Astronomy Variant Answer Set

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User's Guide to the Night Sky

Answer 1: Stellar Triangulation

Baseline:

$$b = 667 \text{ km}, \qquad \Delta\theta = 48''.$$

Convert angle to radians:

$$\theta_{\text{rad}} = \frac{48}{206265}.$$

Distance:

$$d \approx \frac{b}{\theta_{\text{rad}}} = \frac{667}{48/206265} \approx 2.9 \times 10^6 \text{ km}.$$

$$\boxed{d \approx 2.9 \times 10^6 \text{ km}}$$

Answer 2: Parallax Distance and Absolute Magnitude

Star with $m = 7.6$, $p = 0.042''$.

1. Distance:

$$d = \frac{1}{p} = \frac{1}{0.042} \approx 23.8 \text{ pc}.$$

$$\boxed{d \approx 23.8 \text{ pc}}$$

2. Absolute magnitude:

$$M = m + 5 - 5\log_{10}(d) = 7.6 + 5 - 5\log_{10}(23.8) \approx 12.6 - 5(1.376) \approx 12.6 - 6.88 \approx 5.7.$$

$$\boxed{M \approx 5.7}$$

Answer 3: Magnitude and Flux Ratio

Flux scales as:

$$f \propto 10^{-0.4m}.$$

New magnitudes:

$$m'_A = 11.2, \qquad m'_B = 11.0.$$

Flux ratio:

$$\frac{f'_A}{f'_B} = 10^{-0.4(m'_A - m'_B)} = 10^{-0.4(0.2)} = 10^{-0.08} \approx 0.83.$$

$$\boxed{\frac{f'_A}{f'_B} \approx 0.83}$$

Answer 4: Constellation Visibility at Given Latitude

Latitude $\phi = +52^\circ$, declination $\delta = -18^\circ$.

1. Maximum altitude:

   $$h_{\max} = 90^\circ - |\phi - \delta| = 90^\circ - |52^\circ - (-18^\circ)| = 90^\circ - 70^\circ = 20^\circ.$$

   Since $h_{\max} > 0$, the star rises.

   $$\boxed{\text{The star is visible.}}$$

2. $$\boxed{h_{\max} = 20^\circ}$$

Answer 5: Solar Altitude on a Solstice

Latitude $\phi = +35°$;

$$h_{\text{max}} = 90° - |\phi - \delta|.$$

1. Summer solstice: $\delta = +23.5°$,

$$h_{\text{max}} = 90° - |35° - 23.5°| = 90° - 11.5° = 78.5°.$$

$$\boxed{h_{\text{max, summer}} \approx 78.5°}$$

2. Winter solstice: $\delta = -23.5°$,

$$h_{\text{max}} = 90° - |35° - (-23.5°)| = 90° - 58.5° = 31.5°.$$

$$\boxed{h_{\text{max, winter}} \approx 31.5°}$$

Answer 6: RADec and Meridian Crossing Time

Star: RA = $5^{\text{h}}18^{\text{m}}$. Sun on Feb 10: RA $\approx 21^{\text{h}}$.

Sidereal time at midnight:

$$\text{LST}_{\text{mid}} \approx \text{RA}_{\odot} + 12^{\text{h}} = 21^{\text{h}} + 12^{\text{h}} = 33^{\text{h}} \equiv 9^{\text{h}}.$$

Difference:

$$\Delta t = 9^{\text{h}} - 5^{\text{h}}18^{\text{m}} = 3^{\text{h}}42^{\text{m}}.$$

Thus transit occurred 3h 42m before midnight:

$$00{:}00 - 3{:}42 = 20{:}18.$$

$$\boxed{\text{Transit at } \approx 20{:}18}$$

Answer 7: Atmospheric Refraction Shift

True altitude $h = 12°$.

Compute effective angle:

$$h + \frac{10.3°}{h + 5.11°} = 12° + \frac{10.3}{17.11} \approx 12° + 0.60° = 12.6°.$$

Refraction:

$$r = \frac{1.02°}{\tan(12.6°)} \approx \frac{1.02°}{0.223} \approx 4.6'.$$

Apparent altitude:

$$h' = 12° + \frac{4.6'}{60} \approx 12.08°.$$

$$\boxed{h' \approx 12.1°}$$

Answer 8: Sidereal vs Solar Day Timing

Star rises 4 minutes earlier each day.

From Mar 1 to Mar 15:

$$\Delta t = 14 \times 4 \text{ min} = 56 \text{ min}.$$

New rise time:

$$22{:}16 - 0{:}56 = 21{:}20.$$

$$\boxed{t_{\text{rise}} \approx 21{:}20}$$

Answer 9: Lunar SynodicSidereal Relation

$$\frac{1}{P_{\rm sid}} - \frac{1}{P_{\rm syn}} = \frac{1}{P_{\rm year}}.$$

Convert $P_{\rm syn} = 34.7$ hr:

$$P_{\rm syn} = \frac{34.7}{24} \approx 1.4458 \text{ d}.$$

Compute:

$$\frac{1}{P_{\rm sid}} = \frac{1}{1.4458} + \frac{1}{365.25} \approx 0.6915 + 0.00274 = 0.6942 \text{ d}^{-1}.$$

Thus:

$$P_{\rm sid} = \frac{1}{0.6942} \approx 1.440 \text{ d} = 34.56 \text{ hr}.$$

$$\boxed{P_{\rm sid} \approx 34.6 \text{ hr}}$$

Answer 10: Eclipse Geometry and Angular Sizes

Moon radius: $R_m = 1800$ km; distance $D_m = 4.9 \times 10^5$ km. Star radius: $R_s = 6.2 \times 10^5$ km; distance $D_s = 1.4 \times 10^8$ km.

Angular sizes:

$$\theta_m = \frac{R_m}{D_m} = \frac{1800}{4.9 \times 10^5} \approx 0.00367 \text{ rad}.$$

$$\theta_s = \frac{R_s}{D_s} = \frac{6.2 \times 10^5}{1.4 \times 10^8} \approx 0.00443 \text{ rad}.$$

Compare: $\theta_m < \theta_s$, so the moon appears smaller than the star.
Thus eclipse is annular.

$$\boxed{\theta_m \approx 0.00367 \text{ rad}}, \qquad \boxed{\theta_s \approx 0.00443 \text{ rad}}, \qquad \boxed{\text{Annular eclipse}}$$

Answer 11: Planetary Synodic Period and Opposition

Asteroid sidereal period: $P = 4.21$ yr.

Relation for superior planet:

$$\frac{1}{P_{\rm syn}} = \left|\frac{1}{P_\oplus} - \frac{1}{P}\right| = \left|1 - \frac{1}{4.21}\right| = 1 - 0.2377 = 0.7623 \text{ yr}^{-1}.$$

Thus:

$$P_{\rm syn} = \frac{1}{0.7623} \approx 1.31 \text{ yr} \approx 478 \text{ days}.$$

$$\boxed{P_{\rm syn} \approx 1.31 \text{ yr}}, \quad \boxed{\text{Opposition interval} \approx 478 \text{ days}}$$

Answer 12: Meteoroid Impact Rate Estimate

Density:

$$n = 4.0 \times 10^{-8} \text{ m}^{-3}, \qquad v = 29.8 \text{ km/s} = 2.98 \times 10^4 \text{ m/s}.$$

Earth cross-section:

$$A = \pi R^2 = \pi(6.37 \times 10^6)^2 \approx 1.28 \times 10^{14} \text{ m}^2.$$

Flux:

$$\dot{N} = n\,v\,A = (4.0 \times 10^{-8})(2.98 \times 10^4)(1.28 \times 10^{14}) \approx 1.5 \times 10^{11} \text{ s}^{-1}.$$

$$\boxed{\dot{N} \approx 1.5 \times 10^{11} \text{ meteoroids/s}}$$

The Solar System

Answer 1: Orbital Geometry and Inferior Conjunction

For an inferior planet the periods satisfy

$$\frac{1}{P_p} = \frac{1}{P_E} + \frac{1}{S},$$

with $P_E = 365.25$ d and $S = 142$ d.

$$\frac{1}{P_p} = \frac{1}{365.25} + \frac{1}{142} \approx 2.738 \times 10^{-3} + 7.042 \times 10^{-3} \approx 9.780 \times 10^{-3} \text{ d}^{-1},$$

$$P_p \approx \frac{1}{9.780 \times 10^{-3}} \approx 1.02 \times 10^2 \text{ d} \approx 102 \text{ d}.$$

Using Keplers third law in solar units,

$$\left(\frac{P_p}{1 \text{ yr}}\right)^2 = a^3,$$

$$P_p \approx 102.25 \text{ d} \Rightarrow \frac{P_p}{1 \text{ yr}} = \frac{102.25}{365.25} \approx 0.280,$$

$$a^3 \approx (0.280)^2 \approx 0.0784 \Rightarrow a \approx 0.428 \text{ AU}.$$

$$\boxed{P_p \approx 1.02 \times 10^2 \text{ days}}, \qquad \boxed{a \approx 0.43 \text{ AU}}$$

Answer 2: Keplerian Mass Determination in Binary Motion

Circular orbit of radius $r = 880$ km $= 8.80 \times 10^5$ m, period $P = 11.3$ hr $= 4.068 \times 10^4$ s.

From Kepler/centripetal balance:

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{P^2} \Rightarrow GM = \frac{4\pi^2 r^3}{P^2}.$$

Compute:

$$GM \approx \frac{4\pi^2 (8.80 \times 10^5)^3}{(4.068 \times 10^4)^2} \approx 1.63 \times 10^{10}\ \mathrm{m^3 s^{-2}}.$$

Then

$$M = \frac{GM}{G} \approx \frac{1.63 \times 10^{10}}{6.67 \times 10^{-11}} \approx 2.4 \times 10^{20}\ \mathrm{kg}.$$

Escape velocity from radius $R = 420$ km $= 4.20 \times 10^5$ m:

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}} \approx \sqrt{\frac{2 \times 1.63 \times 10^{10}}{4.20 \times 10^5}} \approx 2.8 \times 10^2\ \mathrm{m\ s^{-1}}.$$

$$\boxed{M \approx 2.4 \times 10^{20}\ \mathrm{kg}}, \qquad \boxed{v_{\mathrm{esc}} \approx 2.8 \times 10^2\ \mathrm{m\ s^{-1}}}$$

Answer 3: Atmospheric Scale Height on a Terrestrial World

Scale height

$$H = \frac{k_B T}{mg},$$

with $k_B = 1.38 \times 10^{-23}$ J K^{-1}, $T = 295$ K, $m = 4.65 \times 10^{-26}$ kg, $g = 6.1$ m s^{-2}.

$$H = \frac{1.38 \times 10^{-23} \times 295}{4.65 \times 10^{-26} \times 6.1} \approx \frac{4.07 \times 10^{-21}}{2.84 \times 10^{-25}} \approx 1.44 \times 10^4 \text{ m} = 14.4 \text{ km}.$$

Pressure as function of altitude z:

$$P(z) = P_0 e^{-z/H}.$$

For $z = 42$ km,

$$\frac{z}{H} = \frac{42}{14.4} \approx 2.92, \qquad P(z) = 0.89\, e^{-2.92} \text{ bar} \approx 0.89 \times 0.0537 \approx 4.8 \times 10^{-2} \text{ bar}.$$

$$\boxed{H \approx 1.4 \times 10^4 \text{ m}}, \qquad \boxed{P(42 \text{ km}) \approx 4.8 \times 10^{-2} \text{ bar}}$$

Answer 4: Solar Insolation and Equilibrium Temperature

For a rapidly rotating planet with uniform heat redistribution,

$$T_{\rm eq} \approx 279\,(1-A)^{1/4}\,a^{-1/2},$$

with $A = 0.27$, $a = 2.35$ AU.

$$(1-A)^{1/4} = (0.73)^{1/4} \approx 0.93,$$
$$a^{-1/2} = \frac{1}{\sqrt{2.35}} \approx \frac{1}{1.53} \approx 0.65.$$

Thus

$$T_{\rm eq} \approx 279 \times 0.93 \times 0.65 \approx 279 \times 0.605 \approx 1.68 \times 10^2 \text{ K}.$$

$$\boxed{T_{\rm eq} \approx 1.7 \times 10^2 \text{ K}}$$

Answer 5: Rotation and Oblateness of a Giant Planet

Flattening approximation:

$$f \approx \frac{3R^3}{2GMP^2}.$$

Given $R = 6.4 \times 10^7$ m, $M = 1.9 \times 10^{27}$ kg, $P = 8.1$ hr $= 2.916 \times 10^4$ s.

Compute numerator:

$$3R^3 = 3(6.4 \times 10^7)^3 = 3 \times (2.62 \times 10^{23}) \approx 7.86 \times 10^{23}.$$

Denominator:

$$2GMP^2 = 2 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{27} \times (2.916 \times 10^4)^2.$$

$$2GM \approx 2 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{27} \approx 2.53 \times 10^{17},$$
$$P^2 \approx (2.916 \times 10^4)^2 \approx 8.50 \times 10^8,$$
$$2GMP^2 \approx 2.53 \times 10^{17} \times 8.50 \times 10^8 \approx 2.15 \times 10^{26}.$$

Thus

$$f \approx \frac{7.86 \times 10^{23}}{2.15 \times 10^{26}} \approx 3.6 \times 10^{-3}.$$

$$\boxed{f \approx 3.6 \times 10^{-3}}$$

Answer 6: Surface Cratering and Age Estimation

Crater density:

$$N_A = \frac{185}{10^6\ \mathrm{km}^2} = 1.85 \times 10^{-4}\ \mathrm{km}^{-2}.$$

Impact flux $F = 7.5 \times 10^{-15}\ \mathrm{km}^{-2}\mathrm{yr}^{-1}$.

Assuming steady state,

$$\text{age} \approx \frac{N_A}{F} = \frac{1.85 \times 10^{-4}}{7.5 \times 10^{-15}} \approx 2.47 \times 10^{10}\ \mathrm{yr}.$$

$$\boxed{\text{Surface age} \approx 2.5 \times 10^{10}\ \text{years}}$$

Answer 7: Ring Particle Orbit and Resonances

Ring particle period $P_r = 0.62$ d, moon period $P_m = 1.82$ d.

For a $n\!:\!1$ mean-motion resonance, the ratio of periods is

$$\frac{P_m}{P_r} \approx n.$$

Compute:

$$\frac{P_m}{P_r} = \frac{1.82}{0.62} \approx 2.94.$$

This is much closer to 3 than to 2, so the resonance is approximately 3 : 1:

$$\frac{P_m}{P_r} \approx 3 \Rightarrow \frac{n_m}{n_r} \approx \frac{1}{3}.$$

$$\boxed{\text{The ring is near a } 3\!:\!1 \text{ mean-motion resonance with the moon.}}$$

Answer 8: Meteoroid Ablation and Energy Deposition

Mass $m = 180$ kg, speed $v = 17.5$ km s^{-1} $= 1.75 \times 10^4$ m s^{-1}.

Initial kinetic energy:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 180 \times (1.75 \times 10^4)^2 = 90 \times 3.06 \times 10^8 \approx 2.76 \times 10^{10} \text{ J}.$$

Deposited energy (85%):

$$E_{\text{dep}} = 0.85\, E_k \approx 0.85 \times 2.76 \times 10^{10} \approx 2.34 \times 10^{10} \text{ J}.$$

Convert to tons of TNT (1 ton TNT $= 4.184 \times 10^9$ J):

$$\text{TNT} = \frac{E_{\text{dep}}}{4.184 \times 10^9} \approx \frac{2.34 \times 10^{10}}{4.18 \times 10^9} \approx 5.6 \text{ tons}.$$

$$\boxed{E_{\text{dep}} \approx 2.3 \times 10^{10} \text{ J}}, \qquad \boxed{\text{Equivalent} \approx 5.6 \text{ tons TNT}}$$

Answer 9: Cometary Outgassing and Non-Gravitational Acceleration

Non-gravitational acceleration:

$$a_{\rm ng} = 2.6 \times 10^{-7}\ \mathrm{m\ s^{-2}}, \quad M = 3.4 \times 10^{12}\ \mathrm{kg}, \quad u = 620\ \mathrm{m\ s^{-1}}.$$

Thrust from outgassing:

$$T = M a_{\rm ng} = 3.4 \times 10^{12} \times 2.6 \times 10^{-7} \approx 8.84 \times 10^{5}\ \mathrm{N}.$$

Thrust also $T = \dot{M} u$, so

$$\dot{M} = \frac{T}{u} = \frac{8.84 \times 10^{5}}{620} \approx 1.43 \times 10^{3}\ \mathrm{kg\ s^{-1}}.$$

$$\boxed{\dot{M} \approx 1.4 \times 10^{3}\ \mathrm{kg\ s^{-1}}}$$

Answer 10: Protoplanetary Disk and Accretion Timescale

Mass of a planetesimal:

$$M = \frac{4}{3}\pi r^3 \rho,$$

with $r = 7.0$ km $= 7.0 \times 10^3$ m, $\rho = 1400$ kg m^{-3}.

Accretion rate (2D sweep with gravitational focusing):

$$\frac{dM}{dt} \approx \pi r^2 \Sigma_s \, \Omega \, F_g,$$

where $\Sigma_s = 5.2$ kg m^{-2}, $F_g = 18$, and $\Omega = 2\pi/P$ is the orbital frequency.

Given $P = 5.2$ yr,

$$P = 5.2 \times 365.25 \times 86400 \text{ s} \approx 1.64 \times 10^8 \text{ s},$$

$$\Omega = \frac{2\pi}{P} \approx \frac{6.28}{1.64 \times 10^8} \approx 3.83 \times 10^{-8} \text{ s}^{-1}.$$

Mass-doubling timescale:

$$\tau = \frac{M}{dM/dt} = \frac{\frac{4}{3}\pi r^3 \rho}{\pi r^2 \Sigma_s \Omega F_g} = \frac{4 r \rho}{3 \Sigma_s \Omega F_g}.$$

Compute:

$$\tau = \frac{4 \times 7.0 \times 10^3 \times 1400}{3 \times 5.2 \times 3.83 \times 10^{-8} \times 18} \approx \frac{3.92 \times 10^7}{1.08 \times 10^{-5}} \approx 3.6 \times 10^{12} \text{ s}.$$

Convert to years:

$$\tau_{\text{yr}} = \frac{3.6 \times 10^{12}}{3.16 \times 10^7} \approx 1.2 \times 10^5 \text{ yr}.$$

$$\boxed{\tau \approx 3.6 \times 10^{12} \text{ s} \;\approx\; 1.2 \times 10^5 \text{ years}}$$