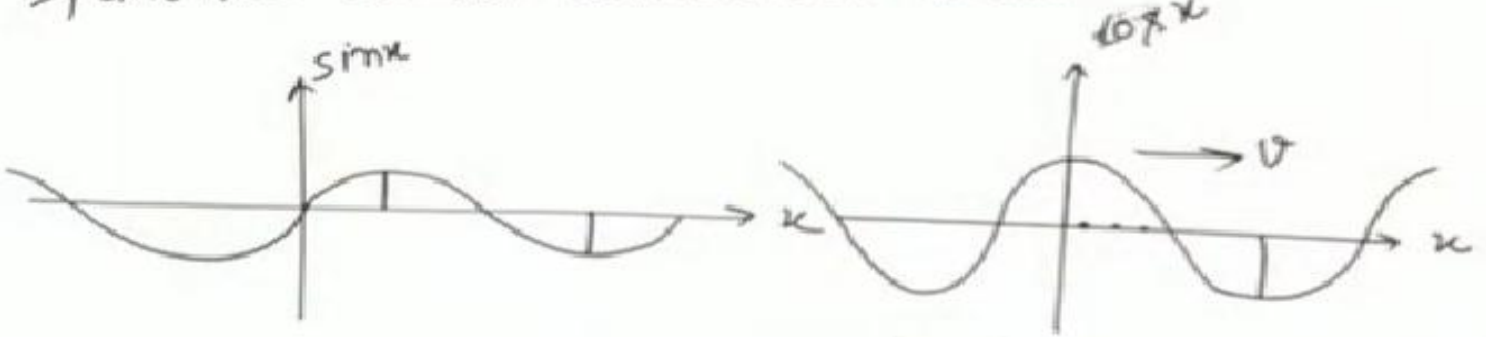
Lecture 5

Wave & Transverse nature of EM wave

Sinusoidal wave →

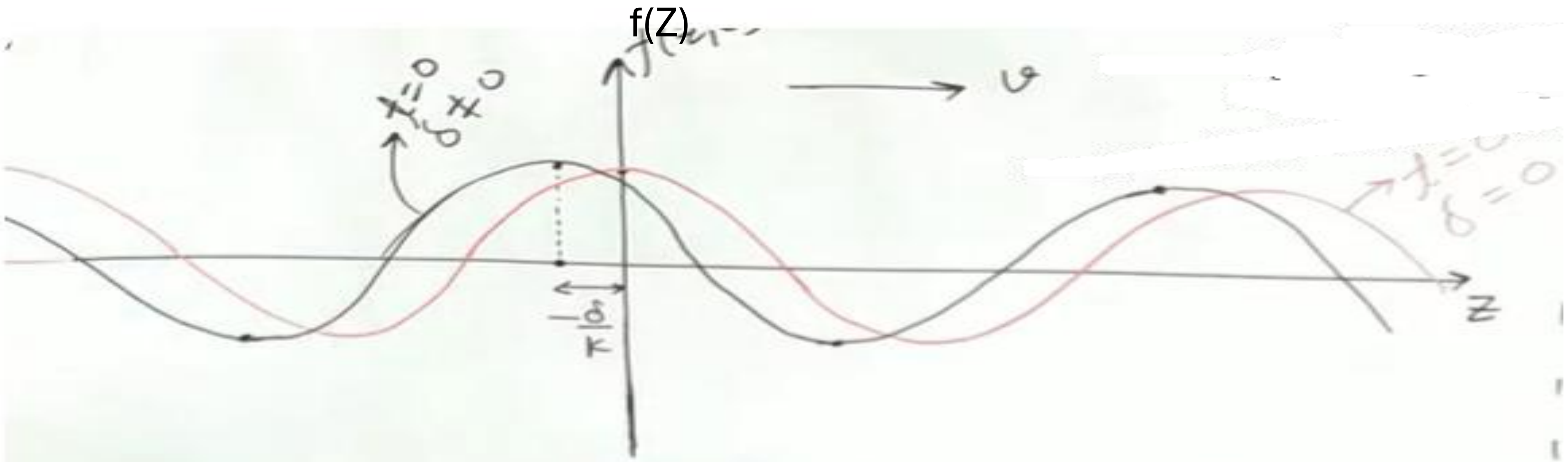
"Any wave which can be represent by any trignomatric Sine or Cosine function called sinusoidal wave"



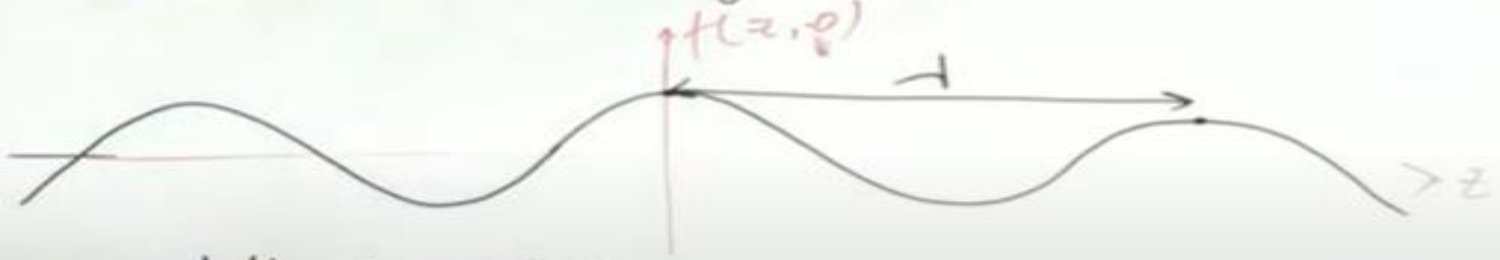
The most common or familear sinusoidal wave is

$$f(z,t) = A\cos\left[k(z - vt) + \delta\right] \quad \text{——— (1)}$$

→ Here A is repersenting amplitude of wave. (Amplitude is maxi. displacement from mean position)



K ≡ wave Number, This wave number is related to wave length (λ)



in following way

$$\boxed{k = \frac{2\pi}{\lambda}} \; *$$

$$\left[\text{Phase} = k(z - vt) + \delta\right]$$ # Sinusoidal wave.

→ δ is called phase constant

[we can add any integer multiple of 2π in δ without changing value of $f(z,t)$]

Ordinarlly $[0 \le \delta \le 2\pi]$

→ if Phase = 0

$$k(z - vt) + \delta = 0$$

$$\boxed{z = vt - \delta/k}$$

at $t=0$ and $\delta = 0$

$$\boxed{z = 0}$$

Position of central maxima

Time period (T) → Time taken to complete one cycle is called time period.

" Time taken by wave to cover distance equa
to wave length "

Distance = speed × time

$$\left[T = \frac{\lambda}{v} = \frac{2\pi}{k v} \right]$$

Frequancy ⇒ No. of revolution in 1 sec.

$$\boxed{\nu = \frac{1}{T} = \frac{v}{\lambda} = \frac{k v}{2\pi}}$$

Angular Frequency

$$\left[\omega = \frac{2\pi}{T} = \frac{\cancel{2\pi}\, k v}{\cancel{2\pi}} = k v = 2\pi\nu \right]$$

Complex form of wave ⇒

$$\because f(z,t) = A\cos[kz - \omega t + \delta] \quad \text{——— (i)}$$

$\because$ Eluear's form

$$e^{i\theta} = \underbrace{\cos\theta}_{Re.} + i\underbrace{\sin\theta}_{Img.}$$

$$\cos\theta = Re[e^{i\theta}]$$

$$\sin\theta = Img[e^{i\theta}]$$

$$f(z,t) = A\,Re\left[e^{i[kz - \omega t + \delta]}\right]$$

$$f(z,t) = Re\left[A\,e^{i(kz - \omega t)} \cdot e^{i\delta}\right]$$

$$f(z,t) = Re\left[\underbrace{A e^{i\delta}\, e^{i(kz - \omega t)}}_{\text{Complex wave funct}^n}\right]$$

$$\left[\tilde{f}(z,t) = \tilde{A}\, e^{i(kz - \omega t)}\right]$$

$$\tilde{A} = A e^{i\delta} \quad (\text{Complex Amp.})$$

Transverse nature of Electromagnetic Wave:

Transverse nature of electromagnetic wave can be proved with the help of Maxwell's equations in free space

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \text{and} \quad \vec{\nabla}\cdot\vec{B} = 0$$

we know that electromagnetic wave equation

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{——(1)}$$

$$\nabla^2 \vec{B} = \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} \quad \text{——(2)}$$

The general solution of this equation

$$\vec{E}(\vec{r}, t) = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

and

$$\vec{B}(\vec{r}, t) = \vec{B}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

where $\vec{E}_0$ and $\vec{B}_0$ are complex amplitude

$\vec{k}$ is a wave propagation vector $\vec{k} = k\hat{n}$, $\vec{k} = \left(\frac{2\pi}{\lambda}\right)\hat{n}$

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial}{\partial x}\left[E_{ox}\,e^{i(k_x x+k_y y+k_z z-\omega t)}\right]+\frac{\partial}{\partial y}\left[E_{oy}\,e^{i(k_x x+k_y y+k_z z-\omega t)}\right]+\frac{\partial}{\partial z}\left[E_{oz}\,e^{i(k_x x+k_y y+k_z z-\omega t)}\right]$$

$$\vec{\nabla}\cdot\vec{E} = E_{ox}\,e^{i(k_x x+k_y y+k_z z-\omega t)}\cdot ik_x + E_{oy}\,e^{i(k_x x+k_y y+k_z z-\omega t)}\cdot ik_y + E_{oz}\,e^{i(k_x x+k_y y+k_z z-\omega t)}\cdot ik_z$$

$$\vec{\nabla}\cdot\vec{E} = i\left(k_x E_{ox}+k_y E_{oy}+k_z E_{oz}\right)e^{i(k_x x+k_y y+k_z z-\omega t)} \text{ ——— } (3)$$

Now we calculate $\vec{K}\cdot\vec{E}$

$$\vec{K}\cdot\vec{E} = \left(\hat{i}k_x+\hat{j}k_y+\hat{k}k_z\right)\cdot\left(\hat{i}E_{ox}+\hat{j}E_{oy}+\hat{k}E_{oz}\right)e^{i(k_x x+k_y y+k_z z-\omega t)}$$

$$\vec{K}\cdot\vec{E} = \left(k_x E_{ox}+k_y E_{oy}+k_z E_{oz}\right)e^{i(k_x x+k_y y+k_z z-\omega t)} \text{ ——— } (4)$$

Using equation (4) in equation (3)

$$\vec{\nabla}\cdot\vec{E} = i(\vec{K}\cdot\vec{E})$$

from Maxwell's equation in free space

$$\vec{\nabla}\cdot\vec{E} = 0$$

then

$$i(\vec{K}\cdot\vec{E}) = 0$$

$$\vec{K}\cdot\vec{E} = 0$$

Similarly $\vec{K}\cdot\vec{B} = 0$

Hence E is perpendicular to K
and B is perpendicular to K

This conclusion indicates that electric and magnetic fields are perpendicular to the direction of propagation vector $\vec{K}$. That is electromagnetic waves are transverse in nature.