EC5.203 Communication Theory I (3-1-0-4):

Lecture 5

Analog Communication Techniques: Amplitude Modulation

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HYDERABAD

References

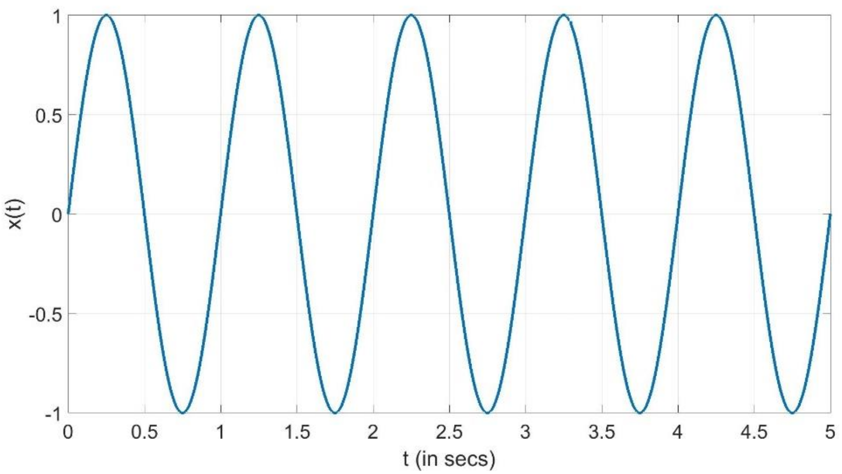
- Chap. 3 (Madhow)

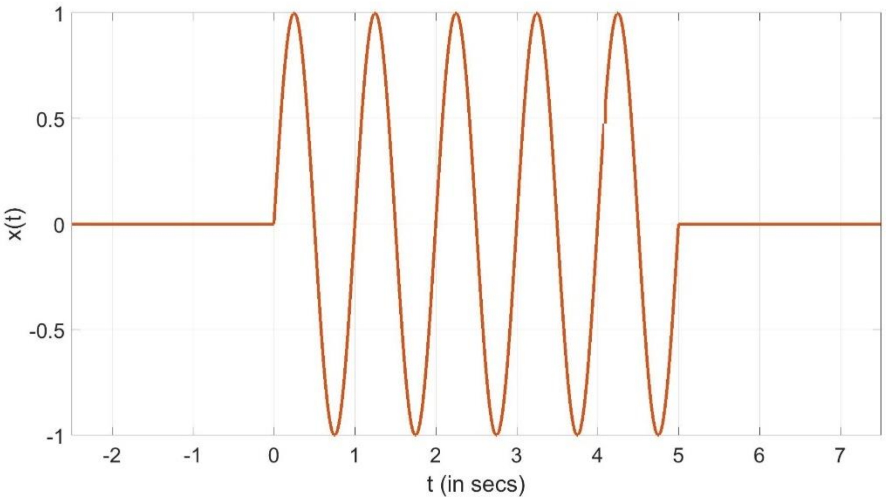
Analog comm techniques: Motivation

- Why bother?
  - After all, the world is going digital
  - Modern comm system designers focused mainly on DSP algorithms for digital comm
- But need to understand the underlying physical analog signals
  - Establishes common language with circuit designers
  - Analog-centric techniques become critical when pushing the limits of carrier frequency, bandwidth and/or power consumption
- Focus of these techniques is on baseband to passband conversion, and back

Terminology and Notations

- Let $m(t)$ denote the message signal with frequency response $M(f)$.
- For a real signal $m(t)$, $M(f) = M^*(-f)$.
- Based on our convenience, we will consider it to be a power or an energy signal.





Terminology and Notations

- Let $m(t)$ denote the message signal with frequency response $M(f)$.
- For a real signal $m(t)$, $M(f) = M^*(-f)$.
- Based on our convenience, we will consider it to be a power or an energy signal.
- Power of the signal is given by

$$\overline{m^2} = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_0^{T_0} m^2(t)\, dt$$

- DC value of the signal is

$$\overline{m} = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_0^{T_0} m(t)\, dt$$

Terminology and Notations

- Let $u_p(t)$ denote the signal transmitted over the channel. Also called passband signal given in term of cartesian coordinates as

$$u_p(t) = \underbrace{\boxed{u_c(t)}}_{\mathrm{I}} \cos(2\pi f_c t) - \underbrace{\boxed{u_s(t)}}_{\mathrm{Q}} \sin(2\pi f_c t)$$

- In Polar coordinates

$$u_p(t) = e(t)\cos(2\pi f_c t + \theta(t))$$

where $e(t)$ is magnitude of the envelope and $\theta(t)$ is the phase.

Key Concepts

- Two ways of encoding info in complex envelope
  - I and Q: amplitude modulation (several variants)
  - Envelope and phase: angle modulation (constant envelope)
- For example, for a sinusoid carrier

$$A_c(t)\cos(2\pi f_c(t)t + \theta_c(t))$$

where $A_c(t)$, $f_c(t)$, $\theta_c(t)$ are the amplitude, frequency, and the phase of the carrier respectively.

Amplitude Modulation

Frequency Modulation

Phase Modulation

Key Concepts

- Up/Down conversion
  - Multiple stages or single stage (superhet or direct conversion)
- Phase locked loop
  - Feedback-based synchronization and tracking

Amplitude Modulation

Example: sinusoidal message

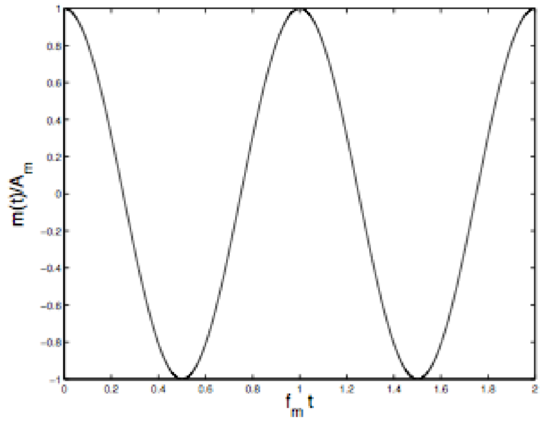
- Consider a sinusoidal message given by

$$m(t) = A_m \cos(2\pi f_m t)$$

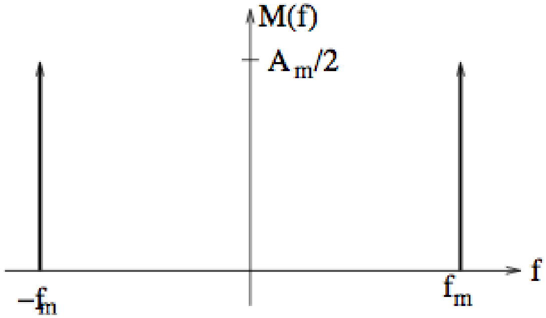
where A_m is magnitude of the envelope and f_m is the signal frequency.

- Fourier transform is given by

$$M(f) = \frac{A_m}{2}(\delta(f + f_m) + \delta(f - f_m))$$



(a) Sinusoidal message waveform



(b) Sinusoidal message spectrum

- Find power and average for this signal. (Assignment)

AM: Double Sideband Suppressed Carrier

- Here the message $m(t)$ modulates the I component of the passband signal $u(t)$ and is given by

$$u_{DSB}(t) = m(t) \cdot A\cos(2\pi f_c t)$$

while the Fourier transform is given by

$$U_{DSB}(f) = \frac{A}{2}(M(f - f_c) + M(f + f_c))$$

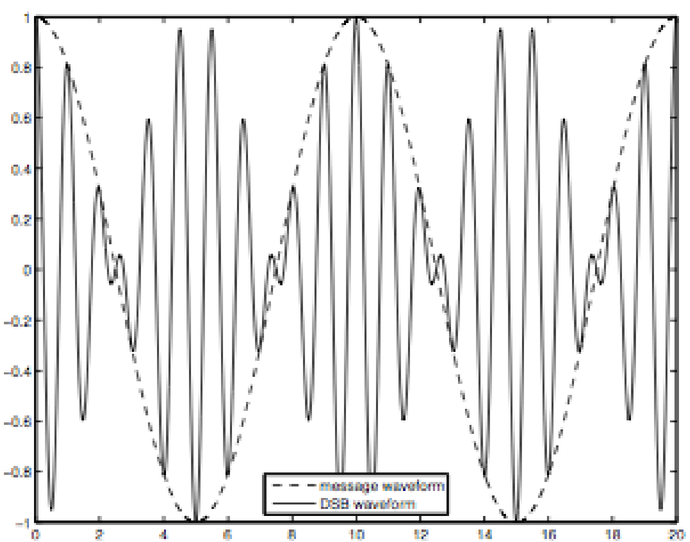
DSB-SC signal for sinusoidal message

Here the signal is given by

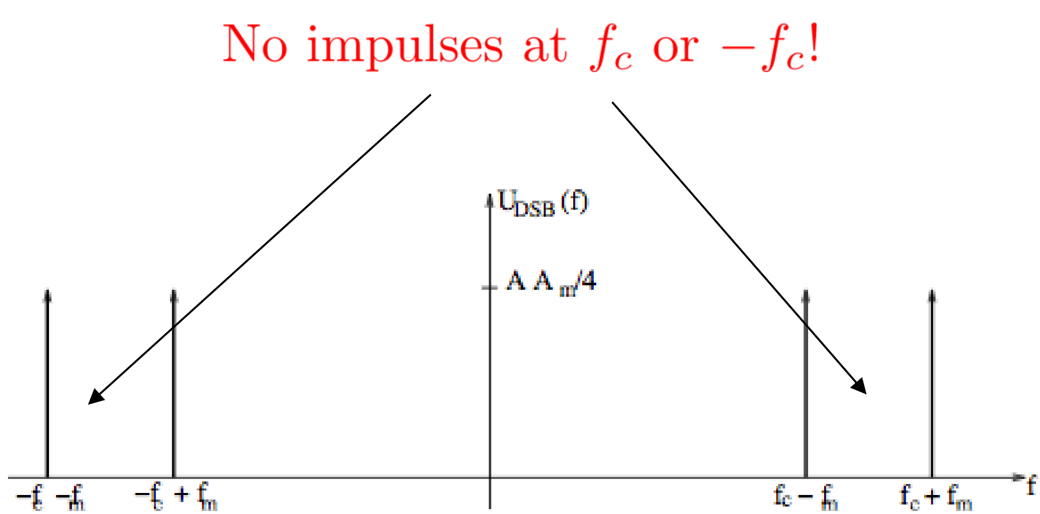
$$u_{DSB}(t) = A_m \cos(2\pi f_m t) \cdot A \cos(2\pi f_c t)$$

while the Fourier transform is given by

$$U_{DSB}(f) = \frac{AA_m}{4}\{\delta(f - f_c - f_m) + \delta(f - f_c + f_m) \\ + \delta(f + f_c + f_m) + \delta(f + f_c - f_m)\}$$



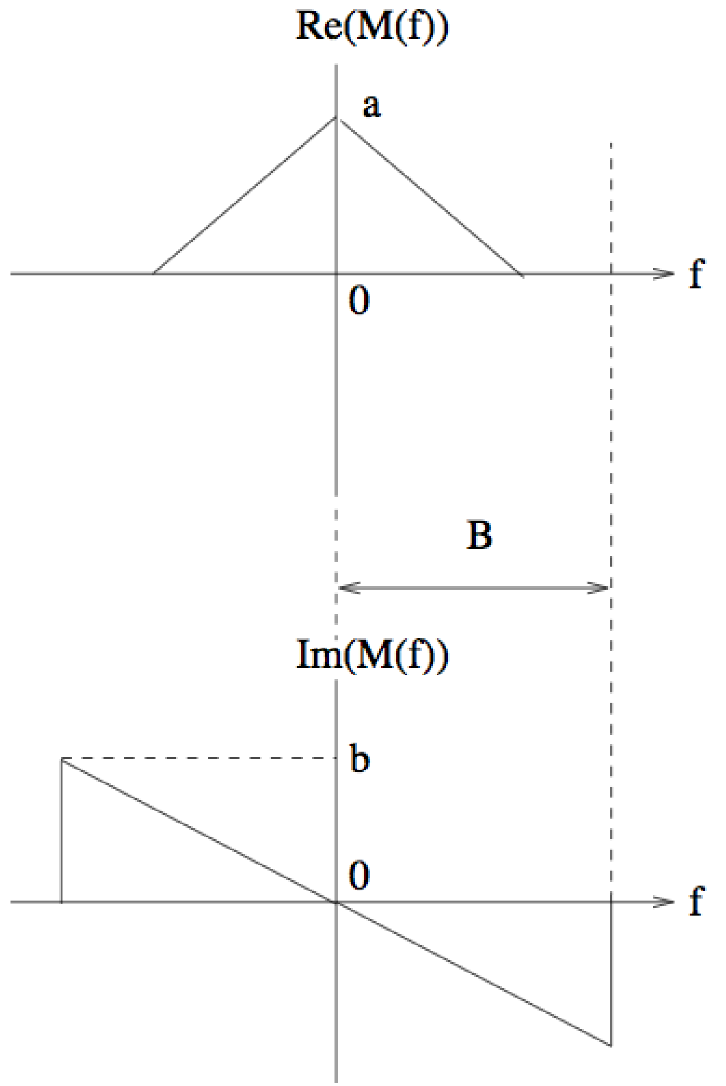
(a) DSB time domain waveform



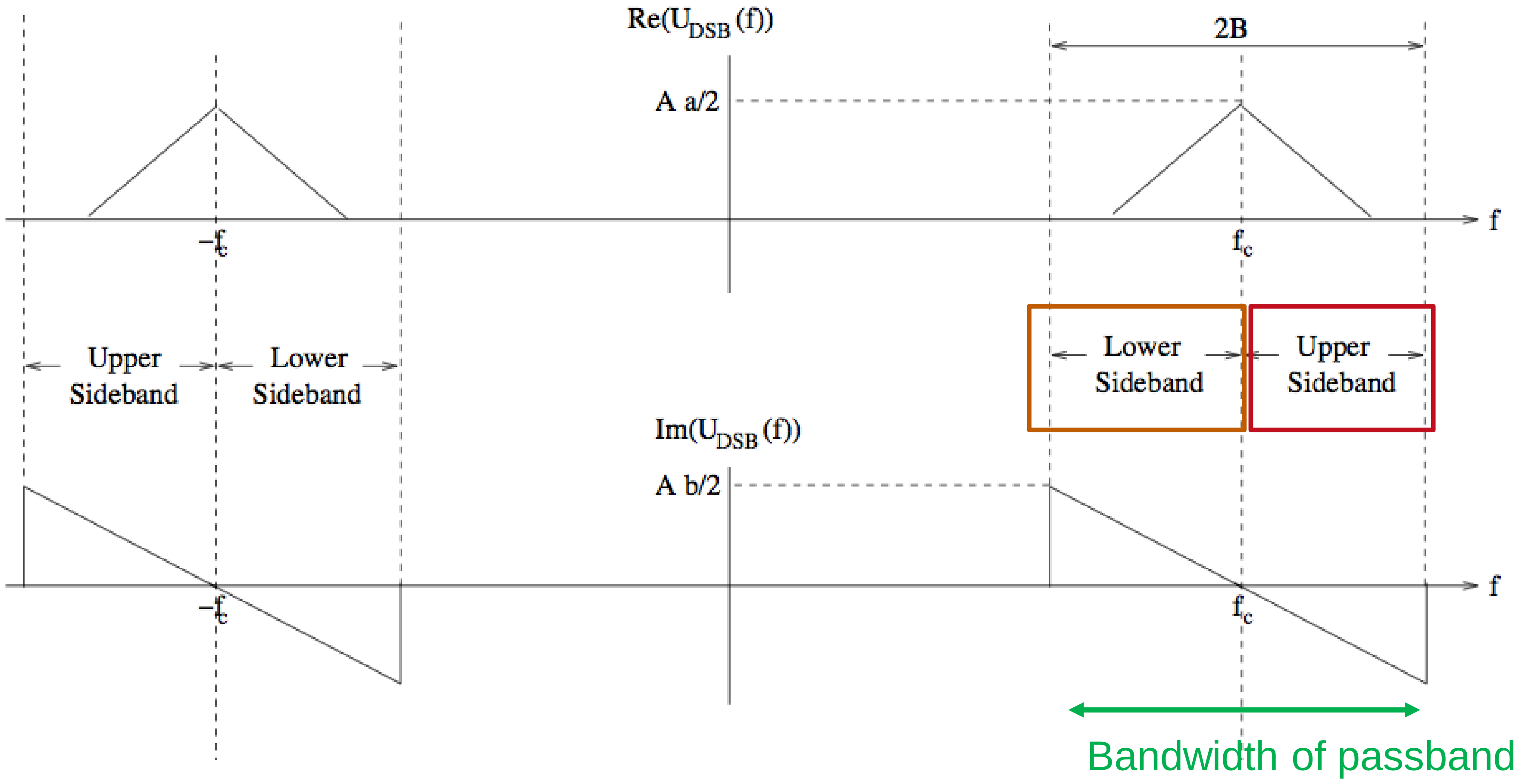
(b) DSB spectrum

Example 2

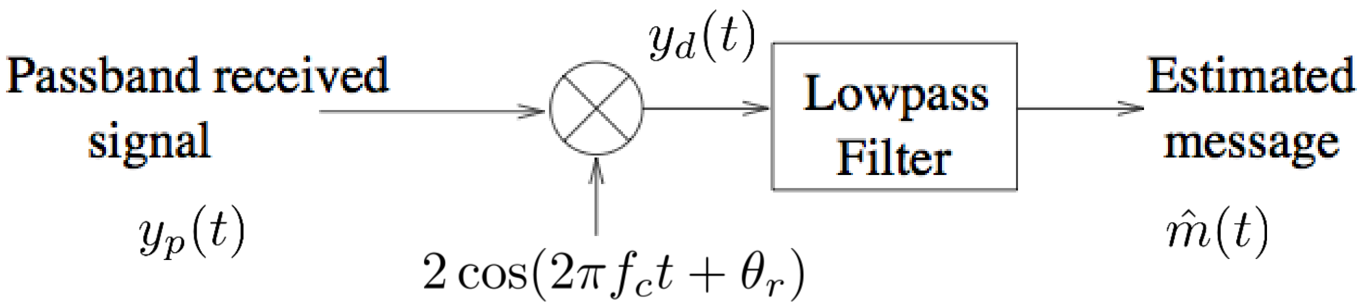
- Consider a message signal $m(t)$ with following frequency response $M(f)$



DSB-SC spectrum for Example 2



Demodulation of DSB-SC



- Standard downconverter to recover I component
- The received signal in the passband is

$$y_p(t) = Am(t)\cos(2\pi f_c t)$$

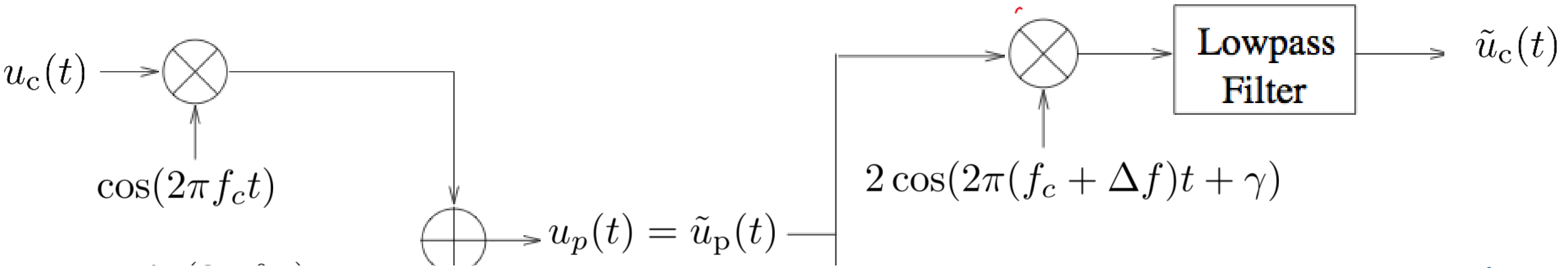
where θ_r is the phase difference arising from the phase offset with respect to local carrier at Rx.

- The output of multiplier followed by low pass filter is

$$\hat{m}(t) = Am(t)\cos\theta_r$$

Try this as a assignment!

Recap: Chapter 2
Effect of Frequency and Phase Offset



Focus in this chapter mostly on I component!

Upconversion (baseband to passband)

Downconversion (passband to baseband)

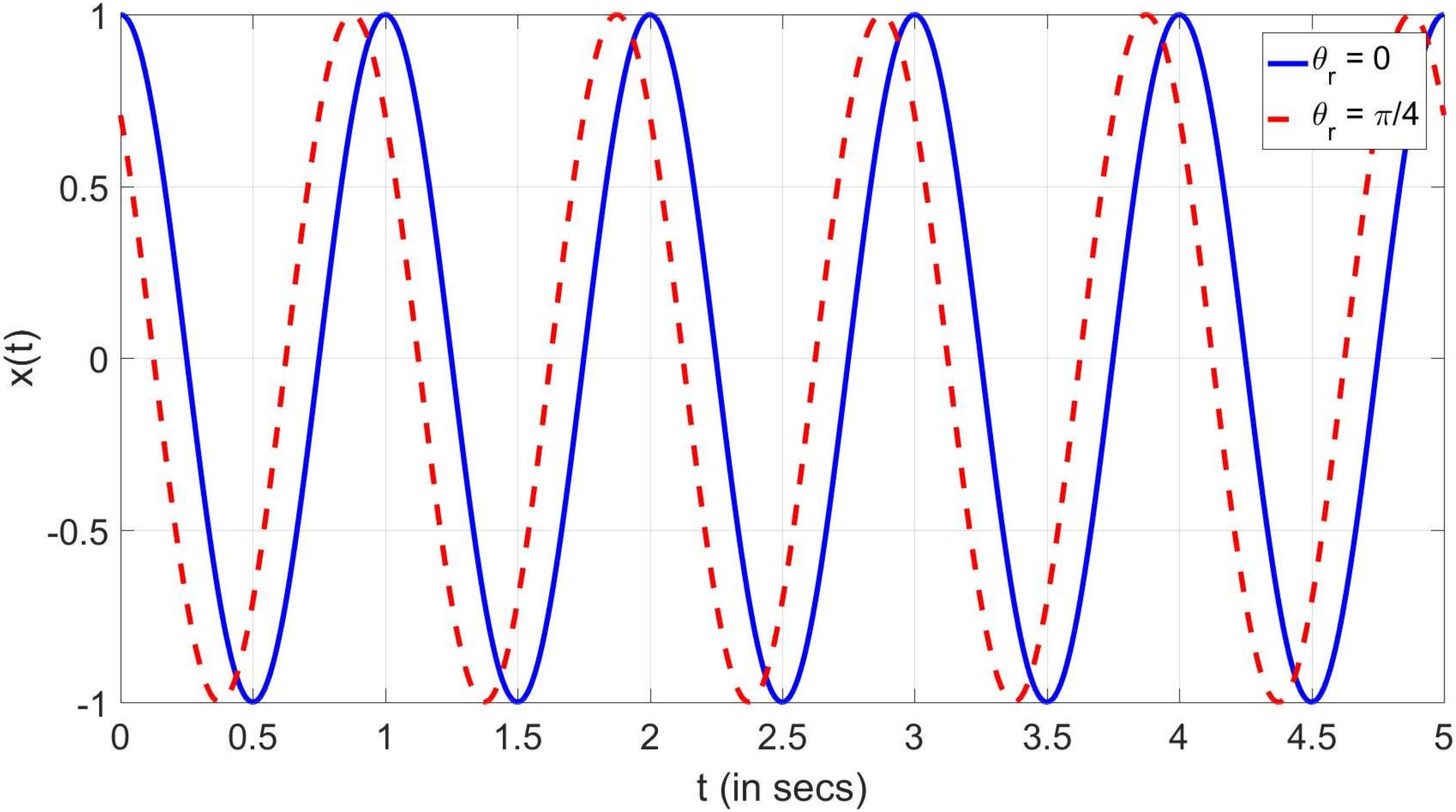
- Show that in this case

$$\tilde{u}_c(t) = u_c(t)\cos\phi(t) + u_s(t)\sin\phi(t)$$

where $\phi(t) = 2\pi\Delta f t + \gamma$ is the phase offset resulting from frequency offset Δf and the phase offset γ. Here $\theta_r = \phi(t)$

Example: Phase Offset

$$x(t) = \cos(2\pi f_c t + \theta_r)$$



Here $\theta = \gamma$

Causes of Phase Offset

- Frequency offset: The local oscillator at the receiver is generating frequency at $f_c + \Delta f$

  $$\theta_r = 2\pi \Delta f t$$

  This happens as the two physical devices cannot be exactly same resulting in slight differences. Here there will be phase difference even if they are same place.

- Timing offset: The transmitter and receiver have slightly different time references or they are separated by distance d resulting in time offset of δt.

  $$\theta_r = 2\pi f_c \Delta t$$

Need of Coherent Detection

- The output of multiplier followed by low pass filter is

$$\hat{m}(t) = Am(t)\cos\theta_r$$

- For $\theta_r = 0$, $\hat{m}(t) = Am(t)$
- For $\theta_r = \pi/2$, $\hat{m}(t) = 0$
- For $\theta_r(t) = 2\pi\Delta f t + \phi$, time varying signal degradation in amplitude and unwanted sign changes.
- Need of synchronization of phase of the local oscillator with the phase of incoming signal:
  - Phased locked loop (PLL)
- Switch to other AM techniques which do not need synchronization
  - Conventional AM or DSB (with carrier)

Conventional AM

Conventional AM

- Add a large carrier component to a DSB-SC signal so that the passband has the following form

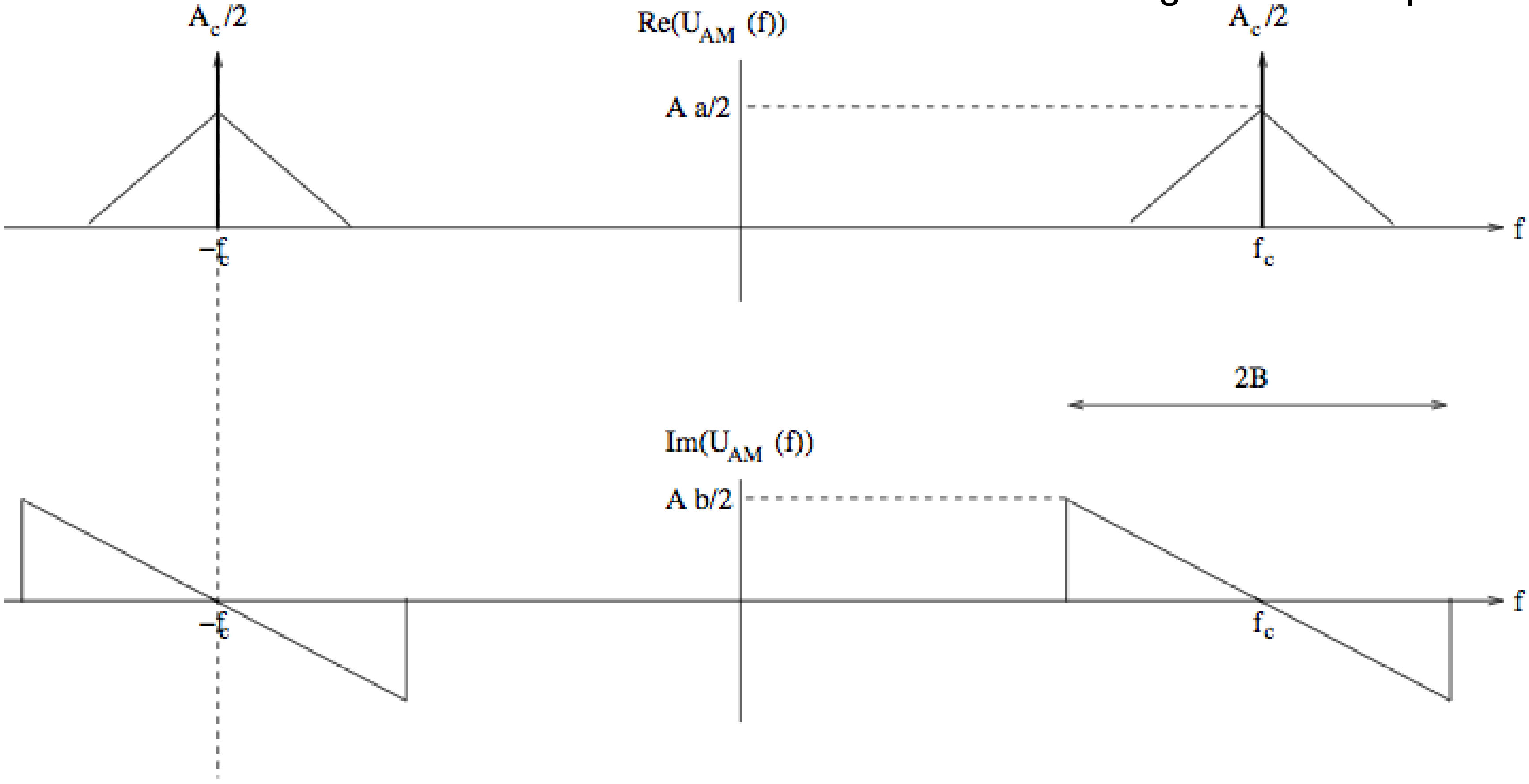
$$\begin{aligned} u_{\mathrm{AM}}(t) &= (Am(t) + A_c)\cos(2\pi f_c t) \\ &= Am(t)\cos(2\pi f_c t) + A_c \cos(2\pi f_c t) \end{aligned}$$

- Taking Fourier transform

$$U_{\mathrm{AM}}(f) = \frac{A}{2}(M(f - f_c) + M(f + f_c)) + \frac{A_c}{2}(\delta(f - f_c) + \delta(f + f_c))$$

Conventional AM: spectrum

Strong carrier component



Envelope and its importance

- Add a large carrier component to a DSB-SC signal so that the passband has the following form

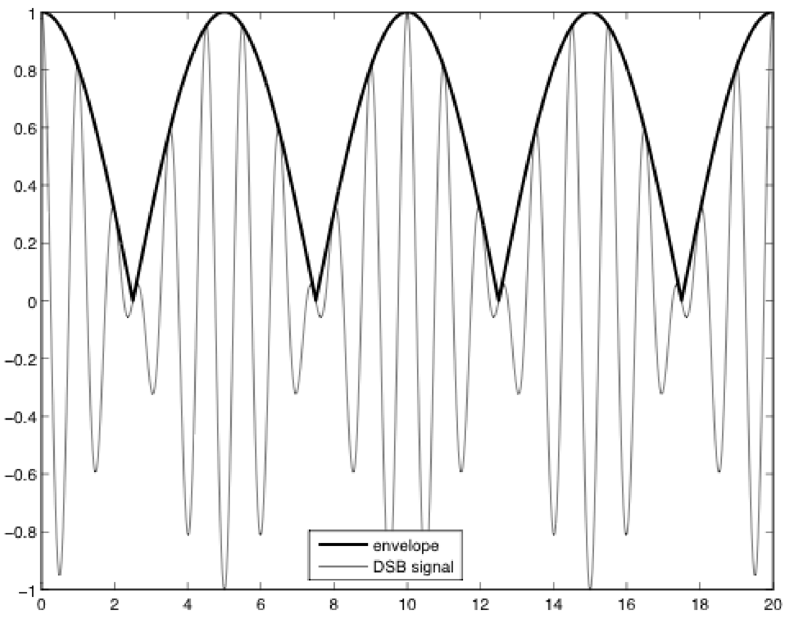
$$\begin{aligned} u_{\mathrm{AM}}(t) &= \boxed{(Am(t) + A_c)} \cos(2\pi f_c t) \\ &= Am(t)\cos(2\pi f_c t) + A_c \cos(2\pi f_c t) \end{aligned}$$

- Envelope is given by $e(t) = |Am(t) + A_c|$.
- If $Am(t) + A_c > 0$, then $e(t) = Am(t) + A_c$. In this case, message $m(t)$ can be recovered from $e(t)$.

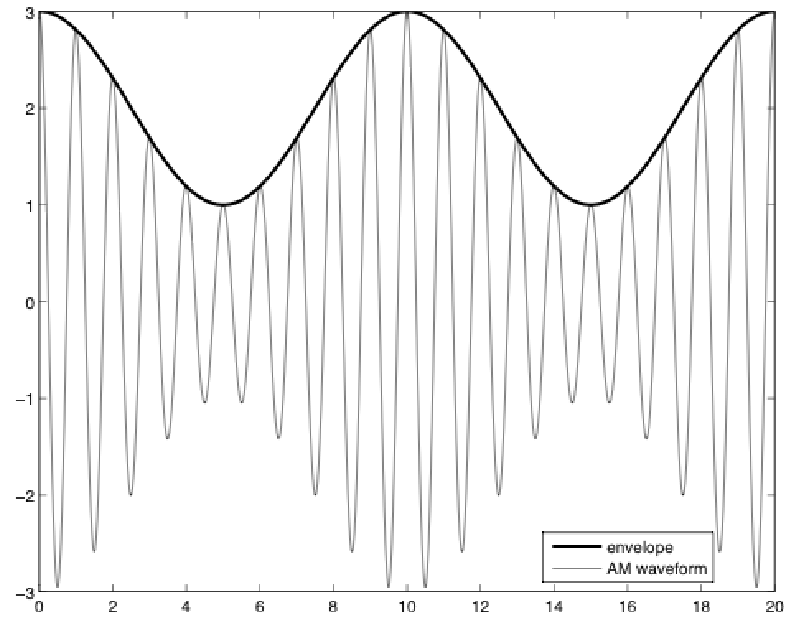
What does the envelope tell us?

- Example: sinusoidal message signal

$$m(t) = A_m \cos(2\pi f_m t)$$



DSB-SC signal
Envelope = message magnitude
→ Envelope detection loses info in message sign.



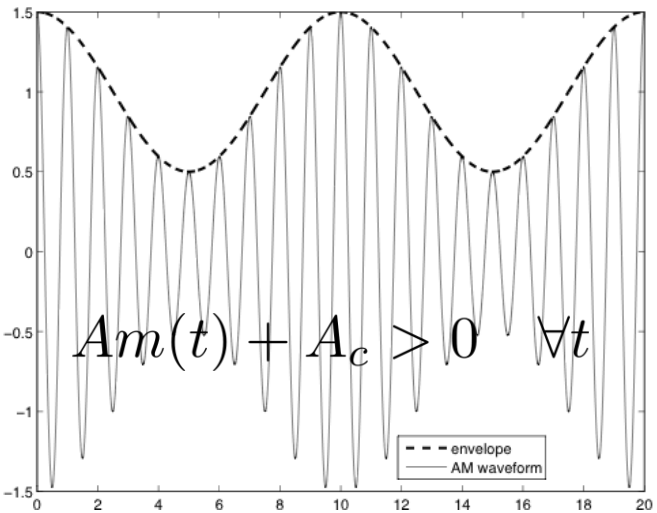
DSB + strong carrier component
Envelope = message + DC
→ Envelope detector + DC block recovers message info

Sidestepping sync requirement

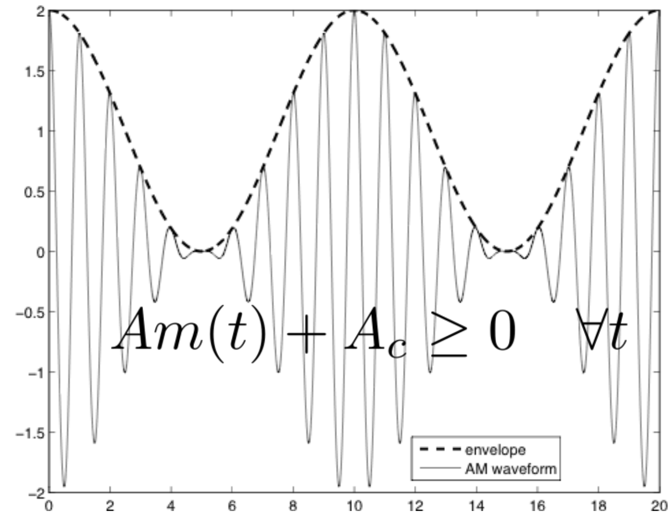
- The envelope (or magnitude of complex envelope) does not depend on carrier phase
- Suppose we can extract the envelope of a passband signal (will soon see a simple circuit for this purpose)
  - Does not require carrier sync
- Can we recover the message?

Constraint for recovering message from envelope

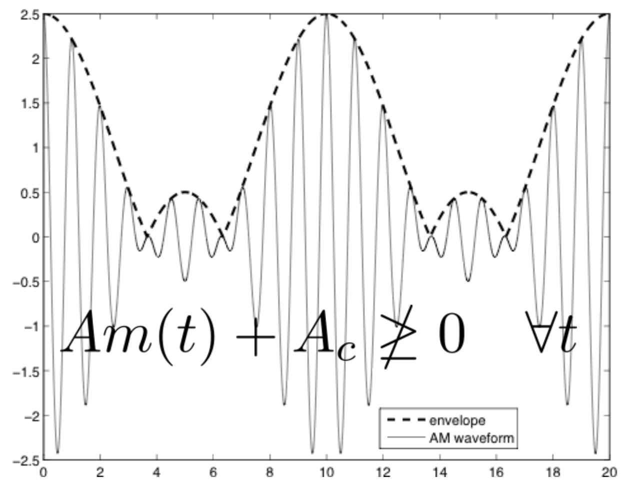
Example of sinusoidal message



Envelope = message + DC

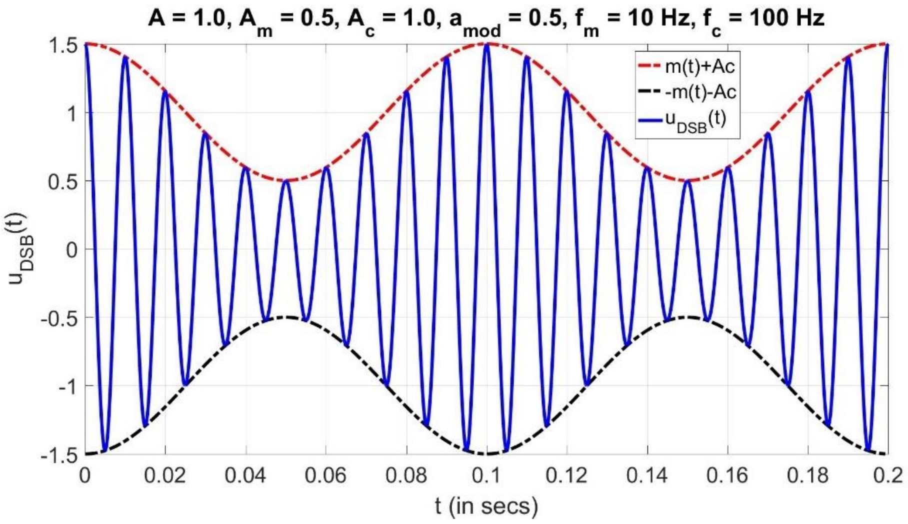


Envelope = message



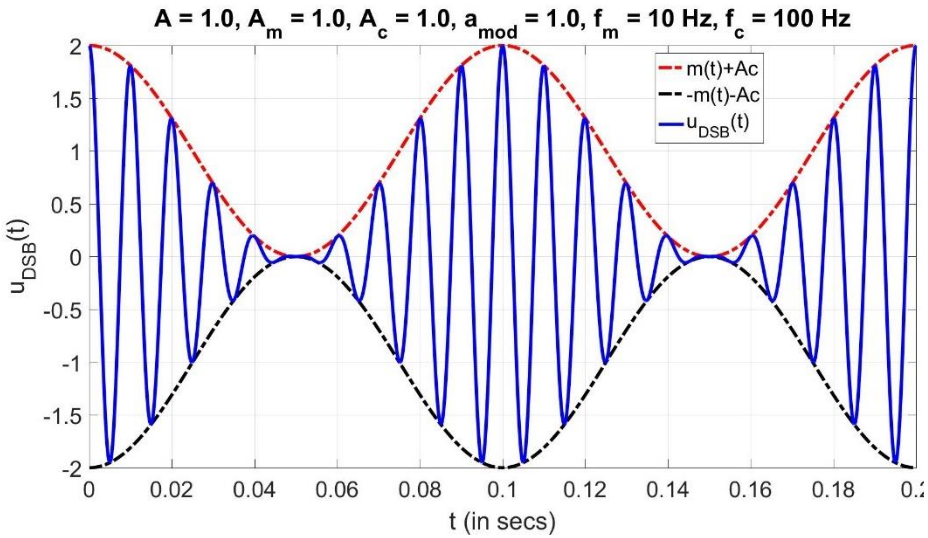
Message info not preserved in envelope

Example of Sinusoidal Message



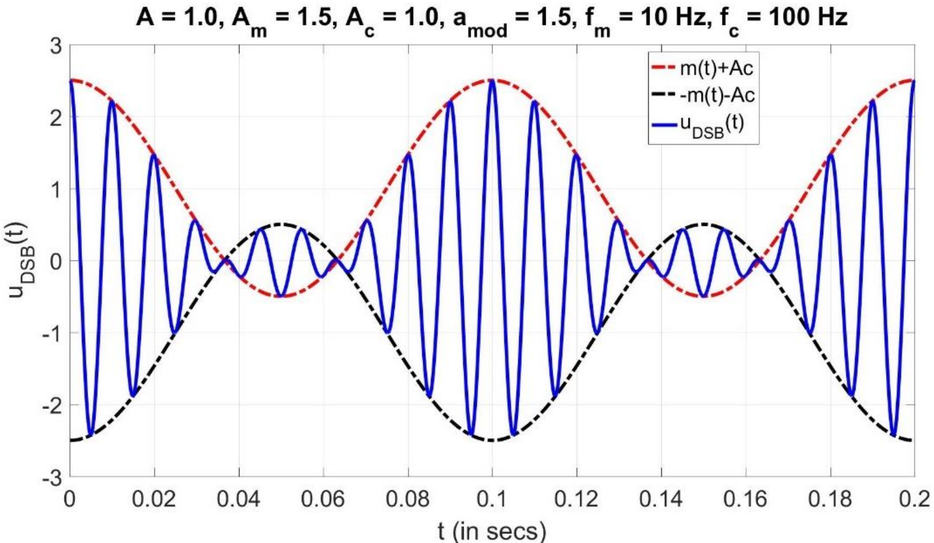
$$Am(t) + A_c > 0 \quad \forall t$$

Envelope = message + DC



$$Am(t) + A_c \geq 0 \quad \forall t$$

Envelope = message + DC



Message info not preserved in envelope

$$Am(t) + A_c \ngeq 0 \quad \forall t$$

Modulation Index

- Condition needed for envelope to preserve message info

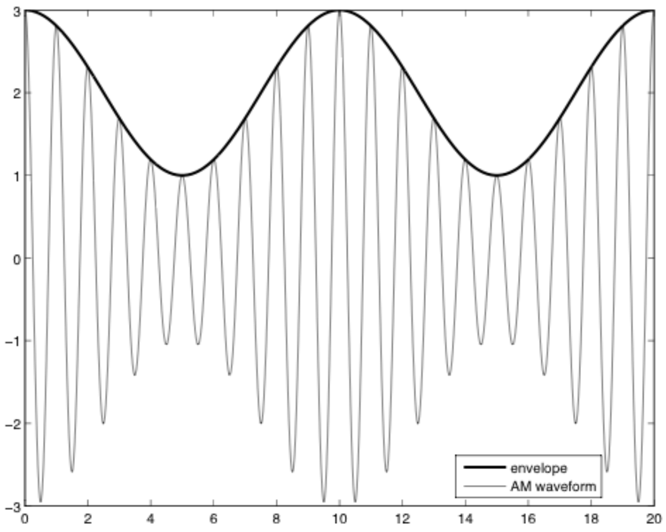
$$A\ m(t) + A_c > 0 \quad \forall t$$

$$A \min_t m(t) + A_c > 0$$

- Can be expressed in terms of modulation index

$$a_{\text{mod}} = \frac{AM_0}{A_c} = \frac{A|\min_t m(t)|}{A_c}$$

- For signal to be recoverable, $a_{\text{mod}} \leq 1$.



AM signal in terms of modulation index

- Convenient to normalize message so that the largest negative swing is -1

$$m_n(t) = \frac{m(t)}{M_0} = \frac{m(t)}{|\min_t m(t)|}$$

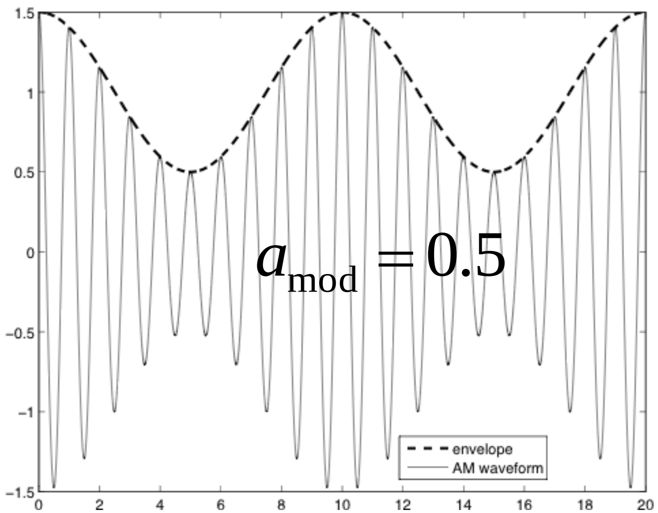
$$\min_t m_n(t) = \frac{\min_t m(t)}{M_0} = -1$$

- AM signal in terms of modulation index and normalized message

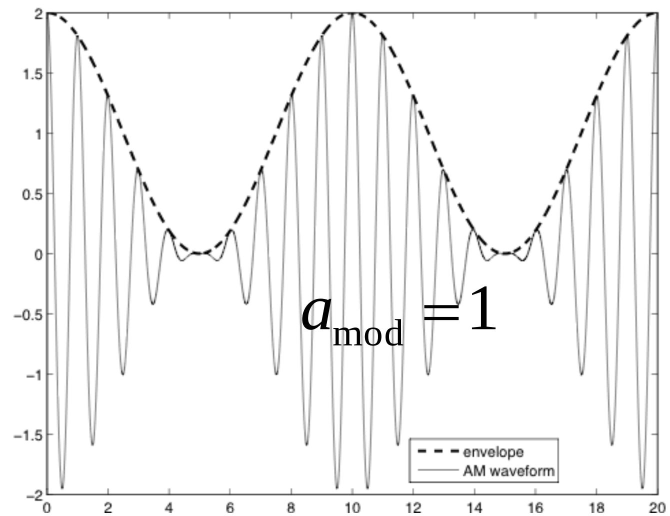
$$y_p(t) = B(1 + a_{\text{mod}} m_n(t)) \cos(2\pi f_c t + \theta_r)$$

Effect of modulation index

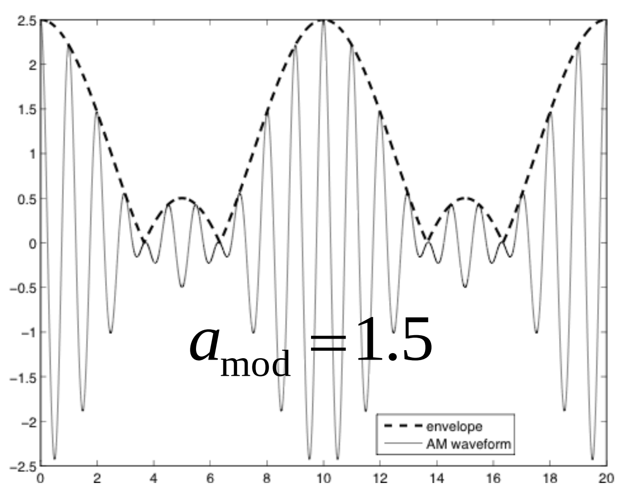
Example of sinusoidal message



Envelope = message + DC

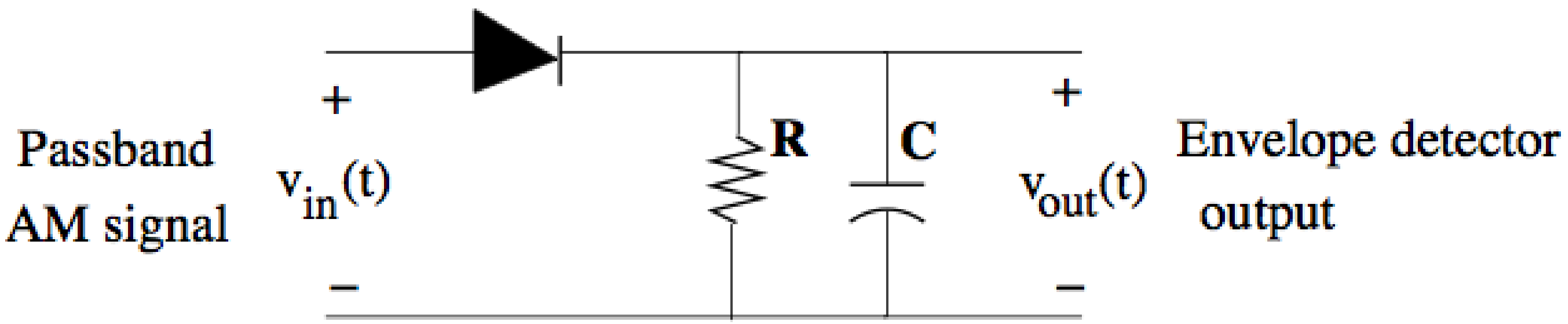


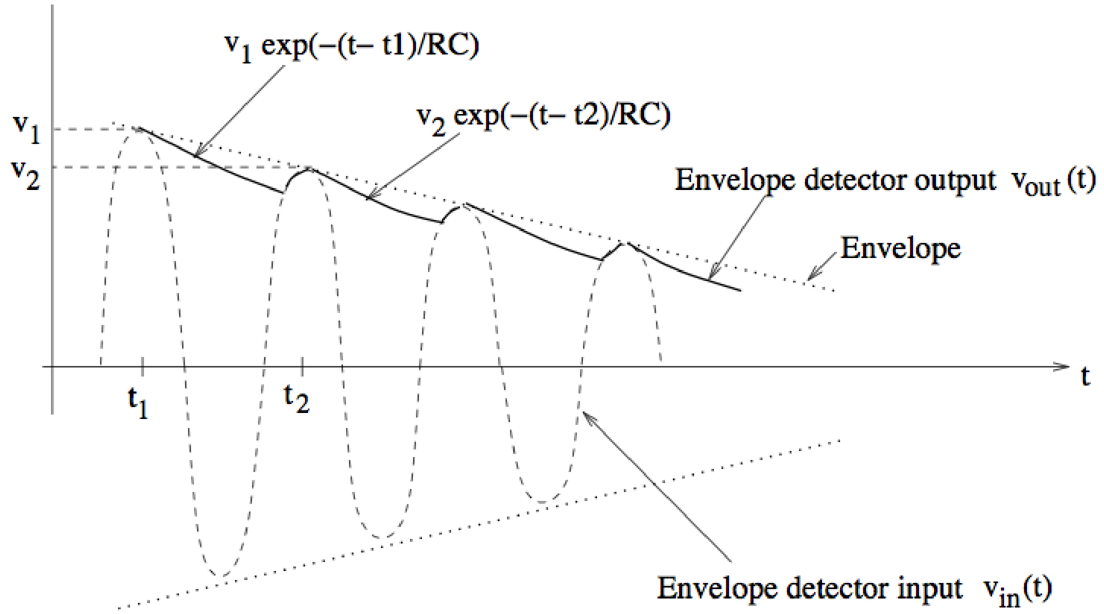
Envelope = message



Message info not preserved in envelope

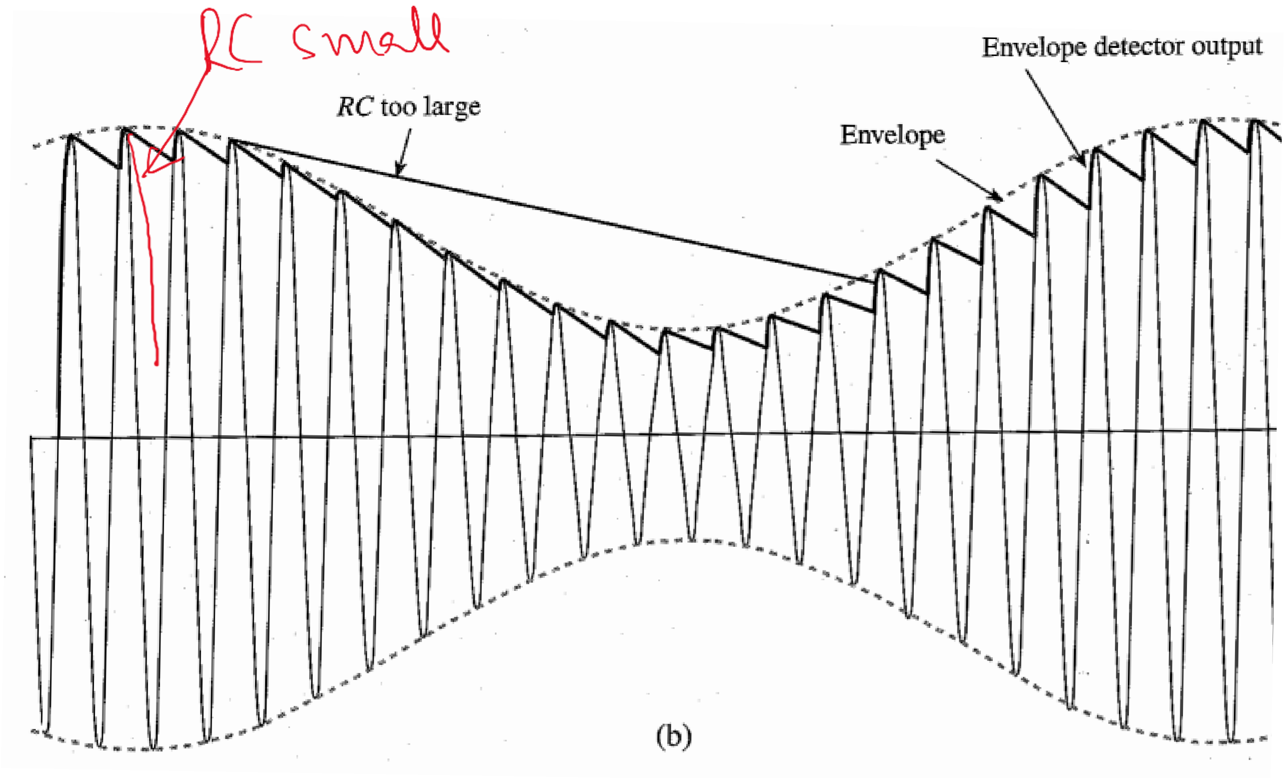
Envelope Detectors





Positive carrier cycle ➔ capacitor charges up (reaches value of envelope)
Negative carrier cycle ➔ capacitor discharges with RC time constant

Envelope detector operation



Positive carrier cycle ➔ capacitor charges up (reaches value of envelope)
Negative carrier cycle ➔ capacitor discharges with RC time constant
Should not discharge too fast during negative cycle
Should react fast enough to follow variations in envelope (which depend on message bandwidth B)

$$\frac{1}{f_c} \ll RC \ll \frac{1}{B}$$