Lecture 8

EM Waves &Radio Frequency-Based Sensors

Intensity of an Electromagnetic Wave:

The **intensity (I)** of an electromagnetic wave is defined as the **energy crossing per unit time per unit area** in a direction perpendicular to the wave propagation.

Mathematical Expression:

$$I = \frac{U}{A \cdot t}$$

Where:

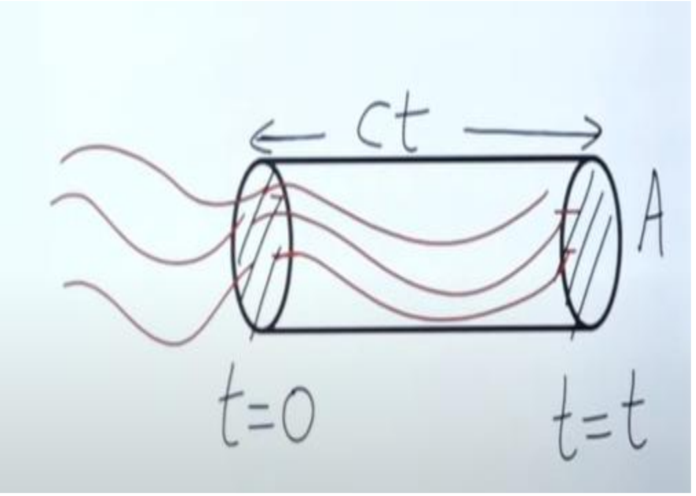
- I = Intensity (W/m^2)
- U = Total energy crossing through the surface
- A = Area through which energy passes
- t = Time duration

Unit Calculation:

$$I = \frac{\mathrm{J}}{m^2 \times s}$$

$$I = \frac{\mathrm{J}}{m^2 \cdot s} \implies \mathrm{W/m^2}$$

The energy (U) inside the cylindrical volume is given by:



$$U = u \times \text{Volume}$$

Where:

- u = **Energy density** of the electromagnetic wave
- Volume = $A \times ct$ (cylinder of cross-sectional area A and length ct)

Substitute Energy Density:

$$U = \left(\frac{1}{2}\epsilon_0 E_0^2\right) \times (A \times ct)$$

$$U = \frac{1}{2}\epsilon_0 E_0^2 Act$$

This formula represents the **total energy** inside the cylindrical volume during the time interval t.

Intensity of Electromagnetic Wave:

The intensity (I) is the **energy per unit area per unit time**:

$$I = \frac{U}{A \times t}$$

Substitute the value of U:

$$I = \frac{\frac{1}{2}\epsilon_0 E_0^2 Act}{A \times t}$$

$$I = \frac{1}{2}\epsilon_0 E_0^2 c$$

This formula shows that the **intensity of an electromagnetic wave** is proportional to the square of the electric field amplitude and the speed of light.

For a plane EM wave, the intensity can also be expressed as:

$$I = \frac{1}{2}\epsilon_0 c E_0^2$$

Where:

- ϵ_0 = Permittivity of free space ($8.85 \times 10^{-12}\,\mathrm{F/m}$)
- c = Speed of light in a vacuum ($3 \times 10^8\,\mathrm{m/s}$)
- E_0 = Peak electric field strength (V/m)

Alternative Expression:

Since the magnetic field B_0 is related to the electric field by $B_0 = \frac{E_0}{c}$, the intensity can also be written as:

$$I = \frac{1}{2}\frac{E_0 B_0}{\mu_0}$$

Where:

- μ_0 = Permeability of free space ($4\pi \times 10^{-7}\,\mathrm{H/m}$)

Electric Displacement Vector (D)

Gauss's Law for the Electric Field:

Gauss's law states that the **electric flux** through a closed surface is proportional to the **total charge enclosed**:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{total}}}{\epsilon_0}$$

Where:

- $\mathbf{E}$ = Electric field
- $d\mathbf{A}$ = Infinitesimal area element
- Q_{total} = Total charge enclosed
- ϵ_0 = Permittivity of free space

The **total charge density** (ρ) is the sum of **free charge density** (ρ_f) and **bound charge density** (ρ_b):

$$\rho = \rho_f + \rho_b$$

Polarization and Bound Charges:

The **polarization vector ($\mathbf{P}$)** represents the **electric dipole moment per unit volume**. The **bound charge density** is given by:

$$\rho_b = -\nabla \cdot \mathbf{P}$$

Hence, the total charge density becomes:

$$\rho = \rho_f - \nabla \cdot \mathbf{P}$$

Substitute in Gauss's Law:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} = \frac{\rho_f - \nabla \cdot \mathbf{P}}{\epsilon_0}$$

Rearranging the equation:

$$\nabla \cdot (\epsilon_0 \mathbf{E} + \mathbf{P}) = \rho_f$$

Defining the Electric Displacement Vector:

The quantity inside the divergence can be expressed as the **electric displacement vector** ($\mathbf{D}$):

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

Thus, Gauss's law in terms of $\mathbf{D}$ becomes:

$$\nabla \cdot \mathbf{D} = \rho_f$$

This equation states that the **divergence of the electric displacement vector equals the free charge density**.

- The electric displacement vector ($\mathbf{D}$) effectively isolates the influence of **free charges** from **bound charges**.
- In a medium, the electric field $\mathbf{E}$ is influenced by **both free and bound charges**, while $\mathbf{D}$ only accounts for the **free charges**.
- This is particularly useful when dealing with **polarized dielectrics**, as it separates **external influences from internal polarization effects**.