lecture 4

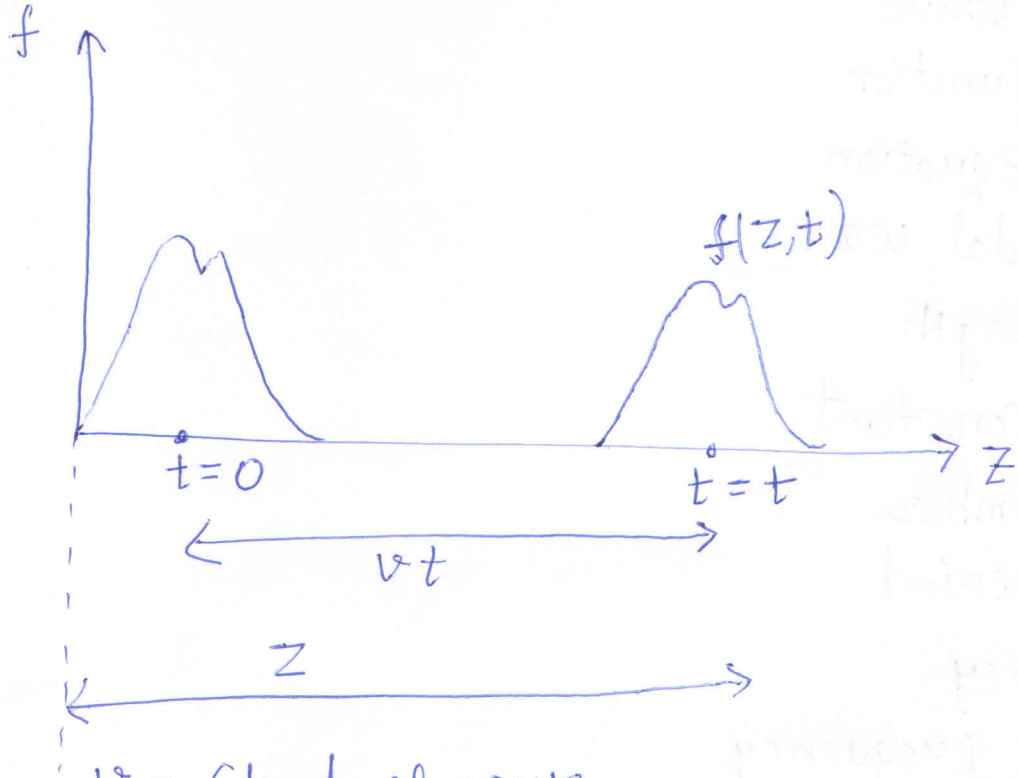
Before we jump into EM wave let us discuss about wave or some imp. points about wave ↓

- What is wave
- Wave function
- Wave Equation
- Sinuosoidal wave
- Wave length
- Phase constant
- Wave Number
- Time period
- Frequency
- Angular frequency
- Complex form of wave.

What is wave

"Wave is a disturbance of continuos medium propogating with fixed shape and constant speed"

Let's take a String



v = Speed of wave

$f(z,t)$ = Displ. of wave at time t

- Displacement of wave at point z and time t is same as Displacement at $(z-vt)$ and time $t=0$

It means

$$\boxed{f(z,t) = f(z-vt, 0)}$$ Wave function

$(z-vt)$ represent wave

* Wave function is mathematical representation of wave.

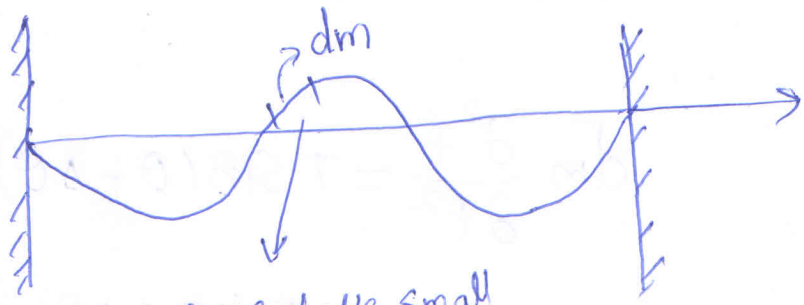
$f(z-vt)$
$f(-z,+vt)$ } Direction of propogation is $+z$ direction

$f(-z-vt)$
$f(z+vt)$ } Direction of propogation is $-z$ direction

Wave Equation

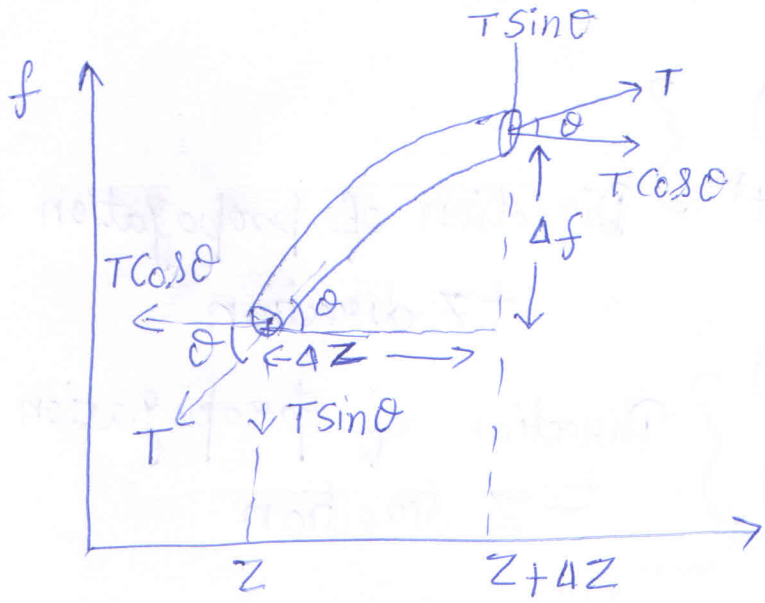
An Equation which satisfies wave-function is called wave Equation

⇒ let us discuss wave in String



Let us take small element dm

μ = mass per unit length.



From the Newton IInd law

$$dm \frac{\partial^2 f}{\partial t^2} = T \sin\theta' - T \sin\theta$$

if $\theta' = \theta + \Delta\theta$ & θ is very-z Small

$$\sin\theta \simeq \theta$$

$$dm \frac{\partial^2 f}{\partial t^2} = T \sin(\theta + \Delta\theta) - T \sin\theta$$

$$= T(\theta + \Delta\theta) - T\theta$$

$$= \cancel{T\theta} + T\Delta\theta - \cancel{T\theta}$$

$$dm \frac{\partial^2 f}{\partial t^2} = T\Delta\theta \quad \text{—— (1)}$$

Here,

$$\tan\theta = \frac{\Delta f}{\Delta Z}$$

$$\tan\theta = \frac{\partial f}{\partial Z}$$

Taking differentiation both side.

$$\cancel{\frac{1}{\cos^2\theta}}^{\,1}\frac{d\theta}{dz} = \frac{\partial^2 f}{\partial z^2}$$

for very small angle

$$\frac{d\theta}{dz} = \frac{\partial^2 f}{\partial z^2}$$

$$\Delta\theta = \frac{\partial^2 f}{\partial z^2}\Delta z \quad \text{——} \quad (2)$$

put (2) in (1)

$$dm\frac{\partial^2 f}{\partial t^2} = T\frac{\partial^2 f}{\partial z^2}\Delta z \qquad [\Delta z \simeq dz] \quad \left[\mu = \frac{dm}{\Delta z}\right]$$

$$\frac{dm}{dz}\frac{\partial^2 f}{\partial t^2} = T\frac{\partial^2 f}{\partial z^2}$$

$$\boxed{\frac{\partial^2 f}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 f}{\partial z^2}}$$

$$f = f(z - vt)$$

$$\frac{\partial^2 f}{\partial t^2} = v^2 f''(z - vt) \quad \text{——} \quad (3)$$

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Electromagnetic waves in Free space or in Vaccum $\longrightarrow$ (For $\vec{E}$ & $\vec{H}$)

let us start with Maxwell Equation $\longrightarrow$

(A) We know for free space

$$\left[\begin{array}{c} \rho_F = 0, \quad \vec{J}_F = 0, \quad \sigma = 0 \\ \varepsilon = \varepsilon_0, \quad \mu = \mu_0 \\ \vec{D} = \varepsilon_0 \vec{E}, \quad \vec{B} = \mu_0 \vec{H} \end{array}\right]$$

(I) $\nabla \cdot \vec{D} = \rho_F$

(II) $\nabla \cdot \vec{B} = 0$

(III) $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

(IV) $\nabla \times \vec{H} = \vec{J}_F + \dfrac{\partial \vec{D}}{\partial t}$

put all above values in the equations

$$\left.\begin{array}{ll} \nabla \cdot \vec{E} = 0 & \text{— (a)} \\ \nabla \cdot \vec{H} = 0 & \text{— (b)} \\ \nabla \times \vec{E} = -\mu_0 \dfrac{\partial \vec{H}}{\partial t} & \text{— (c)} \\ \nabla \times \vec{H} = \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} & \text{— (d)} \end{array}\right] \text{(V)}$$

from Equation (V)(c) we have

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

taking <u>curl</u> on both side

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \frac{\partial (\nabla \times \vec{H})}{\partial t}$$

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$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \frac{\partial}{\partial t}\left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Formula for curl if we have some vector for Ex. $\vec{V}$

$$\left[\nabla \times (\nabla \times \vec{V}) = \nabla(\nabla \cdot \vec{V}) - \nabla^2 \vec{V}\right]$$

= grad of div $\vec{V}$ − laplacian V

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

From V(a)

$$\nabla \cdot \vec{E} = 0$$

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \quad \text{— (VI)}$$

(VI) is Wave Equation for Electric field in free space for Electromagnetic wave

We can write

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

Then

$$\left[\begin{aligned} \nabla^2 E_x &= \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \\ \nabla^2 E_y &= \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \\ \nabla^2 E_z &= \mu_0 \varepsilon_0 \frac{\partial^2 E_z}{\partial t^2} \end{aligned}\right]$$

We can write a generalized Equation

$$\nabla^2 f = \mu_0 \varepsilon_0 \frac{\partial^2 f}{\partial t^2}$$

Where $f = E_x, E_y, E_z$

f = Scalar function

For magnetic field $\vec{H}$

We know from Equation V(d)

$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Taking curl both side

$$\nabla \times (\nabla \times \vec{H}) = \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times \vec{E})$$

$$\cancel{\nabla(\nabla\cdot\vec{H}) - \nabla^2\vec{H} = -\mu_0\varepsilon_0\frac{\partial H}{}}$$

$$\nabla(\nabla\cdot\vec{H}) - \nabla^2\vec{H} = -\mu_0\varepsilon_0\frac{\partial}{\partial t}\left(\frac{\partial\vec{H}}{\partial t}\right)$$

$$0 - \nabla^2\vec{H} = -\mu_0\varepsilon_0\frac{\partial}{\partial t}\left(\frac{\partial\vec{H}}{\partial t}\right)$$

$$\boxed{\nabla^2\vec{H} = \mu_0\varepsilon_0\frac{\partial^2\vec{H}}{\partial t^2}} \quad \text{—(VII)}$$

This is the wave Equation for magnetic field in free space.

Here $\vec{H} = H_x\hat{x} + H_y\hat{y} + H_z\hat{z}$

$$\left[\begin{aligned} \nabla^2 H_x &= \mu_0\varepsilon_0\frac{\partial^2 H_x}{\partial t^2} \\ \nabla^2 H_y &= \mu_0\varepsilon_0\frac{\partial^2 H_y}{\partial t^2} \\ \nabla^2 H_z &= \mu_0\varepsilon_0\frac{\partial^2 H_z}{\partial t^2} \end{aligned}\right] \quad \text{—(VIII)}$$

We can write $\Rightarrow$

$$\nabla^2 f = \mu_0\varepsilon_0\frac{\partial^2 f}{\partial t^2}$$

where $f = E_x, E_y, E_z, H_x, H_y, H_z$

Speed of EM Wave in free space

Wave Equation

$$\nabla^2 f = \mu_0 \varepsilon_0 \frac{\partial^2 f}{\partial t^2}$$

$$f = E_x, E_y, E_z, H_x, H_y, H_z$$

$$\nabla^2 f = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

Compare with wave Equation for $\vec{E}$ & $\vec{H}$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

It shows $v = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\varepsilon_0 = 8.85 \times 10^{-12} \dfrac{c^2}{N-m^2}$ $\quad \mu_0 = 4\pi \times 10^{-7}\ N/A^2$

$$\left[v = 3 \times 10^{8}\ m/Sec \right]$$

equal to Speed of light

$$\boxed{\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial z^2}} \quad \text{— (5)}$$

This is wave Equation

$$\boxed{\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}}$$

In 3-D

$$\boxed{\nabla^2 f = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}}$$