

Limits

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Question 1 (2)

Evaluate the following limit:

$$\lim_{x \rightarrow 3} (2x^3 - 5x^2 + 7x - 1)$$

Question 2 (3)

Evaluate the limit

$$\lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 4}{x^2 - 4}$$

by factoring and canceling common factors where appropriate.

Question 3 (4)

Use the Pinching Theorem to evaluate

$$\lim_{x \rightarrow 0} x^2 \cos \left(\frac{1}{x^2} \right)$$

Justify your choice of bounding functions $g(x)$ and $h(x)$.

Question 4 (3)

A function $f(x)$ satisfies the inequality

$$1 - x^2 \leq f(x) \leq 1 + x^2$$

for all x in a neighborhood of $x = 0$. Use the Pinching Theorem to determine $\lim_{x \rightarrow 0} f(x)$.

Question 5 (4)

Using the fact that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ and appropriate algebraic manipulation, evaluate

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 2x}$$

Question 6 (5)

Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x^2}$$

using the double angle formula (or an appropriate trigonometric identity) and the Calculus of Limits Theorem.

Question 7 (4)

Find the limit

$$\lim_{x \rightarrow \infty} \frac{5x^3 - 2x^2 + 7}{2x^3 + 3x - 1}$$

by dividing numerator and denominator by the highest power of x and applying the Calculus of Limits Theorem.

Question 8 (5)

Evaluate

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 2x}{x^2 + \sin x}$$

Carefully justify each step, including the treatment of the $\sin x$ term in the denominator.

Question 9 (4)

Using an appropriate change of variables, evaluate

$$\lim_{x \rightarrow 0} \frac{\tan 7x}{x}$$

Express your answer in terms of known limits and justify each step.

Question 10 (5)

Using the exponential series and the Calculus of Limits Theorem, prove that

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = 2$$

Show all algebraic steps clearly.