Radio Frequency Based Sensors design: Principles and Applications

Assignment 1

1. $\mathbf{E}$ and $\mathbf{F}$ are vector fields given by:

$$\mathbf{E} = 2x\mathbf{a_x} + \mathbf{a_y} + yz\mathbf{a_z}, \quad \mathbf{F} = xy\mathbf{a_x} - y^2\mathbf{a_y} + xyz\mathbf{a_z}.$$

   Determine:

   (a) $|\mathbf{E}|$ at $(1, 2, 3)$.

   (b) The component of $\mathbf{E}$ along $\mathbf{F}$ at $(1, 2, 3)$.

2. Let $\mathbf{G} = yz\mathbf{a_x} + xz\mathbf{a_y} + xy\mathbf{a_z}$. Transform $\mathbf{G}$ to cylindrical coordinates.

3. Calculate the distance between points $P(4, 30°, 0°)$ and $Q(6, 90°, 180°)$.

4. Calculate the area of the surface defined by $r = 5$, $0 < \theta < \frac{\pi}{4}$, $0 < \phi < \frac{\pi}{2}$.

5. Let $\mathbf{A} = y\mathbf{a_x} + z\mathbf{a_y} + x\mathbf{a_z}$. Find the flux of $\mathbf{A}$ through the surface $y = 1$, $0 < x < 1$, $0 < z < 2$.

6. Calculate the gradient of:

   (a) $V_1 = 6xy - 2xz + z$

   (b) $V_2 = 10\rho \cos\phi - \rho z$

   (c) $V_3 = \frac{2}{r} \cos\phi$

7. Evaluate the divergence of the following vector field:

$$\mathbf{A} = xy\mathbf{a_x} + y^2\mathbf{a_y} - xz\mathbf{a_z}.$$

8. Evaluate $\nabla \times \mathbf{A}$ and $\nabla \cdot (\nabla \times \mathbf{A})$ if:

$$\mathbf{A} = x^2y\mathbf{a_x} + y^2z\mathbf{a_y} - 2xz\mathbf{a_z}.$$

9. A point charge Q is located at $(a, 0, 0)$, while another charge $-Q$ is at $(-a, 0, 0)$. Find the electric field $\mathbf{E}$ at:

   (a) $(0, 0, 0)$

   (b) $(0, a, 0)$

   (c) $(a, 0, a)$

10. Let $\mathbf{D} = 2xy\mathbf{a_x} + x^2\mathbf{a_y}$ C/m^2 and find:

    (a) The volume charge density ρ_v.

    (b) The flux through the surface $0 < x < 1$, $0 < z < 1$, $y = 1$.

    (c) The total charge contained in the region $0 < x < 1$, $0 < y < 1$, $0 < z < 1$.

11. Let the current density be

$$\mathbf{J} = e^{-x} \cos 4y\mathbf{a_x} + e^{-x} \sin 4y\mathbf{a_y} \quad \text{A/m}^2.$$

    Determine the current crossing the surface $x = 2$, $0 < y < \frac{\pi}{3}$, $0 < z < 4$.

12. A 12 V voltage is applied across the ends of a silver wire of length 12.4 m and radius 0.84 mm. Determine the current through the wire.

13. A spherical shell has $r = 1.2$ cm and $r = 2.6$ cm as inner and outer radii, respectively. If

$$P = 4r\mathbf{a_r} \quad \text{pC/m}^2,$$

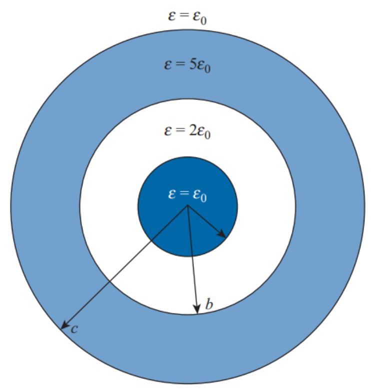
    determine:

    (a) The total bound surface charge on the inner surface.

    (b) The total bound surface charge on the outer surface.

(c) The total bound volume charge.

14. Concentric spheres $r = a$, $r = b$, and $r = c$ have charges 4 C, -6 C, and 10 C, respectively, placed on them. If the regions separating them are filled with different dielectrics as shown in the figure, find $\mathbf{E}$, $\mathbf{D}$, and $\mathbf{P}$ everywhere.



15. If

$$\mathbf{J} = \frac{100}{\rho^2}\mathbf{a}_\rho \quad \text{A/m}^2,$$

find:

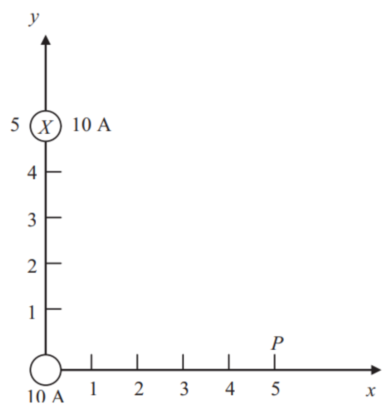
(a) The time rate of increase in the volume charge density.

(b) The total current passing through the surface defined by $\rho = 2$, $0 < z < 1$, $0 < \phi < 2\pi$.

16. (a) State Biot–Savart's law.

(b) The y- and z-axes, respectively, carry filamentary currents 10 A along $\mathbf{a_y}$ and 20 A along $-\mathbf{a_z}$. Find $\mathbf{H}$ at $(-3, 4, 5)$.

17. Two infinitely long wires, placed parallel to the z-axis, carry currents 10 A in opposite directions as shown in the figure. Find $\mathbf{H}$ at point P.



18. The $z = 0$ plane carries current $\mathbf{K} = 10\mathbf{a_x}$ A/m, while a current filament situated at $y = 0$, $z = 6$ carries current I along $\mathbf{a_x}$. Find I (Current) such that $\mathbf{H}(0, 0, 3) = 0$.

19. In a hydrogen atom, an electron revolves at velocity 2.2×10^6 m/s. Calculate the magnetic flux density at the center of the electron's orbit. Assume that the radius of the orbit is $R = 5.3 \times 10^{-11}$ m.

20. Find out which of the following can possibly represent an electrostatic or magnetostatic field in free space:

(a) $\mathbf{D} = y^2 z\mathbf{a_x} + 2(1 + x)yz\mathbf{a_y} - (x + 1)z^2\mathbf{a_z}$

(b) $\mathbf{E} = \left(\frac{z+1}{\rho}\right)\cos\phi\mathbf{a}_\rho + \left(\frac{\sin\phi}{\rho}\right)\mathbf{a_z}$

(c) $\mathbf{F} = \frac{1}{r^2}\left(2\cos\theta\mathbf{a_r} + \sin\theta\mathbf{a}_\theta\right)$

21. A parallel-plate capacitor with a plate area of 5 cm^2 and plate separation of 3 mm has a voltage $50\sin(10^3 t)$ V applied to its plates. Calculate the displacement current assuming $\varepsilon = 2\varepsilon_0$.

22. In free space, $\mathbf{E} = 20\cos(\omega t - 50x)\mathbf{a_y}$ V/m. Calculate:

(a) $\mathbf{J_d}$

(b) ω

23. A rectangular coil has a cross-sectional area of 30 cm^2 and 50 turns. If the coil rotates at 60 rad/s in a magnetic field of 0.2 Wb/m^2 such that its axis of rotation is perpendicular to the direction of the field, determine the induced emf in the coil.

24. In free space, $\mathbf{H} = 10\sin(10^8 t + \beta x)\mathbf{a_y}$ A/m. Find $\mathbf{E}$ and β.

25. In air, $\mathbf{E} = \cos(12\pi x)\sin(10^{11}t - \alpha y)\mathbf{a_z}$ V/m. Find $\mathbf{H}$ and α.

26. An EM wave propagating in a certain medium is described by:

$$\mathbf{E} = 25\sin(2\pi \times 10^6 t - 6x)\mathbf{a_z} \quad \text{V/m.}$$

Determine:

(a) The direction of wave propagation.

(b) Compute the period T, the wavelength λ, and the velocity u.

(c) Sketch the wave at $t = 0$, $T/8$, $T/4$, and $T/2$.

27. An EM wave in free space is described by:

$$\mathbf{H} = 0.4\cos(10^8 t + \beta y) \quad \text{A/m.}$$

Determine:

(a) The angular frequency ω.

(b) The wave number β.

(c) The wavelength λ.

(d) The direction of wave propagation.

(e) The value of $\mathbf{H}(2, 3, 4, 10 \text{ ns})$.

28. In a source-free region, $\mathbf{H} = H_0\cos(\omega t - \beta z)\mathbf{a_x}$ A/m. Find the displacement current density.

29. Assume that dry soil has $\sigma = 10^{-4}$ S/m, $\varepsilon = 3\varepsilon_0$, and $\mu = \mu_0$. Determine the frequency at which the ratio of the magnitudes of the conduction current density and the displacement current density is unity.

30. The electric field intensity of a spherical wave in free space is given by:

$$\mathbf{E} = \frac{10}{r}\sin\theta\cos(\omega t - \beta r)\mathbf{a}_\theta \quad \text{V/m.}$$

Find the corresponding magnetic field intensity $\mathbf{H}$.