EC5.203 Communication Theory I (3-1-0-4):

Lecture 3
Baseband and Passband Representations

Instructor: Dr. Sachin Chaudhari

Email: sachin.chaudhari@iiit.ac.in

Jan. 16, 2025

and Jan. 20, 2025

INTERNATIONAL INSTITUTE OF
INFORMATION TECHNOLOGY

HYDERABAD

Reference

- Chapter 2 (Madhow)
  - Sec. 2.6-2.8: Energy Spectral Density, Bandwidth, Structure of passband signal

Recap

Indicator Function

- S&S: The step function was given by

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

- *CT-1*: $u(t)$ represent CT signal and $u[n]$ represent DT signal while step function is used for

$$I_{[0,\infty)}(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

- Indicator function

$$I_A(x) = \begin{cases} 1 & x \in A \\ 0 & \text{otherwise} \end{cases}$$

Sinusoidal Signal

- Sinusoids

$$s(t) = A\cos(2\pi f_0 t + \theta)$$ **Polar Form**

where $A > 0$ is the amplitude, f_0 is the frequency, and $\theta \in (0, 2\pi]$ is the phase.

- Sinusoids with known A, f_0, and θ cannot carry information.

- Modulation varies one or more of these parameters to convey information.

- Sinusoid can also be written as

Rectangular form

$$s(t) = A_c \cos 2\pi f_0 t - A_s \sin 2\pi f_0 t$$

where $A_c = A\cos\theta$ and $A_s = A\sin\theta$ are real numbers. Using Euler's formula

$$Ae^{j\theta} = A_c + jA_s$$ **Complex number**

where $A = \sqrt{A_c^2 + A_s^2}$ and $\theta = \tan^{-1}(A_s/A_c)$.

Complex Exponential

- Complex exponentials

$$s(t) = Ae^{2\pi f_0 t + \theta} = \alpha e^{2\pi f_0 t}$$

where $\alpha = e^{\theta t}$ a complex number that contains both the amplitude and phase information.

Inner Product

- The inner product for two $m \times 1$ complex vectors $\mathbf{s} = (s[1], \ldots, s[m])^T$ and $\mathbf{r} = (r[1], \ldots, r[m])^T$ is given by

$$< \mathbf{s}, \mathbf{r} >= \sum_{i=1}^{m} s[i] r^*[i] = \mathbf{r}^H \mathbf{s}.$$

  where $(.)^H$ denotes Hermitian operation with vector $(\mathbf{r})^H$ being conjugate transpose of vector $\mathbf{r}$.

- Similarly, we define the inner product of two possibly complex-valued signals $s(t)$ and $r(t)$ as follows

$$< s, r >= \int_{-\infty}^{\infty} s(t) r^*(t) \; dt$$

Energy and Norm

- The energy E_s of a signal

$$E_s = ||s||^2 \; = < s, s > = \int_{-\infty}^{\infty} |s(t)|^2 \; dt$$

- If $E_s = 0$, then it means that $s(t)$ must be zero *almost everywhere* which is also true for the norm of a vector.

Power

- The power of a signal $s(t)$ is defined as the time average of its energy computed over a large time interval

$$P_s = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |s(t)|^2 dt$$

- Finite energy signals have zero power.

Time average

- Time average of signal $g(t)$ is denoted by

$$\bar{g} = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g(t)dt$$

- Using the above notation, power of signal is given by

$$P_s = \overline{|s(t)|^2}$$

while the DC value of $s(t)$ is $\overline{s(t)}$.

Fourier Series

- A periodic signal $x(t)$ can be represented in terms of Fourier series as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where Fourier series coefficients are given by

$$a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

- A periodic signal $u(t)$ can be represented in terms of Fourier series as

$$u(t) = \sum_{n=-\infty}^{\infty} u_n e^{j2\pi n f_0 t}$$

where Fourier series coefficients are given by

$\omega = 2\pi f$

$$u_k = \frac{1}{T_0} \int_{T_0} u(t) e^{-j2\pi k f_0 t} dt$$

Fourier Transform

- Any aperiodic and finite duration signal $x(t)$ can be represented using Fourier transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\, d\omega$$

Inverse Fourier Transform

where

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$

Fourier Transform

- A aperiodic signal $x(t)$ can be represented in terms of Fourier transform as

$$u(t) = \int_{-\infty}^{\infty} U(f)e^{j2\pi ft}df$$

where Fourier transform is given by

$\omega = 2\pi f$

$$U(f) = \int_{-\infty}^{\infty} u(t)e^{-j2\pi ft}dt$$

Properties of Fourier Transform

- Linearity: For arbitrary complex numbers α and β,

$$\alpha u(t) + \beta v(t) \longleftrightarrow \alpha U(f) + \beta V(f)$$

- Time delay

$$u(t - t_0) \longleftrightarrow U(f)e^{-j2\pi f t_0}$$

- Frequency shift

$$U(f - f_0) \longleftrightarrow u(t)e^{j2\pi f_0 t}$$

- Differentiation in time domain

$$x(t) = \frac{du(t)}{dt} \longleftrightarrow j2\pi f U(f)$$

Properties of Fourier Transform

- Step Function: For $v(t) = I_{[0,\infty)}$

$$V(f) \longleftrightarrow \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

- Integration:

$$u(t) = \int_{-\infty}^{t} x(t)\ dt = x(t) * v(t)$$

$$U(f) = X(f)V(f) = \frac{X(f)}{j2\pi f} + \overline{u}\delta(f)$$

Properties of Fourier Transform

- Parseval's identity:

$$< u, v > = \int_{-\infty}^{\infty} u(t)v^*(t)dt = \int_{-\infty}^{\infty} U(f)V^*(f)df$$

- For $u = v$,

$$||u||^2 = < u, u > = \int_{-\infty}^{\infty} |u(t)|^2 dt \ = \int_{-\infty}^{\infty} |U(f)|^2 df$$

Properties of Fourier Transform

- Convolution in time domain:

$$y(t) = u(t) * v(t) = (u * v)(t) \longleftrightarrow Y(f) = U(f)H(f)$$

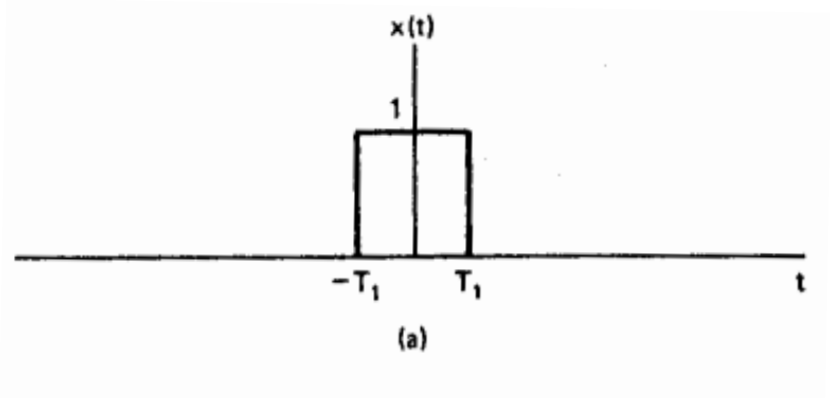
- Multiplication in time domain:

$$y(t) = u(t)v(t) \longleftrightarrow Y(f) = (U * H)(f) = U(f) * V(f)$$

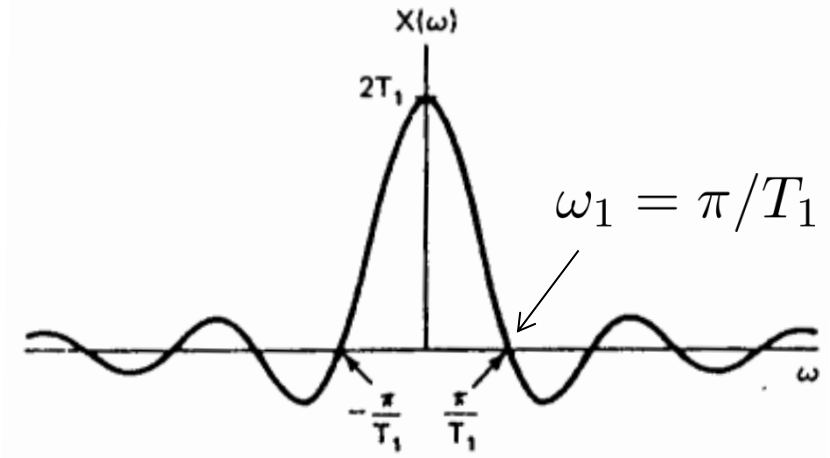
Rectangular pulse and Sinc function: *S&S*

- Find Fourier transform for the signal

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases}$$



$$X(j\omega) = 2\frac{\sin \omega T_1}{\omega}$$



Rectangular pulse and Sinc function

- For a rectangular pulse $u(t) = I_{[-T/2,T/2]}(t)$ of duration T. Its Fourier transform is given by

$$U(f) = T\text{sinc}(fT) = T\frac{\sin(\pi f T)}{\pi f T}$$

- The Fourier transform is denoted by

$$I_{[-T/2,T/2]}(t) \longleftrightarrow T\text{sinc}(fT)$$

- Using duality, the other Fourier transform pair is

$$I_{[-W/2,W/2]}(f) \longleftrightarrow W\text{sinc}(Wt)$$

Today’s Class

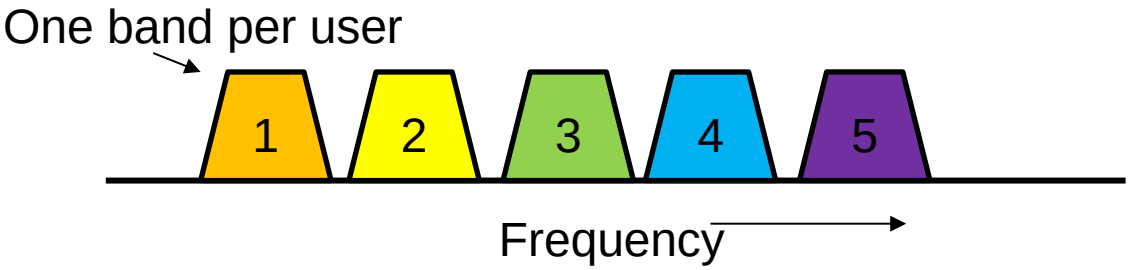
Energy Spectral Density and Bandwidth

Motivation

- Shared physical medium

- Example of Time Division Multiple Access (TDMA)

- Example of Frequency Division Multiple Access (FDMA)

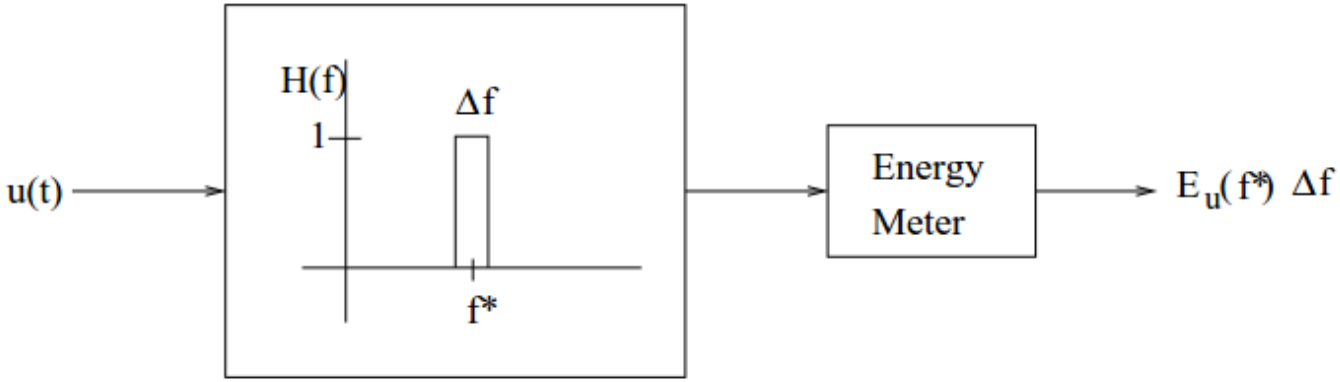


Motivation

- Communication channels have frequency-dependent characteristics
- Wireless spectrum is precious
- A broadcast and a shared medium
- Spectral characterization of the transmitted signal is important
  - Time limited signal has infinite bandwidth and vice-versa.

Energy Spectral Density

- Energy spectral density $E_u(f)$ of a signal $u(t)$ is defined to be the energy at the output of the filter divided by the width Δf in the limit as $\Delta(f) \to 0$.



- Prove that $E_u(f) = |U(f)|^2$

Energy Spectral Density

- Consider an ideal narrowband filter with transfer function

$$H_{f^*}(f) = \begin{cases} 1 & f^* - \Delta f < f < f^* + \Delta f \\ 0 & \text{otherwise} \end{cases}$$

- The frequency response of output $y(t)$

$$Y(f) = U(f)H(f) = \begin{cases} U(f) & f^* - \Delta f < f < f^* + \Delta f \\ 0 & \text{otherwise} \end{cases}$$

- By Parseval's identity, the energy at the output of the filter is

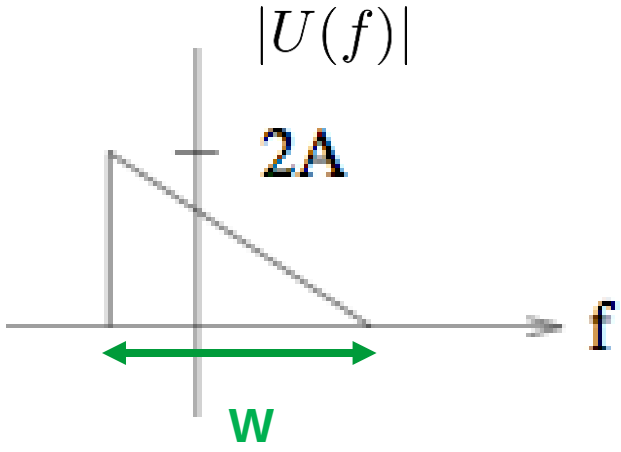
$$\int_{-\infty}^{\infty} |Y(f)|^2 \, df = \int_{f^*-\Delta/2}^{f^*+\Delta/2} |U(f)|^2 \, df \approx |U(f)|^2 \Delta f$$

- Equating this to output of energy meter, we get the desired result.

Bandwidth for bandlimited signals

Several definitions depending on the application

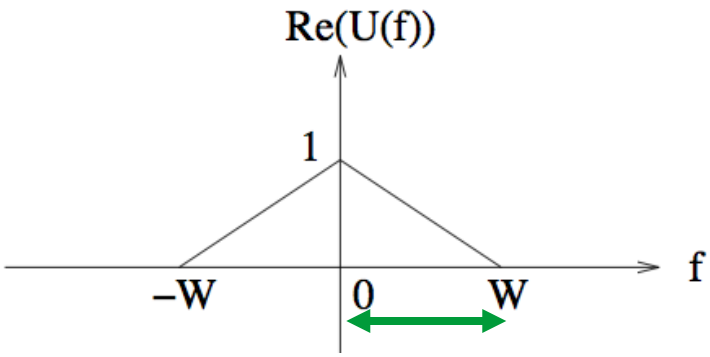
- The bandwidth of a strictly bandlimited signal $u(t)$ is defined to be the size of the band of frequencies occupied by $U(f)$.

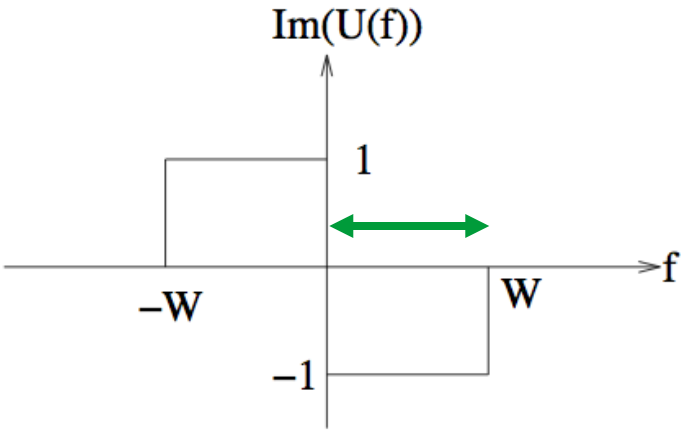


Bandwidth for bandlimited signal: *Real Signal*

Several definitions depending on the application

- For physical signals (real-valued), $U(-f) = U^*(f)$.
- Only positive frequencies count when computing bandwidth for physical (real-valued) signals. Also called one-sided bandwidth.

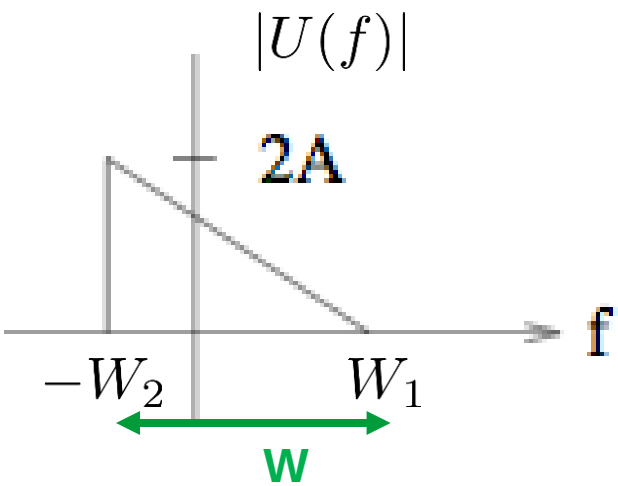




Bandwidth for bandlimited signals: complex

Several definitions depending on the application

- The bandwidth of a strictly bandlimited signal $u(t)$ is defined to be the size of the band of frequencies occupied by $U(f)$.

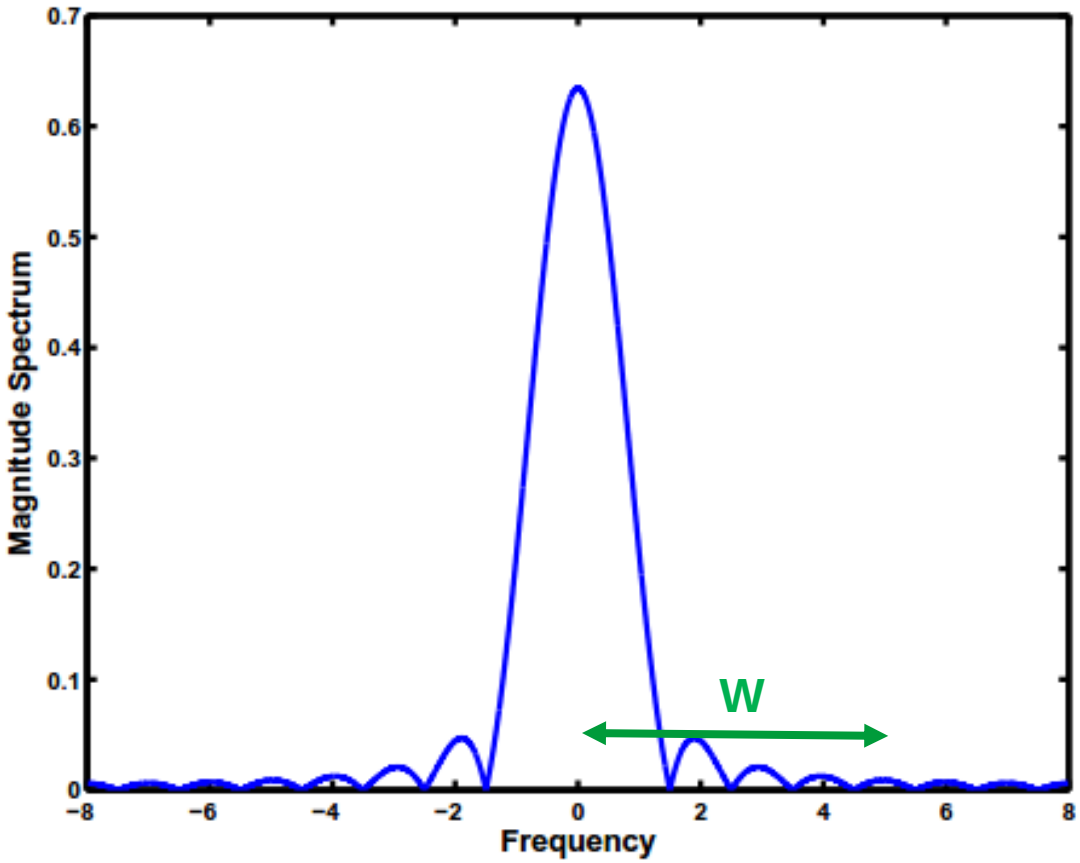


- The bandwidth of complex-valued signals is defined as the size of the frequency band it occupies over both positive and negative frequencies.

Bandwidth for not-bandlimited signal

Several definitions depending on the application

- The bandwidth of signal $u(t)$ which is not bandlimited is defined in terms of energy-containment bandwidth: the size of the smallest band which contains specified fraction of the signal energy.



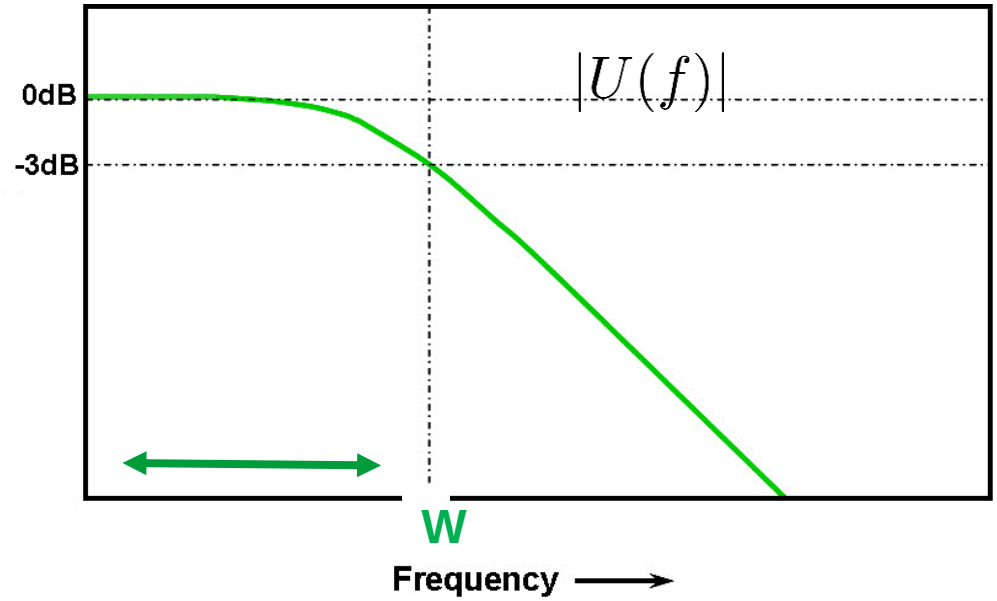
99% Energy

Bandwidth for not-bandlimited signal

Several definitions depending on the application

- The bandwidth of signal $u(t)$ which is not bandlimited can be defined in terms of band over which $|U(f)|^2$ is within some fraction of its peak value.

- For example, if the fraction is 0.5, the bandwidth is called 3-dB bandwidth.



Baseband and Passband

The Complex Baseband Representation

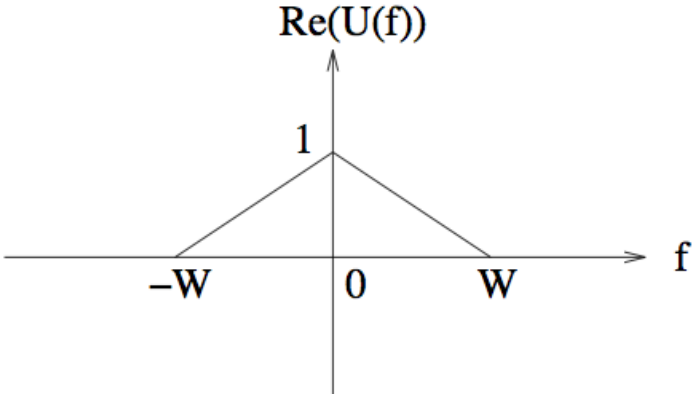
Baseband Signal

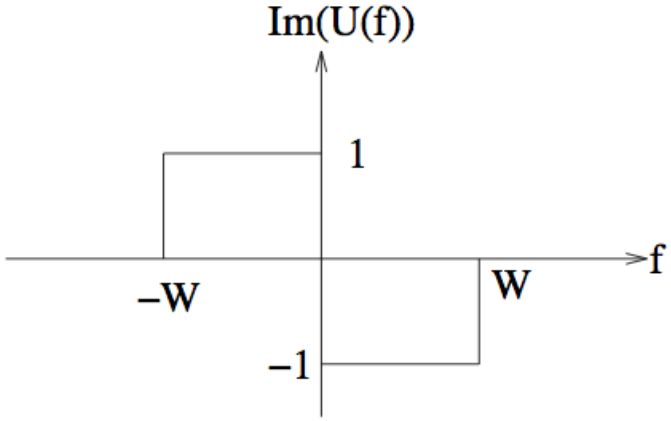
- A signal $u(t)$ is said to be baseband if the signal energy is concentrated in a band around DC and

$$U(f) \approx 0, \quad |f| > W$$

for some W.

- Example: Information sources

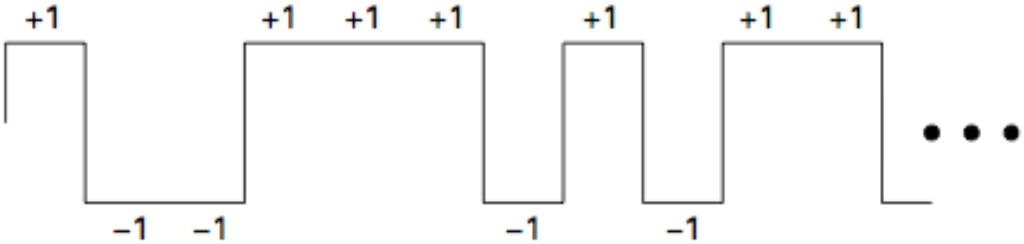




Examples of baseband signals

- Speech and audio are baseband signals

- Two level digital signals over wired connection without modulation (also called line coding)



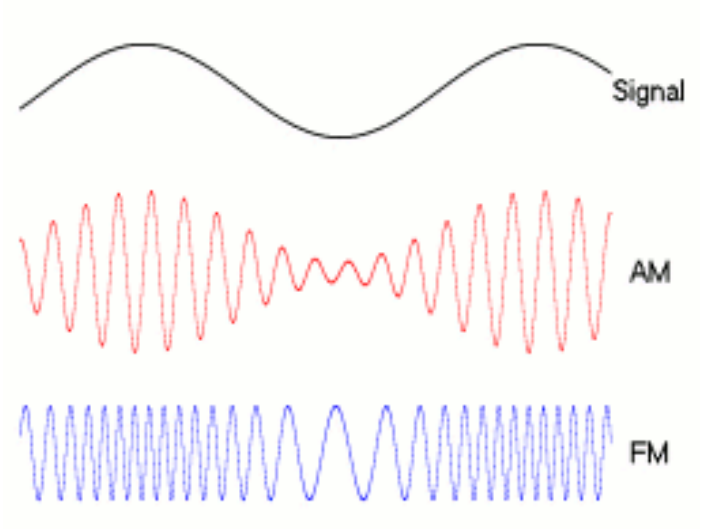
Passband Signal

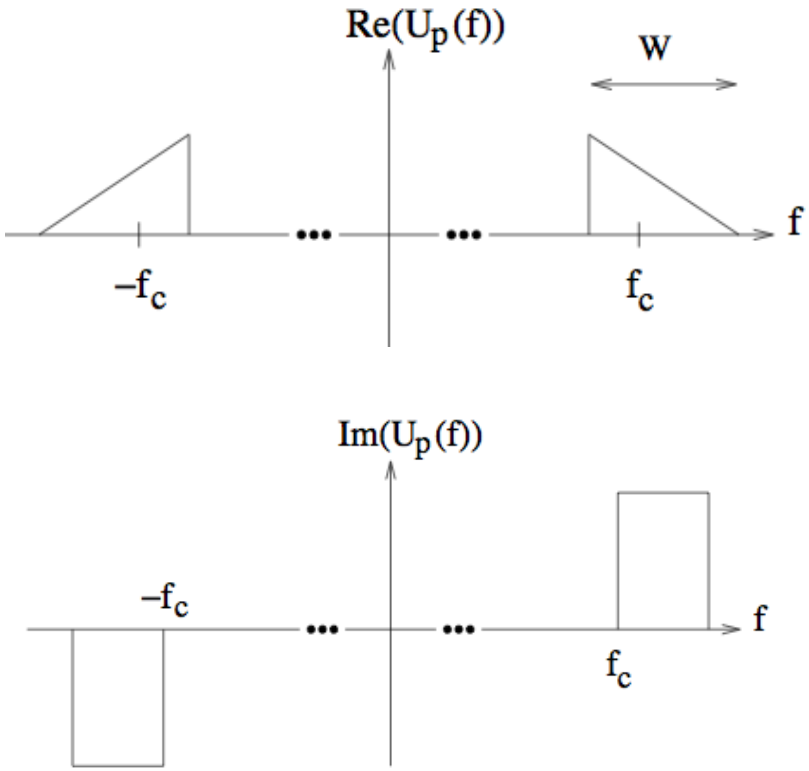
- A signal $u(t)$ is said to be passband if the signal energy is concentrated in a band away from DC with

$$U(f) \approx 0, \quad |f \pm f_c| > W$$

for some $f_c > W > 0$. Typically $f_c \gg W$

- Example: Wireless signal



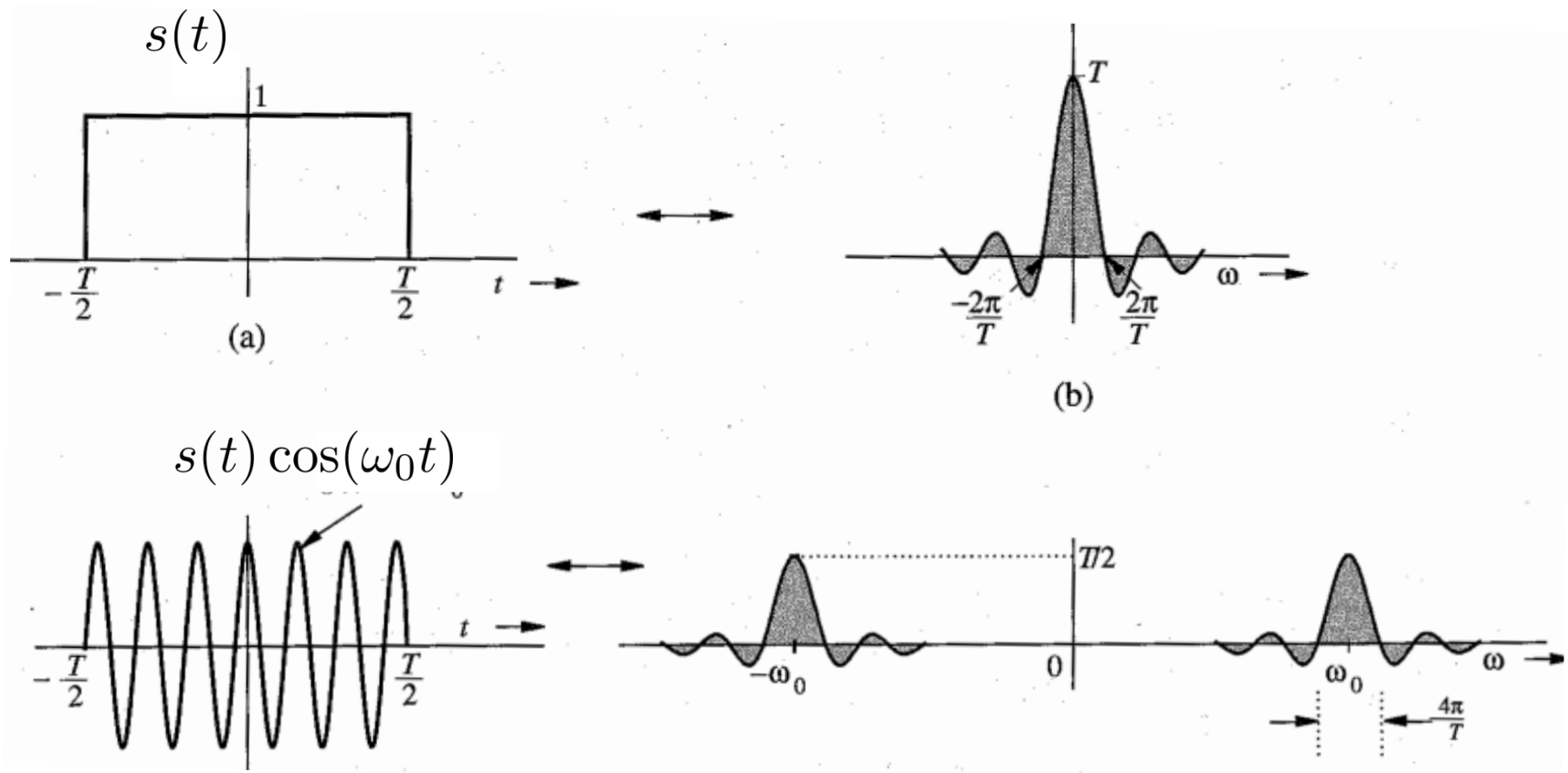


Example of Passband

- Consider this signal

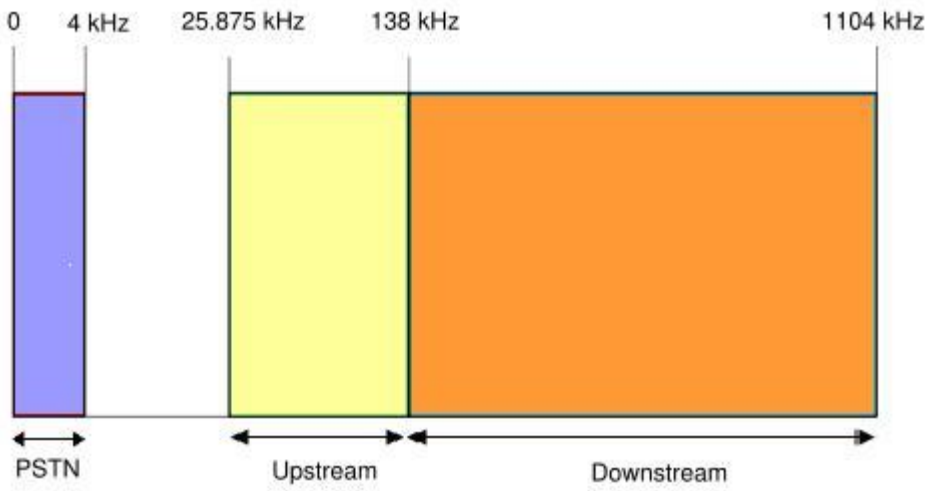
$$u_{\mathrm{p}}(t) = I_{[-T/2,T/2]}(t)\cos(2\pi f_0 t)$$

where $\omega_0 = 2\pi f_0 t$



Example of Passband in Wired Systems

- Digital Subscriber Line (DSL) is a technology, where high-speed data transmission coexists with voice transmissions in 0-4 KHz band
- Physical media is twisted pair copper wire



https://en.wikibooks.org/wiki/Communication_Networks/DSL

Differences between Baseband and Passband

- Baseband
  1. Original signal generated from the message source without any modulation of high frequency carrier
  2. Concentrated near zero frequency
  3. Can carry only one message signal at a time
  4. Examples: Ethernet, landline cables, network cables, coaxial cables.
- Passband
  1. Refers to modulated signal mostly
  2. Away from DC and concentrated around carrier frequency
  3. Multiple signals can be multiplexed simultaneously
  4. Examples: Wireless signal

Baseband Channel

- A channel modeled as a LTI system is said to be baseband if its transfer function $H(f)$ has support concentrated around DC and satisfies

$$H(f) \approx 0, \quad |f| > W$$

  for some W.

- Example: Wired channels

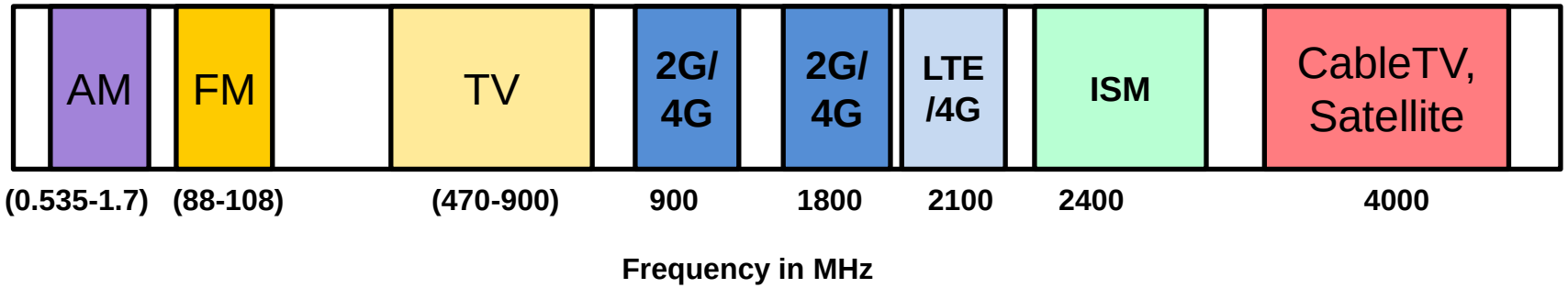
Passband Channel

- A channel modeled as a LTI is said to be passband if its transfer fuction $H(f)$ satisfies

$$H(f) \approx 0, \quad |f \pm f_c| > W$$

for some $f_c > W > 0$.

- Example: Wireless Channel



Baseband and Passband Signals/Channels

- Channels are often modeled as LTI systems: Signal passes through channel and then noise is added
- Channels allocated/described typically in terms of frequency bands: Signals have to be designed for corresponding frequency band
- Complex baseband representation of passband systems: Unified treatment of baseband and passband systems.

Questions?

Structure of Passband Signal

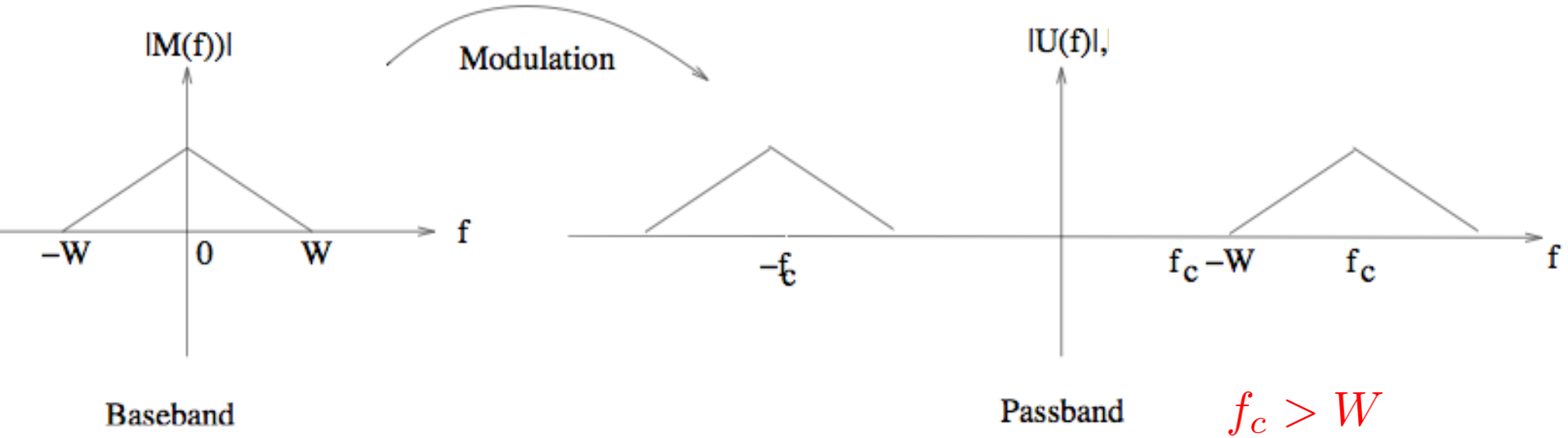
Motivation

- Passband is important for different communication systems
- Need to understand how to design transmitted signal from the information and recover the same from the received signal

Modulation or Upconversion

- Let $m(t)$ be the message signal of bandwidth W to be sent over passband channel around f_c called as carrier frequency.
- One method: multiply by a sinusoid at f_c

$$u_{\mathrm{p}}(t) = m(t)\cos(2\pi f_c t) \longleftrightarrow \frac{1}{2}\left(M(f-f_c) + M(f+f_c)\right)$$



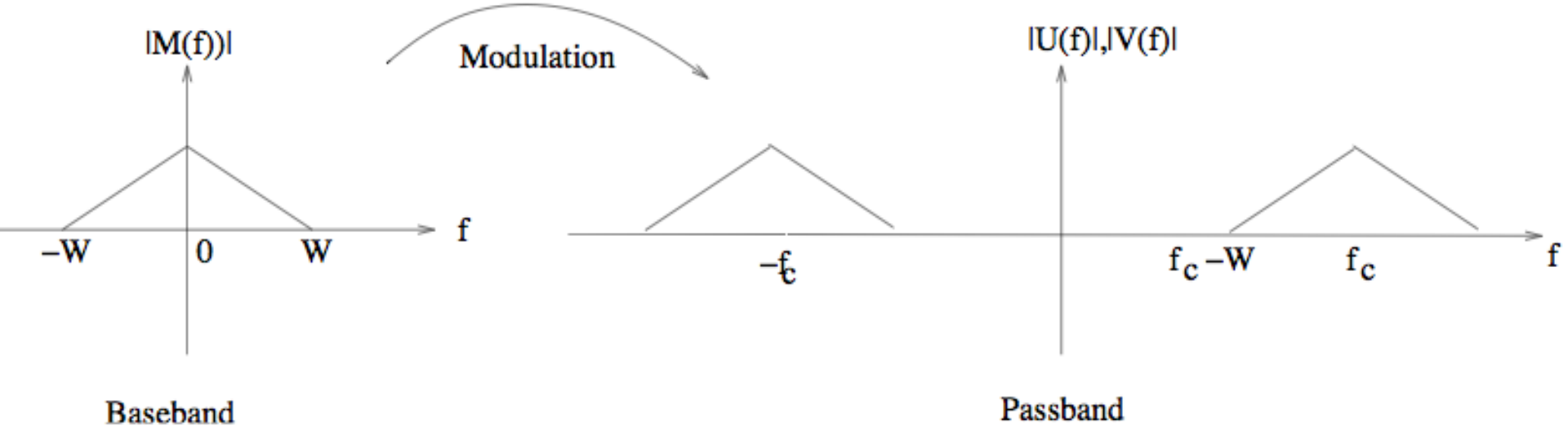
$f_c > W$

Modulation or Upconversion

- You can also use a sine

$$v_{\mathrm{p}}(t) = m(t)\sin(2\pi f_c t) \longleftrightarrow \frac{1}{2j}\left(M(f-f_c) - M(f+f_c)\right).$$

- Magnitude spectrum for $v_{\mathrm{p}}(t)$ will be same as that for $u_{\mathrm{p}}(t)$.



Modulation: *I and Q components*

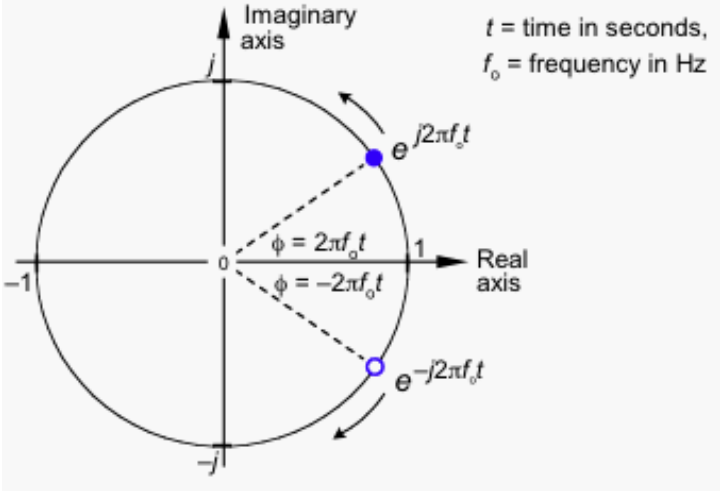
- If we use both cosine and sine carriers, we can construct a passband signal of the form

$$u_{\mathrm{p}}(t) = \boxed{u_{\mathrm{c}}(t)}\cos(2\pi f_c t) - \boxed{u_{\mathrm{s}}(t)}\sin(2\pi f_c t)$$

where $u_c(t)$ and $u_s(t)$ are real baseband signals of bandwidth at most W, with $f_c > W$.

In-phase or I component

Quadrature or Q component



Modulation: *I and Q components*

- If we use both cosine and sine carriers, we can construct a passband signal of the form

$$u_{\mathrm{p}}(t) = \boxed{u_{\mathrm{c}}(t)}\boxed{\cos(2\pi f_c t)} - \boxed{u_{\mathrm{s}}(t)}\boxed{\sin(2\pi f_c t)}$$

where $u_c(t)$ and $u_s(t)$ are real baseband signals of bandwidth at most W, with $f_c > W$.

Carry all the information

Carry NO information

Orthogonality of I and Q channels

- Show that the passband waveforms $a_{\mathrm{p}}(t) = u_{\mathrm{c}}(t)\cos(2\pi f_c t)$ and $u_{\mathrm{b}}(t) = u_{\mathrm{s}}(t)\sin(2\pi f_c t)$ are orthogonal

$$< a_{\mathrm{p}}, b_{\mathrm{p}} >= 0$$

  i.e., I and Q components of $u_{\mathrm{p}}(t)$ are orthogonal.

- Advantage: we can send separate information on I and Q at the same time!.

Demodulation or Downconversion

- If we use both cosine and sine carriers, we can construct a passband signal of the form

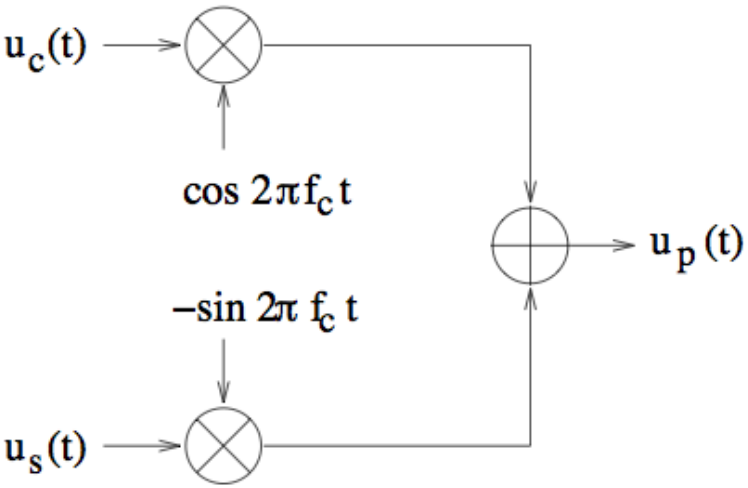
$$u_{\mathrm{p}}(t) = u_{\mathrm{c}}(t)\cos(2\pi f_c t) - u_{\mathrm{s}}(t)\sin(2\pi f_c t)$$

  where $u_c(t)$ and $u_s(t)$ are real baseband signals of bandwidth at most W, with $f_c > W$.

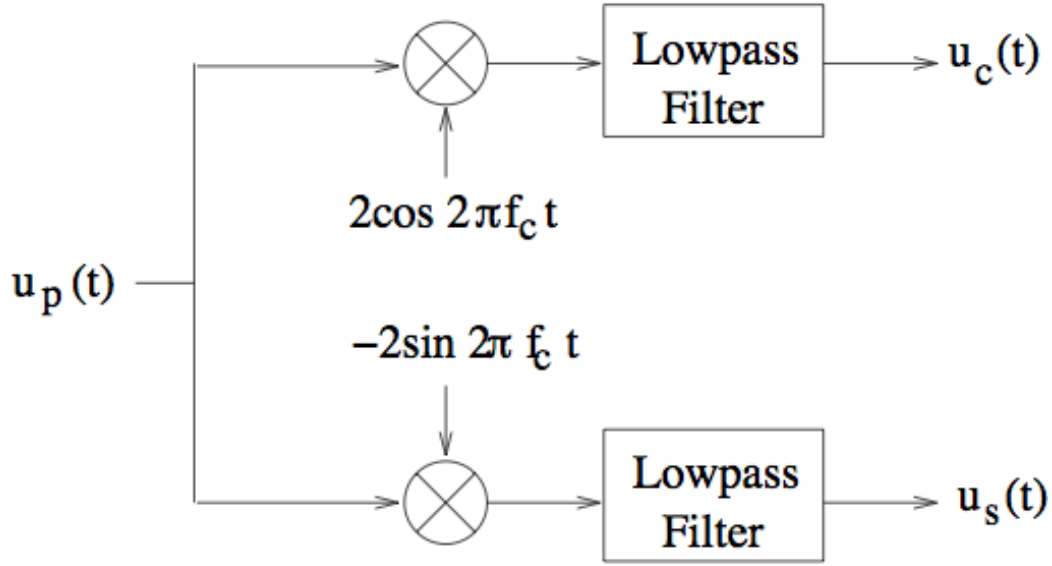
- Show that downconversion from passband to baseband can be done by lowpass filtering of these two components

$$\begin{aligned} &2u_{\mathrm{p}}(t)\cos(2\pi f_c t) \\ &-2u_{\mathrm{p}}(t)\sin(2\pi f_c t) \end{aligned}$$

Upconversion and Downconversion: *Block Diagrams*



Upconversion
(baseband to passband)



Downconversion
(passband to baseband)

Envelope and Phase

- $u_{\mathrm{p}} \equiv (u_{\mathrm{c}}, u_{\mathrm{s}})$ is called two-dimensional modulation.
- I and Q components corresponds to two dimensional waveforms in rectangular coordinates.
- Converting them to polar coorinates, we get

$$e(t) = \sqrt{u_c^2(t) + u_s^2(t)}$$
$$\theta(t) = \tan^{-1}\left(\frac{u_{\mathrm{s}}(t)}{u_{\mathrm{c}}(t)}\right)$$

  where $e(t) \geq 0$ is the envelope and $\theta(t)$ is the phase.

- Here

$$u_{\mathrm{c}}(t) = e(t)\cos\theta(t)$$
$$u_{\mathrm{s}}(t) = e(t)\sin\theta(t)$$

Complex Envelope

- A two dimensional point can be mapped to a complex number.
- The complex envelope $u(t)$ of the passband signal $u_p(t)$ is defined as

$$u(t) = u_c(t) + ju_s(t) = e(t)e^{j\theta(t)}$$

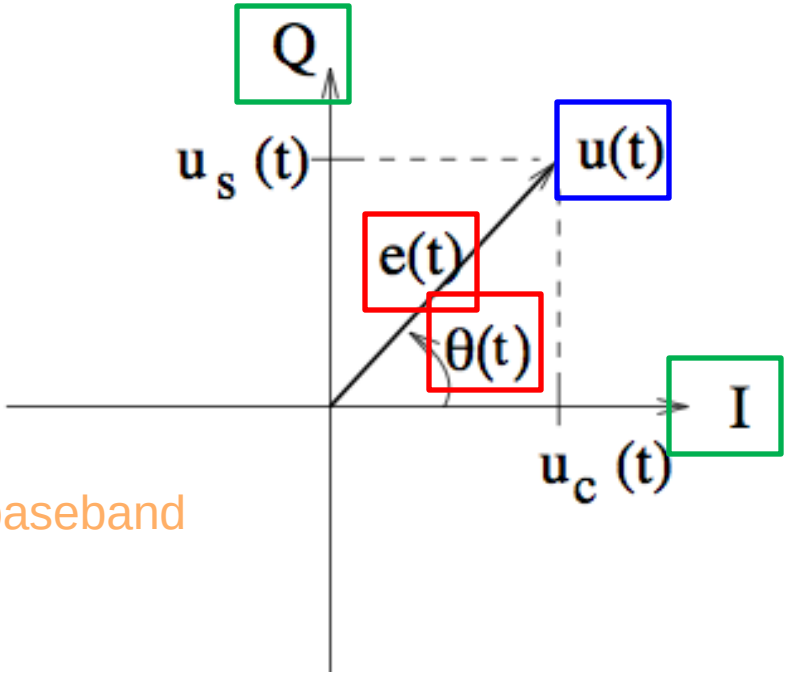
- Show that the passband signal $u_p(t)$ can be expressed in terms of the complex envelope $u(t)$ as

$$u_p(t) = \mathrm{Re}(u(t)e^{j2\pi f_c t})$$

Already proved in previous class

Relationship between three representations

- Two-dimensional modulation: Passband signal u_{p} can be mapped to a pair of real baseband signals.
- The three representations are
  - Rectangular Coordinates I and Q
  - Envelope and Phase $e(t)$ and $\theta(t)$
  - Complex number or envelope $u(t)$



Information resides in complex baseband

Time domain expressions for a passband signal

- In terms of I and Q components

$$u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)$$

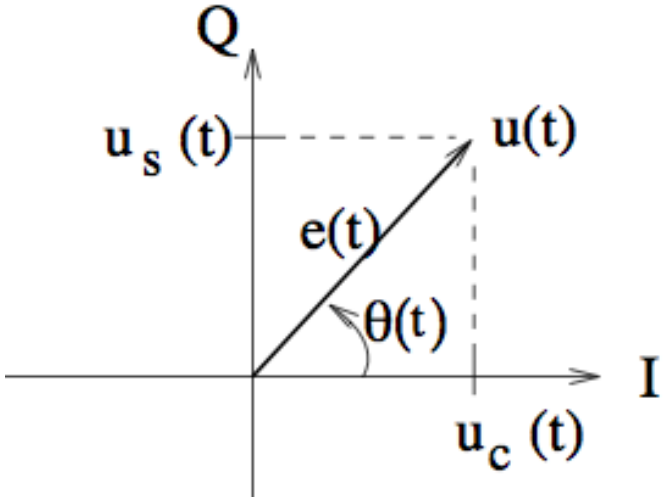
- In terms of envelope and phase

$$u_p(t) = e(t)\cos(2\pi f_c t + \theta(t))$$

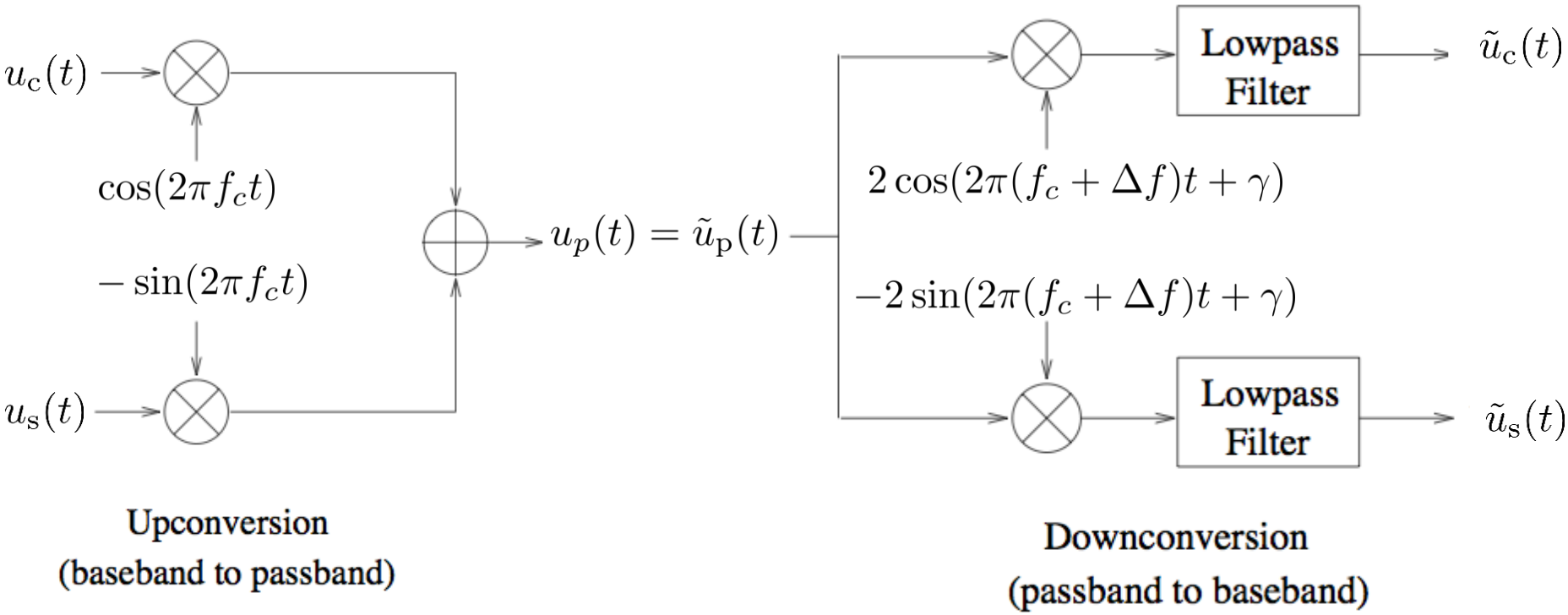
- In terms of complex envelope

$$u_p(t) = \mathrm{Re}(u(t)e^{j2\pi f_c t})$$

Starting from one representation, can derive the rest based on the relations depicted in the figure



Effect of Frequency and Phase Offset



- Show that in this case

$$\tilde{u}_c(t) = u_c(t)\cos\phi(t) + u_s(t)\sin\phi(t)$$
$$\tilde{u}_s(t) = -u_c(t)\sin\phi(t) + u_s(t)\cos\phi(t)$$

where $\phi(t) = 2\pi\Delta f t + \gamma$ is the phase offset resulting from frequency offset Δf and the phase offset γ.

Coherent Detection: Synchronization

- Frequency offset and phase offset cause cross-interference between I and Q components

$$\tilde{u}_{\mathrm{c}}(t) = u_c(t)\cos\phi(t) + u_s(t)\sin\phi(t)$$
$$\tilde{u}_{\mathrm{s}}(t) = -u_c(t)\sin\phi(t) + u_s(t)\cos\phi(t)$$

  where $\phi(t) = 2\pi\Delta t + \gamma$ is the phase offset resulting from frequency offset Δf and the phase offset γ.

- Either have tight synchronization, i.e., $\Delta f \approx 0$ and $\gamma \approx 0$.

- Compensate for the offset $u(t) = \tilde{u}(t)e^{j\phi}$.

Summary of Properties: S&S

- If $u(t)$ is complex with Fourier transform is $U(f)$, then
  - Time reversal: $u(-t) \longleftrightarrow U(-f)$
  - Time conjugation: $u^*(t) \longleftrightarrow U^*(-f)$
  - Frequency shift: $u(t)e^{j2\pi f_0 t} \longleftrightarrow U(f - f_0)$

$U(f) = a + jb$ $\Rightarrow$ $U^*(f) = a - jb$

$\Re e(U(f)) = \Re e(U^*(-f))$

- If $u(t)$ is real
  - Conjugate symmetry in frequency domain $U(f) = U^*(-f)$
  - Even symmetry for real part of $U(f)$: $\Re\{U(f)\} = \Re\{U(-f)\}$
  - Odd symmetry for imaginary part of $U(f)$: $\Im\{U(f)\} = -\Im\{U(-f)\}$
  - Magnitude has even symmetry: $|U(f)| = |U(-f)|$
  - Angle has odd symmetry: $\angle U(f) = -\angle U(-f)$

Frequency Domain Relationship

- Show that the frequency response for

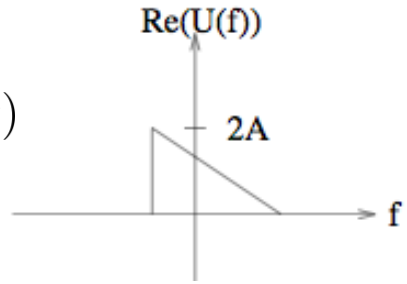
$$u_{\mathrm{p}}(t) = \mathrm{Re}\{u(t)e^{j2\pi f_c t}\}$$

is given by

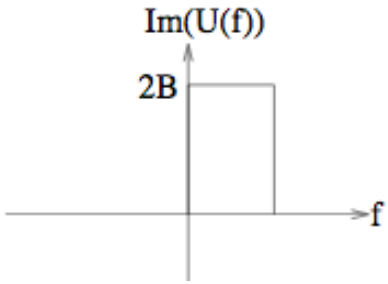
$$U_{\mathrm{p}}(f) = \frac{U(f - f_c) + U^*(-f - f_c)}{2}$$

Construct Passband from Baseband

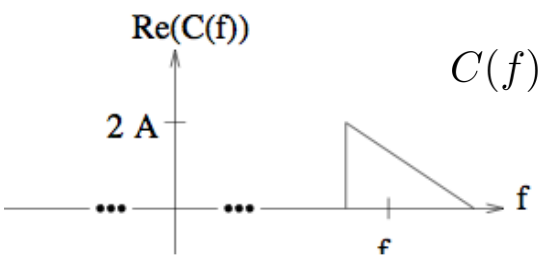
Complex signal $u(t)$



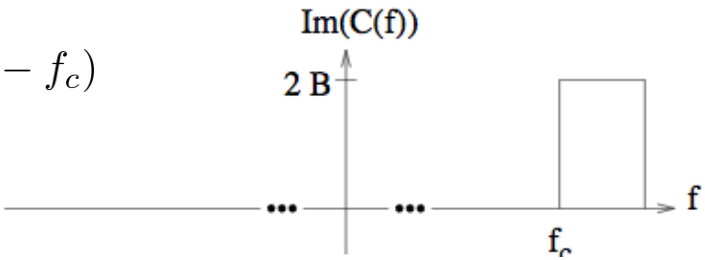
Complex $U(f)$



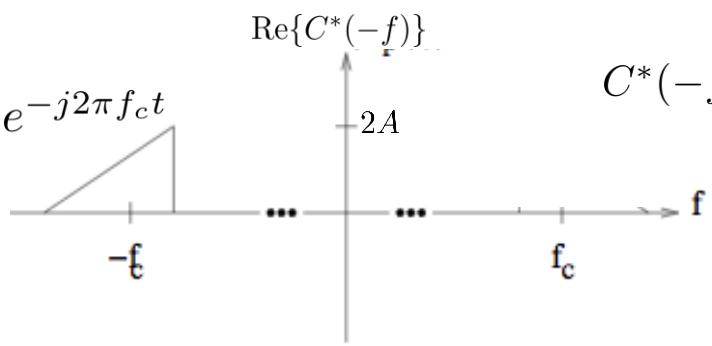
$c(t) = u(t)e^{j2\pi f_c t}$



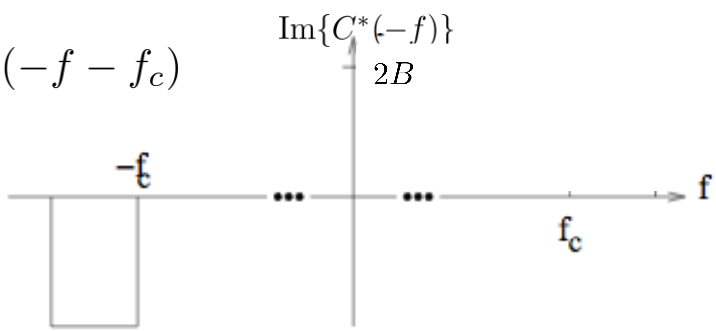
$C(f) = U(f - f_c)$



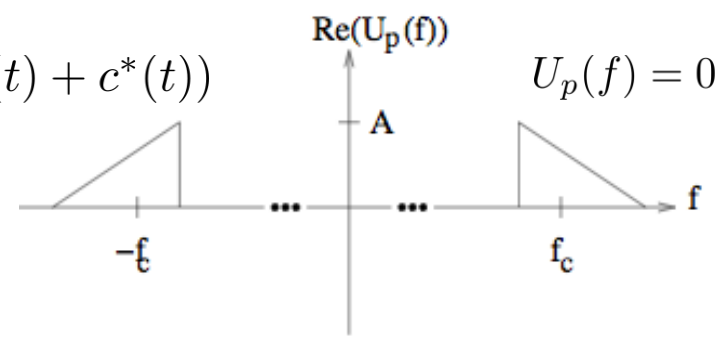
$c^*(t) = u^*(t)e^{-j2\pi f_c t}$



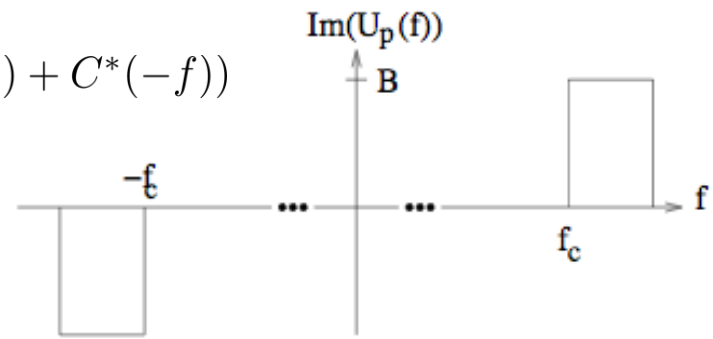
$C^*(-f) = U^*(-f - f_c)$



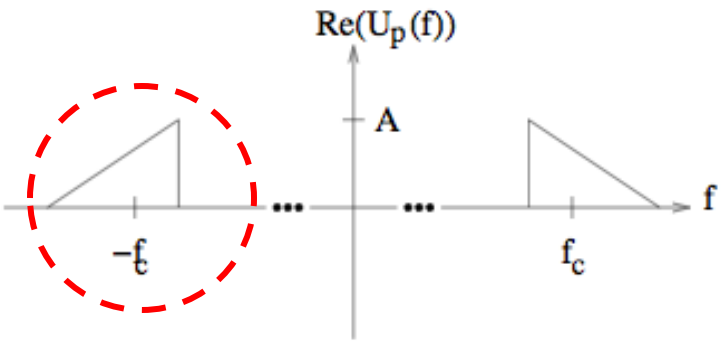
$u_p(t) = 0.5(c(t) + c^*(t))$

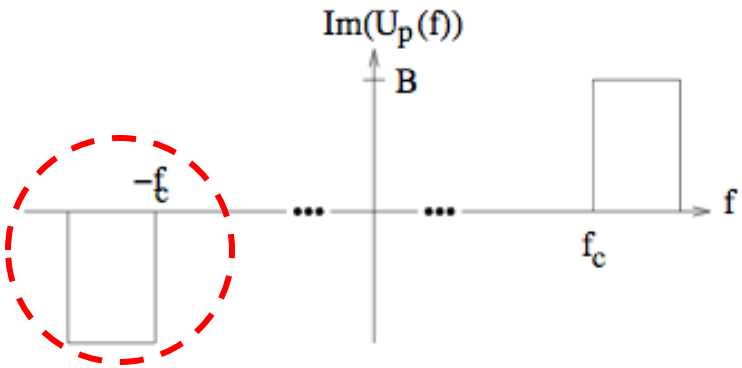


$U_p(f) = 0.5(C(f) + C^*(-f))$

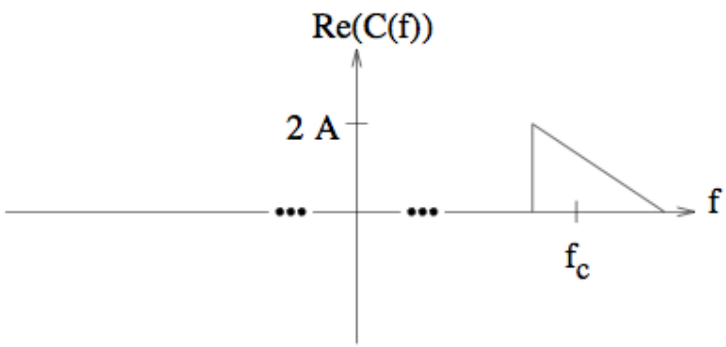


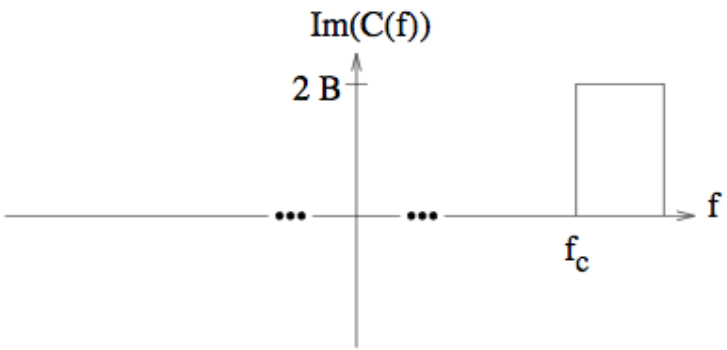
Reconstruct Baseband from Passband



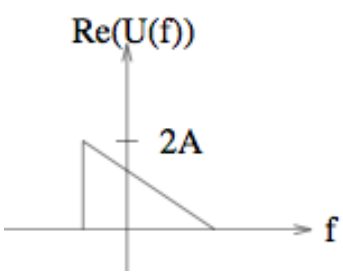


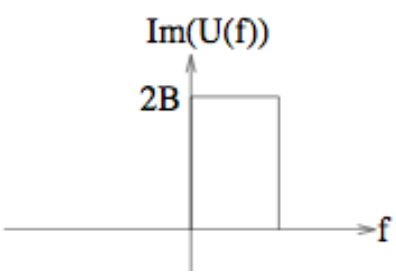
$$C(f) = 2U_p^+(f) = \begin{cases} 2U_p(f)\ , & f > 0 \\ 0\ , & f < 0 \end{cases}$$





$$u(t) = c(t)e^{-j2\pi f_c t} \leftrightarrow U(f) = C(f + f_c) = 2U_p^+(f + f_c)$$





I and Q components in frequency domain

- I and Q in time domain

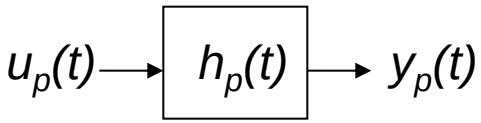
$$u_{\mathrm{c}}(t) = \frac{1}{2}(u(t) + u^*(t))$$
$$u_{\mathrm{s}}(t) = \frac{1}{2j}(u(t) - u^*(t))$$

$$\updownarrow$$

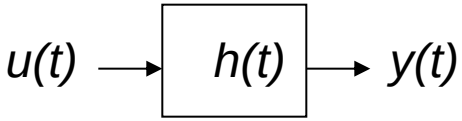
- I and Q in frequency domain

$$U_{\mathrm{c}}(f) = \frac{1}{2}(U(f) + U^*(-f))$$
$$U_{\mathrm{s}}(f) = \frac{1}{2j}(U(f) - U^*(-f))$$

Passband filtering = complex baseband filtering

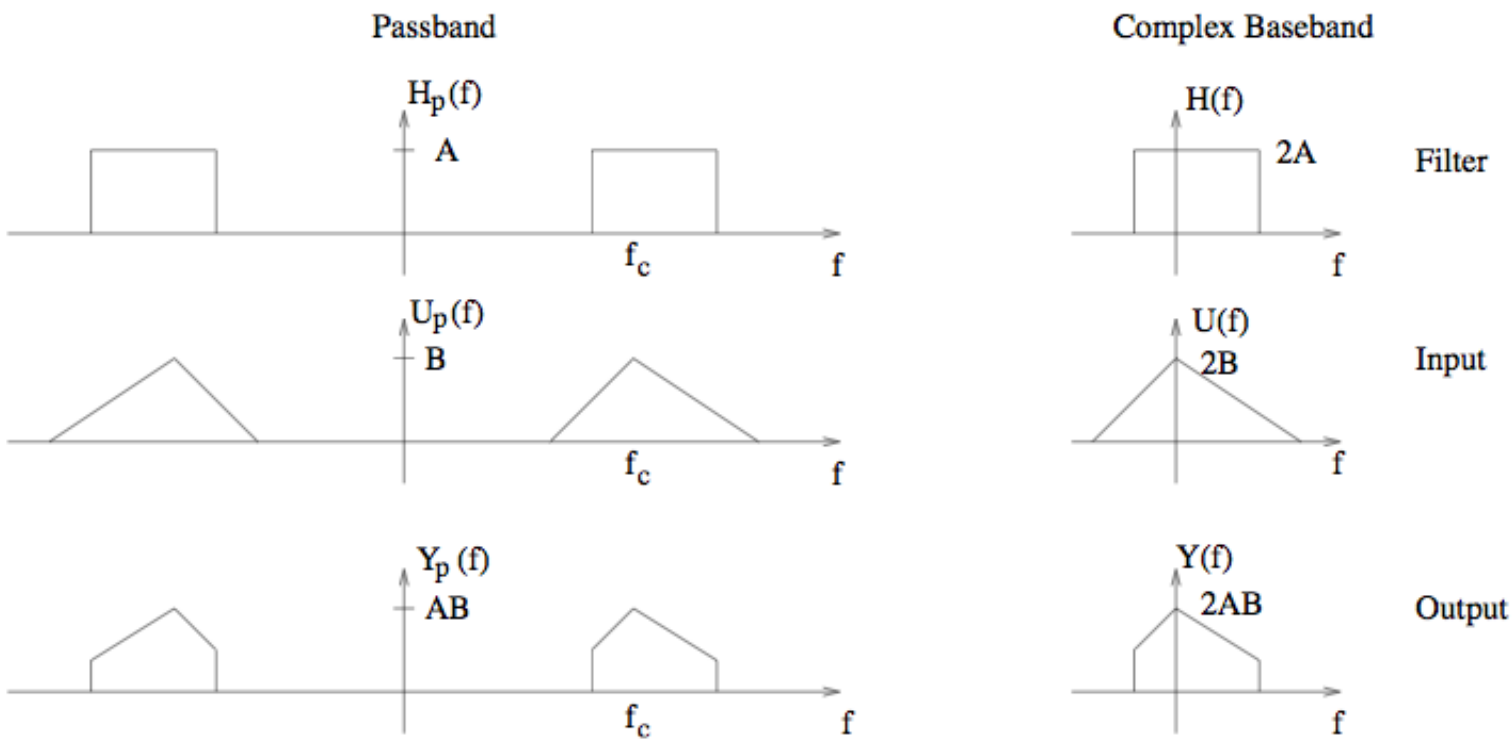


EQUIVALENT TO
(except for a scale factor of 2)



$$u_{\mathrm{p}}(t) = \Re\{u(t)e^{j2\pi f_c t}\}$$
$$h_{\mathrm{p}}(t) = \Re\{h(t)e^{j2\pi f_c t}\}$$
$$y_{\mathrm{p}}(t) = \Re\{y(t)e^{j2\pi f_c t}\}$$

What is complex baseband representation of passband filtering?



$$Y(f) = 2Y_p^+(f+f_c) = 2U_p^+(f+f_c)H_p^+(f+f_c) = \tfrac{1}{2}U(f)H(f)$$

$$\boxed{Y_p(f) = U_p(f)H_p(f) \quad \equiv \quad Y(f) = \tfrac{1}{2}U(f)H(f)}$$

$$\boxed{y_p(t) = u_p(t) * h_p(t) \quad \equiv \quad y(t) = \tfrac{1}{2}u(t) * h(t)}$$

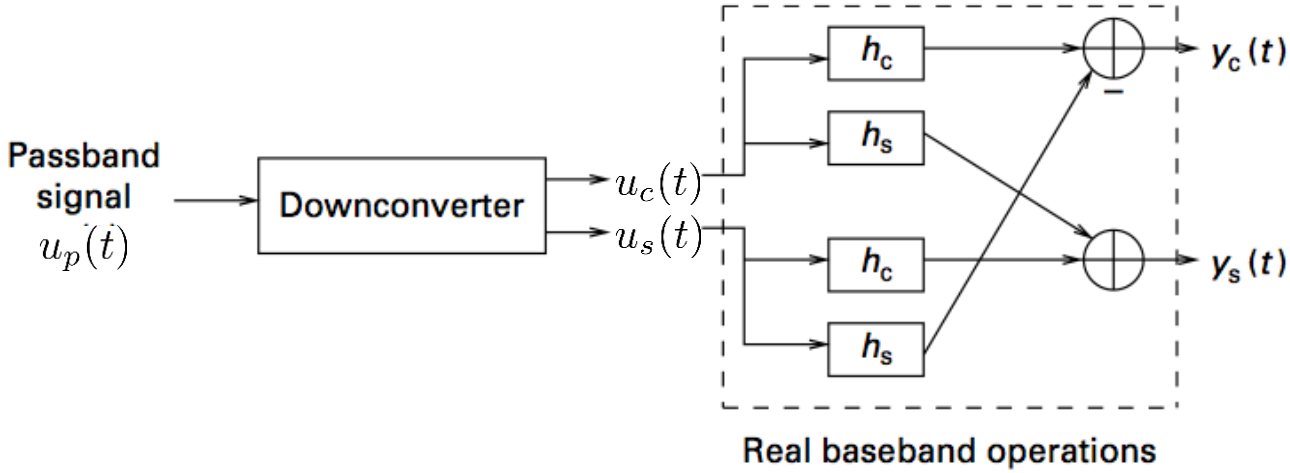
Filtering in complex baseband

- Complex-valued convolution to implement equivalent of passband filtering operation

$$y(t) = \frac{1}{2}(u * h)(t)$$

- Requires four real-valued convolutions

$$y_c = \frac{1}{2}(u_c * h_c - u_s * h_s); \qquad y_s = \frac{1}{2}(u_s * h_c + u_c * h_s);$$



$$x_{\mathrm{p}}(t) = x_{\mathrm{c}}(t)\cos(2\pi f_c t) - x_{\mathrm{s}}(t)\sin(2\pi f_c t)$$
$$x(t) = x_c(t) + jx_s(t)$$

Energy and Power

- Solve: Show that the energy of a passband signal equals that of its complex envelope, i.e.,

$$||u_p||^2 = \frac{1}{2}\left(||u_c||^2 + ||u_s||^2\right) = \frac{1}{2}||u||^2$$

- Since power is computed as time average of energy, similar relation exists for finite-power signals.

Correlation between two signals

- Correlation or inner product of two real-valued passband signals u_p and v_p is defined as

$$< u_p, v_p >= \int_{-\infty}^{\infty} u_p(t) v^*{}_{\mathrm{p}}(t)\ dt$$

- Show that

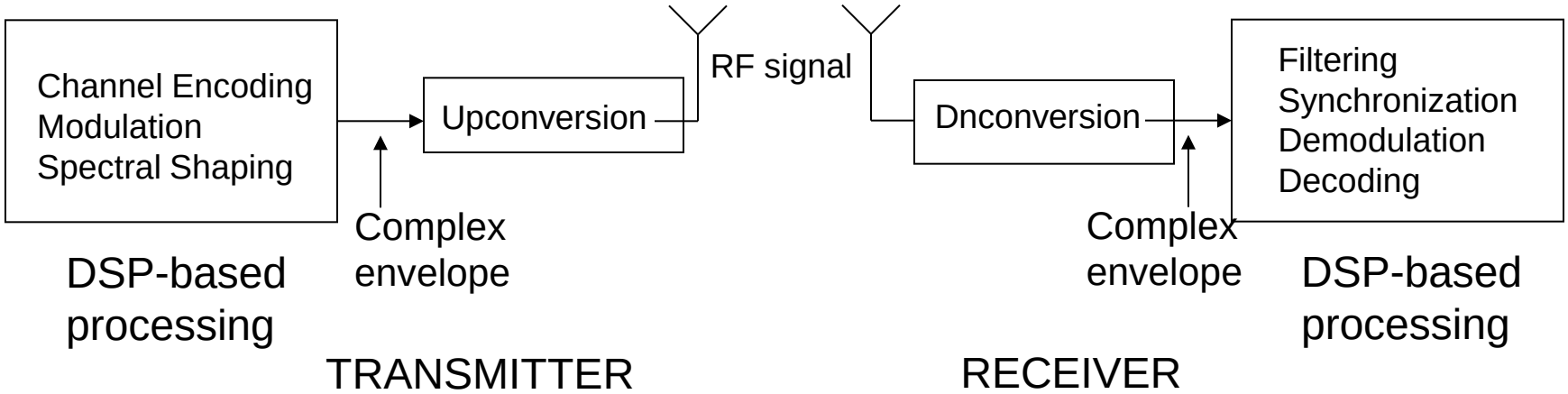
$$< u_p, v_p >= \frac{1}{2} \Re\{< u, v >\}$$

Modern transceiver architectures based on complex baseband

Modern transceiver work with the complex envelope rather than with the passband signal

- Complex baseband signals can be represented accurately by samples at a reasonable sampling rate
- Inexpensive to perform complicated DSP on the samples
- This architecture has been responsible for economics of scale in cellular and WiFi devices.
- Most of the research simulations happen in baseband.

All the action is in complex baseband for a typical wireless transceiver



Complex baseband representation: summary

- Any real-valued passband signal can be represented by a baseband signal which is in general complex-valued. This is called its complex envelope, or complex baseband representation.
- The complex envelope carries all the information in the passband signal
- Passband filtering operations can be equivalently performed in complex baseband
- Two-dimensional representation of complex envelope
  - Cartesian coordinates: A pair of real-valued baseband waveforms called the in-phase (I) and quadrature (Q) components
  - Polar coordinates: Envelope and phase waveforms
- Most of the sophisticated signal processing action in modern transceivers happens in complex baseband