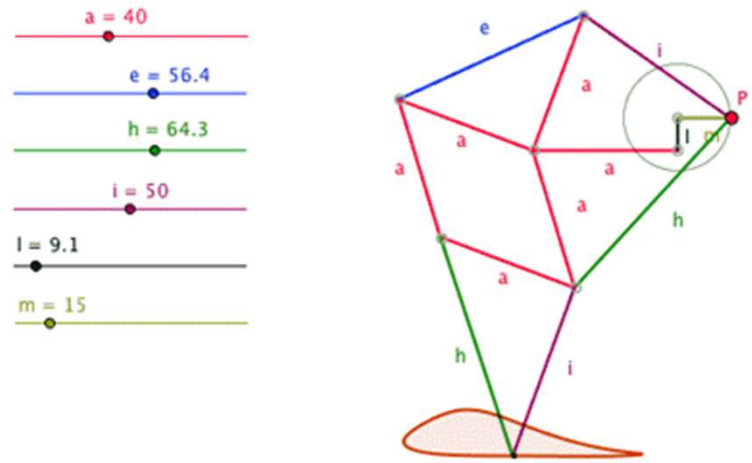
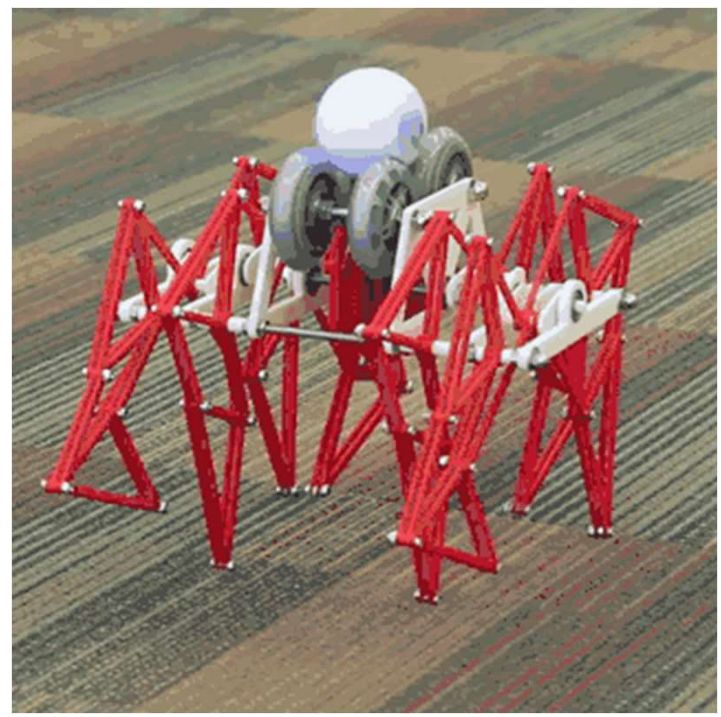
Mechanism design

- Mechanism design involves finding a mechanism that carries out a user-specified task.



- The process involves the selection of joint types and link dimensions.

- Example – Eight-bar **Theo-Jansen** linkage enables robotic walking.

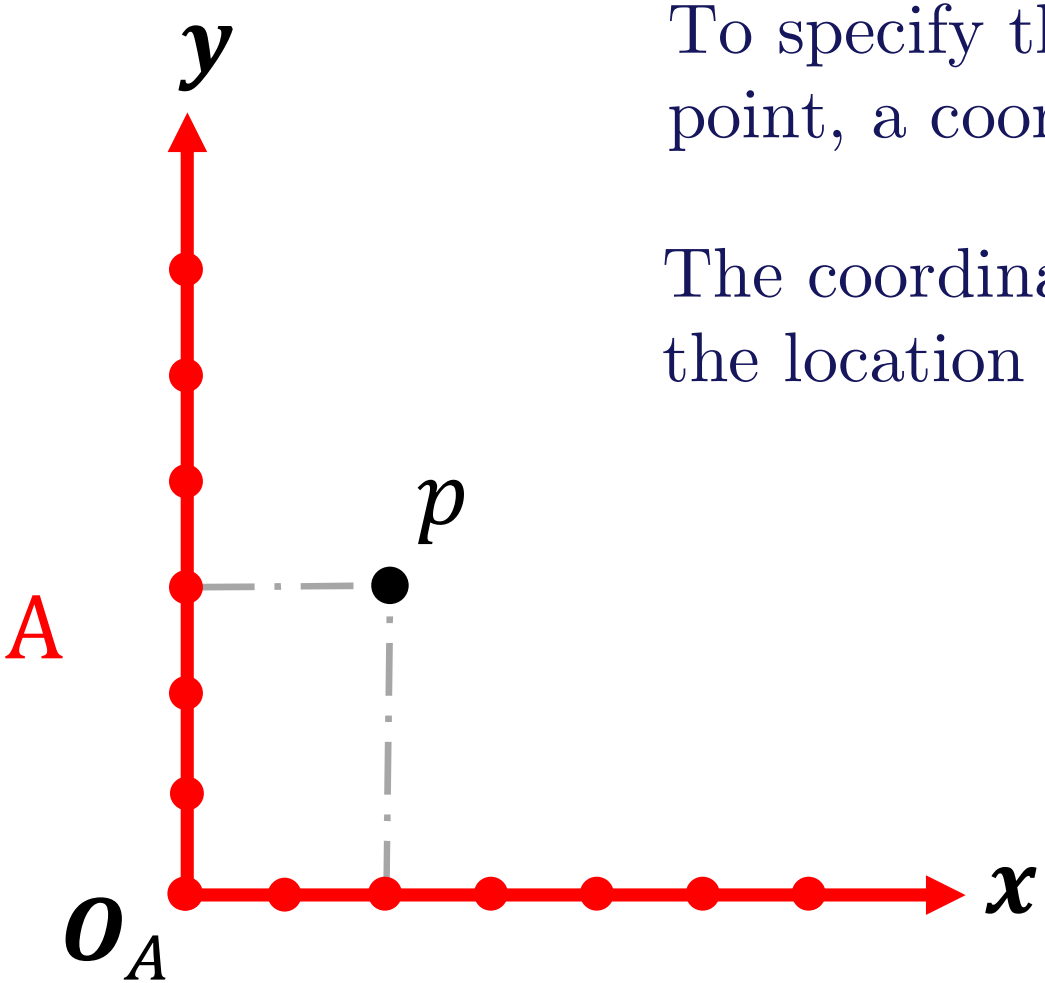


Points

Where is point p?

A point is a 0-dimensional mathematical object which can be specified as n-dimensional space using an $n - tuple$ $(x_1, x_2, \ldots, x_n)$ consisting of n coordinates.

Coordinate Frames



To specify the location of a particle or point, a coordinate frame is needed.

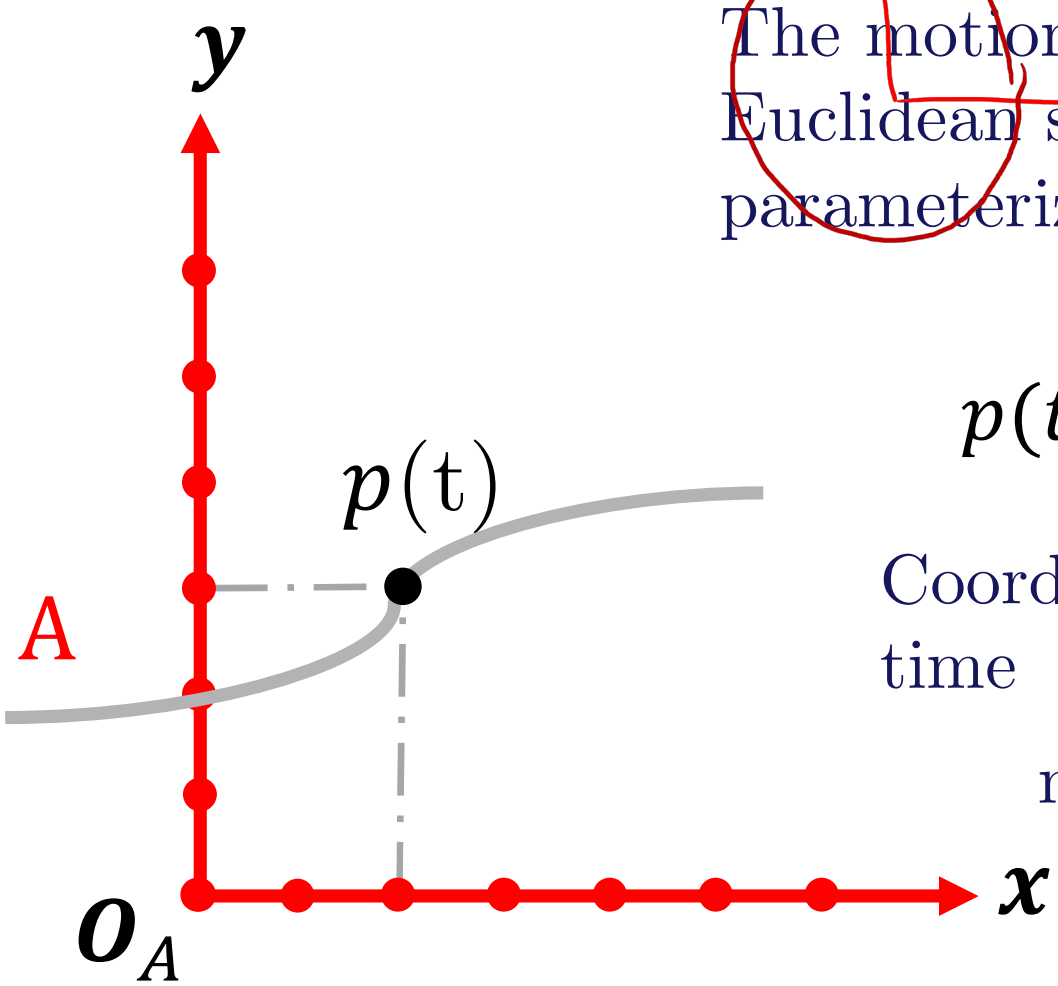
The coordinate frame uniquely describes the location of the point

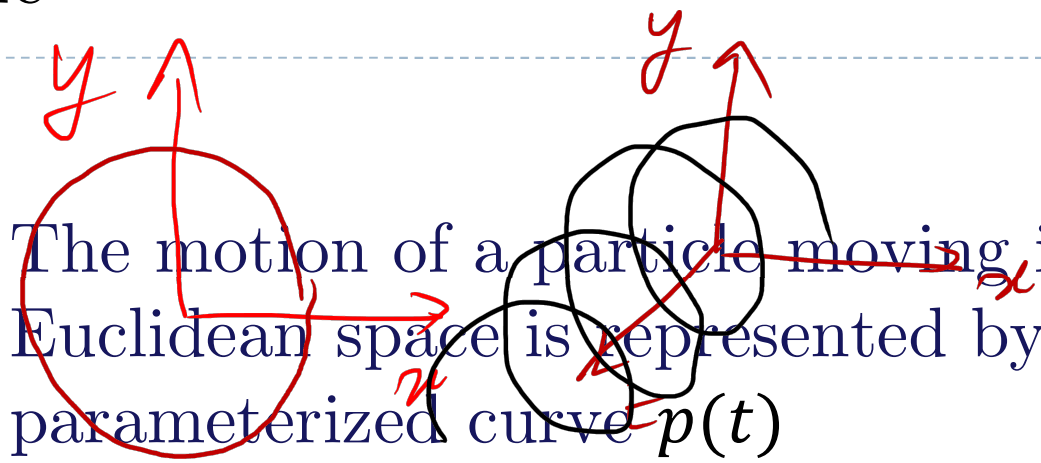
$$p = (x\,, y) \in \mathbb{R}^2$$

no. of coordinates $= 2$

A coordinate system provides us with a measurement system which allows us to measure quantities such as position and orientation, angular and linear velocity, and angular and linear acceleration. We can also describe the forces and torques that act on the rigid body using coordinate systems.

Motion of a particle





The motion of a particle moving in a Euclidean space is represented by a parameterized curve $p(t)$

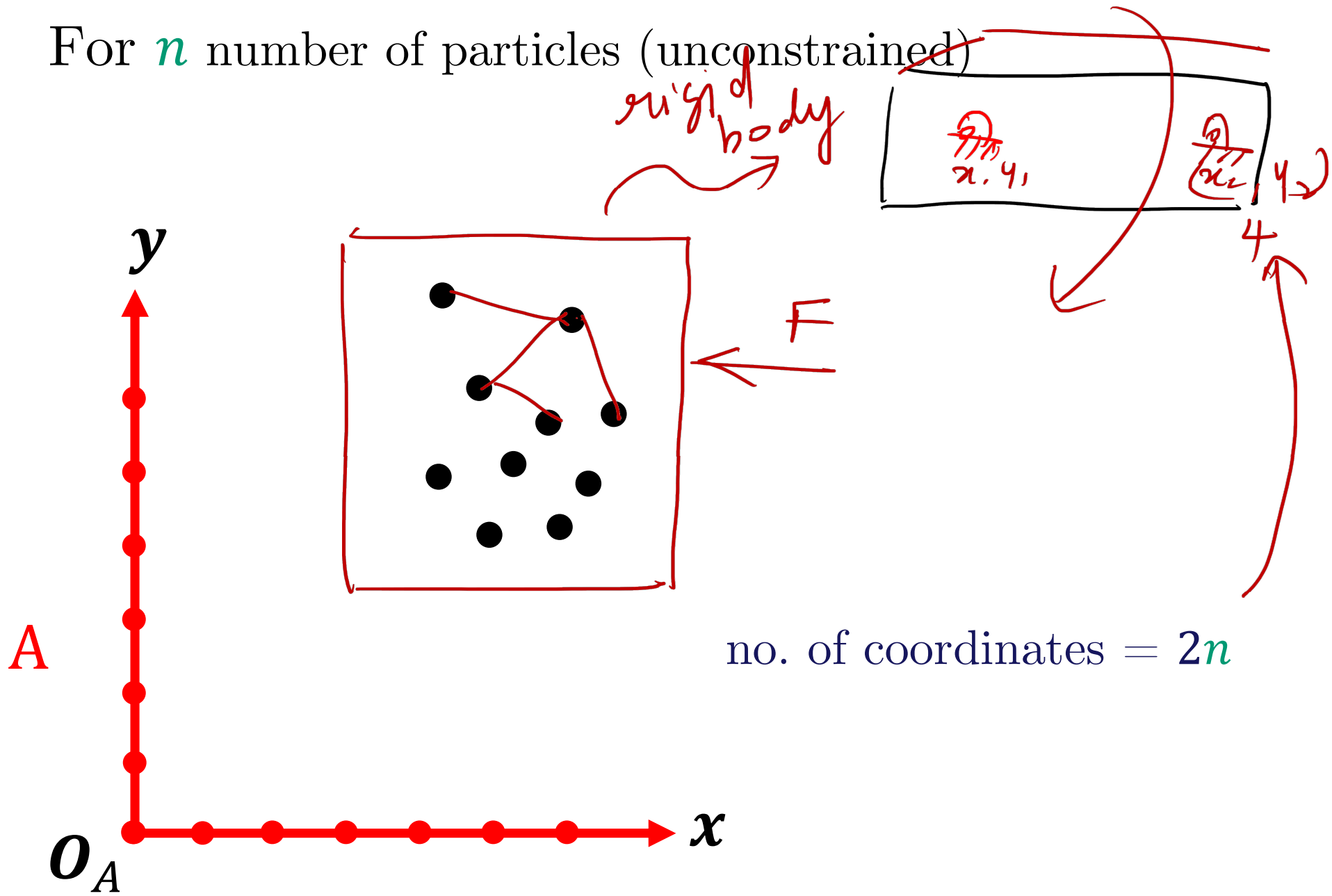
$$p(t) = (x(t)\,, y(t)) \in \mathbb{R}^2$$

Coordinate is a function of time

no. of coordinates $= 2$ and time t

Collection of particles

For n number of particles (unconstrained)

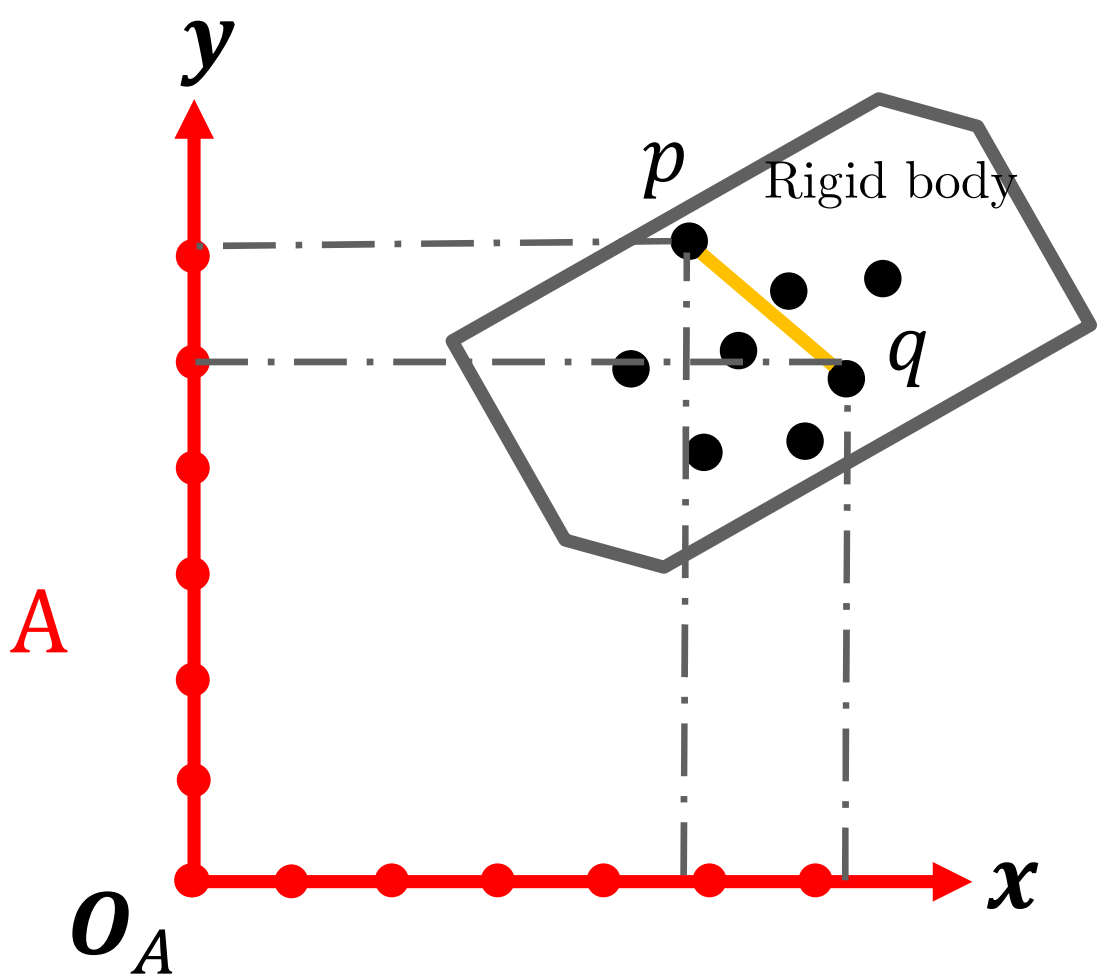


no. of coordinates $= 2n$

Rigid Body

Rigid body is a collection of particles such that the distance between any two particles remains fixed, regardless of any motion or application of forces.

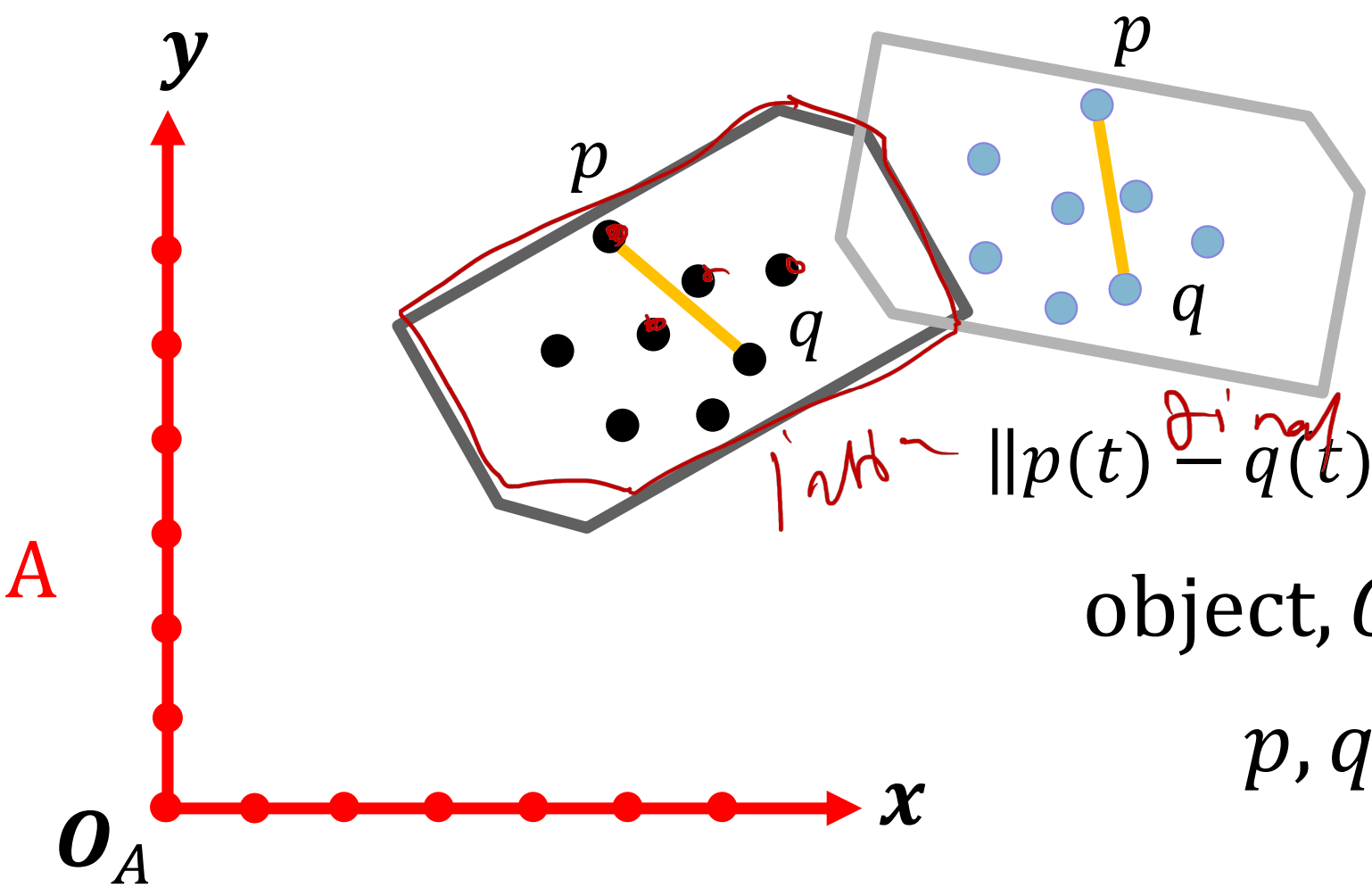
$2n \rightarrow 4$



Length is preserved

$$\|p - q\| = \text{Constant}$$

Rigid Displacement



initial

final

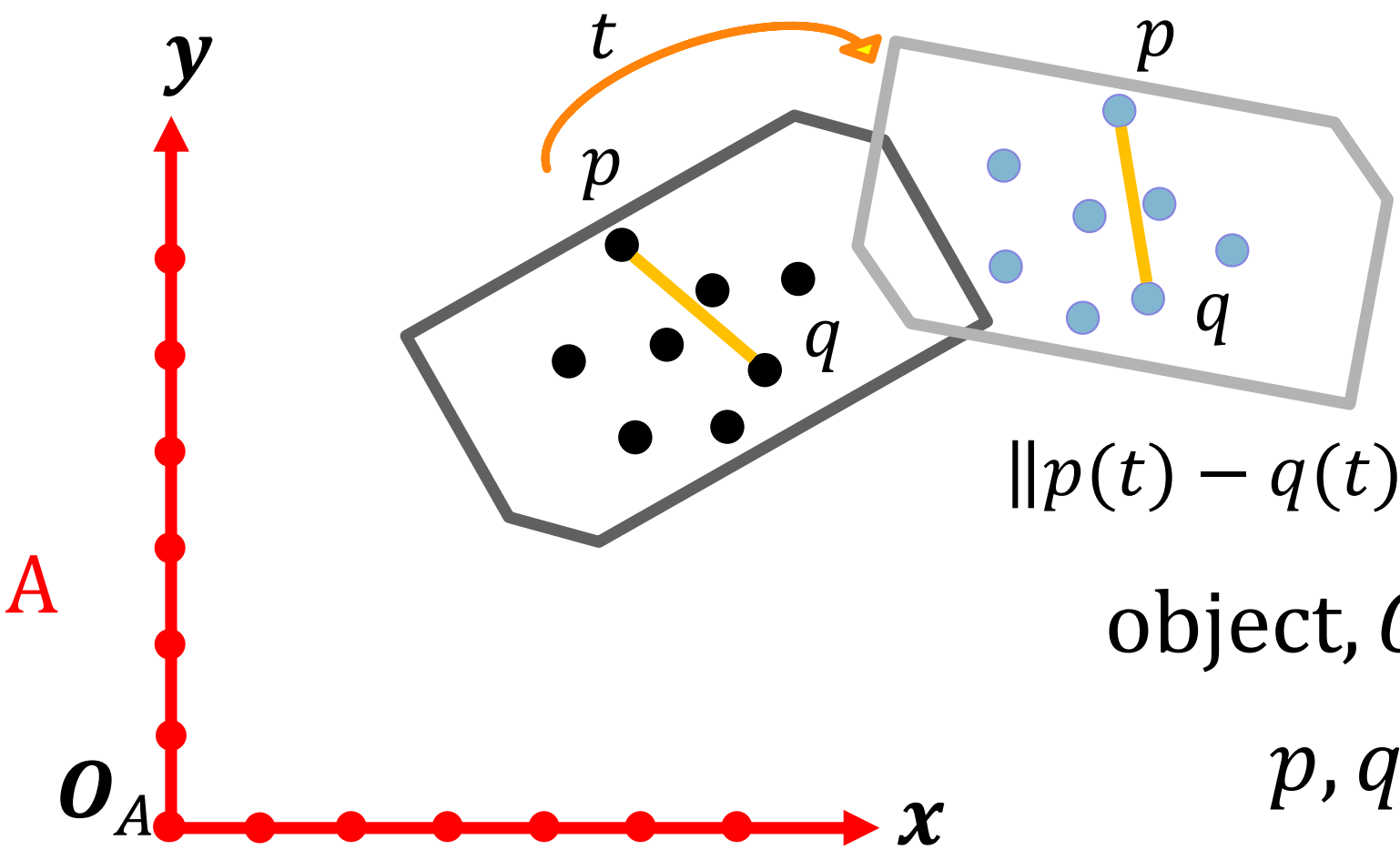
$$\|p(t) - q(t)\| = \text{Constant}$$

$$\text{object}, O \subset \mathbb{R}^2$$

$$p, q \in O$$

Rigid displacement is a single mapping $g: O \to \mathbb{R}^2$ which maps the coordinate of points in the rigid body from initial to final configuration

Rigid Motion



$$\|p(t) - q(t)\| = \text{Constant}$$

$$\text{object}, O \subset \mathbb{R}^2$$

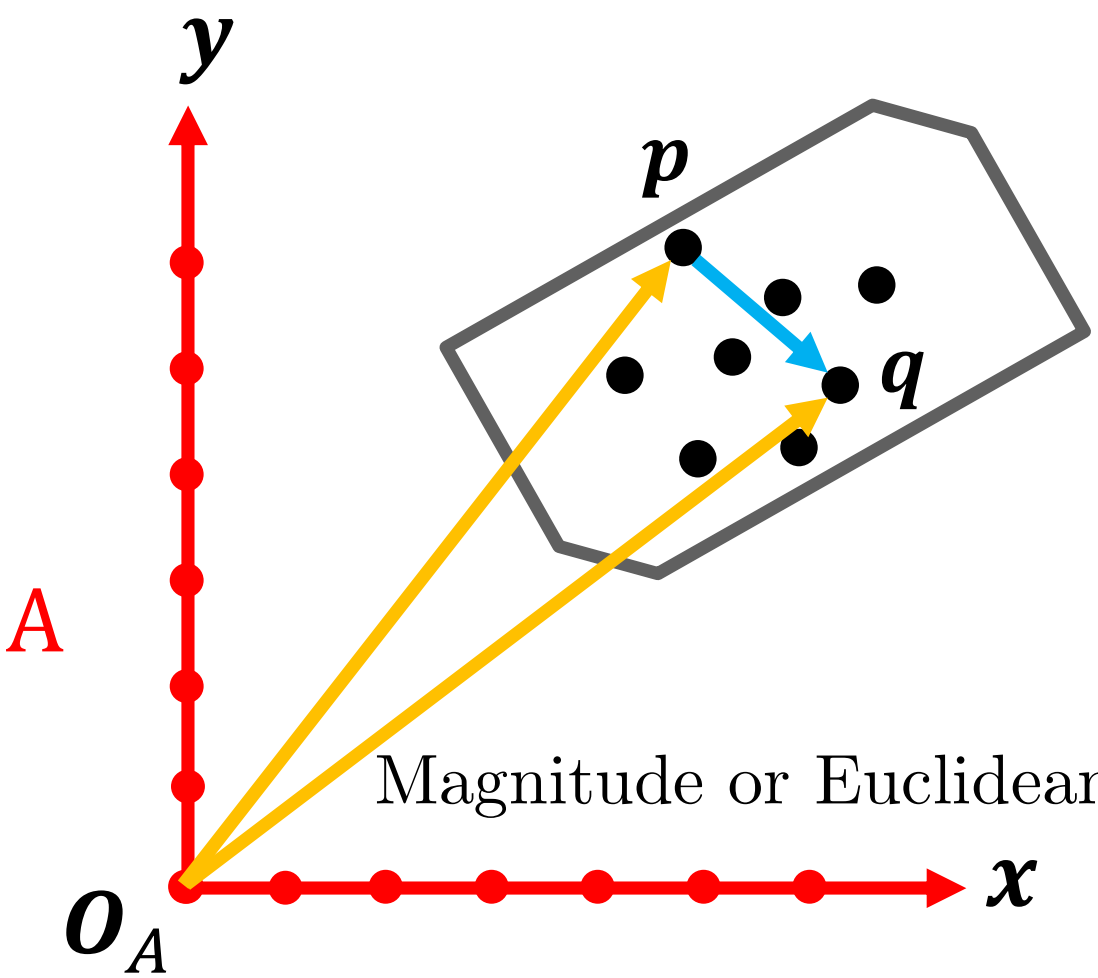
$$p, q \in O$$

Rigid motion is a continuous family of mappings $\boldsymbol{g(t)}: O \to \mathbb{R}^2$ relative to a fixed frame.

$\boldsymbol{g(t)}$ maps the initial coordinates of the points on the body to the coordinates of the same point at $\boldsymbol{t}$

Vectors in $\mathbb{R}^2$

Given two points $p, q \in O$, the vector $\boldsymbol{v} \in \mathbb{R}^2$ is defined to be the directed line segment connecting from p to q



$$\boldsymbol{p} = p_x \hat{\boldsymbol{\imath}} + p_y \hat{\boldsymbol{\jmath}}$$

$$\boldsymbol{p} = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \in \mathbb{R}^2$$

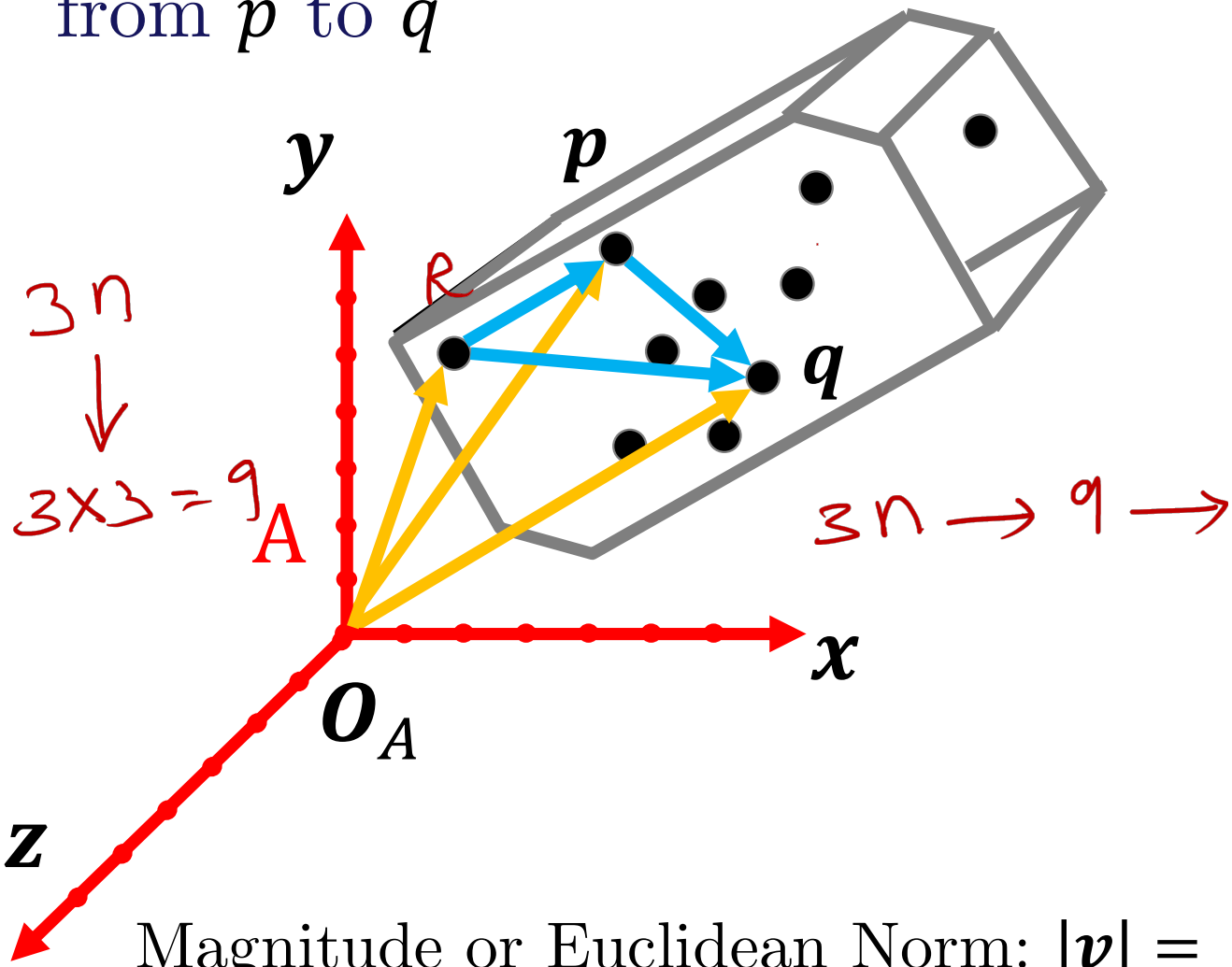
$$\boldsymbol{q} = \begin{bmatrix} q_x \\ q_y \end{bmatrix} \in \mathbb{R}^2$$

$$\boldsymbol{v} = \boldsymbol{q} - \boldsymbol{p}$$

Magnitude or Euclidean Norm: $|\boldsymbol{v}| = \sqrt{{v_x}^2 + {v_y}^2}$

Vectors in $\mathbb{R}^3$

Given two points $p, q \in O$, the vector $\boldsymbol{v} \in \mathbb{R}^3$ is defined to be the directed line segment connecting from p to q



$$\boldsymbol{p} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \in \mathbb{R}^3$$

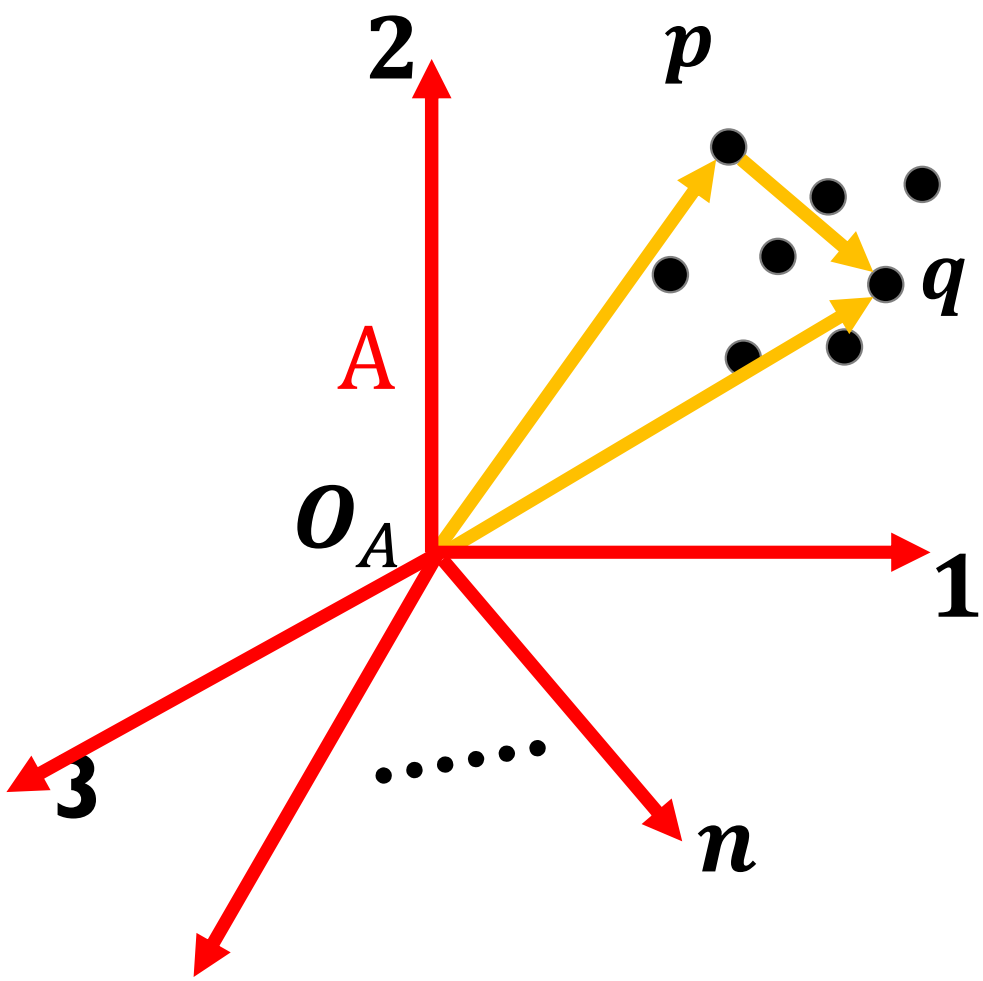
$$\boldsymbol{q} = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} \in \mathbb{R}^3$$

$$\boldsymbol{v} = \boldsymbol{q} - \boldsymbol{p}$$

Magnitude or Euclidean Norm: $|\boldsymbol{v}| = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2}$

Vectors in $\mathbb{R}^n$

Given two points $p, q \in O$, the vector $\boldsymbol{v} \in \mathbb{R}^n$ is defined to be the directed line segment connecting from p to q



$$\boldsymbol{p} = \begin{bmatrix} p_1 \\ p_2 \\ \dots \\ p_n \end{bmatrix} \in \mathbb{R}^n$$

$$\boldsymbol{q} = \begin{bmatrix} q_1 \\ q_2 \\ \dots \\ q_n \end{bmatrix} \in \mathbb{R}^n$$

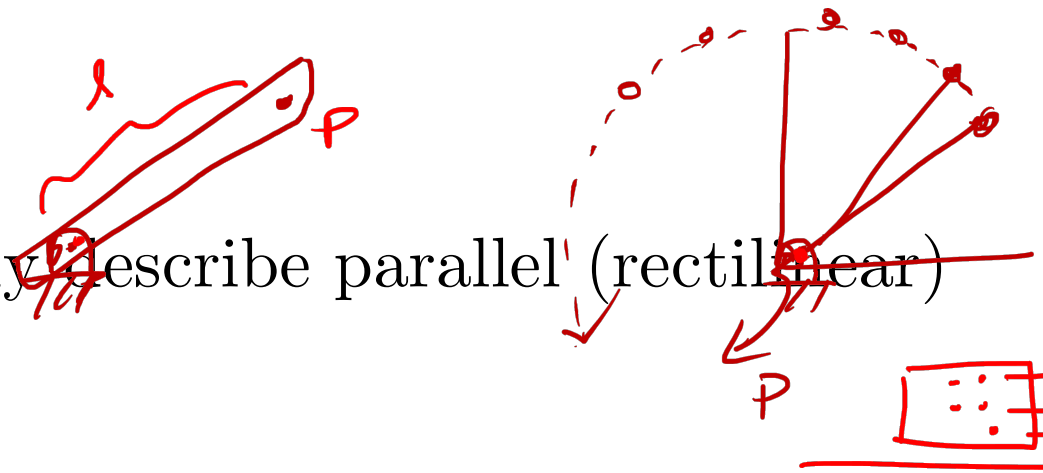
$$\boldsymbol{v} = \boldsymbol{q} - \boldsymbol{p}$$

$$|\boldsymbol{v}| = \sqrt{{v_1}^2 + {v_2}^2 \dots {v_n}^2}$$

TYPES OF MOTION

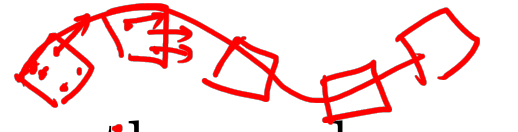
Pure rotation:

- The body possesses one point that has no motion with respect to the "stationary" frame of reference.



Pure translation:

- All points on the body describe parallel (rectilinear) paths.



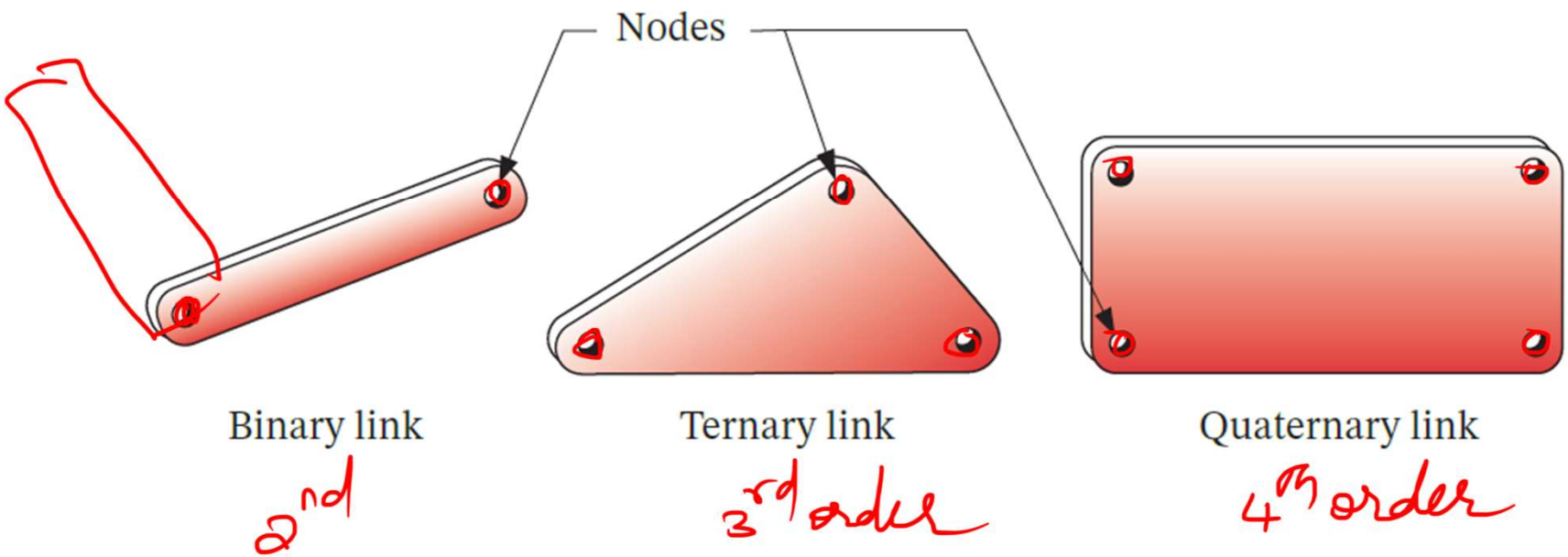
Complex motion:

- Points on the body will travel nonparallel paths, and there will be, at every instant, a center of rotation, which will continuously change location.

Links

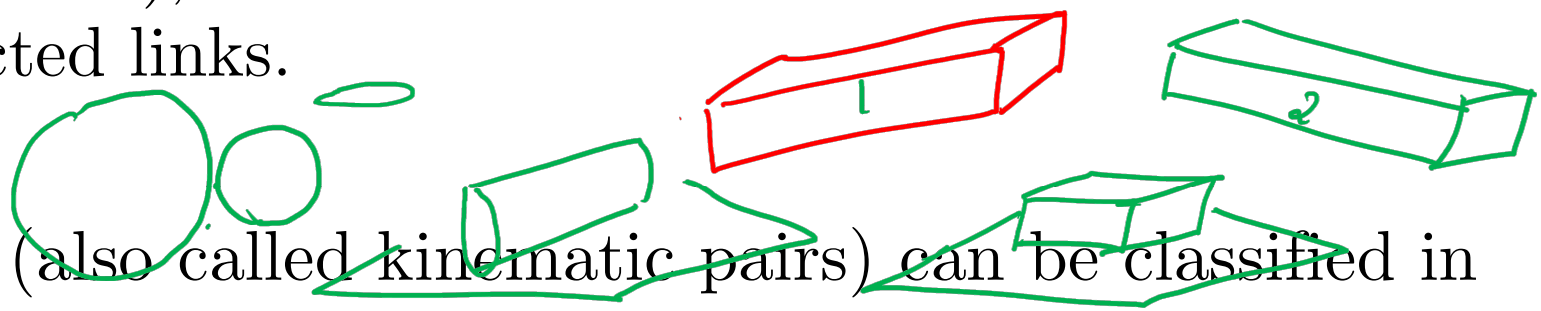
Link: rigid body that possesses at least two nodes that are points for attachment to other links.

- Binary link - one with two nodes.
- Ternary link - one with three nodes.
- Quaternary link - one with four nodes.



Joints

- A joint is a connection between two or more links (at their nodes), which allows some motion between the connected links.



- Joints (also called kinematic pairs) can be classified in several ways
  - By the type of contact between the elements, line, point, or surface.
  - By the number of degrees of freedom allowed at the joint.
  - By the type of physical closure of the joint: either force or form closed.
  - By the number of links joined (order of the joint).

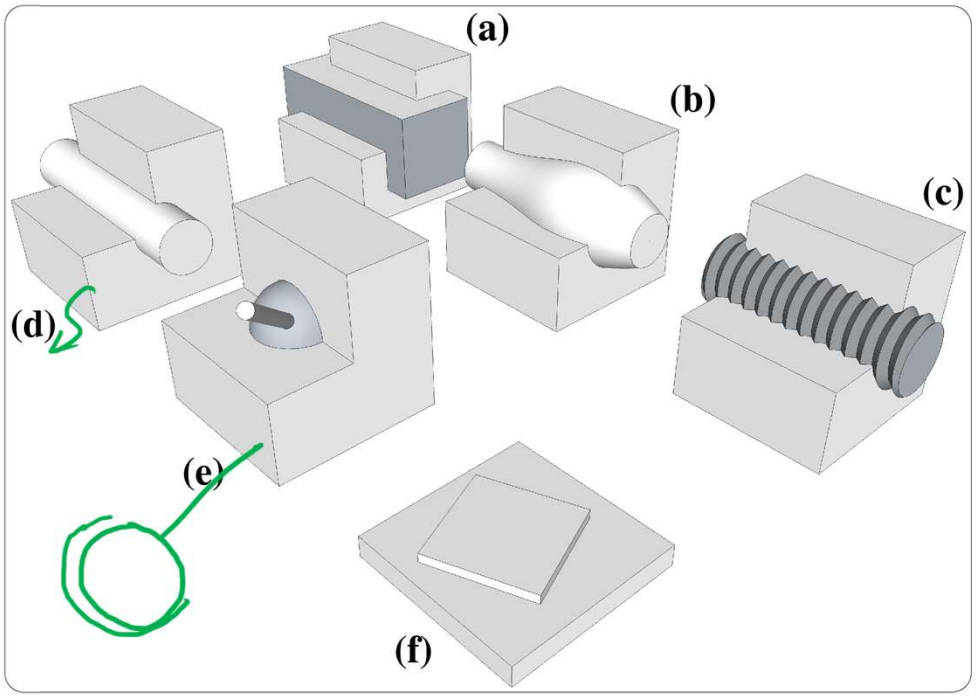
Joints

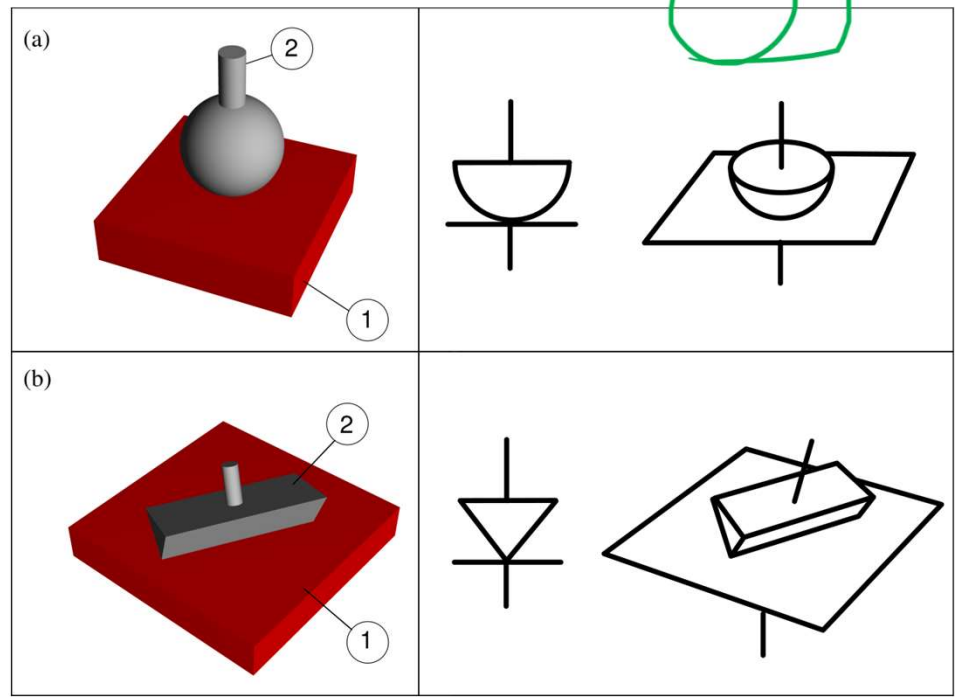
- A joint is a connection between two or more links (at their nodes), which allows some motion between the connected links.

- Joints (also called kinematic pairs) can be classified in several ways
  - **By the type of contact between the elements, line, point, or surface.**
  - **By the number of degrees of freedom allowed at the joint.**
  - By the type of physical closure of the joint: either force or form closed.
  - By the number of links joined (order of the joint).

Joints

- Reuleaux convention
  - **lower pair** joints with surface contact (as with a pin surrounded by a hole), therefore contact stresses are lower
  - **higher pair** to describe joints with point or line contact, therefore contact stresses are higher

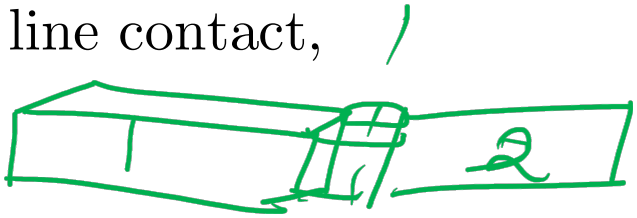




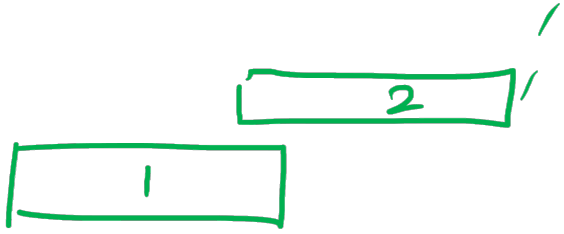
https://research.me.udel.edu/~vroy/AdvancedDynamics/sec_KinPair.html

Joints

- Reuleaux convention
  - **lower pair** joints with surface contact (as with a pin surrounded by a hole), therefore contact stresses are lower
  - **higher pair** to describe joints with point or line contact, therefore contact stresses are higher

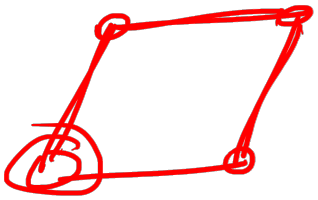


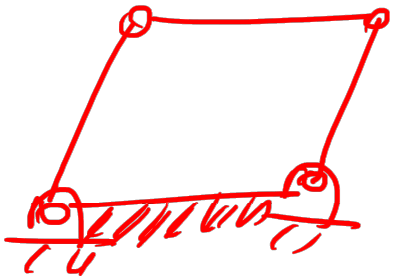
- Six possible **lower pairs** (joints)
  - Revolute (R) pair
  - prismatic (P) pair

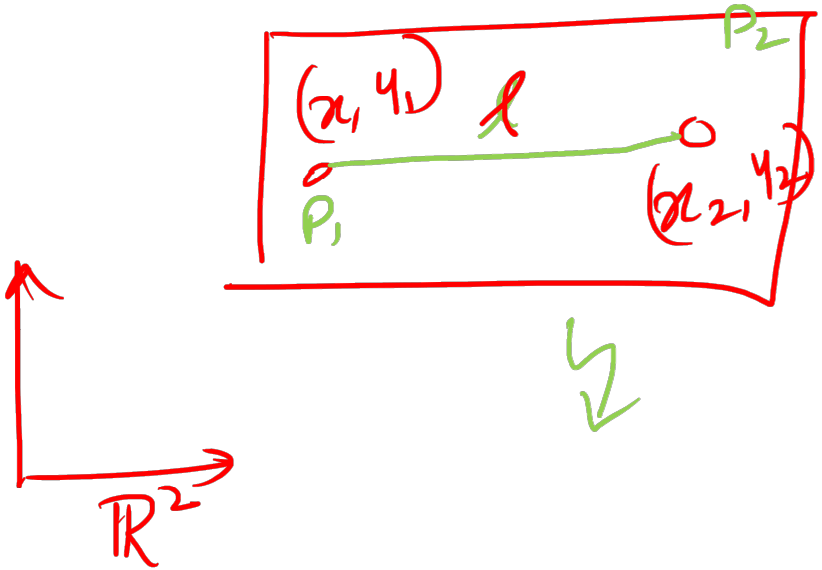


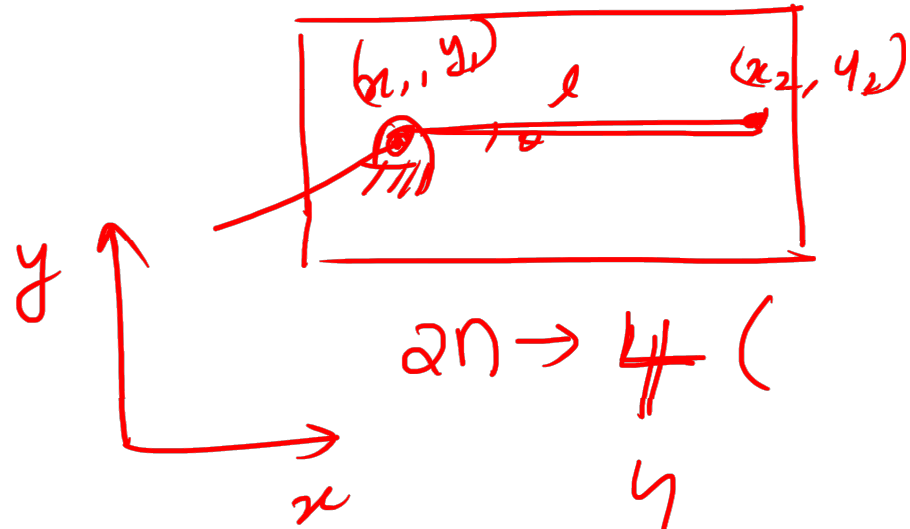
The R and P pairs are the basic building blocks of all other pairs.

  - Screw (H)
  - Cylindrical (C)
  - Spherical (S)
  - Flat (F) pairs



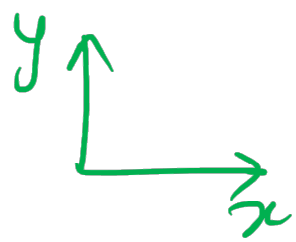


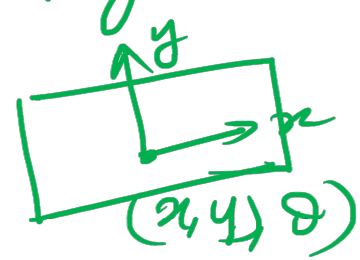


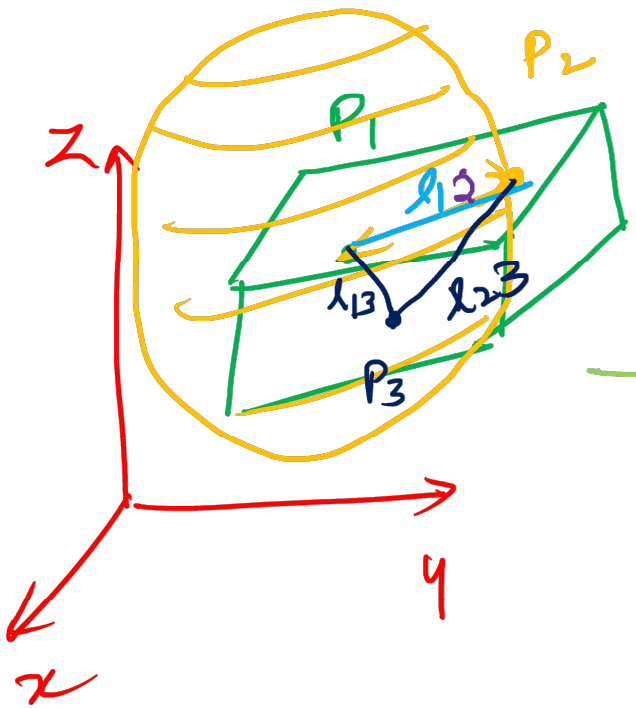


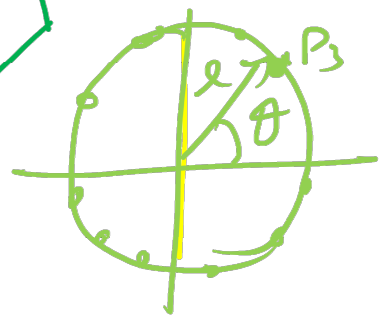
$2n \rightarrow 4$ (

DoF = no. of independent variables $\underline{\underline{3}}$ (x, y, θ) required to define the complete configuration of a system/mechanism.

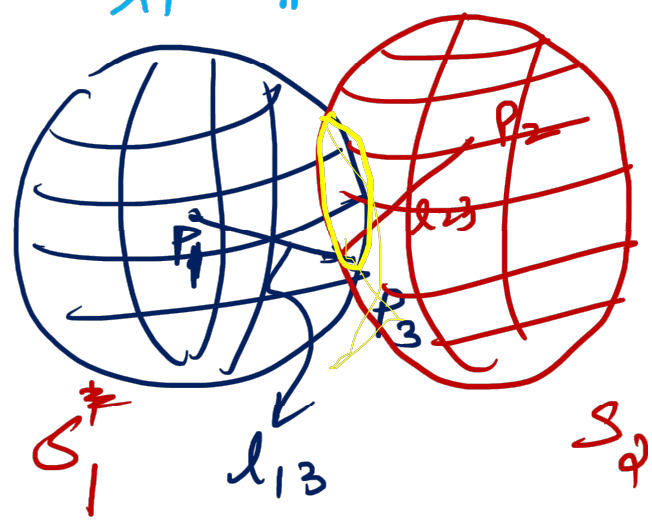




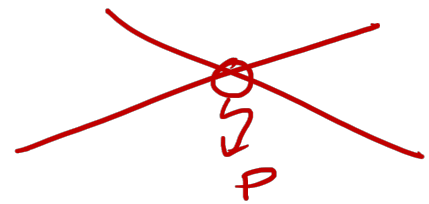


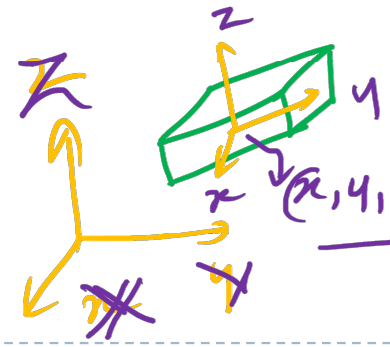


$l_1 = \| P_1 - P_2 \|$



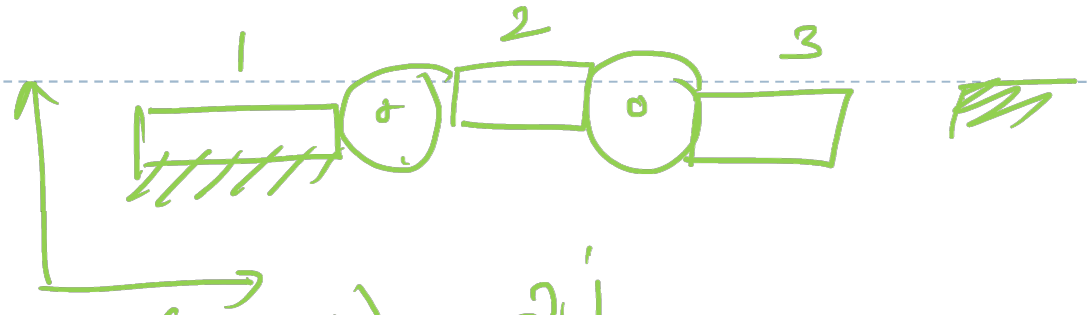
(x_1, y_1, z_1), α, β, γ





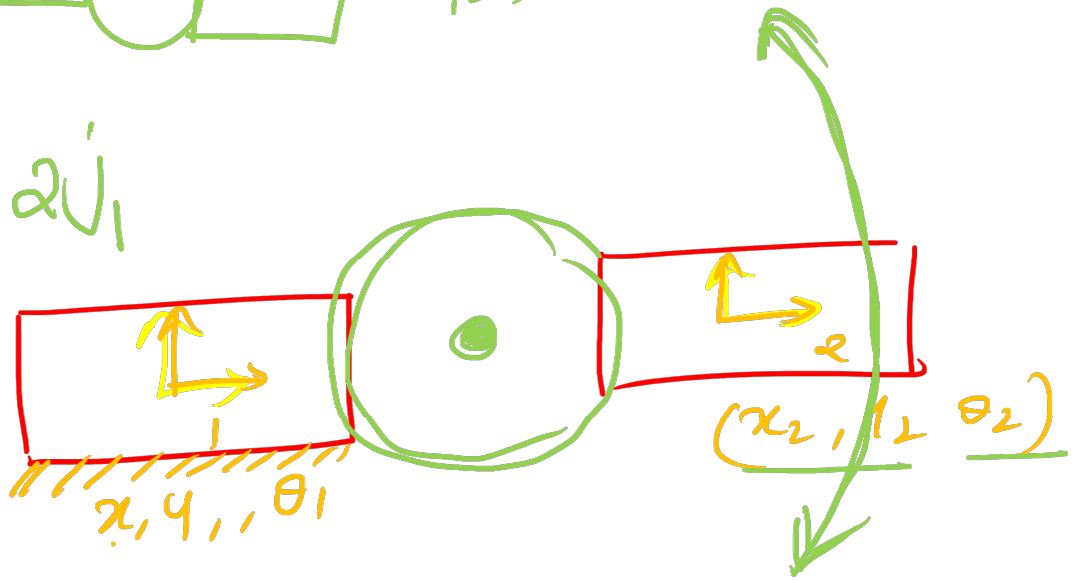
(x, y, z) (α, β, γ)

$3n \longrightarrow 9 \longrightarrow 6$



$$DoF = 3(N-1) - 2J_1$$

$$DoF = 2$$

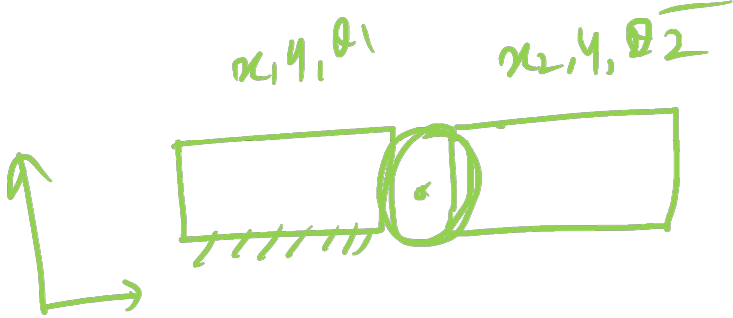


$$2 \times 3$$

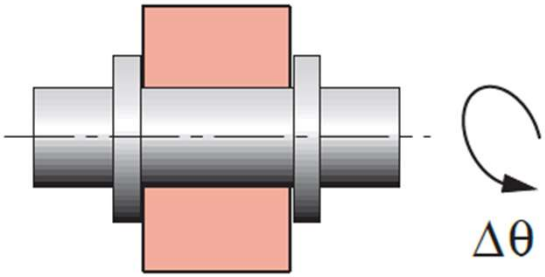
$$DoF = 6 - 3 - 2$$

$$= 1 \ (\theta_2)$$

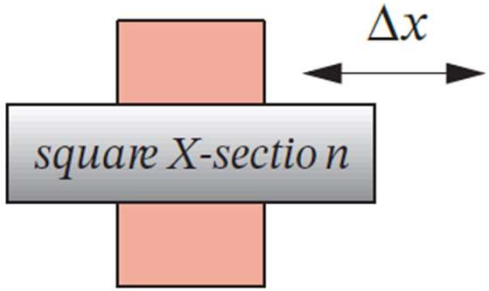
$$6 - 3$$



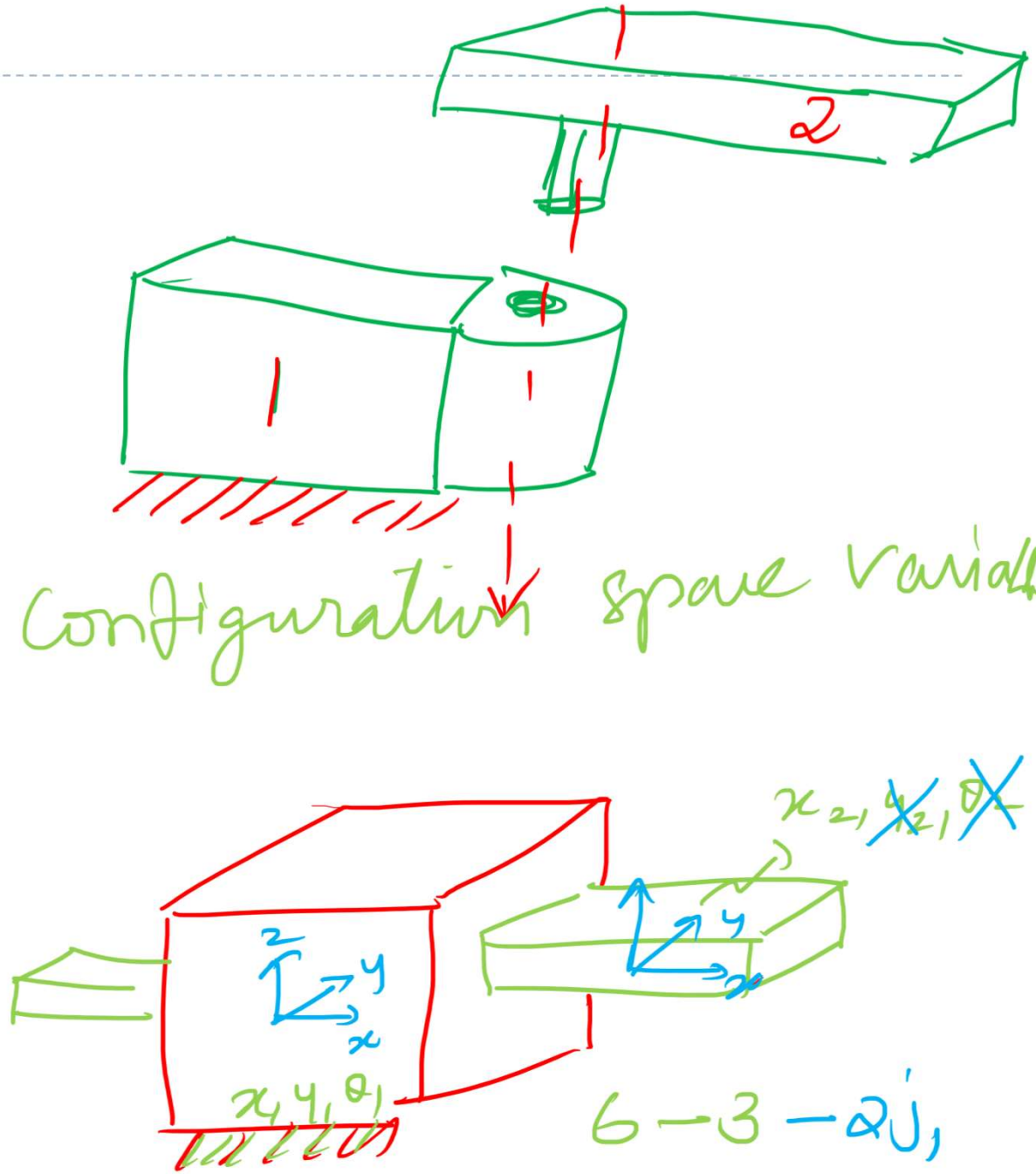
Joints - lower pairs



Revolute (R) joint—1 *DOF*

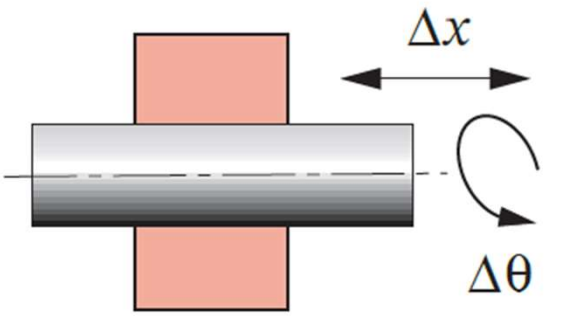


Prismatic (P) joint—1 *DOF*

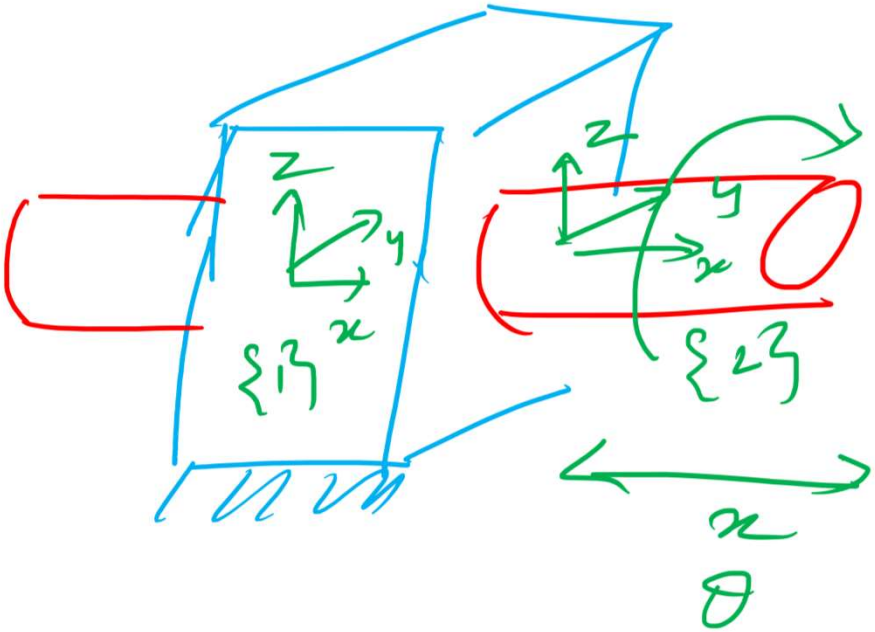


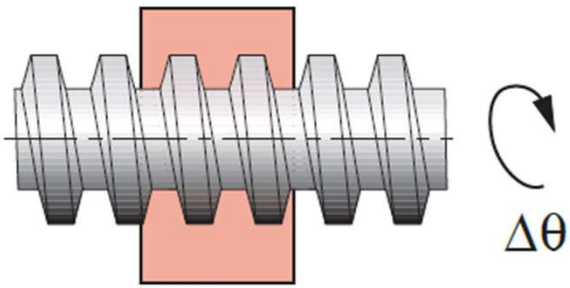
Ref: L. Norton, Design of Machinery "An Introduction to the. Synthesis and Analysis of Mechanisms

Joints - lower pairs

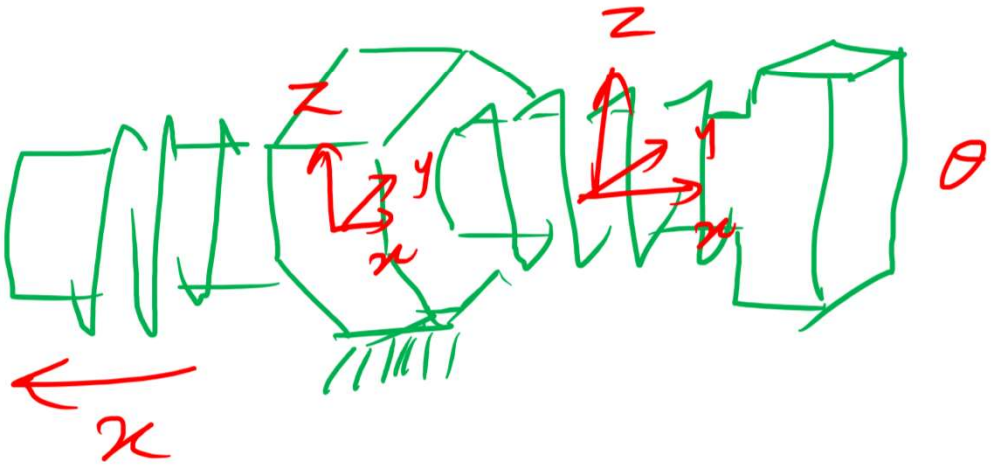


Cylindric (C) joint—2 *DOF*



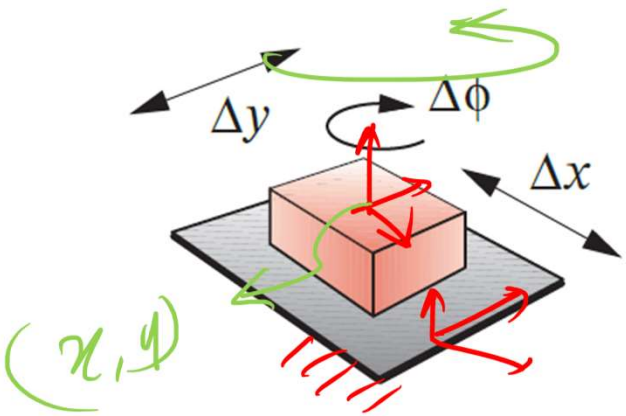


Helical (H) joint—1 *DOF*

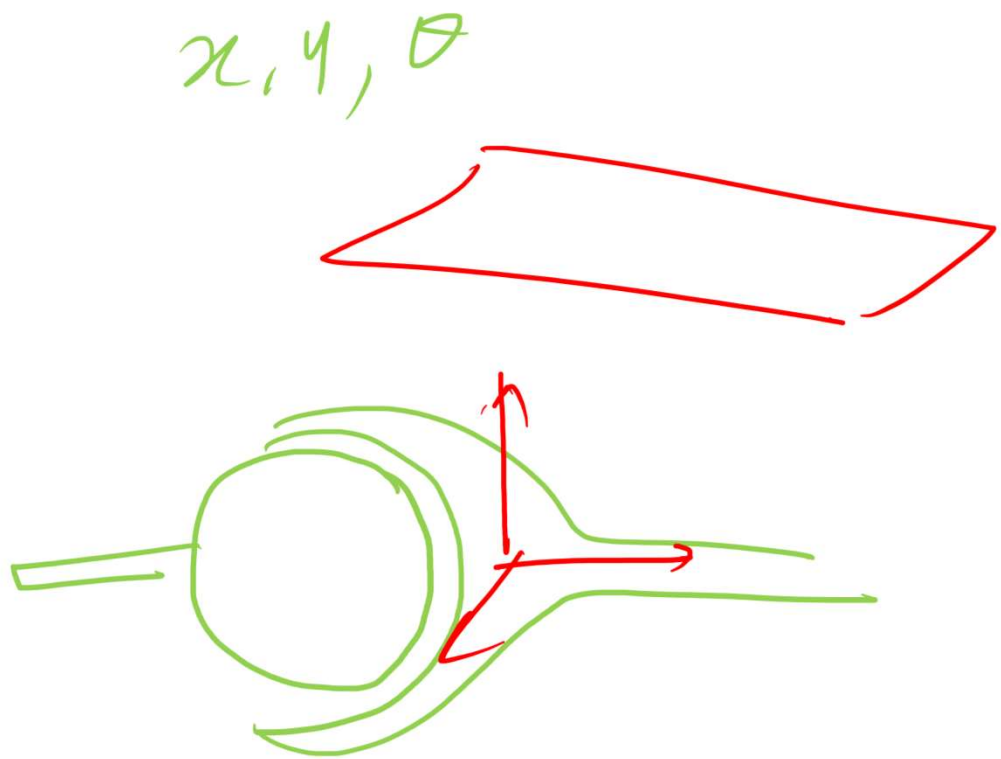


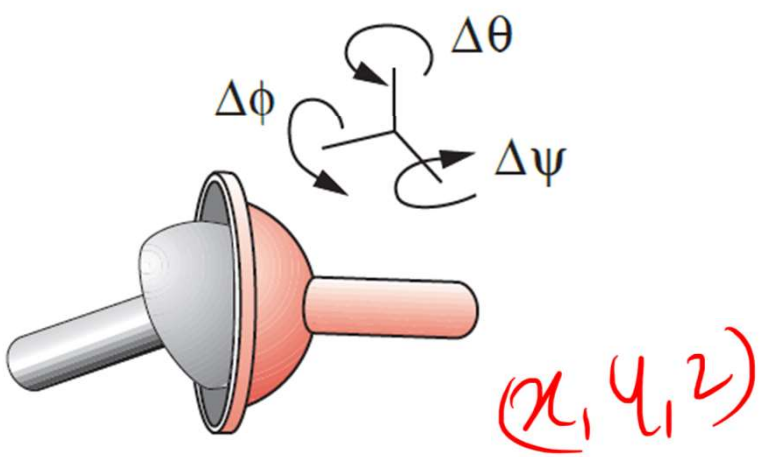
Ref: L. Norton, Design of Machinery "An Introduction to the. Synthesis and Analysis of Mechanisms

Joints - lower pairs



Planar (F) joint—3 *DOF*





Spherical (S) joint—3 *DOF*

Ref: L. Norton, Design of Machinery “An Introduction to the. Synthesis and Analysis of Mechanisms