[illegible]
[illegible]

we want to understand Digital Demodulation

As SNR ↑ it is better to Demodulate

$s(t) = \varphi_1(t) + \varphi_2(t) + \varphi_3(t)$

$= \sum_{i=0}^{3} \varphi_i(t)\, s[i]$

$s[\,] = [s[0], s[1], s[2]]^T$

$\boxed{s = [1,1,1]^T}$

we want from a conti. signal to vector

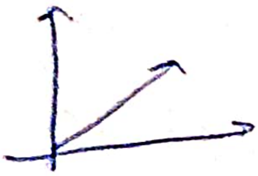
$s_1 = [-1,1,1]$ $\quad s_2 = [0,2,1]$ $\quad s_3 = [1,-1,1]$

orthonormal → energy is "1"

$\varphi_c(t) = p(t)\cos(2\pi f_c t)$ $\quad \varphi_s(t) = -p(t)\sin(2\pi f_c t)$

$\boxed{\|\varphi_c\|^2 = \|\varphi_s\|^2 = \frac{1}{2}\|p\|^2}$

why $n \le M$ → limited dependencies.



no. of signal = 3 = m = N.
no. of dim = 2

$s_i(t) = \sum_{k=0}^{n-1} s_i[k]\, \varphi(k)$

$s_i[k] = \langle s_i, \varphi_k \rangle$

point is to find no. of dimensions.

$$\Phi_k(t) = s_k(t) - \underbrace{\sum_{i=0}^{m} \langle s_k, \psi_i \rangle \psi_i(t)}_{\text{remove the projection}}$$

This is orthogonal!
convert it to orthonormal

If there are 'm' signals we can't go more than 'm' dimensions.

$$\boxed{s_0(t) \longrightarrow \phi_0(t) \longrightarrow \psi_0(t)}$$

Let $\phi_0(t) = s_0(t)$

$$\boxed{\psi_0(t) = \frac{\phi_0(t)}{\|\phi_0(t)\|}}$$

$$\|\phi_0(t)\|^2 = \int_{t=-\infty}^{\infty} |s_0(t)|^2 dt$$

$$= \int_0^3 1 \cdot dt = 3$$

$$\boxed{\psi_0(t) = \frac{s_0(t)}{\sqrt{3}}}$$

$$\psi_0(t) = A\,\phi_0(t) \Rightarrow A = \frac{1}{\sqrt{3}} \qquad \phi_0(t) = s_0(t)$$

$$\boxed{\phi_1(t) = s_1(t) - \sum_{i=0}^{0} \langle s_1, \psi_0 \rangle \psi_0(t)}$$

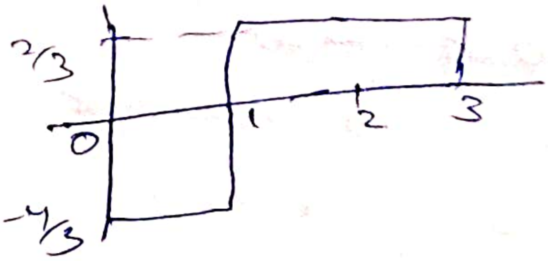
$$\langle s_1, \psi_0 \rangle = \int_{-\infty}^{\infty} s_1(t)\psi_0(t)\,dt = \int_0^3 s_1(t)\,\psi_0(t) = \int_0^1 \frac{-1}{\sqrt{3}}dt + \int_1^3 \frac{1}{\sqrt{3}}dt$$

$$= \frac{-1}{\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\phi_1(t) = s_1(t) - \frac{1}{\sqrt{3}}\psi_0(t)$$

$$\boxed{\phi_1(t) = s_1(t) - \frac{s_0(t)}{3}}$$

$\phi_1(t) =$



$$\|\phi_1(t)\|^2 = \int_0^3 |\phi_1(t)|^2 dt = \frac{16}{9} + \frac{4}{9} + \frac{4}{9} = \frac{24}{9}$$

normalising

$$\boxed{\psi_1(t) = \frac{3\,\phi_1(t)}{\sqrt{24}}}$$

$\varphi_0(t) = \frac{S_0(t)}{\sqrt{3}}$ $\varphi_1(t) = \frac{3\varnothing_1(t)}{\sqrt{24}}$ $\varnothing_1(t) = S_1(t) - \frac{S_0(t)}{3}$

"Will these basis function be unique??"

It wont be unique
If we use diff signal It will be completely
diff so

we got to know how to conv signal → vector.

$s_i(t) = \sum_{k=0}^{n-1} s_i(k)$

for ve

$\langle s_i, s_j \rangle = \int s_i(t)\, s_j(t)\, dt = \sum_{k=0}^{n-1} s_i(k)\, s_j(k) = \langle s_i, s_j \rangle$

$s_i(t) = \sum_{k=0}^{n-1} s_i(k)\, \varphi_k(t)$ $\boxed{s_i[k] = \langle s_i, \varphi_k \rangle}$

$= \int \sum_{k=0}^{n-1} s_i[k]\, \varphi_k(t) \sum_{k'=0}^{n-1} s_j[k']\, \varphi_{k'}(t)\, dt.$

$= \sum_{k=0}^{n-1} \sum_{k'=0}^{n-1} s_i(k)\, s_j(k') \underbrace{\int_{t=0}^{\infty} \varphi_k(t)\, \varphi_{k'}(t)\, dt'}_{1}$

$= \sum_{k=0}^{n-1} \sum_{k'=0}^{n-1} s_i(k)\, s_j(k')$ [∵ If $k = l$. $\underbrace{\int_{-\infty}^{\infty} \varphi_k(t)\, \varphi_l(t)\, dt}_{1}$

$k \neq l$ $\int_{-\infty}^{\infty} \varphi_k(t)\, \varphi_l(t)\, dt = 0$]

$\boxed{s_i(t) = \sum_{k=0}^{n-1} s_i(k)\, s_j(k)}$

"dimension of noise → ∞"

~~As~~ If we take 1000 noise & do Gram-schmidt
we get 1000 φ so 1000 dim.

14th April 2025

converting cont to vector form

which has finite dime 'n'

$\boxed{n \le m}$

$s_i[k] \longrightarrow$ coefficients of Basis function

$N[i] = \langle n, \varphi_i \rangle = \int n(t)\,\varphi_i(t)$

noise

$\boxed{i = 0, 1, \ldots n-1}$

dimen

If $n(t) \rightarrow$ goes to $\varphi_0(t) - - \varphi_k(t) - - \varphi_{n-1}(t)$ we get

sample $\underbrace{N(0), N[1] - - - N[n-1]}$

These are IId Gaussian r.v

$cov(z_1, z_2) = \sigma^2 \langle u_1, u_2 \rangle$ $\boxed{\begin{array}{l} z_1 = \langle n, u_1 \rangle \\ z_2 = \langle n, u_2 \rangle \end{array}}$

$z = (z_1, z_2)^T \sim N(0, C)$ $cov(\langle z_1, z_2 \rangle)$ $cov(\langle z_1, z_2 \rangle)$

$$C = \begin{pmatrix} \sigma^2 \|u_1\|^2 & \sigma^2 \langle u_1, u_2 \rangle \\ \sigma^2 \langle u_1, u_2 \rangle & \sigma^2 \|u_2\|^2 \end{pmatrix}$$

$cov(\langle z_1, z_2 \rangle)$ $cov(\langle z_2, z_2 \rangle)$

$n(t) \longrightarrow$ WGN.

$u_1(t) \longrightarrow$ deterministic signal

$E[\langle n, \varphi \rangle]$

$z_1 \longrightarrow$ Gaussian random variable.

$= E\left[\int n(t)\,\varphi(t)\,dt\right]$

$= \int \varphi(t)\, E[n(t)]\,dt$

Now z_1 is linear f'n of $n(t)$ which is Gaussian random process.

$= 0$

z_1 is a Gaussian random variable (GRV)

Illy is z_2 is also GRV

1 'n'

$n(t)$ is zero mean $\Rightarrow z_1$ & z_2 are zero mean.

$$\mathrm{cov}(z_1, z_2) = \mathrm{cov}(\langle n, u_1\rangle, \langle n, u_2\rangle)$$

$$= E[\langle n, u_1\rangle, \langle n, u_2\rangle]$$

$$= E\left[\left(\int n(t) u_1(t) dt\right)\left(\int n(s) u_2(s) ds\right)\right]$$

$$= \int_t \int_s u_1(t)\, u_2(s)\; E[n(t)\, n(s)]\, dt\, ds.$$

$$= \iint_{t\,s} \sigma^2 \delta(t-s)\, u_1(t)\, u_2(s)$$

$E[z_1] = E\left(\int n(t) u_1(\ldots\right.$

$= \int u(t) E[n(\ldots$

$= \int u(t)(0)\, dt = \ldots$

$$= \sigma^2 \int_t u_1(t)\, u_2(t)\, dt.$$

$$= \sigma^2 \langle u_1, u_2\rangle$$

$$\mathrm{var}(z_1) = \mathrm{cov}(z_1, z_1) = \sigma^2 \langle u_1, u_1\rangle = \sigma^2 \|u_1\|^2.$$

$$\mathrm{var}(z_2) = \mathrm{cov}(z_2, z_2) = \sigma^2 \langle u_2, u_2\rangle = \sigma^2 \|u_2\|^2.$$

$$\mathrm{cov}(N[i], N[j]) = E[\langle n, \varphi_i\rangle, \langle n, \varphi_j\rangle]$$

$$= \sigma^2 \langle \varphi_i, \varphi_j\rangle \qquad \delta_{ij} = 1 \quad i = j$$
$$\qquad\qquad\qquad\qquad\qquad = 0 \quad i \neq j$$

$$= \sigma^2 \delta_{ij}$$

Noise projection are uncorrelated since they are Guassion they are also independent.

Note that all have same variance σ^2.

convert ~~DISC~~ cont $\longrightarrow$ DISC $\Longrightarrow$ and use MAP (or) ML rule and the demod to cont.

$y_1 = s + n_2 \qquad y_2 = n_1$

dont carry inflormation

If $n_1 = n_2$

$y_1 - y_2 = s$.

proof:

under hypothesis $H_i : Y \sim N(\underline{s_i}, \sigma^2 \underline{I})$

Identity matrix.

covariance matrix.

$$P_{Y|i}(y|H_i) = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left(-\frac{\|\underline{y} - \underline{s_i}\|^2}{2\sigma^2}\right) \to \text{HOW}??$$

$\underline{Y} = s_i + \underline{N} \longrightarrow$ mean 0

mean s_i

Bold $\longrightarrow$ vector.

Now MAP rule.

$$\delta_{MAP}(y) = \arg\max_i (\cancel{P(y|H_i)}\ P(H_i|y))$$

$$= \arg\max_i P(y|H_i)\, \pi_i$$

$$= \arg\max_i \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left(\frac{-\|\underline{y} - \underline{s_i}\|^2}{2\sigma^2}\right) \pi_i$$

$$= \arg\max_i \underbrace{-\frac{n}{2} \log 2\pi\sigma^2}_{\text{doesnt depend on } i \text{ so we can remove this}} - \frac{\|\underline{y} - \underline{s_i}\|^2}{2\sigma^2} + \log \pi_i$$

$$= \arg\max_i -\frac{\|y - s_i\|^2}{2\sigma^2} + \log \pi_i$$

This equivalent to

$$= \arg\min_i \frac{\|y - s_i\|^2}{2\sigma^2} - \log \pi_i$$

$$\boxed{\delta_{MAP}(y) = \arg\min_i \|y - s_i\|^2 - 2\sigma^2 \log \pi_i}$$

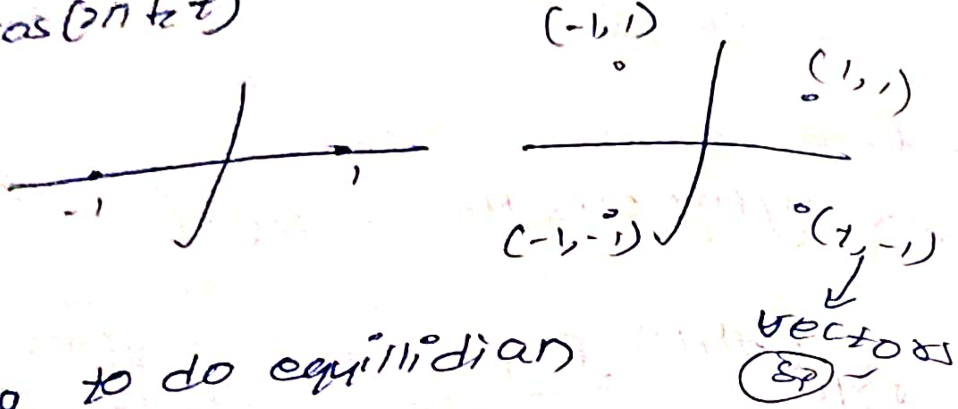
If π_i are equiprobable we will get

$\pi_i = \frac{1}{M}$.

$$\boxed{\delta_{ML}(y) = \arg\min \|y - s_i\|^2} \rightarrow \text{Basis coefficient}$$

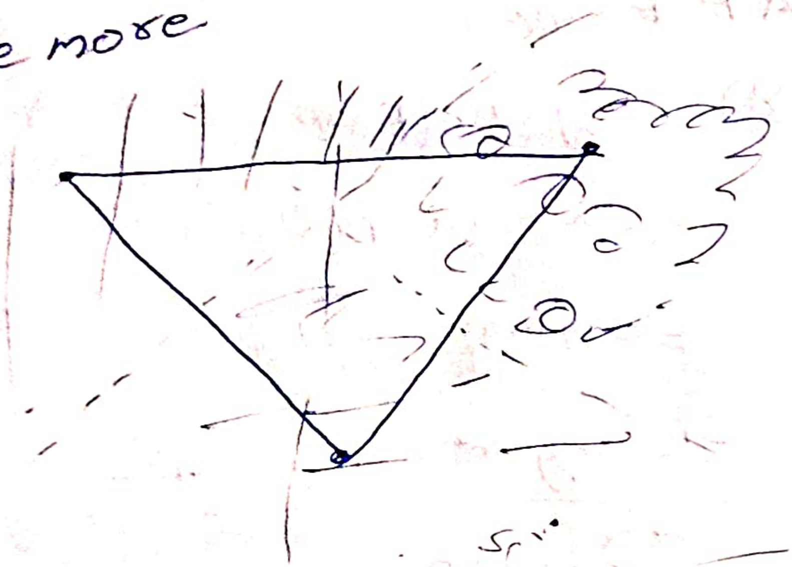
$s_0(t) = \cos 2\pi f_c t$ $\qquad \varphi_1(t) = \sqrt{2}\cos 2\pi f_c t$.

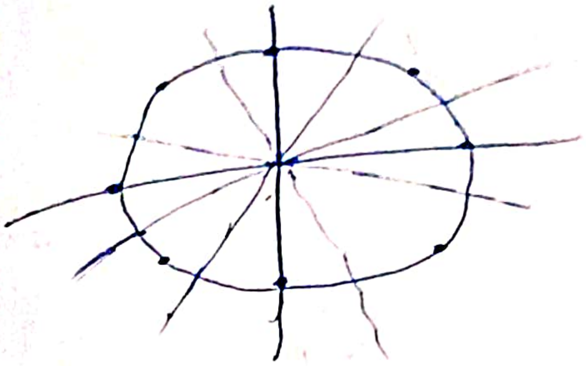
$s_1(t) = -\cos(2\pi f_c t)$

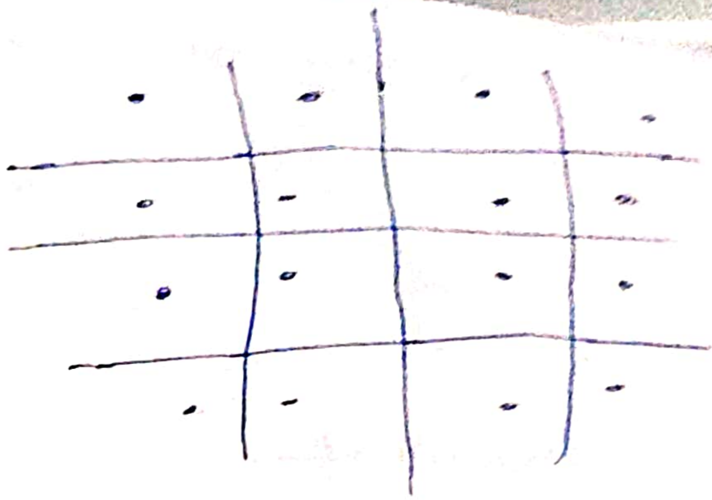


we are trying to do equillidian distance.

If $\pi_0 > \pi_1$ then '0' decision space will be more







In CT as there ~~case~~ is a noise which is ∞ dim. so $\Sigma (x_1 - x_2)^2$

$\|y - s_i\|^2 = (y - s_i)^T (y - s_i)$

$= y^T y - 2 s_i^T y + s_i^T s_i$

max $y^T s_i - \frac{\|s_i\|^2}{2}$.

$\langle y_i, s_i \rangle - \frac{\|s_i\|^2}{2}$.

$\|y - s_i\|^2 \overset{\text{argmin}}{=} \langle y - s_i \rangle \langle y - s_i \rangle$

$= \|y\|^2 - 2\langle y, s_i \rangle + \|s_i\|^2$.

remove the terms without i

$\underset{i}{\text{argmin}} = -2\langle y, s_i \rangle + \|s\|^2$

$= \underset{i}{\text{argmax}} \; \langle y, s_i \rangle - \frac{1}{2}\|s_i\|^2$

$\|y - s_i\|^2 = \|y\|^2 + \|s_i\|^2 - 2\langle y, s_i \rangle$

$\langle y - s_i \rangle \langle y - s_i \rangle$

$\cancel{\|y\|^2}^{\,0} - 2\langle s_i, y \rangle$

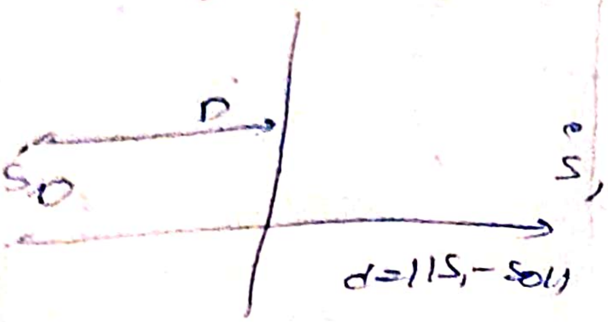
$+ \|s_i\|^2 - \frac{\|s_i\|^2}{2\sigma_0^2}$

$\frac{1}{2\sigma^2}\{\|s_i\|^2$

$-2\langle y, s_i \rangle + \|s_i\|^2 = 2\sigma^2$

$-\langle y, s_i \rangle - \frac{\|s_i\|^2}{2} + \sigma^2$

Geometry for binary signalling

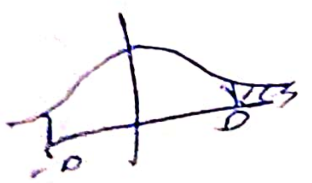


$P(N_{\text{perp}} > D) = \frac{D}{\sigma}$

$x \sim N(\mu, \sigma^2)$

$P(x > \eta) = Q\left(\frac{\eta - \mu}{\sigma}\right)$

$P(N_{\text{perp}} > D) = Q\left(\frac{D}{\sigma}\right) = Q\left(\frac{\|S_1 - S_0\|}{2\sigma}\right) = Q$



CT signal $\longrightarrow s_i$ & also vector $\longrightarrow \underline{s}_i$

ML is given by

$\delta_{ML}(y) = \arg\max_{i=0,1} \langle y, s_i \rangle - \frac{\|s_i\|^2}{2}$ $\quad s_1 = s, \; s_0 = 0$

$\langle y, s \rangle - \frac{\|s\|^2}{2} \underset{H_0}{\overset{H_1}{\gtrless}} \langle y, 0 \rangle - \frac{\|0\|^2}{2}$

$Z \leq \langle y, s \rangle \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\|s\|^2}{2} = \eta$.

error prob

$P_{e|0} = P\left(Z > \frac{\|s\|^2}{2} \middle| H_0\right) \longrightarrow ②$

$P_{e|1} = P\left(Z < \frac{\|s\|^2}{2} \middle| H_1\right) \longrightarrow ①$

Now we need dist. of Z

$Z = \int y(t)\, s(t)\, dt$

linear comb'n of $y(t)$, which is Guassian r.v

$\therefore$ z is Gaussian r.v

Now $E[z|H_0] = E[\langle n,s\rangle] = 0$

$E[z|H_1] = E[\cancel{\langle n,s\rangle}^{0}] + E[\langle s,s\rangle]$

$= E[\langle s,s\rangle] = \|s\|^2 = \int s^2(t)\,dt$

$var(z|H_0) = cov(\langle n,s\rangle, \langle n,s\rangle)$

$= \sigma^2 \|s\|^2$

$var(z|H_1) = cov(\langle s+n,s\rangle, \langle s+n,s\rangle)$

$= cov(\langle n,s\rangle, \langle n,s\rangle) = \sigma^2\|s\|^2$ (4)

$H_0 : z \sim N(0, \sigma^2\|s\|^2)$

$H_1 : z \sim N(\|s\|^2, \sigma^2\|s\|^2)$

$z \triangleq \langle y_i, s_i\rangle \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\|s\|^2}{2} = \gamma$

$P_{e|0} = P(H_1|H_0)$

$= P(z > \gamma | H_0) = Q\left(\frac{\gamma - \mu_0}{\sigma_0}\right) = Q\left(\frac{\|s\|^2}{2\sigma}\right)$ $\quad \mu_0 = 0$, $\gamma = \frac{\|s\|^2}{2}$

$\cancel{Q\left(\frac{\|s\|}{2\sigma\|s\|^2}\right) = Q\left(\frac{1}{2\sigma\|s\|}\right)}$ $\quad \sigma_0 = \sigma\|s\|^2$

$= Q\left(\frac{\|s\|^2}{2\sigma\|s\|}\right)$

$\boxed{P_{e|0} = Q\left(\frac{\|s\|}{2\sigma}\right)}$

$P(z > \gamma)$

$Q($

$P_{e|1} = P(H_0|H_1)$

$\langle s+n, 1\rangle$

$\langle s,1\rangle + \langle n,1\rangle$

$\langle y,s\rangle - \frac{\|s\|^2}{2} \underset{H_{01}}{\overset{H_0}{>}} \langle y,s\rangle - \frac{\|s\|^2}{2}$

$\langle s,s\rangle + \langle n,s\rangle$

$\|s\|^2 - \frac{\|s\|^2}{2}$

$\frac{\|s\|^2}{2}$

$\langle y_0,s\rangle - \frac{\|s\|^2}{2} \underset{H_1}{\overset{H_0}{<}} \langle y,s\rangle - \frac{\|s\|^2}{2}$

$z \triangleq \langle y,s\rangle \underset{H_0}{\overset{H_{01}}{\lessgtr}} \langle y,s\rangle + \frac{\|s\|^2 - 1}{2} = \gamma$

$P(e|_1) = P(H_0|H_1)$

$= P(z < \gamma | H_1)$

$\gamma =$

$= P\left(\frac{\gamma - \mu}{\sigma}\right)$

$1 - P(z > \gamma | H_1)$

$= 1 - Q\left(\frac{\gamma - \mu_1}{\sigma^2}\right)$ $\quad \mu_1 = \|s\|^2$ $\quad \sigma_1 = \sigma\|s_1\|$

$\gamma = \frac{\|s\|^2}{2}$

$= 1 - Q\left(\frac{-\|s\|^2}{2\sigma}\right)$

$= 1 - Q\left(-\frac{\|s\|}{2\sigma}\right)$

$= Q\left(\frac{\|s\|}{2\sigma}\right)$

$P_e = \pi_1 P_{e|1} + \pi_0 P_{e|0}$

$= \frac{1}{2} Q\left(\frac{\|s\|}{2\sigma}\right) + \frac{1}{2} Q\left(\frac{\|s\|}{2\sigma}\right) = Q\left(\frac{\|s\|}{2\sigma}\right).$

$H_0 : y = s_0 + n \qquad H_1 : y = s_1 + n \qquad \tilde{y}\,C$

$\delta_{ML} = \arg\max \langle y, s_i \rangle - \frac{\|s_i\|^2}{2}.$

$\langle y, s_1 \rangle - \frac{\|s_1\|^2}{2} \underset{H_0}{\overset{H_1}{\gtrless}} \langle y, s_0 \rangle - \frac{\|s_0\|^2}{2}.$

$\langle y, s_1 \rangle \underset{H_0}{\overset{H_1}{\gtrless}} \langle y, s_0 \rangle + \left(\frac{\|s_1\|^2 - \|s_0\|^2}{2}\right)$

---

sims!

$\tilde{y}(t) = y(t) - s_0(t)$

$H_1 : \tilde{y}(t) = (s_1(t) - s_0(t)) + n(t)$

$H_2 : \tilde{y}(t) = n(t)$

Q: solve ~~second~~ first part assuming $S_1 - S_0 = SC_{(?)}$.

$$E_b = \frac{1}{2}\left(\|S_0\|^2 + \|S_1\|^2\right)$$

'0' & '1' are equally likely.

$$\boxed{\eta_p = \frac{d^2}{E_b}}$$

$$d = \sqrt{\eta_p E_b}$$

$$P_e = Q\left(\frac{d}{2\sigma}\right) = Q\left(\frac{\sqrt{\eta_p E_b}}{2\sigma}\right)$$

If $\sigma^2 = \frac{N_0}{2} \Rightarrow \sigma = \sqrt{\frac{N_0}{2}}$.

$$= Q\left(\left(\sqrt{\frac{\eta_p E_b 2}{N_0}}\right)\frac{1}{2}\right)$$

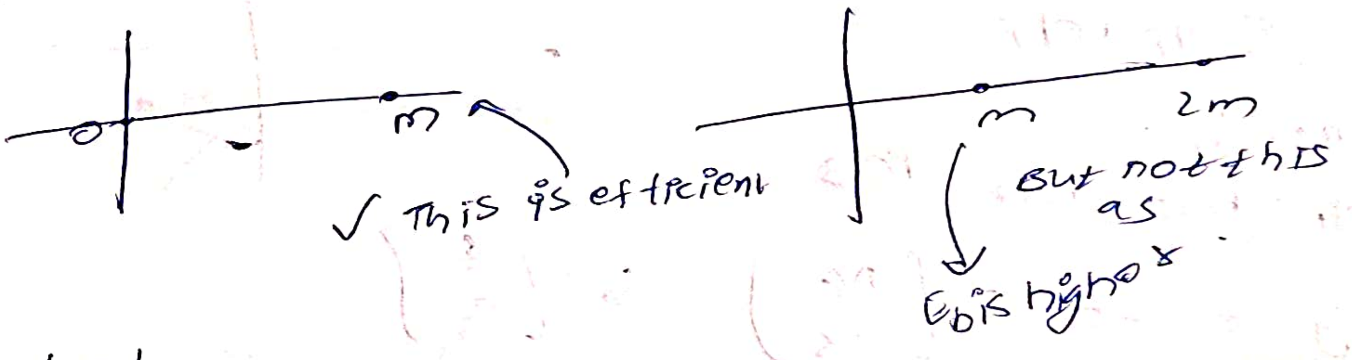
$$\boxed{P_e = Q\left(\sqrt{\frac{\eta_p E_b}{2N_0}}\right)}$$

$\sqrt[n]{x^n}$

$\eta_p = \frac{d^2}{E_b}$ $\quad d = \sqrt{\eta_p E_b}$ $\quad \sigma^2 = \frac{N_0}{2}$ $\quad \sigma = \sqrt{N_0/2}$.

$$\frac{d}{\sigma} = \sqrt{\frac{2\eta_p E_b}{N_0}} \Rightarrow$$

If $d \uparrow$ $P_e \downarrow$. But we need more power.



$d = 1$

$$E_b = \frac{0^2 + 1^2}{2} = \frac{1}{2}$$

$$\eta_p = \frac{d^2}{E_b} = \frac{1}{\frac{1}{2}} = 2$$

Antipodal is better.

2^{st} April 2025

$E_s = \frac{1}{M} \sum_{i=1}^{M} \|S_i\|^2$

$E_b = \frac{E_s}{\log_2 M}$

$\eta_B = \frac{R}{B_{min}}$

M-ary signaling AWGN.

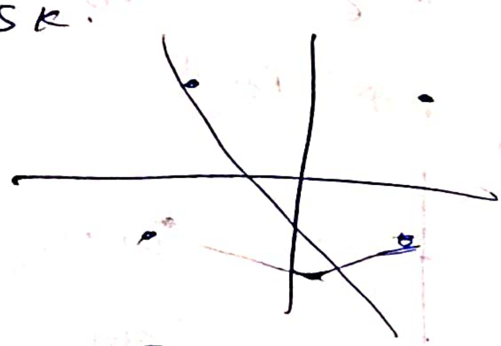
① $E[z|H_i] = \langle S_i, S_j \rangle - \frac{\|S_j\|^2}{2}$

② $\mathrm{cov}\left(\frac{z_j}{\sigma^2}, \frac{z_k}{\sigma^2} \Big| H_i\right) = \frac{\langle S_j, S_k \rangle}{\sigma^2}$

$\frac{\langle S_i, S_j \rangle}{\sigma^2} = \frac{\langle S_i, S_j \rangle}{E_b} \times \frac{E_b}{\sigma^2}$ as $\sigma^2 = \frac{N_0}{2}$.

$= \underbrace{\frac{\langle S_i, S_j \rangle}{E_b}}_{\text{scale-invariant}} \times \frac{2E_b}{N_0}$

exact error prob for QPSK.



$P_e = \sum_{i=0}^{3} \pi_i P_{e|i}$

$\pi_i = \frac{1}{4}$.

$P_{e|1} = P_{e|0} = P_{e|2} = P_{e|3}$

$y = S_0 + \begin{bmatrix} N_c \\ N_s \end{bmatrix}$ $S_0 = \begin{bmatrix} \frac{d}{2} \\ \frac{d}{2} \end{bmatrix}$

$y = \begin{bmatrix} \frac{d}{2} + N_c \\ \frac{d}{2} + N_s \end{bmatrix}$

error happens when $N_c + \frac{d}{2} < 0$ (or) $N_s + \frac{d}{2} < 0$

$P_{e|0} = P[N_c + \frac{d}{2} < 0 \cup N_s + \frac{d}{2} < 0]$

$= P(N_c + \frac{d}{2} < 0) + P(N_s + \frac{d}{2} < 0) - P(N_c + \frac{d}{2} < 0)\, P(N_s + \frac{d}{2} < 0)$

$= P(N_c < -\frac{d}{2}) + P(N_s < -\frac{d}{2}) - P(N_c < -\frac{d}{2})\, P(N_s < -\frac{d}{2})$

$= (1 - P(N_c > -\frac{d}{2})) + (1 - P(N_s > -\frac{d}{2})) - (1 - P(N_c > -\frac{d}{2}))(1 - P(N_s > -\frac{d}{2}))$

$= \underbrace{1 - Q(\frac{-d}{2\sigma})}_{Q(\frac{d}{2\sigma})} + (1 - Q(\frac{-d}{2\sigma}))$

$= Q(\frac{d}{2\sigma}) + Q(\frac{d}{2\sigma}) - Q(\frac{d}{2\sigma})\, Q(\frac{d}{2\sigma})$

$= 2\, Q(\frac{d}{2\sigma}) - Q^2(\frac{d}{2\sigma}) \longrightarrow (4)$

$d =$

energy per symbol

$E_s = \frac{1}{m} \sum \|S_i\|^2$

$= \|S_i\|^2 = (\frac{d}{2})^2 + (\frac{d}{2})^2 = \frac{d^2}{2}$

Energy per bit

$E_b = \frac{E_s}{\log_2 m} = \frac{\frac{d^2}{2}}{2} = \frac{d^2}{4}$

$d = \sqrt{4 E_b} \longrightarrow (5)$

$\sigma^2 = \frac{N_0}{2} \Rightarrow \sigma = \sqrt{\frac{N_0}{2}}$

$2\sigma = \sqrt{2 N_0} \longrightarrow (6)$

put (5) & (6) in (4)

$P(A_1 \cup A_2 \cup \dots A_n) \le P(A_1) + P(A_2) + \dots P(A_n)$

$P_{e|0} \le Q\left(\frac{\|S_1 - S_0\|}{2\sigma}\right) + Q\left(\frac{\|S_2 - S_0\|}{2\sigma}\right) + Q\left(\frac{\|S_3 - S_0\|}{2\sigma}\right)$

4

$P_{e|}$

$\frac{1}{16}\left(4(2) + 8(3) + 4(4)\right)$

$\frac{4}{16}[2+6+4] = \frac{12}{4} = 3$

$\boxed{\bar{N}_{dmin} = 3}$

$\frac{1}{4}(2(2)+4)$

4

$\boxed{P_e = 3Q\left(\frac{d_{min}}{2\sigma}\right)}$

$P_e = 3Q\left(\frac{1}{\sigma}\right)$

$E_s = \frac{1}{16}\{4(1^2+1^2)+4(3^2+3^2)+8(3^2+1^2)\}$

$= \frac{160}{16} = 10$

$E_b = \frac{E_s}{\log_2 M} = \frac{10}{4} = \frac{5}{2}$

$\eta_p = \frac{d_{min}^2}{E_b} = \frac{4}{\frac{5}{2}} = \frac{8}{5}$

$\boxed{\eta_p = \frac{8}{5}}$

N

$P_e = \bar{N}_{dmin} Q\left(\frac{d_{min}}{2\sigma}\right)$

B W efficiency

$$\eta_B = \frac{\log_2^m}{N_D} \rightarrow \text{compendim}$$

$x \& y \rightarrow 1$

$x \rightarrow \frac{1}{2}$

for QPSK

$$\eta_B = \log_2^4 = 2/1$$

$$\eta_P = \frac{d^2}{E_b} = 4$$

$E_s = 2 \cdot \frac{1}{4}$ EE

$$E_b = \frac{E_s}{\log_2^4} = \frac{2}{2} = 1$$

16 - QAM

$$\eta_P = \frac{8}{5} = 1.6 \qquad \eta_B = \log_2^M = 4$$

$W \rightarrow$ B.W

$R \rightarrow$ rate.

$$P_b = \frac{P_s}{\log_2^m}$$