Q1 (i) Define the terms- [4×2=8]

(a) Stimulus, (b) Sensor, (c) Actuator, and (d) Transducer with definition and examples.?

(a) Stimulus

Answer: A **stimulus** refers to the external physical condition, change, or event that causes a sensor to respond and produce a measurable output.

Example:

Detectable Phenomenon

Stimulus	Quantity
Acoustic	Wave (amplitude, phase, polarization), Spectrum, Wave Velocity
Biological & Chemical	Fluid Concentrations (Gas or Liquid)
Electric	Charge, Voltage, Current, Electric Field (amplitude, phase, polarization), Conductivity, Permittivity
Magnetic	Magnetic Field (amplitude, phase, polarization), Flux, Permeability
Optical	Refractive Index, Reflectivity, Absorption
Thermal	Temperature, Flux, Specific Heat, Thermal Conductivity
Mechanical	Position, Velocity, Acceleration, Force, Strain, Stress, Pressure, Torque

Any Example from the list is fine or apart from that, which shows that concept.

(b). Sensor

Function: A sensor is a device that detects and measures physical properties (such as temperature, pressure, light, motion, etc.) and converts this information into a readable signal, usually electrical, that can be processed or displayed.

Example: A temperature sensor (thermocouple)

(b) Actuator -Function: An actuator is a device that takes input signals (typically electrical) and converts them into mechanical movement or action. It is responsible for moving or controlling a mechanism or system.

Example: A motor that receives an electrical signal to rotate a shaft or a valve that opens or closes based on an electrical signal.

(c) Transducer:

Function: A transducer is a broader term for any device that converts one form of energy into another. Both sensors and actuators can be considered transducers as they transform energy. A sensor converts physical properties into electrical signals, while an actuator converts electrical signals into mechanical motion.

Example: A piezoelectric transducer converts mechanical pressure into electrical voltage (sensor) or electrical voltage into mechanical vibration (actuator).

Q2.

Find the **divergence** of **F**, i.e. $\nabla \cdot \mathbf{F}$

(a)

The divergence of a vector field $\mathbf{F} = F_x\mathbf{a}_x + F_y\mathbf{a}_y + F_z\mathbf{a}_z$ is:

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

- $F_x = xy \Rightarrow \frac{\partial F_x}{\partial x} = y$
- $F_y = y^2 \Rightarrow \frac{\partial F_y}{\partial y} = 2y$
- $F_z = xz \Rightarrow \frac{\partial F_z}{\partial z} = x$

So,

$$\nabla \cdot \mathbf{F} = y + 2y + x = 3y + x$$

(b)

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & y^2 & xz \end{vmatrix}$$

Expanding the determinant:

- **x-component**:

$$\frac{\partial(xz)}{\partial y} - \frac{\partial(y^2)}{\partial z} = 0 - 0 = 0$$

- **y-component**:

$$-\left(\frac{\partial(xz)}{\partial x} - \frac{\partial(xy)}{\partial z}\right) = -(z - 0) = -z$$

- **z-component**:

$$\frac{\partial(y^2)}{\partial x} - \frac{\partial(xy)}{\partial y} = 0 - x = -x$$

(c)

Determine if $\vec{F}$ is Solenoidal or Rotational

- Solenoidal field $\Rightarrow \nabla \cdot \vec{F} = 0$
  Here, $\nabla \cdot \vec{F} = 3y + x \neq 0 \Rightarrow$ Not solenoidal
- Rotational field $\Rightarrow \nabla \times \vec{F} \neq 0$
  Here, $\nabla \times \vec{F} = -z\,\hat{a}_y - x\,\hat{a}_z \neq 0 \Rightarrow$ Yes, rotational

Q4. Attempted any 2.

(a) Explain the concept of (a) wave impedance and (b) Poynting Vector in an electromagnetic wave and derive its value for free space.

(b): Explain the Energy density of the EM wave and prove that the energy density associated with the electric field is equal to that of the magnetic field in a vacuum.

(c). Write down Maxwell's Equations in a Dielectric Medium: also derive the wave equation for electric and magnetic fields.

(d) Derive the differential form of Ampère's law with Maxwell's correction from its integral form. Explain the role of the displacement current.

4 (a)

Electromagnetic Wave Impedance (Z)

Electromagnetic (EM) wave impedance is a fundamental parameter that describes the relationship between the **electric field** (E) and the **magnetic field** (H) in a medium. It plays a crucial role in **wave propagation, reflection, and transmission**.

The intrinsic **wave impedance** of a medium is given by:

$$Z = \frac{E}{H}$$

where:

- E is the electric field intensity (V/m),
- H is the magnetic field intensity (A/m).

It determines how an EM wave propagates in a given medium.

Wave Impedance in Free Space

For free space (μ_0, ε_0):

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Using the values:

- $\mu_0 = 4\pi \times 10^{-7}$ H/m,
- $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m,

we get:

$$Z_0 \approx 377\,\Omega$$

This is the **characteristic impedance of free space**, an important constant in EM theory.

Poynting Vector – Electromagnetic Power Flow

The **Poynting vector** represents the **power flow per unit area** in an electromagnetic (EM) wave. It tells us **how energy moves** in space due to electric ($\mathbf{E}$) and magnetic ($\mathbf{H}$) fields.

The instantaneous **Poynting vector** is given by:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

where:

- $\mathbf{E}$ is the **electric field** (V/m),
- $\mathbf{H}$ is the **magnetic field** (A/m),
- $\mathbf{S}$ has units of **watts per square meter** (W/m^2).

🔹 **Interpretation**: The direction of $\mathbf{S}$ gives the **direction of energy flow**, and its magnitude represents the **power density**.

Poynting Vector in Free Space (Plane Wave Case)

For a **uniform plane wave** traveling in the $+z$-direction:

$$\mathbf{E} = E_0 e^{i(kz-\omega t)}\hat{x}, \quad \mathbf{H} = H_0 e^{i(kz-\omega t)}\hat{y}$$

Using $\mathbf{S} = \mathbf{E} \times \mathbf{H}$:

$$\mathbf{S} = (E_0 e^{i(kz-\omega t)}\hat{x}) \times (H_0 e^{i(kz-\omega t)}\hat{y})$$

Since $\hat{x} \times \hat{y} = \hat{z}$, we get:

$$\mathbf{S} = (E_0 H_0) e^{2i(kz-\omega t)}\hat{z}$$

Taking the **time average**:

$$\langle \mathbf{S} \rangle = \frac{1}{2} E_0 H_0 \hat{z}$$

Since **wave impedance** is $Z = E/H$, we can express this as:

$$\langle \mathbf{S} \rangle = \frac{E_0^2}{2Z}\hat{z}$$

(b): Explain the Energy density of the EM wave and prove that the energy density associated with the electric field is equal to that of the magnetic field in a vacuum.

The **energy density** of an electromagnetic (EM) wave refers to the **amount of energy stored in the electric and magnetic fields per unit volume**. It's the sum of the energy densities of the electric field and the magnetic field.

Formula:

$$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

Where:

- u = total energy density (in joules per cubic meter, J/m^3)
- ε_0 = vacuum permittivity ≈ $8.85 \times 10^{-12}\,\mathrm{F/m}$
- μ_0 = vacuum permeability ≈ $4\pi \times 10^{-7}\,\mathrm{H/m}$
- E = electric field amplitude (V/m)
- B = magnetic field amplitude (T)

To Show:

$$(\mu_B)_{avg} = (\mu_E)_{avg}$$

Electric Field Energy Density (Average):

$$(\mu_E)_{avg} = \frac{1}{4}\epsilon_0 E_0^2$$

Substituting the relation:

$$E_0 = C \cdot B_0$$

$$(\mu_E)_{avg} = \frac{1}{4}\epsilon_0 (B_0 \cdot C)^2$$

$$= \frac{1}{4}\epsilon_0 B_0^2 C^2$$

We know

$$C = \frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Put this value in equation (a)

We will get

$$= \frac{1}{4}\frac{B_0^2}{\mu_0}$$

Which is equal to $(\mu_B)_{avg}$

Thus,

$$(\mu_B)_{avg} = (\mu_E)_{avg}$$

(c). Write down Maxwell's Equations in a Dielectric Medium: also derive the wave equation for electric and magnetic fields.

- A dielectric medium is an insulating material with no free charges and no free currents. Examples include glass, plastic, and air.

Maxwell's Equations in a Dielectric Medium

$$\nabla \cdot \mathbf{E} = \frac{\rho_f}{\varepsilon} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu\varepsilon\frac{\partial \mathbf{E}}{\partial t}$$

Taking the curl of **Faraday's Law**:

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B})$$

Using **Ampère's Law**:

$$\nabla \times \mathbf{B} = \mu\varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

Substituting:

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Using the vector identity:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

Substituting:

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Using the vector identity:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

Since $\nabla \cdot \mathbf{E} = 0$ in a dielectric:

$$\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

A similar equation holds for **magnetic field**:

$$\nabla^2 \mathbf{B} = \mu\varepsilon \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

From the wave equation, the wave speed is:

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

(d) Derive the differential form of Ampère's law with Maxwell's correction from its integral form. Explain the role of the displacement current.

Fourth Equation :- Ampere's law ⇒ (3.6)

The Line integral of the magnetic field around any closed path is equal to μ_0 times the current enclosed with in the path.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I \quad \text{— (1)}$$

$$= \mu_0 \int_S J \, ds$$

Apply Stokke's theorem

$$\int_S \vec{\nabla} \times \vec{B} \, ds = \mu_0 \int_S J \, ds$$

It shows

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}} \quad \text{— (2)}$$

$$\vec{B} = \mu_0 \vec{H}$$

$$\boxed{\nabla \times \vec{H} = \vec{J}} \quad \text{— (3)}$$

hold good for steady current only

From (2) $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

Taking divergence

$$\vec{\nabla} \cdot (\nabla \times \vec{B}) = \vec{\nabla} \cdot (\mu_0 \vec{J})$$

[Div of curl = 0]

$$0 = \mu_0 (\vec{\nabla} \cdot \vec{J})$$

$$\boxed{\vec{\nabla} \cdot \vec{J} = 0}$$ hold good only for steady current

(7)

We need a correction in this equation for (3.7) time varying field

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

from maxwell's first Equation

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\rho = \varepsilon_0 (\vec{\nabla} \cdot \vec{E})$$

$$\nabla \cdot \vec{J} = -\frac{\partial}{\partial t}(\varepsilon_0 \vec{\nabla} \cdot \vec{E})$$

$$\nabla \cdot \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) = 0$$

$$\varepsilon_0 \frac{\partial \vec{E}}{\partial t} = J_D$$

Displacement current density due to time varying electric field.

Then

$$\nabla \times \vec{B} = \mu_0 (J + J_D)$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \left(J + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)}$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \left[J + \frac{\partial \vec{D}}{\partial t}\right]}$$

$B = \mu_0 H$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

in free space $\vec{D} = \varepsilon_0 \vec{E}$

D = Electric displacement vector.

- Displacement current allows a **time-varying electric field to generate a magnetic field**, even in free space.
- This mutual generation of $\vec{E}$ and $\vec{B}$ fields leads to **self-sustaining electromagnetic waves** traveling through space.

Q5 ANS (A)(i) Radio waves typically have frequencies in the range of:

Answer: 3 kHz to 300 GHz

(ii) Which represents the correct conversion from Cartesian coordinates (x, y) to polar coordinates (r, θ)?
ANS (A)

$r = \sqrt{(x^2 + y^2)}$

$\theta = \tan^{-1}(y / x)$

(iii) Which of the following represents the correct conversion from Cartesian coordinates (x, y, z) to spherical coordinates (r, θ, φ)?
Answer:
ANS (A)

$\mathbf{r = \sqrt{(x^2 + y^2 + z^2)}}$

$\boldsymbol{\theta = \cos^{-1}(z / r)}$ **(polar angle)**

$\boldsymbol{\varphi = \tan^{-1}(y / x)}$ **(azimuthal angle).**

(iv) Two electromagnetic waves have intensities in the ratio of 4:1. What is the ratio of their electric field amplitudes?
Answer: B) 2:1
(Because intensity $\propto$ (Electric field amplitude)^2 $\Rightarrow$ $E_1/E_2 = \sqrt{(I_1/I_2)} = \sqrt{4/1} = 2/1$)

(v) Which statement correctly describes the physical significance of bound and free charges in a dielectric material?
Answer: A) Bound charges are associated with the alignment of molecular dipoles, while free charges move freely in a conductor.

(vi) What happens to the frequency of light when it travels from air to glass?
Answer: C) Remains constant
(Only speed and wavelength change; frequency stays the same across media)