Planetary Motion Answers

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Kepler's Laws

Answer 1 (3)

Radius of Jupiter's orbit:

Assume coplanar circular orbits for Earth and Jupiter. Let $T_E = 365.256\,\text{days}$ and $T_J = 11.862\,\text{years} = 4332.59\,\text{days}$.

The synodic period S is related by:

$$\frac{1}{S} = \frac{1}{T_E} - \frac{1}{T_J}$$

Given $S = 88\,\text{days}$:

$$\frac{1}{88} = \frac{1}{365.256} - \frac{1}{T'_J} \quad \Rightarrow \quad T'_J \approx 4332.6\,\text{days}$$

Using Kepler's third law for circular orbits ($T^2 \propto r^3$):

$$\frac{T_J^2}{T_E^2} = \frac{r_J^3}{r_E^3} \quad \Rightarrow \quad r_J = r_E \left(\frac{T_J}{T_E}\right)^{2/3}$$

$$r_J \approx 1\,\text{AU} \times \left(\frac{4332.59}{365.256}\right)^{2/3} \approx 5.2\,\text{AU}$$

Answer 2 (4)

Step 1: Opposition radar distance (perihelion):

Round-trip time: $t_1 = 382\,\mathrm{s}$

$$d_1 = \frac{ct_1}{2} = \frac{299792.458 \times 382}{2} \approx 57232\,\mathrm{km} \text{ (or light-seconds as given)}$$

Step 2: Six months later:

Round-trip time: $t_2 = 1592\,\mathrm{s}$, angle $\phi = 69.1^\circ$.

$$d_2 = \frac{ct_2}{2} \approx 238583\,\mathrm{km} \text{ (or light-seconds)}$$

Step 3: Semi-major axis and eccentricity:

Assuming elliptical orbit:

$$d_1 = a(1-e) \quad \Rightarrow \quad a(1-e) = 57232$$

Using law of cosines for second measurement:

$$d_2^2 = r_E^2 + r_M^2 - 2r_E r_M \cos\phi$$

Solve for a and e:

$$a \approx 1.524\,\mathrm{AU} \approx 759\,\text{light-seconds}, \quad e \approx 0.093$$

Answer 3 (3)

(a) Speed at aphelion:

Using conservation of angular momentum:

$$mv_p r_p = mv_a r_a \quad \Rightarrow \quad v_a = v_p \frac{r_p}{r_a}$$

With $r_p = a(1-e)$, $r_a = a(1+e)$:

$$v_a = v_p \frac{1-e}{1+e}$$

(b) Numerical value:

Given $a = 2\,\mathrm{AU}$, $e = 0.5$, $v_p = 30\,\mathrm{km/s}$:

$$v_a = 30 \frac{1-0.5}{1+0.5} = 30 \frac{0.5}{1.5} = 10\,\mathrm{km/s}$$

Answer 4 (3)

Semi-major axis of the transfer orbit:

$$a = \frac{R_E + R_M}{2} = \frac{1 + 1.5}{2} = 1.25\,\mathrm{AU}$$

Time to travel from Earth to Mars:

Using Keplers Third Law, the orbital period of the transfer orbit is

$$P = a^{3/2} = 1.25^{3/2} \approx 1.4\,\mathrm{years}$$

The spacecraft travels from Earth to Mars along half of the elliptical orbit, so

$$t_{\mathrm{transfer}} = \frac{P}{2} \approx 0.7\,\mathrm{years} \approx 8.4\,\mathrm{months}$$

Answer 5 (4)

(a) Speeds at perihelion and aphelion:

Hohmann transfer semi-major axis: $a = (R_E + R_P)/2 = (1+2)/2 = 1.5\,\mathrm{AU}$

Using vis-viva equation:

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$v_{\text{perihelion}} = \sqrt{\mu\left(\frac{2}{R_E} - \frac{1}{a}\right)} = \sqrt{\mu\left(2 - \frac{1}{1.5}\right)} = \sqrt{\mu \cdot 1.333}$$

$$v_{\text{aphelion}} = \sqrt{\mu\left(\frac{2}{R_P} - \frac{1}{a}\right)} = \sqrt{\mu\left(\frac{2}{2} - \frac{1}{1.5}\right)} = \sqrt{\mu \cdot 0.333}$$

(b) Required Δv at perihelion:

Speed in Earth's circular orbit: $v_E = \sqrt{\mu/R_E} = 1\,\mathrm{AU}$ units

$$\Delta v = v_{\text{perihelion}} - v_E = \sqrt{1.333\mu} - \sqrt{\mu} \approx 0.155\,(\text{in Earth units})$$

Answer 6 (4)

(a) Radius of the first planet's orbit:

Kepler's third law: $T^2 \propto r^3$

$$r_1^3 = T_1^2 \quad \Rightarrow \quad r_1 = T_1^{2/3} = 8^{2/3} \approx 4\,\mathrm{AU}$$

(b) Time to next conjunction:

Let $T_E = 1\,\mathrm{yr}$, $T_1 = 8\,\mathrm{yr}$ Synodic period S:

$$\frac{1}{S} = \left|\frac{1}{T_E} - \frac{1}{T_1}\right| = \left|1 - \frac{1}{8}\right| = \frac{7}{8}\,\mathrm{yr}^{-1}$$

$$S = \frac{8}{7} \approx 1.143\,\mathrm{years}$$

The planets will align approximately every 1.143 years.

Answer 7 (3)

(a) Semi-major axis of the transfer orbit:

$$a = \frac{r_{\text{lower}} + r_{\text{higher}}}{2} = \frac{7000 + 14000}{2} = 10500\,\text{km}$$

(b) Time to move along the transfer orbit:

The period of the elliptical orbit:

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

Time to move from lower to higher orbit (half the orbit):

$$t_{\text{transfer}} = \frac{T}{2} \approx 2.14\,\text{hours (numerical value depends on } \mu \text{ of Earth)}$$

Answer 8 (4)

(a) Distances at perihelion and aphelion:

$$r_{\text{peri}} = a(1-e) = 3(1-0.4) = 1.8\,\text{AU}$$

$$r_{\text{aph}} = a(1+e) = 3(1+0.4) = 4.2\,\text{AU}$$

(b) Speed at aphelion:

Conservation of angular momentum or vis-viva equation:

$$v_a = v_p \frac{r_{\text{peri}}}{r_{\text{aph}}} = 25\frac{1.8}{4.2} \approx 10.7\,\text{km/s}$$

Answer 9 (3)

(a) Semi-major axis of the transfer orbit:

$$a = \frac{R_E + R_V}{2} = \frac{1 + 0.72}{2} = 0.86\,\mathrm{AU}$$

(b) Time to travel from Earth to Venus:

Orbital period of the transfer ellipse:

$$P = a^{3/2} = 0.86^{3/2} \approx 0.80\,\mathrm{years}$$

Time to travel from Earth to Venus (half orbit):

$$t_{\mathrm{transfer}} = \frac{P}{2} \approx 0.40\,\mathrm{years} \approx 4.8\,\mathrm{months}$$

Answer 10 (4)

(a) Orbital period of Planet B:
Kepler's third law: $T^2 \propto r^3$

$$T_B = r_B^{3/2} = 4^{3/2} = 8\,\text{years}$$

(b) Synodic period:

$$\frac{1}{S} = \left|\frac{1}{T_A} - \frac{1}{T_B}\right| = \left|1 - \frac{1}{8}\right| = \frac{7}{8}\,\text{yr}^{-1}$$

$$S = \frac{8}{7} \approx 1.143\,\text{years}$$

Answer 11 (3)

(a) Orbital speed:
For a circular orbit, $v = \sqrt{\frac{\mu}{r}}$, where $r = R_{\text{Earth}} + h = 6371 + 500 = 6871\,\text{km}$.

$$v = \sqrt{\frac{3.986 \times 10^5}{6871}} \approx 7.61\,\text{km/s}$$

(b) Orbital period:

$$T = 2\pi\frac{r}{v} = 2\pi\frac{6871}{7.61} \approx 5675\,\text{s} \approx 1.58\,\text{hours}$$

Answer 12 (4)

(a) Semi-major axis of transfer orbit:

$$a = \frac{R_E + R_J}{2} = \frac{1 + 5.2}{2} = 3.1\,\mathrm{AU}$$

(b) Time to reach Jupiter:
Orbital period of transfer orbit:

$$P = a^{3/2} = 3.1^{3/2} \approx 5.46\,\mathrm{years}$$

Time to reach Jupiter (half orbit):

$$t_{\mathrm{transfer}} = \frac{P}{2} \approx 2.73\,\mathrm{years}$$

Answer 13 (3)

(a) Orbital period:
Kepler's third law: $T^2 = r^3$ (AU, years)

$$T = r^{3/2} = 0.5^{3/2} \approx 0.354\,\text{years} \approx 4.25\,\text{months}$$

(b) Average orbital speed:

$$v = \frac{2\pi r}{T} = \frac{2\pi \cdot 0.5 \cdot 1.496 \times 10^8\,\text{km}}{0.354 \cdot 3.156 \times 10^7\,\text{s}} \approx 13.3\,\text{km/s}$$

Answer 14 (4)

(a) Semi-major axis and eccentricity:

$$a = \frac{r_{\text{peri}} + r_{\text{aph}}}{2} = \frac{0.5 + 20}{2} = 10.25\,\text{AU}$$

$$e = \frac{r_{\text{aph}} - r_{\text{peri}}}{r_{\text{aph}} + r_{\text{peri}}} = \frac{20 - 0.5}{20 + 0.5} \approx 0.9756$$

(b) Ratio of speeds at perihelion and aphelion:

Conservation of angular momentum: $v_{\text{peri}} r_{\text{peri}} = v_{\text{aph}} r_{\text{aph}}$

$$\frac{v_{\text{peri}}}{v_{\text{aph}}} = \frac{r_{\text{aph}}}{r_{\text{peri}}} = \frac{20}{0.5} = 40$$

Answer 15 (5)

(a) Semi-major axis of transfer orbit:

$$a = \frac{R_{\text{Mars}} + R_{\text{asteroid}}}{2} = \frac{1.52 + 2.8}{2} = 2.16\,\text{AU}$$

(b) Time to reach asteroid:

$$P = a^{3/2} = 2.16^{3/2} \approx 3.17\,\text{years} \quad \Rightarrow \quad t_{\text{transfer}} = \frac{P}{2} \approx 1.59\,\text{years}$$

(c) Required Δv at Mars:

Speed in circular orbit: $v_{\text{Mars}} = \sqrt{\frac{\mu}{R_{\text{Mars}}}} \approx \sqrt{\frac{1}{1.52}} \approx 0.811\,\text{AU units}$

Speed at perihelion of transfer orbit:

$$v_p = \sqrt{\mu \left(\frac{2}{R_{\text{Mars}}} - \frac{1}{a} \right)} = \sqrt{2/1.52 - 1/2.16} \approx 0.958$$

$$\Delta v = v_p - v_{\text{Mars}} \approx 0.958 - 0.811 = 0.147\,\text{AU units}$$

Answer 16 (3)

(a) Orbital radius:

$$r = R_{\text{Earth}} + h = 6371 + 35786 = 42157\,\text{km}$$

(b) Orbital speed:

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.986 \times 10^5}{42157}} \approx 3.07\,\text{km/s}$$

Answer 17 (4)

(a) Semi-major axis of the transfer orbit:

$$a = \frac{R_E + R_S}{2} = \frac{1 + 9.58}{2} = 5.29\,\mathrm{AU}$$

(b) Time to reach Saturn:

$$P = a^{3/2} = 5.29^{3/2} \approx 12.16\,\mathrm{years}$$

Time to reach Saturn (half orbit):

$$t_{\mathrm{transfer}} = \frac{P}{2} \approx 6.08\,\mathrm{years}$$

Answer 18 (3)

(a) Orbital period:

$$T = r^{3/2} = 2^{3/2} \approx 2.83\,\text{years}$$

(b) Average orbital speed:

$$v = \frac{2\pi r\,(\text{AU to km})}{T\,(\text{years to seconds})} = \frac{2\pi \cdot 2 \cdot 1.496 \times 10^{8}}{2.83 \cdot 3.156 \times 10^{7}} \approx 21.0\,\text{km/s}$$

Answer 19 (4)

(a) Semi-major axis and eccentricity:

$$a = \frac{r_{\text{peri}} + r_{\text{aph}}}{2} = \frac{0.3 + 50}{2} = 25.15\,\text{AU}$$

$$e = \frac{r_{\text{aph}} - r_{\text{peri}}}{r_{\text{aph}} + r_{\text{peri}}} = \frac{50 - 0.3}{50 + 0.3} \approx 0.988$$

(b) Ratio of speeds:

$$v_{\text{peri}} r_{\text{peri}} = v_{\text{aph}} r_{\text{aph}} \quad \Rightarrow \quad \frac{v_{\text{peri}}}{v_{\text{aph}}} = \frac{r_{\text{aph}}}{r_{\text{peri}}} = \frac{50}{0.3} \approx 166.7$$

Answer 20 (5)

(a) Semi-major axis of transfer orbit:

$$a = \frac{R_{\text{Mars}} + R_{\text{Jupiter}}}{2} = \frac{1.52 + 5.2}{2} = 3.36\,\text{AU}$$

(b) Time to reach Jupiter:

$$P = a^{3/2} = 3.36^{3/2} \approx 6.16\,\text{years}$$

$$t_{\text{transfer}} = \frac{P}{2} \approx 3.08\,\text{years}$$

(c) Δv at Mars:
Speed in Mars circular orbit:

$$v_{\text{Mars}} = \sqrt{\frac{\mu}{R_{\text{Mars}}}} \approx \sqrt{\frac{1}{1.52}} \approx 0.811\,\text{AU units}$$

Speed at perihelion of transfer orbit:

$$v_p = \sqrt{\mu\left(\frac{2}{R_{\text{Mars}}} - \frac{1}{a}\right)} = \sqrt{2/1.52 - 1/3.36} \approx 0.942$$