Proof By Induction

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Summation Results

Question 1 (3)

Prove by induction that

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

Question 2 (3)

Prove by induction that

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

Question 3 (4)

Prove by induction that

$$\sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Question 4 (4)

Prove by induction that

$$\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$$

Question 5 (4)

Prove by induction that

$$\sum_{r=1}^{n} r \cdot r! = (n+1)! - 1$$

Question 6 (5)

Prove by induction that

$$\sum_{r=1}^{n} r^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

Question 7 (5)

Prove by induction that

$$\sum_{r=1}^{n}(2r-1) = n^2$$

Question 8 (5)

Prove by induction that

$$\sum_{r=1}^{n} r(r+1)(r+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

Question 9 (5)

Prove by induction that

$$\sum_{r=1}^{n} r \cdot 2^r = (n-1)2^{n+1} + 2$$

Question 10 (5)

Prove by induction that

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$$

Divisibility Results

Question 1 (3)

Let $f(n) = 7^n - 1$, $n \in \mathbb{N}$. Prove by induction that $f(n)$ is divisible by 6 for all $n \in \mathbb{N}$.

Question 2 (3)

Let $f(n) = 3^{2n} - 4^n$, $n \in \mathbb{N}$. Prove by induction that $f(n)$ is divisible by 5 for all $n \in \mathbb{N}$.

Question 3 (3)

Let $f(n) = 4^{n+1} - 1$, $n \in \mathbb{N}$. Prove by induction that $f(n)$ is divisible by 3 for all $n \in \mathbb{N}$.

Question 4 (3)

Let $f(n) = 5^n - 1$, $n \in \mathbb{N}$. Prove by induction that $f(n)$ is divisible by 4 for all $n \in \mathbb{N}$.

Question 5 (4)

Let $f(n) = 2^{2n} - 1$, $n \in \mathbb{N}$. Prove by induction that $f(n)$ is divisible by 3 for all $n \in \mathbb{N}$.

Question 6 (3)

Let $f(n) = 3^{2n+1} + 1$, $n \in \mathbb{N}_0$. Prove by induction that $f(n)$ is divisible by 4 for all $n \in \mathbb{N}_0$.

Question 7 (3)

Let $f(n) = 2^{2n} + 1$, $n \in \mathbb{N}$. Prove by induction that $f(n)$ is divisible by 5 for all $n \in \mathbb{N}$.

Question 8 (3)

Let $f(n) = 7^n - 2^n$, $n \in \mathbb{N}$. Prove by induction that $f(n)$ is divisible by 5 for all $n \in \mathbb{N}$.

Question 9 (3)

Let $f(n) = 3^{2n} - 2^{2n}$, $n \in \mathbb{N}$. Prove by induction that $f(n)$ is divisible by 5 for all $n \in \mathbb{N}$.

Question 10 (4)

Let $f(n) = 2^{2n+1} + 3 \cdot 3^{n-1}$, $n \in \mathbb{N}$. Prove by induction that $f(n)$ is divisible by 5 for all $n \in \mathbb{N}$.