3rd order VS 4th order Band-pass filter

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***Abstract*—we are comparing 3rd order and 4th order band-pass filters in butter-worth model and Chebyshev model**

I. Introduction

we need to build a 3rd order and 4th order band-pass filter in both butter-wort and Chebyshev models and compare them in terms of slopes and ripples in the frequency response.

II. Specifications

we are designing the filters for ECG signal. The range of frequencies of ECG signal is from 0.5Hz to 150 Hz in adults and up to 250Hz in infants and new born babies.

III. Components used

For building the filters we required UA741 OPAMP, resistors and capacitors. The exact values of resistors and not available in lab so we had to adjust few values from our simulations in LT Spice.

IV. What is the need for higher order filters

1) Sharper Frequency Selectivity:
   - Higher-order band-pass filters provide narrower bandwidths compared to first-order filters.
   - They can selectively pass a specific range of frequencies while attenuating others more effectively.
   - Applications such as audio equalization, communication systems, and instrumentation often require precise frequency selectivity.
2) Improved Attenuation:
   - As the filter order increases, the roll-off rate (rate of attenuation beyond the pass-band) becomes steeper.
   - This improved attenuation helps suppress unwanted frequencies outside the desired pass-band.
   - For example, in radio receivers, higher-order band-pass filters help reject adjacent channel interference.
3) Reduced Sensitivity to Component Tolerances:
   - Higher-order filters are less sensitive to variations in component values (resistors, capacitors, etc.).
   - This robustness ensures that the filter's performance remains consistent even with manufacturing tolerances.
4) Reduced Noise and Interference:
   - A higher-order band-pass filter can effectively suppress out-of-band noise and interference.
   - In applications like biomedical signal processing or seismic data analysis, noise reduction is critical.
5) Better Signal-to-Noise Ratio (SNR):
   - By narrowing the bandwidth, higher-order filters allow more signal energy to pass through while attenuating noise.
   - This improves the SNR, especially in communication systems and audio applications.
6) Filter Cascading and Equalization:
   - Cascading multiple lower-order filters can achieve the desired higher-order response.
   - For instance, combining second-order filters can create a fourth-order band-pass filter.
   - Equalization in audio systems often requires higher-order filters to shape the frequency response.

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V. Schematics for low-pass and high pass filters

- unity gain 2nd order low-pass filter

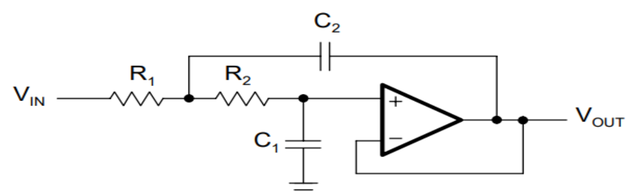


Fig. 1. schematic for a 2nd order unity gain LPF.

- unity gain 2nd order high-pass filter

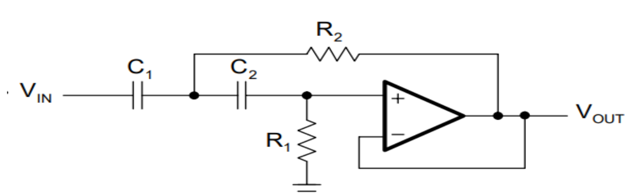


Fig. 2. schematic for a 2nd order unity gain HPF.

- a higher order filter can be made by cascading the first and second order filters in series

VI. Characteristics of the filter

the frequency response of the filter should be close to expected i.e allow only frequencies between 10 Hz to 250 Hz but we can't perfectly implement the cutoff values because lab have only few resistors that are close to our requirements but the cutoffs are almost perfect, we followed filter designs.

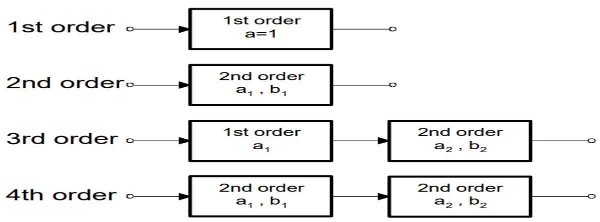


Fig. 3. combinations to make higher order filter.

VII. What is the need to use standard filter designs ?

we can make filters without following any designs but we need to follow some standard designs and use the constants from the designs because of the following reasons

1) Design Consistency and Predictability :
- Filter configurations provide a consistent framework for designing filters.
- By following established models, we can predict the filter's behavior and performance.
- Using known filter types ensures that the resulting circuit meets specific requirements.
2) Desired Frequency Response :
- Different filter types (Butter-worth, Bessel, Chebyshev, etc.) have distinct frequency responses.
- we can choose a filter based on the desired pass-band flatness, roll-off rate, and phase characteristics.
- For example: - Butter-worth filters offer a flat pass-band response.
- Bessel filters prioritize linear phase response.
- Chebyshev filters allow controlled ripple in the pass-band.
3) Trade-Offs and Application-Specific Goals :
- Each filter type has trade-offs like phase linearity vs. steep roll-off.
- we can select a filter based on the specific application requirements:
- Audio systems may prioritize linear phase.
- Communication systems may need sharp roll-off.
- Video processing may require constant group delay.
we can chose the trade-off that doesn't affect our
4) Efficiency and Time Savings :
- Designing filters directly from scratch can be time-consuming and we will obviously make some mistakes.
- Using established models with predefined constants saves time and can promise accuracy.
- These constants are derived from mathematical analysis and optimization.
5) Consistent Performance Across Systems :
- Following filter configurations ensures consistent performance across different systems.
- we can replicate the same filter behavior in various applications.
- This consistency simplifies system integration and maintenance.
-If we make the filter from scratch it may not be useful for other circuits in case if we want to extend the use of our filter.

Using filter configurations and constants from established models provides a systematic approach to filter design, ensures predictable behavior, and allows us to meet specific performance goals. It's about achieving reliable and consistent results while optimizing for the desired application.

VIII. Butter-worth and Chebyshev filters

In this project we are making 3rd order and 4th order band-pass filters in both butter-worth and Chebyshev models. There are few characteristics for both of these models

1) **Butter-worth:**
   - The Butter-worth filter is commonly referred to as the "maximally flat" option because it offers a flat frequency response in the pass-band.
   - Flat Pass-band Response: The Butter-worth filter provides a relatively flat frequency response within the pass-band.
   - Linear Phase Response: It maintains a more linear phase response than alternative filters.
   - Ideal for High-Quality Audio: Due to its linear phase and minimal overshoot, it's ideal for high-quality audio applications.
   - Better Selectivity: The Butter-worth filter has better selectivity than some other filter types.
2) **Chebyshev:**
   - Ripple in the Pass-band:
     Unlike Butter-worth filters (which have a smooth pass-band response), Chebyshev filters introduce ripple within the pass-band.
     - The ripple can be either in the form of type I (maximum ripple in the pass-band) or type II (maximum ripple in the stop-band).
   - Steeper Roll-Off Rate:
     Chebyshev filters exhibit a faster roll-off beyond the pass-band compared to Butter-worth filters.
     - This means that they attenuate frequencies outside the desired range more aggressively.
   - Trade-Off Between Ripple and Roll-Off: The amount of ripple in the pass-band is a design parameter.
     - By adjusting the ripple level, we can achieve a trade-off between ripple and roll-off rate.
     - Higher ripple results in a steeper roll-off.
   - Selectivity and Bandwidth Control:
     - Chebyshev filters allow precise control over the bandwidth and selectivity.
     - They are useful when you need to isolate a specific frequency range with minimal transition band.

IX. Structure of the Active Band-Pass Filters

simulation model for filters

- 3rd order filter

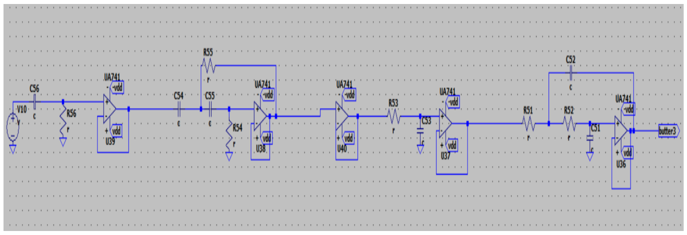


Fig. 4. simulation for 3rd order band-pass filter.

- 4th order filter

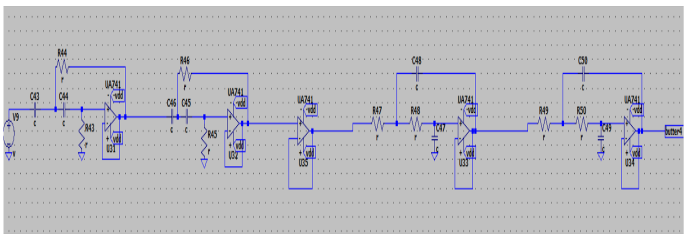


Fig. 5. simulation for 4th order band-pass filter.

- **what is the problem in cascading LPF and HPF?** When we cascade LPF and HPF there is a difference in impedance which will alter the cutoff frequencies of the filters ,by using the OPAMP as a buffer we can solve this problem of difference in impedance.
- By following the methods as said in fig 3 we constructed active 3rd order and 4th order band-pass filters, now we need to to find the values of R,C according to our requirements by following butter-worth and Chebyshev filter methods for reliability.

X. Determining the Values of R,C

Manually calculating the values id time consuming.
so we wrote a Matlab code to calculate the values quickly the code goes as in fig 6.
we tried calculations using pen and paper but the results not accurate because it included complex numbers also.
so we decided to write a simple code in Matlab for this job.
The resistors we used are not what we exactly as per calculations there are few adjustments because of the availability in lab.

Given C_1 and C_2, the resistor values for R_1 and R_2 are calculated through:

$$R_{1,2} = \frac{a_1C_2 \mp \sqrt{a_1{}^2C_2{}^2 - 4b_1C_1C_2}}{4\pi f_cC_1C_2}$$

Fig. 6. To calculate R for LPF.

In order to obtain real values under the square root, C_2 must satisfy the following condition:

$$C_2 \geq C_1\frac{4b_1}{a_1{}^2}$$

Fig. 7. for obtaining real values we must satisfy this criteria.

Given C, the resistor values for R_1 and R_2 are calculated through:

$$R_1 = \frac{1}{\pi f_c C a_1}$$

$$R_2 = \frac{a_1}{4\pi f_c C b_1}$$

Fig. 8. to calculate R for HPF.

```
C1_value = 10;
C2_value = 20;


[R1_result, R2_result] = calculate_resistors(C1_value, C2_value);

disp(['Resulting R1: ', num2str(R1_result)]);
disp(['Resulting R2: ', num2str(R2_result)]);


function [R1, R2] = calculate_resistors(c1,c2)

    a1 = 1;
    b1 = 4;
    fc= 10;


    R1 = ((a1 * c2) -sqrt (a1*a1*c2*c2 -4*b1*c1*c2))/(4*pi*fc*c1*c2);
   R2 = ((a1 * c2) +sqrt (a1*a1*c2*c2 -4*b1*c1*c2))/(4*pi*fc*c1*c2);

    disp(['R1: ', num2str(R1)]);
    disp(['R2: ', num2str(R2)]);
end
```

Fig. 9. Matlab code for determining RC values.

XI. Simulations

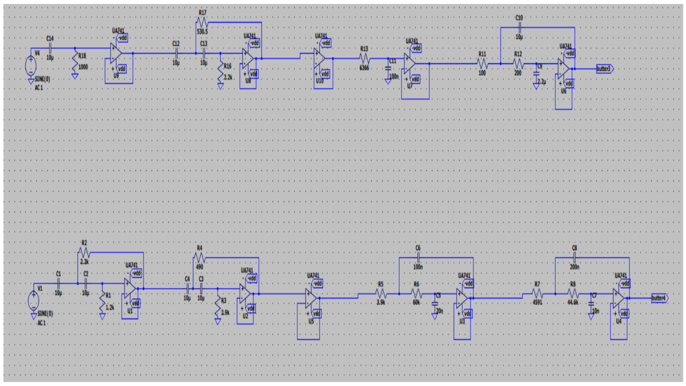


Fig. 10. simulation for 4th order and 3rd order band-pass filter butter-worth method.

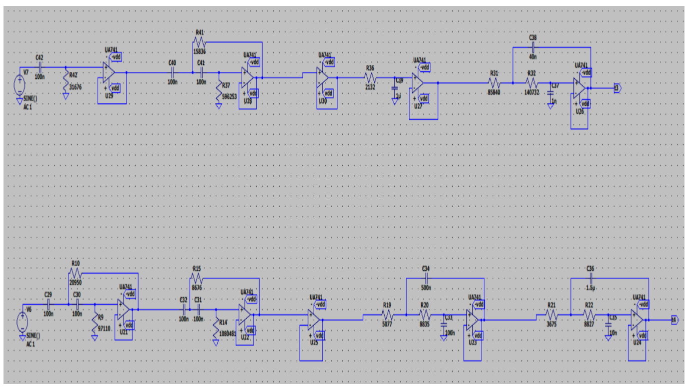


Fig. 11. simulation for 4th order and 3rd order band-pass filter Chebyshev method.

XII. Simulation Results

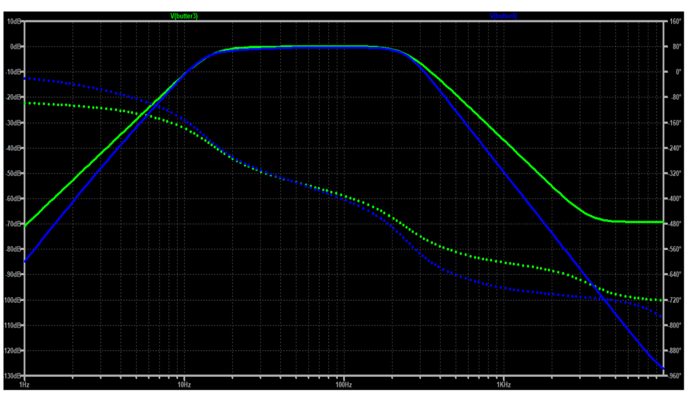


Fig. 12. simulation results for 4th order and 3rd order band-pass filter butter-worth method.

We can clearly observe from the simulations that the filters are allowing only frequencies between 10 Hz to 250 Hz as intended.

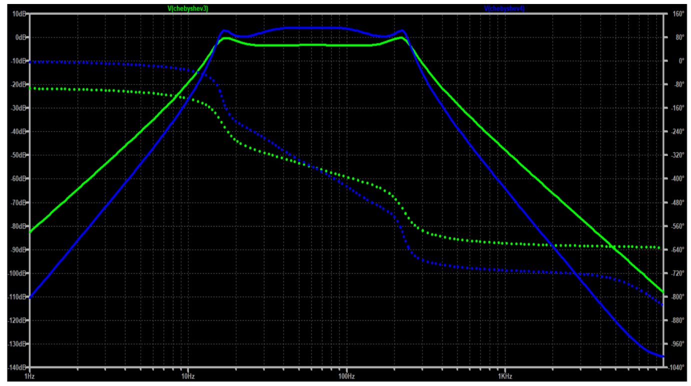


Fig. 13. simulation results for 4th order and 3rd order band-pass filter Chebyshev method.

A. Observations

we can observe some significant difference in the simulation output

1) 3 rd order VS 4 th order:

- 3 rd order filter
  - The 3rd-order filter has a gentler roll-off beyond the pass-band compared to higher-order filters.
  - It provides moderate attenuation in the stop-band.
  - The transition from pass-band to stop-band is not as steep as in higher-order filters.
- 4 th order filter
  - The 4th order filter exhibits a steeper roll-off beyond the pass-band.
  - It attenuates frequencies outside the desired range more aggressively.
  - The transition from pass-band to stop-band is sharper than in the 3rd-order filter the angle is increased by 20 degrees from 3rd order to 4th order.

2) Butter-worth model filter vs Chebyshev model filter:

- Butter- worth model filter
  - Maximally flat response within the pass-band. Moderate phase distortion.
  - All poles are located on the unit circle with equal angles.
  - No ripple in the pass-band.
  - Steeper roll-off as the order increases.
- Chebyshev model filter
  - Ripple in the pass-band (type I or type II).
  - Sharper transition band compared to Butter-worth.
  - Non-uniform group delay.
  - All poles are located on an ellipse inside the unit circle.

B. Ripples in Chebyshev model filter

- There are significant effects on the filter response due to location of poles.
  1) poles inside unit circle(stable)

- If all poles are inside the unit circle, the filter is stable.
- Stable filters tend to have smoother frequency responses with minimal ripples.

2) On the Unit Circle:
   - Poles on the unit circle can lead to oscillations in the frequency response.
   - These oscillations result in ripples.
3) outside unit circle(Unstable)
   - Filters with poles outside the unit circle are unstable.
   - Unstable filters exhibit erratic behavior, including severe ripples.

- The difference in filter models lies only in the constants that directly effects the location of poles.
- In Chebyshev filter model the constants intentionally introduces ripples in to the frequency response in order to generate a steeper slope.
- For a steeper slope we are allowing ripples in the pass band this is a Trade-off in Chebyshev -3dB filter model.

XIII. Results from the actual physical circuit

A. Butter-worth filter

1) 3rd order

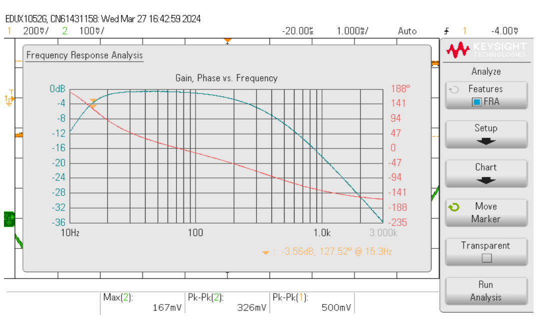


Fig. 14. result observed in oscilloscope for 3rd order butter worth band pass filter.

The observed cutoffs for pass band is from 15.3Hz to 251 Hz. the cutoffs are very close to the expected values.

2) 4th order

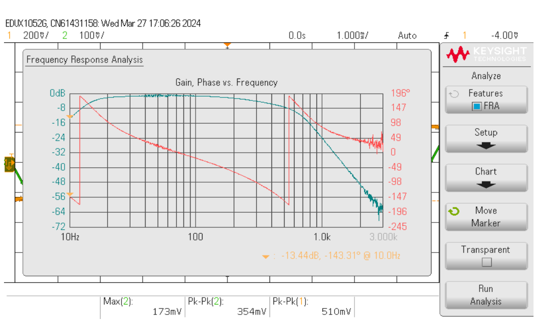


Fig. 15. result observed in oscilloscope for 4th order butter worth band pass filter.

The observed cutoffs for pass band is from 15 Hz to 250.2 Hz the cutoff is closer to the desired frequency.

B. Chebyshev filter

1) 3rd order

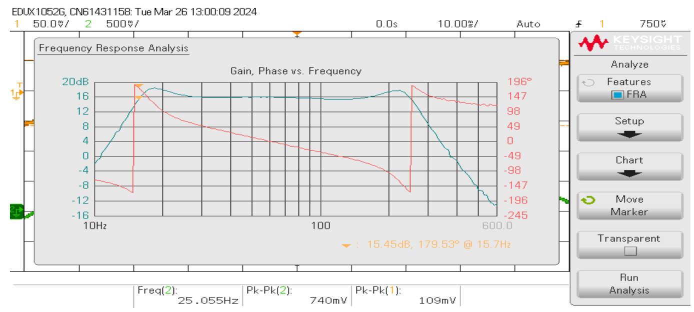


Fig. 16. result observed in oscilloscope for 3rd order Chebyshev filter.

2) 4th order

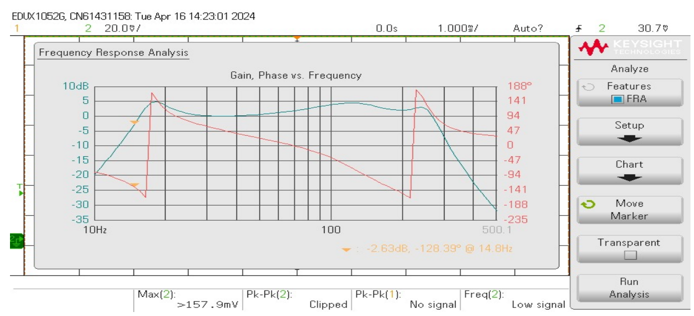


Fig. 17. result observed in oscilloscope for 4th order Chebyshev filter.

XIV. Total Harmonic distortion

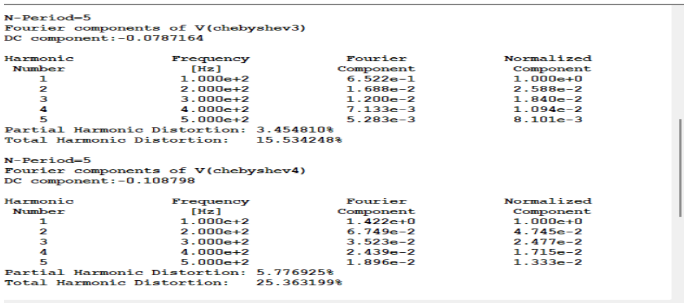
```
N-Period=5
Fourier components of V(butter3)
DC component:-0.234686

Harmonic        Frequency       Fourier         Normalized
 Number            [Hz]         Component        Component
    1           1.000e+2        9.688e+0        1.000e+0
    2           2.000e+2        1.927e-1        1.989e-2
    3           3.000e+2        1.086e-1        1.121e-2
    4           4.000e+2        1.095e-1        1.130e-2
    5           5.000e+2        9.610e-2        9.920e-3
Partial Harmonic Distortion: 2.733939%
Total Harmonic Distortion:    13.156655%

N-Period=5
Fourier components of V(butter4)
DC component:-0.0274928

Harmonic        Frequency       Fourier         Normalized
 Number            [Hz]         Component        Component
    1           1.000e+2        9.327e-1        1.000e+0
    2           2.000e+2        3.023e-2        3.241e-2
    3           3.000e+2        1.939e-2        2.079e-2
    4           4.000e+2        1.218e-2        1.306e-2
    5           5.000e+2        9.119e-3        9.777e-3
Partial Harmonic Distortion: 4.181667%
Total Harmonic Distortion:    16.605721%
```

Fig. 18. Total harmonic distortion of butter worth filters



```
N-Period=5
Fourier components of V(chebyshev3)
DC component:-0.0787164

Harmonic      Frequency      Fourier      Normalized
 Number         [Hz]        Component     Component
    1        1.000e+2       6.522e-1      1.000e+0
    2        2.000e+2       1.688e-2      2.588e-2
    3        3.000e+2       1.200e-2      1.840e-2
    4        4.000e+2       7.133e-3      1.094e-2
    5        5.000e+2       5.283e-3      8.101e-3
Partial Harmonic Distortion: 3.454810%
Total Harmonic Distortion:   15.534248%

N-Period=5
Fourier components of V(chebyshev4)
DC component:-0.108798

Harmonic      Frequency      Fourier      Normalized
 Number         [Hz]        Component     Component
    1        1.000e+2       1.422e+0      1.000e+0
    2        2.000e+2       6.749e-2      4.745e-2
    3        3.000e+2       3.523e-2      2.477e-2
    4        4.000e+2       2.439e-2      1.715e-2
    5        5.000e+2       1.896e-2      1.333e-2
Partial Harmonic Distortion: 5.776925%
Total Harmonic Distortion:   25.363199%
```

Fig. 19. Total harmonic distortion of Chebyshev filters

from this we can conclude that total harmonic distortion in butter worth is less compared to Chebyshev filter , this is because Chebyshev filter allows ripples in pass band so the distortion is a little higher than butter worth.

XV. Temperature analysis

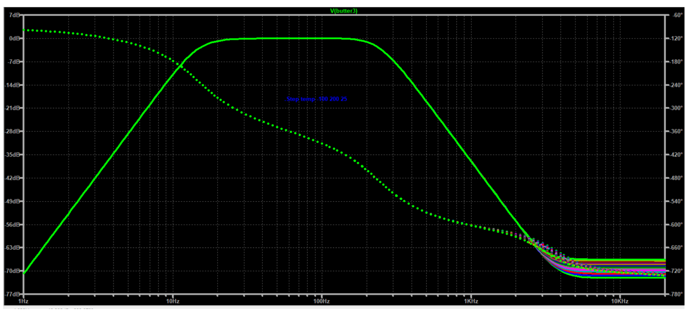


Fig. 20. Temperature analysis of 3rd order butter worth filter from -100 to 200 degrees .

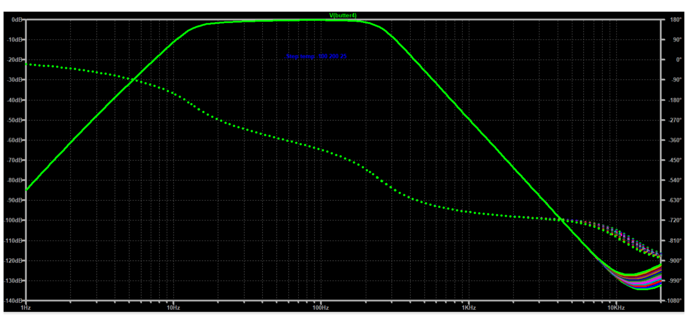


Fig. 21. Temperature analysis of 4 th order butter worth filter from -100 to 200 degrees .

From the simulation results we can observe that our filters are stable on range of temperature that we usually encounter. If the temperature is increased we can observe some variations in stop band, but this doesn't effect our pass band so we can use the filters without any mistakes.

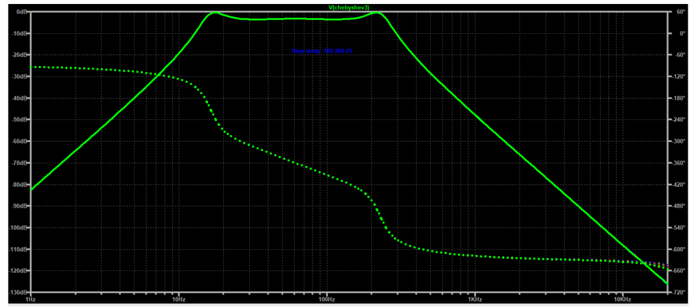


Fig. 22. Temperature analysis of 3rd order Chebyshev filter from -100 to 200 degrees .

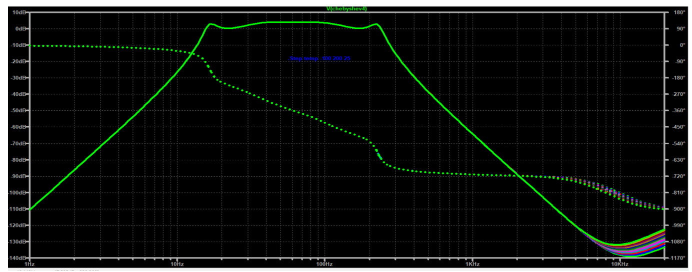


Fig. 23. Temperature analysis of 4th order Chebyshev filter from -100 to 200 degrees .

XVI. Input and output Impedance

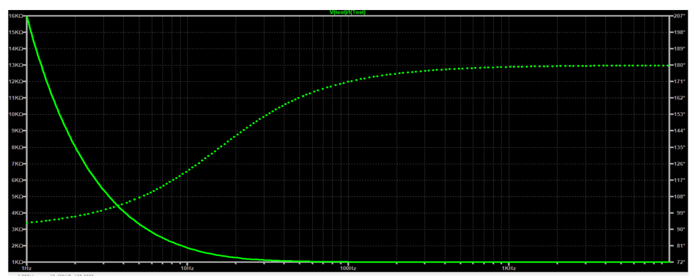


Fig. 24. Input impedance of Butter 3 rd order filter is 1K Ohms .

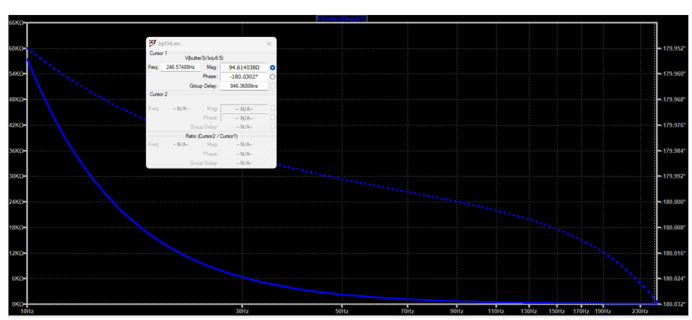


Fig. 25. output impedance of Butter 3 rd order filter is 94 Ohms.

The input impedance of 4th order butter worth filter is 2.57 K Ohms.
The output impedance of 4th order butter worth filter is 100 Ohms.

XVII. Power analysis

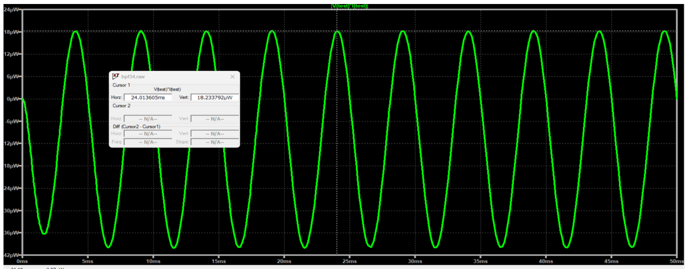


Fig. 26. Input power of 3rd order butter worth filter is 18.2 µW.

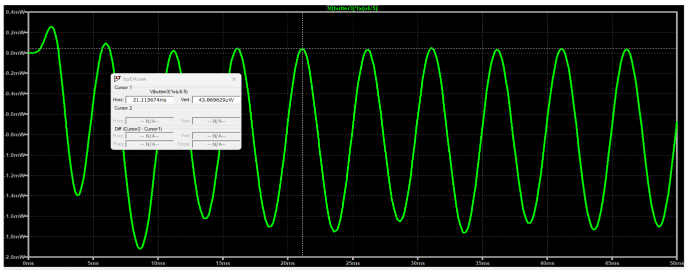


Fig. 27. output power of 3rd order butter worth filter is 43.8 µW.

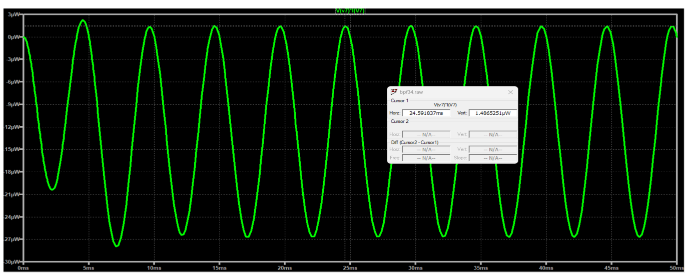


Fig. 28. Input power of 3rd order Chebyshev filter is 1.4865251 µW.

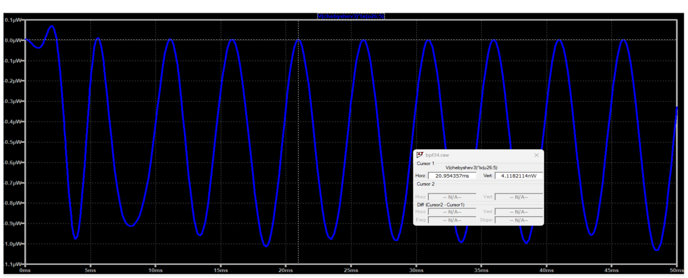


Fig. 29. output power of 4th order Chebyshev filter is 4.1182114 nW.

XVIII. Achieved Characteristics of filter

1) lower CUT-OFF frequency : 15Hz.
2) upper CUT-OFF frequency : 250Hz.
3) Band-width : 235 Hz.
4) Increase in slope between 3rd order and 4 th order : 21 degrees.
5) Results from temperature analysis prove that the filter is stable for varying temperatures.
6) Total harmonic distortion is in desired and controllable range.
7) filters are consuming very less power in the order of micro watts we can observe from power analysis.
8) input and output impedance are in desired ranges.

XIX. Data points

we saved data points from filter outputs from oscilloscope below are the links for the sheets.
Click on the link embedded text below to open Excel sheet corresponding to that filter's data points.

1) 3rd order butter worth
2) 4th order butter worth
3) 3rd order Chebyshev
4) 4th order Chebyshev

We have successfully created 3rd order and 4th order filters in butter worth and Chebyshev models and compared them in various analysis (Temperature , Total Harmonic distortion , power analysis etc.