De Moivre's Theorem Answers

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Answer 1 (3)

Express $\cos(3x)$ as a polynomial in $\cos(x)$.

Step 1. Recall De Moivre's Theorem:

$$(\cos x + i \sin x)^n = \cos(nx) + i \sin(nx)$$

Step 2. Expand $(\cos x + i \sin x)^3$:

$$(\cos x + i \sin x)^3 = \cos^3 x + 3i \cos^2 x \sin x - 3 \cos x \sin^2 x - i \sin^3 x$$

Step 3. Extract the real part:

$$\cos(3x) = \cos^3 x - 3 \cos x \sin^2 x$$

Step 4. Use $\sin^2 x = 1 - \cos^2 x$:

$$\cos(3x) = \cos^3 x - 3 \cos x(1 - \cos^2 x)$$

$$\cos(3x) = 4 \cos^3 x - 3 \cos x$$

$$\boxed{\cos(3x) = 4 \cos^3 x - 3 \cos x}$$

Answer 2 (3)

Express $\sin(3x)$ as a polynomial in $\sin(x)$ and $\cos(x)$.

Step 1. Use De Moivre's Theorem:

$$(\cos x + i \sin x)^3 = \cos(3x) + i \sin(3x)$$

Step 2. Expand:

$$(\cos x + i \sin x)^3 = \cos^3 x + 3i \cos^2 x \sin x - 3 \cos x \sin^2 x - i \sin^3 x$$

Step 3. Extract the imaginary part:

$$\sin(3x) = 3 \cos^2 x \sin x - \sin^3 x$$

Step 4. Factor out $\sin x$:

$$\sin(3x) = \sin x(3 \cos^2 x - \sin^2 x)$$

Step 5. Use $\sin^2 x = 1 - \cos^2 x$:

$$\sin(3x) = \sin x(3 \cos^2 x - (1 - \cos^2 x)) = \sin x(4 \cos^2 x - 1)$$

$$\boxed{\sin(3x) = \sin x(4 \cos^2 x - 1)}$$

Answer 3 (3)

Express $\cos(4x)$ as a polynomial in $\cos(x)$.

Step 1. De Moivre's Theorem:

$$(\cos x + i \sin x)^4 = \cos(4x) + i \sin(4x)$$

Step 2. Expand using binomial theorem:

$$(\cos x + i \sin x)^4 = \cos^4 x + 4i \cos^3 x \sin x - 6 \cos^2 x \sin^2 x - 4i \cos x \sin^3 x + \sin^4 x$$

Step 3. Real part gives $\cos(4x)$:

$$\cos(4x) = \cos^4 x - 6 \cos^2 x \sin^2 x + \sin^4 x$$

Step 4. Replace $\sin^2 x = 1 - \cos^2 x$:

$$\cos(4x) = \cos^4 x - 6 \cos^2 x(1 - \cos^2 x) + (1 - \cos^2 x)^2$$

Step 5. Simplify:

$$\cos(4x) = 8 \cos^4 x - 8 \cos^2 x + 1$$

$$\boxed{\cos(4x) = 8 \cos^4 x - 8 \cos^2 x + 1}$$

Answer 4 (3)

Express $\sin(4x)$ as a polynomial in $\sin(x)$ and $\cos(x)$.

Step 1. Use De Moivre:

$$(\cos x + i\sin x)^4 = \cos(4x) + i\sin(4x)$$

Step 2. Imaginary part:

$$\sin(4x) = 4\cos^3 x \sin x - 4\cos x \sin^3 x$$

Step 3. Factor $\sin x$:

$$\sin(4x) = 4\sin x(\cos^3 x - \cos x \sin^2 x)$$

Step 4. Replace $\sin^2 x = 1 - \cos^2 x$:

$$\sin(4x) = 4\sin x(\cos^3 x - \cos x(1 - \cos^2 x)) = 4\sin x(2\cos^3 x - \cos x)$$

$$\boxed{\sin(4x) = 4\sin x(2\cos^3 x - \cos x)}$$

Answer 5 (4)

Express $\cos(5x)$ as a polynomial in $\cos(x)$.

Step 1. De Moivre:

$$(\cos x + i\sin x)^5 = \cos(5x) + i\sin(5x)$$

Step 2. Expand and extract real part:

$$\cos(5x) = \cos^5 x - 10\cos^3 x\sin^2 x + 5\cos x\sin^4 x$$

Step 3. Replace $\sin^2 x = 1 - \cos^2 x$ and simplify:

$$\cos(5x) = 16\cos^5 x - 20\cos^3 x + 5\cos x$$

$$\boxed{\cos(5x) = 16\cos^5 x - 20\cos^3 x + 5\cos x}$$

Answer 6 (4)

Express $\sin(5x)$ as a polynomial in $\sin(x)$ and $\cos(x)$.

Step 1. Imaginary part of De Moivre expansion:

$$\sin(5x) = 5\cos^4 x \sin x - 10\cos^2 x \sin^3 x + \sin^5 x$$

Step 2. Factor $\sin x$:

$$\sin(5x) = \sin x(5\cos^4 x - 10\cos^2 x \sin^2 x + \sin^4 x)$$

Step 3. Replace $\sin^2 x = 1 - \cos^2 x$ and simplify:

$$\sin(5x) = \sin x(16\cos^4 x - 12\cos^2 x + 1)$$

$$\boxed{\sin(5x) = \sin x(16\cos^4 x - 12\cos^2 x + 1)}$$

Answer 7 (4)

Express $\cos(6x)$ as a polynomial in $\cos(x)$.

Step 1. Use De Moivre's Theorem:

$$(\cos x + i \sin x)^6 = \cos(6x) + i \sin(6x)$$

Step 2. Real part gives:

$$\cos(6x) = \cos^6 x - 15 \cos^4 x \sin^2 x + 15 \cos^2 x \sin^4 x - \sin^6 x$$

Step 3. Replace $\sin^2 x = 1 - \cos^2 x$:

$$\cos(6x) = \cos^6 x - 15 \cos^4 x(1 - \cos^2 x) + 15 \cos^2 x(1 - \cos^2 x)^2 - (1 - \cos^2 x)^3$$

Step 4. Expand and simplify:

$$\cos(6x) = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

$$\boxed{\cos(6x) = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1}$$

Answer 8 (4)

Express $\sin(6x)$ as a polynomial in $\sin(x)$ and $\cos(x)$.

Step 1. Use De Moivre's Theorem:

$$(\cos x + i \sin x)^6 = \cos(6x) + i \sin(6x)$$

Step 2. Extract imaginary part:

$$\sin(6x) = 6 \cos^5 x \sin x - 20 \cos^3 x \sin^3 x + 6 \cos x \sin^5 x$$

Step 3. Factor $\sin x$:

$$\sin(6x) = \sin x(6 \cos^5 x - 20 \cos^3 x \sin^2 x + 6 \cos x \sin^4 x)$$

Step 4. Replace $\sin^2 x = 1 - \cos^2 x$ and simplify:

$$\sin(6x) = \sin x(32 \cos^5 x - 32 \cos^3 x + 6 \cos x)$$

$$\boxed{\sin(6x) = \sin x(32 \cos^5 x - 32 \cos^3 x + 6 \cos x)}$$

Answer 9 (5)

Express $\cos(7x)$ as a polynomial in $\cos(x)$.

Step 1. Use De Moivre's Theorem:

$$(\cos x + i\sin x)^7 = \cos(7x) + i\sin(7x)$$

Step 2. Real part using binomial expansion:

$$\cos(7x) = \cos^7 x - 21\cos^5 x\sin^2 x + 35\cos^3 x\sin^4 x - 7\cos x\sin^6 x$$

Step 3. Replace $\sin^2 x = 1 - \cos^2 x$ and simplify:

$$\cos(7x) = 64\cos^7 x - 112\cos^5 x + 56\cos^3 x - 7\cos x$$

$$\boxed{\cos(7x) = 64\cos^7 x - 112\cos^5 x + 56\cos^3 x - 7\cos x}$$

Answer 10 (5)

Express $\sin(7x)$ as a polynomial in $\sin(x)$ and $\cos(x)$.

Step 1. Imaginary part of De Moivre's expansion:

$$\sin(7x) = 7\cos^6 x \sin x - 35\cos^4 x \sin^3 x + 21\cos^2 x \sin^5 x - \sin^7 x$$

Step 2. Factor $\sin x$:

$$\sin(7x) = \sin x(7\cos^6 x - 35\cos^4 x \sin^2 x + 21\cos^2 x \sin^4 x - \sin^6 x)$$

Step 3. Replace $\sin^2 x = 1 - \cos^2 x$ and simplify:

$$\sin(7x) = \sin x(64\cos^6 x - 112\cos^4 x + 56\cos^2 x - 7)$$

$$\boxed{\sin(7x) = \sin x(64\cos^6 x - 112\cos^4 x + 56\cos^2 x - 7)}$$