Waves Question Set

Prepared by Salkaro

Question 1 (3)

A particle moves along the x-axis with displacement

$$x(t) = A\cos(\omega t + \phi) + B,$$

where A, ω, ϕ and B are constants.

1. By direct differentiation, show that the motion satisfies a second-order differential equation of the form $\ddot{x} = -\omega^2(x - B)$.
2. State clearly under what condition on B the motion is simple harmonic about the origin.
3. If $A = 4.0$ cm and $\omega = 12.0$ rad s^{-1}, calculate the maximum speed of the particle.

Question 2 (3)

A transverse travelling wave on a stretched string is described by

$$y(x, t) = A\cos(kx - \omega t).$$

1. Show that the wavelength λ and period T are related to k and ω by $\lambda = 2\pi/k$ and $T = 2\pi/\omega$.
2. Hence show that the wave speed is $v = \omega/k$.
3. If $k = 12.5$ m^{-1} and $\omega = 220$ rad s^{-1}, calculate v, λ, and f.

Question 3 (4)

A standing wave on a string fixed at both ends is described by

$$y(x, t) = A \sin(kx) \sin(\omega t),$$

for $0 \leq x \leq L$.

1. Show that the boundary conditions require $k = n\pi/L$, where n is a positive integer.
2. Hence derive an expression for the allowed angular frequencies ω_n in terms of n, L, and the wave speed v.
3. For $L = 0.80$ m and $v = 120$ m s^{-1}, calculate the frequencies of the first three normal modes.

Question 4 (4)

A stretched string of linear mass density $\mu = 2.0 \times 10^{-3}$ kg m^{-1} is under a tension $T = 90$ N.

1. Derive an expression for the speed of transverse waves on the string.
2. Calculate the wave speed for the given values.
3. The string is fixed at both ends with length $L = 0.75$ m. Determine the fundamental frequency.
4. If the tension is increased by a factor of four, by what factor does the fundamental frequency change?

Question 5 (5)

A guitar string of length L is plucked at its midpoint and released from rest, producing an initial triangular displacement with maximum amplitude A.

1. Explain qualitatively why the subsequent motion can be expressed as a superposition of standing wave modes.
2. Show that only odd harmonics are present in the Fourier series representation of the initial shape.
3. If the fundamental frequency is $f_1 = 110$ Hz, calculate the frequencies of the next three modes that are actually present.
4. The sound produced by the string is transmitted through air to a stationary listener. State whether the wavelength and frequency of the sound in air are equal to those of the string wave, and justify your answer.

Question 6 (4)

An ambulance siren emits sound at a constant frequency of $f_s = 950$ Hz while moving in a straight line toward a stationary observer with speed $v_s = 28$ m s^{-1}. The speed of sound in air is $v = 343$ m s^{-1}.

1. Derive an expression for the frequency heard by the observer while the source is approaching.
2. Calculate the observed frequency.
3. Determine the frequency heard immediately after the ambulance has passed and is receding with the same speed.

Question 7 (4)

A cyclist moves at $v_L = 6.0$ m s^{-1} directly toward a stationary loudspeaker emitting a tone of frequency $f_s = 720$ Hz. The speed of sound in air is 343 m s^{-1}.

1. Derive the Doppler shift formula for a moving listener and stationary source.
2. Calculate the frequency heard by the cyclist.
3. If the cyclist turns around and rides directly away at the same speed, determine the new observed frequency.

Question 8 (5)

A train moves at constant speed v_s along a straight track while blowing its horn at frequency $f_s = 500$ Hz. A stationary observer measures the approaching frequency to be $f_{\text{app}} = 620$ Hz.

1. Derive an expression for the observed frequency in terms of v_s, v, and f_s for an approaching source.
2. Determine the speed of the train.
3. Calculate the frequency heard after the train has passed the observer.
4. Determine the percentage change in frequency between the approaching and receding cases.

Question 9 (5)

Two cars travel along the same straight road in opposite directions. Car A emits a steady tone of frequency $f_s = 880$ Hz while moving at $v_A = 20$ m s^{-1}. Car B moves toward Car A at $v_B = 25$ m s^{-1}. Take the speed of sound to be 343 m s^{-1}.

1. Derive the general Doppler shift formula for both moving source and moving observer.
2. Calculate the frequency heard by the driver of Car B.
3. If both cars reverse direction but keep the same speeds, calculate the new observed frequency.

Question 10 (5)

A distant galaxy emits light at a rest-frame wavelength of $\lambda_0 = 486$ nm and is observed on Earth at $\lambda = 648$ nm.

1. State whether the galaxy is approaching or receding.
2. Using the relativistic Doppler formula

$$\frac{\lambda}{\lambda_0} = \sqrt{\frac{1 + v/c}{1 - v/c}},$$

   calculate the recession speed as a fraction of c.

3. Using Hubbles law $v = Hd$ with $H = 70$ km s^{-1} Mpc^{-1}, estimate the distance to the galaxy.