EC5.203 Communication Theory I (3-1-0-4):

Lecture 18:
Optimal Demodulation

24 March 2025

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References

- Chap. 6 (Madhow)

Optimal Demodulation

- In 16 QAM, one of 16 passband waveforms corresponding to 16 symbols is sent, where each passband waveform is given by

$$s_i(t) = s_{b_c,c_s} = b_c p(t) \cos(2\pi f_c t) - b_s p(t) \sin(2\pi f_c t)$$

  where b_c, b_s each takes value in $\{\pm 1, \pm 3\}$

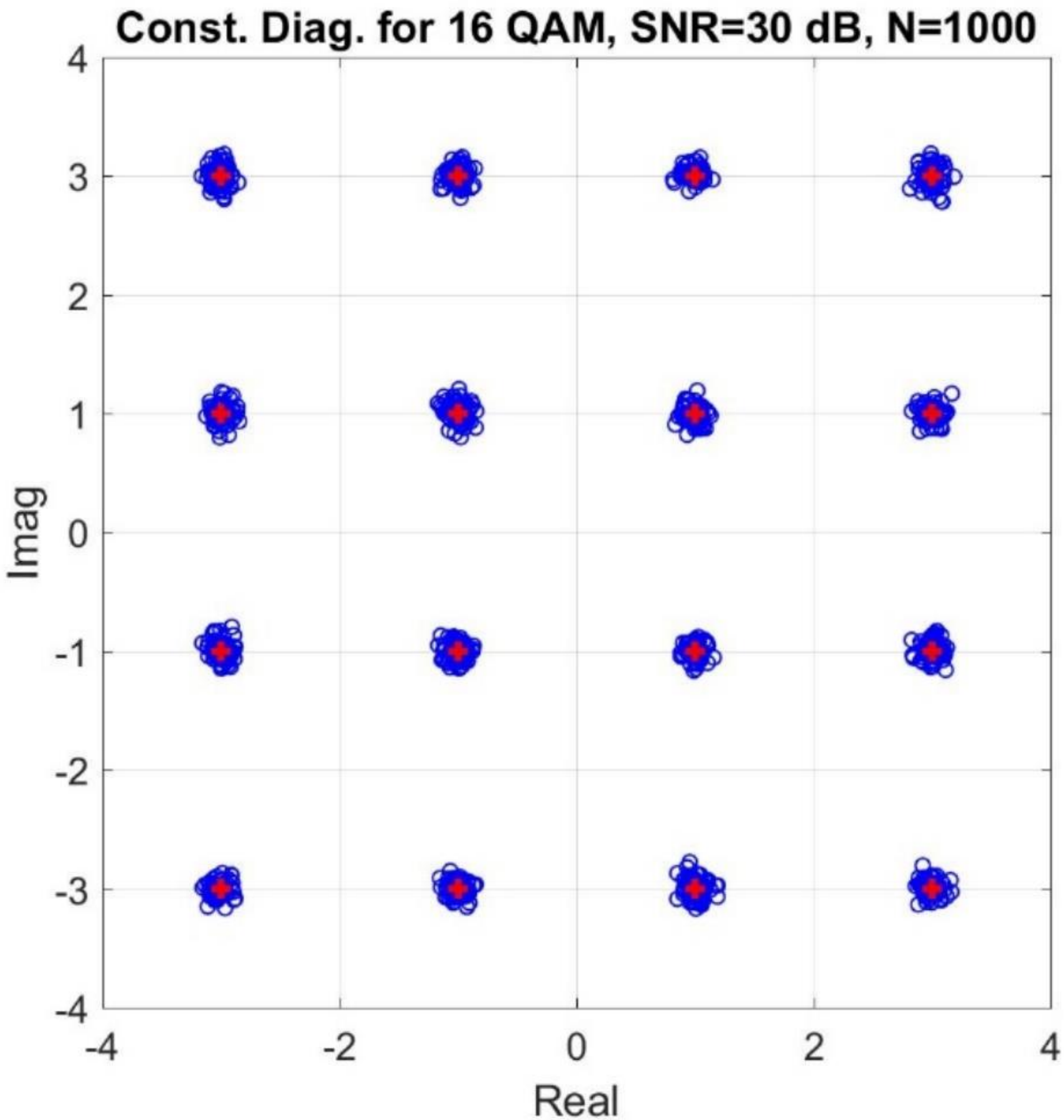
- At the receiver, we have noisy observations

$$y(t) = s_i(t) + n(t)$$

- Now, we are faced with a hypothesis testing problem at the receiver: we have M possible hypotheses about which signal was sent.

- Hypothesis: Possible cause of an event.

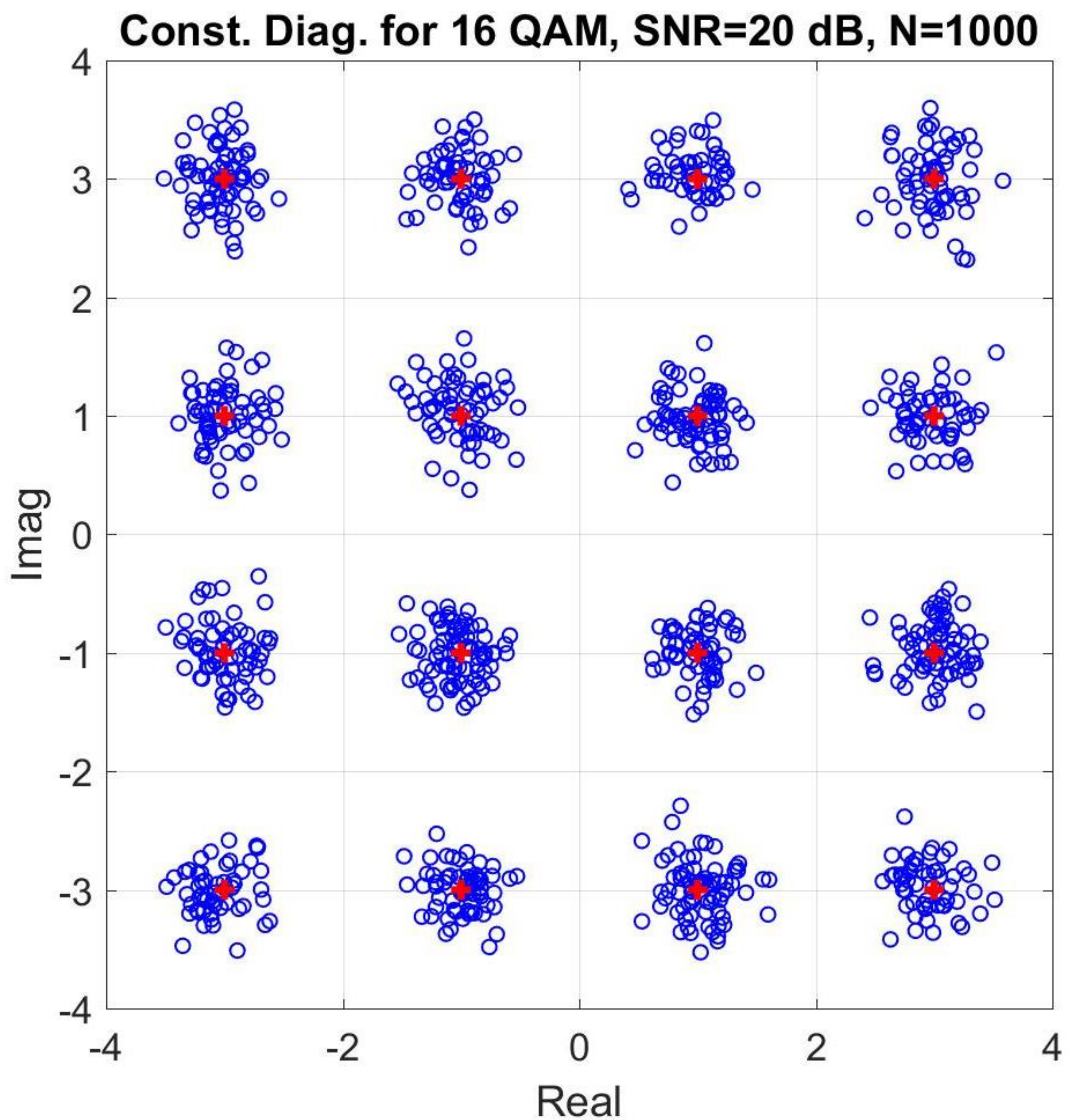
Received Data at SNR = 30 dB



$y(t) = s_i(t) + n(t)$

$n(t)$ is AWGN

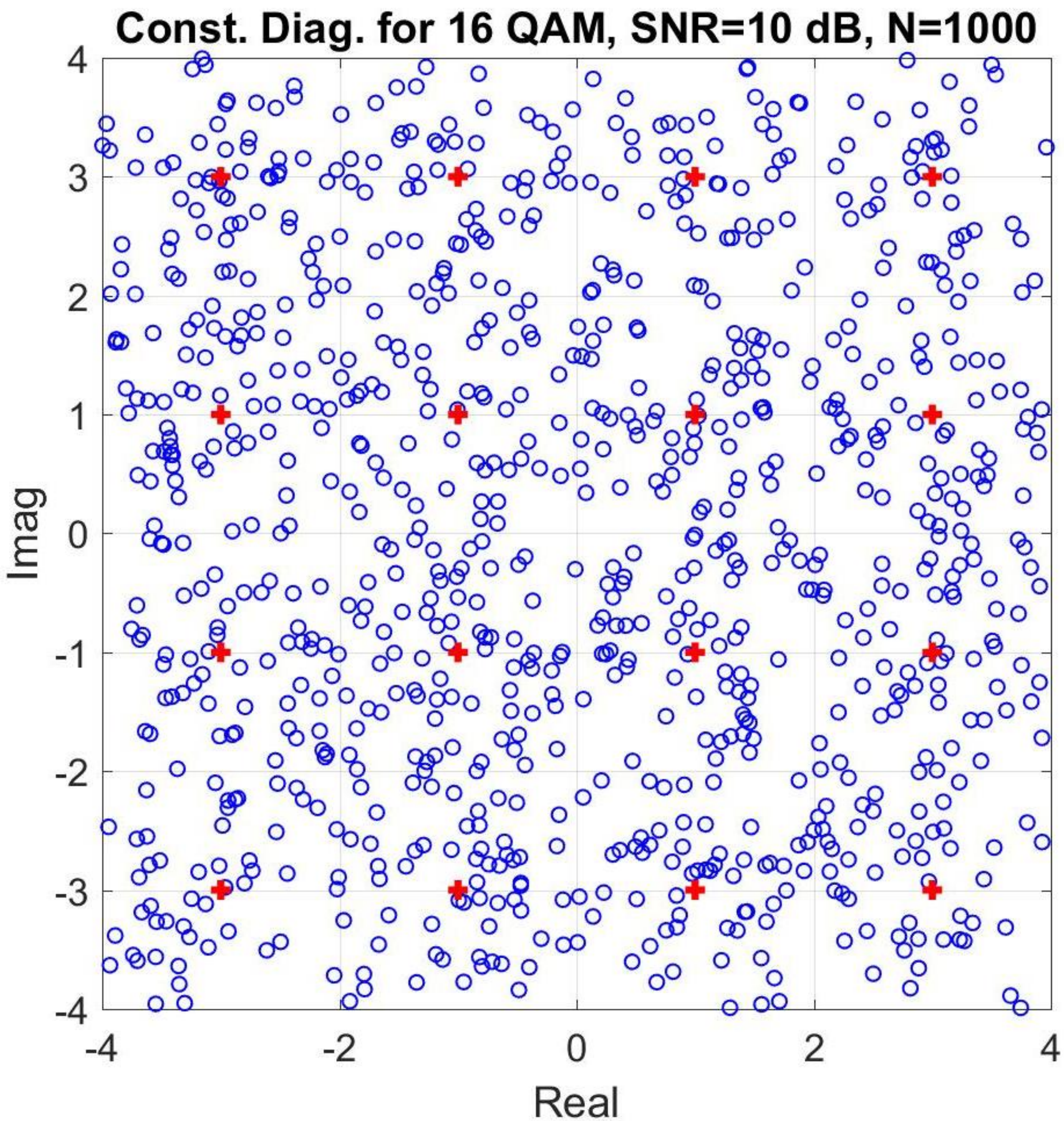
Received Data at SNR = 20 dB



$y(t) = s_i(t) + n(t)$

$n(t)$ is AWGN

Received Data at SNR=10dB



$$y(t) = s_i(t) + n(t)$$

$n(t)$ is AWGN

Optimal Demodulation

- In 16 QAM, one of 16 passband waveforms corresponding to 16 symbols is sent, where each passband waveform is given by

$$s_i(t) = s_{b_c,c_s} = b_c p(t)\cos(2\pi f_c t) - b_s p(t)\sin(2\pi f_c t)$$

  where b_c, b_s each takes value in $\{\pm 1, \pm 3\}$.

- At the receiver, we are faced with a hypothesis testing problem: we have M possible hypotheses about which signal was sent.

- Based on the observations

$$y(t) = s_i(t) + \boxed{n(t)} \quad \text{AWGN}$$

  we are interested in finding a decision rule to make a best guess which hypothesis was sent.

- For communications applications, performance criteria is to minimize the probability of error (i.e., the probability of making a wrong guess).

S&S Recap: Signal Energy

- The energy in a CT signal $x(t)$ over time interval (t_1, t_2)

$$\int_{t_1}^{t_2} |x(t)|^2 \, dt$$

  where $x(t)$ is a complex signal.

- The energy in a DT signal $x[n]$ over sample interval $[n_1, n_2]$ is

$$\sum_{n_1}^{n_2} |x[n]|^2$$

- The energy in a CT $x(t)$ and a DT signal $x[n]$ over infinite time interval respectively are

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \qquad \text{and} \qquad E_\infty = \sum_{-\infty}^{\infty} |x[n]|^2$$

S&S Recap: Signal Power

- The power in a continuous-time signal $x(t)$ over time interval (t_1, t_2)

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, \mathrm{d}t$$

- The power in a discrete-time signal $x[n]$ over samples (n_1, n_2) is

$$\frac{1}{n_2 - n_1 + 1} \sum_{n_1}^{n_2} |x[n]|^2$$

- The power in $x(t)$ and $x[n]$ over infinite time interval respectively are

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \qquad \text{and} \qquad P_\infty = \lim_{N \to \infty} \frac{1}{2N} \sum_{-N}^{N} |x[n]|^2$$

S&S Recap: Note on Dimension of Energy and Power Definitions

- Consider an example where $v(t)$ and $i(t)$ are the instantaneous voltage and current across a resistor R, then the instantaneous power across the resistor is

$$p(t) = v(t)i(t) = \frac{v^2(t)}{R}$$

- The total energy dissipated over the time interval (t_1, t_2) is

$$E = \int_{t_1}^{t_2} p(t)\ dt$$

- The average power dissipated over the time interval (t_1, t_2) is

$$P = \frac{E}{t_2 - t_1}$$

The definitions used earlier are generic and may have wrong dimensions and scaling. The advantage is the convenience and wide applicability irrespective of where the signal is coming from.

Example 5.6.3

- **Binary on-off keying in Gaussian noise**

$$
\begin{aligned}
Y &= m + n \quad && \text{if 1 is sent} \\
Y &= n && \text{if 0 is sent}
\end{aligned}
$$

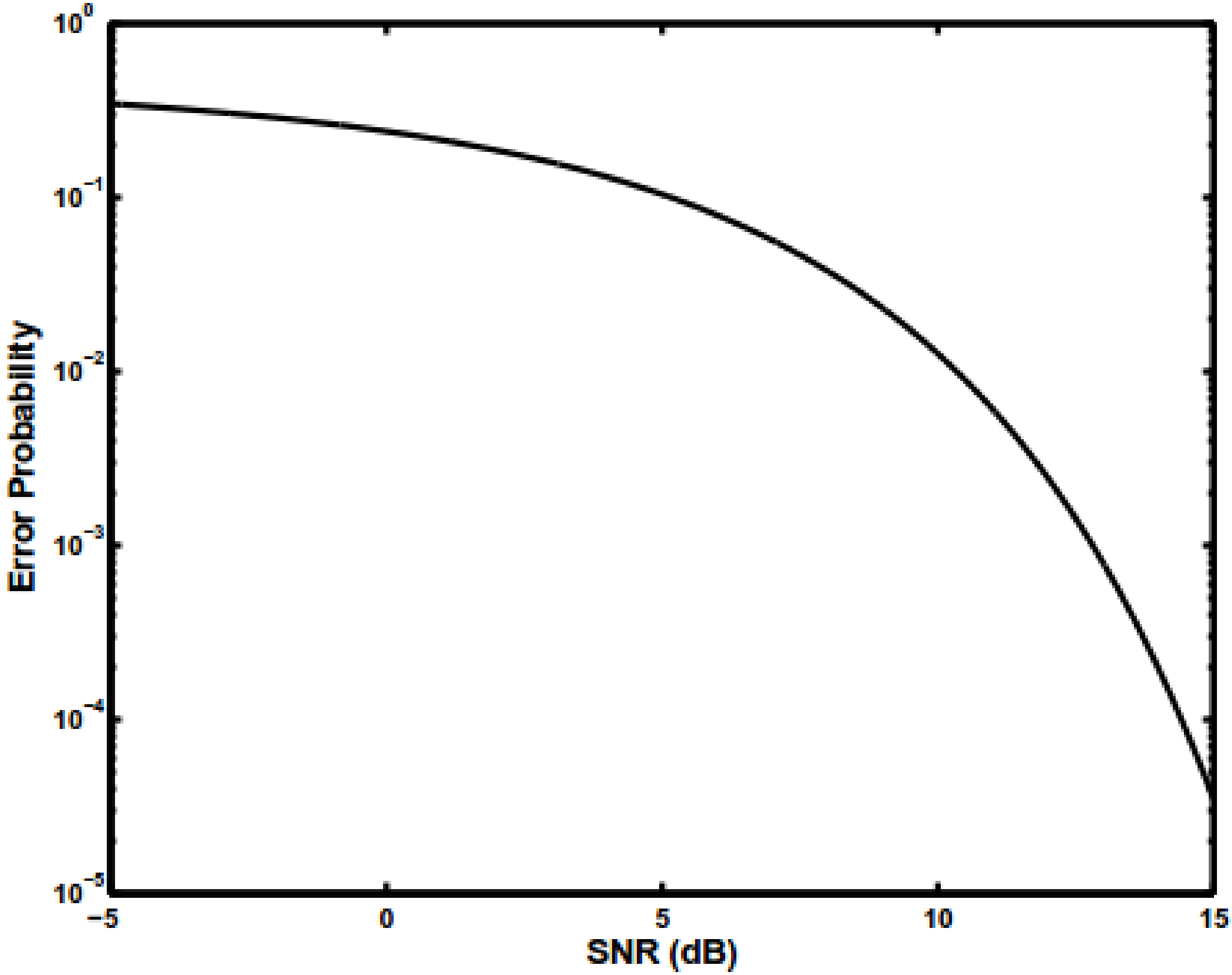
  Here Y is the received sample, $m > 0$ is some constant and n is AWGN sample with $\mathcal{N}(0, v^2)$.

- At the receiver, the detection strategy is

$$
\begin{aligned}
Y &> m/2 \quad && \text{Decide 1 is sent} \\
Y &\leq m/2 && \text{Decide 0 is sent}
\end{aligned}
$$

- Assuming that both 0 and 1 are equally likely,
  - Find the average signal power
  - Find the conditional probability of error conditioned on 0 being sent
  - Find the conditional probability of error conditioned on 1 being sent
  - Find average error probability
  - Find the probability of error for SNR of 13 dB?

(Bit) Error Probability vs SNR for Example



Example 5.6.3: Poll

- **Binary on-off keying in Gaussian noise**

$$\begin{aligned} Y &= m + n && \text{if 1 is sent} \\ Y &= n && \text{if 0 is sent} \end{aligned}$$

  Here Y is the received sample, $m > 0$ is some constant and n is AWGN sample with $\mathcal{N}(0, v^2)$.

- What is a optimal guessing strategy if $\pi_0 = P(H_0) = 1$?

- What is the average P_e?