Circular Motion Answers

Prepared by Salkaro

Constant Angular Velocity

Answer 1 (4)

(a) Angular velocity:

$$L = mr^2\omega = \text{constant} \quad \Rightarrow \quad \omega_2 = \omega_1 \left(\frac{r_1}{r_2}\right)^2$$

(b) Kinetic energy ratio:

$$K = \frac{1}{2}mr^2\omega^2 \quad \Rightarrow \quad \frac{K_2}{K_1} = \frac{r_2^2\omega_2^2}{r_1^2\omega_1^2} = \left(\frac{r_1}{r_2}\right)^2$$

Answer 2 (5)

Equilibrium angle:

$$N\cos\theta = mg, \quad N\sin\theta = m\omega^2 R\sin\theta \quad \Rightarrow \quad \cos\theta = \frac{g}{\omega^2 R}, \quad \theta = \arccos\left(\frac{g}{\omega^2 R}\right)$$

Stability condition:

$$\omega^2 R > g$$

Answer 3 (4)

(a) Normal reaction at top:

$$N = mg - m\frac{v^2}{R}$$

(b) Maximum speed:

$$N = 0 \quad \Rightarrow \quad v_{\max} = \sqrt{gR}$$

Answer 4 (5)

(a) Frictional force magnitude:

$$f = mr\omega^2 \quad \text{(radially inward)}$$

(b) Range of ω:

$$f \leq \mu mg \quad \Rightarrow \quad 0 \leq \omega \leq \sqrt{\frac{\mu g}{r}}$$

Answer 5 (4)

(a) Angular speed:

$$T\cos\theta = mg, \quad T\sin\theta = m\omega^2 L\sin\theta \quad \Rightarrow \quad \omega = \sqrt{\frac{g}{L\cos\theta}}$$

(b) Numerical θ:

$$\cos\theta = \frac{g}{L\omega^2} = \frac{9.8}{1.2\cdot 9} \approx 0.9074 \quad \Rightarrow \quad \theta \approx 25^\circ$$

Answer 6 (5)

(a) Angular velocity:

$$mr\omega^2 = \frac{k}{r^2} \quad \Rightarrow \quad \omega = \sqrt{\frac{k}{mr^3}}$$

(b) Total energy:

$$K = \frac{1}{2}mr^2\omega^2 = \frac{k}{2r}, \quad U = -\frac{k}{r} \quad \Rightarrow \quad E = K + U = -\frac{k}{2r}$$

Answer 7 (5)

(a) Speed at bottom:

$$\frac{1}{2}mv^2 = mgR(1 - \cos\theta) \quad \Rightarrow \quad v_{\text{bottom}} = \sqrt{2gR(1 - \cos\theta)}$$

(b) Minimum height to complete loop:

$$\frac{1}{2}mv_{\text{top}}^2 + mg(2R) = mgh_{\text{min}}, \quad v_{\text{top}}^2 = gR \quad \Rightarrow \quad h_{\text{min}} = 2.5R$$

Answer 8 (4)

(a) Kinetic energy of rod:

$$I = \frac{1}{3}mL^2, \quad K = \frac{1}{2}I\omega^2 = \frac{1}{6}mL^2\omega^2$$

(b) Tension at mid-point:

$$T(x) = \int_x^L \frac{m}{L} s\omega^2 ds = \frac{m\omega^2}{2L}(L^2 - x^2) \quad \Rightarrow \quad T_{\text{mid}} = T(L/2) = \frac{3}{8}mL\omega^2$$

Answer 9 (5)

(a) Kinetic energy:

$$L = mr^2\omega \quad \Rightarrow \quad \omega = \frac{L}{mr^2}, \quad K = \frac{1}{2}m(r\omega)^2 = \frac{L^2}{2mr^2}$$

(b) Work done by tension:

$$W = \Delta K = \frac{L^2}{2m}\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)$$

Answer 10 (4)

(a) Angular velocity:

$$N_{\text{top}} = mg - m\frac{v^2}{R} = 0.8mg, \quad N_{\text{bottom}} = mg + m\frac{v^2}{R} = 1.2mg$$

$$v^2 = 0.2gR \quad \Rightarrow \quad \omega = \frac{v}{R} = \sqrt{\frac{0.2g}{R}} \approx 0.404 \text{ rad/s}$$

(b) Period:

$$T = \frac{2\pi}{\omega} \approx 15.6 \text{ s}$$