

08.01.2019

# Statistical Methods in AI (CSE/ECE 471)

## Lecture-3: Decision Tree Learning

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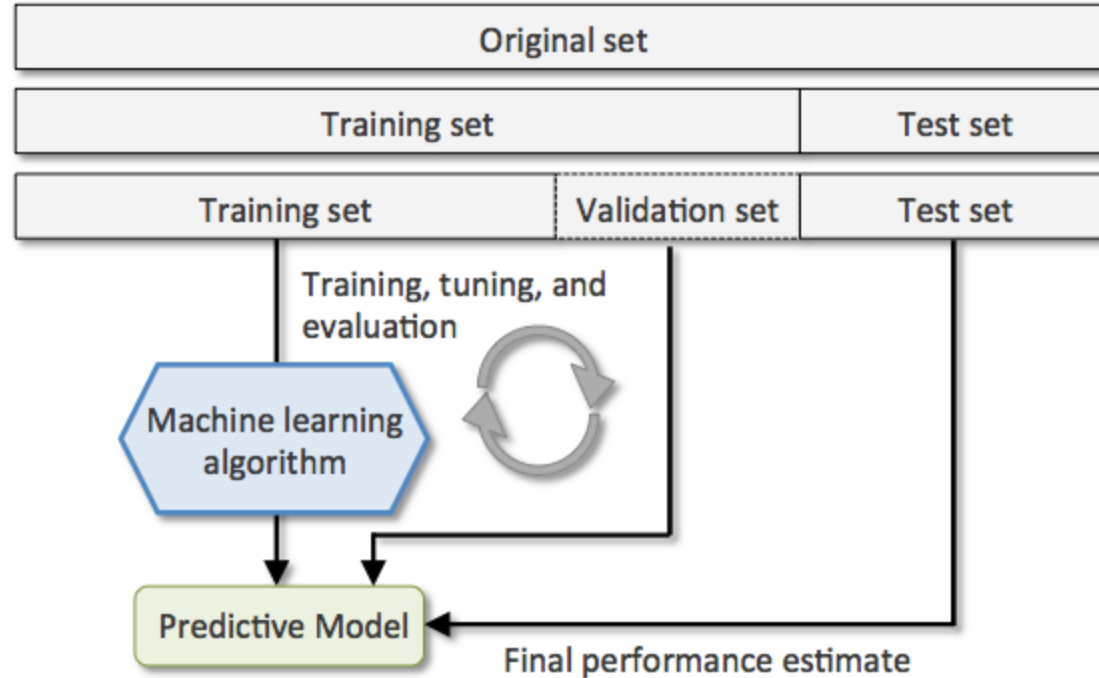
# Announcements

- IMPORTANT: All assignments/projects will need to be in Python.
- This week's tutorial: Probability recap, ML datasets, visualization approaches. **Bring your laptops.**
- Ask questions. Take notes.

# Announcements

- Assignments – Questions involving equations/mathematical derivation
  - Write up in latex [overleaf.com] → submit
  - Write neatly on paper → scan as photo/pdf [camscanner] → submit
- TA office hours, locations have been announced.

# The Train-Validation-Test paradigm



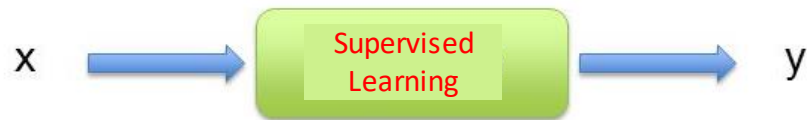
# Supervised Learning

```
graph TD; A[Supervised Learning] --> B[Classification]; A --> C[Regression]; A --> D[Reinforcement Learning];
```

Classification

Regression

Reinforcement  
Learning



## Classification

Binary

$\{0,1\}$

Multi-class

1-of-K

Multi-label

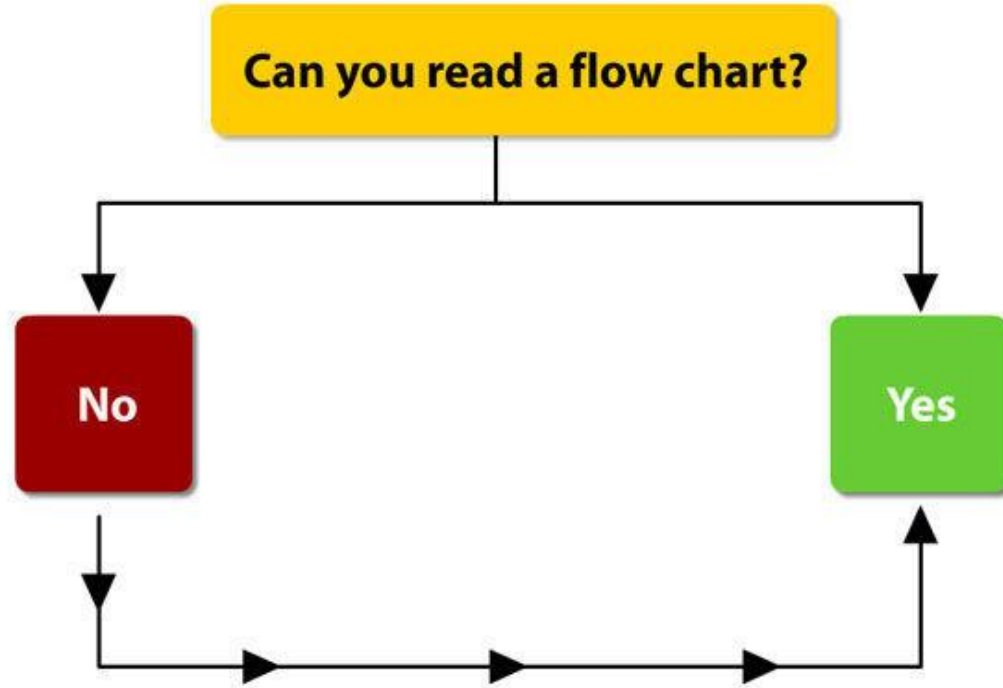
n-of-K

Structure

E.g. graph/sequence



# Flowcharts



# Trees

□ Node

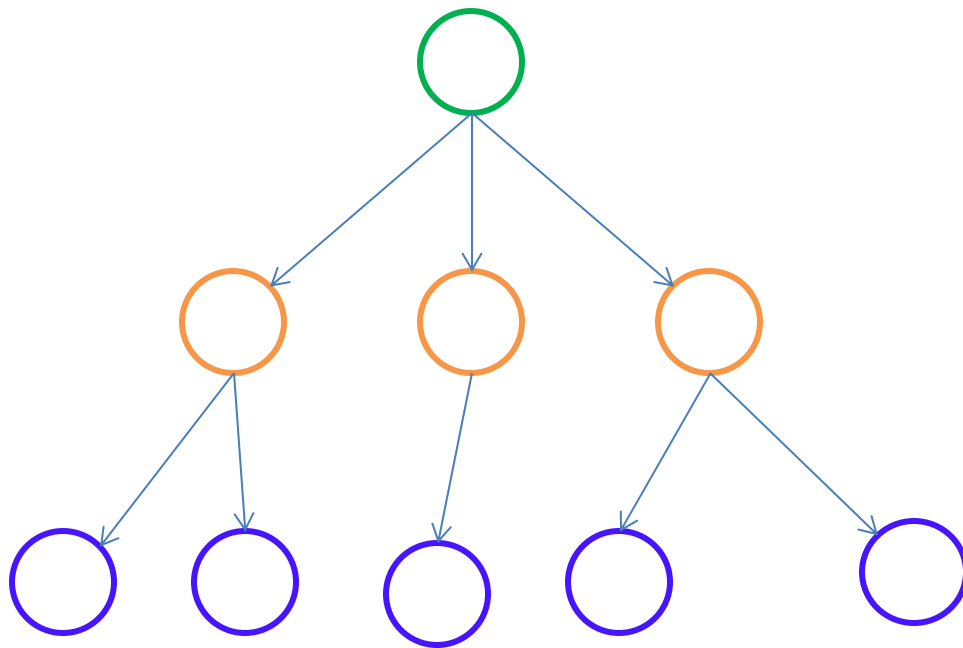
□ Root

□ Leaf

□ Edge/Branch

□ Path

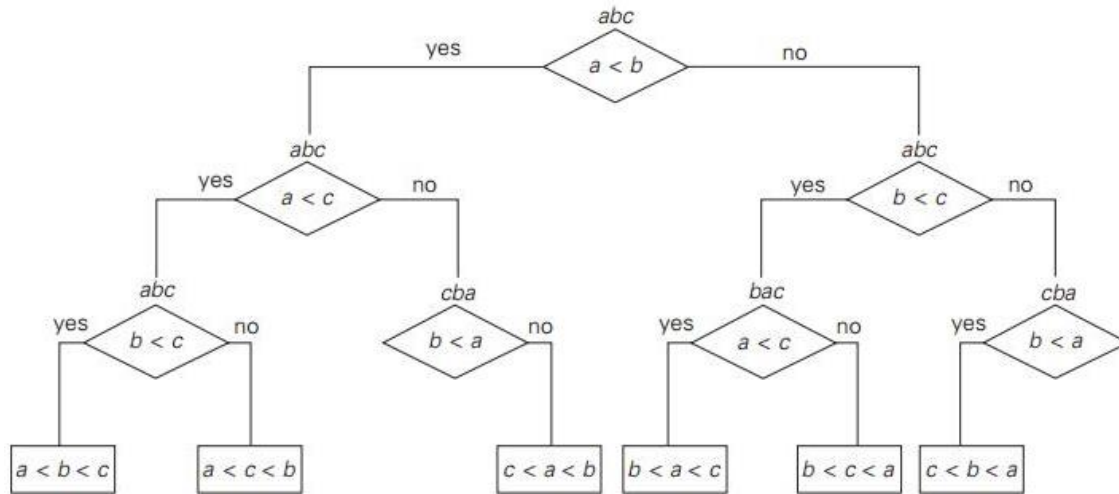
□ Depth



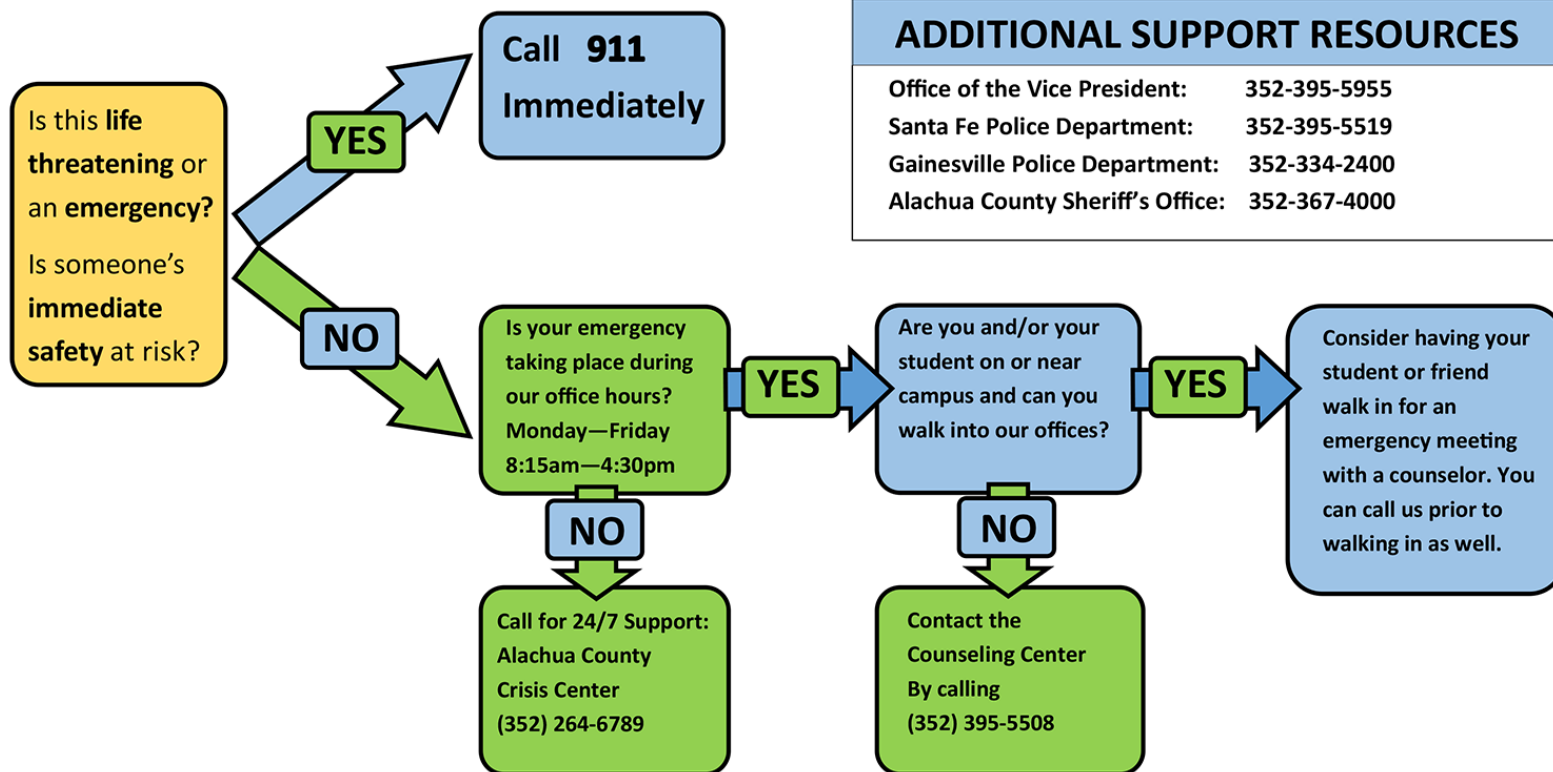


# if/then

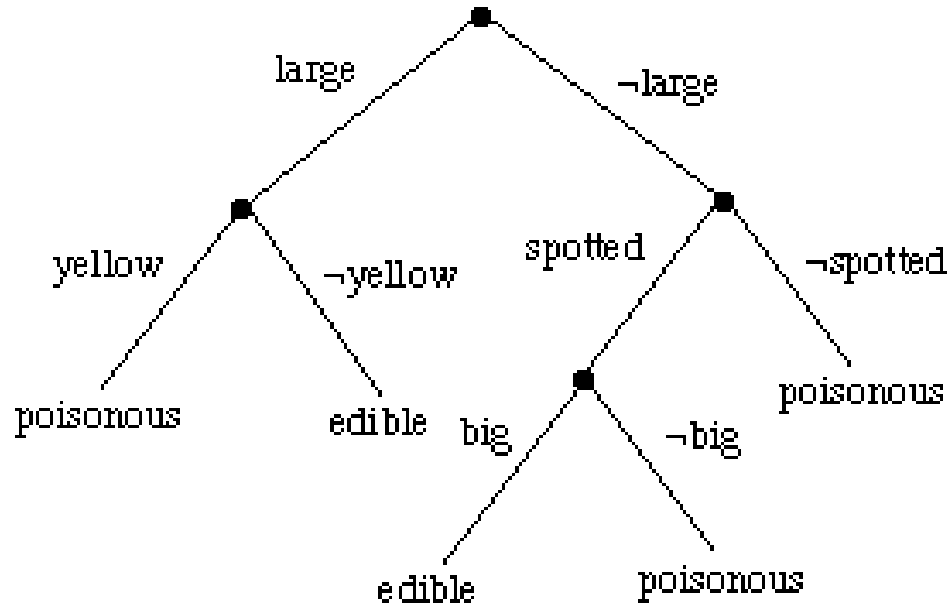
## if/else/then



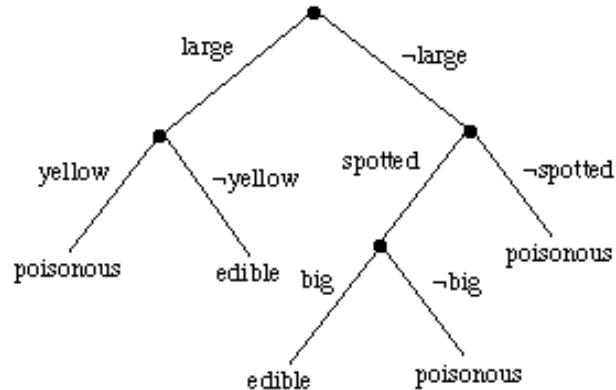
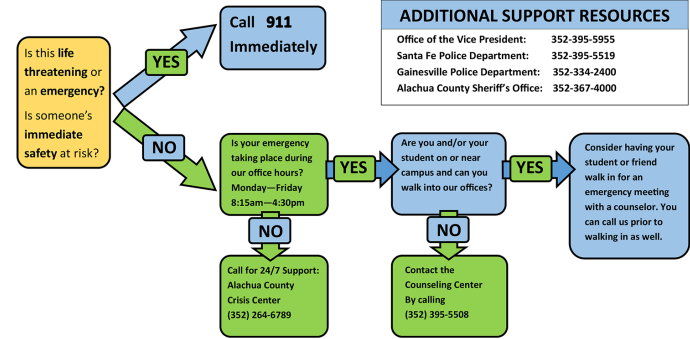
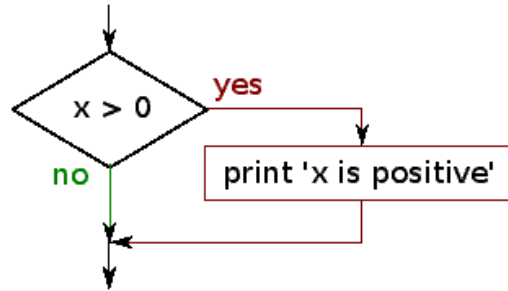
# Emergency Response



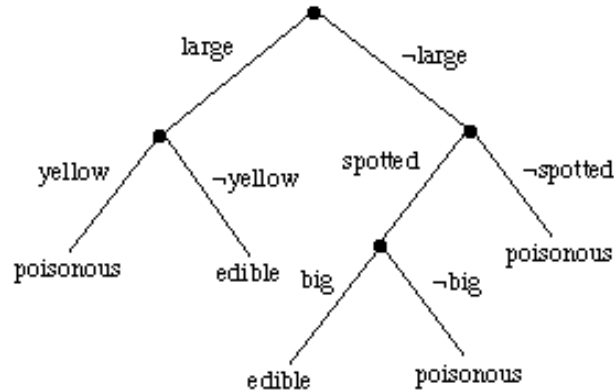
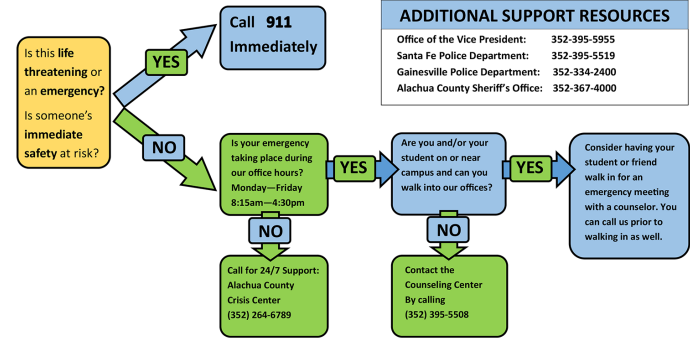
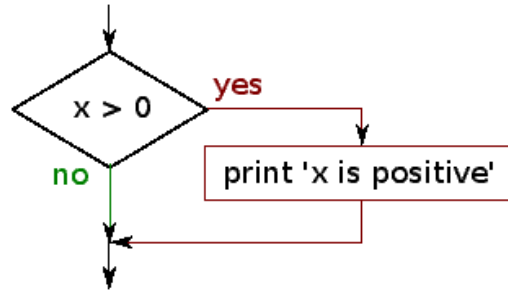
# Edible Mushroom



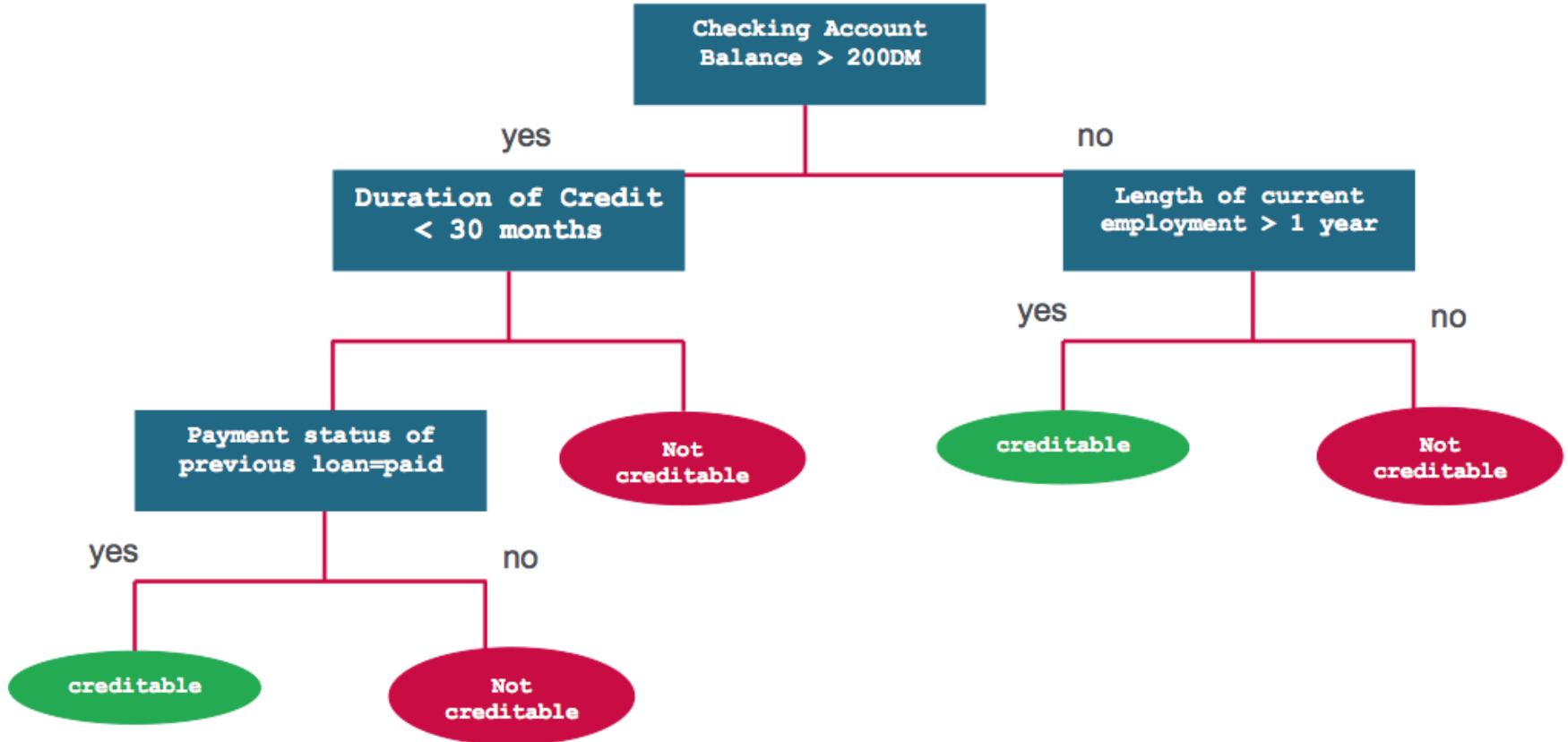
# Hand-crafted, fixed trees



# Hand-crafted, fixed trees



# Credit Approval



# Credit Approval (Raw Data)

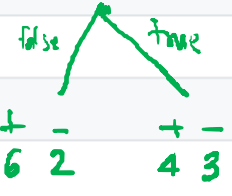
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
64	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	class
65	a	20.42	0.835	u	g	q	v	1.585	t	t	1	f	g	0	0	+
66	b	26.67	4.25	u	g	cc	v	4.29	t	t	1	t	g	120	0	+
67	b	34.17	1.54	u	g	cc	v	1.54	t	t	1	t	g	520	50000	+
68	a	36	1	u	g	c	v	2	t	t	11	f	g	0	456	+
69	b	25.5	0.375	u	g	m	v	0.25	t	t	3	f	g	260	15108	+
70	b	19.42	6.5	u	g	w	h	1.46	t	t	7	f	g	80	2954	+
71	b	35.17	25.125	u	g	x	h	1.625	t	t	1	t	g	515	500	+
72	b	32.33	7.5	u	g	e	bb	1.585	t	f	0	t	s	420	0	-
73	b	34.83	4	u	g	d	bb	12.5	t	f	0	t	g		0	-
74	a	38.58	5	u	g	cc	v	13.5	t	f	0	t	g	980	0	-
75	b	44.25	0.5	u	g	m	v	10.75	t	f	0	f	s	400	0	-
76	b	44.83	7	y	p	c	v	1.625	f	f	0	f	g	160	2	-
77	b	20.67	5.29	u	g	q	v	0.375	t	t	1	f	g	160	0	-
78	b	34.08	6.5	u	g	aa	v	0.125	t	f	0	t	g	443	0	-

	outlook	temp	humidity	windy	play
1	sunny	hot	high	false	no
2	sunny	hot	high	true	no
3	overcast	hot	high	false	yes
4	rainy	mild	high	false	yes
5	rainy	cool	normal	false	yes
6	rainy	cool	normal	true	no
7	overcast	cool	normal	true	yes
8	sunny	mild	high	false	no
9	sunny	cool	normal	false	yes
10	rainy	mild	normal	false	yes
11	sunny	mild	normal	true	yes
12	overcast	mild	high	true	yes
13	overcast	hot	normal	false	yes
14	rainy	mild	high	true	no
15					

X

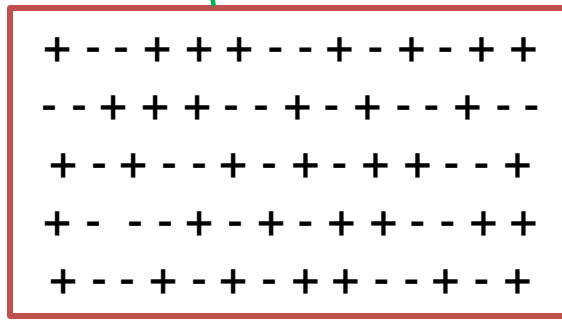
(10+5-)

Y



X



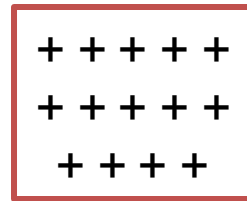
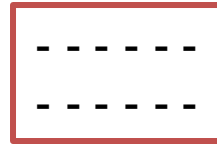


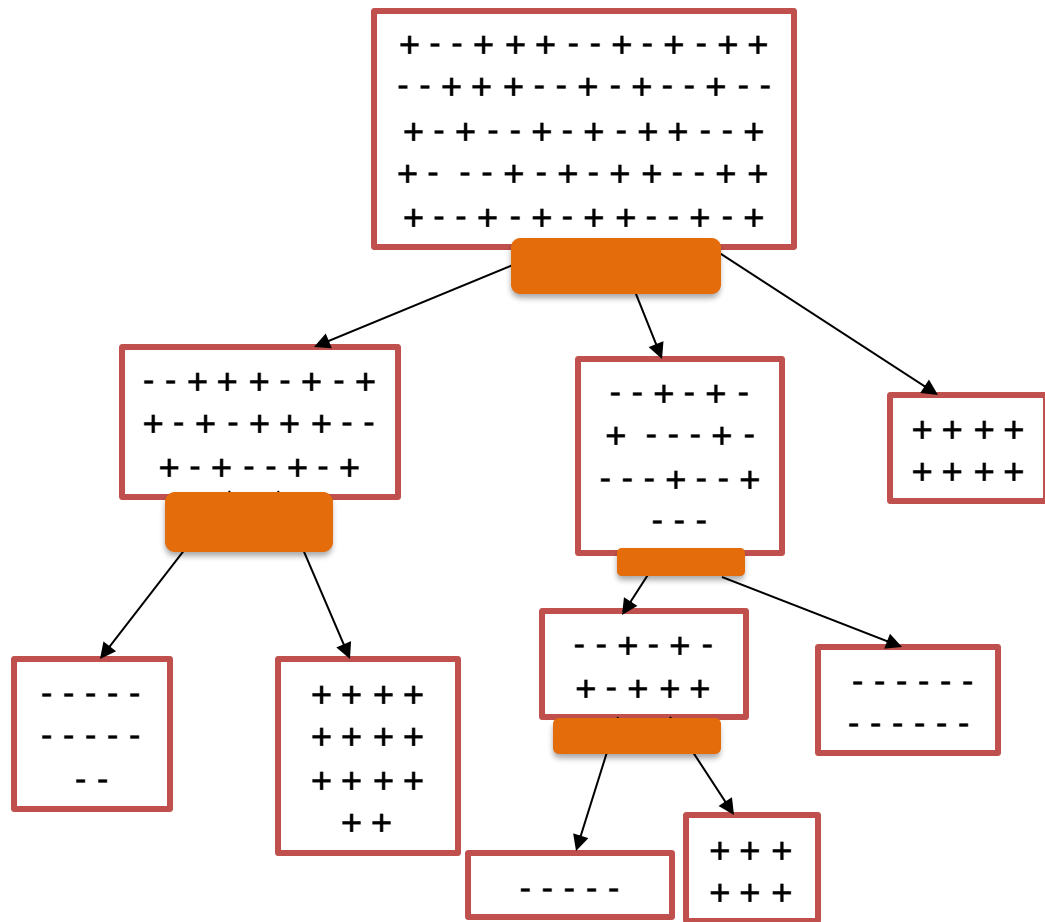
ATTRIBUTE  $A$

$v_1$

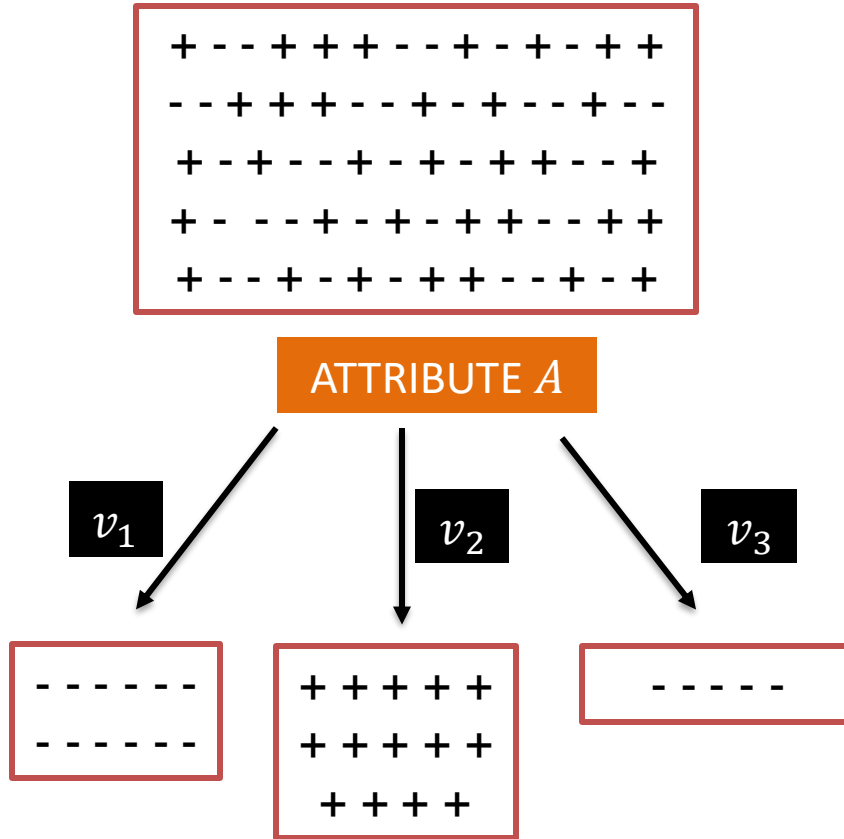
$v_2$

$v_3$

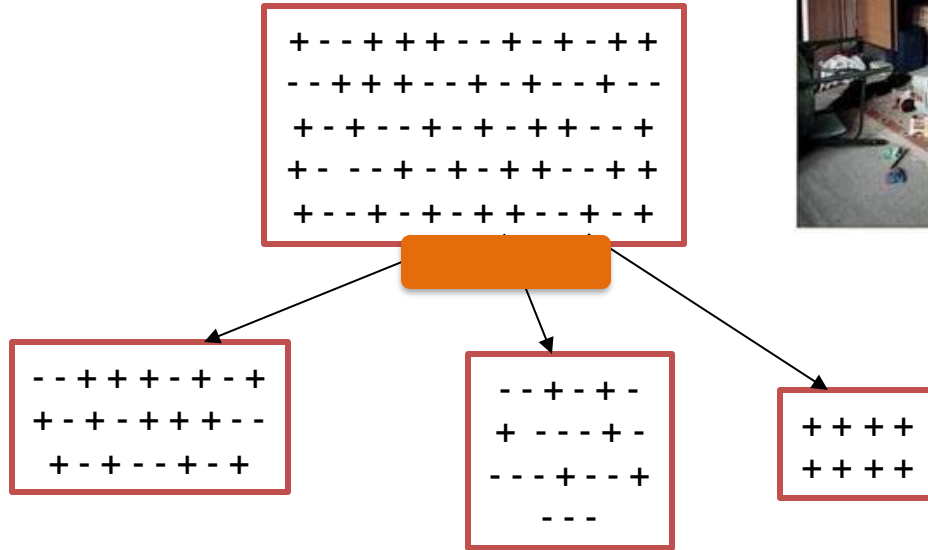




How much 'impurity' does this attribute decrease ?

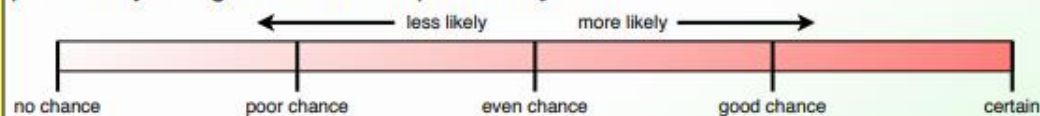


How much 'impurity' does this attribute decrease ?



# PROBABILITY

Probability is the chance (or likelihood) of an event happening. We can describe probability using words from a probability scale:



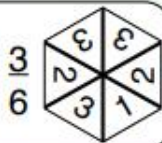
We can also describe probability using fractions.

The probability of a flipped coin landing on heads is one out of two.

$$\frac{1}{2}$$



The probability of this spinner landing on '3' is three out of six.



The probability of rolling a '2' on a die is one out of six.

$$\frac{1}{6}$$



What is the probability of rolling two sixes?

What is the probability of a flipped coin landing on tails?



What is the probability of picking a white marble out of a jar?



What is the probability of picking the Queen of Spades from a deck of cards?



What is the probability of a ball falling in a black pocket on a roulette wheel?



What does 'random' mean?  
If you put your hand into a bag of marbles and took one out without looking, you picked it at **random**. You didn't choose that one - you got it by chance.

# Discrete probability distribution

- Variables that have only a finite number of possible outcomes
- For example ...a six-sided die is thrown

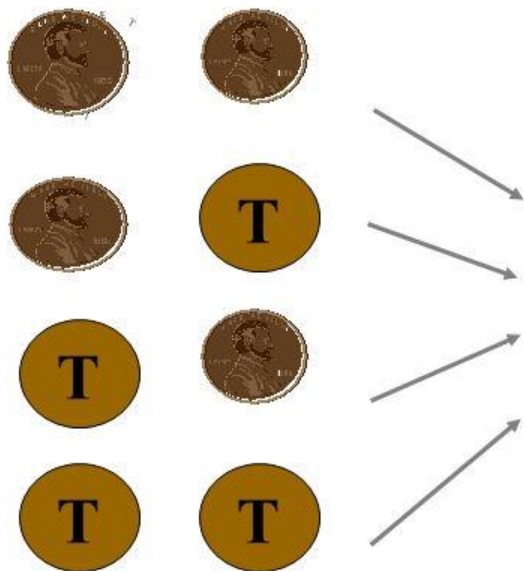
Possibilities $r=1$	1	2	3	4	5	6
Probability that $Z=r$	1/6	1/6	1/6	1/6	1/6	1/6

- $0 \leq P(X_j) \leq 1 \quad \sum P(X_j) = 1$

# Discrete probability distribution

Event: Toss two coins

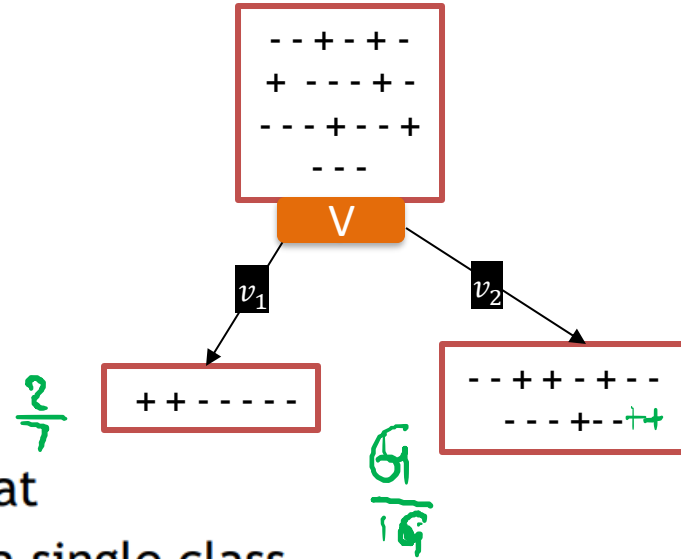
Count the number of tails



Probability Distribution	
<u>Values</u>	<u>Probability</u>
0	$1/4 = .25$
1	$2/4 = .50$
2	$1/4 = .25$

# Properties of an impurity measure

- Class labels: Binary  $\{+1, -1\}$
- $q_{v_1} = \frac{(\# \text{ of instances for which } V=v_1, y=1)}{(\# \text{ of instances for which } V=v_1)}$



An **impurity measure** is a function  $i(V)$  such that

- $i(V) \geq 0$ , with  $i(V) = 0$  iff  $V$  consists of a single class
- a larger value of  $i(V)$  indicates that the distribution defined by  $(q, (1 - q))$  is closer to the uniform distribution

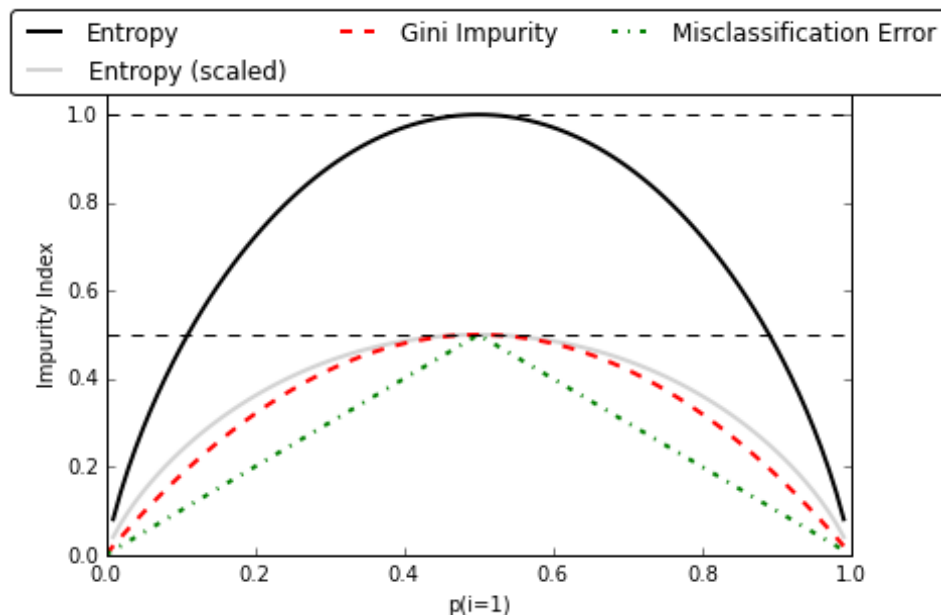


# Impurity function: candidates

**Entropy:**  $i(V) = -(q \log q + (1 - q) \log(1 - q))$

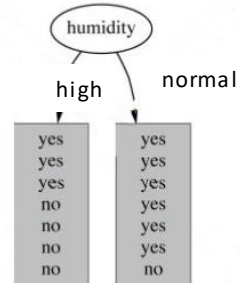
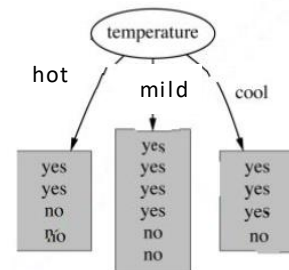
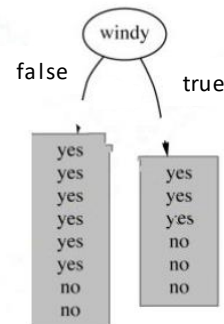
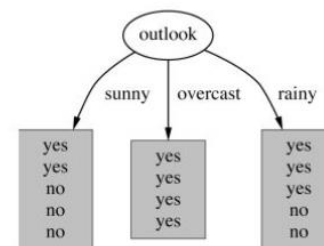
**Gini index:**  $i(V) = 2q(1 - q)$

**Misclassification rate:**  $i(V) = \min(q, 1 - q)$



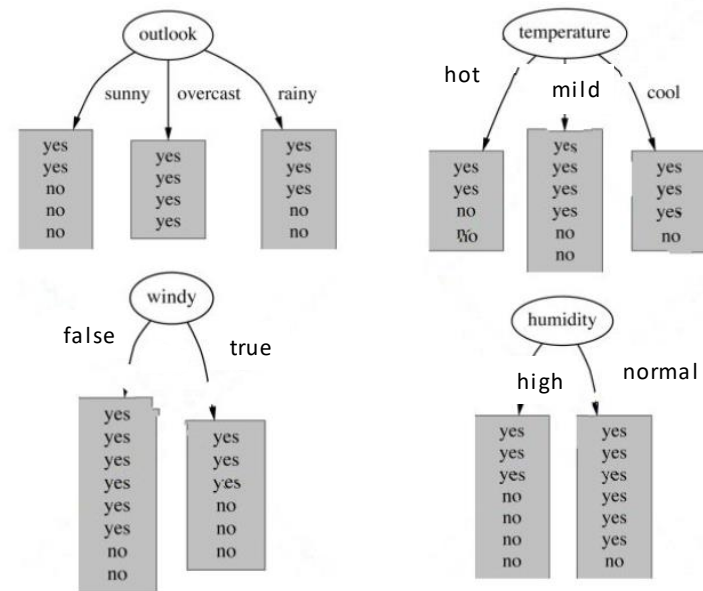
# Example

Day	Temperature	Outlook	Humidity	Windy	Play Golf?
07-05	hot	sunny	high	false	no
07-06	hot	sunny	high	true	no
07-07	hot	overcast	high	false	yes
07-09	cool	rain	normal	false	yes
07-10	cool	overcast	normal	true	yes
07-12	mild	sunny	high	false	no
07-14	cool	sunny	normal	false	yes
07-15	mild	rain	normal	false	yes
07-20	mild	sunny	normal	true	yes
07-21	mild	overcast	high	true	yes
07-22	hot	overcast	normal	false	yes
07-23	mild	rain	high	true	no
07-26	cool	rain	normal	true	no
07-30	mild	rain	high	false	yes



# Step-1: Compute impurity score of training label distribution

Day	Temperature	Outlook	Humidity	Windy	Play Golf?
07-05	hot	sunny	high	false	no
07-06	hot	sunny	high	true	no
07-07	hot	overcast	high	false	yes
07-09	cool	rain	normal	false	yes
07-10	cool	overcast	normal	true	yes
07-12	mild	sunny	high	false	no
07-14	cool	sunny	normal	false	yes
07-15	mild	rain	normal	false	yes
07-20	mild	sunny	normal	true	yes
07-21	mild	overcast	high	true	yes
07-22	hot	overcast	normal	false	yes
07-23	mild	rain	high	true	no
07-26	cool	rain	normal	true	no
07-30	mild	rain	high	false	yes



**Entropy:**  $i(V) = -(q \log q + (1 - q) \log(1 - q))$

$$E(S) = -\left(\frac{9}{14} \log\left(\frac{9}{14}\right) + \frac{5}{14} \log\left(\frac{5}{14}\right)\right) = 0.94$$

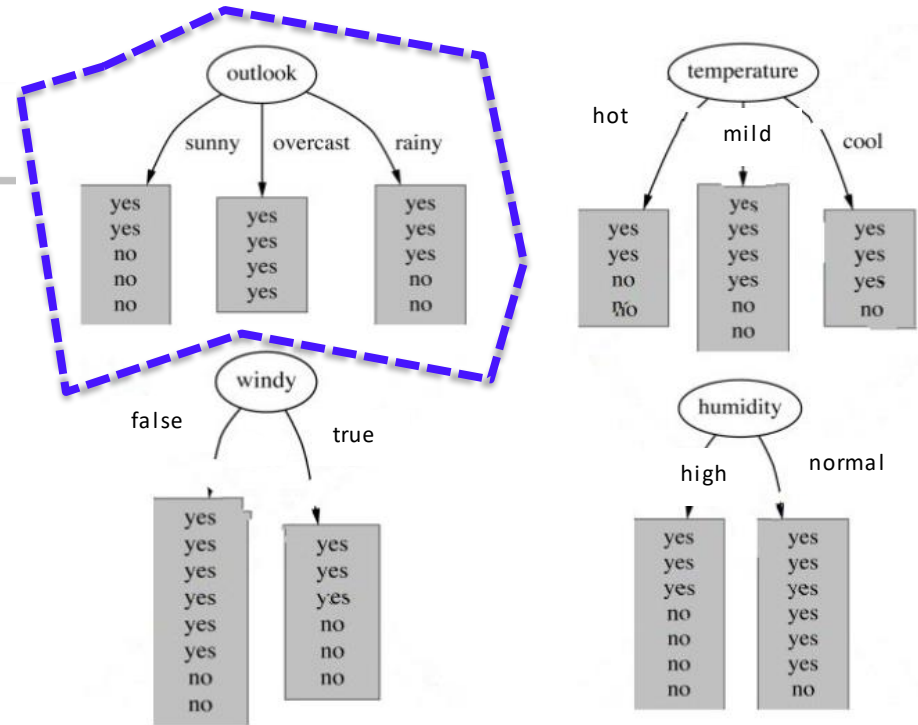
## Step-2: Compute impurity score for each unique value of candidate attributes

### Example: Attribute Outlook

**Entropy:**  $i(V) = -(q \log q + (1 - q) \log(1 - q))$

- **Outlook = rainy** 3 examples yes, 2 examples no

$$E(\text{Outlook}=\text{sunny}) = -\frac{2}{5} \log\left(\frac{2}{5}\right) - \frac{3}{5} \log\left(\frac{3}{5}\right) = 0.971$$



## Step-2: Compute impurity score for each unique value of candidate attributes

### Example: Attribute Outlook

**Entropy:**  $i(V) = -(q \log q + (1 - q) \log(1 - q))$

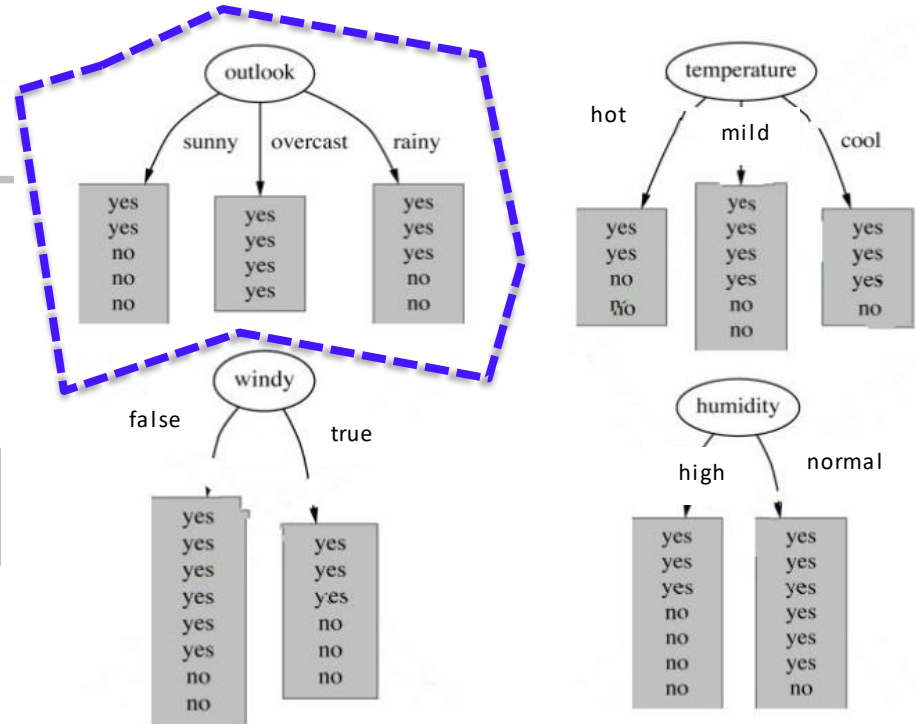
- **Outlook = rainy** 3 examples yes, 2 examples no

$$E(\text{Outlook}=\text{sunny}) = -\frac{2}{5} \log\left(\frac{2}{5}\right) - \frac{3}{5} \log\left(\frac{3}{5}\right) = 0.971$$

- **Outlook = overcast:** 4 examples yes, 0 examples no

$$E(\text{Outlook}=\text{overcast}) = -1 \log(1) - 0 \log(0) = 0$$

**Note:** this is normally undefined. Here: = 0



## Step-2: Compute impurity score for each unique value of candidate attributes

### Example: Attribute Outlook

**Entropy:**  $i(V) = -(q \log q + (1 - q) \log(1 - q))$

- **Outlook = rainy** 3 examples yes, 2 examples no

$$E(\text{Outlook}=\text{sunny}) = -\frac{2}{5} \log\left(\frac{2}{5}\right) - \frac{3}{5} \log\left(\frac{3}{5}\right) = 0.971$$

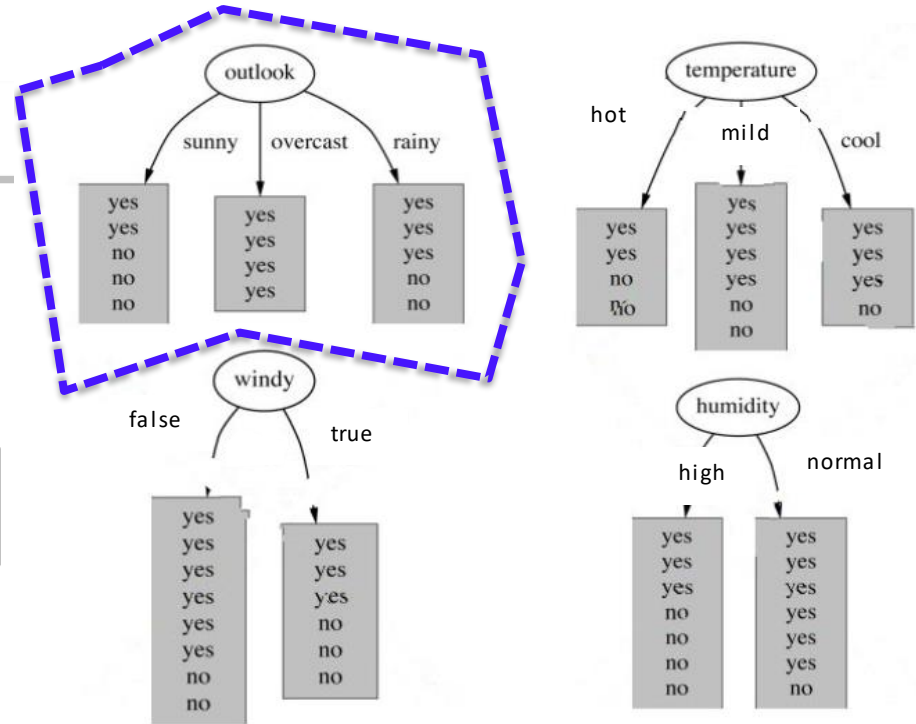
- **Outlook = overcast:** 4 examples yes, 0 examples no

$$E(\text{Outlook}=\text{overcast}) = -1 \log(1) - 0 \log(0) = 0$$

**Note:** this is normally undefined. Here: = 0

- **Outlook = sunny** 2 examples yes, 3 examples no

$$E(\text{Outlook}=\text{rainy}) = -\frac{3}{5} \log\left(\frac{3}{5}\right) - \frac{2}{5} \log\left(\frac{2}{5}\right) = 0.971$$



## Step-3: Compute impurity score for candidate attribute

- **Outlook = rainy** 3 examples yes, 2 examples no

$$E(\text{Outlook}=\text{sunny}) = -\frac{2}{5} \log\left(\frac{2}{5}\right) - \frac{3}{5} \log\left(\frac{3}{5}\right) = 0.971$$

- **Outlook = overcast:** 4 examples yes, 0 examples no

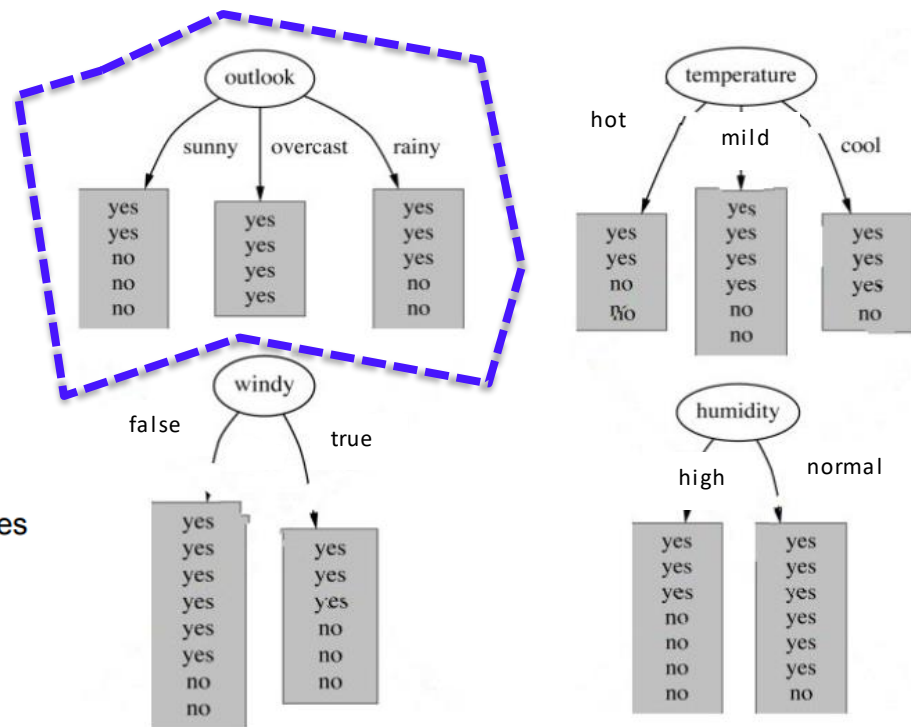
$$E(\text{Outlook}=\text{overcast}) = -1 \log(1) - 0 \log(0) = 0$$

**Note:** this is normally undefined. Here: = 0

- **Outlook = sunny** 2 examples yes, 3 examples no

$$E(\text{Outlook}=\text{rainy}) = -\frac{3}{5} \log\left(\frac{3}{5}\right) - \frac{2}{5} \log\left(\frac{2}{5}\right) = 0.971$$

- Entropy only computes the quality of a single (sub-)set of examples
  - corresponds to a single value
- How can we compute the quality of the entire split?
  - corresponds to an entire attribute



## Step-3: Compute impurity score for candidate attribute

- **Outlook = rainy** 3 examples yes, 2 examples no

$$E(\text{Outlook}=\text{sunny}) = -\frac{2}{5} \log\left(\frac{2}{5}\right) - \frac{3}{5} \log\left(\frac{3}{5}\right) = 0.971$$

- **Outlook = overcast:** 4 examples yes, 0 examples no

$$E(\text{Outlook}=\text{overcast}) = -1 \log(1) - 0 \log(0) = 0$$

**Note:** this is normally undefined. Here: = 0

- **Outlook = sunny** 2 examples yes, 3 examples no

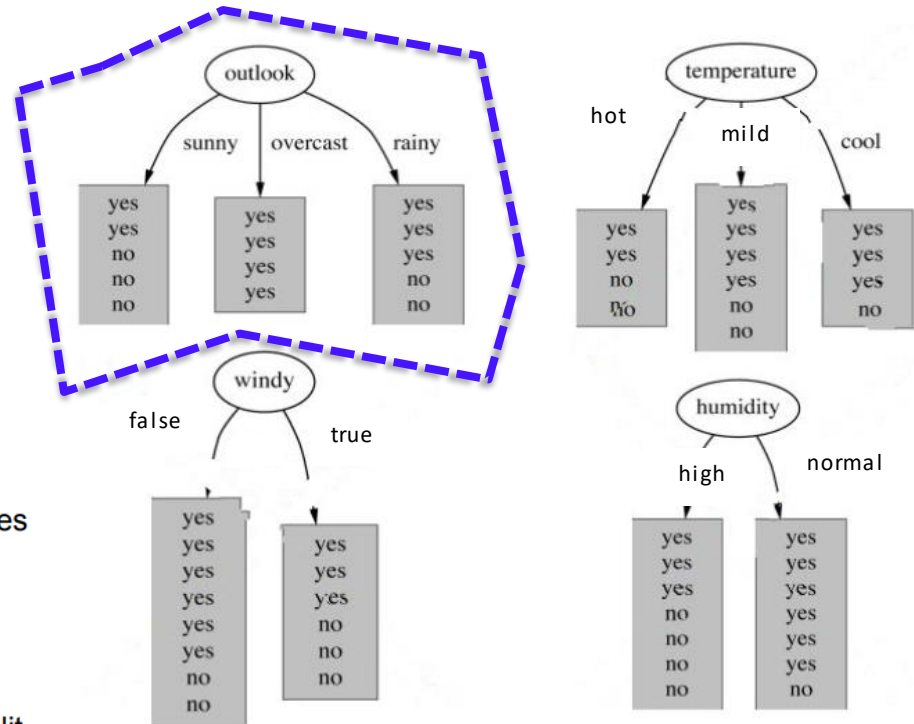
$$E(\text{Outlook}=\text{rainy}) = -\frac{3}{5} \log\left(\frac{3}{5}\right) - \frac{2}{5} \log\left(\frac{2}{5}\right) = 0.971$$

- Entropy only computes the quality of a single (sub-)set of examples
  - corresponds to a single value
- How can we compute the quality of the entire split?
  - corresponds to an entire attribute

### Solution:

- Compute the weighted average over all sets resulting from the split
  - weighted by their size

$$I(S, A) = \sum_i \frac{|S_i|}{|S|} \cdot E(S_i)$$





## Step-3: Compute impurity score for candidate attribute

- **Outlook = rainy** 3 examples yes, 2 examples no

$$E(\text{Outlook}=\text{sunny}) = -\frac{2}{5} \log\left(\frac{2}{5}\right) - \frac{3}{5} \log\left(\frac{3}{5}\right) = 0.971$$

- **Outlook = overcast:** 4 examples yes, 0 examples no

$$E(\text{Outlook}=\text{overcast}) = -1 \log(1) - 0 \log(0) = 0$$

**Note:** this is normally undefined. Here: = 0

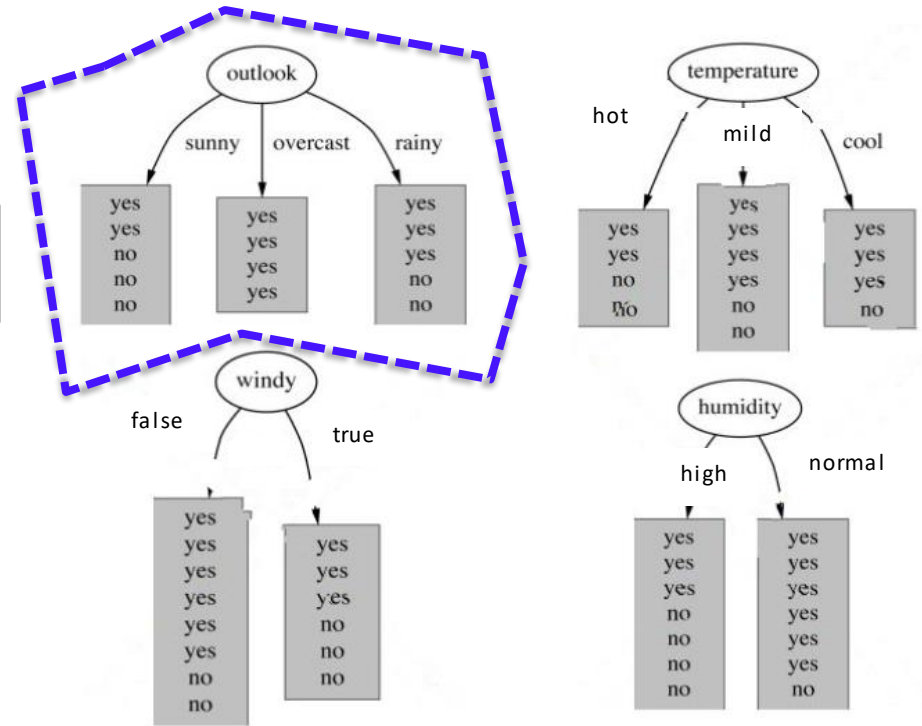
- **Outlook = sunny** 2 examples yes, 3 examples no

$$E(\text{Outlook}=\text{rainy}) = -\frac{3}{5} \log\left(\frac{3}{5}\right) - \frac{2}{5} \log\left(\frac{2}{5}\right) = 0.971$$

$$I(S, A) = \sum_i \frac{|S_i|}{|S|} \cdot E(S_i)$$

- Average entropy for attribute *Outlook*:

$$I(\text{Outlook}) = \frac{5}{14} \cdot 0.971 + \frac{4}{14} \cdot 0 + \frac{5}{14} \cdot 0.971 = 0.693$$



## Step-4: Compute Information Gain (reduction in impurity score) provided by candidate attribute

$$I(S, A) = \sum_i \frac{|S_i|}{|S|} \cdot E(S_i)$$

- Average entropy for attribute *Outlook*:

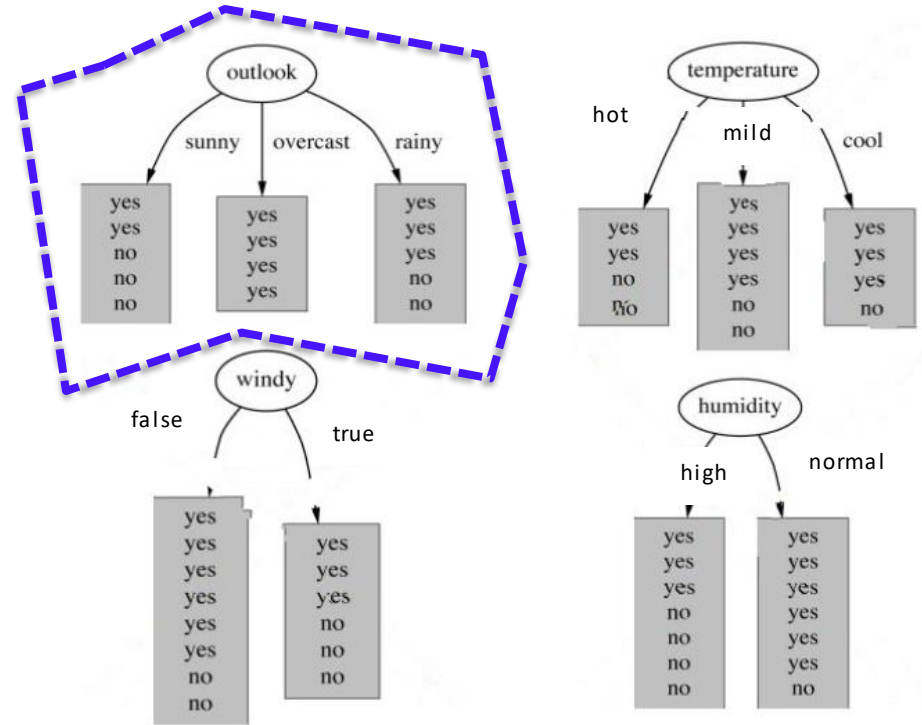
$$I(\text{Outlook}) = \frac{5}{14} \cdot 0.971 + \frac{4}{14} \cdot 0 + \frac{5}{14} \cdot 0.971 = 0.693$$

$$E(S) = -\left(\frac{9}{14} \log\left(\frac{9}{14}\right) + \frac{5}{14} \log\left(\frac{5}{14}\right)\right) = 0.94$$

Information Gain for Attribute *A*

$$\text{Gain}(S, A) = E(S) - I(S, A) = E(S) - \sum_i \frac{|S_i|}{|S|} \cdot E(S_i)$$

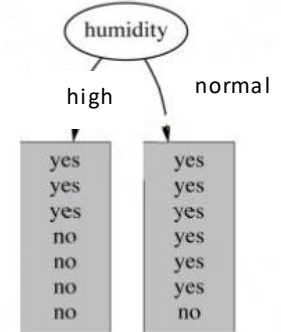
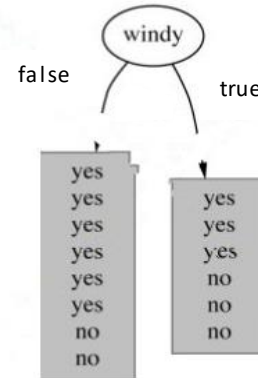
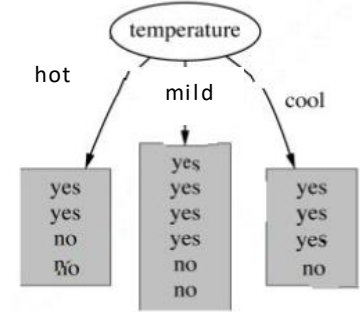
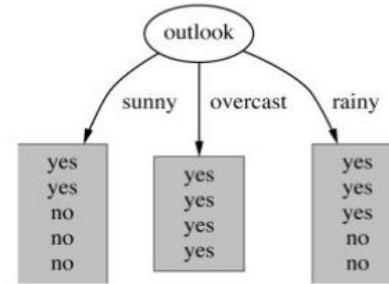
$$\text{Gain}(S, \text{Outlook}) = 0.246$$



## Step-5: Compare Information Gain provided by all candidates

Information Gain for Attribute  $A$

$$Gain(S, A) = E(S) - I(S, A) = E(S) - \sum_i \frac{|S_i|}{|S|} \cdot E(S_i)$$



$Gain(S, Humidity)$

$$= .940 - (7/14).985 - (7/14).592$$

$$= .151$$

$Gain(S, Wind)$

$$= .940 - (8/14).811 - (6/14)1.0$$

$$= .048$$

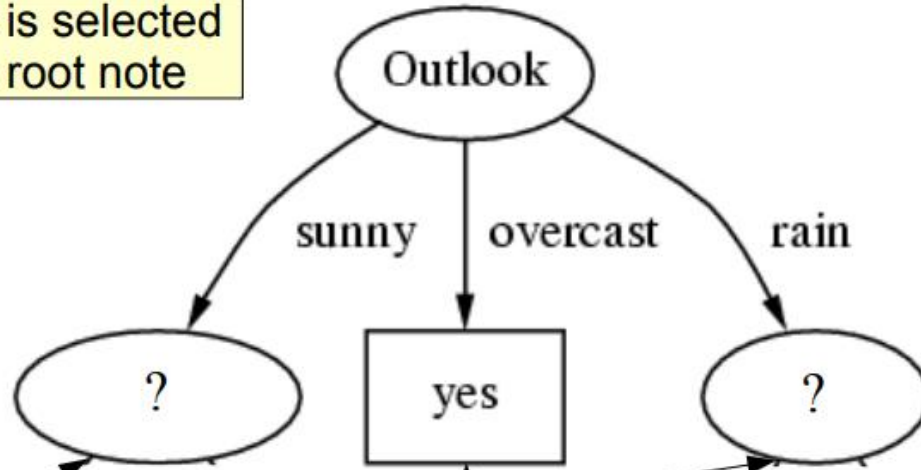
$Gain(S, Outlook) = 0.246$

$Gain(S, Temperature) = 0.029$

Select attribute which provides largest 'impurity reduction'/Information Gain

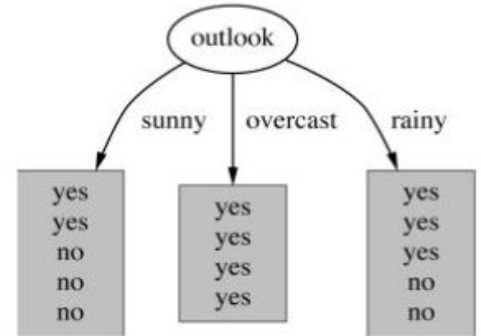
# Step-6: Assign root node

**Outlook** is selected  
as the root node

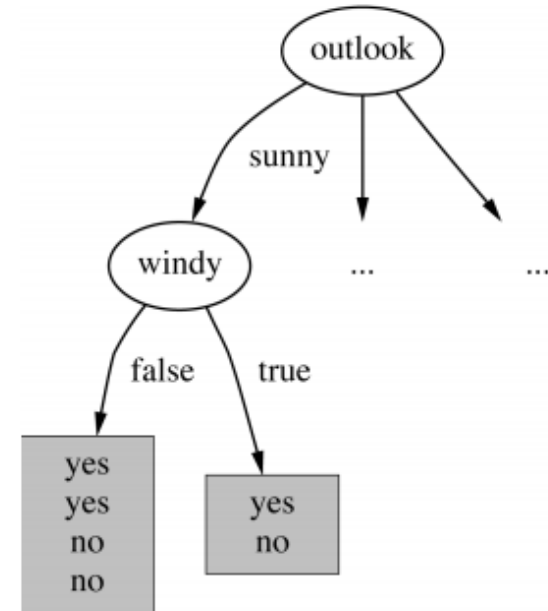
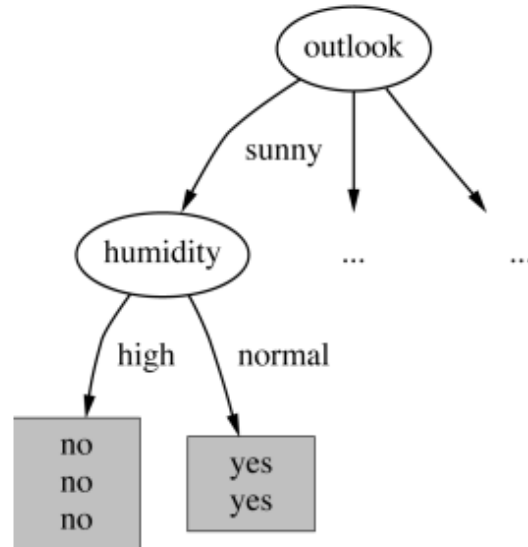
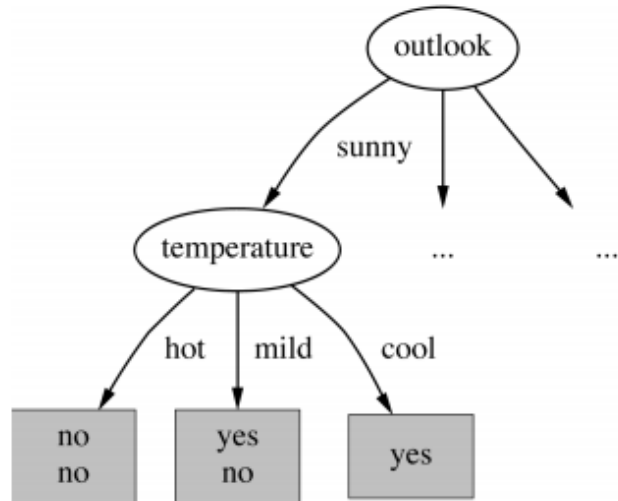


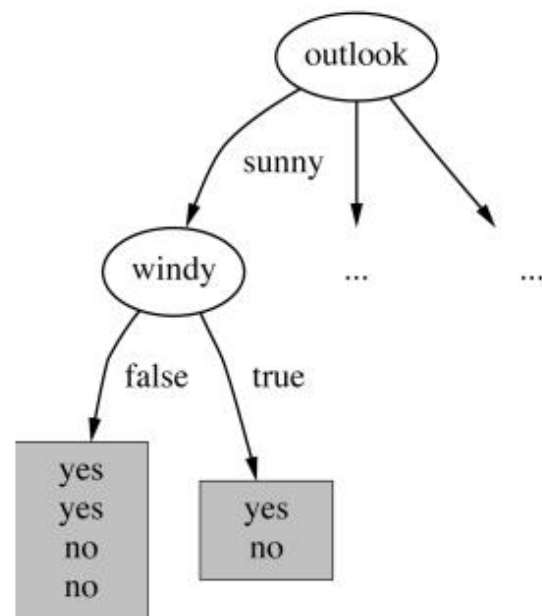
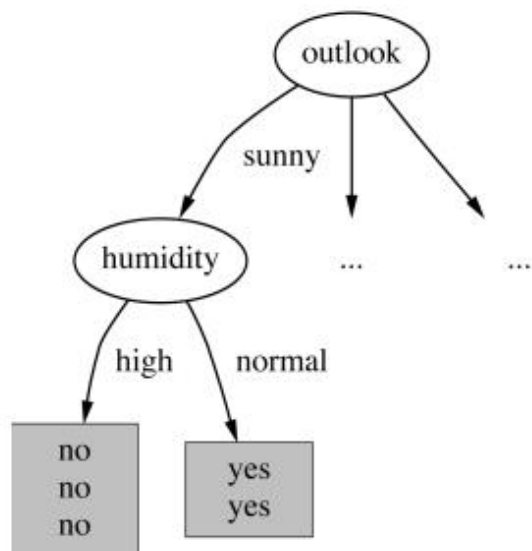
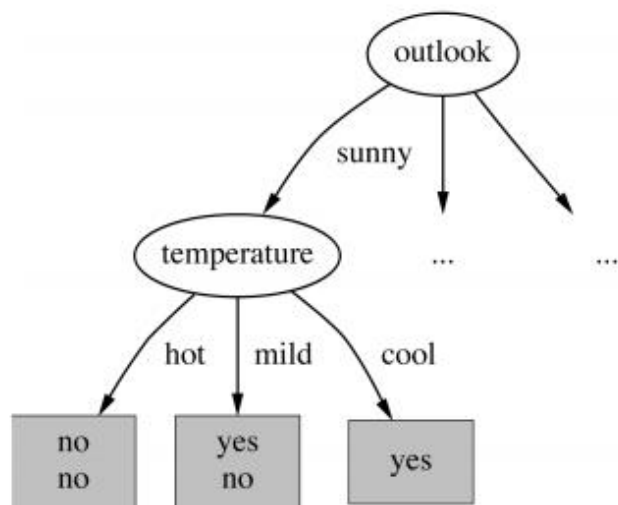
further splitting  
necessary

**Outlook = overcast**  
contains only  
examples of class **yes**



# Recurse and repeat Steps 1-6





$\text{Gain}(\text{Temperature})$

$= 0.571 \text{ bits}$

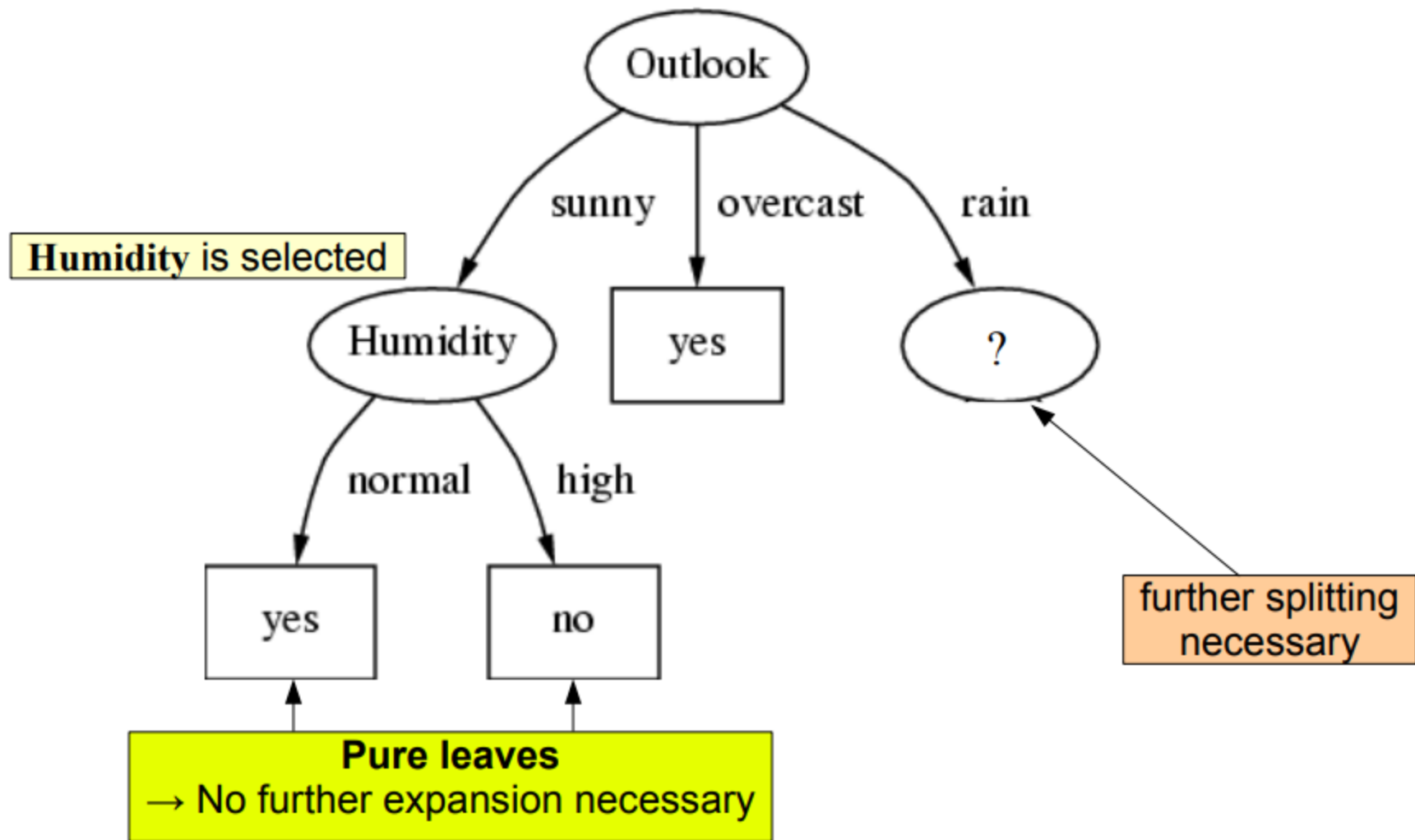
$\text{Gain}(\text{Humidity})$

$= 0.971 \text{ bits}$

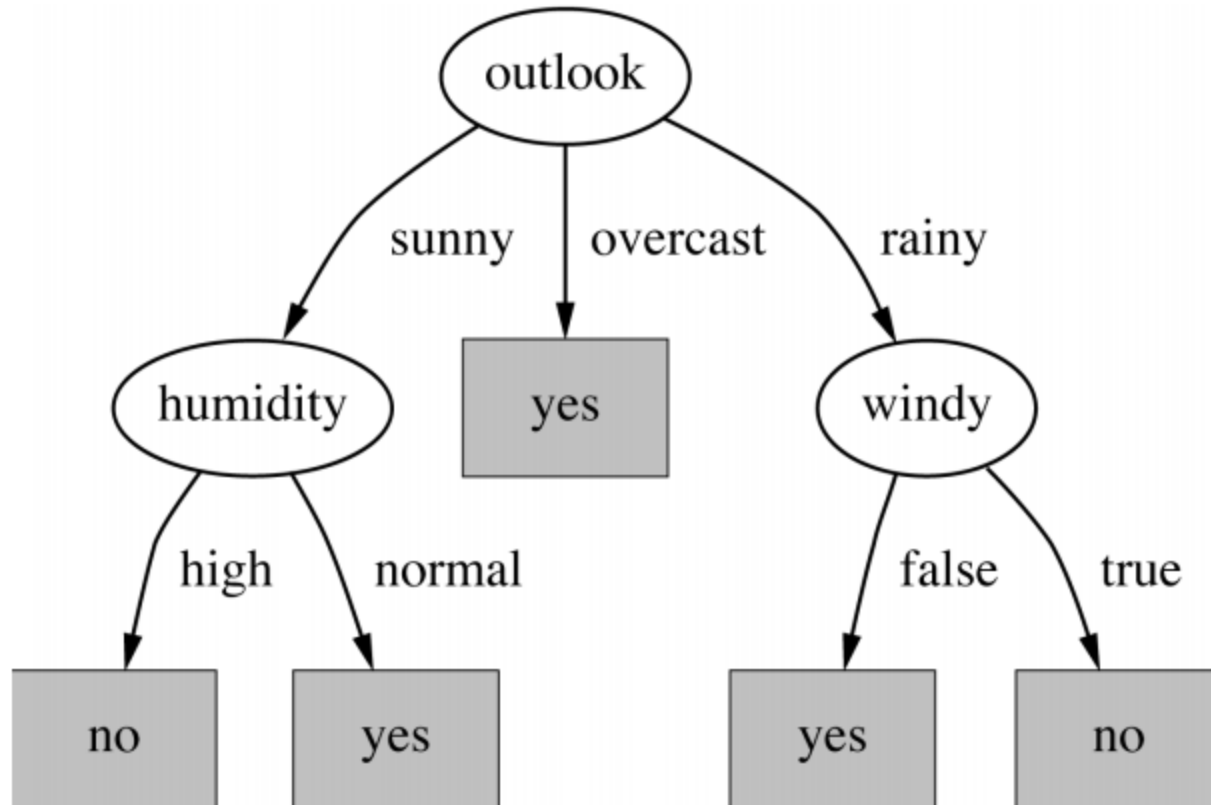
$\text{Gain}(\text{Windy})$

$= 0.020 \text{ bits}$

**Humidity is selected**



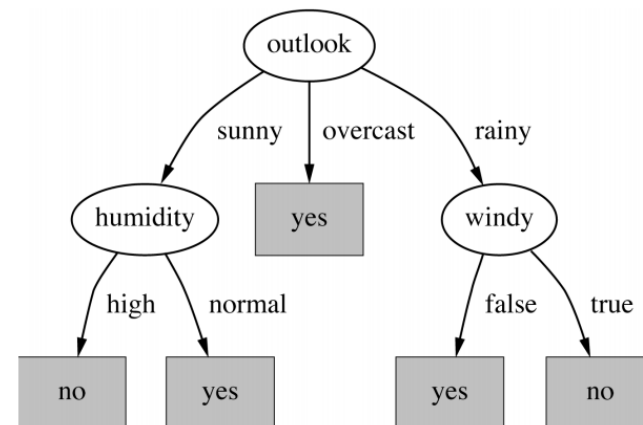
# Final Decision Tree





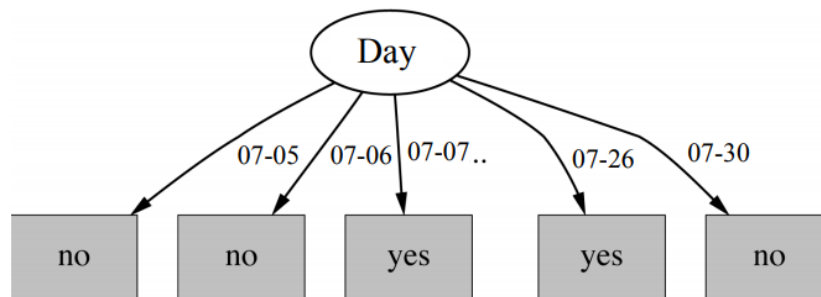
# Final Decision Tree

Day	Temperature	Outlook	Humidity	Windy	Play Golf?
07-05	hot	sunny	high	false	no
07-06	hot	sunny	high	true	no
07-07	hot	overcast	high	false	yes
07-09	cool	rain	normal	false	yes
07-10	cool	overcast	normal	true	yes
07-12	mild	sunny	high	false	no
07-14	cool	sunny	normal	false	yes
07-15	mild	rain	normal	false	yes
07-20	mild	sunny	normal	true	yes
07-21	mild	overcast	high	true	yes
07-22	hot	overcast	normal	false	yes
07-23	mild	rain	high	true	no
07-26	cool	rain	normal	true	no
07-30	mild	rain	high	false	yes



- Problematic: attributes with a large number of values
  - extreme case: each example has its own value
    - e.g. example ID; Day attribute in weather data

- Problematic: attributes with a large number of values
  - extreme case: each example has its own value
    - e.g. example ID; Day attribute in weather data
- Subsets are more likely to be pure if there is a large number of different attribute values
  - Information gain is biased towards choosing attributes with a large number of values



- Entropy of split:

$$I(\text{Day}) = \frac{1}{14} (E([0,1]) + E([0,1]) + \dots + E([0,1])) = 0$$

- Information gain is maximal for Day (0.940 bits)

$$\begin{aligned} \text{Gain}(S, \text{Humidity}) &= .940 - (7/14).985 - (7/14).592 \\ &= .151 \end{aligned}$$

$$\begin{aligned} \text{Gain}(S, \text{Wind}) &= .940 - (8/14).811 - (6/14)1.0 \\ &= .048 \end{aligned}$$

$$\text{Gain}(S, \text{Temperature}) = 0.029$$

$$\text{Gain}(S, \text{Outlook}) = 0.246$$

# Attributes with large # of values

- This may cause several problems:
  - *Overfitting*
    - selection of an attribute that is non-optimal for prediction
  - *Fragmentation*
    - data are fragmented into (too) many small sets

# Attributes with large # of values – measure

- Intrinsic information of a split
  - entropy of distribution of instances into branches
  - i.e. how much information do we need to tell which branch an instance belongs to

$$IntI(S, A) = - \sum_i \frac{|S_i|}{|S|} \log \left( \frac{|S_i|}{|S|} \right)$$

- Example:
  - Intrinsic information of Day attribute:

$$IntI(\text{Day}) = 14 \times \left( -\frac{1}{14} \cdot \log \left( \frac{1}{14} \right) \right) = 3.807$$

- Observation:
  - Attributes with higher intrinsic information are less useful

# Gain Ratio

- modification of the information gain that reduces its bias towards multi-valued attributes
- takes number and size of branches into account when choosing an attribute
  - corrects the information gain by taking the *intrinsic information* of a split into account
- Definition of Gain Ratio:

$$GR(S, A) = \frac{Gain(S, A)}{IntI(S, A)}$$

- Example:
  - Gain Ratio of Day attribute

$$GR(\text{Day}) = \frac{0.940}{3,807} = 0.246$$

# Handling numerical attributes

- Standard method: binary splits
  - E.g. Temperature  $< 78$
- Multiple split points possible
- Computationally more demanding

# Handling numerical attributes – some optimizations

- Assume a numerical attribute for Temperature
- First step:
  - Sort all examples according to the value of this attribute
  - Could look like this:

64	65	68	69	70	71	72	72	75	75	80	81	83	85
Yes	No	Yes	Yes	Yes	No	No	Yes	Yes	Yes	No	Yes	Yes	No



# Handling numerical attributes – some optimizations

- Assume a numerical attribute for Temperature
- First step:
  - Sort all examples according to the value of this attribute
  - Could look like this:

64	65	68	69	70	71	72	72	75	75	80	81	83	85
Yes	No	Yes	Yes	Yes	No	No	Yes	Yes	Yes	No	Yes	Yes	No

- One split between each pair of values
  - E.g.            Temperature < 71.5: yes/4, no/2  
                    Temperature ≥ 71.5: yes/5, no/3

$$I(\text{Temperature @ } 71.5) = \frac{6}{14} \cdot E(\text{Temperature} < 71.5) + \frac{8}{14} \cdot E(\text{Temperature} \geq 71.5) = 0.939$$

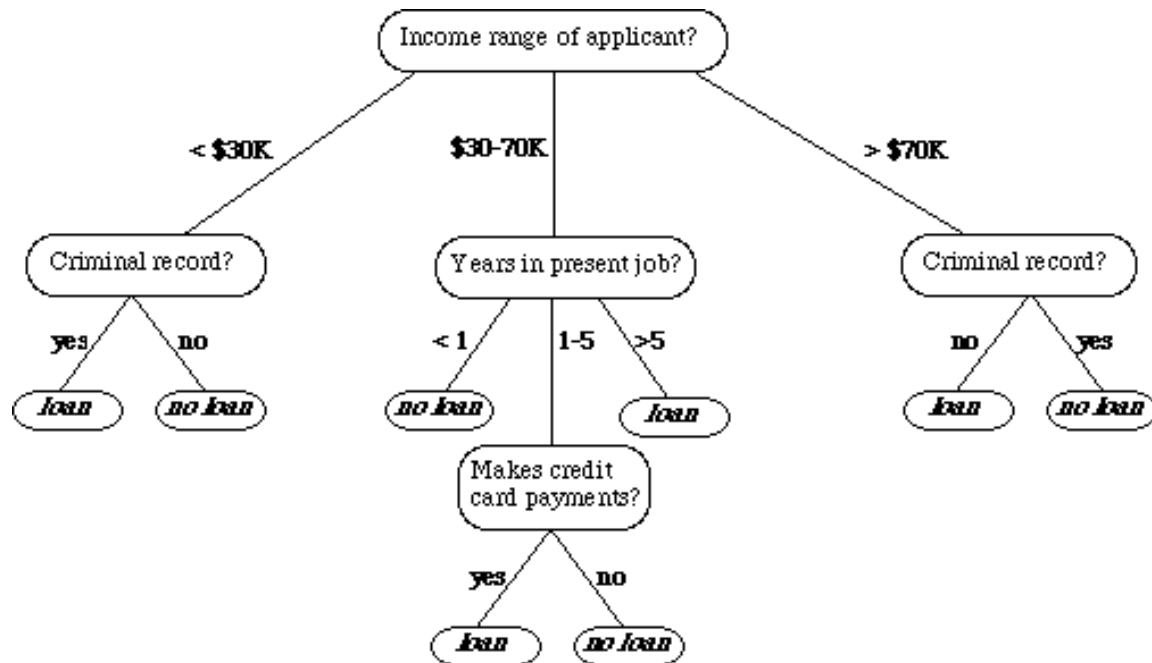
- Split points can be placed between values or directly at values

# Handling numerical attributes

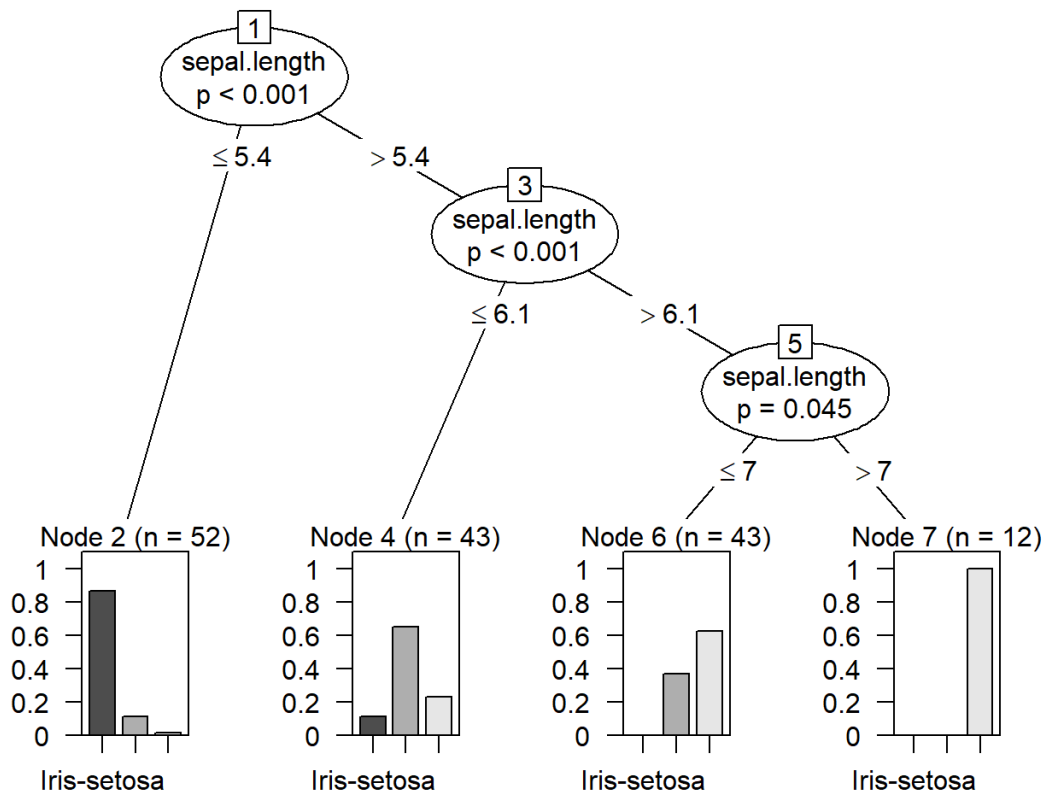
- Splitting (multi-way) on a nominal attribute exhausts all information in that attribute
  - Nominal attribute is tested (at most) once on any path in the tree
- Not so for binary splits on numerical attributes (why ?)
- ➔ Attribute may be tested multiple times in the tree
- ➔ Tree may become hard to read

# Handling numerical attributes

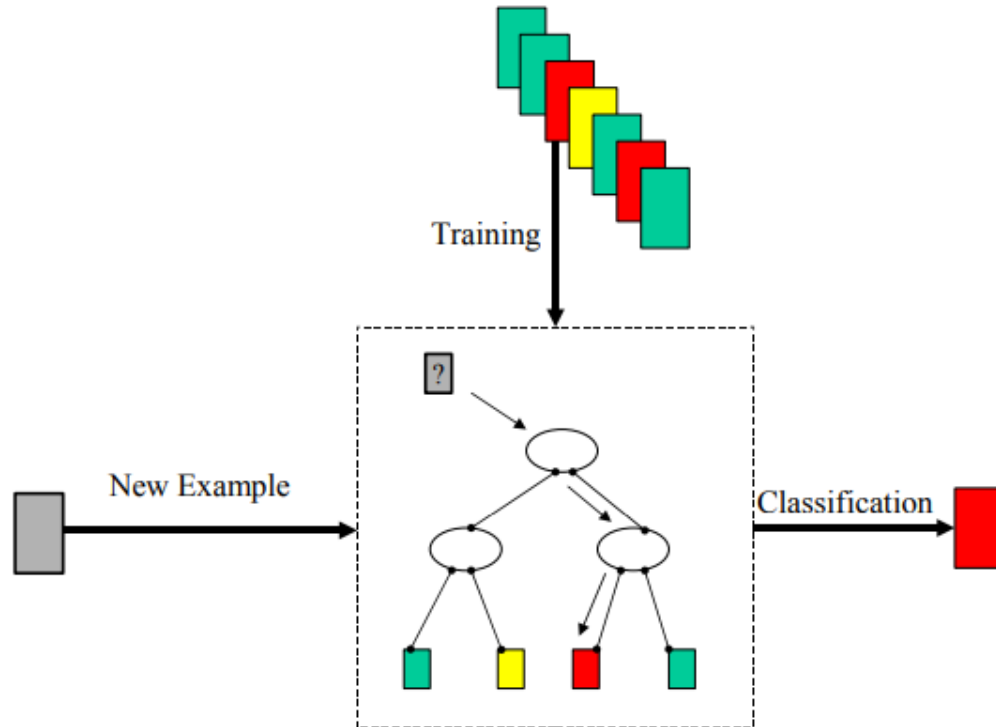
- Discretization / Clustering



# Decision Tree with numerical attribute



# Deployment



# Learning Algorithm for Decision Trees

$$S = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_N, y_N)\}$$

$$\mathbf{x} = (x_1, \dots, x_d)$$
$$x_j, y \in \{0, 1\}$$

GROWTREE( $S$ )

**if** ( $y = 0$  for all  $\langle \mathbf{x}, y \rangle \in S$ ) **return** new leaf(0)

**else if** ( $y = 1$  for all  $\langle \mathbf{x}, y \rangle \in S$ ) **return** new leaf(1)

**else**

choose best attribute  $x_j$

$S_0 =$  all  $\langle \mathbf{x}, y \rangle \in S$  with  $x_j = 0$ ;

$S_1 =$  all  $\langle \mathbf{x}, y \rangle \in S$  with  $x_j = 1$ ;

**return** new node( $x_j$ , GROWTREE( $S_0$ ), GROWTREE( $S_1$ ))

DT algs differ on this choice!

- ID3
- CAT4.5
- CART

# Other issues to address

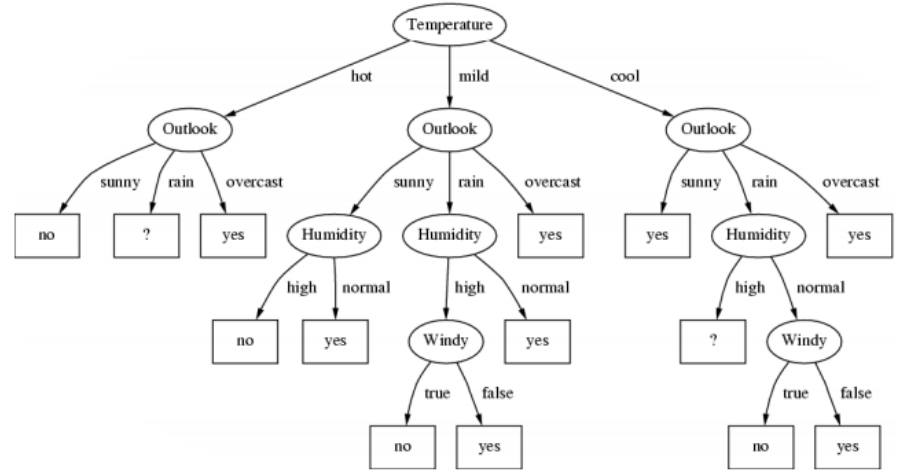
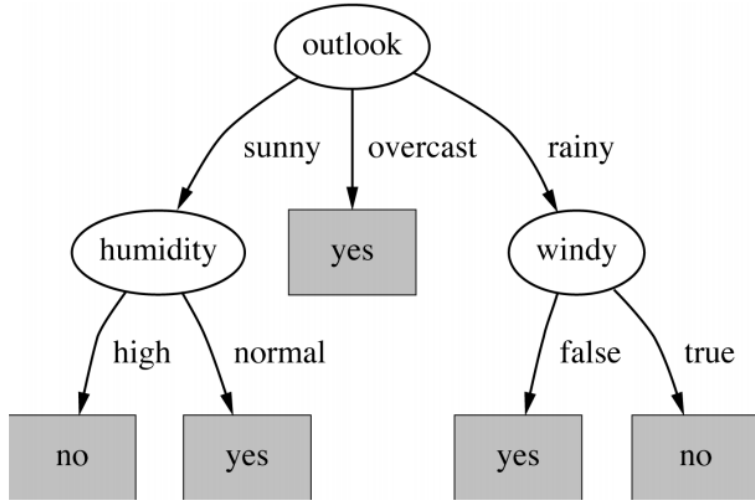
- Missing attributes
- Attribute values not seen during tree induction (construction)
- Attribute missing in 'test phase'
  - Divide into pieces etc.

I, Donald J. Trump, am currently inclined to use a position of power to modify the location that is above the country of Mexico and below the country of Canada to be held up to the standards it cemented to be of excellent status, at an earlier point in the country's lifetime.





# Small is often better



# The Smallest Decision Tree

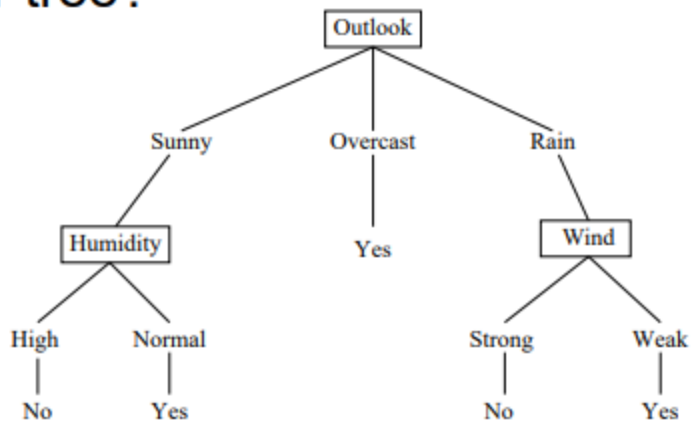
- Learning the smallest DT is NP-hard (Hyafil & Rivest '76)
- Greedy Heuristic
  - Start from empty decision tree
  - Split on next best attribute (feature)
  - Recurse

# Overfitting in Decision Trees

- Consider adding noisy training example #15:

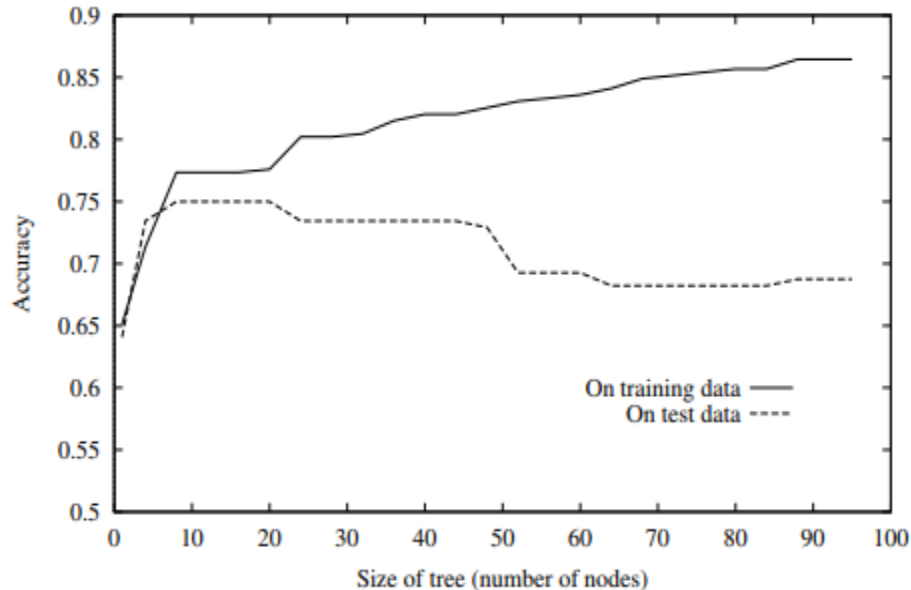
*Sunny, Hot, Normal, Strong, PlayTennis = No*

- What effect on earlier tree?



# Overfitting in Decision Trees

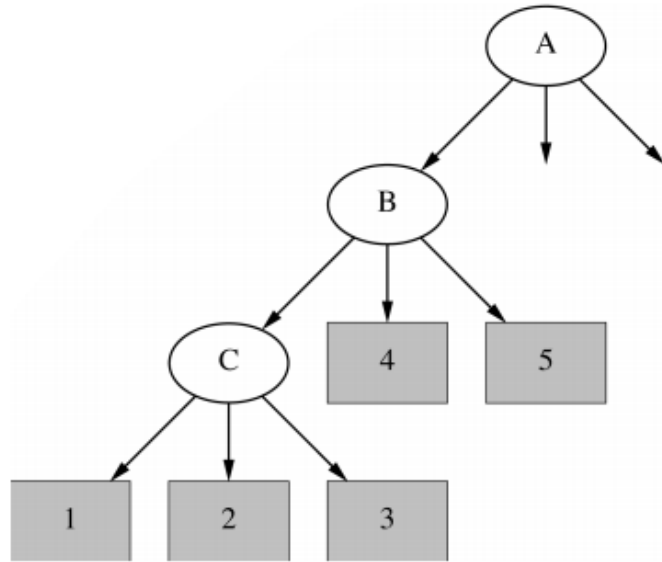
- Overfitting can occur with noisy training examples, and also when small numbers of examples are associated with leaf nodes ( $\rightarrow$  coincidental or accidental regularities)



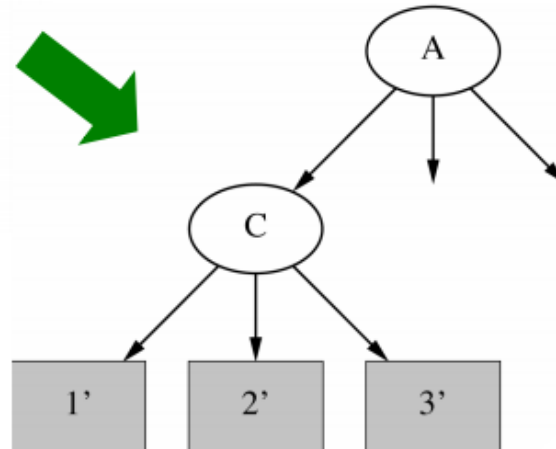
# Avoiding overfitting

- Pre-pruning : stop growing tree based on statistical tests of significance
- Post-pruning : Grow full tree, then prune

# Post-pruning by subtree raising



- Delete node B
- Redistribute instances of leaves 4 and 5 into C



# Decision Trees → Code

rec	Age	Income	Student	Credit_rating	<i>Buys_computer(CLASS)</i>
r1	<=30	High	No	Fair	No
r2	<=30	High	No	Excellent	No
r3	31...40	High	No	Fair	Yes
r4	>40	Medium	No	Fair	Yes
r5	>40	Low	Yes	Fair	Yes
r6	>40	Low	Yes	Excellent	No
r7	31...40	Low	Yes	Excellent	Yes
r8	<=30	Medium	No	Fair	No
r9	<=30	Low	Yes	Fair	Yes
r10	>40	Medium	Yes	Fair	Yes
r11	<=30	Medium	Yes	Excellent	Yes
r12	31...40	Medium	No	Excellent	Yes
r13	31...40	High	Yes	Fair	Yes
r14	>40	Medium	No	Excellent	No

IF *age* = "<=30" AND *student* = "no" THEN  
*buys\_computer* = "no"

IF *age* = "<=30" AND *student* = "yes" THEN  
*buys\_computer* = "yes"

IF *age* = "31...40" THEN  
*buys\_computer* = "yes"

IF *age* = ">40" AND *credit\_rating* = "excellent" THEN  
*buys\_computer* = "no"

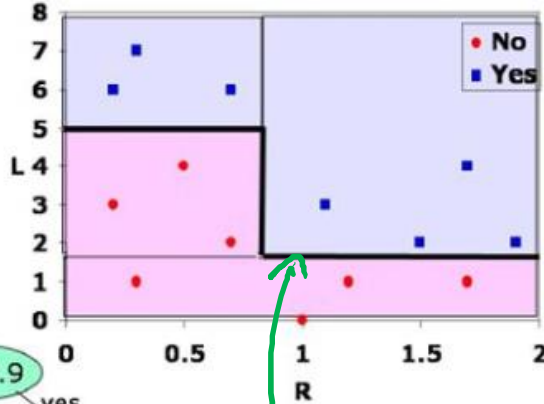
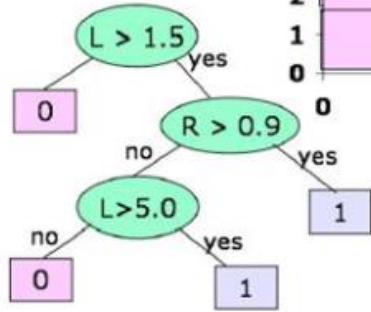
IF *age* = ">40" AND *credit\_rating* = "fair" THEN  
*buys\_computer* = "yes"

# Attributes with Differing Costs

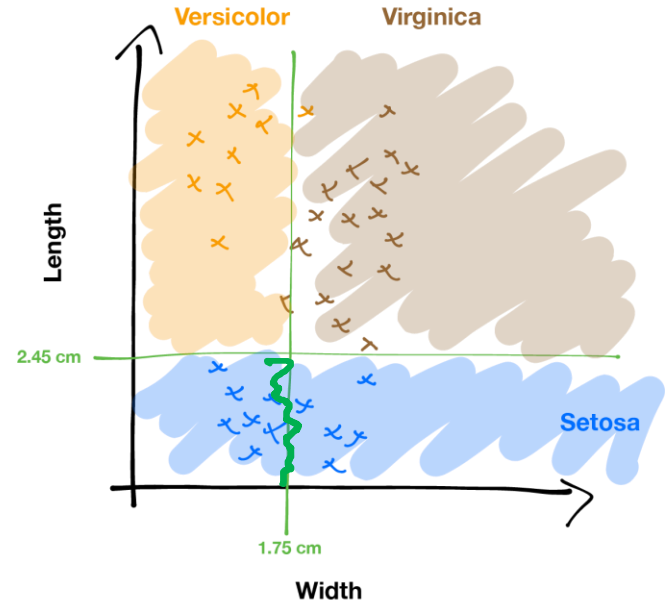
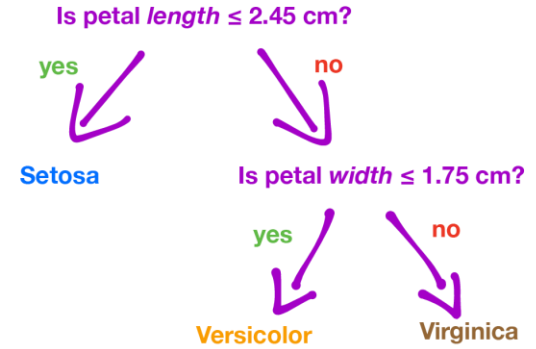
- Measuring attribute costs something
  - prefer cheap ones if possible
  - use costly ones only if good gain
- Introduce cost term in selection measure
  - no guarantee in finding optimum, but give bias towards cheapest
- Example applications
  - robot & sonar: time required to position
  - medical diagnosis: cost of a laboratory test



# Decision Boundaries



decision boundary



## Decision trees for classification

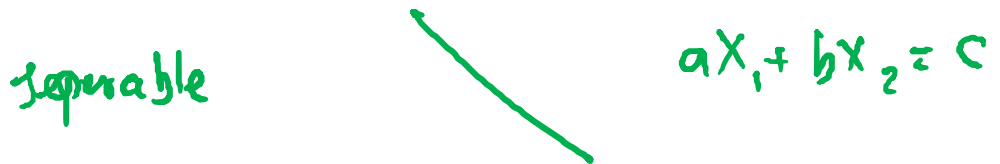
Some real examples (from Russell & Norvig, Mitchell)

- BP's GasOIL system for separating gas and oil on offshore platforms - decision trees replaced a hand-designed rules system with 2500 rules. C4.5-based system outperformed human experts and saved BP millions. (1986)
- learning to fly a Cessna on a flight simulator by watching human experts fly the simulator (1992)
- can also learn to play tennis, analyze C-section risk, etc.

# Advantages of DT

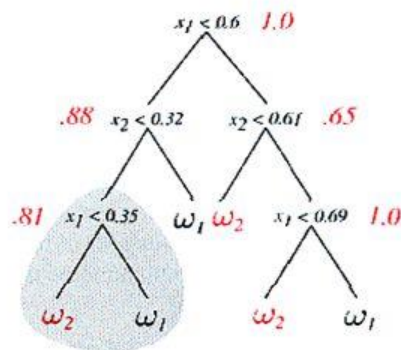
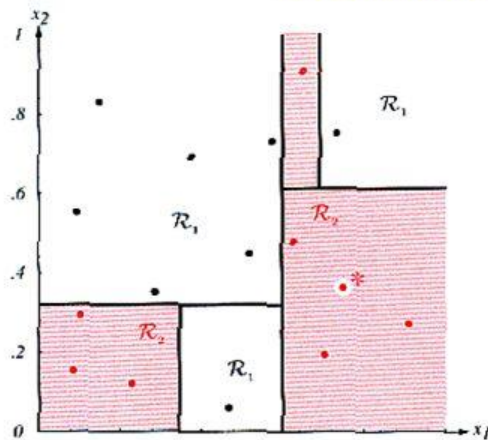
- Easy to use, understand
- Produce rules that are easy to interpret & implement
- Variable selection & reduction is automatic
- Do not require the assumptions of statistical models
- Can work without extensive handling of missing data

# Disadvantages

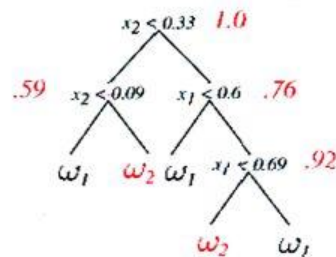
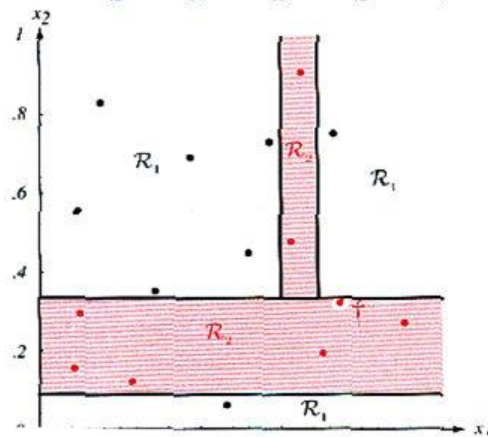
- May not perform well where there is structure in the data that is not well captured by horizontal or vertical splits  


separable  $aX_1 + bX_2 = c$
- Since the process deals with one variable at a time, no way to capture interactions between variables

## Decision Trees are not stable



Moving just one example slightly may lead to quite different trees and space partition!



Lack of stability against small perturbation of data.

Figure from  
Duda, Hart & Stork,  
Chap. 8

# Decision Tree Demo

- <http://www.r2d3.us/visual-intro-to-machine-learning-part-1/>

# Reading

- <https://www-users.cs.umn.edu/~kumar001/dmbook/ch4.pdf>