Determining the Charge on an Electron Using Millikan's Oil Drop Experiment

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PHYSICS HL, Electricity and Magnetism

Author Note

Using different methods to determine the charge on an electron using Millikan's oil drop experiment. While exploring different aspects and properties of magnetic fields and electrons

The Electric Charge

*Elementary charge (*or fundamental charge*)* is the electrical charge possessed by a single electron. It is also known to be the smallest of the charged objects – on a Nanoscale[1].

According to the *"CODATA Value, The NIST Reference on Constants, Units, and Uncertainty. US National Institute of Standards and Technology"* The charge possessed by a single electron – to which other charges are multiples to equates to: $\mathbf{1.602\,176\,6208 \times 10^{-19}}$ **C** (Coulombs)[2]. There are many ways to achieve this seemingly arbitrary and enigmatic value. This experiment will take into account, and refer to the *ways* [of achieving the value] from an experimental perspective.

Historical Groundwork

Robert Millikan (from this point fourth referred to as 'Millikan') and *Harvey Fletcher* (from this point fourth referred to as 'Fletcher') were the pioneers of an ingenious experiment which led to this [charge of an electron] breakthrough. In large, the experiment involved measuring the force on oil droplets inside a glass chamber, between two electrodes.

At that point in time the existence of subatomic particles was widely disputed, despite electrons (or as they were called then: ***'corpuscles'***) already been discovered as negatively charged particles by J.J Thompson through his *Plum Pudding Model.*

Figure 1[3] shows the apparatus that was originally used by Millikan and Fletcher. The basis for the experiment lied in observing falling oil droplets within an electric field which was induced through the two electrodes between which the oil drop was allowed to fall.

By bending the electron in a charged field, Millikan and investigators were able to deduce that the electron was negatively charged and that that the mass to charge ratio for all electrons was the same (and about 1700 times smaller than that of an ionized hydrogen atom). By measuring the charge on clouds of water droplets, Millikan and others were able to achieve a crude value for the charge of electrons in the selected cloud, i.e. 10^{-19} C (an approx. value).

Later they set out find the charge on an individual electron, and that is when Millikan and others started experimenting with oil droplets, and came to achieve a highly precise value for the charge on an electron. One of the biggest factors which led to this success was the temporal duration of oil droplets in the electric field, which was much larger than water, since, water would quickly evaporate. This enabled them to study the falling oil droplets for significantly larger time periods. Enabling them to draw sounder conclusions.

Research Question

How does the derived charge on an oil droplet help determine the charge on an electron by varying voltages and finding the value at which an oil drop would remain stationary between two charged plates?

** The methodology for the experiment is divided into two parts and each explanation is a part of the method. Once a step is complete its relevant explanation with calculations is included just below that step. Numbers steps have not been included to conserve the flow and coherency of the exploration.

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[1] Villanueva, John Carl. "Charge of Electron." *Universe Today*, Universe Today, 25 Dec. 2015, www.universetoday.com/38394/charge-of-electron/.

[2] Mohr, Peter J., et al. "CODATA Recommended Values of the Fundamental Physical Constants: 2006." National Institute of Standards and Technology, 6 June 2008.

Research Rational

The aim of my investigation is to determine the charge on an electron using Millikan's original oil drop experiment, only this time using a simulation. There are two ways Millikan's oil drop experiment simulation can help determine the charge on an electron: measuring the voltage at which the oil drop is stationary and measuring the terminal velocity of the oil drop at a particular voltage – kept constant. The complete forthcoming investigation is a step-by-step walkthrough of the procedure where each step is accompanied by it's details right after it.

The reason why I chose a simulation for this experiment was due to the experiment requiring equipment not available to me. Also, such an experiment would require extensive calibration and fine tuning which is not possible in my context. Furthermore, a simulation would provide me with the required accuracy, thus, enabling me to find the precise value of the charge on an electron.

Introduction to Millikan's Oil Drop Experiment

Figure 2[4] depicts Millikan's oil drop experiment setup.

At the heart of the setup are two oppositely charged metal plates which create a magnetic field when the power source is switched on. An atomizer is used to spray a fine layer of oil droplets on the positively charged metal plate, which has a pinhole cut through it to allow a meniscal number of oil drops to pass through. Once the oil droplets are between the charged metal plates an X-ray source emits radiation which ionizes the oil droplets. Once the oil droplets are ionized, they can be controlled by the voltage across the metal plates, therefore, enabling us to alter their speed. If sufficient voltage was provided, the oil drops could also start moving upwards.

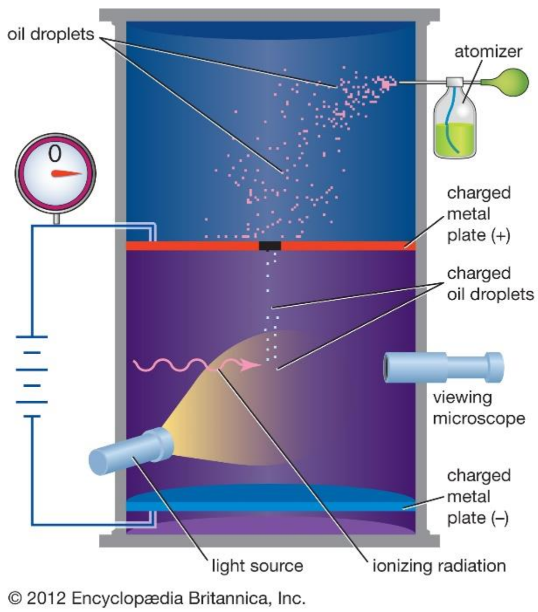

Figure 1: Millikan's Apparatus

Contextually, when an oil droplet is falling between the two plates without any electric field, due to constant velocity, the weight of the oil droplet would be equal to the upthrust and the air resistance (forces acting on the oil droplet). *Figure 3*[5] shows the forces acting on an oil droplet without an electric field. The buoyant force is also called the upthrust and the drag force resembles the air resistance.

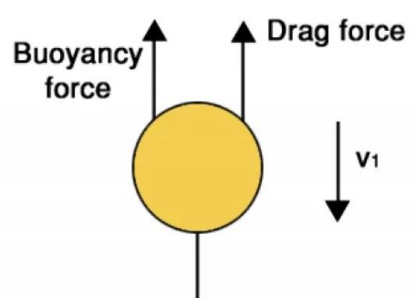

Figure 2: Forces on a free-falling oil drop

With the introduction of an electric field the oil drop would begin moving upwards – this would cause the drag force (aka the air resistance) to reverse in the downward direction. Image on the forthcoming page.

[4] "Millikan's Oil Drop Experiment." *Encyclopædia Britannica*, Encyclopædia Britannica, www.britannica.com/science/Millikan-oil-drop-experiment/media/382908/19493.

[5] "13617382_f1024." *Owlcation*, Owlcation, usercontent1.hubstatic.com/13617382_f1024.jpg.

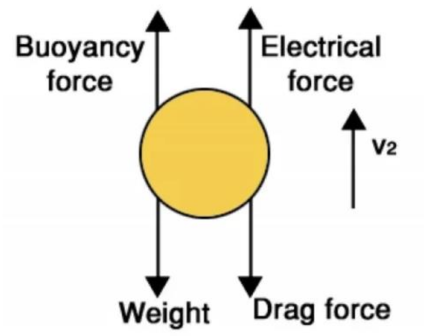

Figure 3: Forces acting on an oil droplet influenced by an electric field

Figure 4[6] shows the reversed drag force and the direction of the applied electrical field. Millikan then went on to find the voltage at which the oil droplet would be suspended in air between the two plates. In this scenario, the upward forces – buoyant and electric – would cancel out the downward forces – weight and drag.

The voltage required to suspend the droplet is then used to calculate the charge on the oil droplet. Millikan, after repeating this experiment several times, noticed that the charges on the individual oil droplets were always a multiple of the lowest possible charge: 1.602×10^{-19}. This lowest value being considered as the charge of an electron.

The discovery of the charge on an electron was groundbreaking since this was then used to identify the mass of an individual electron using the charge of mass ratio that was established by J.J Thompson in late 19th century.

Determining the best simulation

While finding simulations for Millikan's Oil Drop experiment, I stumbled across many samples. My idea of a good simulation consisted of a setup which provided me with an experience which most closely resembled the original experiment and was similar to hands-on. Certain simulations which I came across were very simplistic and did now allow scope for independent manipulation of certain factors. For instance, the only changeable factor in such simulations was the voltage between the two plates. I finally came across a simulation by the University of Boniface which resembled most closely to the real apparatus. This simulation gave me the ability to manipulate the distance between the two plates, gave me information on the Air and Oil density as well as the distance between the two plates. It can be, thus, said that this simulation fit best to my criteria.

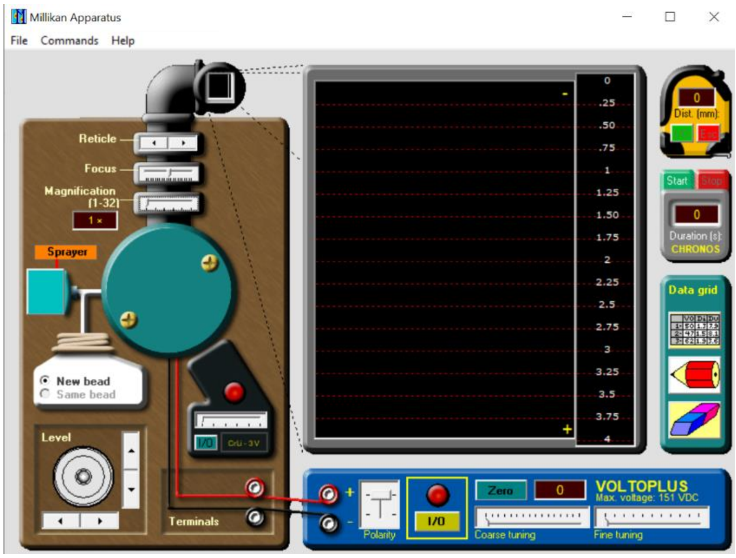

Figure 4: Image of the simulation by University of Boniface

Figure 5[7] is an image of the simulation. As it is visible it gives all the apparatus required for the experiment such that many factors can be manipulated and adjusted.

In brief, the first method consists of finding the voltage at which different oil drops remain stationary and using that voltage finding the charge on the electrons. Once this charge is found investigating weather it really is a multiple of the basic unit of the elementary charge.

[6] "13617382_f1024." *Owlcation*, Owlcation, usercontent1.hubstatic.com/13617382_f1024.jpg.

[7] https://ustboniface.ca/physique/simulateurs_maison-appareil_de_millikan

Method One: Elementary charge multiple

Recreating Millikan's original oil drop experiment.

Figure 6 shows the forces acting on the oil drop with the influence of an electric field on it. In Millikan's original oil drop experiment he calibrated the voltage until the oil drop was stationary in the chamber – i.e. all forces acting on the oil drop were cancelling each other out:

$$Buoyant\ Force + Electrical\ Force\ = Weight + Drag\ Force$$

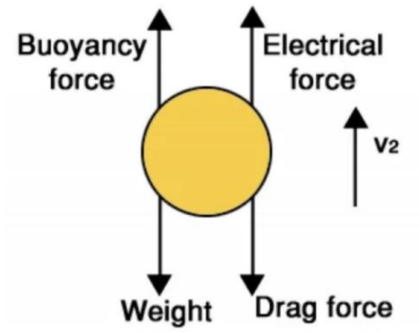

Figure 5: Forces acting on oil drop in electric field

The $weight$ of the droplet is $\boldsymbol{W = mg}$, the values in which could be found by the density of the oil droplet:

$$Density\ = \frac{mass}{volume}$$

$$\Rightarrow mass\ (m) = density\ \times volume$$

$$\Rightarrow m = \frac{4}{3}\pi r^3 \rho_{oil}$$

Hence, the weight of the oil droplet, using the equations derived above, is:

$$W = \frac{4}{3}\pi r^3 \rho_{oil} g$$

Such that, $\boldsymbol{\rho_{oil}}$ is the density of the oil and $\boldsymbol{g}$ is the gravity that is acting on the oil droplet and $\boldsymbol{r}$ is the radius of the oil droplet.

Since the drop is moving in air, we have an upthrust, or buoyant force, acting on the oil droplet and since the oil droplet is in air, we will consider $\boldsymbol{\rho_{air}}$ instead of $\boldsymbol{\rho_{oil}}$. Therefore, making the equation:

$$B_f = \frac{4}{3}\pi r^3 \rho_{air} g$$

The electric force is determined by:

$$E_f = qE$$

Where $\boldsymbol{E}$ can be replaced by $\boldsymbol{\frac{V}{d}}$[8]**.** Such that $\boldsymbol{V}$ is the voltage across the plates and $\boldsymbol{d}$ is the distance between the two plates. The drag force – also called the frictional force – can be found out using Stokes' law which determines the force acting on small spherical objects moving through viscous fluids. Stokes' law is stated in equation as[9]:

$$D_f = 6\pi\eta r v$$

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[8] Tsokos, K. A. *Physics for the IB Diploma*. Cambridge University Press, 2015.

[9] "Dropping the Ball (Slowly)." *Stokes' Law,* galileo.phys.virginia.edu/classes/152.mf1i.spring02/Stokes_Law.htm.

In the Stokes' law equation, η is the viscosity of the fluid, r is the radius of the oil drop, v is velocity relative to the object – i.e. because it is the interaction between the fluid and the particle. The SI unit for D_f is in *newtons* $(kg\ m\ s^{-2})$.

The program I used provided me with the properties of the simulation, properties which can be input into the equation derived below:

- Density of oil $(\boldsymbol{\rho_{oil}}) = 1050\ kg\ m^{-3}$
- Density of air $(\boldsymbol{\rho_{air}}) \approx 1.2\ kg\ m^{-3}$
- Radius of droplet $(\boldsymbol{r}) = 4.57 \times 10^{-7}\ m$
- Distance between the plates $(\boldsymbol{d}) = 4 \times 10^{-3}\ m$

Since the oil drop is suspended in air and has a velocity of zero, the drag force on the oil droplet in an electric field will be zero. Therefore, combining all the equations we get:

$$\frac{4}{3}\pi r^3 \rho_{air} g + qE = \frac{4}{3}\pi r^3 \rho_{oil} g$$

Making q the subject of the equation:

$$\frac{4}{3}\pi r^3 \rho_{air} g + qE = \frac{4}{3}\pi r^3 \rho_{oil} g$$

$$\Rightarrow \frac{4}{3}\pi r^3 \rho_{oil} g - \frac{4}{3}\pi r^3 \rho_{air} g = qE$$

$$\Rightarrow \frac{4}{3}\pi r^3 (\rho_{oil} - \rho_{air}) g = qE$$

$$\Rightarrow \frac{4}{3E}\pi r^3 (\rho_{oil} - \rho_{air}) g = q$$

Substituting E as $\frac{V}{d}$:

$$q = \frac{4}{3E}\pi r^3 (\rho_{oil} - \rho_{air}) g$$

$$\Rightarrow q = \frac{4d}{3V}\pi r^3 (\rho_{oil} - \rho_{air}) g$$

Collection of raw data

My raw data will consist of 10 readings for different beads. For all of these beads I've found the voltage at which they are absolutely stationary, using the magnifying tool given in the simulation. Before I conducted the experiment, for a more real hands-on, I adjusted the reticle level as well as the scaling lines in the simulation. I used the sprayer to spray a drop of oil onto the main area – the reticle – where I used the simulation's coarse tuning and fine-tuning dials to precisely adjust the voltage at which the oil droplet was stationary on the provided scale. To achieve more precision, I used the simulation magnification tool to make sure the oil drop was completely stationary. The data collected could then be then saved into a grid from where I exported it onto a proper tabular format giving me all the readings I need to process. I took 10 readings to give me enough data to come to a proper conclusion and identify a trend in the readings.

Raw Data

Trial Number	Voltage (± 0.01) V
1	25.73
2	11.43
3	51.40
4	102.90
5	51.40
6	34.30
7	20.57
8	17.14
9	10.28
10	34.33

Processed data

With the formula for charge derived, using the raw data I had, I calculated the charge for each trial. The uncertainty for the voltage is not accounted while calculating the charge since it becomes negligible once the operations are performed. Substituting the voltage for each trial into the derived charge equation would yield the charge on each oil droplet.

Sample calculation for trial number 1:

$$q = \frac{4d}{3V}\pi r^3(\rho_{oil} - \rho_{air})g$$

$$q = \frac{4 \times 4 \times 10^{-3}}{3 \times 25.73} \times \pi \times (4.57 \times 10^{-7})^3 \times (1050 \ - 1.2) \times 9.8 = \ 6.39 \times 10^{-19}$$

The table below shows the same calculation for all the trials.

Trial	Voltage (± 0.01) V	Charge (10^{-19}) C
1	25.73	6.39
2	11.43	1.44
3	51.40	3.20
4	102.90	1.60
5	51.40	3.20
6	34.30	4.80
7	20.57	8.00
8	17.14	9.59
9	10.28	1.60
10	34.33	4.79

Variables

There are certain variables which were kept constant throughout the whole calculation process: the density of air, set to $1.2\ kg\ m^{-3}$; density of oil, set to $1050\ kg\ m^{-3}$; radius of the oil droplet, set to 4.57×10^{-7} and the distance between the two plates set to $4 \times 10^{-3}\ m$. The simulation helped the radius of the oil droplet remain constant since in real hands-on that would be very difficult to do. All of the above-mentioned factors were kept constant by the simulation itself, hence allowing no scope for error.

The independent variable for this experiment was the voltage that was carefully calibrated for each trial. The difference is that the final value at which the simulation shows the oil drop to be stationary is arbitrary, meaning it cannot be predetermined. This omits the scope for any ranges that can be considered when measuring the voltage at which the oil drop appears stationary. The voltage was manipulated at two stages, coarse turning allowed the gross voltage to be set, i.e. the voltage scale adjusted to tens, fine tuning enabled the voltage to be adjusted to the nearest tenths. All of these tools were provided in the simulation itself.

The dependent variable which could be identified is the times a charge is a multiple of the lowest possible elementary charge. As the voltage changes the times a charge is a multiple of the elementary charge also changes. This relationship between the charge calculated using the equation and the multiple can be used to identify the lowest possible value for the elementary charge – which is the charge on a single electron.

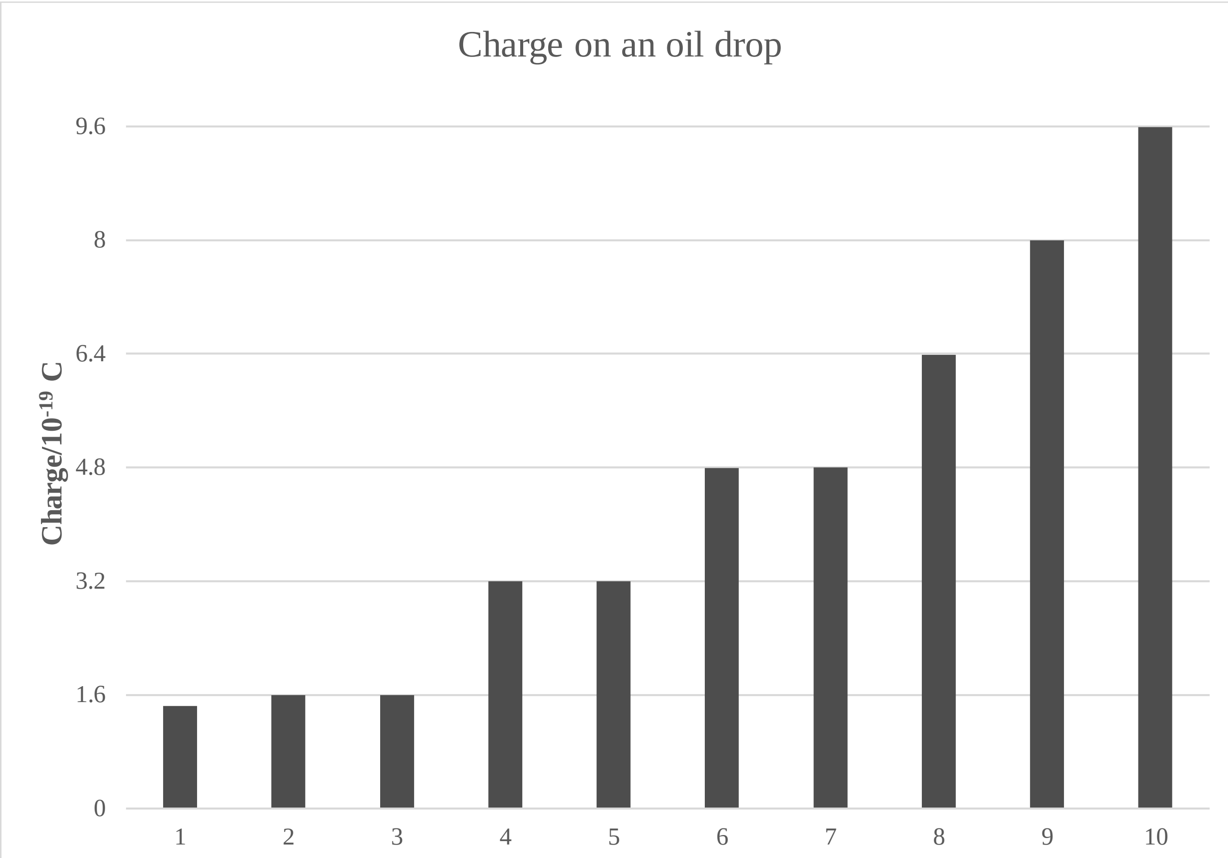
Sorted data

Voltage (± 0.01) V	**Charges (10^{-19}) C**
11.43	1.44
102.9	1.60
10.28	1.60
51.4	3.20
51.4	3.20
34.33	4.79
34.30	4.80
25.73	6.39
20.57	8.00
17.14	9.59

The table above shows a sorted, from lowest to highest, list of charges and their voltages.

Safety, ethical, and environmental issues are not applicable to my investigation due to contact of the simulation and with the simulation is strictly limited to the virtual space. Ethical and environmental concerns need not be addressed.

By graphing these trials and charges on a bar graph a distinct stair pattern appears, which upon calculation has a factor of 1.6, approximately the magnitude of the charge on an electron.

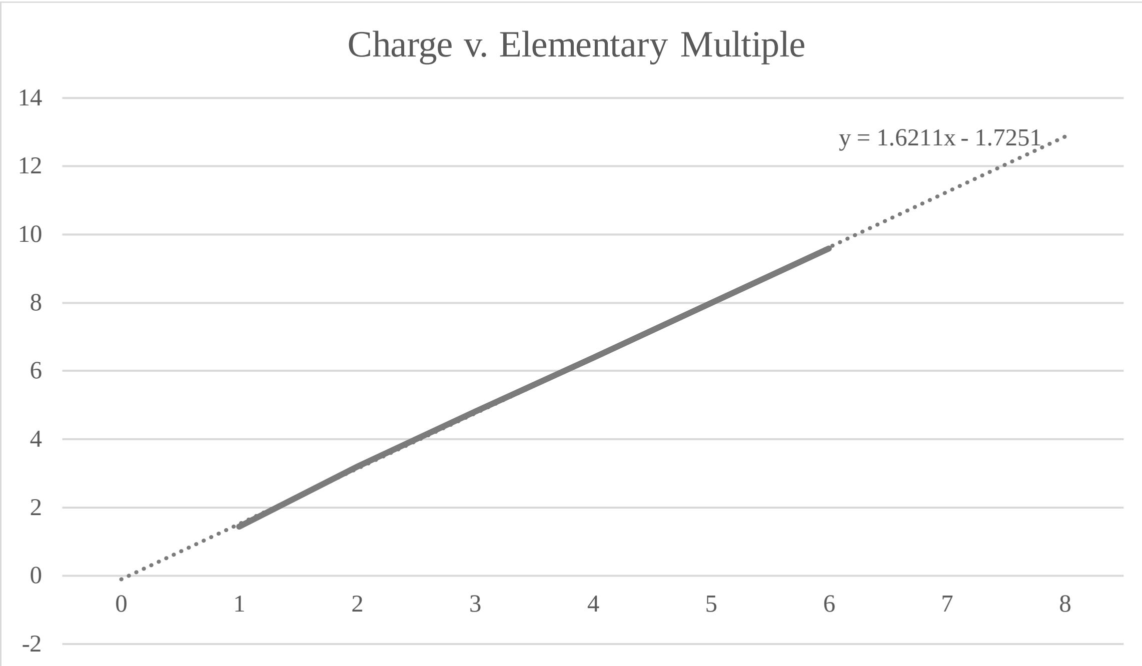


Each trial appears to be a multiple of the lowest value of 1.6×10^{-19} C. Since they appear to be multiples, graphing the experimental charges vs. the magnitude of the multiple could give us the elementary charge, a value which is more precise. This could be done using the equation.

$$EXP_C = multiple\ of\ e\ \times elementary\ charge$$
$$multiple\ of\ e = \frac{EXP_C}{elementary\ charge}$$

Voltage (± 0.01) V	**Charge (10^{-19}) C**	**Multiple of elementary charge**
11.43	1.44	1
102.9	1.60	1
10.28	1.60	1
51.4	3.20	2
51.4	3.20	2
34.33	4.79	3
34.30	4.80	3
25.73	6.39	4
20.57	8.00	5
17.14	9.59	6

Using the values for the charge and the multiples of e we could create a linear graph which's gradient would indicate the value for the elementary charge. This value should be precise to a large extent.



According to the gradient of the graph, the elementary charge on a electron is coming to approximately 1.6211×10^{-19} C.

An Alternative Method

There is another approach to finding the elementary charge using Millikan's oil drop simulation. At a certain voltage the oil beads start to rise up and attain terminal velocity, by comparing this terminal velocity with that of a free fall of oil beads with no electric field one could determine the charge on the oil droplet and therefore determine the charge on an electron. Though, certain equations, different from the previously derived ones, will be required to determine this:

Constants:

- Density of oil ($\boldsymbol{\rho_{oil}}$) $= 1050\ kg\ m^{-3}$
- Density of air ($\boldsymbol{\rho_{air}}$) $\approx 1.2\ kg\ m^{-3}$
- Radius of droplet ($\boldsymbol{r}$) $= 4.57 \times 10^{-7}\ m$
- Distance between the plates ($\boldsymbol{d}$) $= 4 \times 10^{-3}\ m$

Fundamental equations:

- Weight of oil drop: $W = \frac{4}{3}\pi r^3 \rho_{oil} g$
- Upthrust: $B_f = \frac{4}{3}\pi r^3 \rho_{air} g$
- Drag force: $D_f = 6\pi\eta r v$
- Electric force: $E_f = qE$

Since we are dealing with free fall and terminal velocity the forces acting on the oil drop will also cancel out under an electric field and as well as during free fall. Therefore, the equation for the forces acting on the oil drop can be put as:

$$Weight\ (W) + Drag\ \left(D_f\right) = Upthrust\ \left(B_f\right) + Electric\ \left(E_f\right)$$

$$\frac{4}{3}\pi r^3 \rho_{oil} g + \ 6\pi\eta r v_f \ = \ \frac{4}{3}\pi r^3 \rho_{air} g + \ qE$$

Since we have no information about the viscosity of the air surrounding the oil drop, we can calculate it using the equations – derived from stokes law[10]:

$$\eta = \frac{2}{9} \cdot \frac{r^2 g(\rho_{oil} - \rho_{air})}{v_t}$$

In the equation above, v_t is the velocity under free fall. Substituting this equation for η in the equation for the forces acting on an oil drop, we could determine the charge on the oil droplet by comparing the value for velocity under free fall (v_t) and v which is the terminal velocity in an electric field.

Substituting η in the original equation:

$$\frac{4}{3}\pi r^3 \rho_{oil} g + \ 6\pi \left(\frac{2}{9} \cdot \frac{r^2 g(\rho_{oil} - \rho_{air})}{v_t}\right) r v_f \ = \ \frac{4}{3}\pi r^3 \rho_{air} g + \ qE$$

Solving for q:

$$\Rightarrow \ qE = \ \frac{4}{3}\pi r^3 \rho_{oil} g + \ 6\pi \left(\frac{2}{9} \cdot \frac{r^2 g(\rho_{oil} - \rho_{air})}{v_t}\right) r v_f - \ \frac{4}{3}\pi r^3 \rho_{air} g$$

$$\Rightarrow \ qE = \ \frac{4}{3}\pi r^3 (\rho_{oil} - \rho_{air}) g + 6\pi \left(\frac{2}{9} \cdot \frac{r^2 g(\rho_{oil} - \rho_{air})}{v_t}\right) r v_f$$

$$\Rightarrow \ qE = \ \frac{4}{3}\pi r^3 (\rho_{oil} - \rho_{air}) g + \ \frac{12\pi r^3 (\rho_{oil} - \rho_{air}) g v_f}{9 v_t}$$

$$\Rightarrow \ qE = \ \frac{4}{3}\pi r^3 (\rho_{oil} - \rho_{air}) g + \ \frac{4\pi r^3 (\rho_{oil} - \rho_{air}) g v_f}{3 v_t}$$

$$\Rightarrow \ qE = \ \frac{4}{3}\pi r^3 (\rho_{oil} - \rho_{air}) g + \frac{4}{3}\pi r^3 (\rho_{oil} - \rho_{air}) g \cdot \frac{v_f}{v_t}$$

$$\Rightarrow \ qE = \ \frac{4}{3}\pi r^3 (\rho_{oil} - \rho_{air}) g \ \cdot \left(1 + \ \frac{v_f}{v_t}\right)$$

$$\Rightarrow \ q = \ \frac{4}{3E}\pi r^3 (\rho_{oil} - \rho_{air}) g \ \cdot \left(1 + \ \frac{v_f}{v_t}\right)$$

$$\Rightarrow \ q = \ \frac{4d}{3V}\pi r^3 (\rho_{oil} - \rho_{air}) g \ \cdot \left(1 + \ \frac{v_f}{v_t}\right) \ldots Substituted\ E\ as\ \frac{V}{d}$$

Therefore, the final equation to determine the charge on a falling oil droplet with respect to the free fall velocity (v_f) and the terminal velocity of each individual droplet (v_t) is:

$$q = \frac{4d}{3V}\pi r^3(\rho_{oil} - \rho_{air})g \cdot \left(1 + \frac{v_f}{v_t}\right)$$

The next step is to determine the velocity of free fall using the simulation for different oil beads. Their average would give us the average velocity of free fall. This average can then be used for individual calculations while determining charge on oil reaching their terminal velocity under the influence of an electric field.

Raw data

To find v_f I sprayed an oil bead on the screen and measured it's time on the screen and its distance using the digital distance measuring functionality provided in the simulation. Using this I determined the free-falling velocity for five oil droplets. The results of this were then averaged and the value of v_f determined. In all of the following readings the voltage of the setup was set to zero. All other constants regarding the oil droplets and the chamber conditions persist, as mentioned above.

Trial	Distance ($\pm$ $\mathbf{0.001}$) $\boldsymbol{mm}$	Duration ($\pm$ $\mathbf{0.01}$) $\boldsymbol{s}$
1	1.583	1.80
2	1.078	1.21
3	1.105	1.24
4	1.213	1.35
5	1.141	1.28

Processed data

To calculate the velocity from the data above, I used the equation: $Velocity = \frac{Distance}{Duration}$. I also calculated the error for each reading, though very meniscal. One sample error calculation is shows below. All the other values, for the free fall velocity and the error were automatically calculated by inputting the data above in a spreadsheet software and applying a formula on every cell.

Trial	Distance ($\pm$ $\mathbf{0.001}$) $\boldsymbol{mm}$	Duration ($\pm$ $\mathbf{0.01}$) $\boldsymbol{s}$	Velocity ($\boldsymbol{mm\ s^{-1}}$)	$\boldsymbol{\Delta Velocity}$ ($\pm$)
1	1.583	1.80	0.88	0.0062
2	1.078	1.21	0.89	0.0092
3	1.105	1.24	0.89	0.0090
4	1.213	1.35	0.90	0.0082
5	1.141	1.28	0.89	0.0087

The uncertainty for each trial velocity (ΔV) = $\frac{\Delta Distance}{Distance} + \frac{\Delta Duration}{Duration}$. This is basically dividing the constant uncertainty given and dividing it with each individual trial value. As we can see the calculated uncertainty is extremely small, near negligible. This is the reason why the uncertainty was not calculated for the previous method.

Based on the readings above, the average free fall velocity (v_f) is **0.89** $\boldsymbol{mm\ s^{-1}}$.

The next step is to experimentally find the terminal velocity for oil beads under the influence of a voltage. To do this, I sprayed an oil bead on the screen, one which was going up due to the voltage, and measured its distance covered and the time it took to travel that distance. Using these two values I would be able to calculate the terminal velocity (v_t) for a voltage applied. This can then be inserted into the equation for each individual v_t and the charge for each individual oil droplet can be calculated.

For this part of the experiment I kept the voltage constant at **36 V** which is also the average voltage at which most of the droplets in the previous method were stationary. This number would ensure that I get a range of values for the oil beads and their terminal velocity. I am not going to calculate the uncertainties for the values due to them, largely, being negligible and do not contribute to any future calculations.

Raw data

Trial	Distance ($\pm$ 0.001) mm	Duration ($\pm$ 0.01) s
1	1.243	1.79
2	1.382	0.87
3	1.015	0.64
4	1.326	1.92
5	0.965	1.40
6	0.926	0.97
7	1.472	2.14
8	1.033	1.07
9	1.061	0.81
10	1.470	1.11

Processed data

Using the same equation as used previously to calculate the velocity: $v = \frac{Distance}{Duration}$ the velocity was calculated for each individual reading. This will be the v_t for each individual oil bead trial. Once again, the value for the uncertainty for the distance and duration is not included.

Trial	Distance ($\pm$ 0.001) mm	Duration ($\pm$ 0.01) s	Velocity ($mm\ s^{-1}$)
1	1.243	1.79	0.694
2	1.382	0.87	1.589
3	1.015	0.64	1.586
4	1.326	1.92	0.691
5	0.965	1.40	0.689
6	0.926	0.97	0.955
7	1.472	2.14	0.688
8	1.033	1.07	0.965
9	1.061	0.81	1.310
10	1.470	1.11	1.324

On the forthcoming page I have calculated the charge for each trial above. The free fall velocity has been taken as a constant. One calculation has been done for one trial. In the same table the beads multiple of the elementary charge is also stated.

Velocity ($mm\ s^{-1}$)	Charge (C)	Multiple of elementary charge
0.694	$\cancel{1.402 \times 10^{-18}}$	
1.589	7.123×10^{-19}	4.45
1.586	7.127×10^{-19}	4.45
0.691	$\cancel{1.044 \times 10^{-18}}$	
0.689	$\cancel{1.046 \times 10^{-18}}$	
0.955	8.820×10^{-19}	5.51
0.688	$\cancel{1.047 \times 10^{-18}}$	
0.965	8.776×10^{-19}	5.50
1.310	7.667×10^{-19}	4.80
1.324	7.634×10^{-19}	4.80

Constants:

- Density of oil ($\boldsymbol{\rho_{oil}}$) $= 1050\ kg\ m^{-3}$
- Density of air ($\boldsymbol{\rho_{air}}$) $\approx 1.2\ kg\ m^{-3}$
- Radius of droplet ($\boldsymbol{r}$) $= 4.57 \times 10^{-7}\ m$
- Distance between the plates ($\boldsymbol{d}$) $= 4 \times 10^{-3}\ m$

Equation:

$$q = \frac{4d}{3V}\pi r^3(\rho_{oil} - \rho_{air})g\ \cdot\left(1 + \frac{v_f}{v_t}\right)$$

The results from this method show scope for error. The multiples of the elementary charge are in decimals. This scope for error is addressed in the limitations of this simulation later on in the report. Below is the graph for the elementary charge multiple and the charge on the oil droplets to, nevertheless of the inaccurate results, determine the charge on the electron using this method. The quantitative deviation from the original value will give me an error percentage which I can then use to evaluate this method.

It is not fruitful to draw a bar chart to represent the trials and their elementary charge multiples since a large number of them are extensively deviated and would yield no identifiable patters. The same can be said about the line chart which could represent the charges and the multiples on a single plane. Because of the inconsistencies that chart would, also, not be feasible. However, we can still calculate the elementary charge that could be derived from this test data. This can be simply done by calculating the gradient of the graph, which can be easily done without even plotting the data, using the equation:

$$Gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

$$Gradient = \frac{(7.127) - (8.776)}{4.45 - 5.50} = 1.57$$

According to the slope of the data, the elementary charge value is 1.57×10^{-19} C. This value can now be used to determine and evaluate the validity of the second method and its effectiveness in determining the elementary charge.

Limitations

Despite being a digital simulation, there are certain avenues for speculation, avenues which could have impacted the accuracy and the data collection in the experiment.

Method one entailed an aspect of the simulation equipment which could have led way for unaccounted uncertainties. The magnification of the viewing screen was, to some extent, insufficient. Once the oil bead "appeared" to be stationary it was that in some instances it was still moving but at extraordinarily slow speeds. To account for these speeds the simulation must introduce a higher magnification to observe the oil droplets that are truly stationary. This must've had an impact on my noted voltage. Therefore, this can be said as one of the factors which are responsible for the deviation in the final smallest elementary charge value determined from the graph. There was more scope for imprecision in the first method. The fact that the voltage could only be adjusted to two decimal places sacrificed certainty, the values provided for the constants by the simulation, such as the density of oil, air and radius, could have included more significant figures for they would have increased the experiment's precision. In the first method the culmination of all the aforementioned improvements could have resulted in a more precise value for the elementary charge, using the graph.

In conjunction with method one, method two also entailed certain errors. One of the biggest points of flaw was this methods inability to determine the smallest possible elementary charge using the data and elementary charge multiples. There could be certain reasons for this, the voltage that was kept constant was not right to calculate the smallest possible elementary charge, even though it being the average of the voltages from method one. There were other lacunas with this method too, the reaction time to measure the time and distance was a very qualitative factor – it was solely based on human observation which introduces scope from random error in individual readings. Another possible error related to human reaction time is the point where the oil droplet reached its terminal velocity. It must be that when I started the timer the drop may not have reached its terminal velocity. This error may have crept into other readings, speculatively, the ones which had to be discarded due to being clear outliers. Again, an increase in the magnification could help me with this. More precision could be achieved in this experiment with an increase in the significant figures provided for each trial recorded. All the aforementioned errors are likely the reason for the outlier values which were extremely deviated from the value of the elementary charge. It is also worthwhile acknowledging the deviation in the multiples of the elementary charge. The charges on oil drops were, in decimals, multiples of the elementary charge – which should not be the case. It can be deduced from this the precision and the distance-duration measuring system to measure the terminal velocity needs to be better implemented and its utility should be increased.

Conclusion

This simulation gave me a glimpse into Millikan's apparatus. This endeavor was an exploration into the origins of discovering the charge of an electron, it gave me a hands-on experience of operating the setup and managing its different aspects. This experiment also gave me an opportunity to explore the simulation from two different perspectives to approach to the same result. Values for the elementary charge calculated through method 1 and 2 were considerably accurate.

The percent deviation from the original value can be determined for the two values I determined through the different methods implemented above. The accepted value for the elementary charge is $1.602\ 176\ 6208\ \times 10^{-19}$ C. M1 elementary charge value is $\frac{1.6211\times 10^{-19} - 1.602\ 176\ 6208\ \times 10^{-19}}{1.602\ 176\ 6208\ \times 10^{-19}} \times 100 = 1.18\ \%$ deviated from the original value. M2 elementary charge value is $\frac{1.57\times 10^{-19} - 1.602\ 176\ 6208\ \times 10^{-19}}{1.602\ 176\ 6208\ \times 10^{-19}} \times 100 = 2.008\%$ deviated from the accepted value. This shows the uncertainties of the second method and its shortcomings.

There are certain improvements that could be implemented in the design of this experiment to yield better and more accurate results for the elementary charge. In the second method I would like to explore the terminal velocity with a wide variety of voltages to find an optimum voltage at which the terminal velocity achieved by the oil drops will result in a charge which would be a precise multiple of the elementary charge. An extension to this experiment would be to actually perform this experiment with real equipment to actually experience the physics behind the Millikan's oil drop experiment. An improvement to this simulation would be a greater magnification factor. This is very plausible since it is a computer simulation, zooming in on the oil drop while falling would increase the measurement precision by a great deal, therefore, yielding a value much closer to the accepted value relative to the ones achieve in this exploration.

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