EC5.203 Communication Theory I (3-1-0-4):

Lecture 6:
Analog Communication Techniques: Amplitude Modulation

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Recap

Key Concepts

- Two ways of encoding info in complex envelope
  - I and Q: amplitude modulation (several variants)
  - Envelope and phase: angle modulation (constant envelope)
- For example, for a sinusoid carrier

$$A_c(t)\cos(2\pi f_c(t)t + \theta_c(t))$$

where $A_c(t)$, $f_c(t)$, $\theta_c(t)$ are the amplitude, frequency, and the phase of the carrier respectively.

Amplitude Modulation

Frequency Modulation

Phase Modulation

AM: Double Sideband Suppressed Carrier

- Here the message $m(t)$ modulates the I component of the pass-band signal $u(t)$ and is given by

$$u_{DSB}(t) = m(t) \cdot A\cos(2\pi f_c t)$$

while the Fourier transform is given by

$$U_{DSB}(f) = \frac{A}{2}(M(f - f_c) + M(f + f_c))$$

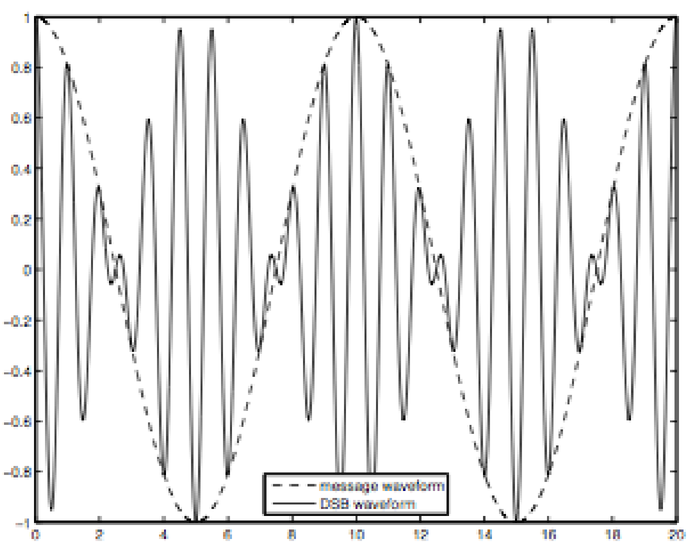
DSB-SC signal for sinusoidal message

Here the signal is given by

$$u_{DSB}(t) = A_m \cos(2\pi f_m t) \cdot A \cos(2\pi f_c t)$$

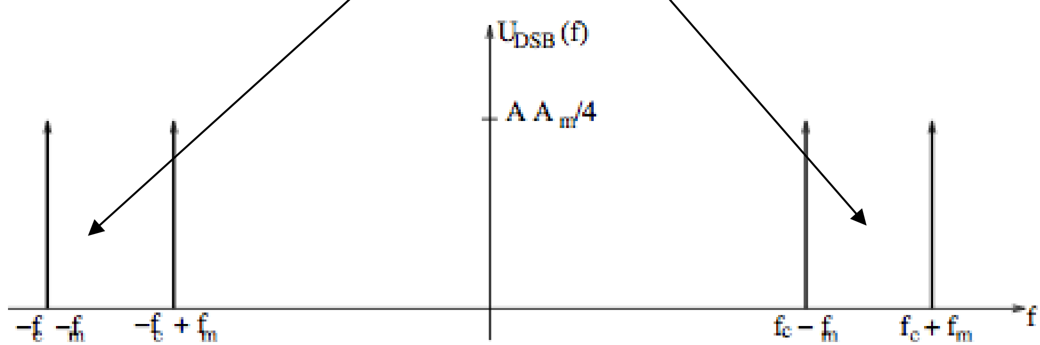
while the Fourier transform is given by

$$U_{DSB}(f) = \frac{AA_m}{4}\{\delta(f - f_c - f_m) + \delta(f - f_c + f_m) + \delta(f + f_c + f_m) + \delta(f + f_c - f_m)\}$$



(a) DSB time domain waveform

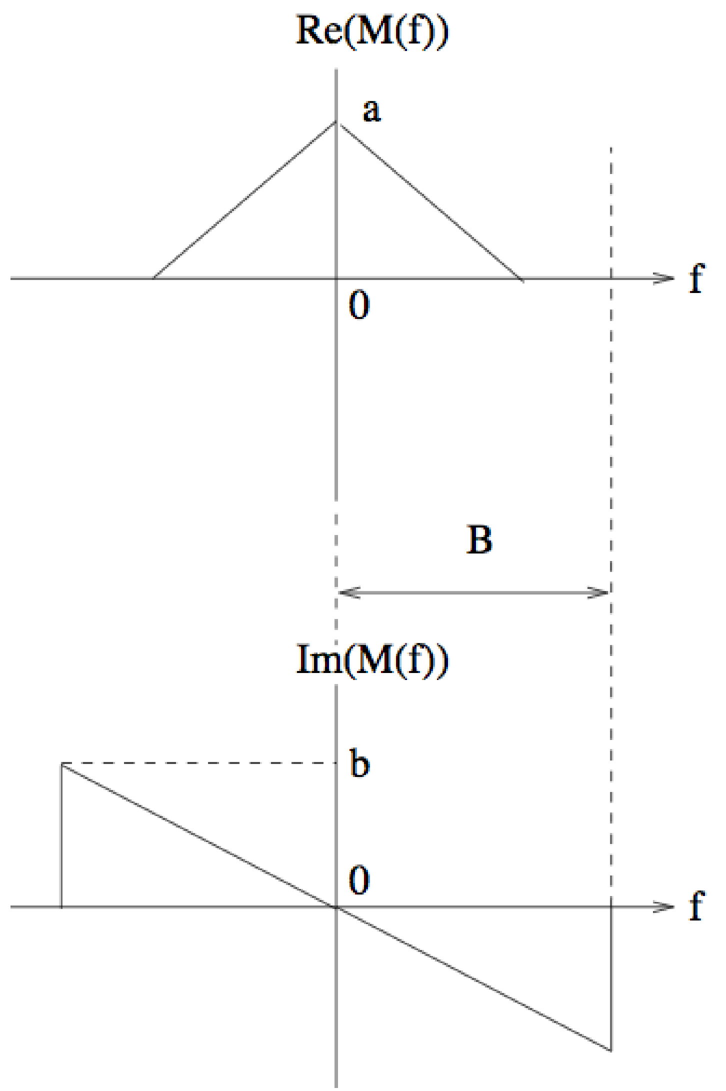
No impulses at f_c or $-f_c$!



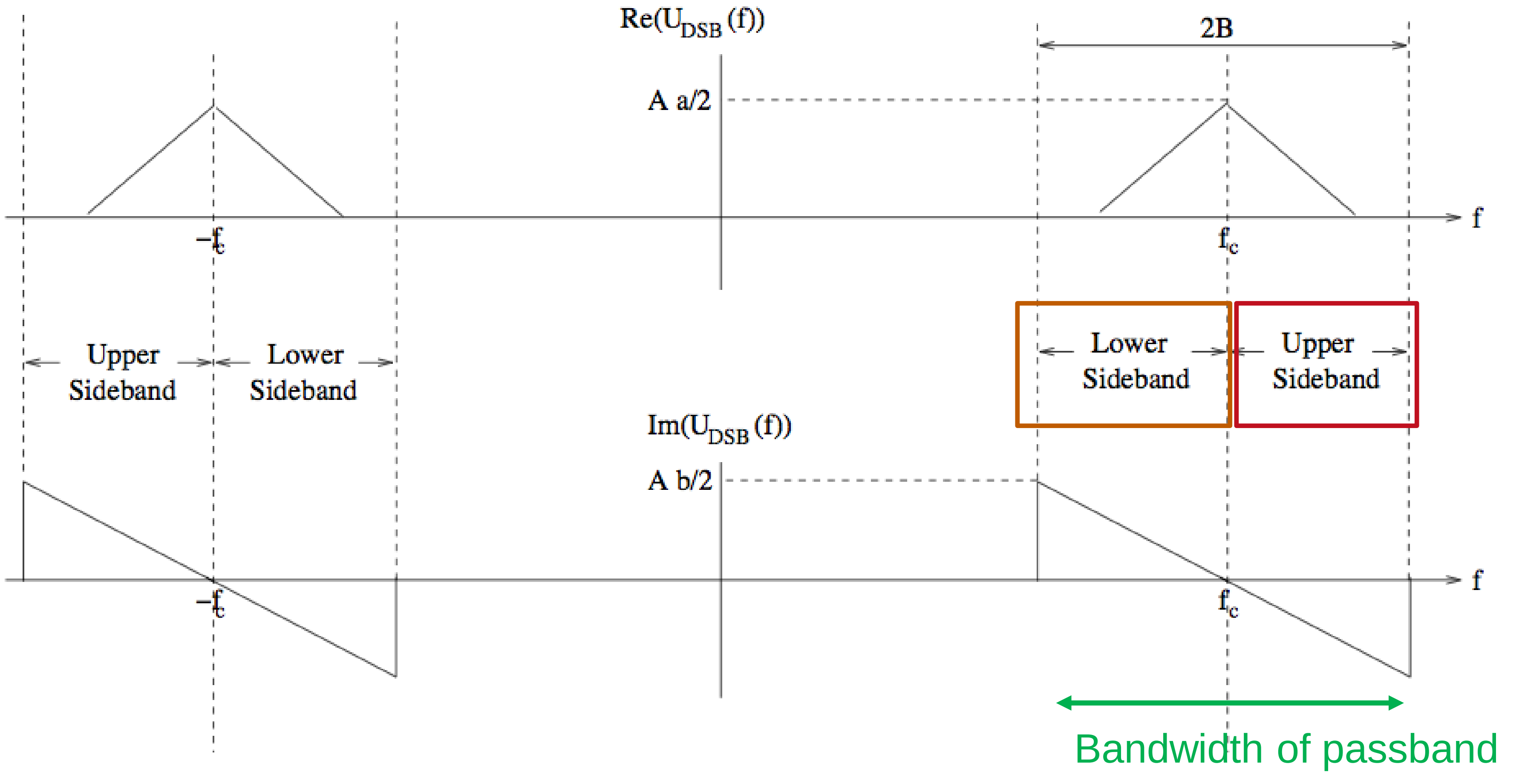
(b) DSB spectrum

Example 2

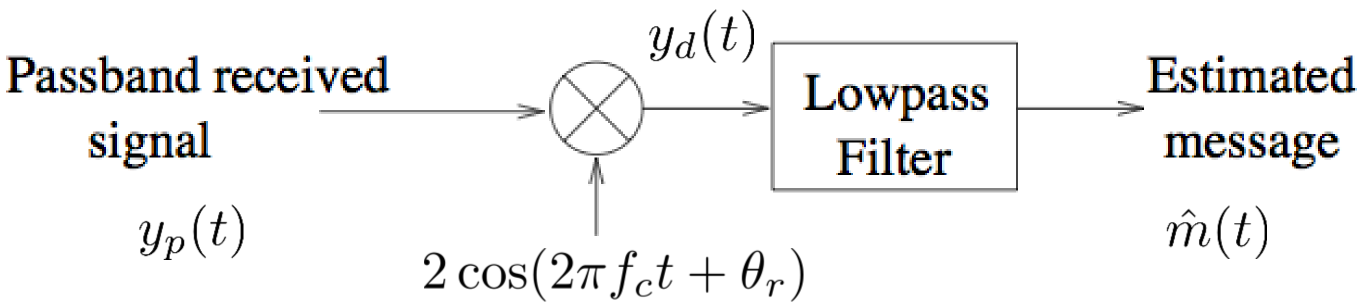
- Consider a message signal $m(t)$ with following frequency response $M(f)$



DSB-SC spectrum for Example 2



Demodulation of DSB-SC



- Standard downconverter to recover I component
- The received signal in the passband is

$$y_p(t) = Am(t)\cos(2\pi f_c t)$$

where θ_r is the phase difference arising from the phase offset with respect to local carrier at Rx.

- The output of multiplier followed by low pass filter is

$$\hat{m}(t) = Am(t)\cos\theta_r$$

Causes of Phase Offset

- Frequency offset: The local oscillator at the receiver is generating frequency at $f_c + \Delta f$

  $$\theta_r = 2\pi \Delta f t$$

  This happens as the two physical devices cannot be exactly same resulting in slight differences. Here there will be phase difference even if they are same place.

- Timing offset: The transmitter and receiver have slightly different time references or they are separated by distance d resulting in time offset of δt.

  $$\theta_r = 2\pi f_c \Delta t$$

Need of Coherent Detection

- The output of multiplier followed by low pass filter is

$$\hat{m}(t) = Am(t)\cos\theta_r$$

- For $\theta_r = 0$, $\hat{m}(t) = Am(t)$
- For $\theta_r = \pi/2$, $\hat{m}(t) = 0$
- For $\theta_r(t) = 2\pi\Delta f t + \phi$, time varying signal degradation in amplitude and unwanted sign changes.
- Need of synchronization of phase of the local oscillator with the phase of incoming signal:
  - Phased locked loop (PLL)
- Switch to other AM techniques which do not need synchronization
  - Conventional AM or DSB (with carrier)

Conventional AM

- Add a large carrier component to a DSB-SC signal so that the passband has the following form

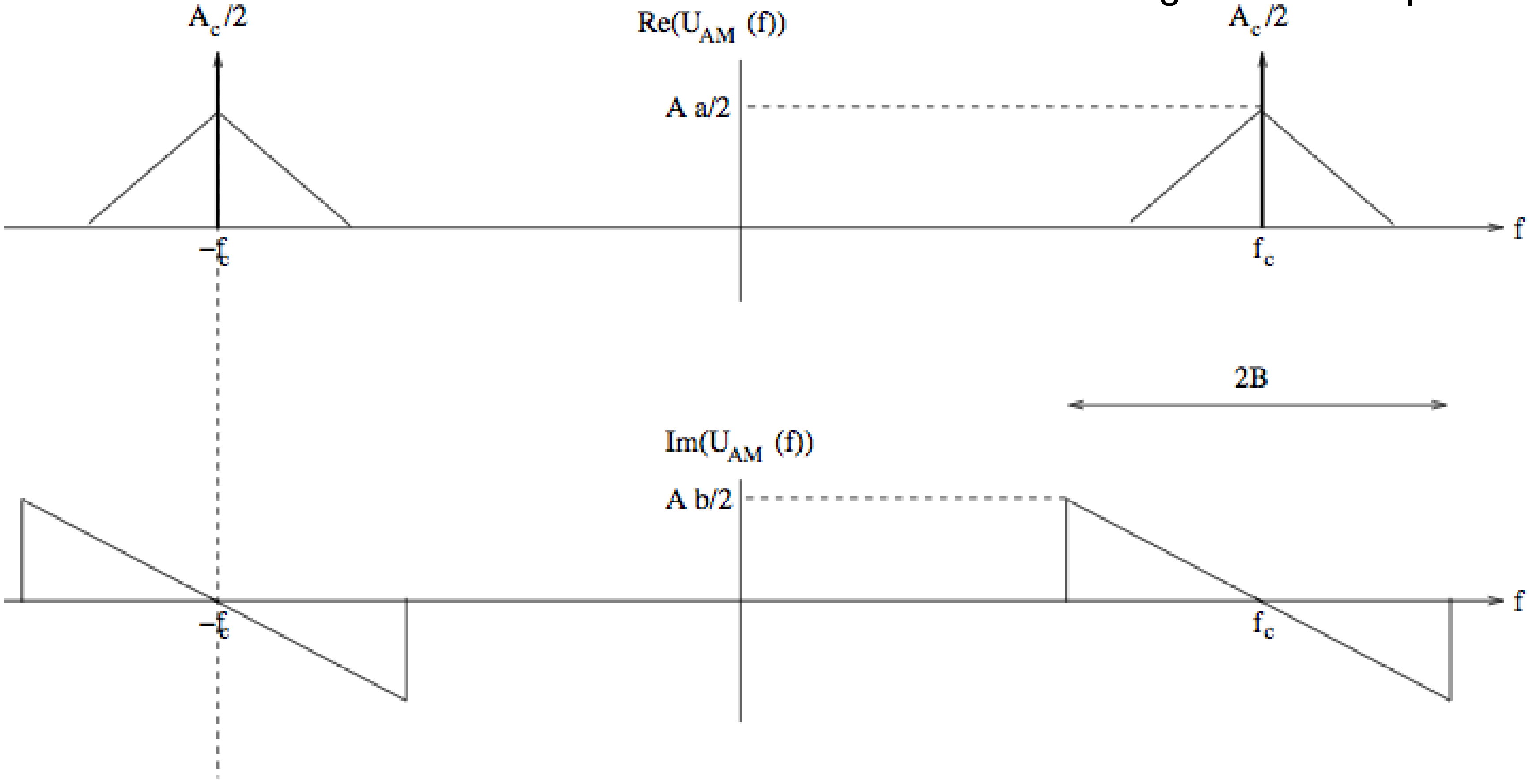
$$\begin{aligned} u_{\mathrm{AM}}(t) &= (Am(t) + A_c)\cos(2\pi f_c t) \\ &= Am(t)\cos(2\pi f_c t) + A_c\cos(2\pi f_c t) \end{aligned}$$

- Taking Fourier transform

$$U_{\mathrm{AM}}(f) = \frac{A}{2}(M(f-f_c)+M(f+f_c)) + \frac{A_c}{2}(\delta(f-f_c)+\delta(f+f_c))$$

Conventional AM: spectrum

Strong carrier component



Envelope and its importance

- Add a large carrier component to a DSB-SC signal so that the passband has the following form

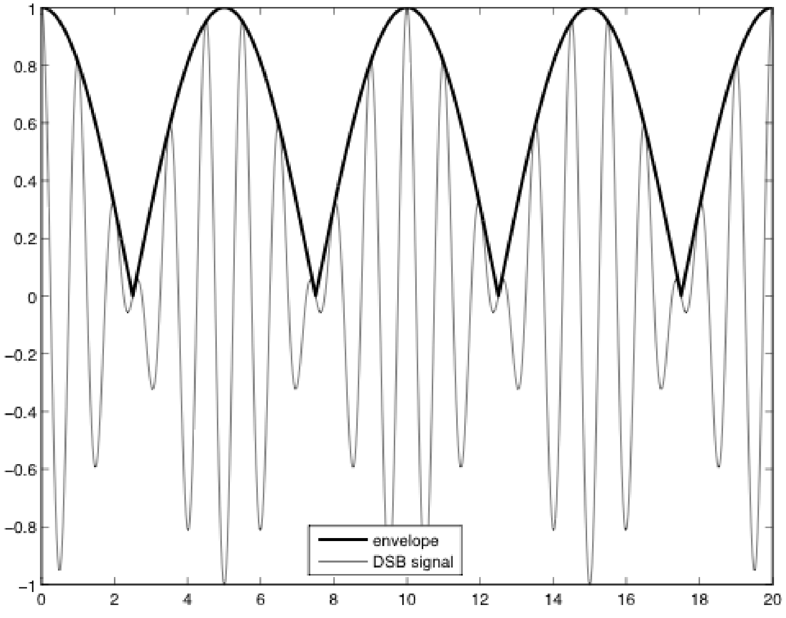
$$\begin{aligned} u_{\mathrm{AM}}(t) &= \boxed{(Am(t) + A_c)} \cos(2\pi f_c t) \\ &= Am(t)\cos(2\pi f_c t) + A_c \cos(2\pi f_c t) \end{aligned}$$

- Envelope is given by $e(t) = |Am(t) + A_c|$.
- If $Am(t) + A_c > 0$, then $e(t) = Am(t) + A_c$. In this case, message $m(t)$ can be recovered from $e(t)$.

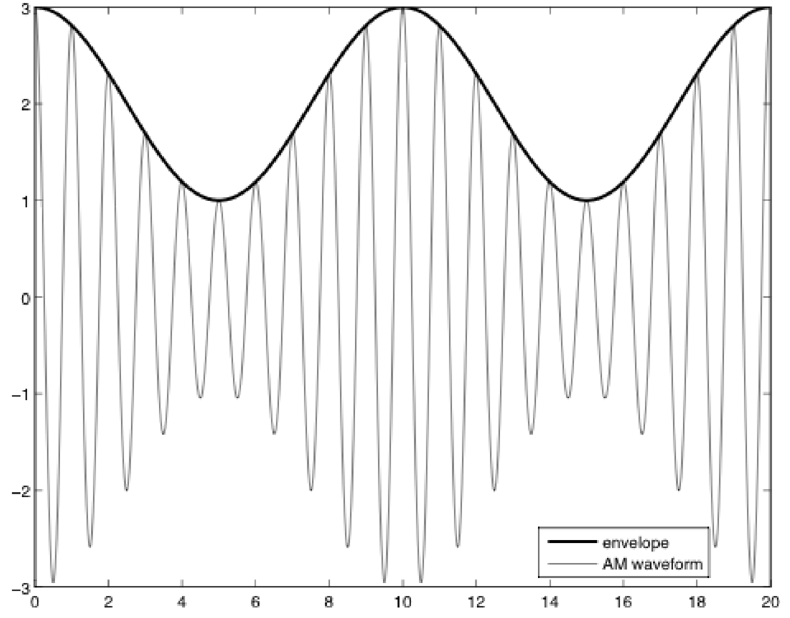
What does the envelope tell us?

- Example: sinusoidal message signal

$$m(t) = A_m \cos(2\pi f_m t)$$



DSB-SC signal
Envelope = message magnitude
$\rightarrow$ Envelope detection loses info in message sign.



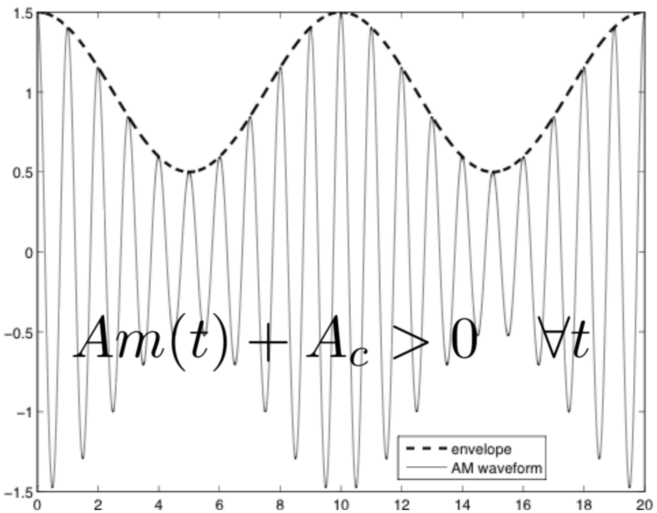
DSB + strong carrier component
Envelope = message + DC
$\rightarrow$ Envelope detector + DC block recovers message info

Sidestepping sync requirement

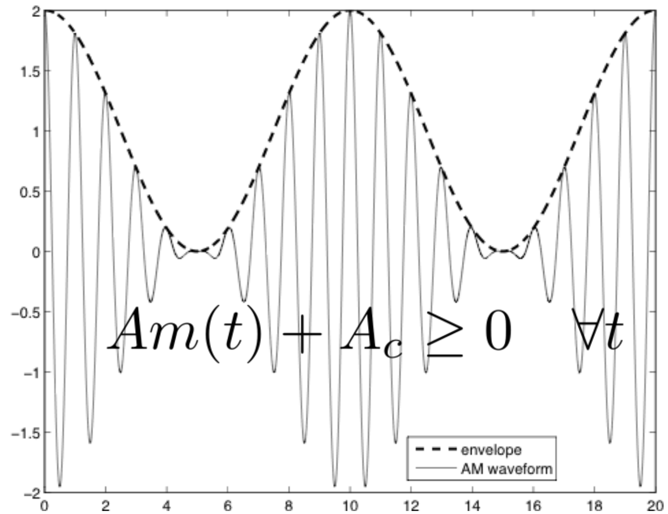
- The envelope (or magnitude of complex envelope) does not depend on carrier phase
- Suppose we can extract the envelope of a passband signal (will soon see a simple circuit for this purpose)
  - Does not require carrier sync
- Can we recover the message?

Constraint for recovering message from envelope

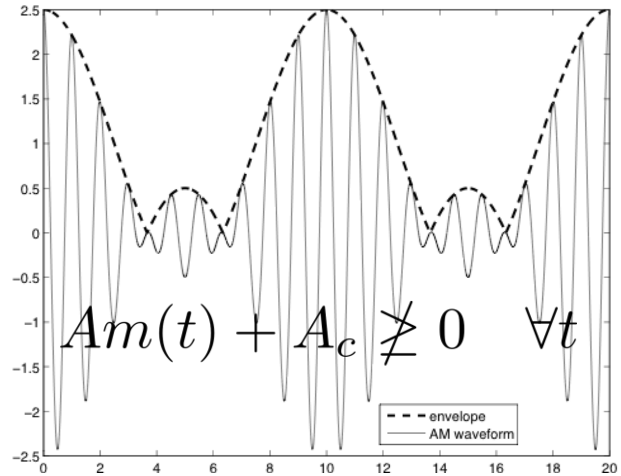
Example of sinusoidal message



Envelope = message + DC

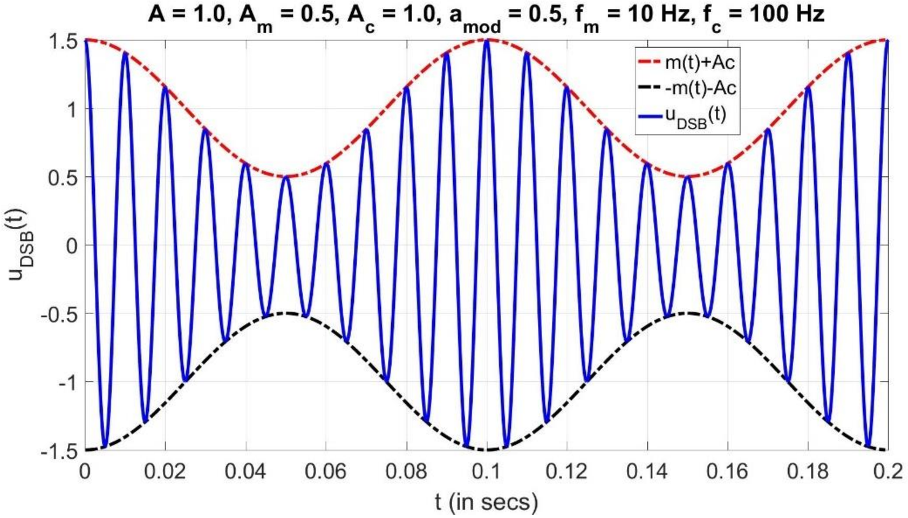


Envelope = message



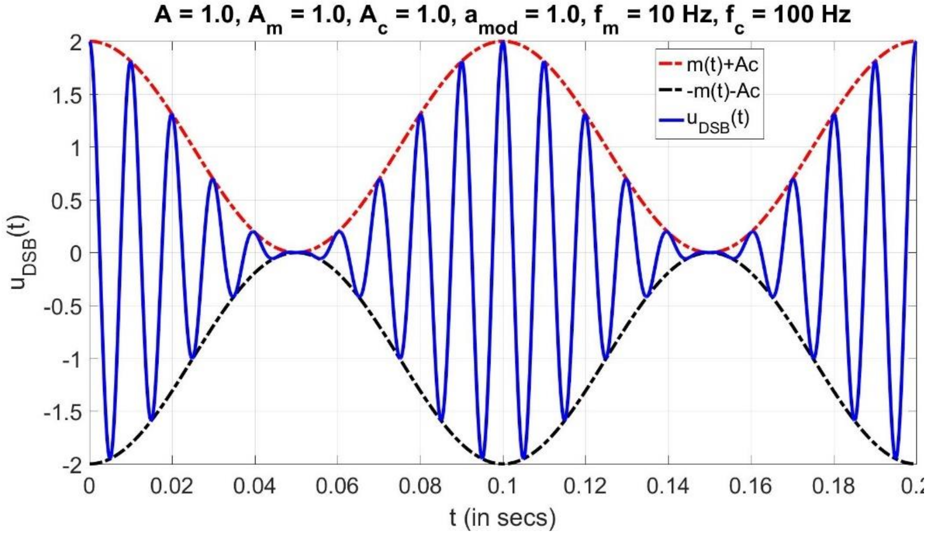
Message info not preserved in envelope

Example of Sinusoidal Message



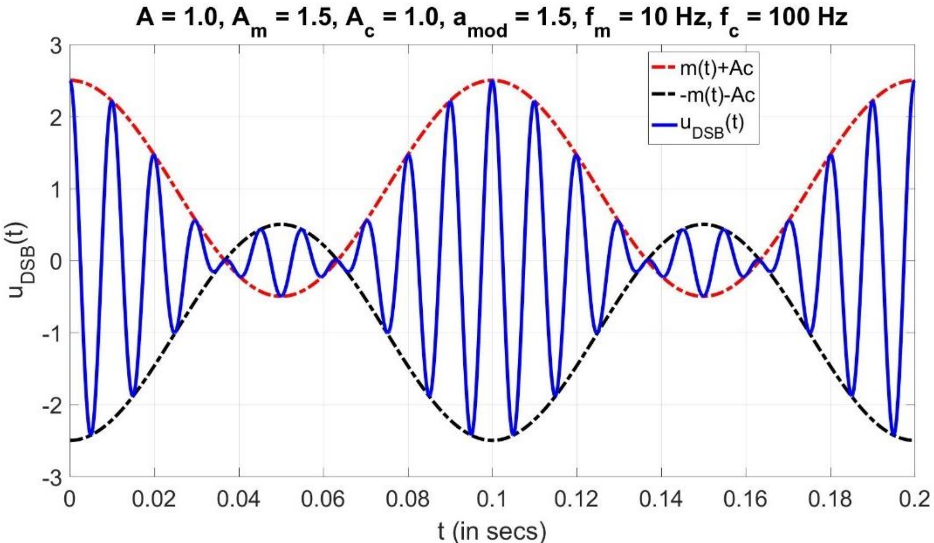
$$Am(t) + A_c > 0 \quad \forall t$$

Envelope = message + DC



$$Am(t) + A_c \geq 0 \quad \forall t$$

Envelope = message



Message info not preserved in envelope

$$Am(t) + A_c \ngeq 0 \quad \forall t$$

Modulation Index

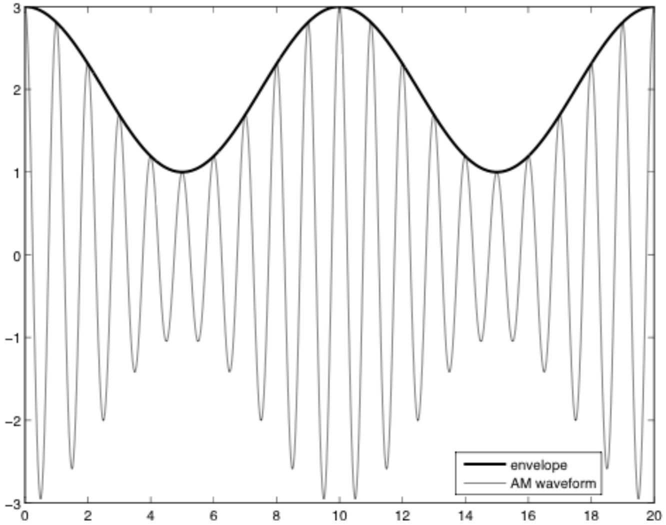
- Condition needed for envelope to preserve message info

$$A\ m(t) + A_c > 0 \quad \forall t$$
$$A \min_t m(t) + A_c > 0$$

- Can be expressed in terms of modulation index

$$a_{\mathrm{mod}} = \frac{AM_0}{A_c} = \frac{A|\min_t m(t)|}{A_c}$$

- For signal to be recoverable, $a_{\mathrm{mod}} \leq 1$.



AM signal in terms of modulation index

- Convenient to normalize message so that the largest negative swing is -1

$$m_n(t) = \frac{m(t)}{M_0} = \frac{m(t)}{|\min_t m(t)|}$$

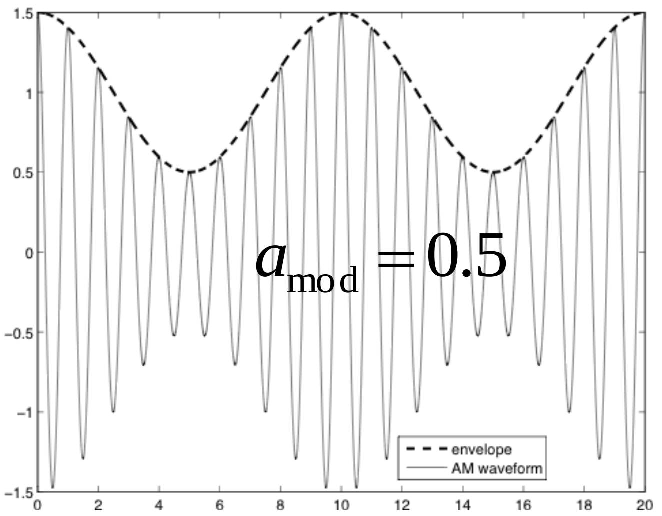
$$\min_t m_n(t) = \frac{\min_t m(t)}{M_0} = -1$$

- AM signal in terms of modulation index and normalized message

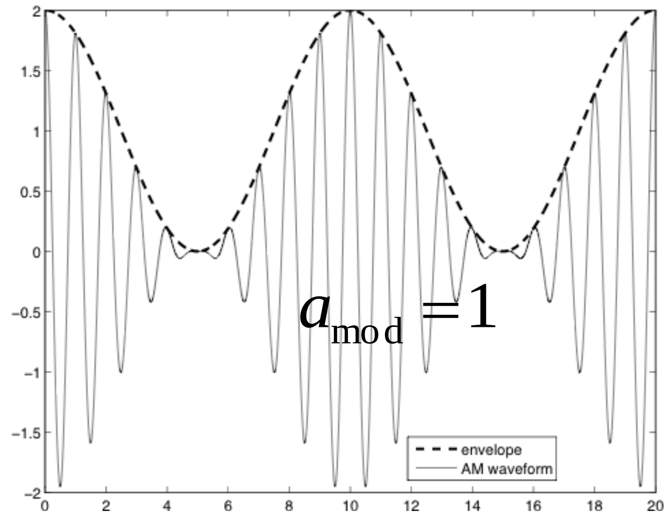
$$y_p(t) = B(1 + a_{\text{mod}} m_n(t)) \cos(2\pi f_c t + \theta_r)$$

Effect of modulation index

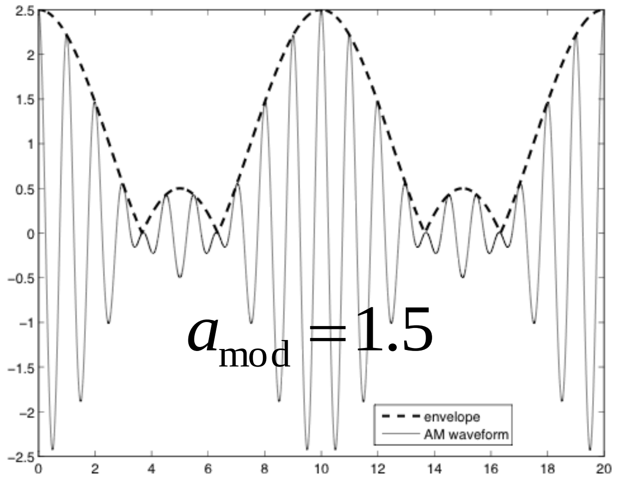
Example of sinusoidal message



Envelope = message + DC

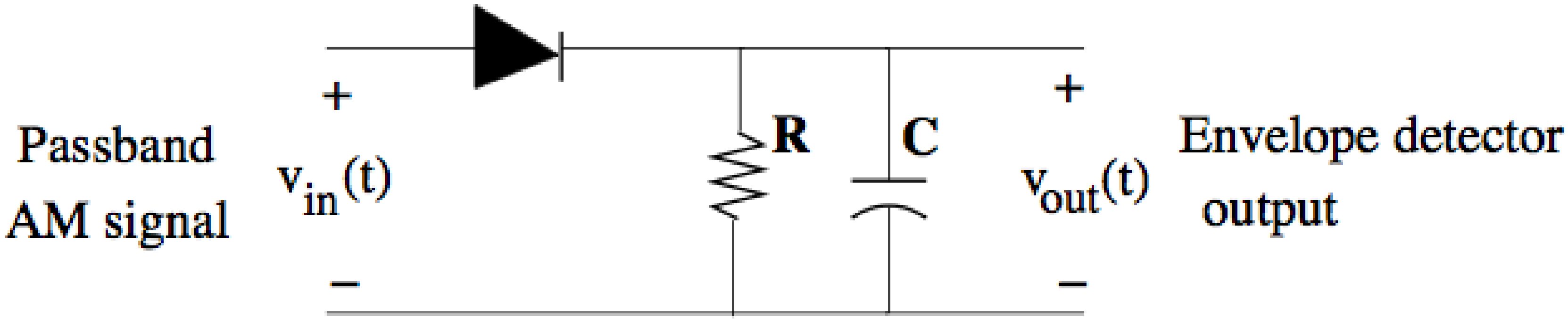


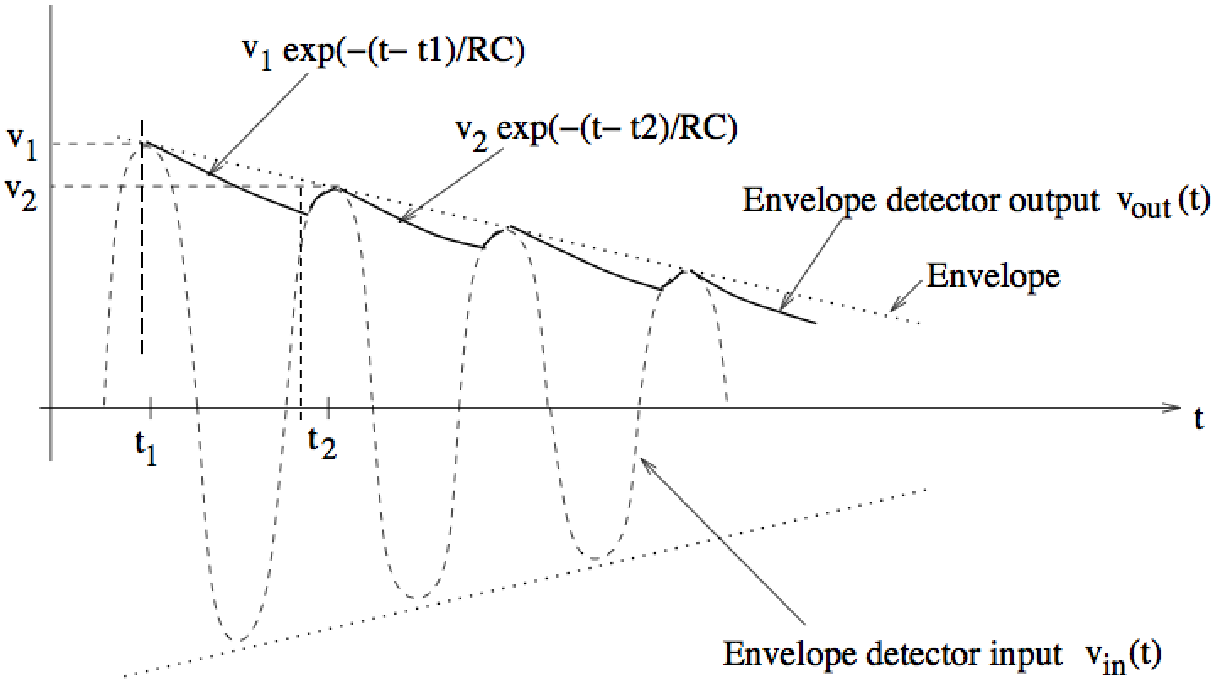
Envelope = message



Message info not preserved in envelope

Envelope Detectors

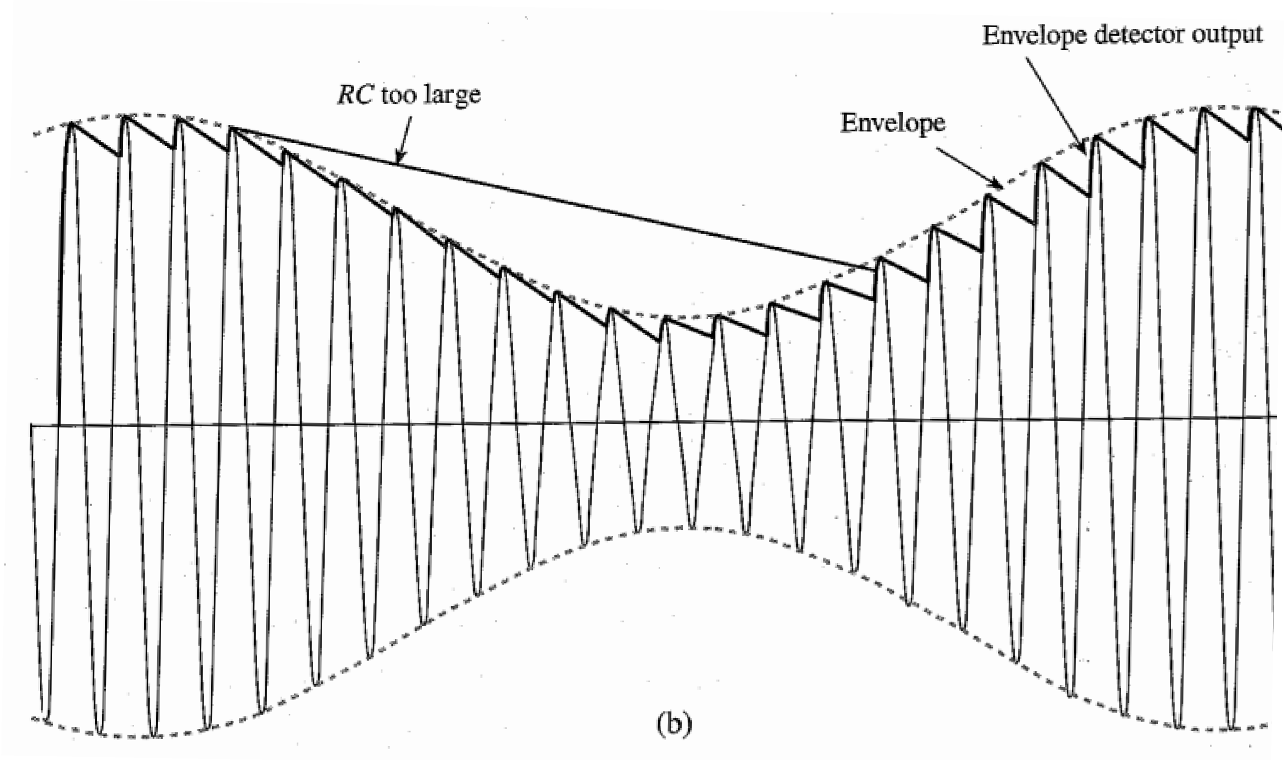




- At $t = t_1$, diode reverse biases and capacitor starts discharging.
- When $t = t_2'$, diode forward biases and capacitor starts charging.

Positive carrier cycle ➔ capacitor charges up (reaches value of envelope)
Negative carrier cycle ➔ capacitor discharges with RC time constant

Envelope detector operation



Positive carrier cycle ➔ capacitor charges up (reaches value of envelope)
Negative carrier cycle ➔ capacitor discharges with RC time constant

Should not discharge too fast during negative cycle
Should react fast enough to follow variations in envelope (which depend on message bandwidth B)

$$\frac{1}{f_c} \ll RC \ll \frac{1}{B}$$

Todays' Class

References

- Chap. 3 (Madhow)
- BP Lathi

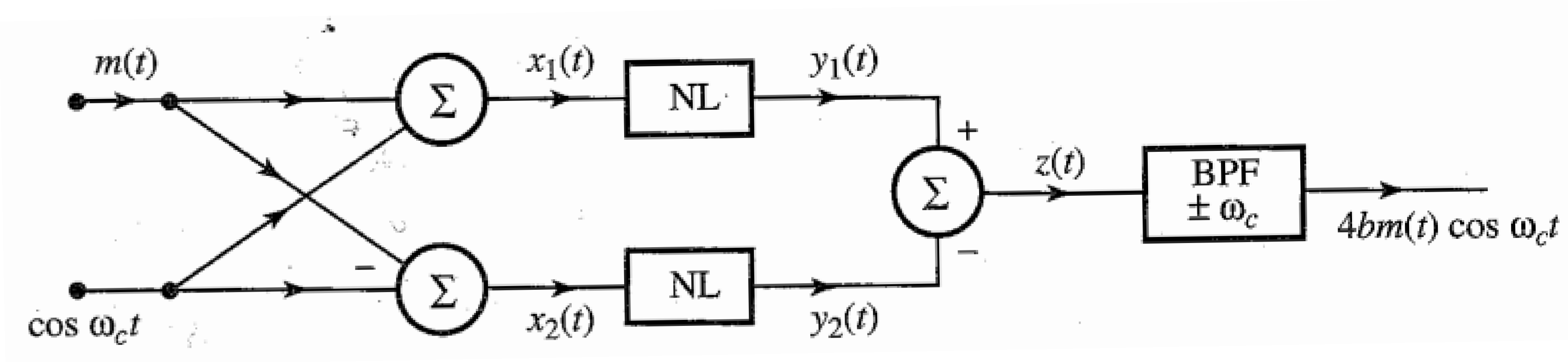
Conventional AM Modulators

B.P.Lathi pages: 155-159

How do we do conventional AM modulation?

- Use of multiplier
  - Several ways: Analog multiplier such as Sheingold, Variable gain amplifier, etc
  - It is rather difficult to maintain linearity in this kind of amplifier
  - They are expensive
- Few of other simple yet practical methods
  - Non-linear modulators
  - Switching modulators

Non-linear Modulators

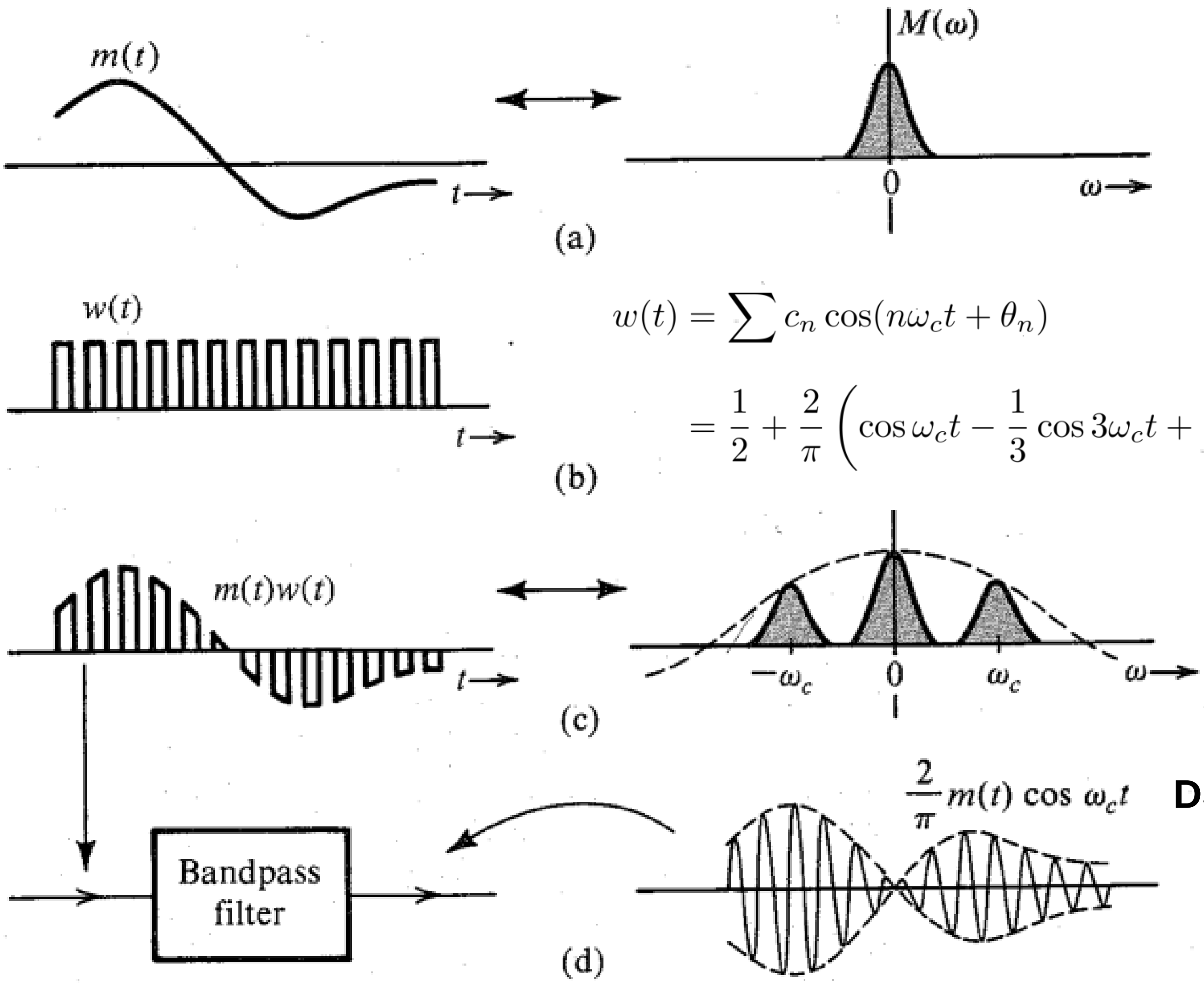


- Assuming the input-output characteristics of the nonlinear (NL) elements be approximated by a power series

$$y(t) = ax(t) + bx^2(t)$$

show that the output of the above ciccuit is $4bm(t)\cos\omega_c t$.

Switching Modulators

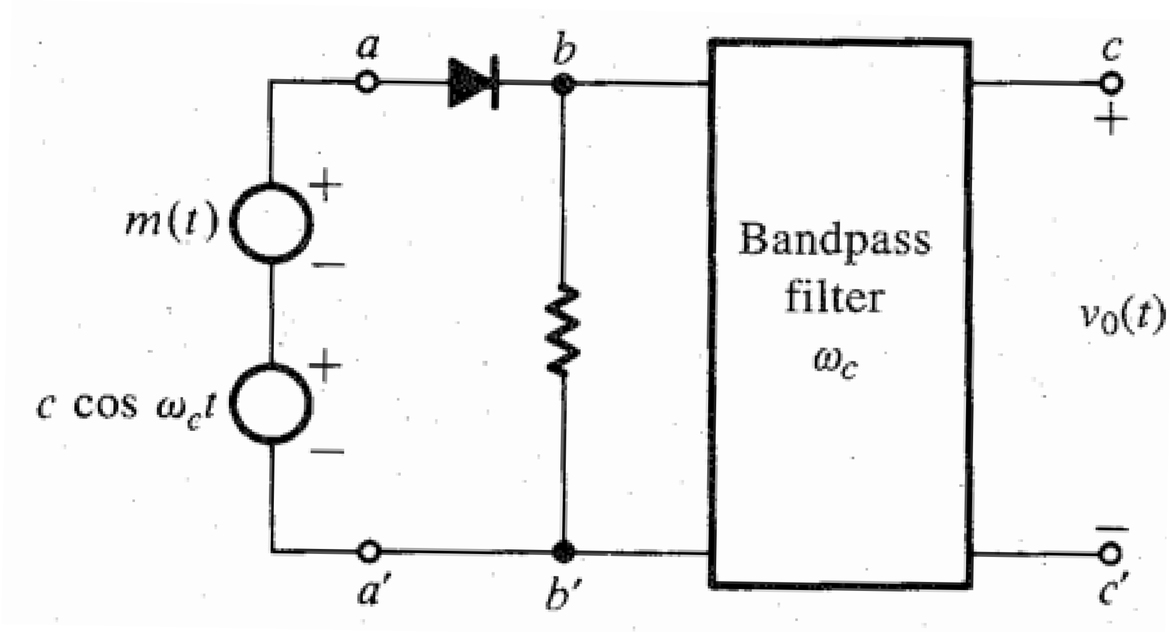


$$w(t) = \sum c_n \cos(n\omega_c t + \theta_n)$$

$$= \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \ldots\right)$$

DSB-SC

Switching Modulators



Conventional DSB

$$w(t) = \sum c_n \cos(n\omega_c t + \theta_n) \;=\; \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \dots\right)$$

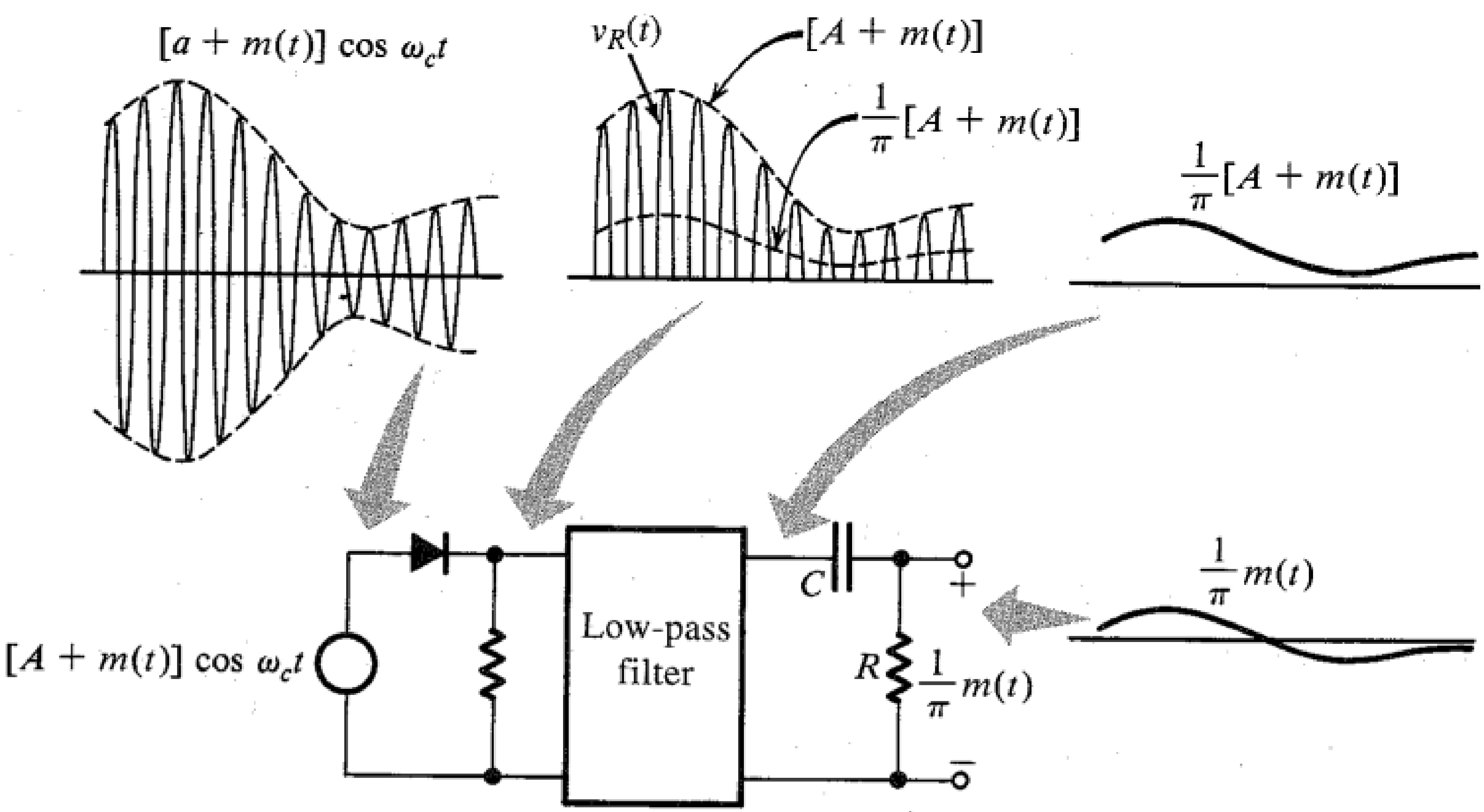
$$\begin{aligned} v_{bb'} &= (c\cos\omega_c t + m(t))w(t) \\ &= (c\cos\omega_c t + m(t))\{\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \dots\right)\} \\ &= \frac{c}{2}\cos\omega_c t + \frac{2}{\pi}m(t)\cos\omega_c(t) + \boxed{\text{Other terms}} \end{aligned}$$

Suppressed by BPF

Conventional AM Demodulators

B.P.Lathi pages: 166-169

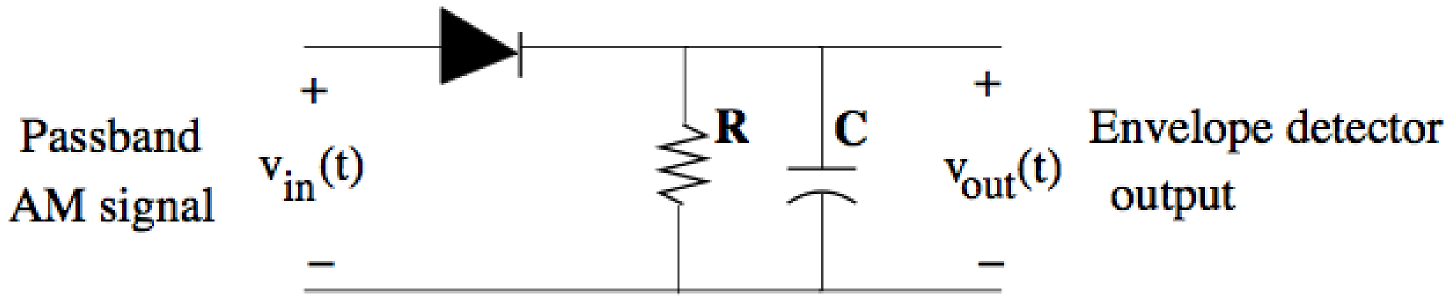
Rectifier Detector

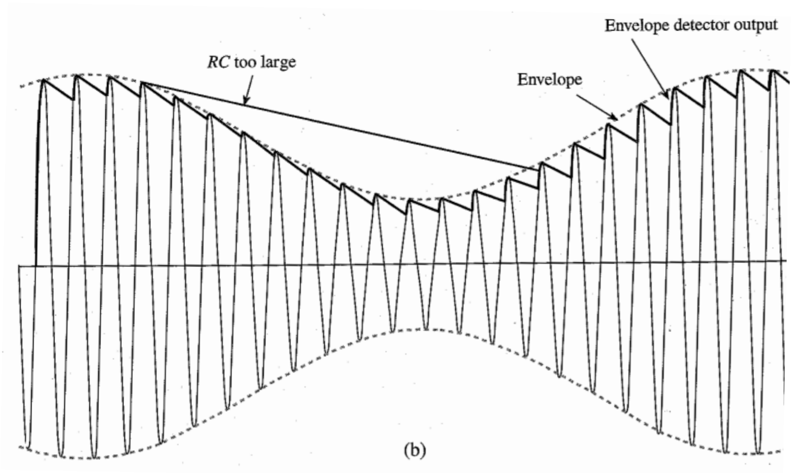


Lathi

$$
\begin{aligned}
v_{bb'} &= \{(A + m(t))\cos\omega_c t\}w(t) \\
&= \{(A + m(t))\cos\omega_c t\}\{\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \ldots\right)\} \\
&= \frac{1}{\pi}[A + m(t)] + \text{Other terms of higher frequencies}
\end{aligned}
$$

Envelope detector operation





$$\frac{1}{f_c} \ll RC \ll \frac{1}{B}$$

Madhow

Positive carrier cycle → capacitor charges up (reaches value of envelope)
Negative carrier cycle → capacitor discharges with RC time constant
Should not discharge too fast during negative cycle
Should react fast enough to follow variations in envelope (which depend on message bandwidth B)

Power efficiency of conventional AM

Power efficiency of conventional AM

- DSB expression

$$u_{\mathrm{AM}}(t) = Am(t)\cos(2\pi f_c t) + \boxed{A_c \cos(2\pi f_c t)}$$

Extra Non-information carrying component

- Power efficiency is given by

$$\eta = \frac{\text{Power in information carrying signal}}{\text{Power in total signal}}$$

- Prove that power efficiency for conventional AM is given by

$$\eta_{\mathrm{AM}} = \frac{a_{\mathrm{mod}}^2 \overline{m_n^2}}{1 + a_{\mathrm{mod}}^2 \overline{m_n^2}}$$

- Further prove that

$$\eta_{\mathrm{AM}} \leq 50\%$$

- Solve: Find η_{AM} for sinusoidal message signal $m(t) = A_m \cos(2\pi f_m t)$

Comments on Conventional AM

- Conventional AM trades-off synchoronous requirement with power efficiency.
- Suitable for broadcasting application
- Note that coherent detector is also possible for this!

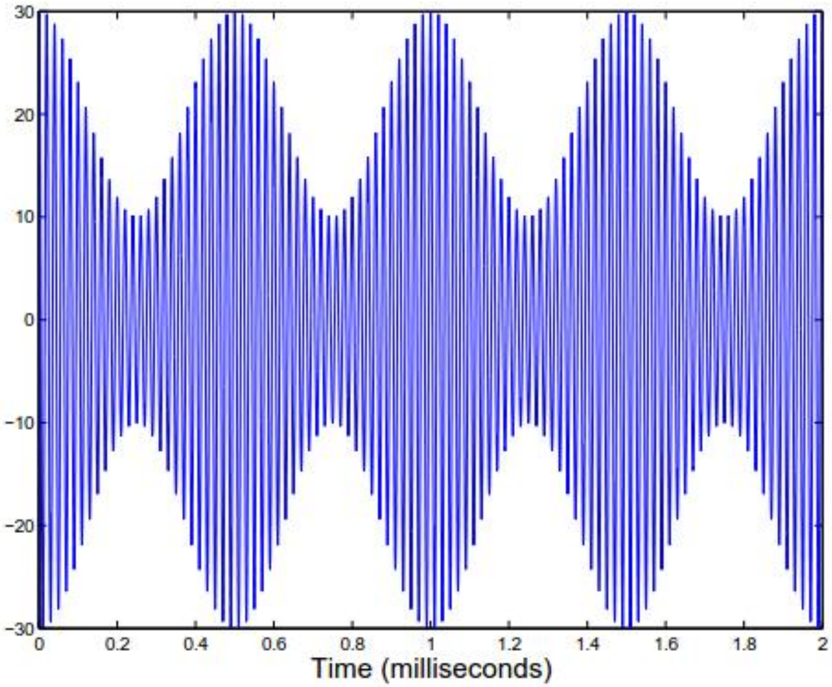
Example on power efficiency computation

The message $m(t) = 2\sin(2000\pi t) - 3\cos(4000\pi t)$ is used in AM system with a modulation index of 70% and carrier frequency of 580 KHz.

- What is the power efficiency?
- If the net transmitted power is 10 W, find magnitude spectrum of the transmitted signal.

Tutorial

Example: Tutorial!



$$u_{\mathrm{AM}}(t) = (A_c + m(t))\cos 2\pi f_c t$$

Fig. above shows a signal obtained after amplitude modulation by a sinusoidal message. The carrier frequency is difficult to determine from the figure and is not required for answering following questions

- Find the modulation index
- Find the signal power
- Find the bandwidth of the AM signal

Questions?