c, while c<2, the profit will be 0. **So he has no motivation to change the bid.**
- **For advertiser 3,** in this situation, he will make profits $0. If the other 2 do not change, however, he has to bid more than $4 to get a slot. But then the price will be $4, which is more than his value. **So his better choice is to remain at zero profit.**