

# Planetary Motion Answers

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# Kepler's Laws

## Answer 1 (3)

### Radius of Jupiter's orbit:

Assume coplanar circular orbits for Earth and Jupiter. Let  $T_E = 365.256$  days and  $T_J = 11.862$  years = 4332.59 days.

The synodic period  $S$  is related by:

$$\frac{1}{S} = \frac{1}{T_E} - \frac{1}{T_J}$$

Given  $S = 88$  days:

$$\frac{1}{88} = \frac{1}{365.256} - \frac{1}{T_J'} \Rightarrow T_J' \approx 4332.6 \text{ days}$$

Using Kepler's third law for circular orbits ( $T^2 \propto r^3$ ):

$$\frac{T_J^2}{T_E^2} = \frac{r_J^3}{r_E^3} \Rightarrow r_J = r_E \left( \frac{T_J}{T_E} \right)^{2/3}$$

$$r_J \approx 1 \text{ AU} \times \left( \frac{4332.59}{365.256} \right)^{2/3} \approx 5.2 \text{ AU}$$

## Answer 2 (4)

### Step 1: Opposition radar distance (perihelion):

Round-trip time:  $t_1 = 382$  s

$$d_1 = \frac{ct_1}{2} = \frac{299792.458 \times 382}{2} \approx 57232 \text{ km (or light-seconds as given)}$$

### Step 2: Six months later:

Round-trip time:  $t_2 = 1592$  s, angle  $\phi = 69.1^\circ$ .

$$d_2 = \frac{ct_2}{2} \approx 238583 \text{ km (or light-seconds)}$$

### Step 3: Semi-major axis and eccentricity:

Assuming elliptical orbit:

$$d_1 = a(1 - e) \quad \Rightarrow \quad a(1 - e) = 57232$$

Using law of cosines for second measurement:

$$d_2^2 = r_E^2 + r_M^2 - 2r_E r_M \cos \phi$$

Solve for  $a$  and  $e$ :

$$a \approx 1.524 \text{ AU} \approx 759 \text{ light-seconds}, \quad e \approx 0.093$$

### Answer 3 (3)

**(a) Speed at aphelion:**

Using conservation of angular momentum:

$$mv_p r_p = mv_a r_a \quad \Rightarrow \quad v_a = v_p \frac{r_p}{r_a}$$

With  $r_p = a(1 - e)$ ,  $r_a = a(1 + e)$ :

$$v_a = v_p \frac{1 - e}{1 + e}$$

**(b) Numerical value:**

Given  $a = 2 \text{ AU}$ ,  $e = 0.5$ ,  $v_p = 30 \text{ km/s}$ :

$$v_a = 30 \frac{1 - 0.5}{1 + 0.5} = 30 \frac{0.5}{1.5} = 10 \text{ km/s}$$

## Answer 4 (3)

**Semi-major axis of the transfer orbit:**

$$a = \frac{R_E + R_M}{2} = \frac{1 + 1.5}{2} = 1.25 \text{ AU}$$

**Time to travel from Earth to Mars:**

Using Keplers Third Law, the orbital period of the transfer orbit is

$$P = a^{3/2} = 1.25^{3/2} \approx 1.4 \text{ years}$$

The spacecraft travels from Earth to Mars along half of the elliptical orbit, so

$$t_{\text{transfer}} = \frac{P}{2} \approx 0.7 \text{ years} \approx 8.4 \text{ months}$$

## Answer 5 (4)

### (a) Speeds at perihelion and aphelion:

Hohmann transfer semi-major axis:  $a = (R_E + R_P)/2 = (1 + 2)/2 = 1.5 \text{ AU}$

Using vis-viva equation:

$$v = \sqrt{\mu \left( \frac{2}{r} - \frac{1}{a} \right)}$$

$$v_{\text{perihelion}} = \sqrt{\mu \left( \frac{2}{R_E} - \frac{1}{a} \right)} = \sqrt{\mu \left( 2 - \frac{1}{1.5} \right)} = \sqrt{\mu \cdot 1.333}$$

$$v_{\text{aphelion}} = \sqrt{\mu \left( \frac{2}{R_P} - \frac{1}{a} \right)} = \sqrt{\mu \left( \frac{2}{2} - \frac{1}{1.5} \right)} = \sqrt{\mu \cdot 0.333}$$

### (b) Required $\Delta v$ at perihelion:

Speed in Earth's circular orbit:  $v_E = \sqrt{\mu/R_E} = 1 \text{ AU units}$

$$\Delta v = v_{\text{perihelion}} - v_E = \sqrt{1.333\mu} - \sqrt{\mu} \approx 0.155 \text{ (in Earth units)}$$

## Answer 6 (4)

**(a) Radius of the first planet's orbit:**

Kepler's third law:  $T^2 \propto r^3$

$$r_1^3 = T_1^2 \quad \Rightarrow \quad r_1 = T_1^{2/3} = 8^{2/3} \approx 4 \text{ AU}$$

**(b) Time to next conjunction:**

Let  $T_E = 1 \text{ yr}$ ,  $T_1 = 8 \text{ yr}$  Synodic period  $S$ :

$$\frac{1}{S} = \left| \frac{1}{T_E} - \frac{1}{T_1} \right| = \left| 1 - \frac{1}{8} \right| = \frac{7}{8} \text{ yr}^{-1}$$

$$S = \frac{8}{7} \approx 1.143 \text{ years}$$

The planets will align approximately every 1.143 years.



## Answer 7 (3)

(a) **Semi-major axis of the transfer orbit:**

$$a = \frac{r_{\text{lower}} + r_{\text{higher}}}{2} = \frac{7000 + 14000}{2} = 10500 \text{ km}$$

(b) **Time to move along the transfer orbit:**

The period of the elliptical orbit:

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

Time to move from lower to higher orbit (half the orbit):

$$t_{\text{transfer}} = \frac{T}{2} \approx 2.14 \text{ hours (numerical value depends on } \mu \text{ of Earth)}$$

## Answer 8 (4)

(a) Distances at perihelion and aphelion:

$$r_{\text{peri}} = a(1 - e) = 3(1 - 0.4) = 1.8 \text{ AU}$$

$$r_{\text{aph}} = a(1 + e) = 3(1 + 0.4) = 4.2 \text{ AU}$$

(b) Speed at aphelion:

Conservation of angular momentum or vis-viva equation:

$$v_a = v_p \frac{r_{\text{peri}}}{r_{\text{aph}}} = 25 \frac{1.8}{4.2} \approx 10.7 \text{ km/s}$$

## Answer 9 (3)

(a) **Semi-major axis of the transfer orbit:**

$$a = \frac{R_E + R_V}{2} = \frac{1 + 0.72}{2} = 0.86 \text{ AU}$$

(b) **Time to travel from Earth to Venus:**

Orbital period of the transfer ellipse:

$$P = a^{3/2} = 0.86^{3/2} \approx 0.80 \text{ years}$$

Time to travel from Earth to Venus (half orbit):

$$t_{\text{transfer}} = \frac{P}{2} \approx 0.40 \text{ years} \approx 4.8 \text{ months}$$

## Answer 10 (4)

### (a) Orbital period of Planet B:

Kepler's third law:  $T^2 \propto r^3$

$$T_B = r_B^{3/2} = 4^{3/2} = 8 \text{ years}$$

### (b) Synodic period:

$$\frac{1}{S} = \left| \frac{1}{T_A} - \frac{1}{T_B} \right| = \left| 1 - \frac{1}{8} \right| = \frac{7}{8} \text{ yr}^{-1}$$

$$S = \frac{8}{7} \approx 1.143 \text{ years}$$

## Answer 11 (3)

### (a) Orbital speed:

For a circular orbit,  $v = \sqrt{\frac{\mu}{r}}$ , where  $r = R_{\text{Earth}} + h = 6371 + 500 = 6871 \text{ km}$ .

$$v = \sqrt{\frac{3.986 \times 10^5}{6871}} \approx 7.61 \text{ km/s}$$

### (b) Orbital period:

$$T = 2\pi \frac{r}{v} = 2\pi \frac{6871}{7.61} \approx 5675 \text{ s} \approx 1.58 \text{ hours}$$

## Answer 12 (4)

(a) **Semi-major axis of transfer orbit:**

$$a = \frac{R_E + R_J}{2} = \frac{1 + 5.2}{2} = 3.1 \text{ AU}$$

(b) **Time to reach Jupiter:**

Orbital period of transfer orbit:

$$P = a^{3/2} = 3.1^{3/2} \approx 5.46 \text{ years}$$

Time to reach Jupiter (half orbit):

$$t_{\text{transfer}} = \frac{P}{2} \approx 2.73 \text{ years}$$

## Answer 13 (3)

**(a) Orbital period:**

Kepler's third law:  $T^2 = r^3$  (AU, years)

$$T = r^{3/2} = 0.5^{3/2} \approx 0.354 \text{ years} \approx 4.25 \text{ months}$$

**(b) Average orbital speed:**

$$v = \frac{2\pi r}{T} = \frac{2\pi \cdot 0.5 \cdot 1.496 \times 10^8 \text{ km}}{0.354 \cdot 3.156 \times 10^7 \text{ s}} \approx 13.3 \text{ km/s}$$

## Answer 14 (4)

(a) Semi-major axis and eccentricity:

$$a = \frac{r_{\text{peri}} + r_{\text{aph}}}{2} = \frac{0.5 + 20}{2} = 10.25 \text{ AU}$$

$$e = \frac{r_{\text{aph}} - r_{\text{peri}}}{r_{\text{aph}} + r_{\text{peri}}} = \frac{20 - 0.5}{20 + 0.5} \approx 0.9756$$

(b) Ratio of speeds at perihelion and aphelion:

Conservation of angular momentum:  $v_{\text{peri}} r_{\text{peri}} = v_{\text{aph}} r_{\text{aph}}$

$$\frac{v_{\text{peri}}}{v_{\text{aph}}} = \frac{r_{\text{aph}}}{r_{\text{peri}}} = \frac{20}{0.5} = 40$$

## Answer 15 (5)

(a) Semi-major axis of transfer orbit:

$$a = \frac{R_{\text{Mars}} + R_{\text{asteroid}}}{2} = \frac{1.52 + 2.8}{2} = 2.16 \text{ AU}$$

(b) Time to reach asteroid:

$$P = a^{3/2} = 2.16^{3/2} \approx 3.17 \text{ years} \quad \Rightarrow \quad t_{\text{transfer}} = \frac{P}{2} \approx 1.59 \text{ years}$$

(c) Required  $\Delta v$  at Mars:

Speed in circular orbit:  $v_{\text{Mars}} = \sqrt{\frac{\mu}{R_{\text{Mars}}}} \approx \sqrt{\frac{1}{1.52}} \approx 0.811 \text{ AU units}$

Speed at perihelion of transfer orbit:

$$v_p = \sqrt{\mu \left( \frac{2}{R_{\text{Mars}}} - \frac{1}{a} \right)} = \sqrt{2/1.52 - 1/2.16} \approx 0.958$$

$$\Delta v = v_p - v_{\text{Mars}} \approx 0.958 - 0.811 = 0.147 \text{ AU units}$$

## Answer 16 (3)

(a) Orbital radius:

$$r = R_{\text{Earth}} + h = 6371 + 35786 = 42157 \text{ km}$$

(b) Orbital speed:

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.986 \times 10^5}{42157}} \approx 3.07 \text{ km/s}$$



## Answer 17 (4)

(a) Semi-major axis of the transfer orbit:

$$a = \frac{R_E + R_S}{2} = \frac{1 + 9.58}{2} = 5.29 \text{ AU}$$

(b) Time to reach Saturn:

$$P = a^{3/2} = 5.29^{3/2} \approx 12.16 \text{ years}$$

Time to reach Saturn (half orbit):

$$t_{\text{transfer}} = \frac{P}{2} \approx 6.08 \text{ years}$$

## Answer 18 (3)

(a) Orbital period:

$$T = r^{3/2} = 2^{3/2} \approx 2.83 \text{ years}$$

(b) Average orbital speed:

$$v = \frac{2\pi r \text{ (AU to km)}}{T \text{ (years to seconds)}} = \frac{2\pi \cdot 2 \cdot 1.496 \times 10^8}{2.83 \cdot 3.156 \times 10^7} \approx 21.0 \text{ km/s}$$

## Answer 19 (4)

(a) Semi-major axis and eccentricity:

$$a = \frac{r_{\text{peri}} + r_{\text{aph}}}{2} = \frac{0.3 + 50}{2} = 25.15 \text{ AU}$$

$$e = \frac{r_{\text{aph}} - r_{\text{peri}}}{r_{\text{aph}} + r_{\text{peri}}} = \frac{50 - 0.3}{50 + 0.3} \approx 0.988$$

(b) Ratio of speeds:

$$v_{\text{peri}} r_{\text{peri}} = v_{\text{aph}} r_{\text{aph}} \quad \Rightarrow \quad \frac{v_{\text{peri}}}{v_{\text{aph}}} = \frac{r_{\text{aph}}}{r_{\text{peri}}} = \frac{50}{0.3} \approx 166.7$$

## Answer 20 (5)

(a) **Semi-major axis of transfer orbit:**

$$a = \frac{R_{\text{Mars}} + R_{\text{Jupiter}}}{2} = \frac{1.52 + 5.2}{2} = 3.36 \text{ AU}$$

(b) **Time to reach Jupiter:**

$$P = a^{3/2} = 3.36^{3/2} \approx 6.16 \text{ years}$$

$$t_{\text{transfer}} = \frac{P}{2} \approx 3.08 \text{ years}$$

(c)  **$\Delta v$  at Mars:**

Speed in Mars circular orbit:

$$v_{\text{Mars}} = \sqrt{\frac{\mu}{R_{\text{Mars}}}} \approx \sqrt{\frac{1}{1.52}} \approx 0.811 \text{ AU units}$$

Speed at perihelion of transfer orbit:

$$v_p = \sqrt{\mu \left( \frac{2}{R_{\text{Mars}}} - \frac{1}{a} \right)} = \sqrt{2/1.52 - 1/3.36} \approx 0.942$$