Planetary Motion

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Kepler's Laws

Question 1 (3)

88 days after being observed at opposition, Jupiter is observed at eastern quadrature. Assume coplanar circular orbits for both the Earth and Jupiter, and determine the radius of Jupiters orbit in units of the radius of the Earths orbit, i.e., 1 AU.

The sidereal orbital periods of the Earth and Jupiter are 365.256 days and 11.862 years respectively.

Question 2 (4)

The following question uses made-up data to illustrate how the semi-major axis and eccentricity of Mars' elliptical orbit can be determined from two radar distance measurements.

First, consider the configuration where Mars is at opposition when also at perihelion (positions E_1 and M_1). At this time, a radar signal is sent from Earth to Mars. It takes 382 seconds for the round trip.

Six months after the opposition date (E_2 and M_2), when the angle between the Sun and Mars on the sky is observed to be 69.1°, a radar signal is sent again. In this case, the round trip takes 1592 seconds.

Assume a circular orbit for the Earth with a radius of 1 AU (499 light-seconds), and use the measurements to derive:

1. The semi-major axis of Mars' orbit (in light-seconds).
2. The eccentricity of Mars' orbit.

Question 3 (3)

A planet orbits a star in an elliptical orbit with a semi-major axis a and eccentricity e. At perihelion, the planet's orbital speed is v_p.

1. Using conservation of angular momentum and energy, derive an expression for the speed of the planet at aphelion v_a in terms of a, e, and v_p.
2. If $a = 2\,\mathrm{AU}$, $e = 0.5$, and $v_p = 30\,\mathrm{km/s}$, calculate v_a.

Question 4 (3)

A spacecraft is planned to travel from Earth to Mars using a Hohmann transfer orbit. Assume that both Earth and Mars move in circular coplanar orbits with radii $R_E = 1\,\mathrm{AU}$ and $R_M = 1.5\,\mathrm{AU}$.

1. Determine the semi-major axis of the Hohmann transfer orbit.
2. Calculate the time required for the spacecraft to travel from Earth to Mars along this orbit.

Question 5 (4)

A spacecraft initially orbits the Earth at a low circular orbit of radius $R_E = 1\,\mathrm{AU}$. It is desired to send the spacecraft to a planet in a circular orbit of radius $R_P = 2\,\mathrm{AU}$ using a Hohmann transfer orbit.

1. Determine the speed of the spacecraft at perihelion and aphelion of the transfer orbit.
2. Calculate the required Δv at perihelion to inject the spacecraft into the transfer orbit from Earth's circular orbit.

Question 6 (4)

A hypothetical planet orbits its star with a period of 8 Earth years. A second planet in the same plane has a circular orbit of radius 1 AU.

1. Using Kepler's third law, determine the radius of the orbit of the first planet in AU.
2. If the planets are initially aligned, after how many Earth years will they next be in conjunction (aligned again with the star)?

Question 7 (3)

A spacecraft orbits Earth in a low circular orbit of radius 7000 km. It performs a Hohmann transfer to a higher circular orbit of radius 14000 km.

1. Determine the semi-major axis of the transfer orbit.

2. Calculate the time required to move from the lower orbit to the higher orbit along the transfer path.

Question 8 (4)

A planet orbits a star in an elliptical orbit with semi-major axis $a = 3\,\mathrm{AU}$ and eccentricity $e = 0.4$.

1. Determine the distance of the planet from the star at perihelion and aphelion.

2. If the planets speed at perihelion is $v_p = 25\,\mathrm{km/s}$, calculate its speed at aphelion.

Question 9 (3)

A spacecraft is sent from Earth to Venus using a Hohmann transfer orbit. Assume circular coplanar orbits with $R_E = 1\,\text{AU}$ and $R_V = 0.72\,\text{AU}$.

1. Calculate the semi-major axis of the transfer orbit.
2. Determine the time taken to travel from Earth to Venus along the transfer orbit.

Question 10 (4)

Two planets orbit the same star. Planet A has an orbital radius of 1 AU, and Planet B has an orbital radius of 4 AU.

1. Using Kepler's third law, determine the orbital period of Planet B in Earth years.
2. If both planets are initially aligned, calculate the synodic period (time until next alignment) in Earth years.

Question 11 (3)

A satellite is in a circular orbit around Earth at an altitude of 500 km.

1. Calculate the orbital speed of the satellite.
2. Determine the orbital period.

Question 12 (4)

A spacecraft is planned to travel from Earth to Jupiter using a Hohmann transfer orbit. Assume circular coplanar orbits with radii $R_E = 1\,\mathrm{AU}$ and $R_J = 5.2\,\mathrm{AU}$.

1. Determine the semi-major axis of the transfer orbit.
2. Calculate the time required to reach Jupiter.

Question 13 (3)

A planet orbits a star with an orbital radius of 0.5 AU.

1. Calculate the orbital period of the planet in Earth years.
2. Determine its average orbital speed.

Question 14 (4)

A comet moves in a highly elliptical orbit around the Sun with a perihelion distance of 0.5 AU and an aphelion distance of 20 AU.

1. Determine the semi-major axis and eccentricity of the orbit.
2. Using conservation of angular momentum, find the ratio of the comet's speed at perihelion to its speed at aphelion.

Question 15 (5)

A spacecraft is to travel from Mars to an asteroid in the asteroid belt using a Hohmann transfer orbit. The orbital radius of Mars is 1.52 AU, and the target asteroid's orbit is 2.8 AU.

1. Determine the semi-major axis of the transfer orbit.

2. Calculate the time required to reach the asteroid.

3. Determine the Δv required at Mars for injection into the transfer orbit (assume circular initial orbit and use vis-viva).

Question 16 (3)

A satellite is in a geostationary orbit around Earth at an altitude of 35,786 km.

1. Calculate the orbital radius from the center of the Earth.
2. Determine the orbital speed.

Question 17 (4)

A spacecraft is to transfer from Earth to Saturn using a Hohmann transfer orbit. Assume circular coplanar orbits with radii $R_E = 1\,\mathrm{AU}$ and $R_S = 9.58\,\mathrm{AU}$.

1. Determine the semi-major axis of the transfer orbit.
2. Calculate the time required to reach Saturn.

Question 18 (3)

A planet orbits a star with an orbital radius of 2 AU.

1. Determine the orbital period of the planet in Earth years.
2. Calculate its average orbital speed.

Question 19 (4)

A comet follows a highly elliptical orbit around the Sun with a perihelion distance of 0.3 AU and aphelion distance of 50 AU.

1. Determine the semi-major axis and eccentricity of the comet's orbit.
2. Using conservation of angular momentum, calculate the ratio of the comet's speed at perihelion to its speed at aphelion.

Question 20 (5)

A spacecraft is sent from Mars to Jupiter using a Hohmann transfer orbit. Assume circular coplanar orbits with radii $R_{\text{Mars}} = 1.52\,\text{AU}$ and $R_{\text{Jupiter}} = 5.2\,\text{AU}$.

1. Determine the semi-major axis of the transfer orbit.
2. Calculate the time required to reach Jupiter.
3. Determine the Δv required at Mars to inject the spacecraft into the transfer orbit.