EC4.404: Mechatronics System Design

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General Information

Mechatronics: Study of the integration of mechanical hardware, electrical/electronic hardware with computer hardware and software. Named by Tetsuro Mori from Japan when working with Yaskawa Electric Coorporation.
Applications: Robotics, Aerospace industry, automotive industry, process industry etc.

Course Objective: To introduce the design and development of a mechatronic system.
Instructors: Harikumar Kandath and Nagamanikandan Govindan.

Sensors in Ground Robot

- Wheel Encoder
- Magnetometer
- Inertial Measurement Unit (IMU): contains Accelerometer and Gyroscope.
- Global Positioning System (GPS)
- Range measuring sensor (LIDAR, ultrasonic, camera)

Sensors in UAV

- Inertial Measurement Unit (IMU) contains Accelerometer and Gyroscope.
- Altimeter
- Airspeed sensor
- Magnetometer
- Global Positioning System (GPS)
- Range measuring sensor (LIDAR, ultrasonic, RADAR, camera)

Sensors in Robotic Manipulator

- IMU
- Encoder
- Force-Torque sensor
- Camera

Position Estimation (Odometry)

$$\frac{dx}{dt} = v_x = V\, cos\, \psi \tag{1}$$

$$x(t) = \int_0^t V\, cos\, \psi\, dt + x(0) \tag{2}$$

$$\frac{dy}{dt} = v_y = V\, sin\, \psi \tag{3}$$

$$y(t) = \int_0^t V\, sin\, \psi\, dt + y(0) \tag{4}$$

NB: Errors in the measurement of V and ψ will lead to error in the above-mentioned position estimate.

Limitations of using only wheel encoder and magnetometer for odometry

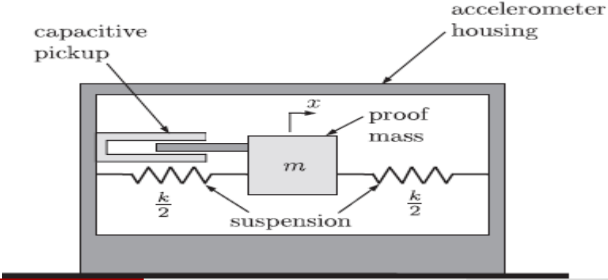
- The angle ψ will not change when computed using magnetometer if the robot can move sideways without changing the orientation.
- Error in wheel encoder based speed estimation during wheel slip. Slip is a condition where wheel rotates without changing the position of the robot as observed in the normal case (i.e. one complete rotation of the wheel does not corresponds to a distance of $2\pi r$) .

MEMS Accelerometer

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F(= ma) \tag{5}$$

For a step input, at steady-state $\frac{d^2x}{dt^2} = 0$, $\frac{dx}{dt} = 0$ making $x \propto F$.

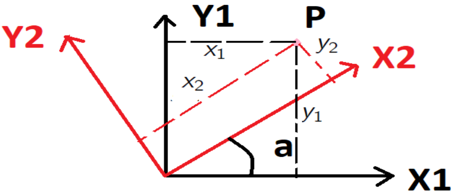
$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k} \tag{6}$$



Definition of position, velocity, acceleration and orientation

- Inertial Frame: position (x, y), velocity (v_x, v_y), acceleration (a_x, a_y).
- Body Frame: velocity (v_{xb}, v_{yb}), acceleration (a_{xb}, a_{yb})

Rotation Matrix



$P = [x_1,\ y_1,\ z_1]^T$ in $X1 - Y1 - Z1$.
$P = [x_2,\ y_2,\ z_2]^T$ in $X2 - Y2 - Z2$.
Rotation about Z axis $(R(a)_1^2)$, i.e. $Z1 = Z2 \implies z_1 = z_2$.

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} cos(a) & sin(a) & 0 \\ -sin(a) & cos(a) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \tag{7}$$

Accelereometer measurement

Accelerometer measures acceleration along body-axis denoted by a_{xb} and a_{yb} respectively.

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} cos\psi & -sin\psi \\ sin\psi & cos\psi \end{pmatrix} \begin{pmatrix} v_{xb} \\ v_{yb} \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} cos\psi & -sin\psi \\ sin\psi & cos\psi \end{pmatrix} \begin{pmatrix} a_{xb} \\ a_{yb} \end{pmatrix} \tag{9}$$

$$\dot{\psi} = r \tag{10}$$

$$a_{xb} = \dot{v}_{xb} - rv_{yb} \tag{11}$$

$$a_{yb} = \dot{v}_{yb} + rv_{xb} \tag{12}$$

MEMS Gyroscope - 1

Coriolis acceleration

$$a_c = 2\omega \times v \tag{13}$$

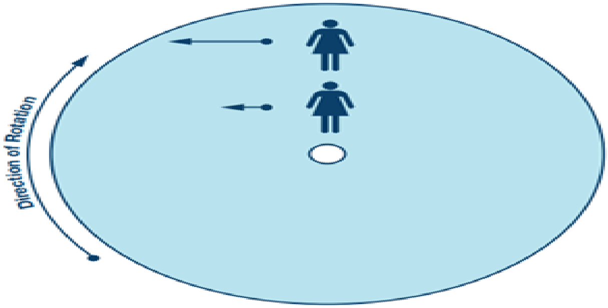


Figure: Motion along a rotating platform

MEMS Gyroscope - 2

Coriolis acceleration

$$a_c = 2\omega \times v \tag{14}$$

$$|a_c| = 2|\omega||v| \tag{15}$$

For a known '$|v|$', $|\omega|$ can be obtained after measuring $|a_c|$.

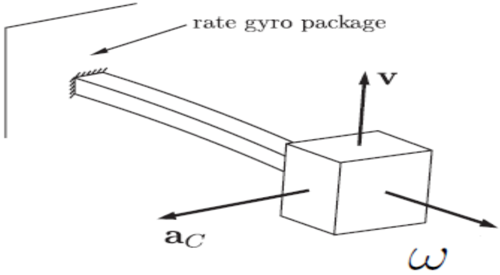


Figure: Working of a gyro

IMU-Encoder Data Fusion for Position Estimation

Wheel encoder based position estimate $(x_w(t),\ y_w(t))$

$$x_w(t) = \int_0^t V(t)\, cos\,\psi(t)\, dt + x(0),\ \ y_w(t) = \int_0^t V(t)\, sin\,\psi(t)\, dt + y(0) \quad (16)$$

IMU based position estimate $(x_i(t),\ y_i(t))$

$$\psi(t) = \int_0^t r(t)\, dt + \psi(0) \quad (17)$$

$$\begin{pmatrix} a_x(t) \\ a_y(t) \end{pmatrix} = \begin{pmatrix} cos\psi(t) & -sin\psi(t) \\ sin\psi(t) & cos\psi(t) \end{pmatrix} \begin{pmatrix} a_{xb}(t) \\ a_{yb}(t) \end{pmatrix} \quad (18)$$

$$v_x(t) = \int_0^t a_x(t)\, dt + v_x(0),\ \ x_i(t) = \int_0^t v_x(t)\, dt + x_i(0) \quad (19)$$

$$v_y(t) = \int_0^t a_y(t)\, dt + v_y(0),\ \ y_i(t) = \int_0^t v_y(t)\, dt + y_i(0) \quad (20)$$

$$x(t) = \alpha x_w(t) + (1-\alpha)x_i(t),\ \ y(t) = \alpha y_w(t) + (1-\alpha)y_i(t),\ \ 0 \le \alpha \le 1 \quad (21)$$

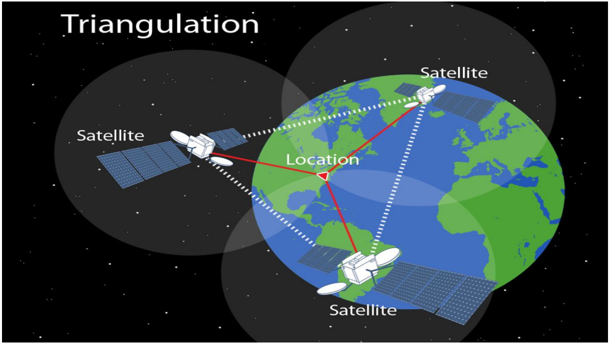
Position

- GPS- Global Positioning System (satellite)

Signal frequency $L_1 = 1575.42$ MHz, $L_2 = 1227.60$ MHz.
Accuracy within a few meters (expressed in CEP (circular error probability).

Triangulation

$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = d_1^2 \tag{22}$$
$$(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 = d_2^2 \tag{23}$$
$$(x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 = d_3^2 \tag{24}$$



Differential GPS

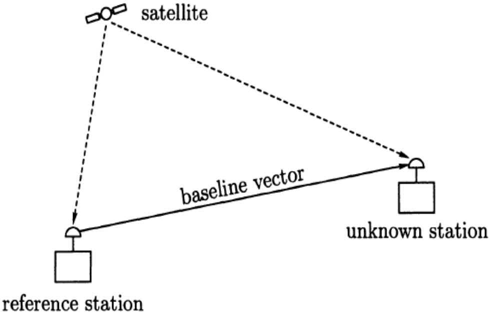


Figure: Relative position estimation

Ellipsoidal and ECEF

ECEF - earth centered earth fixed frame

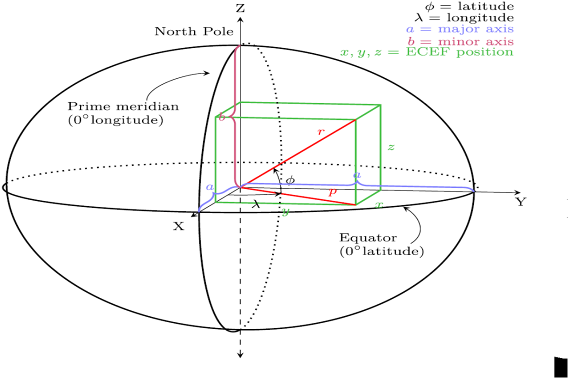
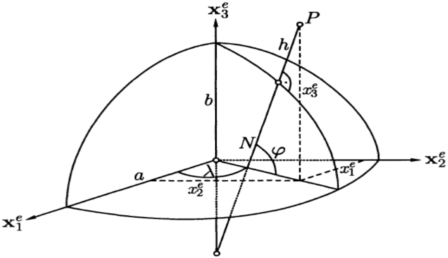


Figure: Ellipsoidal and ECEF frame

Ellipsoidal to ECEF transformation



NB: Here $(\mathbf{x}_1^e, \mathbf{x}_2^e, \mathbf{x}_3^e)$ frame denote the (X, Y, Z) frame (ECEF frame shown in the previous slide).

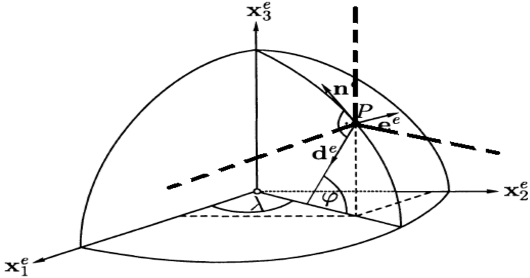
$N = \frac{a^2}{\sqrt{a^2 cos^2\phi + b^2 sin^2\phi}}$, $a = 6378137m$, $b = 6356752m$.

$$\begin{pmatrix} x_1^e \\ x_2^e \\ x_3^e \end{pmatrix} = \begin{pmatrix} (N+h)cos\phi cos\lambda \\ (N+h)cos\phi sin\lambda \\ (\frac{b^2}{a^2}N + h)sin\phi \end{pmatrix} \quad (25)$$

ECEF to NED frame

NED - North, East, Down.

$$\begin{pmatrix} -sin\phi cos\lambda & -sin\lambda & -cos\phi cos\lambda \\ -sin\phi sin\lambda & cos\lambda & -cos\phi sin\lambda \\ cos\phi & 0 & -sin\phi \end{pmatrix} \tag{26}$$



THANK YOU