EC5.203 Communication Theory (3-1-0-4):

Introductory Class

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Jan. 06, 2025

and

Jan. 09, 2025

INTERNATIONAL INSTITUTE OF
INFORMATION TECHNOLOGY

HYDERABAD

My Background: *Academics and Industry*

BE (Electronics)
1998-2002

ME (Telecommunications)
2002-2004

Sr. Wireless Engg.
2004-2007

ESQUBE

IISc Startup

PhD (Signal Processing for Communications) (2007-2012)
Post-Doc (2013-2014)

Aalto University
School of Engineering

INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
HYDERABAD

Assistant Professor since Dec. 2014
Associate Professor since July 2021

I am also a Senior IEEE member

IEEE
HYDERABAD SECTION

Aalto University

- Formerly famous as Helsinki University of Technology (TKK)
- Founded in 1849, got the university status in 1908
- Best in Finland and a top ranking university in Europe, especially in ICT
- Famous Alumni:
  - **Alvar Aalto (World Famous Designer),**
  - Founders of **Linux, AngryBirds, Supercell, SSH, F-Secure**
  - CEOs of **Nokia, Kone, MySQL**
  - **Noble Laurete** in Chemistry

- Merged with Helsinki School of Business and University of Art and Design in Helsinki to form Aalto University in 2010
- Motivation behind Aalto: A unique interdisciplinary university to create new innovative thought

My Background: *Research Areas*

- **Signal Processing and Machine Learning for Wireless Communication**
  - Internet of Things (IoT)
  - 5G and Beyond
  - Satellite Communications
  - Cognitive Radios
- During PhD and post-doc
  - Spectrum Sensing for Cognitive Radios
  - **Encor2** (2013-2014): TEKES funded project titled *Enabling methods for dyNamic spectrum access and COgnitive Radios*
  - **SENDORA** (2008-2010): An EU FP7 project titled *SEnsor Networks for Dynamic and cOgnitive Radio Access*
- At IIITH
  - Cognitive Radios for 5G and Beyond (2015-2019)
  - IoT for Smart Cities (2018-ongoing)
    - IoT-enabled Smart Cities: Pollution, Health and Governance *(DST and PRIF)*
    - CoE on IoT for Smart Cities *(India-EU Collaboration Project for Standardization)*
    - Smart City Living Lab (*MEITY and Smart City Mission*)
    - 5G Use Case Lab at IIITH (*DoT*)
  - Satellite Communication for IoT (2020-ongoing)

Teaching Assistants

- Baly Naganjani (MS, ECE)
- Nikhil Singh Parihar (MS, ECE)
- M. Manasa (MS, ECE)
- Vishnupriya (UG4, ECE)

Outline

- Course Intro
  - What is a communication system?
    - Channel Issues
    - Analog Communication
    - Digital Communication
  - Motivation and Importance

- Course Administration
  - Syllabus
  - Resources
  - Evaluation
  - Assignments
  - Tutorials

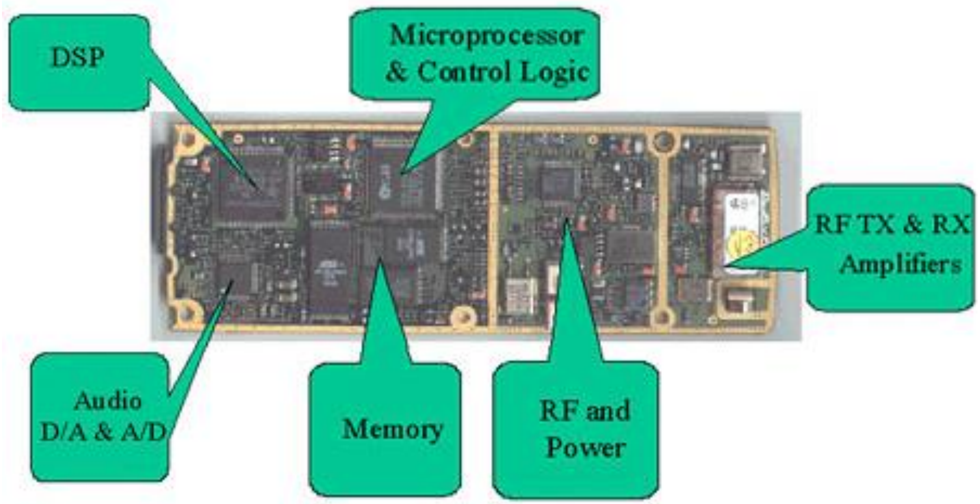
Introduction and Motivation

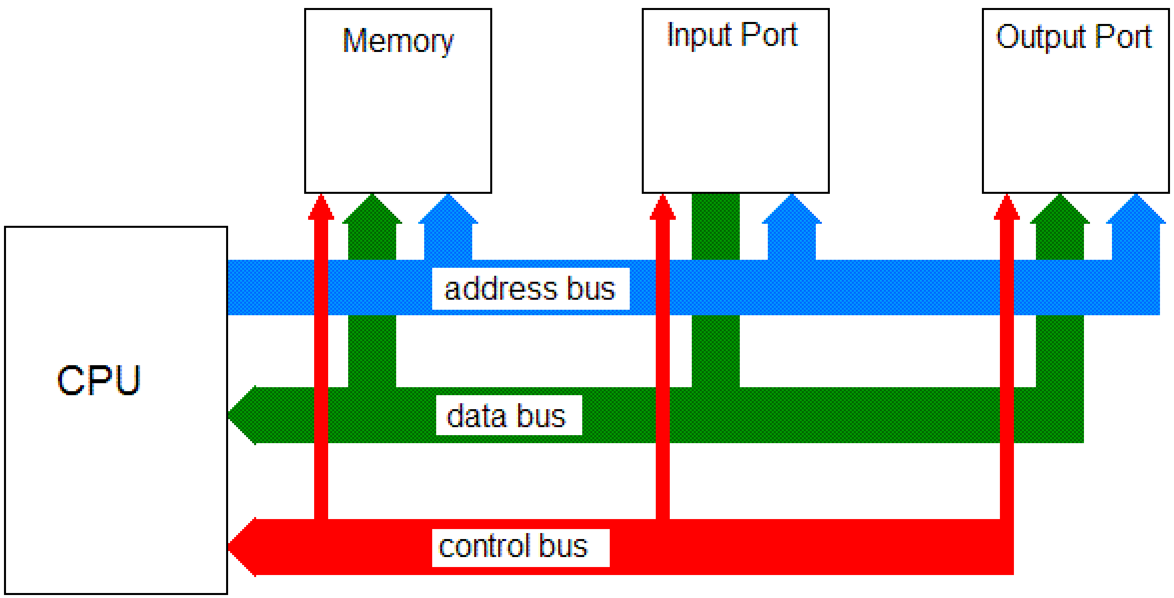
What is Communication?

- Communication is defined as transmission of information from source to destination via a transmission medium

- Examples:
  - AM and FM Radios
  - TV
  - Phone call/Whatsapp Message/Internet
  - Accessing Intranet over Ethernet
  - Microprocessor and peripherals

Examples: Microprocessor and Peripherals





Communication: A slightly different perspective

- Information transfer can be across space or time
  - Separated in space (telephony, web browsing,…)
  - Separated in time ??
    - *Storage of information - CDs, DVDs, hard drives, cloud*

Popular Communication Systems

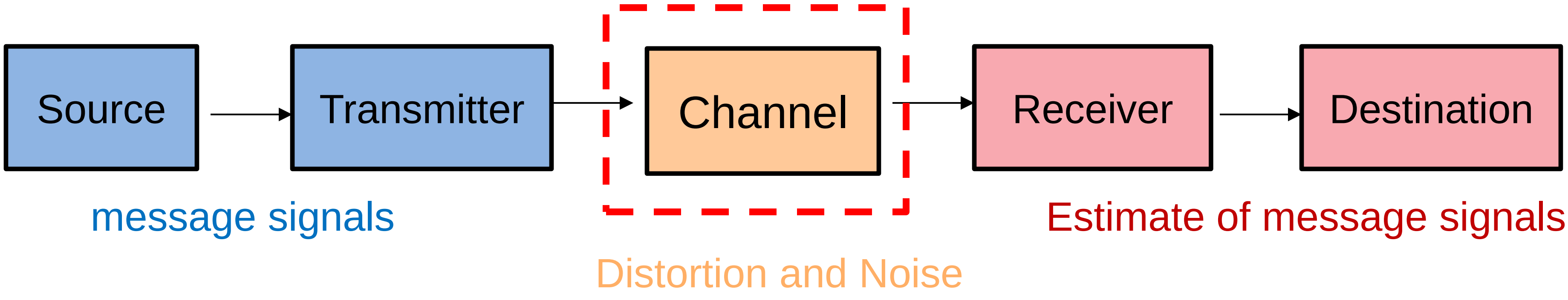
- Cellular Communication (2G/3G/4G/5G)
- LAN
  - Ethernet
  - WLAN
- Broadcasting: radio and TV
- Bluetooth
- Sensor networks
- Satellite communication
- Many more……..

Why Wireless Communication?

- Wired communication will always be better than wireless
- Wireless provides “Anytime Anywhere communication”
- A necessity in today’s world
- Mobile phone has become an addiction
  - Teenagers and technology: 'I'd rather give up my kidney than my phone’
  - Digital communication is not just prevalent in teenagers' lives. It IS teenagers' lives
  - https://www.theguardian.com/lifeandstyle/2010/jul/16/teenagers-mobiles-facebook-social-networking

Key Steps in Communication Link

- Insertion of information into a signal, termed the transmitted signal, compatible with the physical medium of interest.
- Propagation of the signal through the physical medium (termed the channel) in space or time
- Extraction of information from the signal (termed the received signal) obtained after propagation through the medium.



Questions?

CHANNEL

The Main Resource and

the Main Challenge!

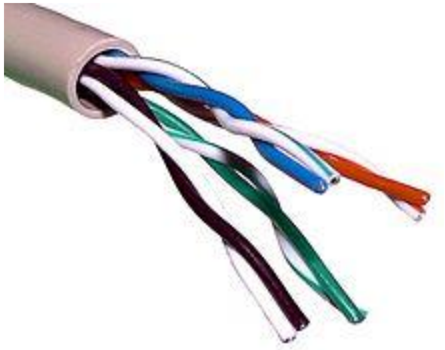
Types of Channel

Classification based on Medium

- Guided channels
  - Copper wire (twisted pair)
  - Coaxial cable
  - Optical fibre
  - microwave guides

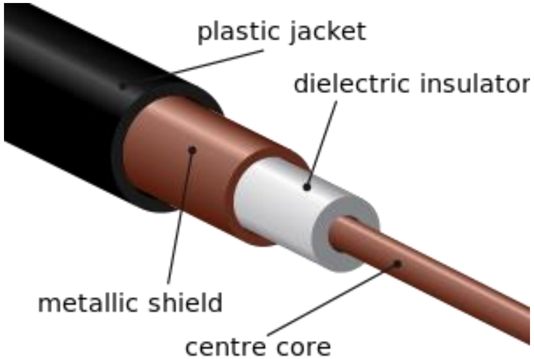
- Unguided channels
  - wireless channel
  - underwater acoustic channel

Wired Medium



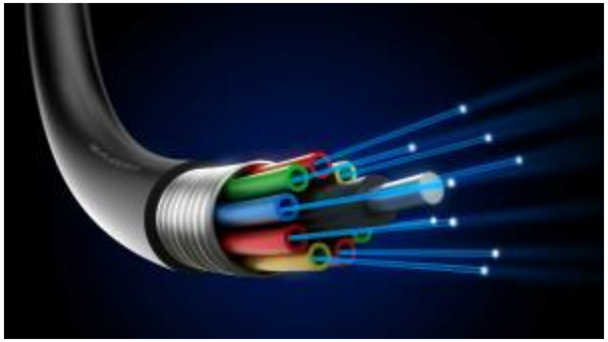
Unshielded Twisted Pair

https://en.wikipedia.org/wiki/Twisted_pair



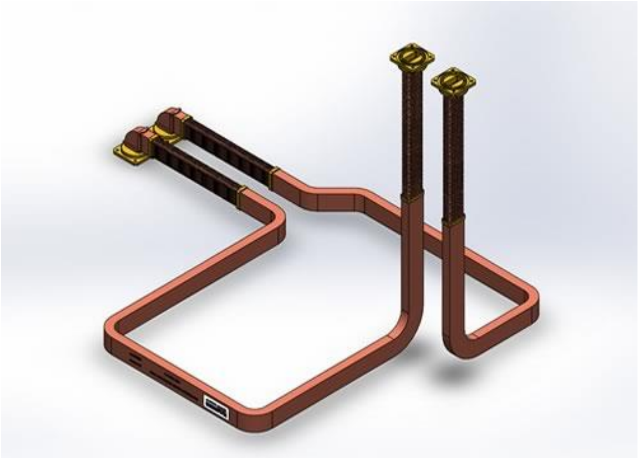
Coaxial Cable

https://en.wikipedia.org/wiki/Coaxial_cable



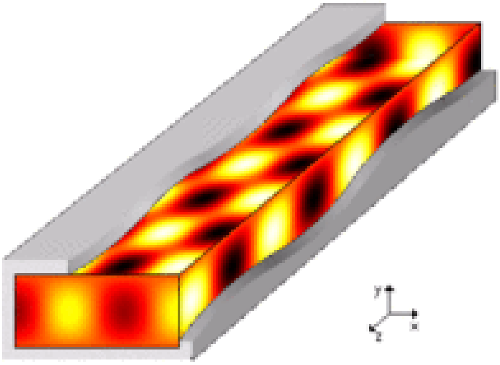
Fiber Optics

http://www.slideshare.net/subrata11/optical-fiber-55382806



Waveguide

http://www.megaind.com/



Waveguide

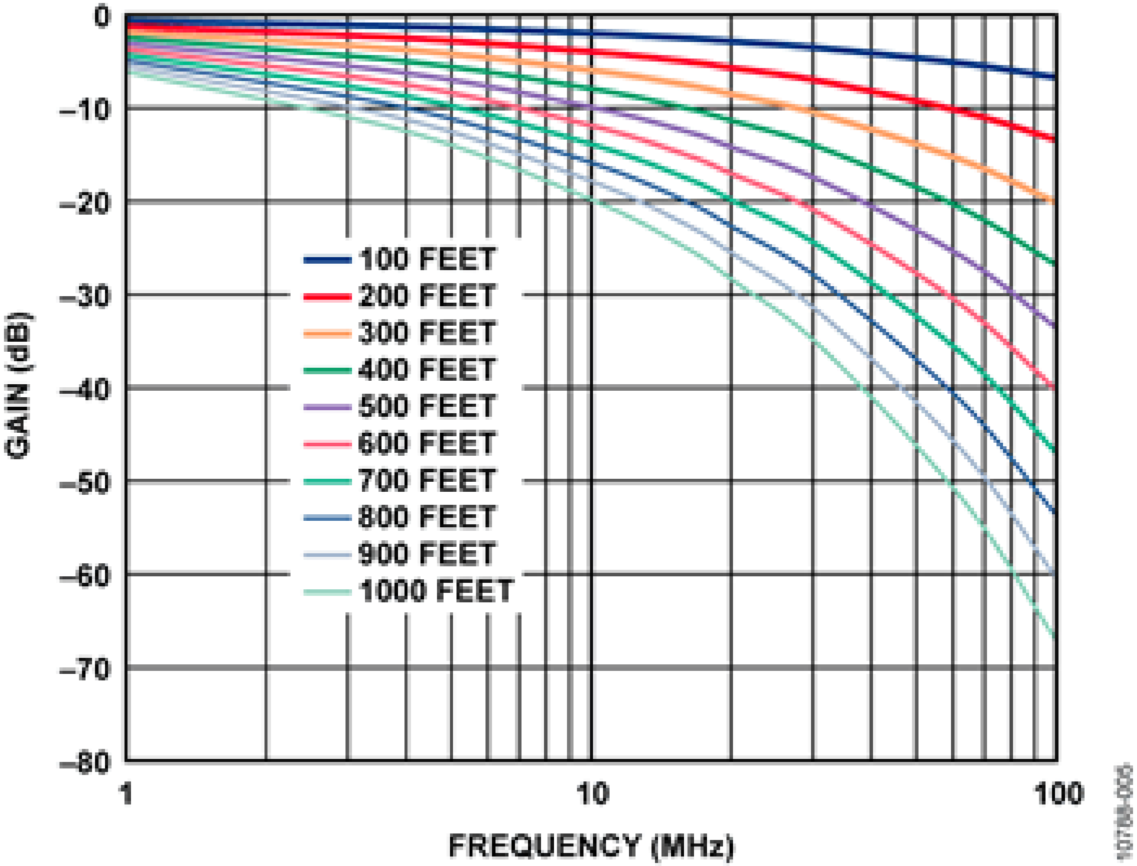
https://en.wikipedia.org/wiki/Waveguide

Channel: the challenges!

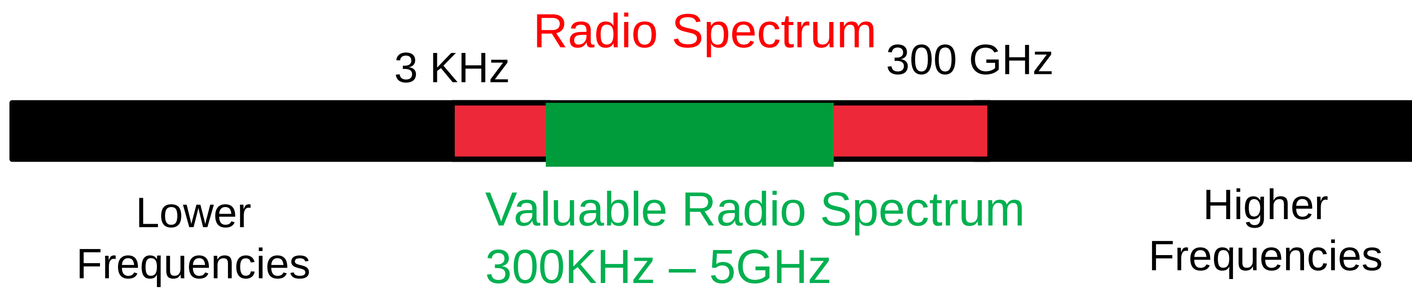
- Limited Spectrum
  - Limited wireless spectrum: High frequencies high attenuation, at low frequencies: large antenna sizes and low bandwidth
  - Wired spectrum: the medium has inherent spectrum of bands it will allow.
  - Cannot transmit in all bands as it may not be allowed.
- Needs to be shared by many users
  - Congestion, Collision, Call Drops
- Distorts the signal
  - Attenuation at different frequencies
  - Noise/Interference
  - Multipath
- Main resource or infrastructure (such as road or pipe)
  - Limits the amount of information that can be transmitted

Wired: *Limited Spectrum*

- Channel is the most challenging part in communication
  - Wired spectrum: the medium has inherent spectrum of bands it will allow



Wireless: *Limited Spectrum*



- Low radio frequencies: Large antenna heights
- High radio frequencies
  - High attenuation
  - Above 5 GHz only line of sight

Channel: the challenges!

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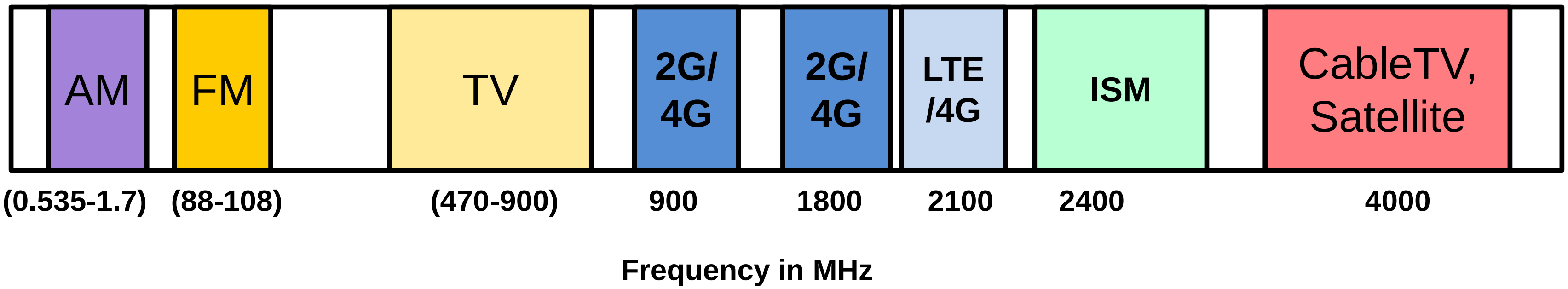
Wireless: *Need for sharing spectrum*

- Shared spectrum across different technologies



- Shared spectrum across same technology
  - By the end of 2012, the number of **mobile-connected devices** exceeded the number of people on earth (approx. 7 billion).
  - Leads to congestion, collisions, call drops

Channel: the challenges!

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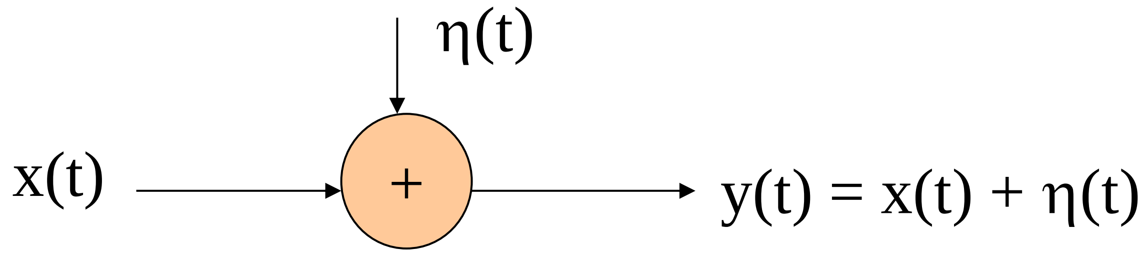
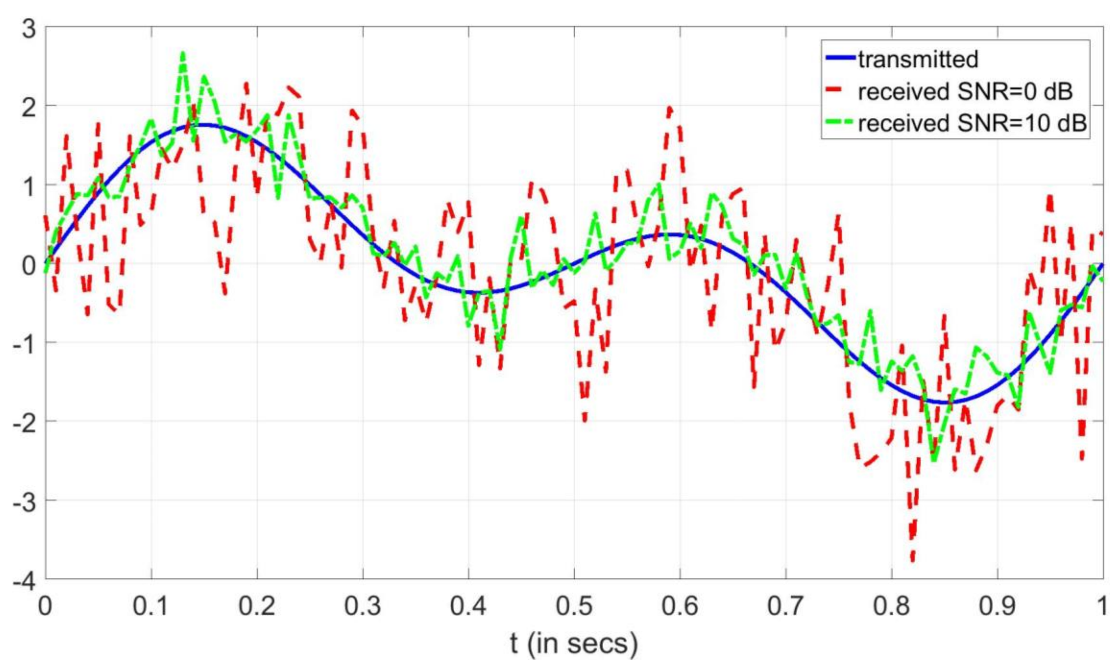
Channel Distortion: Noise

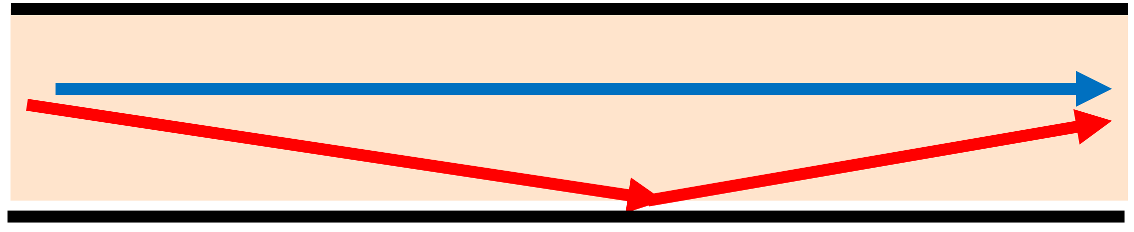
- Sources: Thermal noise at the receiver, Interference from other sources, man-made EMI, powerline interference
- Modelled as AWGN (additive White Gaussian noise)

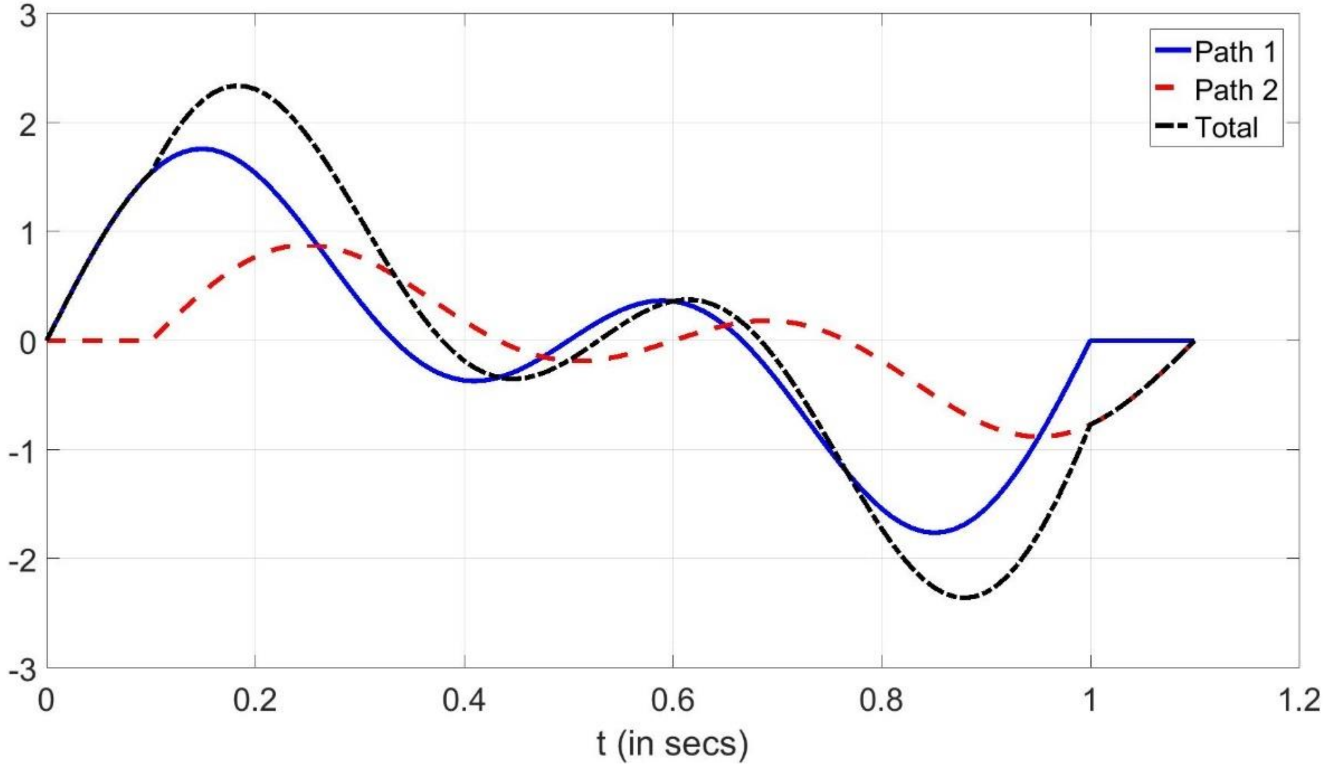




Channel Distortion: *Multipath*

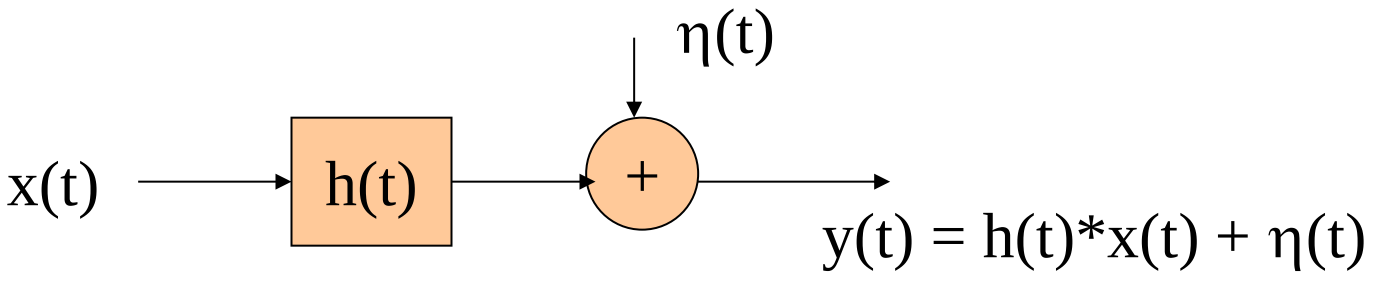
- Guided Medium: Time Invariant





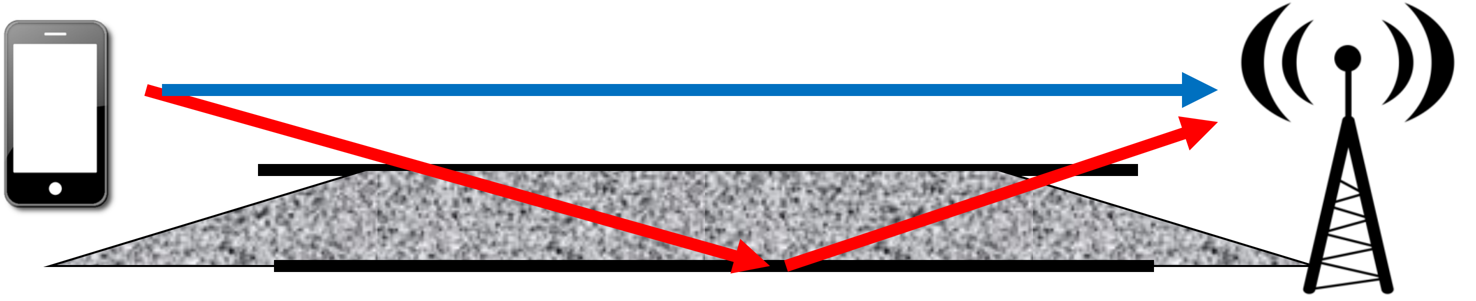
Channel: Linear Time-Invariant filter

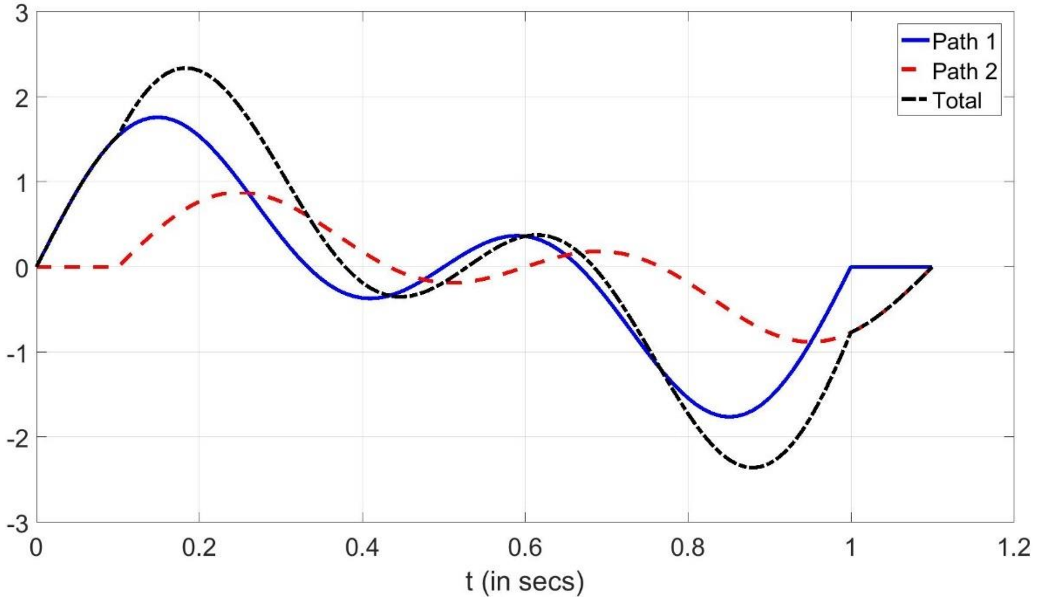
- Channels whose time/ frequency characteristics does not change in time are modeled in this fashion.
- E.g: Wired Channels



Channel Distortion: *Multipath*

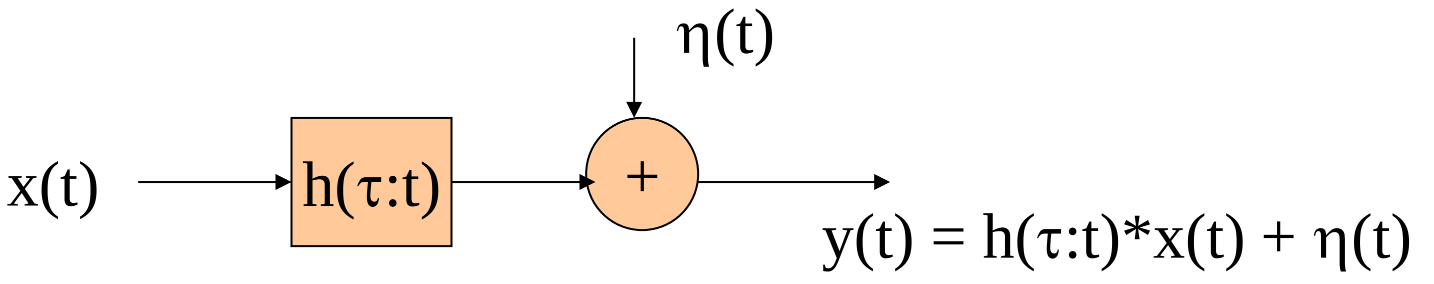
- Wireless Channel: Delay and Time Variance





Channel: Linear Time-Variant filter

- Channels whose time/ frequency characteristics change in time are modeled in this fashion.
- h(τ : t) is the time varying impulse response of the filter. E.g: Wireless Channels with multi-path.



Channel: the challenges!

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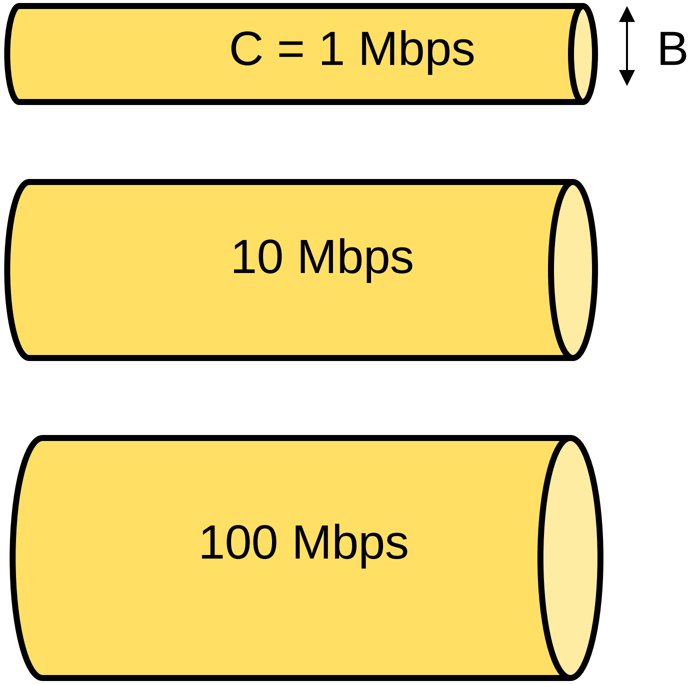
Limited Capacity: *Shannon's Theorem*

- Capacity of AWGN channel is given by

$$C = B \log_2(1 + \mathrm{SNR})$$

where B is the channel bandwidth, $\mathrm{SNR} = P/N$ is the signal to noise ratio, P is the signal power, N is the noise power.



Typical Wired Media Bandwidth

Typical Media	Max. Bandwidth	Max. Speed	Max. Physical Distance
50-Ohm coaxial cable (Thinnet)	Few MHz	10 Mbps	185m
75-Ohm coaxial cable (thicknet)	Few MHz	10 Mbps	500m
CAT 5 100 Base-TX Ethernet	100 MHz	100 Mbps	100m
CAT 5e 1000 Base-TX Ethernet	100 MHz	1000 Mbps	100m
Fiber 100 Base-FX Ethernet	Few GHz	100 Mbps	2000m
Fiber 1000 base LX Ethernet	Few GHz	1000 Mbps (1 Gbps)	5000m

Typical Wireless Bandwidth

Typical Media	**Max. Theoretical Bandwidth**	**Max. Speed**
WLAN 802.11a	20 MHz	54 Mbps
LTE 3G	20 MHz	100 Mbps
LTE-Advanced 4G	20-100 MHz	1 Gbps
WLAN 802.11n	20-40 MHz	600 Mbps

Channel: the bright side!

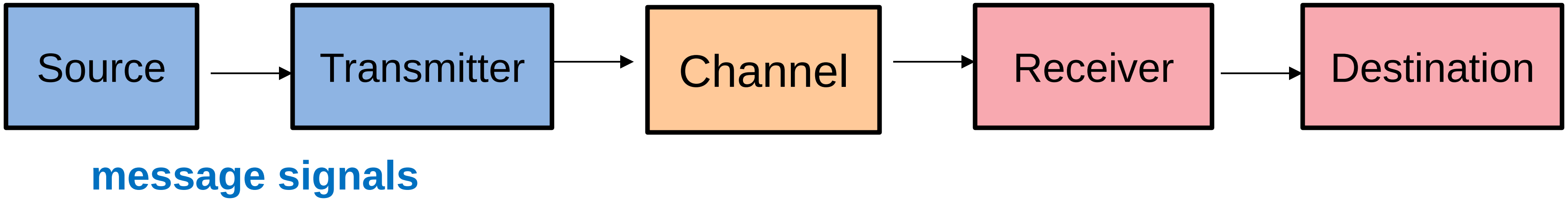
- Without the issues and challenges in the channel, there would not had been any ECE branch and several billions dollar companies such as Qualcomm, AT&T, Bell Labs, Nokia,

Questions?

Types of Communication Systems: Analog and Digital

Type of Communication Systems

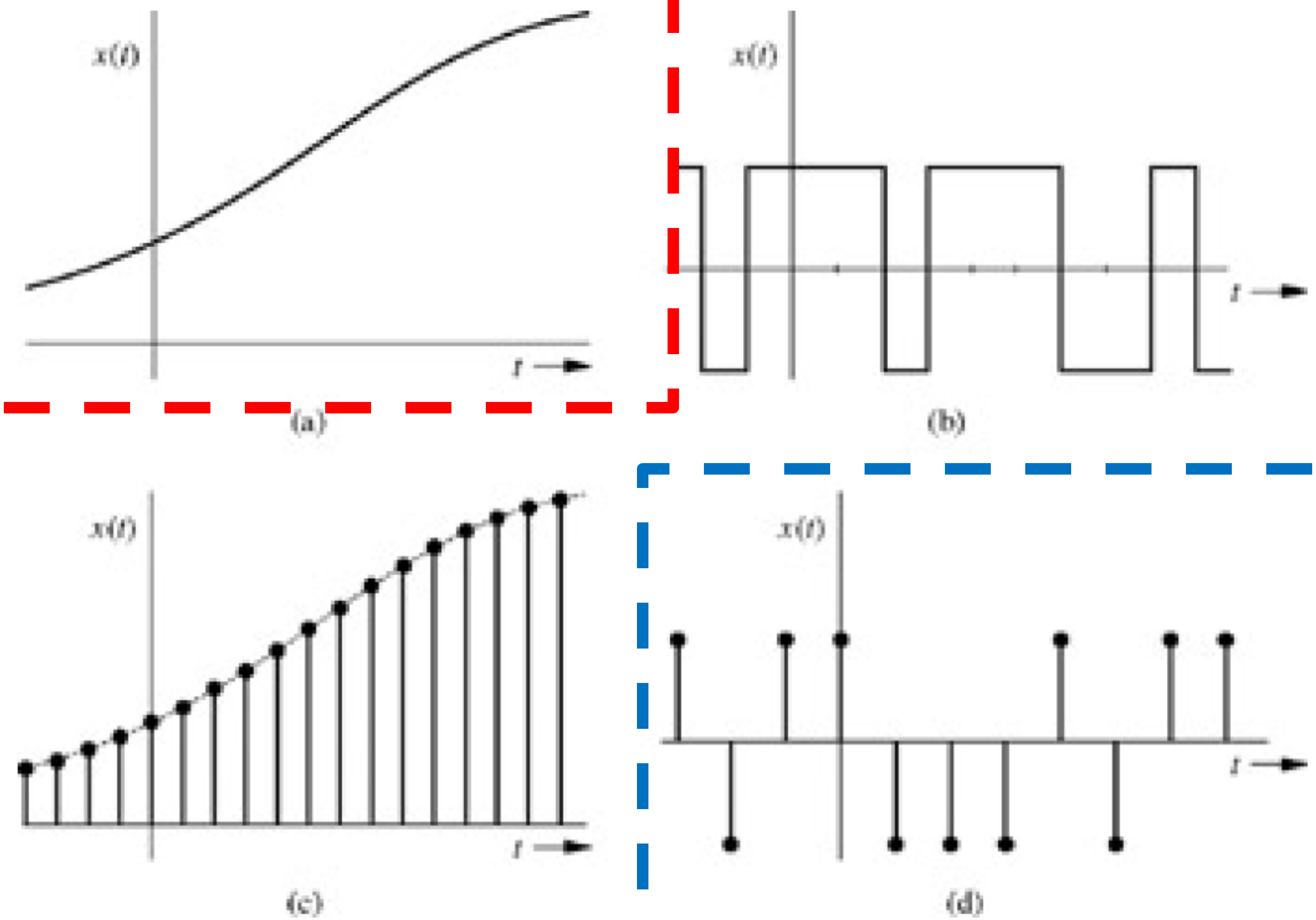
- Depends on the message signals



Analog and Digital: S&S

Analog and CT signal

Digital and CT signal

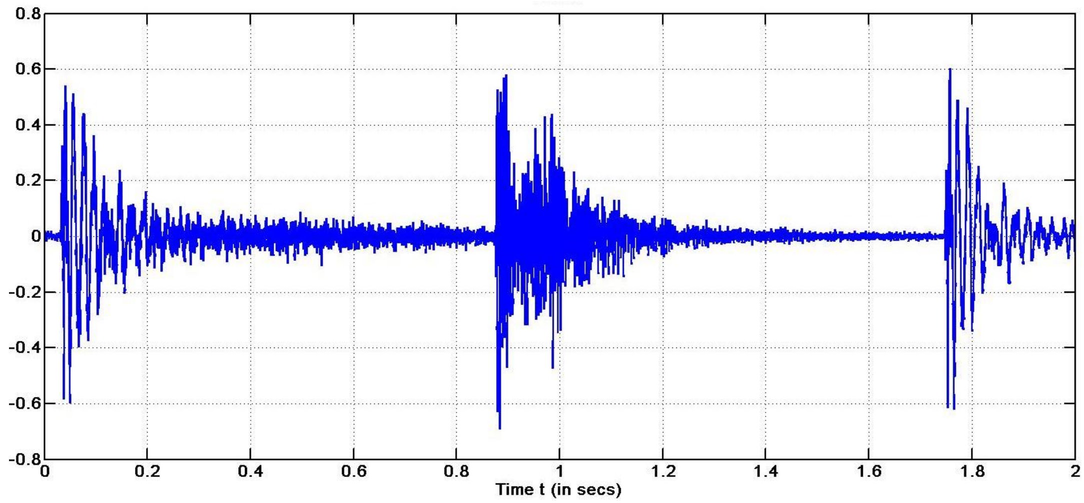


Analog and DT signal

Discrete and DT signal

Analog Communications

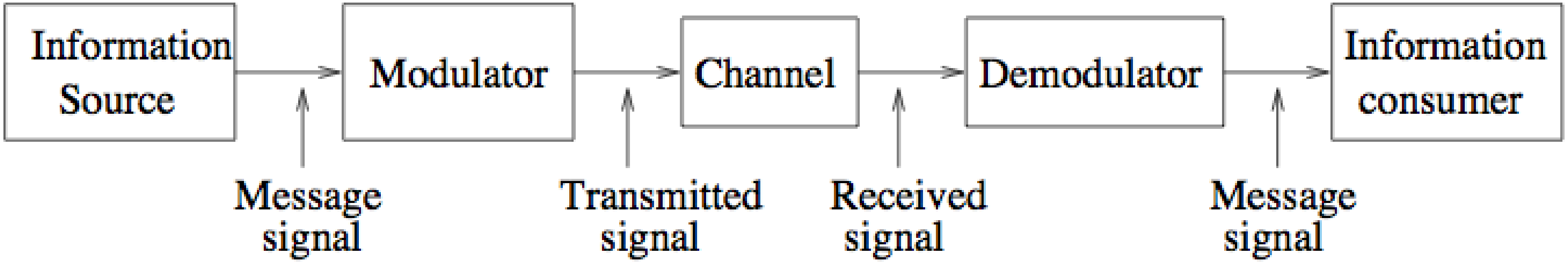
- Message signal is analog
  - Continuous time signal which takes continuum of values.
- Example: Audio signals, Speech,
- Transmitted signals over physical medium are also analog



Analog Communication Systems

- Examples
  - AM (amplitude modulation) and FM (frequency modulation) radios
  - Analog television
  - first generation cellular technology (AMPS),
  - vinyl records, audio cassettes, and VHS

Analog Communication Systems



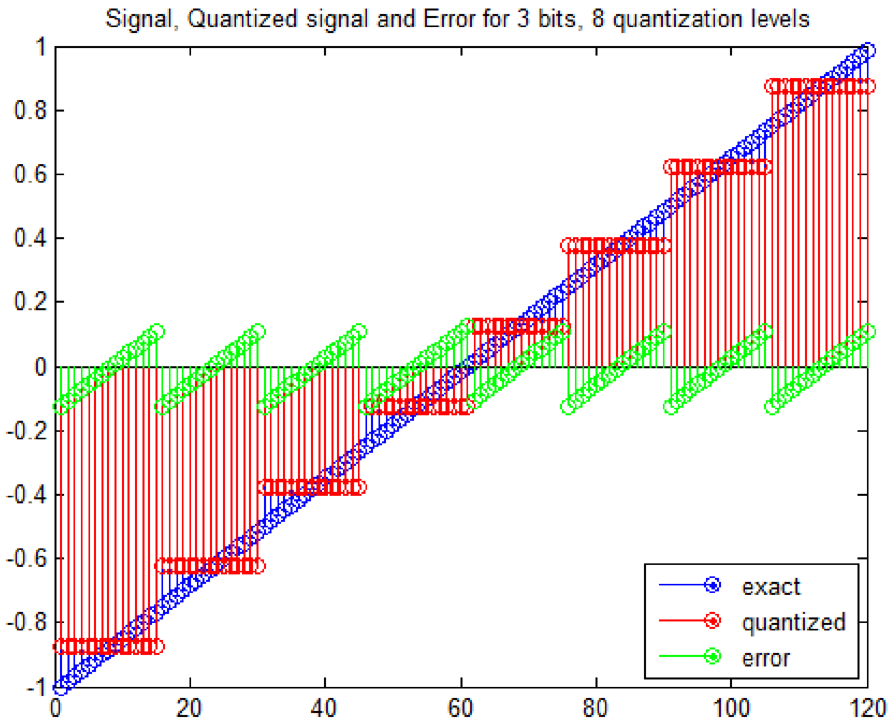
- Modulator: Moves the signal to higher frequency for transmission and multiplexing
- The “obvious” thing to do
  - Message waveforms are analog
  - Waveforms sent over the channel must be analog
- But not the right thing to do
  - Not efficient
  - Analog communication is rendered obsolete by digital communication

Digital Communications

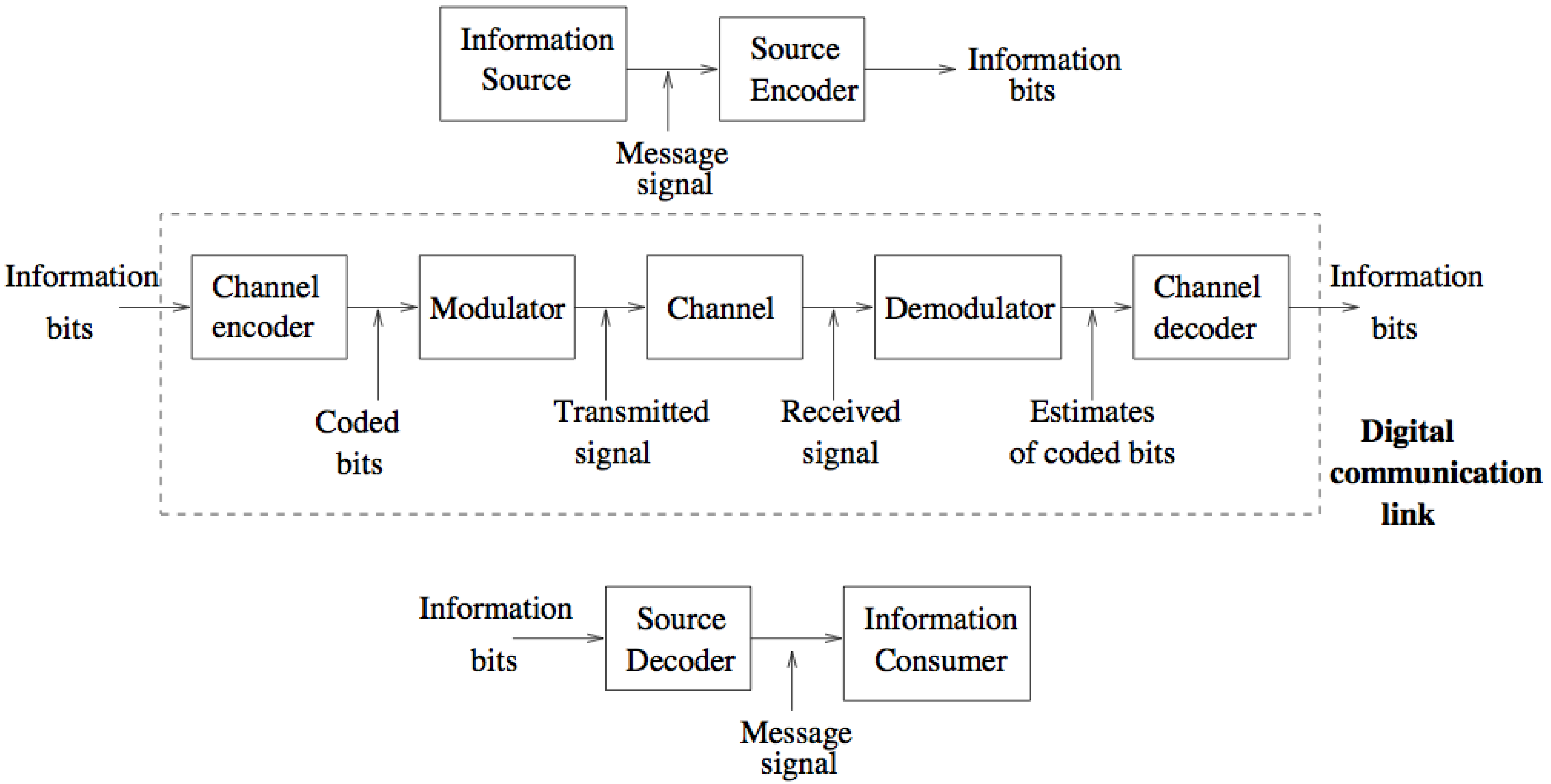
- **Message signal is digital**
  - Discrete time signal which takes only discrete level of values
  - Any signal which can be converted to 0’s and 1’s
- Example
  - Demographic data: Census data (Number of people in family, Gender, Age, Income (rounded to some decimal),
  - Text in any language (ASCII code: A 0041H, B0042H,…)
  - Data generated or stored by computer and mobiles

Digital Communications

- Analog signal can be converted to digital signal or sequence by sampling and quantization
  - Songs/Movies stored in CD/DVD, Data in hard-drive



Digital Communication Systems



Digital Communications: *Transmitter*

- **Source Encoder**
  - Finite number of messages to bits
  - Source compression
    - Obtains a digital representation of source signals using minimum binary digits. This leads to source compression. Removes Redundancy
    - Compression based on statistics: Example Huffman coding

Message	Binary Mapping
m_1	00
m_2	01
m_3	10
m_4	11

Message	Probability	Huffman Coding
m_1	0.8	0
m_2	0.1	11
m_3	0.08	100
m_4	0.02	101

      - Avg. number of bits/sample = 2
      - Avg. number of bits/sample = 0.8(1) + 0.1(2)+0.08(3) +0.02(3) = 1.3
    - Compression based on redundancy For e.g.: ZIP, PNG, MPEG, JPEG
      - 2000000000 (2 billion) can be represented by 2E9 (3 characters for 10)
  - Information bit rate depends on the message and nature of application

Digital Communications: *Transmitter*

- **Channel Encoder**
  - Introduces controlled redundancy to combat channel errors.

Example of repetition code (3,1)

- Let P_b be bit error probability that a bit will be flipped.
- Using repetition code, we send 111 for 1 and 000 for 0.
- Decide 1 was sent if majority of bits are 1
- Probability of error for this repetition code is

$$\begin{aligned} P_e &= P(2 \text{ bits in error}) + P(3 \text{ bits in error}) \\ &= \binom{3}{2} P_b^2 (1 - P_b) + \binom{3}{3} P_b^3 = 3P_b^2 - 2P_b^3 \\ &\approx 3P_b^2 \end{aligned}$$

- For $P_b = 0.1$, $P_e \approx 0.03$; For $P_b = 0.01$, $P_e \approx 0.0003$.

Digital Communications: *Transmitter*

- **Digital Modulator**
  - Converts the bit stream into a waveform suitable for transmission over the channel
    - For e.g: If the channel has band-pass response, then the modulator up-converts the frequency band of the information source to the necessary band.
  - Power and bandwidth constraints

Digital Communication: *Receiver*

- **Digital Demodulator**
  - It obtains the corrupt waveform and converts it to a bit-stream. For e.g. It down-coverts the frequency of the waveform and converts it in bits
- **Channel Decoder**
  - Obtains an estimate of the information bits. Uses the “Redundancy” to combat channel variations
- **Source Decoder**
  - It obtains an estimate of the actual information transmitted
  - Decompress or unzip the signal
  - Bits to message mapping

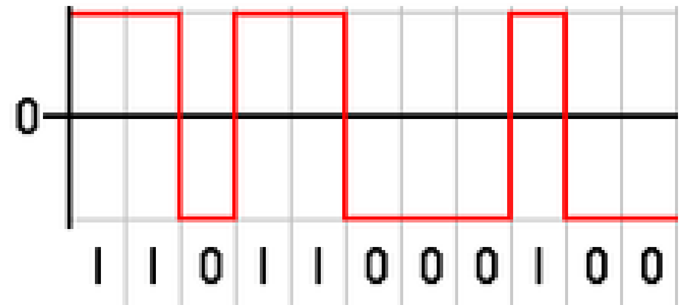
Questions?

Digital Communication Systems

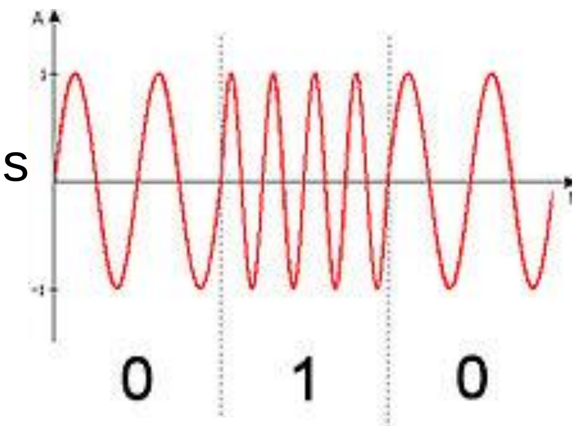
- Examples
  - Cellular 2G/3G/4G/5G
  - TV and Radio broadcasting (DVB-T and DRM)
  - CD/DVD
  - Hard drive

Digital Communications

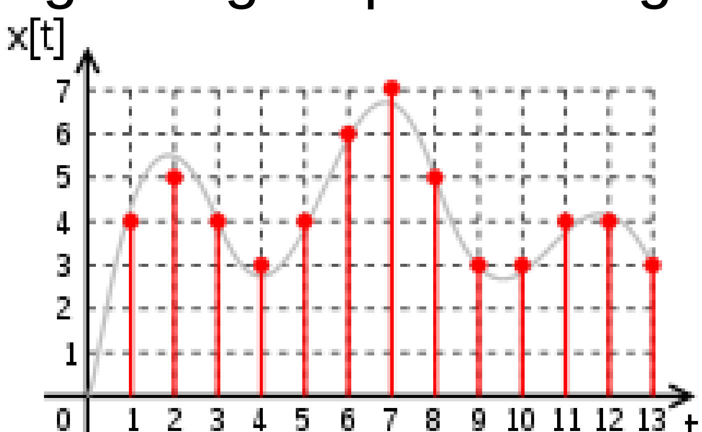
- In digital electronics, digital signal transmitted as a pulse train in baseband



- Digital communication in passband
  - Distinct finite number of analog waveforms



- Digital signal processing and storage



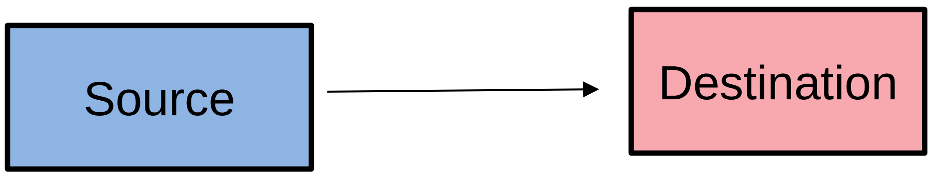
Quantization Levels	Binary mapping
l_1	000
l_2	001
l_3	010
l_4	011
l_5	100
l_6	101
l_7	110
l_8	111

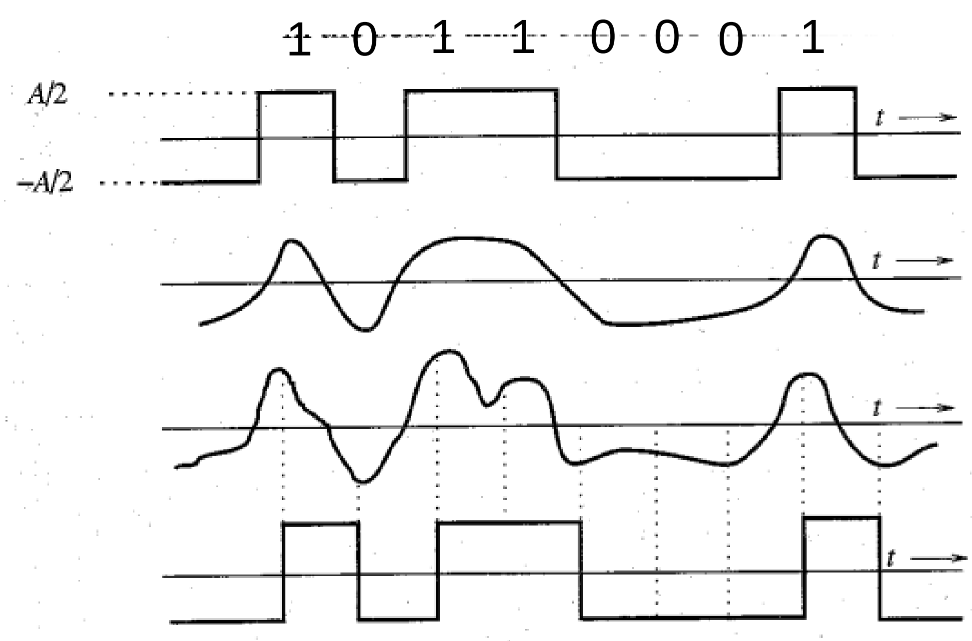
Transition from Analog to Digital

- Content is often analog (speech, image, video)
- Signals sent over physical channels are analog
  - Currents, voltages, EM waves are continuous-valued, continuous-time functions
- Many communication systems have shifted
  - Analog cellular to digital cellular (CDMA, GSM, OFDM)
  - Analog TV/radio to Digital TV/radio
  - VR and Audio casettes to CDs, VHS to DVD

Why Digital?

- Robust against distortion and noise





Transmitted Signal

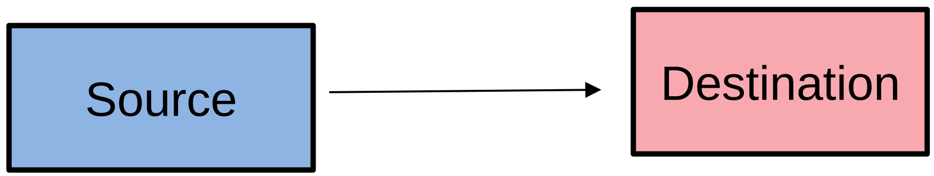
Distorted Signal

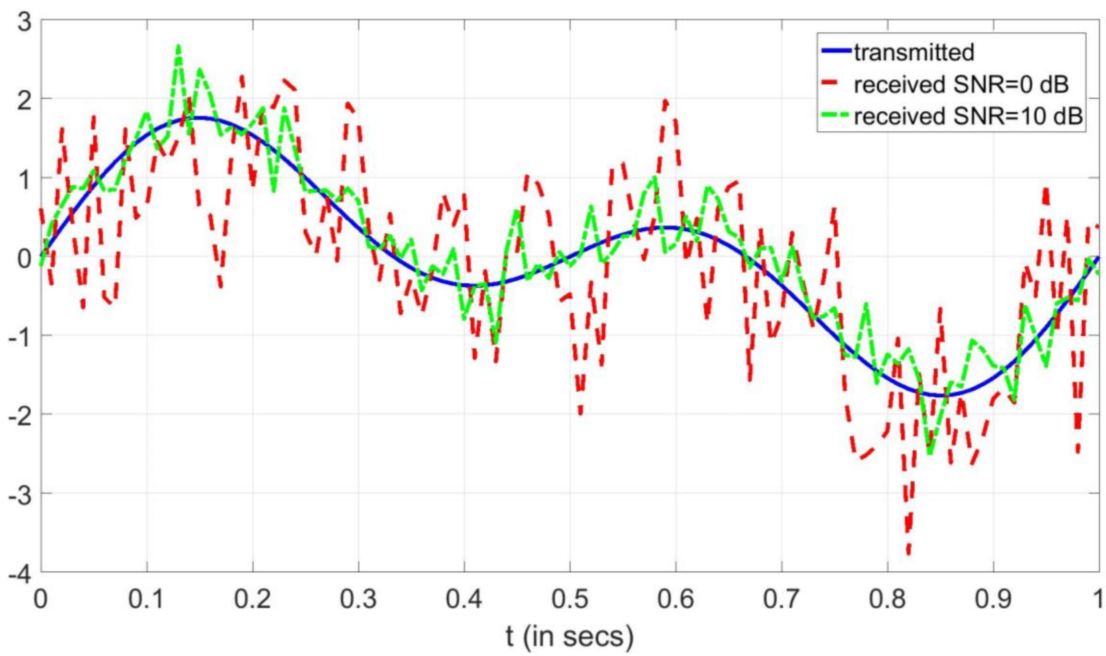
Received Signal=
Distorted Signal with Noise

Recovered Signal

Why Digital?

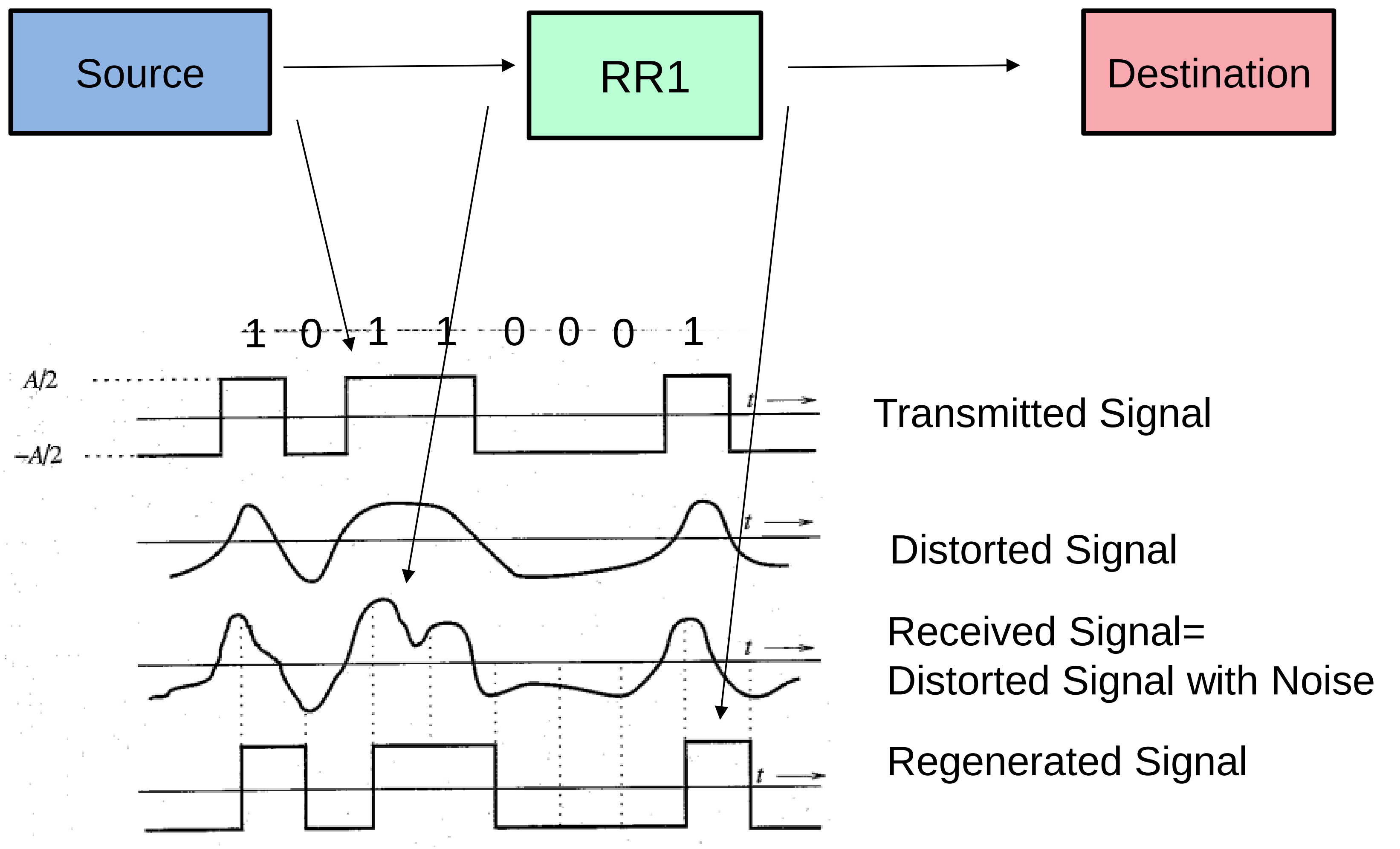
- Robust against distortion and noise
- Case of Analog





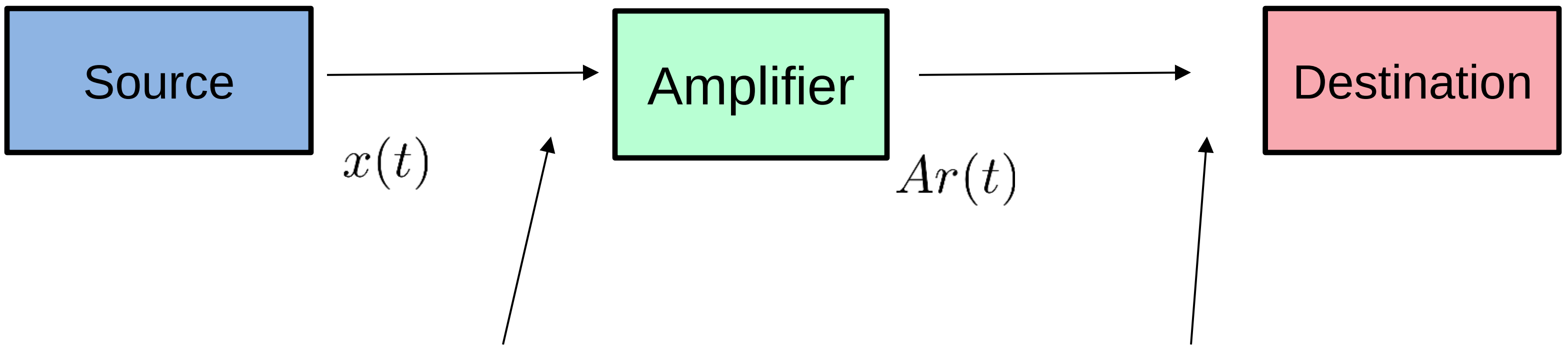
Why Digital?

- Viability of regenerative repeaters for digital comm.



Why Digital?

- Case of Analog and AWGN channels



$$r(t) = \alpha_1 x(t) + w_1(t)$$

$$\begin{aligned} y(t) &= \alpha_2 A r(t) + w_2(t) \\ &= \alpha_2 A(\alpha_1 x(t) + w_1(t)) + w_2(t) \\ &= \alpha_2 A \alpha_1 x(t) + \alpha_2 A w_1(t) + w_2(t) \end{aligned}$$

- Signal strength increases after each amplifier. SNR keeps decreasing after each stage of amplification.

Why Digital?

- Robust as compared to analog communication
  - Viability of regenerative repeaters for digital comm.
- Design of digital communication systems based on source-channel separation principle
  - Source-independent and channel-optimized: Huge Gains
  - Not possible in Analog
- Digital hardware is flexible and allows the use of microprocessors and VLSI circuits
  - Scalability
- Digital hardware is much cheaper and more robust
- Digital signals can be coded for arbitrary low error rates
  - Shannon’s theorem
- Possibility of encryption
- Digital signal storage is easy and inexpensive to store
- And Many more…

Is there still a role for analog?

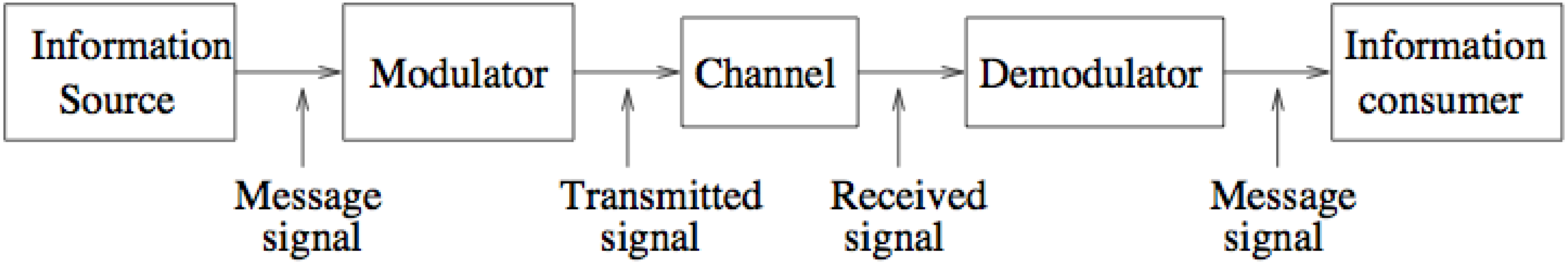
- **Of course! The physical world is analog**
- Signal transmission: Need to convert digital data to analog signals that can be sent over the physical channel
- Signal reception: Need to convert analog received signals back into digital data
- RF signal processing still a challenge

Today's Class

Importance of this subject

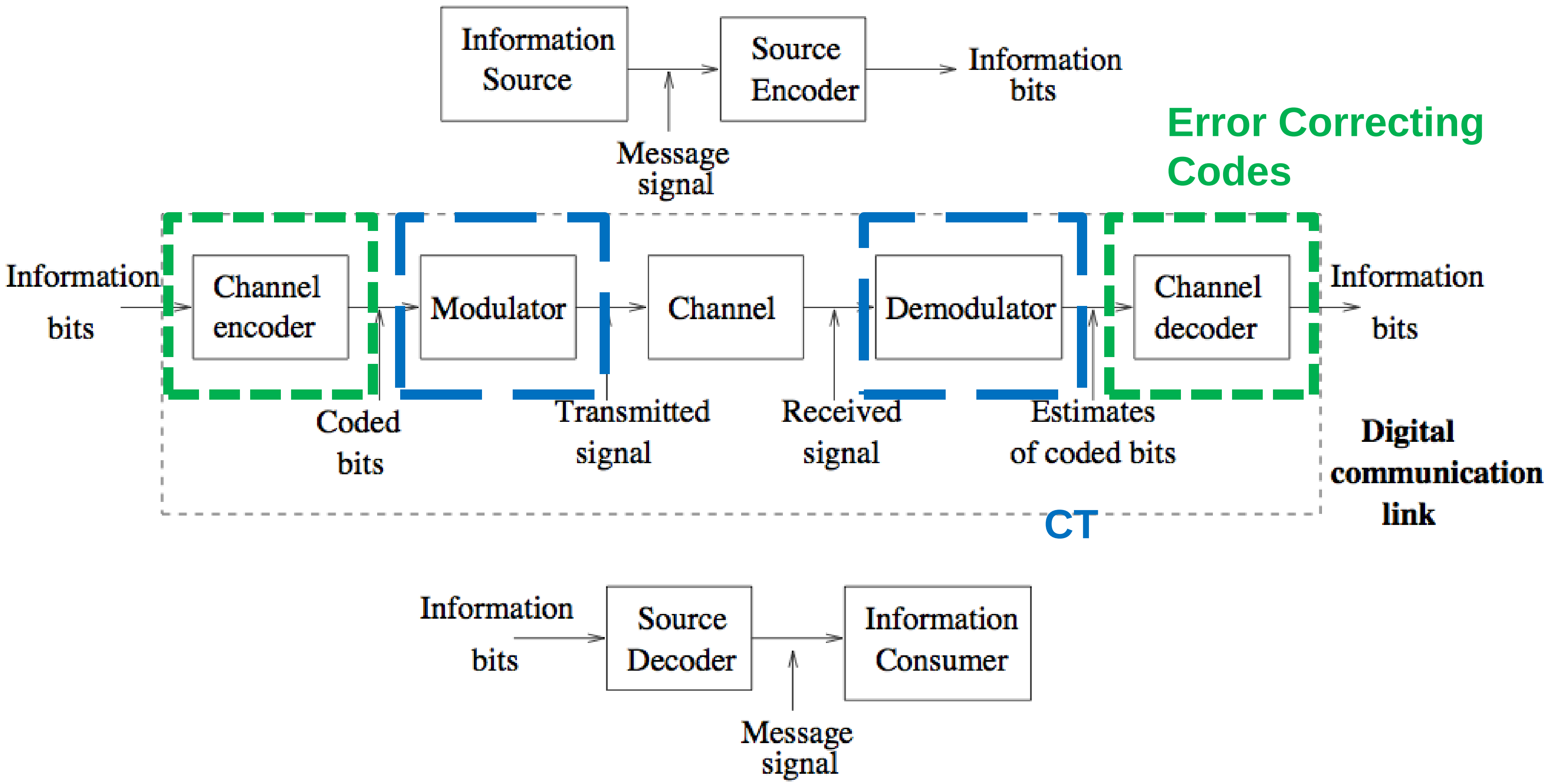
- Core subject
  - Form foundation for communication theory
  - Prerequisite for higher level subjects
    - Wireless Communications
    - Error Correcting Codes
    - Information Theory
    - Machine Learning for Communications

Analog Communication Systems

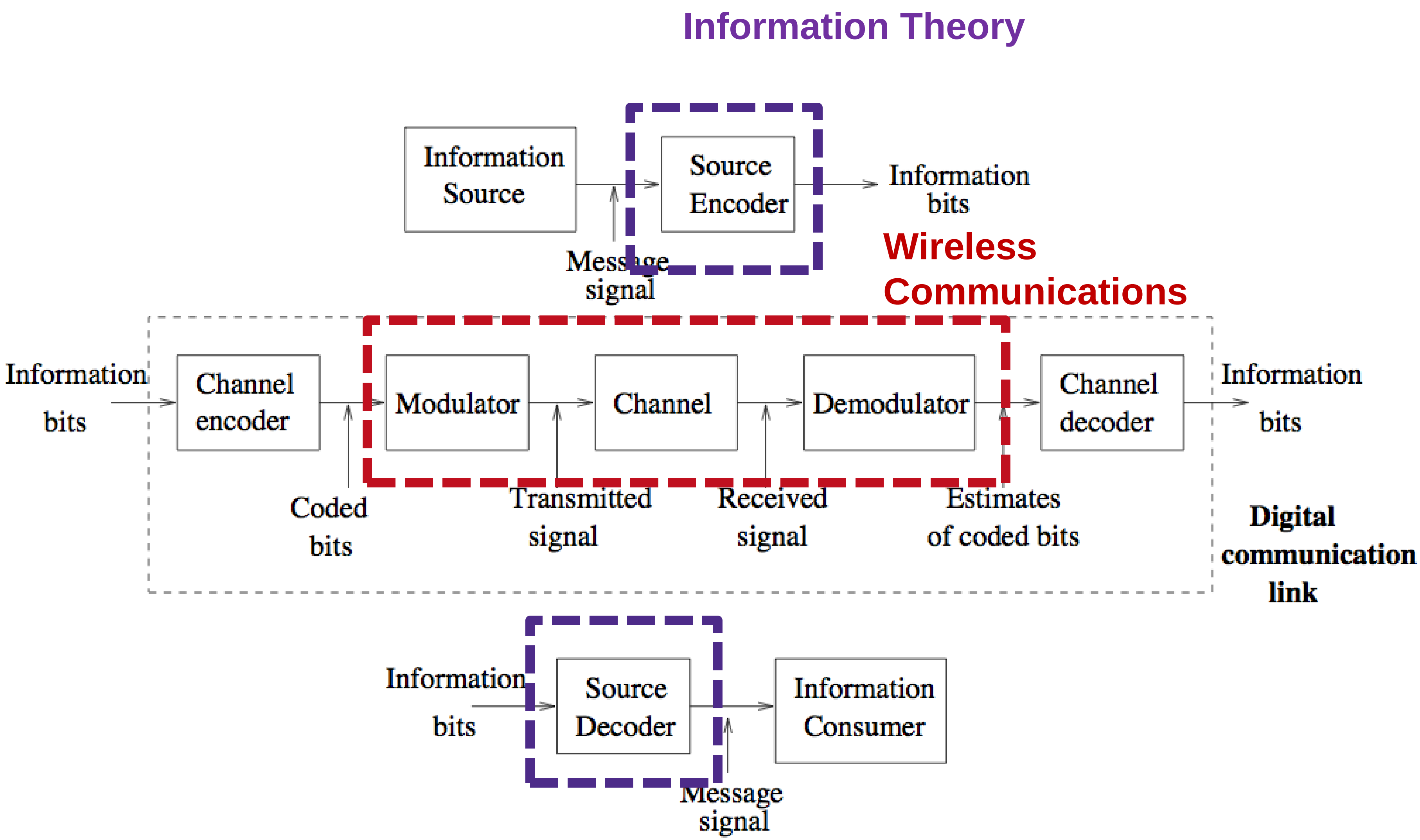


CT

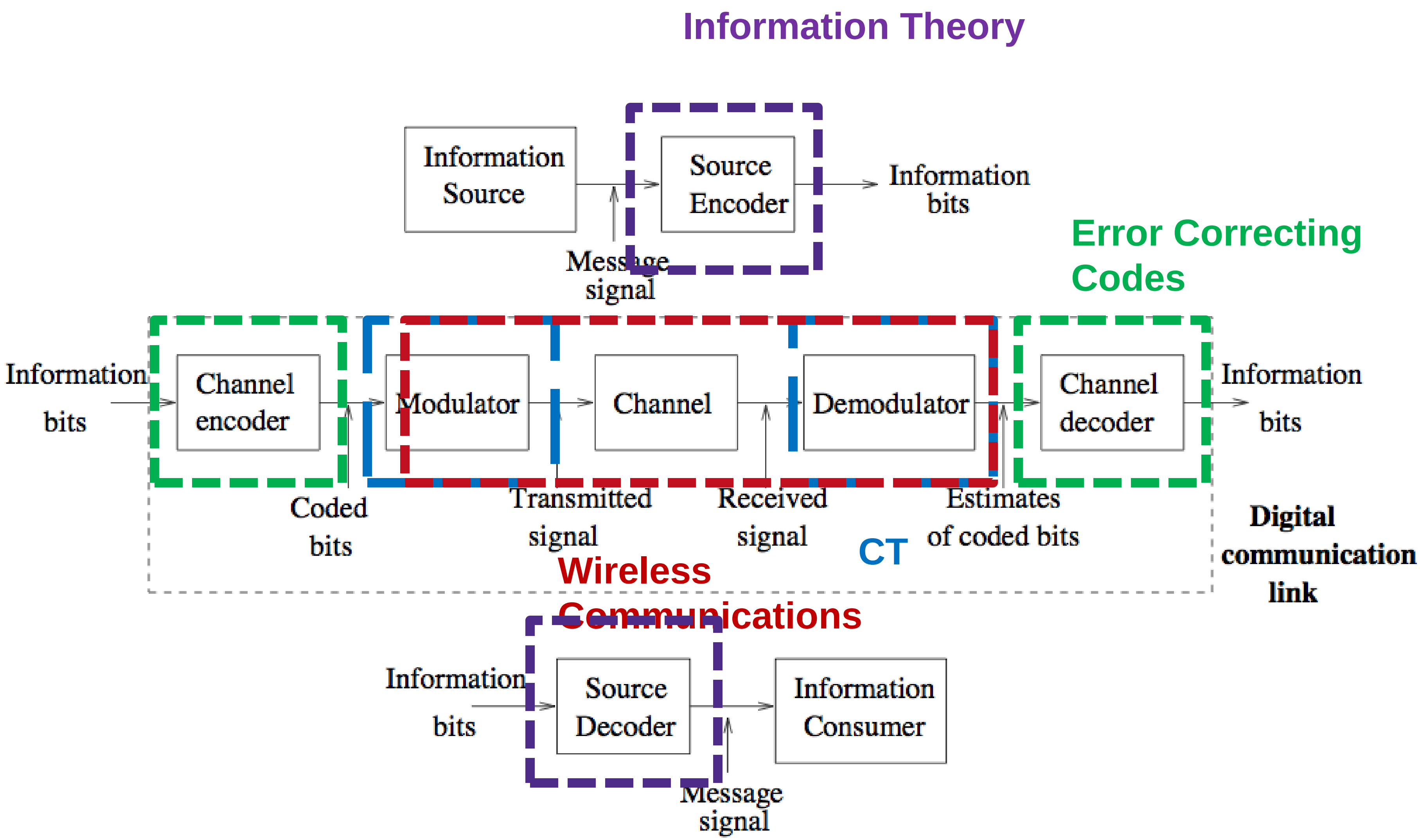
Digital Communication Systems



Digital Communication Systems



Digital Communication Systems



Importance of this subject

- Core subject
  - Form foundation for communication theory
  - Prerequisite for higher level subjects
    - Wireless Communications
    - Error Correcting Codes
    - Information Theory
    - Machine Learning for Communications
- Industry and Academic
  - Decades of ECE: 5G, 6G, IoT, ML for Comm., mmWave, Tactile internet (Autonomous vehicles, remote surgery, AR/VR tours)
- Problem solving and analytical skills
- Opportunity to work in SPCRC

Questions?

Course Details/ Logistics

Syllabus (Tentative)

- Representation of bandpass signals and systems
  - lowpass equivalent of bandpass signals and systems
- Analog Communication Methods
  - AM-DSB and SSB; FM-narrowband and wideband; Demodulation of AM and PM/FM, Phased locked loop (PLL);
  - Brief overview of Line Coding and PWM
- Digital Modulation
  - Representation of Digitally Modulated Signals; Memoryless modulation methods: PAM, PSK, QAM, Orthogonal Multi-Dimensional Signals
- Random Processes
  - Review of Correlation, ESP and PSD; Noise Modelling, Thermal Noise, AWGN
- Performance of Digital methods in the presence of AWGN
  - Hypothesis testing, Signal Space Concepts, Performance analysis of ML reception, Bit error probability, Link budget analysis

Resources

BOOKS

- **U. Madhow: Introduction to Communication Systems**
  - **http://www.ece.ucsb.edu/wcsl/Publications/intro_comm_systems_madhow_jan2014b.pdf**
- B.P.Lathi, “Modern Digital and Analog Communication Systems”, 3rd Edition, Oxford University Press, 2007.
- J.G.Proakis, M.Salehi, “Fundamentals of Communication Systems”, Pearson Education 2006

VIDEOS

- National Programme on Technology Enhanced Learning (NPTEL)
  - Analog Communication http://nptel.ac.in/courses/117102059/
  - Digital Communication: http://nptel.ac.in/courses/117101051/

VIRTUAL LAB

- Virtual lab on Systems, Communications and Control, IIT Guwahati

  http://iitg.vlab.co.in/?sub=59&brch=163

Course Portal

MOODLE: https://courses.iiit.ac.in/

Under Spring 2025

If you are not already enroled, email me.

- Assignments
- News
- Discussion Forum

Approach

- Slides + Blackboard
- Active participation in class
- Regular Attendance
- Expected: Read-Attend-Revise-Assignments

Guidelines

- No disturbance in class
- No mobile phones/tablets in class
- No recording

Exams and Evaluation

- Mark Distribution
  - Quiz 1 (10)
  - Quiz 2 (10)
  - MidSem (20)
  - Assignments + Quiz (20)
  - Final Exam (40-50)

- Grading: TBD
  - Mostly Gaussian with fixed cut/off for A (>=85) and F(<40) grades)

Assignments

- 6-8 handwritten assignments + 1-2 Matlab
- Due in one week
- Firm Deadlines
  - One late homework assignment allowed without penalty
  - 2 marks will be deducted on other late assignments
- Quiz based on Assignment!
  - There are 2-2.5 marks for each assignment
  - These marks will be given only if the assignment is completed and quiz answers are correct.
- Cooperative learning is encouraged not copying!!!
  - Discussion of concepts in homework is encouraged
  - Sharing of homework or code is not permitted and will be penalized

Tutorials

- Time: Tuesdays 2-3.30 pm
- Venue: H204
- Attendance: Mandatory if informed in advance!
- Quizes, Problem Solving, Queries
- Use of discussion forum
- TAs: Naganjani, Nikhil, Manasa and Vishnu Priya

EC5.203 Communication Theory I (3-1-0-4):

Lecture 2
Overview of Signals And Systems

Instructor: Dr. Sachin Chaudhari

Email: sachin.chaudhari@iiit.ac.in

Jan. 9, 2025

INTERNATIONAL INSTITUTE OF
INFORMATION TECHNOLOGY

HYDERABAD

Reference

- Chapter 2 (Madhow)
  - Sec. 2.1-2.5 : Overview of Signals and Systems, Notations
  - Sec. 2.6-2.8: Energy Spectral Density, Bandwidth, Structure of passband signal

Quick Review of Signals and Systems and Notations

Indicator Function

- S&S: The step function was given by

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

- *CT-1*: $u(t)$ represent CT signal and $u[n]$ represent DT signal while step function is used for

$$I_{[0,\infty)}(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

- Indicator function

$$I_A(x) = \begin{cases} 1 & x \in A \\ 0 & \text{otherwise} \end{cases}$$

Sinusoidal Signal

- Sinusoids

$$s(t) = A\cos\left(2\pi f_0 t + \theta\right) \quad \text{Polar Form}$$

  where $A > 0$ is the amplitude, f_0 is the frequency, and $\theta \in (0, 2\pi]$ is the phase.

- Sinusoids with known A, f_0, and θ cannot carry information.

- Modulation varies one or more of these parameters to convey information.

Sinusoidal Signal

- Sinusoids

$$s(t) = A\cos(2\pi f_0 t + \theta)$$ **Polar Form**

where $A > 0$ is the amplitude, f_0 is the frequency, and $\theta \in (0, 2\pi]$ is the phase.

- Sinusoid can also be written as

Rectangular form

$$s(t) = A_c \cos 2\pi f_0 t - A_s \sin 2\pi f_0 t$$

where $A_c = A\cos\theta$ and $A_s = A\sin\theta$ are real numbers. Using Euler's formula

Complex number

$$Ae^{j\theta} = A\cos\theta + jA\sin\theta = A_c + jA_s$$

where $A = \sqrt{A_c^2 + A_s^2}$ and $\theta = \tan^{-1}(A_s/A_c)$.

Complex Exponential

- Complex exponentials

$$s(t) = Ae^{j(2\pi f_0 t + \theta)} = \alpha e^{j2\pi f_0 t}$$

where $\alpha = Ae^{j\theta}$ a complex number that contains both the amplitude and phase information.

Inner Product

- The inner product for two $m \times 1$ complex vectors $\mathbf{s} = (s[1], \ldots, s[m])^T$ and $\mathbf{r} = (r[1], \ldots, r[m])^T$ is given by

$$< \mathbf{s}, \mathbf{r} >= \sum_{i=1}^{m} s[i] r^*[i] = \mathbf{r}^H \mathbf{s}.$$

  where $(.)^H$ denotes Hermitian operation with vector $(\mathbf{r})^H$ being conjugate transpose of vector $\mathbf{r}$.

- Similarly, we define the inner product of two possibly complex-valued signals $s(t)$ and $r(t)$ as follows

$$< s, r >= \int_{-\infty}^{\infty} s(t) r^*(t) \, dt$$

Energy and Norm

- The energy E_s of a signal

$$E_s = ||s||^2 \; = < s, s > = \int_{-\infty}^{\infty} |s(t)|^2 \; dt$$

- If $E_s = 0$, then it means that $s(t)$ must be zero *almost everywhere* which is also true for the norm of a vector.

Example 1: Solve!

- Find energy for signal

  (a) $s(t) = 2I_{[0,T]} + jI_{[T/2,2T]}$

  (b) $s(t) = e^{-3|t|+j2\pi t}$

Power

- The power of a signal $s(t)$ is defined as the time average of its energy computed over a large time interval

$$P_s = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |s(t)|^2 dt$$

- Finite energy signals have zero power.

Time average

- Time average of signal $g(t)$ is denoted by

$$\bar{g} = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g(t) dt$$

- Using the above notation, power of signal is given by

$$P_s = \overline{|s(t)|^2}$$

while the DC value of $s(t)$ is $\overline{s(t)}$.

Example 2

- Compute E_s, P_s, and DC value for $s(t) = Ae^{j(2\pi f_0 t+\theta)}$ where $A > 0$ is the amplitude and $\theta \in [0, 2\pi]$ and f_0 is the real-valued frequency.

- Next, compute P_s and DC value for a real valued sinusoid $s(t) = A\cos(2\pi f_0 t + \theta)$, where $A > 0$ is the amplitude and $\theta \in [0, 2\pi]$ and f_0 is the real-valued frequency.

Convolution

- Convolution of two signals $u_1(t)$ and $u_2(t)$ is given by

$$v(t) = u_1(t) * u_2(t) = (u_1 * u_2)(t)$$

Fourier Series

- A periodic signal $x(t)$ can be represented in terms of Fourier series as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where Fourier series coefficients are given by

$$a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

- A periodic signal $u(t)$ can be represented in terms of Fourier series as

$$u(t) = \sum_{n=-\infty}^{\infty} u_n e^{j2\pi n f_0 t}$$

where Fourier series coefficients are given by

$\omega = 2\pi f$

$$u_k = \frac{1}{T_0} \int_{T_0} u(t) e^{-j2\pi k f_0 t} dt$$

Fourier Transform

- Any aperiodic and finite duration signal $x(t)$ can be represented using Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} \, d\omega$$

Inverse Fourier Transform

where

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt$$

Fourier Transform

- A aperiodic signal $x(t)$ can be represented in terms of Fourier transform as

$$u(t) = \int_{-\infty}^{\infty} U(f) e^{j2\pi f t} df$$

where Fourier transform is given by

$\omega = 2\pi f$

$$U(f) = \int_{-\infty}^{\infty} u(t) e^{-j2\pi f t} dt$$

Properties of Fourier Transform

- Linearity: For arbitrary complex numbers α and β,

$$\alpha u(t) + \beta v(t) \longleftrightarrow \alpha U(f) + \beta V(f)$$

- Time delay

$$u(t - t_0) \longleftrightarrow U(f) e^{-j2\pi f t_0}$$

- Frequency shift

$$U(f - f_0) \longleftrightarrow u(t) e^{j2\pi f_0 t}$$

- Differentiation in time domain

$$x(t) = \frac{du(t)}{dt} \longleftrightarrow j2\pi f U(f)$$

Properties of Fourier Transform

- Step Function: For $v(t) = I_{[0,\infty)}$

$$V(f) \longleftrightarrow \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

- Integration:

$$u(t) = \int_{-\infty}^{t} x(t)\ dt = x(t) * v(t)$$

$$U(f) = X(f)V(f) = \frac{X(f)}{j2\pi f} + \overline{u}\delta(f)$$

Properties of Fourier Transform

- Parseval's identity:

$$< u, v > = \int_{-\infty}^{\infty} u(t)v^*(t)dt = \int_{-\infty}^{\infty} U(f)V^*(f)df$$

- For $u = v$,

$$||u||^2 = < u, u > = \int_{-\infty}^{\infty} |u(t)|^2 dt \ = \int_{-\infty}^{\infty} |U(f)|^2 df$$

Properties of Fourier Transform

- Convolution in time domain:

$$y(t) = u(t) * v(t) = (u * v)(t) \longleftrightarrow Y(f) = U(f)H(f)$$

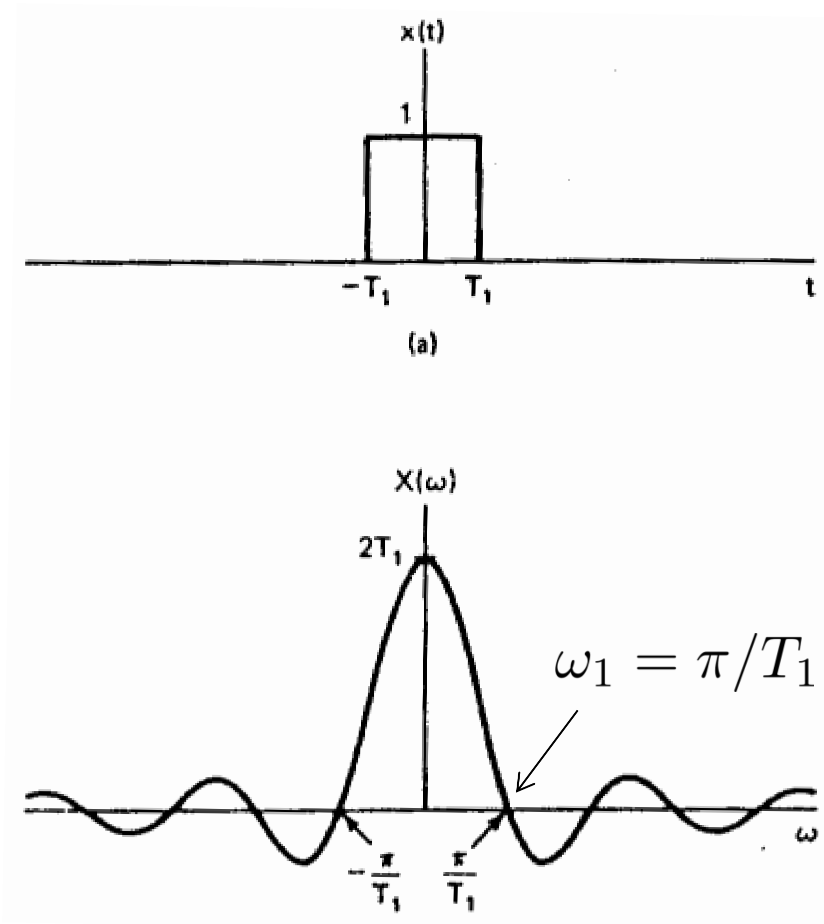
- Multiplication in time domain:

$$y(t) = u(t)v(t) \longleftrightarrow Y(f) = (U * H)(f) = U(f) * V(f)$$

Rectangular pulse and Sinc function: *S&S*

- Find Fourier transform for the signal

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases}$$



$$X(j\omega) = 2\frac{\sin \omega T_1}{\omega}$$

Rectangular pulse and Sinc function

- For a rectangular pulse $u(t) = I_{[-T/2,T/2]}(t)$ of duration T. Its Fourier transform is given by

$$U(f) = T\mathrm{sinc}(fT) = T\frac{\sin(\pi f T)}{\pi f T}$$

$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

- The Fourier transform is denoted by

$$I_{[-T/2,T/2]}(t) \longleftrightarrow T\mathrm{sinc}(fT)$$

- Using duality, the other Fourier transform pair is

$$I_{[-W/2,W/2]}(f) \longleftrightarrow W\mathrm{sinc}(Wt)$$

Summary of Properties: S&S

Property	Aperiodic signal	Fourier transform
	$x(t)$	$X(j\omega)$
	$y(t)$	$Y(j\omega)$
Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time Reversal	$x(-t)$	$X(-j\omega)$
Time and Frequency Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega) * Y(j\omega)$
Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$

Summary of Properties: S&S

Property	Aperiodic signal	Fourier transform
	$x(t)$	$X(j\omega)$
	$y(t)$	$Y(j\omega)$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \mathcal{Re}\{X(j\omega)\} = \mathcal{Re}\{X(-j\omega)\} \\ \mathcal{Im}\{X(j\omega)\} = -\mathcal{Im}\{X(-j\omega)\} \\ \lvert X(j\omega)\rvert = \lvert X(-j\omega)\rvert \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{Ev}\{x(t)\}$ $[x(t)$ real$]$ $x_o(t) = \mathcal{Od}\{x(t)\}$ $[x(t)$ real$]$	$\mathcal{Re}\{X(j\omega)\}$ $j\mathcal{Im}\{X(j\omega)\}$

Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$