Communication Theory

Assignment - 3

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① Given $p(t) = I_{[-1/2, 1/2]}(t)$

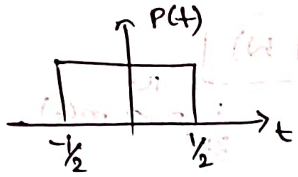
$$m(t) = \sum_{n=-\infty}^{\infty} (-1)^n p(t-n)$$

$$u(t) = 20\cos(2\pi f_c t + \phi(t))$$ where

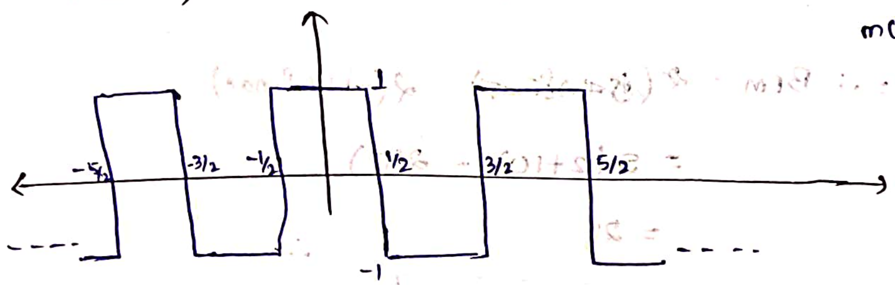
$$\phi(t) = 20\pi \int_{-\infty}^{t} m(\tau)\,d\tau + a$$

⇒ a choosen s.t $\phi(0) = 0$. #

(a) $p(t) \rightarrow$ sketch:



Sketch the m(t) ⇒



$m(t) \rightarrow$ periodic

Given $\phi(0) = 0$

$$\therefore \phi(0) = 0 = 20\pi \int_{-\infty}^{0} m(\tau)\,d\tau + a.$$

$$\therefore a = -20\pi \int_{-\infty}^{0} m(\tau)\,d\tau$$

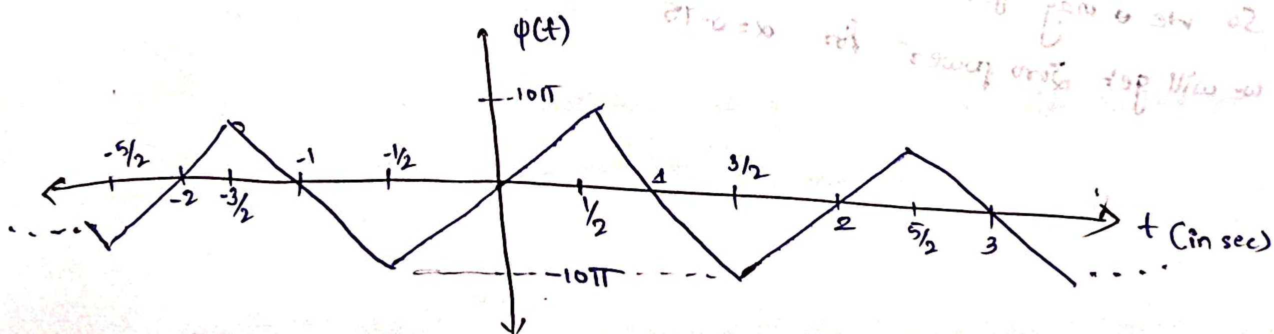
$$= -20\pi \left[\cancel{\int_{-\infty}^{-1/2} m(\tau)\,d\tau}^{\,0} + \int_{-1/2}^{0} m(\tau)\,d\tau \right]$$

$$\therefore a = -20\pi \int_{-1/2}^{0} (1)\,d\tau$$

$$= -20\pi [\tau]_{-1/2}^{0} = -\overset{10}{\cancel{20}}\pi \left[\frac{1}{\cancel{2}}\right]$$

$$\boxed{\therefore a = -10\pi}$$

Sketch of $\phi(t)$:



(b) We can observe from part (a) sketch that $f(t) = \frac{1}{2\pi}\frac{d\phi}{dt}$ takes value -10 and +10, so $\Delta f_{max} = 10$.

$$f(t) = \frac{1}{2\pi}\frac{d\phi}{dt}$$

$\Rightarrow$ Method:

$$\max(f(t)) = \max\left(\frac{1}{2\pi}\frac{d\phi}{dt}\right)$$

$$= \frac{\max\left(\frac{d\phi}{dt}\right)}{2\pi} = \frac{1}{2\pi}\cdot\max\left(\frac{d\phi}{dt}\right)$$

$$= \frac{20\pi}{2\pi} = 10$$

$\therefore$ Deviation of $f(t)$ at max by 10.#

$$\uparrow 10+f \quad -f \quad \downarrow 10f-10$$

$\therefore$ using Carson's formulae, we get the Estimation of Bandwidth,

$$\boxed{\text{Bandwidth} = 2(\Delta f_{max} + W)}$$

$$\Delta f_{max} = \max|f(t)| = \frac{1}{2\pi}\,20\pi\, m(t)$$

$$= \max|10(m(t))| = 10$$

$$\boxed{W = 2 \quad \hookrightarrow \text{given}}$$

Bandwidth of $m(t)$

Bandwidth of $u(t)$

$$\therefore B_{FM} = \cancel{2(B+\Delta f_{max})}\ 2(W+\Delta f_{max})$$

$$= 2(2+10) = 2(12)$$

$$= 24$$

$$\boxed{\therefore \text{Bandwidth of } u(t) = 24}$$

(c)

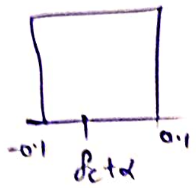
The message $m(t)$ is periodic with 2 and hence $\phi(t)$ and complex envelope $e^{j\phi(t)}$.

$m(t) \Rightarrow$ Periodic with period "2".

Fundamental frequency $= \frac{1}{2}$ Hz $= f_0$

We expect to find Fourier series components for complex envelope at nf_m, where $f_m = \frac{1}{2}$ is fundamental frequency and 'n' takes integer values, so that the spectrum of passband signal has discrete components $f_c + nf_m$.

$\Rightarrow$ So we may get non-zero power at $f_c + \alpha$ # for $\alpha = 0.5, 1$ but we will get zero power for $\alpha = 0.75$



$$f_c+\alpha-0.1 < f_c+nf_0 < f_c+\alpha+0.1$$

$$\alpha-0.1 < nf_0 < \alpha+0.1$$

$$\alpha \approx nf_0 \qquad \alpha \approx n(\tfrac{1}{2})$$

$m(t) \rightarrow$ period $= 2$

$\hookrightarrow f_m = \frac{1}{2}$ Hz

So, strong components will be $f_c,\ f_c \pm \frac{1}{2},\ f_c \pm 1,\ f_c \pm \frac{3}{2}$

$\Rightarrow \alpha = 0.5$ & $\alpha = 1$ $\rightarrow$ both will have non-zero power

$\alpha = 0.75 \Rightarrow$ zero power

So, If $\alpha = 0.5 \rightarrow$ then $f_c \pm \frac{1}{2}$ Component is present after BPF similarly for $\alpha = 1$ $\boxed{f_c \pm 1}$ $\rightarrow$ component is present. So there will be some non-zero power but for $\alpha = 0.75$ there is no component present in freq domain at $f_c \pm 0.75$ so there is zero power present here.

(2)

(a) $\beta = \dfrac{\Delta f_{max}}{B}$ $\qquad \theta(t) = \beta \sin(2\pi f_m t)$

$\pi^c = 180°$ $\qquad 100° < \beta < 600°$

$1° = \left(\dfrac{\pi}{180}\right)^c$ $\qquad$ & $\beta = n\pi$

$$\frac{400}{180}\pi < n\pi < \frac{600}{180}\pi$$

$$2.2 < n < 3.3$$

$$\boxed{\therefore n = 3}$$

since $\beta = n\pi$

$$\boxed{\beta = 3\pi}$$

for a sinusoidal message at frequency f_m, we have seen the phase deviation of FM signal is given by $\beta \sin(2\pi f_m t)$ & $\beta = \dfrac{\Delta f_{max}}{B}$ is modulation Index.

From Figure We can observe max deviation between $400 < \beta < 600$ so as β is Integer multiple of π so we set $\boxed{\beta = 3\pi}$ radians.

(b) $m(t) = \frac{1}{2\pi}\frac{d\theta}{dt}$

From figure we can observe timeperiod $(T) = 0.2ms$
$= 0.2\times10^{-3} sec$

So, $f_m = \frac{1}{T}$

$= \frac{1}{0.2\times10^{-3}} = \frac{10}{2}\times10^{3} = 5\times10^{3}$ Hz

$= 5KHz$

So, Bandwidth $(B) = 5KHz$.#

(single sided Bandwidth)

(c) $B_{FM} = 2(\Delta f_{max} + B)$

$= 2(\beta+1)f_m$

as $\beta = 3\pi$ and $f_m = 5KHz$

$B_{FM} = 2(3\pi+1)5\times10^{3}$

$\Rightarrow \boxed{B_{FM} \cong 104 KHz}$

$\boxed{B_{FM} = 104.247 KHz}$

(4)

(a)

$$M(f) = \begin{cases} j2\pi f & ; |f|<1 \\ 0 & ; \text{otherwise} \end{cases}$$

$$\phi_m(t) = A\cos(2\pi f_c t + \phi(t))$$

$$m(t) = \frac{1}{2\pi}\frac{d}{dr}(\phi(t))$$

$$\phi(t) = 2\pi \int_{-\infty}^{t} m(\tau)\,d\tau$$

$$m(t) \longrightarrow \boxed{\int \cdot H(t)} \longrightarrow \phi(t)$$

$$\Rightarrow 2\pi\,(M(f)\cdot H(f)) = \Phi(f)$$

$\rightsquigarrow$ $H(f)$ is the impulse response of integration

$$\boxed{H(f) = \frac{1}{j2\pi f}}$$

$$\phi(t) = 2\pi k_f \int_{-\infty}^{\tau} m(\tau)\,d\tau$$

$$\Phi(f) = 2\pi k_f \left(\frac{1}{j2\pi f}\right) M(f)$$

$$= 2\pi k_f \left(\frac{1}{\cancel{j2\pi f}}\right)(\cancel{j2\pi f})\, I_{[-1,1]}(f)$$

$$= 2\pi k_f\, I_{[-1,1]}(f) \qquad -1<f<1$$

{assume ($k_f = 1$)}

$$\Phi(f) = 2\pi\, I_{[-1,1]}(f)$$

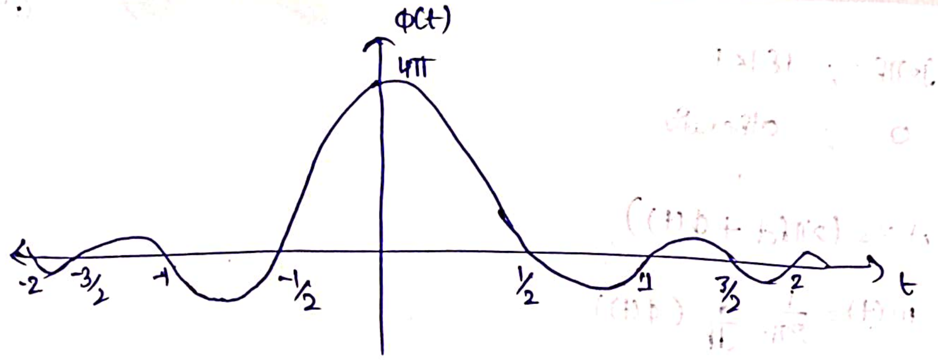
$$\Phi(f) \longleftrightarrow \phi(t) \qquad I_{[-a/2,a/2]}(f) \longleftrightarrow a\,\text{sinc}(at)$$

$$I_{[-T/2,T/2]} \longleftrightarrow T\,\text{sinc}(fT)$$

$$\therefore I_{[-1,1]}(f) \longleftrightarrow 2\,\text{sinc}(2t)$$

$$,\ \Phi(f) = 2\pi I_{[-1,1]}(f) \longleftrightarrow \phi(t) = 2\pi(2)\,\text{sinc}(2t)$$

$$\boxed{\therefore \phi(t) = 4\pi\,\text{sinc}(2t)}$$



(b) $\phi(t) = 4\pi\,\text{sinc}(2t)$

$$\phi_{FM}(t) = A\cos(2\pi f_c t + 4\pi\,\text{sinc}(2t))$$

Here at $t \to \infty$

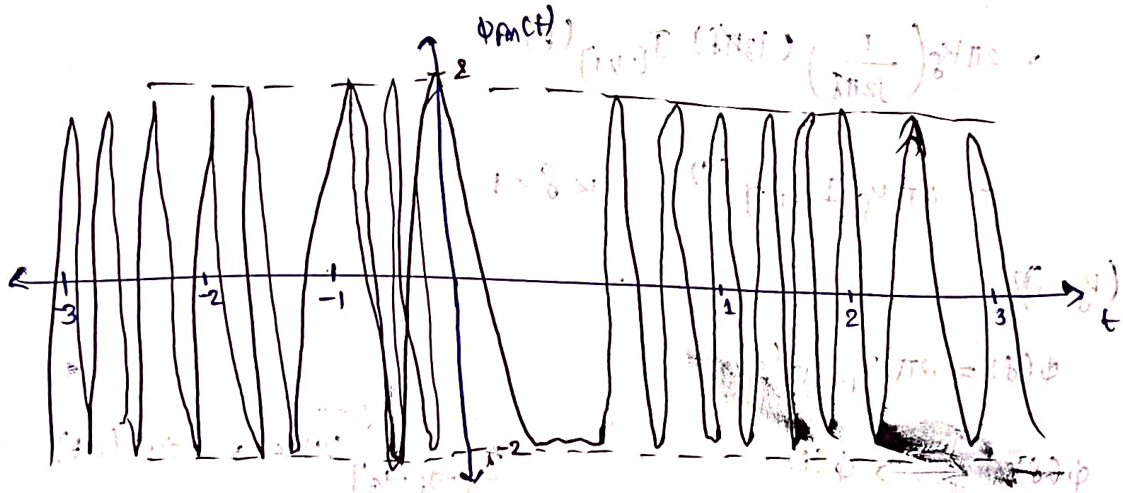
$$\text{sinc}(2t) \to 0$$

$$\boxed{\phi_{FM}(t) = A\cos(2\pi f_c t)}$$

at $t \to 0$

$$\phi_{FM}(t) = A\cos(2\pi f_c t + 4\pi)$$

$$\boxed{\phi_{FM}(t) = A\cos(2\pi f_c t)}$$



(C) The instantaneous frequency deviation is given by

$$\Delta f(t) = k_f m(t)$$

$$= \frac{1}{2\pi}\frac{d(\phi(t))}{dt} = 2\frac{d}{dt}\,\text{sinc}(2t)$$

$$= \cancel{2}\frac{d}{dt}\left[\frac{\sin(2\pi t)}{\cancel{2}\pi t}\right]$$

$$= \frac{1}{\pi}\frac{d}{dt}\left(\frac{\sin(2\pi t)}{t}\right)$$

$$= \frac{1}{\pi}\,\frac{\cos(2\pi t)(2\pi)t - \sin(2\pi t)}{t^2}$$

$$= \frac{1}{\pi}\left[\frac{\cos(2\pi t)(2\pi t)}{t^2} - \frac{\sin(2\pi t)}{\cancel{4}t^2}\times\frac{2}{2}\right]$$

$$= \frac{1}{\cancel{\pi}}\,\frac{\cos(2\pi t)(2\cancel{\pi})}{t} - \frac{\sin(2\pi t)}{\pi t^2}\times\frac{2}{2}$$

$$= \frac{2\cos(2\pi t)}{t} - \frac{2\,\text{sinc}(2\pi t)}{t}$$

At $t = 1/4$

$$|\Delta f(t = 1/4)| = \left|2\cos\left(\cancel{2}\frac{\pi}{\cancel{4}_2}\right)\times 4 - 2\,\text{sinc}\left(\cancel{2}\frac{\pi}{\cancel{4}_2}\right)\times 4\right|$$

$\because \sin(\pi/2) = 1$

$$= \left|-2\,\frac{\sin(\pi/2)}{\pi/2}\times 4\right|$$

$$= \left|-\frac{16}{\pi}\sin(\pi/2)\right|$$

$$= \left|\frac{16}{\pi}\times 1\right| = \frac{16}{\pi} = 5.09\ Hz$$

$$|\Delta f(t = 1/4)| = 5.09\ Hz$$

(d) · ~~B~~ · $B \rightarrow$ is the Bandwidth of $m(t)$.

so as $M(f) = j2\pi f \quad |f| < 1$

so $\boxed{B = 1}$

$B_{FM} = 2(B + \Delta f_{max}) \longrightarrow$ carson's formula

$\Delta f_{max} = 5.4812$ [From Desmos graph]

$$= 2(1 + 5.4812)$$

$$= 2(6.482)$$

$$= 12.9624$$

$$\boxed{\therefore B_{FM} = 12.9624}$$

③ $u_p(t) = 10\cos(f_c + 5\sin(3000t) + 10\sin(2000\pi t))$

(a) Here we need to find power

$$\overline{u_p^2(t)} = \overline{10^2 \cos^2(f_c + 5\sin(3000t) + 10\sin(2000\pi t))}$$

$$= 100\left[\frac{1 + \overline{\cos(2f_c + 10\sin(3000t) + 20\sin(2000\pi t))}}{2}\right]$$

$$= 100\left(\tfrac{1}{2}\right) + \cancel{100(0)} \to 0$$

$$\boxed{\overline{u_p^2(t)} = 50}$$

(b)

$$f_i(t) = \frac{1}{2\pi}\frac{d\phi}{dt} = \frac{1}{2\pi}\frac{d}{dt}\left[5\sin(3000t) + 10\sin(2000\pi t)\right]$$

$$= \frac{1}{2\pi}\left[5(3000)\cos(3000t) + 10(2000\pi)\cos(2000\pi t)\right]$$

$$= \frac{\overset{7500}{\cancel{15000}}\cos(3000t)}{\cancel{2}\pi} + \frac{\overset{10000}{\cancel{20000}}\cos(2000\pi t)}{\cancel{2}\pi}$$

$$= \frac{7500}{\pi}\cos(3000t) + 10000\cos(2000\pi t)$$

$$\max(f_i(t)) = \max\left(\frac{7500}{\pi}\cos(3000t) + 10000\cos(2000\pi t)\right)$$

$\Rightarrow$ This is when $t=0$ We will get the max value at $t=0$

$$\Delta f_{max} = \frac{7500}{\pi} + 10000$$

$$= 2387.32 + 10000$$

$$\boxed{\Delta f_{max} = 12{,}387.32 \text{ Hz}}$$

$$= 12.38732 \text{ KHz}$$

(c) Modulation index (β) :

$$\beta = \frac{\Delta f_{max}}{B}$$

$$\beta = \frac{\frac{7500}{\pi} + 10000}{B}$$

We need to find Bandwidth (B) of msg signal. The message signal Bandwidth will be same as $5\sin(3000t) + 10\sin(2000\pi t)$

So Bandwidth $= \max\left(\frac{3000}{2\pi}, \frac{2000\pi}{2\pi}\right)$

$= \max(477.46, 1000)$

$$\boxed{B = 1000}$$

$\Rightarrow \quad \beta = \frac{\frac{7500}{\pi} + 10000}{1000} \quad \Rightarrow \quad \frac{7.5}{\pi} + 10$

$\beta = 2.387 + 10 \quad \Rightarrow \boxed{\beta = 12.387}$

(d)

Bandwidth of $\varphi(t)$ $\quad B_{PM} = 2(\Delta f_{max} + B)$

$= 2(1000 + 12387.324)$

$= 2(13387.324)$

$\boxed{B_{FM} = 26774.648}$ Hz