EC5.203 Communication Theory I (3-1-0-4):

Lecture 8:
Analog Communication Techniques: Frequency Modulation - 1

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INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
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Recap

Key Concepts

- Two ways of encoding info in complex envelope
  - I and Q: amplitude modulation (several variants)
  - Envelope and phase: angle modulation (constant envelope)
- For example, for a sinusoid carrier

$$A_c(t)\cos(2\pi f_c(t)t + \theta_c(t))$$

where $A_c(t)$, $f_c(t)$, $\theta_c(t)$ are the amplitude, frequency, and the phase of the carrier respectively.

Amplitude Modulation

Frequency Modulation

Phase Modulation

AM: Double Sideband Suppressed Carrier

- Here the message $m(t)$ modulates the I component of the passband signal $u(t)$ and is given by

$$u_{DSB}(t) = m(t) \cdot A\cos(2\pi f_c t)$$

while the Fourier transform is given by

$$U_{DSB}(f) = \frac{A}{2}(M(f - f_c) + M(f + f_c))$$

Conventional AM

- Add a large carrier component to a DSB-SC signal so that the passband has the following form

$$\begin{aligned} u_{\mathrm{AM}}(t) &= (Am(t) + A_c)\cos(2\pi f_c t) \\ &= Am(t)\cos(2\pi f_c t) + A_c \cos(2\pi f_c t) \end{aligned}$$

- Taking Fourier transform

$$U_{\mathrm{AM}}(f) = \frac{A}{2}(M(f-f_c)+M(f+f_c))+\frac{A_c}{2}(\delta(f-f_c)+\delta(f+f_c))$$

Power efficiency of conventional AM

- DSB expression

$$u_{\rm AM}(t) = Am(t)\cos(2\pi f_c t) + \boxed{A_c \cos(2\pi f_c t)}$$

Extra Non-information carrying component

- Power efficiency is given by

$$\eta = \frac{\text{Power in information carrying signal}}{\text{Power in total signal}}$$

- Prove that power efficiency for conventional AM is given by

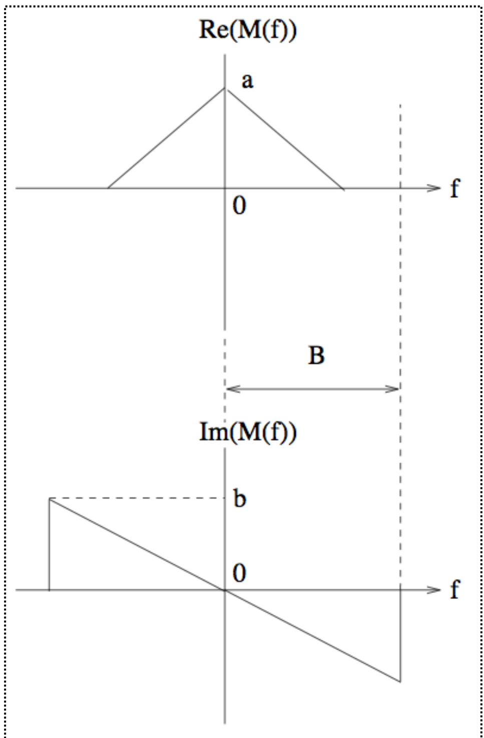
$$\eta_{\rm AM} = \frac{a_{\rm mod}^2 \overline{m_n^2}}{1 + a_{\rm mod}^2 \overline{m_n^2}}$$

- Further prove that

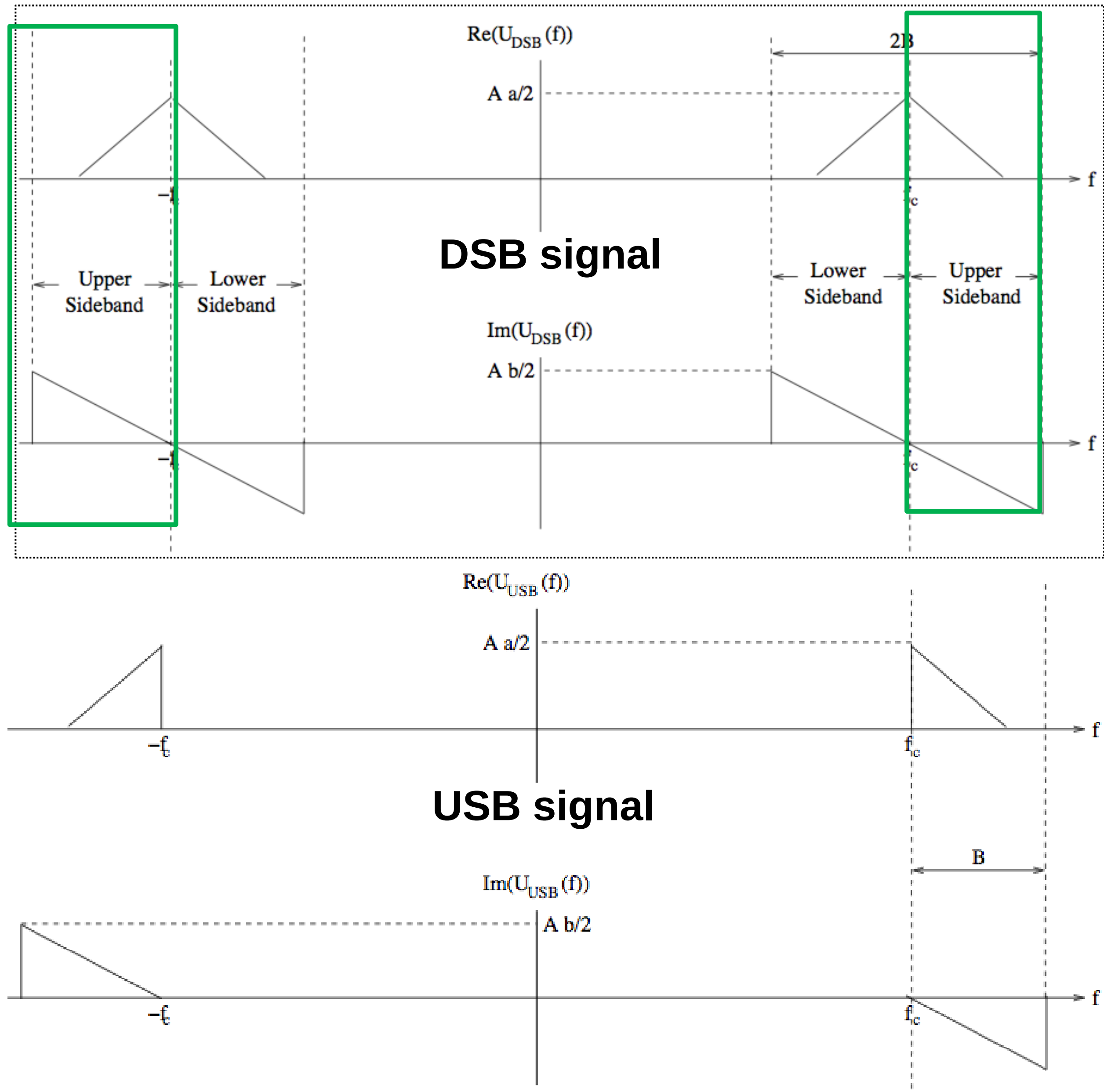
$$\eta_{\rm AM} \leq 50\%$$

- Solve: Find $\eta_{\rm AM}$ for sinusoidal message signal $m(t) = A_m \cos(2\pi f_m t)$

DSB ➔ USB (SSB)



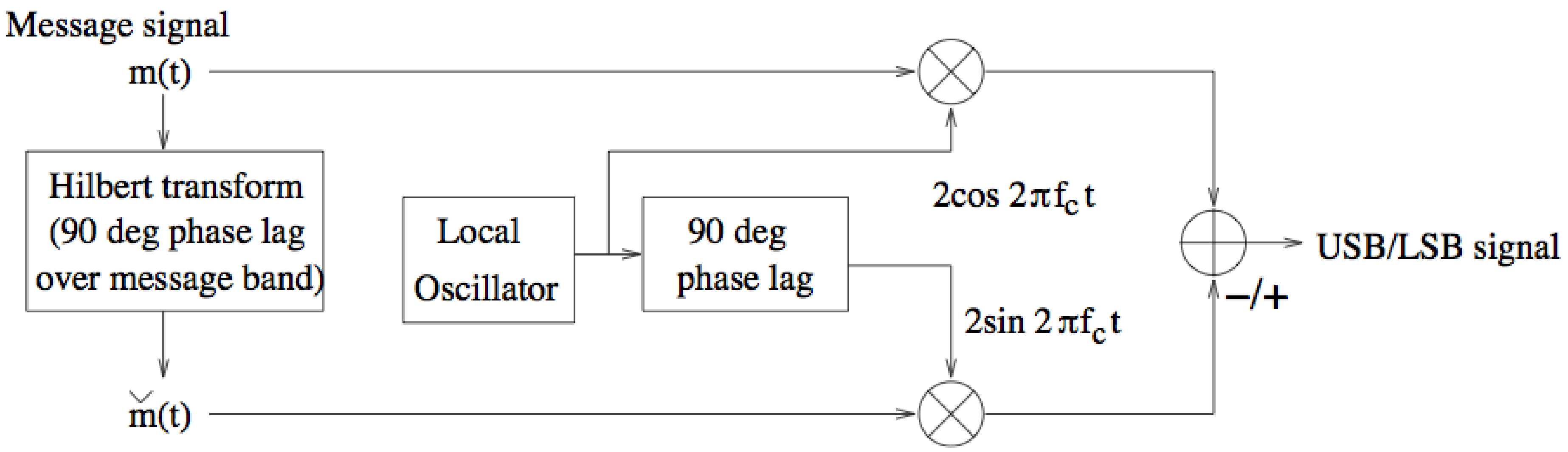
Message signal



DSB signal

USB signal

SSB in baseband using Hilbert Transform



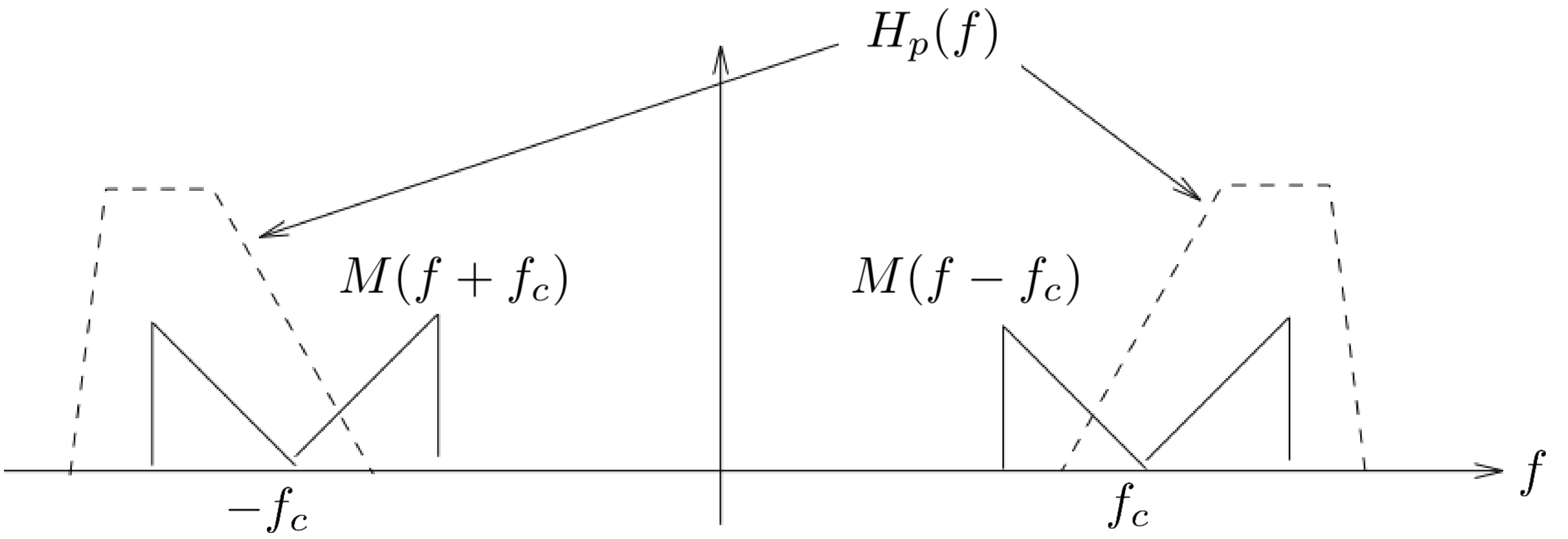
- Implementing Hilbert transform in baseband avoids need for sharp filtering at passband!
- In next few slides, we will see why it works!

Todays' Class

Amplitude Modulation:

Vestigial Side Band (VSB)

VSB signaling



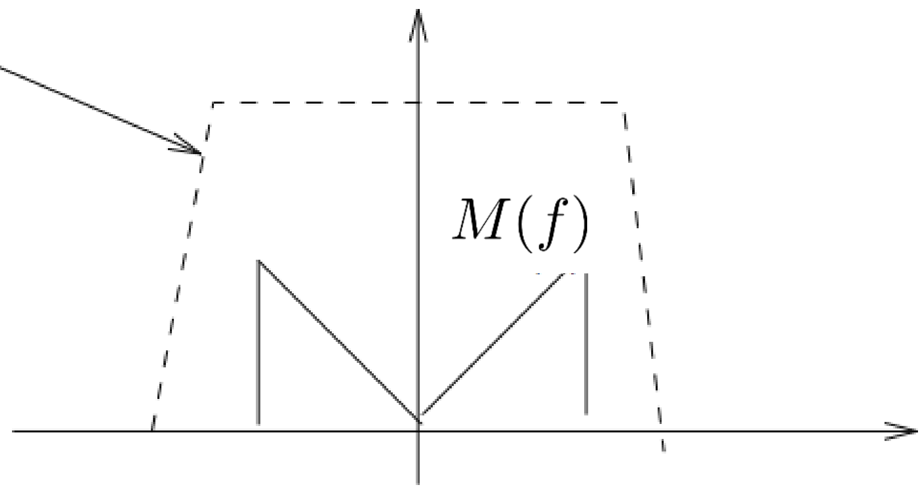
- Dictionary meaning of vestige: *a trace or remnant of something that is disappearing or no longer exists*

- VSB is general form of SSB: Filter DSB signal so as to leave vestige of one sideband

- Trade-off of ease of filtering requirements and bandwidth

How to choose VSB filter?

$H_p(f - f_c) + H_p(f + f_c)$ constant over message band (Prove!)

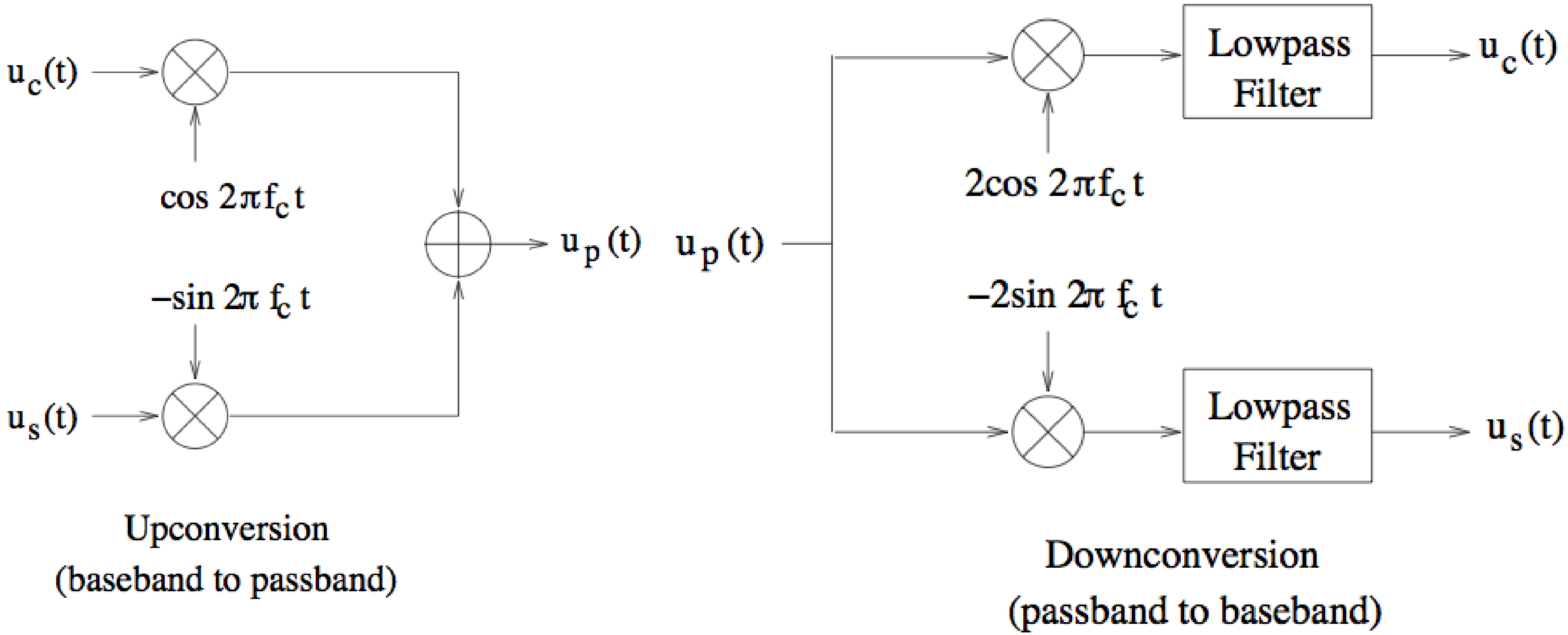


I component = message
Q component = filtered version of message that cancels portion of spectrum

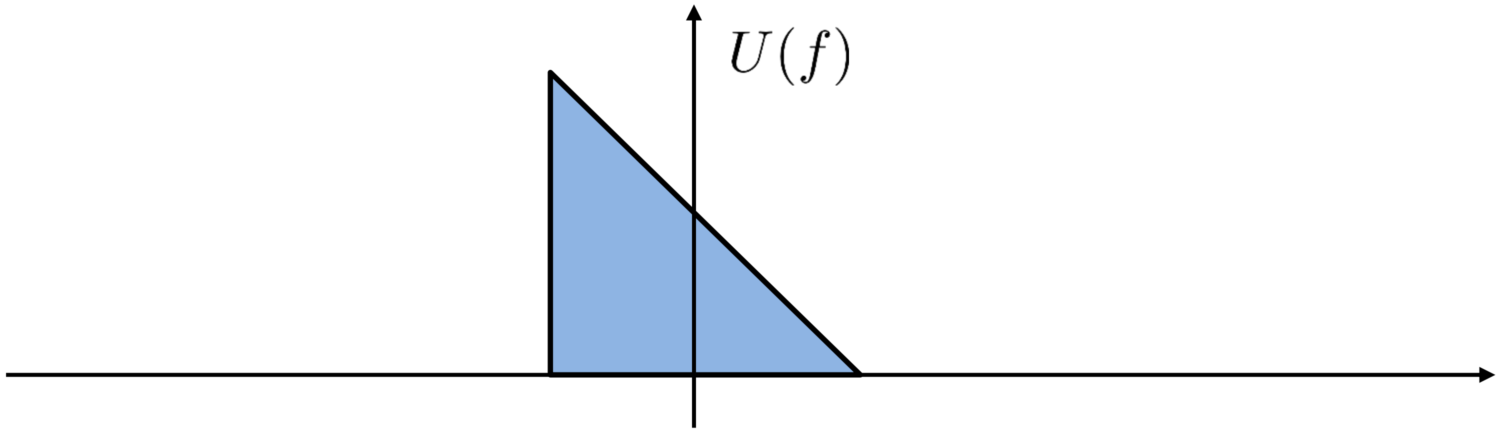
Quadrature Amplitude Modulation

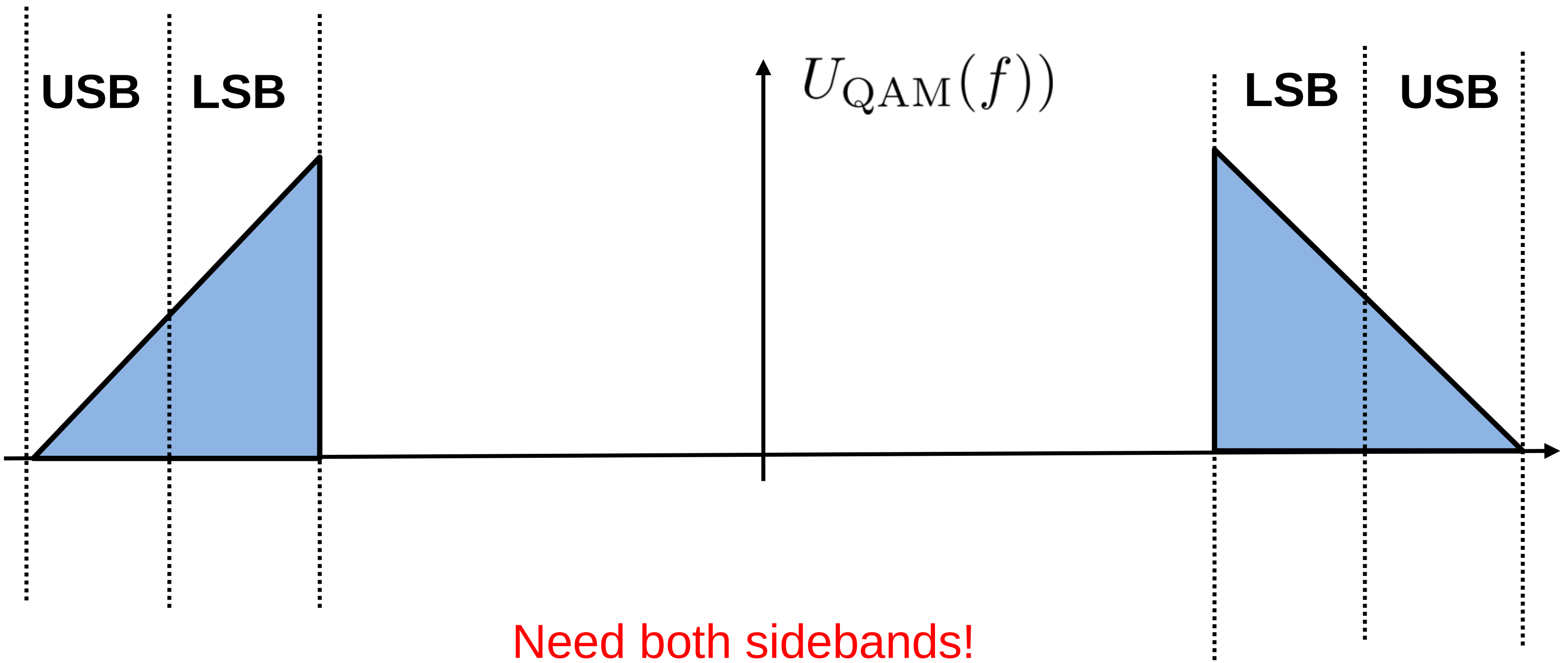
QAM

$$
\begin{aligned}
u(t) &= u_c(t) + j u_s(t) \\
u_{\mathrm{QAM}}(t) &= \mathrm{Re}\{u(t) e^{j2\pi f_c t}\} \\
&= u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)
\end{aligned}
$$



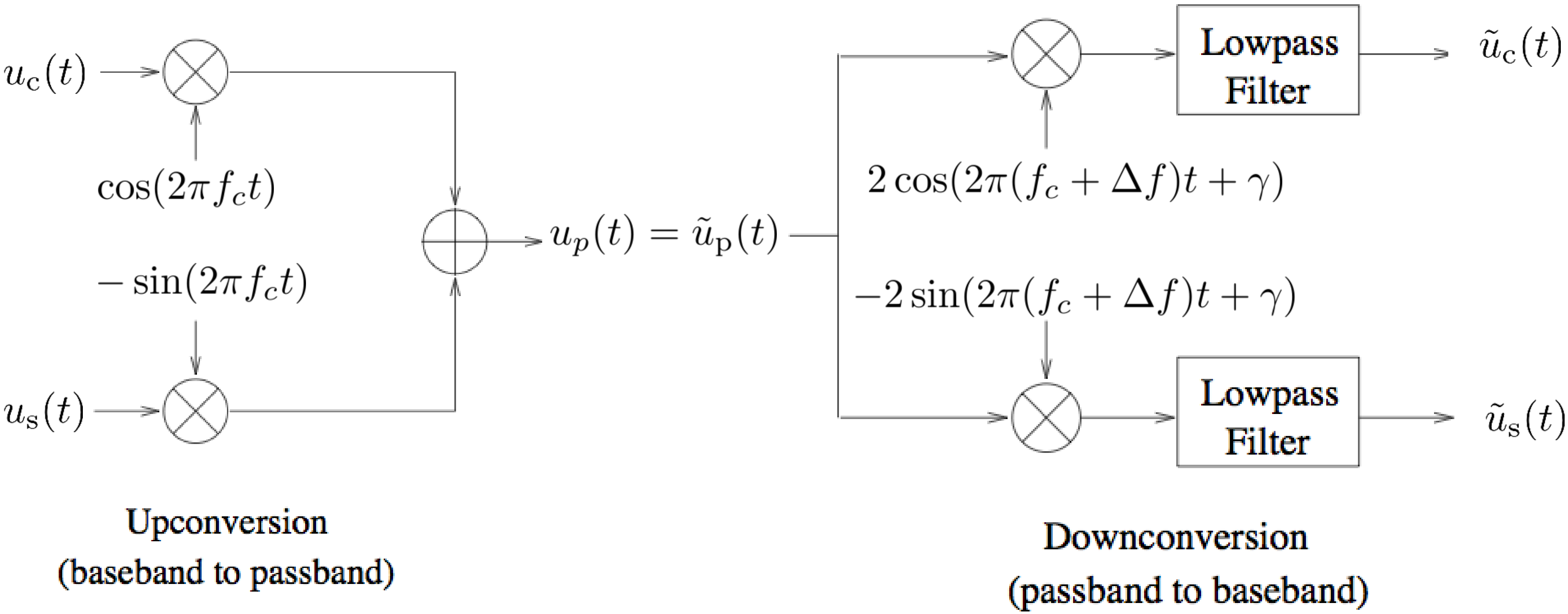
QAM





Need both sidebands!

Effect of Frequency and Phase Offset



- We have already seen this in Ch. 2: In this case

$$\tilde{u}_c(t) = u_c(t)\cos\phi(t) + u_s(t)\sin\phi(t)$$
$$\tilde{u}_s(t) = -u_c(t)\sin\phi(t) + u_s(t)\cos\phi(t)$$

where $\phi(t) = 2\pi\Delta f t + \gamma$ is the phase offset resulting from frequency offset Δf and the phase offset γ.

Coherent Detection: Synchronization

- Frequency offset and phase offset cause cross-interference between I and Q components

$$\tilde{u}_{\mathrm{c}}(t) = u_c(t)\cos\phi(t) + u_s(t)\sin\phi(t)$$
$$\tilde{u}_{\mathrm{s}}(t) = -u_c(t)\sin\phi(t) + u_s(t)\cos\phi(t)$$

  where $\phi(t) = 2\pi\Delta t + \gamma$ is the phase offset resulting from frequency offset Δf and the phase offset γ.

- Either have tight synchronization, i.e., $\Delta f \approx 0$ and $\gamma \approx 0$.

- Compensate for the offset $u(t) = \tilde{u}(t)e^{j\phi}$.

Questions?

Frequency Modulation

Recap: *Different Modulations*

- Two ways of encoding info in complex envelope
  - I and Q: amplitude modulation (several variants)
  - Envelope and phase: angle modulation (constant envelope)
- For example, for a sinusoid carrier

$$A_c(t)\cos(2\pi f_c(t)t + \theta_c(t))$$

where $A_c(t)$, $f_c(t)$, $\theta_c(t)$ are the amplitude, frequency, and the phase of the carrier respectively.

Amplitude Modulation

Frequency Modulation

Phase Modulation

Frequency Modulation

- The transmitted signal is given as

$$u_{\mathrm{FM}}(t) = A_c \cos(2\pi(\underline{f_c + f(t)})t + \phi)$$

- Here $f(t) = k_f m(t)$ is the frequency offset relative to the carrier, $m(t)$ is the message signal while k_f (design constant), A_c (amplitude) and ϕ (phase) are constants.

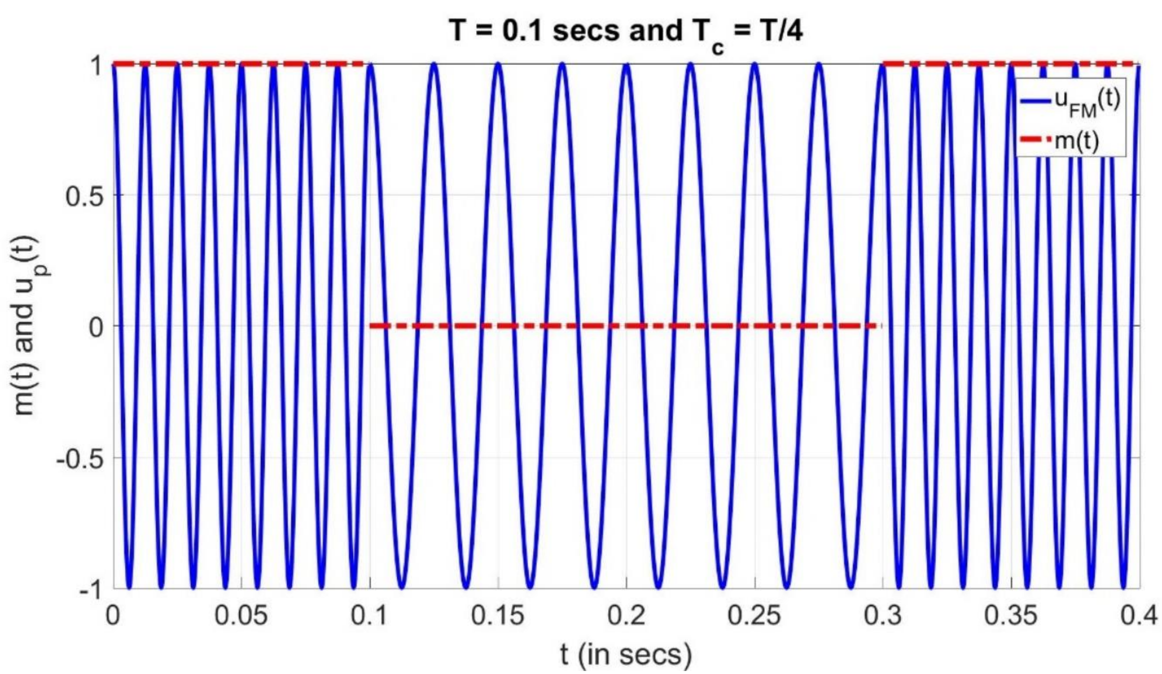
- The instantaneous frequency is given by

$$f_i(t) = f_c + f(t) = f_c + k_f m(t)$$

Example of FM Wave

- The instantaneous frequency is given by

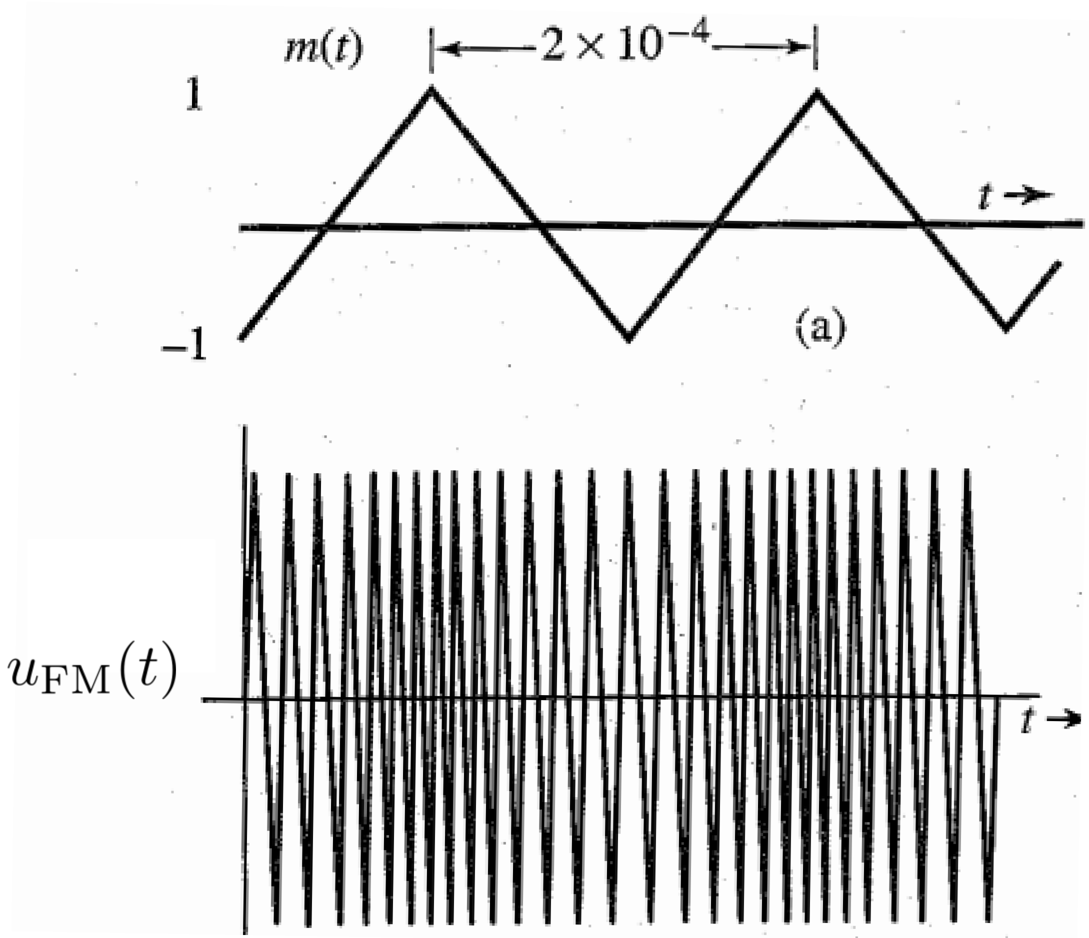
$$f_i(t) = f_c(1 + m(t))$$



Example 2 of FM Wave

- The instantaneous frequency is given by

$$f_i(t) = f_c + f(t) = f_c + k_f m(t)$$



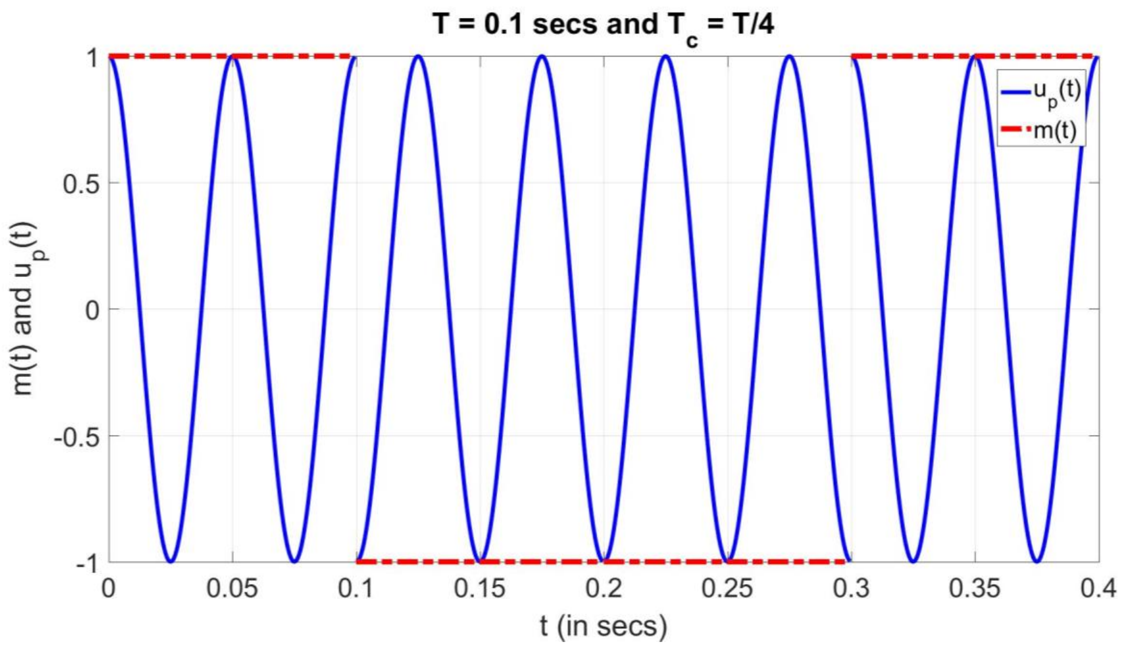
Phase Modulation

- The transmitted signal is given as

$$u_{\mathrm{PM}}(t) = A_c \cos(2\pi f_c t + \boxed{\theta(t)} + \phi)$$

- Here $\theta(t) = k_p m(t)$ while k_p, A_c, ϕ and f_c are constants.

Example of PM Wave



$$
\begin{aligned}
u(t) &= A_c \cos(2\pi f_c t + 0.5(m(t) - 1)\pi) \\
&= A_c \cos(2\pi f_c t) \qquad \color{red}{m(t) = 1} \\
&= A_c \cos(2\pi f_c t - \pi) \quad \color{red}{m(t) = -1}
\end{aligned}
$$

Generalized Model: Angle Modulation

- The transmitted signal is given as

$$u_{\mathrm{p}}(t) = A_c \cos(2\pi f_c t + \boxed{\theta(t)})$$

$$\theta(t) = g(m(t))$$

- Angle modulation is a general form
  - Phase modulation

$$\theta(t) = \theta(0) + k_p m(t)$$

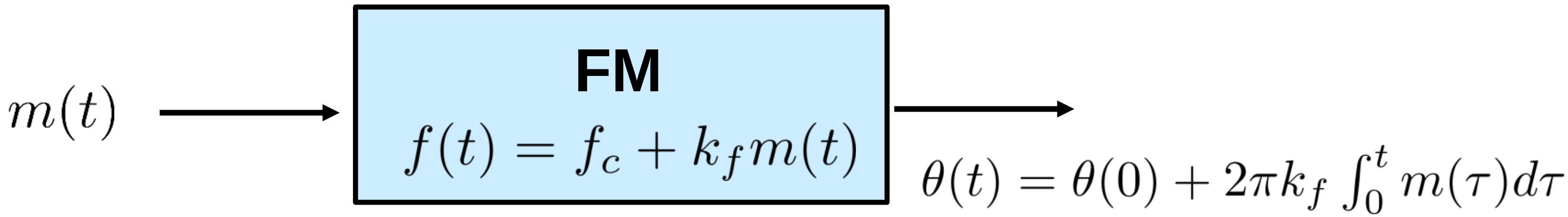
  - Frequency modulation

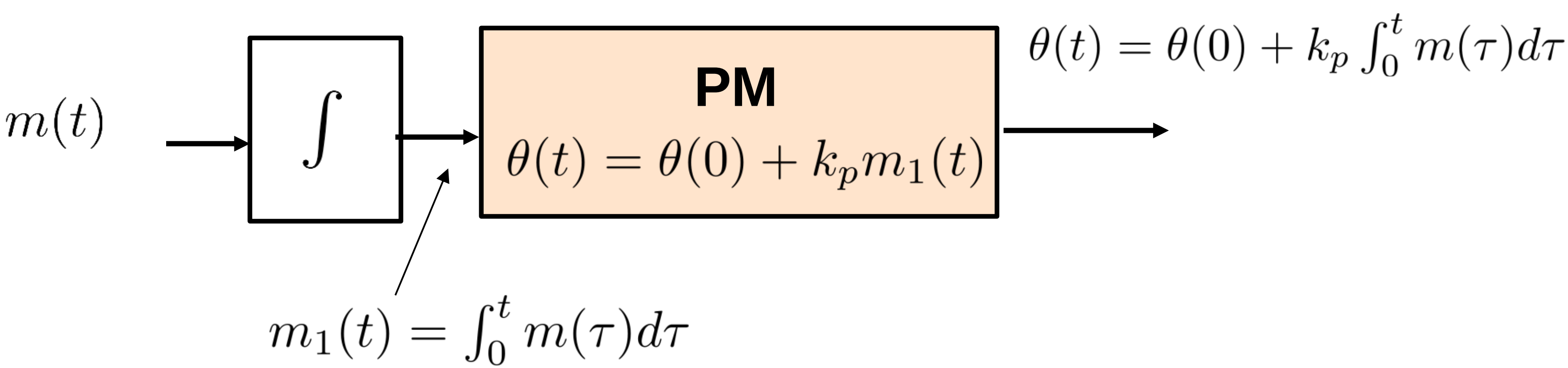
$$\frac{1}{2\pi}\frac{d\theta(t)}{dt} = f(t) = k_f m(t)$$

$$\theta(t) = \theta(0) + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

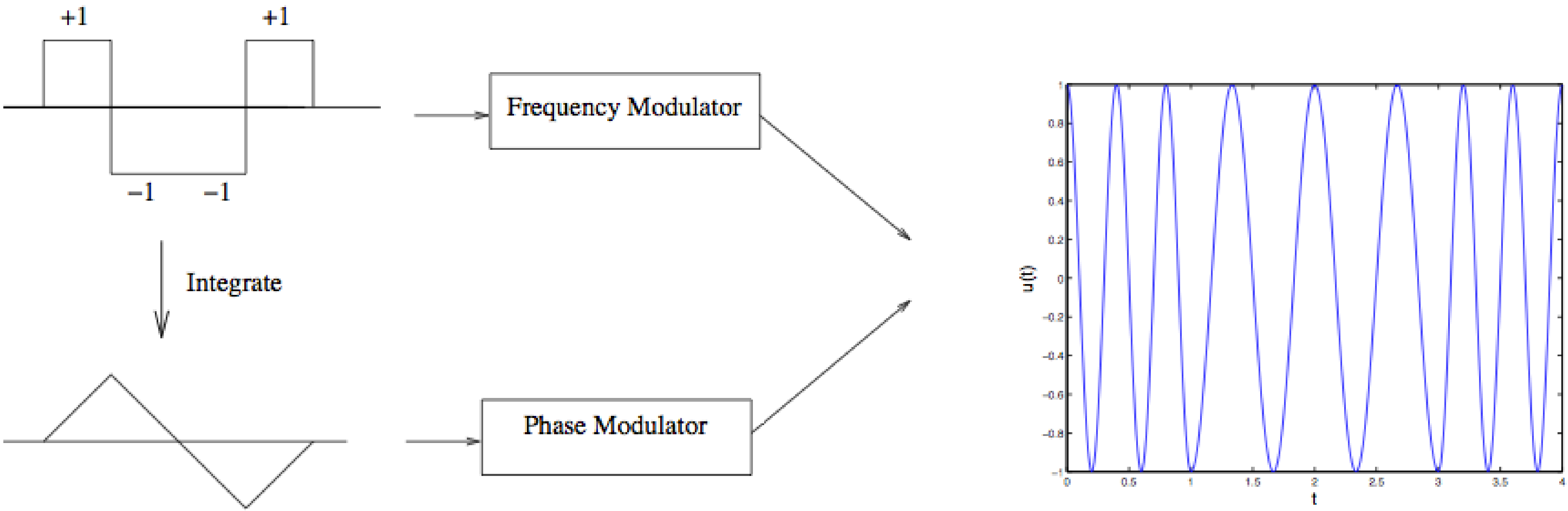
where k_p and k_f are constants while $f(t)$ is the frequency offset relative to the carrier. Also $\phi = \theta(0)$ where $t = 0$ is used as reference point.

Equivalence of PM and FM: *FM using PM*

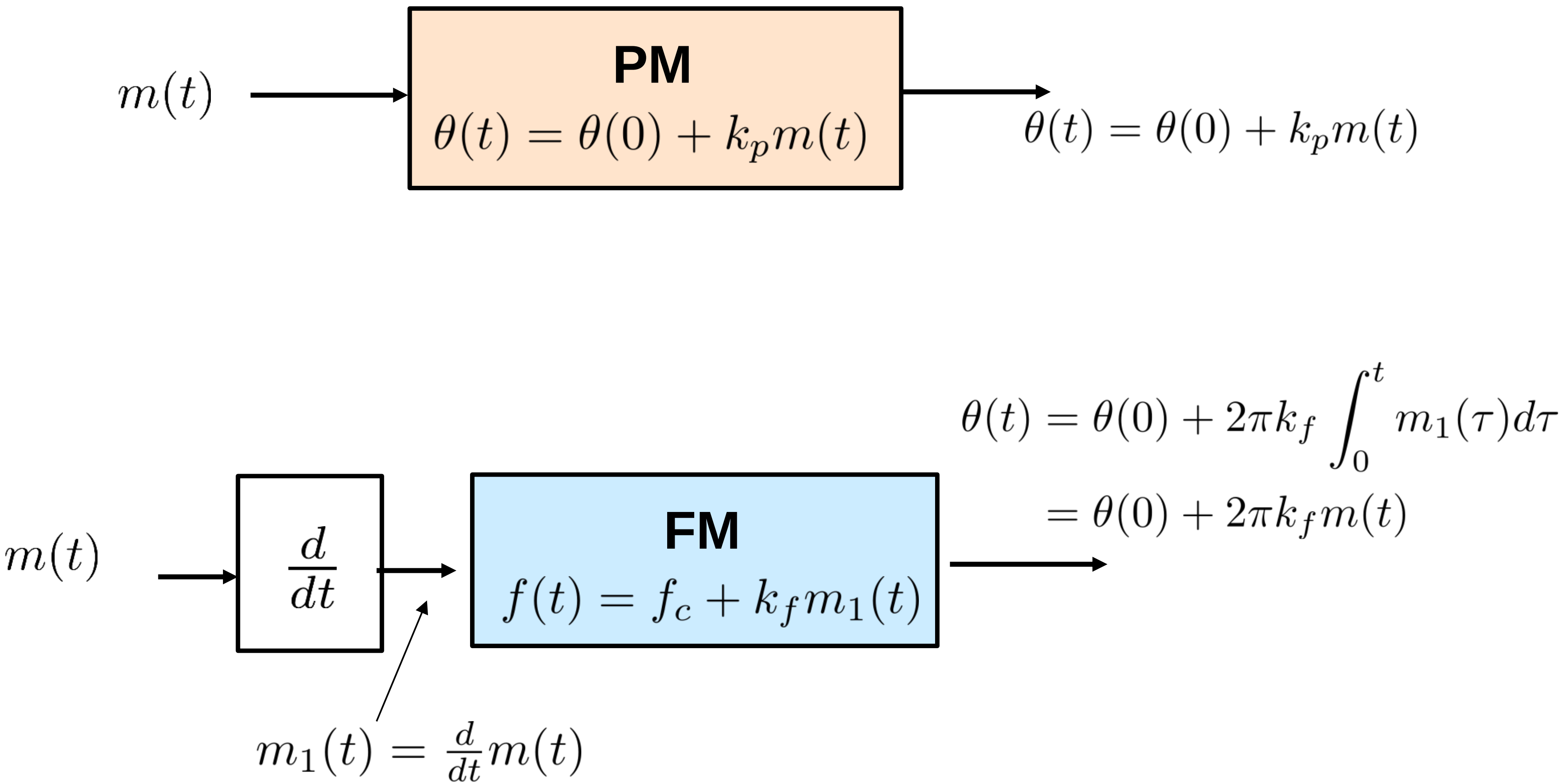




Equivalence of FM and PM: *FM using PM*



Equivalence of PM and FM: *PM using FM*



Poll

Which of the following are non-linear modulations?

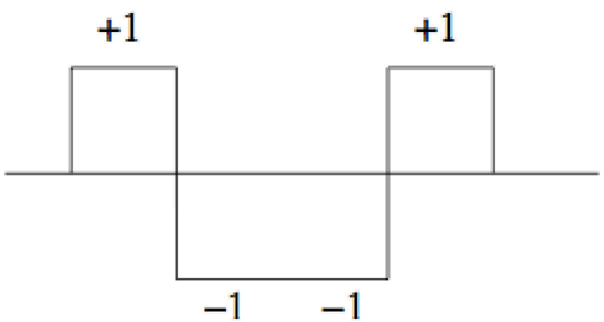
- AM
- PM
- FM
- None of the above

Non-linearity of Angle or Phase Modulation

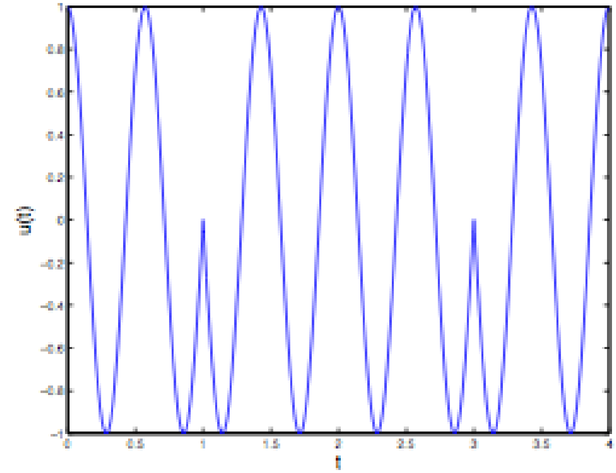
- Prove that angle modulation is a non-linear operation while amplitude modulation is a linear operation.

PM versus FM

Digital message



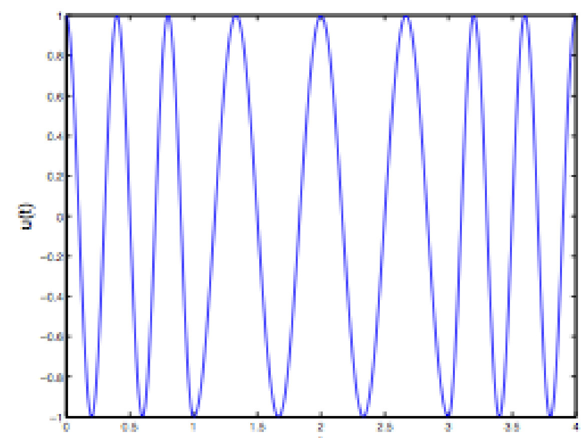
PM
(PSK for digital messages)



Discontinuous phase ➔ vulnerable to nonlinearities, poorer frequency containment

Continuous phase ➔ more robust to nonlinearities, better frequency containment

FM



PM versus FM in practice

- Legacy analog communication → no control over message signal → FM preferred
  - Integration of message prior to phase modulation leads to smooth phase which leads to better bandwidth containment.
  - Most famous application: radio broadcasting
  - FM has been used in 2G GSM (Gaussian MSK, a form of FM); Optimal demodulation more complicated
  - Lately being used in power limited systems: FSK is used in LoRaWAN
- Digital communication → can design message signal → PM (PSK specifically) often preferred
  - Easier to implement optimal demodulator
  - Use bandwidth-efficient pulses rather than rectangular pulses to create smoother signals with better frequency containment
  - Used in modern digital communication systems

Focus on FM in this chapter. PSK studied in Chapter 4 and beyond.

Frequency Modulation

- The transmitted signal is given as

$$u_{\mathrm{FM}}(t) = A_c \cos(2\pi(\underline{f_c + f(t)})t + \phi)$$

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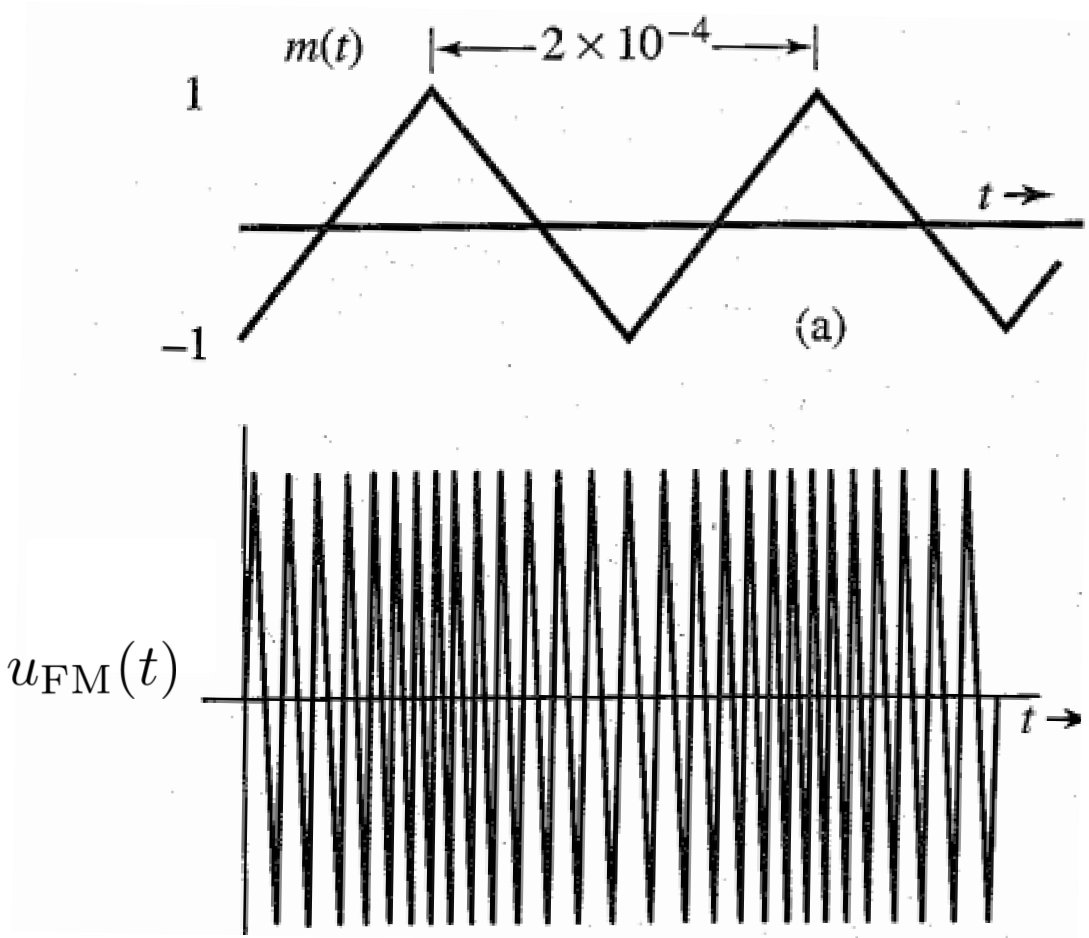


FM Modulation: Effect on Phase

- The transmitted signal is given as

$$u_{\mathrm{p}}(t) = A_c \cos(2\pi f_c t + \theta(t))$$

- Instantaneous phase $\theta(t)$

$$\frac{1}{2\pi}\frac{d\theta(t)}{dt} = f(t) = k_f m(t)$$

$$\theta(t) = \theta(0) + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

where k_f are constants while $f(t)$ is the frequency offset relative to the carrier.

FM Modulation Index

- Modulation index for FM is given by

$$\beta = \frac{\Delta f_{\max}}{B}$$

  where the frequency deviation $\Delta f_{\max} = k_f \max_t |m(t)|$ and B is the bandwidth of the signal..

- Narrowband FM: $\beta < 1$

- Wideband FM: $\beta > 1$

- Solve: For sinusoidal message $m(t) = A_m \cos(2\pi f_m t)$, find β. Also find $\theta(t)$ in terms of β assuming $\theta(0) = 0$.

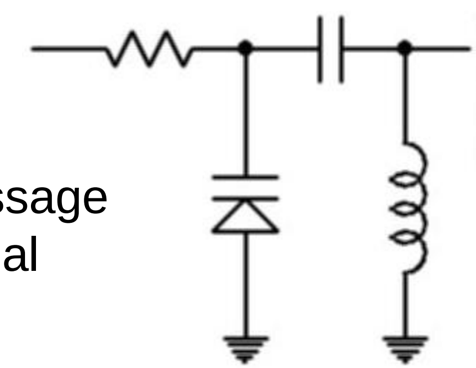
Modulation

- Direct method
  - Voltage controlled oscillator (VCO)
  - Use of varacter diode which provides voltage controlled capacitance in LC tuned circuits
  - Directly generates passband
  - Both narrow and wideband
- Indirect method
  - An alternative method for wideband FM signal generation when direct method is infeasible or costly
  - First generate narrowband signal (using PM modulation) and then increase the frequency shift and frequency by using several stages of multipliers (non-linearity)
  - Not used nowadays as direct FM methods are now feasible and cost-effective.

Example of VCO

- Use of varacter diode which provides voltage controlled capacitance in LC tuned circuits
- Directly generates passband
- Both narrow and wideband

Message signal



Passband Sinusoid with varying freq.

Source: Globe Collector - Own work, CC0, https://commons.wikimedia.org/w/index.php?curid=18036362

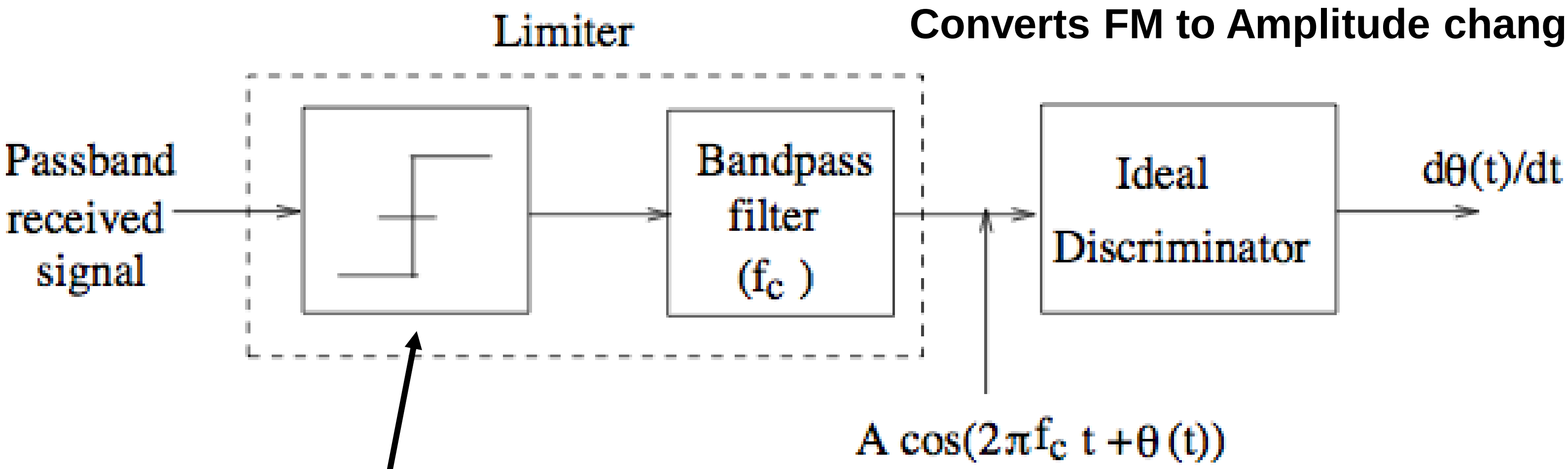
FM Demodulation

- There are several methods
  - Limiter discriminator
  - Phase locked loop (PLL) (in detail later in this chapter)

Limiter Discriminator

Enforces constant envelope

Converts FM to Amplitude changes

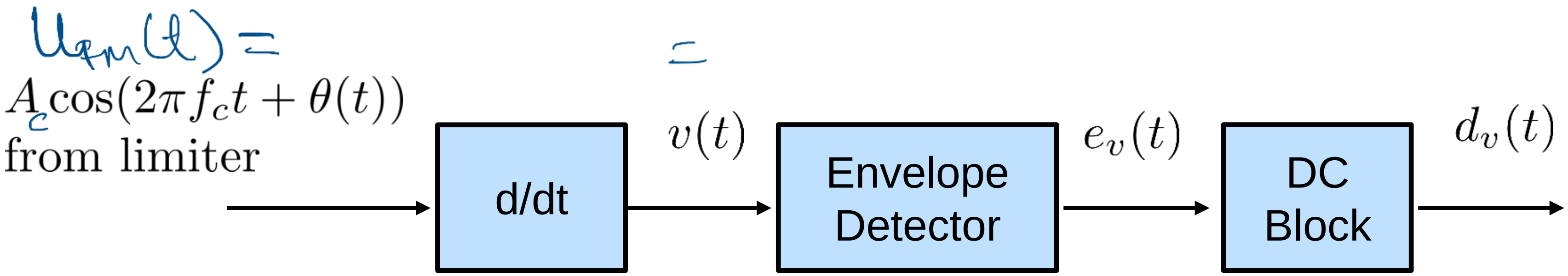


Removes amplitude fluctuations caused by noise and channel

Limiting enduces harmonics since it is a non-linear operation

$y(t) = a x(t) + b x^2(t) + c x^3(t) + \ldots$

A crude discriminator



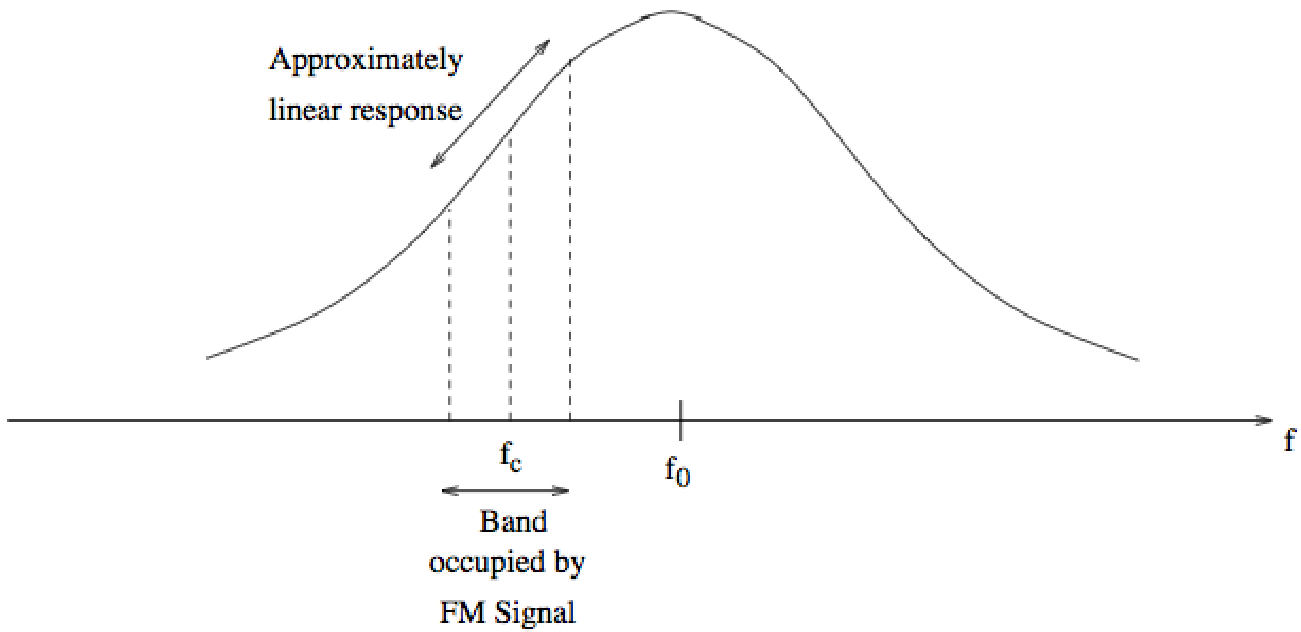
- Here $f(t) = \frac{d\theta(t)}{dt} = 2\pi k_f m(t)$.
- Show that the output of the crude discriminator shown above is a scaled version of the message signal $m(t)$.

Approximate differentiation

- For Differentiation, Fourier transform pair is $\frac{dx(t)}{dt} \stackrel{\mathcal{F}}{\longleftrightarrow} j2\pi f X(f)$

$$X(f) \longrightarrow \boxed{H(f) = j2\pi f} \longrightarrow j2\pi f X(f)$$

- Can use linear slope region of filter response



Questions