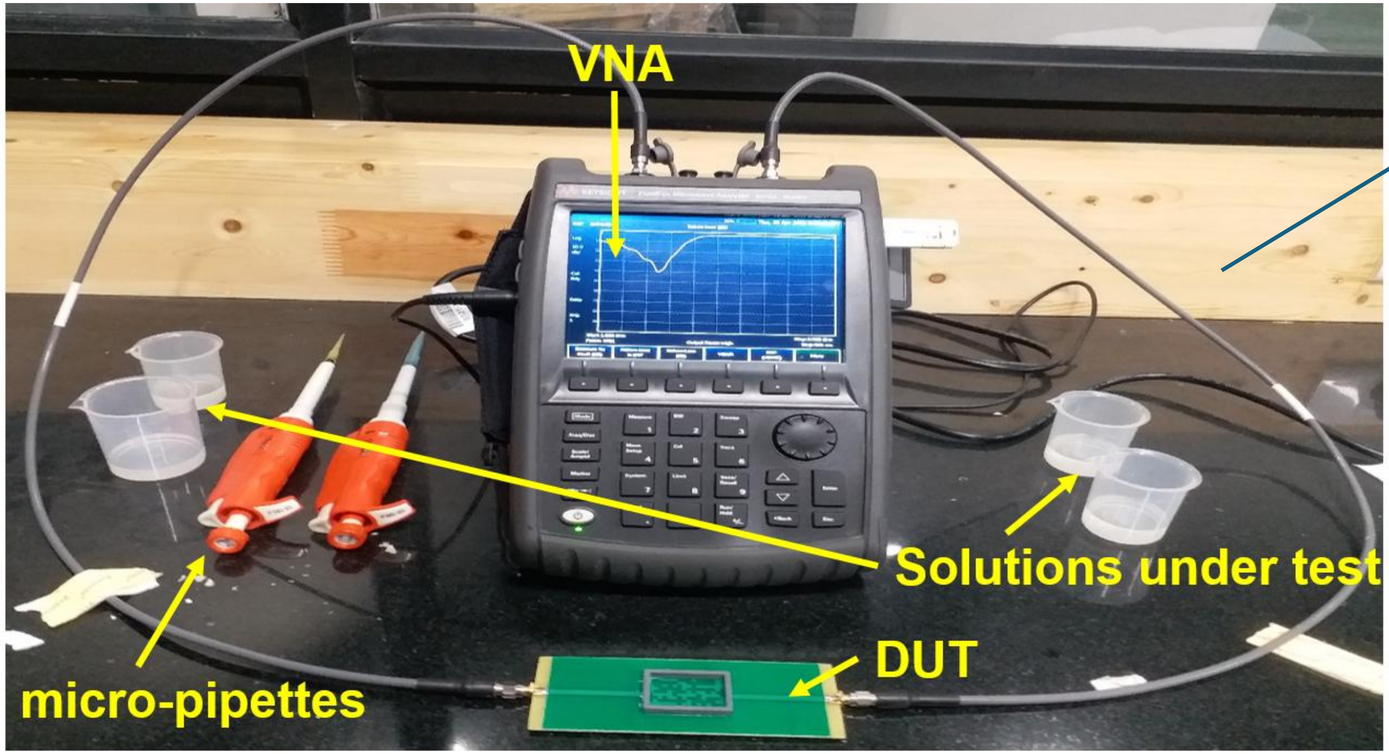
Lecture 10

RF Sensors

RF Sensor testing setup-

SMA cable



DUT=Device Under Test

Vector Network Analyzer (VNA)

- A **Vector Network Analyzer (VNA)** is a crucial instrument used in RF (Radio Frequency) and microwave engineering to measure the network parameters of electrical networks. It is primarily used to characterize components like antennas, filters, cables, RF Sensors, and other devices in terms of their reflection and transmission characteristics.

Key Functions of a VNA:

- S-Parameter Measurement: A VNA measures S-parameters (Scattering parameters), which describe how RF signals are reflected or transmitted through a network.
- The most common S-parameters for 2-port networks include:
- **S11: Reflection coefficient at port 1 (input reflection).**
- **S21: Transmission coefficient from port 1 to port 2 (forward transmission).**
- **S12: Transmission coefficient from port 2 to port 1 (reverse transmission).**
- **S22: Reflection coefficient at port 2 (output reflection).**

Scattering Parameters (S parameters)

- ***Analysis of microwave components, RF circuits, and antennas is done by Scattering Parameters.***

Why S-Parameters?

At **microwave frequencies**, it is **not possible** to have **Open Circuit (O.C.)** or **Short Circuit (S.C.)** conditions.

Equipment is not available to directly measure **voltage and current** at microwave frequencies.

Active devices (e.g., **microwave transistors**, **tunnel diodes**, **TRAPATT diodes**, etc.) have **issues of stability**.

Properties of Scattering Parameters

1. **S-Matrix is a square matrix** of order $n \times n$,
   where n = number of ports.

2. **Unitary Property (Lossless condition):**

$$[S][S]^* = [I]$$

   (Where $*$ denotes conjugate transpose and I is the identity matrix.)

3. **Symmetry:**

$$S_{ij} = S_{ji}$$

   (Holds for reciprocal networks.)

4. **Orthogonality Condition (Lossless + Unitary):**

$$\sum_{i=1}^{n} S_{ij} S_{ik}^* = 0, \quad j \neq k, \quad i, j, k = 1, 2, \ldots, n$$

Scattering parameters of 2 port Network

$$\rightarrow \quad a_1 = \frac{V_1^+}{\sqrt{Z_{01}}}, \quad b_1 = \frac{V_1^-}{\sqrt{Z_{01}}}, \quad a_2 = \frac{V_2^+}{\sqrt{Z_{02}}}, \quad b_2 = \frac{V_2^-}{\sqrt{Z_{02}}}$$

→ As we have two port Network

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

→ Basic form of S_{ij}

$$S_{ij} = \frac{\text{Normalized Reflected wave } i^{th} \text{ Port}}{\text{Normalized Incident wave } j^{th} \text{ Port}} = \frac{b_i}{a_j}$$

So, How to measure S parameters?

$$S_{ij} = \frac{\text{Normlized Reflected wave at } i^{th} \text{ Port}}{\text{Normalized Incident wave at } j^{th} \text{ Port}} = \frac{b_i}{a_j}$$

$$b_i = \text{Reflected Voltage at } i^{th} \text{ port} / \sqrt{Z_{0i}}$$

$$a_j = \text{Incident Voltage at } j^{th} \text{ port} / \sqrt{Z_{0j}}$$

→ As we have two port Network

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

→ Basic form of S_{ij}

$$S_{ij} = \frac{\text{Normalized Reflected wave } i^{th} \text{ Port}}{\text{Normalized Incident wave } j^{th} \text{ Port}} = \frac{b_i}{a_j}$$

→ $S_{11} = \frac{b_1}{a_1}$, $S_{12} = \frac{b_1}{a_2}$, $S_{21} = \frac{b_2}{a_1}$, $S_{22} = \frac{b_2}{a_2}$

→ So, total Reflected Voltage at port 1 & 2

$$b_1 = a_1 S_{11} + a_2 S_{12}$$

$$b_2 = a_1 S_{21} + a_2 S_{22}$$

$$S_{11} = \frac{b_1}{a_1} = \frac{V_1^- / \sqrt{Z_{01}}}{V_1^+ / \sqrt{Z_{01}}} = \frac{V_1^-}{V_1^+} = \Gamma_1$$

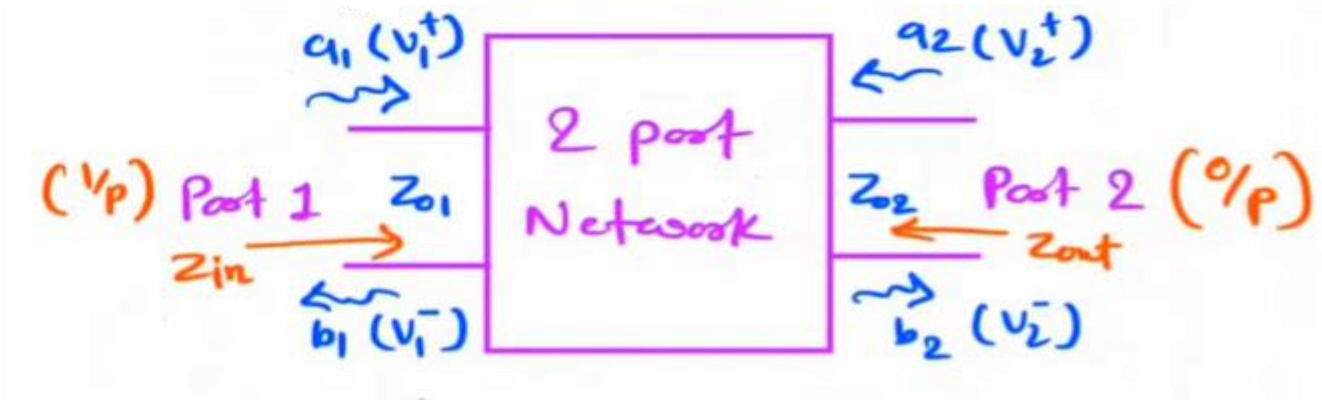
$$S_{12} = \frac{b_1}{a_2} = \frac{V_1^- / \sqrt{Z_{01}}}{V_2^+ / \sqrt{Z_{02}}} = \sqrt{\frac{Z_{02}}{Z_{01}}} \left(\frac{V_1^-}{V_2^+} \right)$$

$$S_{21} = \frac{b_2}{a_1} = \frac{V_2^- / \sqrt{Z_{02}}}{V_1^+ / \sqrt{Z_{01}}} = \sqrt{\frac{Z_{01}}{Z_{02}}} \left(\frac{V_2^-}{V_1^+} \right)$$

$$S_{22} = \frac{b_2}{a_2} = \frac{V_2^- / \sqrt{Z_{02}}}{V_2^+ / \sqrt{Z_{02}}} = \frac{V_2^-}{V_2^+} = \Gamma_2$$

$$Z_{01} = Z_{02}$$

$$S_{11} = \frac{V_1^-}{V_1^+}, \quad S_{12} = \frac{V_1^-}{V_2^+}, \quad S_{21} = \frac{V_2^-}{V_1^+} \quad S_{22} = \frac{V_2^-}{V_2^+}$$



$$r_1 = \frac{Z_{in} - Z_{01}}{Z_{in} + Z_{01}} = \frac{b_1}{a_1}, \quad r_2 = \frac{Z_{out} - Z_{02}}{Z_{out} + Z_{02}} = \frac{b_2}{a_2}$$

Feature	Characteristic Impedance Z_0	Input Impedance Z_{in}
What it is	Property of the transmission line	Impedance seen at input
Depends on	Line's physical/electrical makeup	Load, length, frequency, Z_0
Typical value	50Ω (standard)	Varies with load & conditions
Affects	Wave propagation	Signal reflection/matching

Why 50 Ohms?

- **50 ohms** is a standard value for the **characteristic impedance** of most RF transmission lines, like coaxial cables. It’s chosen because it provides a good balance between **power handling** (ability to transmit power) and **loss** (energy dissipation).
  - **Lower impedance (e.g., 25 ohms)** allows more current to flow and better power handling but increases losses.
  - **Higher impedance (e.g., 75 ohms)** minimizes losses but can’t handle as much power.
- **50 ohms** is a compromise that minimizes reflection and loss, while also providing good power handling.

Parameter	Measured As	Meaning
S_{11}	$\frac{\text{Reflected Power at Port 1}}{\text{Incident Power at Port 1}}$	Input reflection
S_{21}	$\frac{\text{Transmitted Power to Port 2}}{\text{Incident Power at Port 1}}$	Forward gain/transmission
S_{12}	$\frac{\text{Transmitted Power to Port 1}}{\text{Incident Power at Port 2}}$	Reverse transmission
S_{22}	$\frac{\text{Reflected Power at Port 2}}{\text{Incident Power at Port 2}}$	Output reflection

1. dB to Linear (Voltage / Amplitude Ratio)

$$|S|_{\text{linear}} = 10^{\frac{S_{\text{dB}}}{20}}$$

Use this when converting S-parameters (which are voltage-based) to linear **amplitude** values.

2. Linear (Amplitude) to dB

$$S_{\text{dB}} = 20 \cdot \log_{10}(|S|_{\text{linear}})$$

3. dB to Linear Power Ratio

$$\text{Power Ratio} = 10^{\frac{S_{\text{dB}}}{10}}$$

Use this when you're converting dB into **power** reflection or transmission (like % reflected power).

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4. Linear Power to dB

$$S_{\text{dB}} = 10 \cdot \log_{10}(\text{Power Ratio})$$

In dB:

$$S_{11} = -10\,\text{dB}$$

This is a **logarithmic value**. To understand what it means in real-world terms, convert it to **linear** scale.

Convert –10 dB to linear values:

1. Amplitude reflection (voltage ratio):

$$|S_{11}| = 10^{\frac{-10}{20}} = 0.316$$

This means **31.6% of the voltage** is reflected back.

2. Power reflection:

$$|S_{11}|^2 = (0.316)^2 = 0.1$$

This means 10% of the input power is reflected back.

- **S_{11} = –10 dB**
  → **Amplitude** of reflected signal = **0.316**
  → **Power** of reflected signal = **10%**

So when you hear "**–10 dB reflection means 10% is reflected**", they're talking about **power**, not voltage amplitude. Both are true — just different perspectives.

S_{11} (dB) at dip	Meaning
–10 dB	10% reflected, **acceptable match**
–20 dB	1% reflected, **excellent match**
–30 dB	0.1% reflected, **near ideal**

Common Sensor Parameters

Parameter	Symbol	Description
Sensitivity	S	Change in output per unit change in input (e.g., frequency shift per unit concentration). Higher = better.
Selectivity	–	Ability to distinguish the target analyte from other substances.
Resonant Frequency	f_0	The frequency at which the sensor resonates. Changes when analyte is present.
Quality Factor (Q-Factor)	$Q = \frac{f_0}{\Delta f}$	Describes the sharpness of resonance. Higher Q = better resolution and sensitivity.
Return Loss	S_{11} in dB	Indicates how well the sensor is matched. More negative = better matching.
Insertion Loss	S_{21} in dB	Indicates how much signal is lost when passing through the sensor. Lower = better.
Response Time	t_{resp}	Time taken for the sensor to respond to an input change.
Recovery Time	t_{rec}	Time for the sensor to return to baseline after removing the analyte.

Limit of Detection (LOD)	–	Minimum concentration or amount of analyte that can be reliably detected.
Linearity	–	How linear the sensor response is with respect to input changes. Important for calibration.
Repeatability	–	Ability to produce the same result under identical conditions.
Reproducibility	–	Ability to reproduce results across different devices or environments.

In RF Planar Sensors, the most commonly observed parameters are:

- **Resonance Frequency Shift** (Δf) due to loading (e.g., biological samples)
- **Change in S-Parameters** (especially S_{11}, S_{21})
- **Q-Factor** for sharpness of the resonance curve

General Sensitivity Formula

$$\boxed{S = \frac{\Delta f}{\Delta X}}$$

Where:

- $\Delta f = f_{\text{unloaded}} - f_{\text{loaded}}$: Change in resonance frequency (GHz or MHz)
- ΔX: Change in the quantity being measured (e.g., concentration, dielectric constant)

Units of Sensitivity

Type of Sensing	Sensitivity Unit
Concentration	MHz/(mg/mL), GHz/ppm
Dielectric constant	MHz/ε_r
Mass	kHz/ng

Example 1: Sensitivity to Concentration

Let's say you're using a CSRR sensor to detect glucose:

- Baseline $f_0 = 2.45\,\mathrm{GHz}$
- After applying 1 mg/mL glucose: $f_0 = 2.42\,\mathrm{GHz}$
- $\Delta f = 30\,\mathrm{MHz}$

$$S = \frac{30\,\mathrm{MHz}}{1\,\mathrm{mg/mL}} = \boxed{30\,\mathrm{MHz/(mg/mL)}}$$

Example 2: Sensitivity to Dielectric Constant

Suppose you're testing two samples:

- Air ($\varepsilon_r = 1$) $\rightarrow f = 2.45\,\mathrm{GHz}$
- Ethanol ($\varepsilon_r = 5$) $\rightarrow f = 2.35\,\mathrm{GHz}$

Then:

$$S = \frac{2.45 - 2.35}{5 - 1} = \frac{0.10}{4} = \boxed{25\,\mathrm{MHz}/\varepsilon_r}$$

Important Notes:

- Sensitivity can be **linear** or **non-linear**. Always plot frequency vs. analyte to confirm.
- Use **multiple measurements** for accuracy and to calculate **average sensitivity**.

Why Selectivity Matters:

In real applications (like blood, water, air), the sensor isn't just exposed to the target — it sees **many substances**. A good sensor must distinguish between:

- Target molecule (e.g., glucose, creatinine)
- Other similar molecules (e.g., urea, sodium, water)

What is Resonance Frequency (f_0)?

It's the frequency at which a sensor (or resonator) naturally oscillates with **maximum energy transfer** and **minimum impedance** (or maximum transmission in some cases).

At this frequency:

- Energy **builds up** in the sensor's reactive elements (capacitive + inductive).
- The sensor is most **sensitive to changes** in its environment (like dielectric loading).

The resonance frequency of a sensor, such as an RF biosensor, depends on several factors related to its physical design and material properties. In the case of sensors using structures like Interdigitated Capacitive (IDC) sensors or Complementary Split Ring Resonators (CSRR), the resonance frequency can be estimated using different formulas based on the specific sensor geometry and material properties.

1. **For an IDC sensor:** The resonance frequency of an IDC sensor can be approximated by the formula:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where:

- L is the inductance of the sensor,
- C is the capacitance between the fingers.

The inductance and capacitance will depend on the design parameters such as finger width, separation, and length, as well as the dielectric constant of the material between the fingers.

2. **For a CSRR (Complementary Split Ring Resonator):** The resonance frequency for a CSRR can be approximated using the formula for an LC circuit, as it behaves similarly to an LC resonator:

$$f_0 = \frac{1}{2\pi\sqrt{L_{\text{eff}}C_{\text{eff}}}}$$

where:

- L_{eff} is the effective inductance of the resonator, which depends on the geometry of the split rings,
- C_{eff} is the effective capacitance, which depends on the size of the gap and the surrounding dielectric material.

The effective inductance and capacitance of CSRR can be calculated numerically based on the split ring geometry (outer and inner radius, width, gap size) and the dielectric material between the rings.

Factors Affecting Resonance Frequency:

- **Sensor Geometry:** Changes in dimensions (e.g., finger width or gap size) or material properties (e.g., dielectric constant) will affect the resonance frequency.
- **Material Properties:** The dielectric constant of the material used between the IDC fingers or the material surrounding the CSRR can significantly influence the resonance.
- **External Environmental Conditions:** Factors like temperature, pressure, and the presence of target analytes in biosensing applications can shift the resonance frequency.

Quality Factor (Q factor)

The **Quality Factor (Q factor)** of a resonant sensor is a measure of how “sharp” or “selective” the resonance is. In simpler terms, it tells you how well the sensor can distinguish between small changes near its resonance frequency — which is especially important in sensing applications like biochemical detection.

Definition:

$$Q = \frac{f_0}{\Delta f}$$

Where:

- f_0 = Resonance frequency
- Δf = Bandwidth (the frequency range over which the power falls to half its peak, i.e., –3 dB from the peak) (or Full Width at Half Maximum, FWHM)

Return loss

- In RF (Radio Frequency) sensors and systems, return loss is a measure of how much power is reflected back from the sensor or device port due to impedance mismatch. It tells you how well the sensor is matched to the transmission line or source it's connected to.

Return Loss (RL) is the ratio of the power sent into the sensor to the power reflected back from it, expressed in decibels (dB).

Mathematically:

$$\text{Return Loss (dB)} = -20\log_{10}|\Gamma|$$

Where Γ (Gamma) is the **reflection coefficient**, which is:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Z_L = load impedance (sensor)
- Z_0 = characteristic impedance of the system (usually 50 Ω)

High Return Loss = Good Matching

- **High return loss** (e.g., −30 dB) means **low reflected power** → most of the power is absorbed by the sensor.
- **Low return loss** (e.g., −5 dB) means **high reflected power** → poor matching and inefficient sensing.

Why It Matters in RF Sensors:

In RF biosensors (like IDC or DCSRR types):

- Return loss gives insight into **resonance behavior**.
- At resonance, if the sensor is well matched, **return loss dips (S11 minimum)**.
- Changes in the sensed material (e.g., permittivity due to chemical binding) shift this resonance and affect return loss.
- So, **tracking return loss over frequency** helps detect biochemical interactions.

Example:

Say your RF biosensor shows a deep return loss dip at 2.4 GHz. If you apply a chemical that shifts the dip to 2.35 GHz and makes the dip shallower, this implies:

- The **resonance frequency has shifted**.
- The **matching condition has changed** due to the material properties.
- This shift is used for **analyte detection**.

What is Insertion Loss (IL)?

Insertion loss is the amount of signal power lost (or attenuated) when a device (e.g., a sensor) is inserted in the path of an RF signal.

It is also expressed in **decibels (dB)**.

Low Insertion Loss = Good Transmission

- A **small IL** (e.g., −0.5 dB) means most of the signal **passes through** the sensor.
- A **high IL** (e.g., −5 dB or worse) means more power is **lost**—due to absorption, mismatch, or material effects.

Mathematical Definition:

$$\text{Insertion Loss (dB)} = -10\log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right)$$

Where:

- P_{in} = power entering the sensor
- P_{out} = power exiting the sensor

Or in terms of **S-parameters** (from a VNA):

$$\text{Insertion Loss} = -20\log_{10}|S_{21}|$$

Where S_{21} measures how much of the signal **gets through** from port 1 to port 2.

- **What is Response Time?** Response time is when a sensor reacts to a change in the input or environment and produces a measurable output.
- **In the Context of RF Sensors:** For RF (Radio Frequency) sensors, response time refers to how quickly the sensor's resonance characteristics (like frequency, amplitude, or phase) change after the introduction of a target.
- **Technically,** it is usually defined as the time taken for the sensor to reach a certain percentage (commonly 90% or 95%) of the final steady-state value after the stimulus is applied.

Response Time vs Recovery Time:

- **Response time**: Time taken to detect the change.
- **Recovery time**: Time taken to return to baseline after removing the stimulus.

In RF Sensors: In RF sensors, recovery time refers to how quickly the resonant frequency, amplitude, or phase returns to its initial value after the target analyte is no longer present.