EC5.203 Communication Theory I (3-1-0-4):

Lecture 7:
Analog Communication Techniques: Amplitude Modulation - 3

Feb. 06, 2025

INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY

HYDERABAD

Recap

Key Concepts

- Two ways of encoding info in complex envelope
  - I and Q: amplitude modulation (several variants)
  - Envelope and phase: angle modulation (constant envelope)
- For example, for a sinusoid carrier

$$A_c(t)\cos(2\pi f_c(t)t + \theta_c(t))$$

where $A_c(t)$, $f_c(t)$, $\theta_c(t)$ are the amplitude, frequency, and the phase of the carrier respectively.

Amplitude Modulation

Frequency Modulation

Phase Modulation

AM: Double Sideband Suppressed Carrier

- Here the message $m(t)$ modulates the I component of the passband signal $u(t)$ and is given by

$$u_{DSB}(t) = m(t) \cdot A\cos(2\pi f_c t)$$

while the Fourier transform is given by

$$U_{DSB}(f) = \frac{A}{2}(M(f - f_c) + M(f + f_c))$$

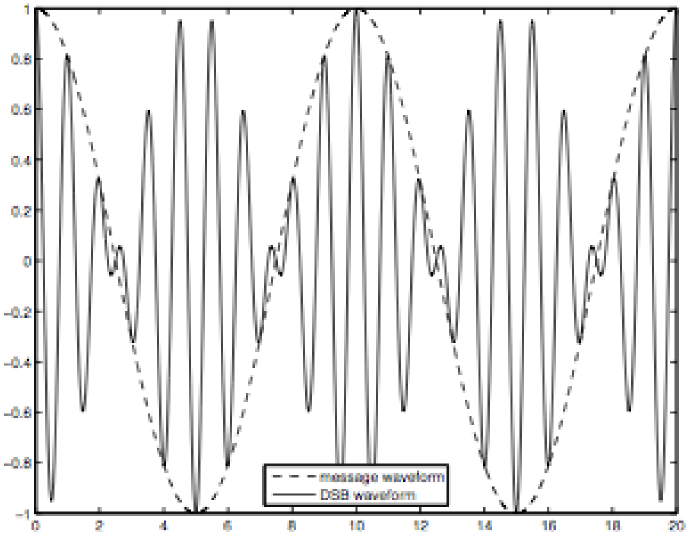
DSB-SC signal for sinusoidal message

Here the signal is given by

$$u_{DSB}(t) = A_m \cos(2\pi f_m t) \cdot A \cos(2\pi f_c t)$$

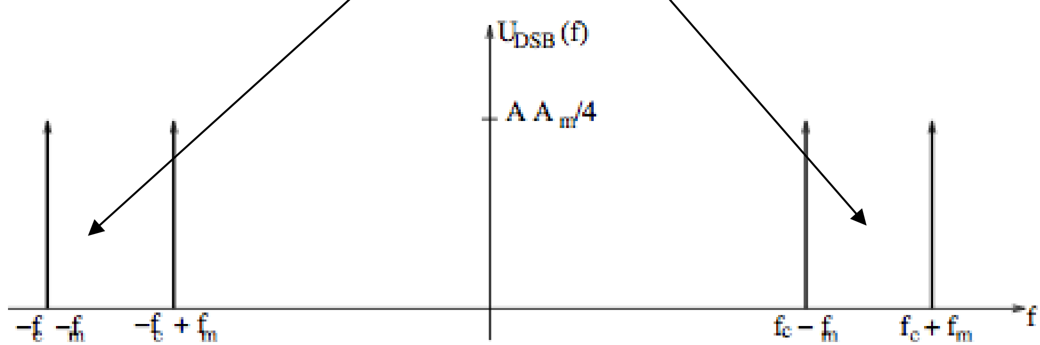
while the Fourier transform is given by

$$U_{DSB}(f) = \frac{AA_m}{4}\{\delta(f - f_c - f_m) + \delta(f - f_c + f_m) + \delta(f + f_c + f_m) + \delta(f + f_c - f_m)\}$$



(a) DSB time domain waveform

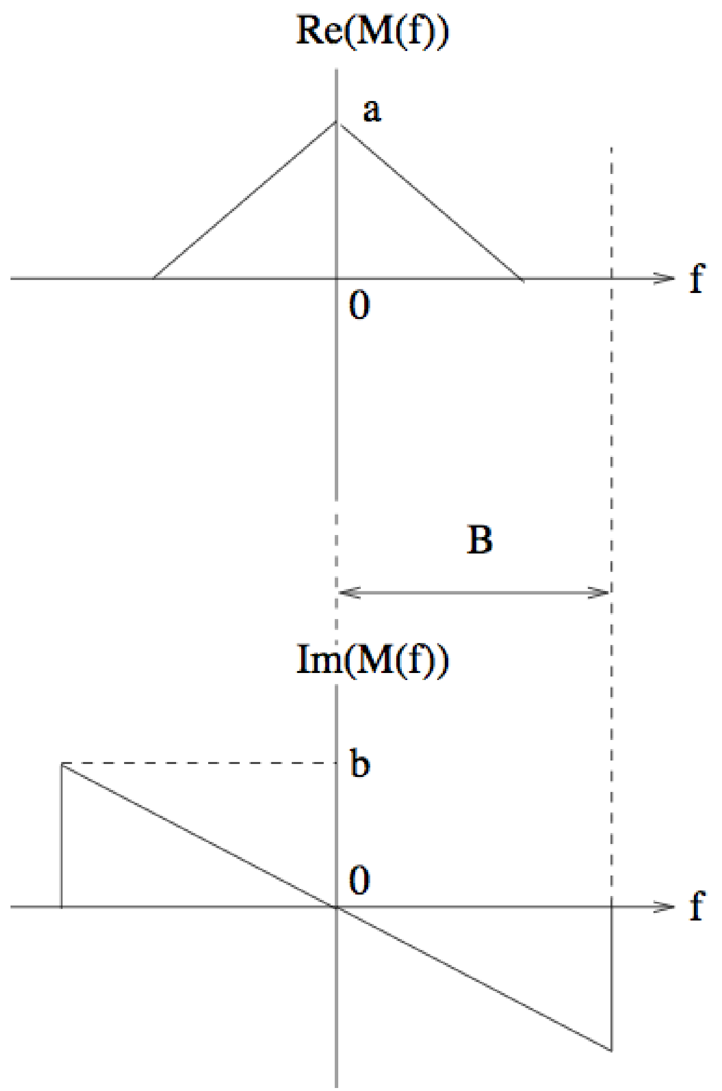
No impulses at f_c or $-f_c$!



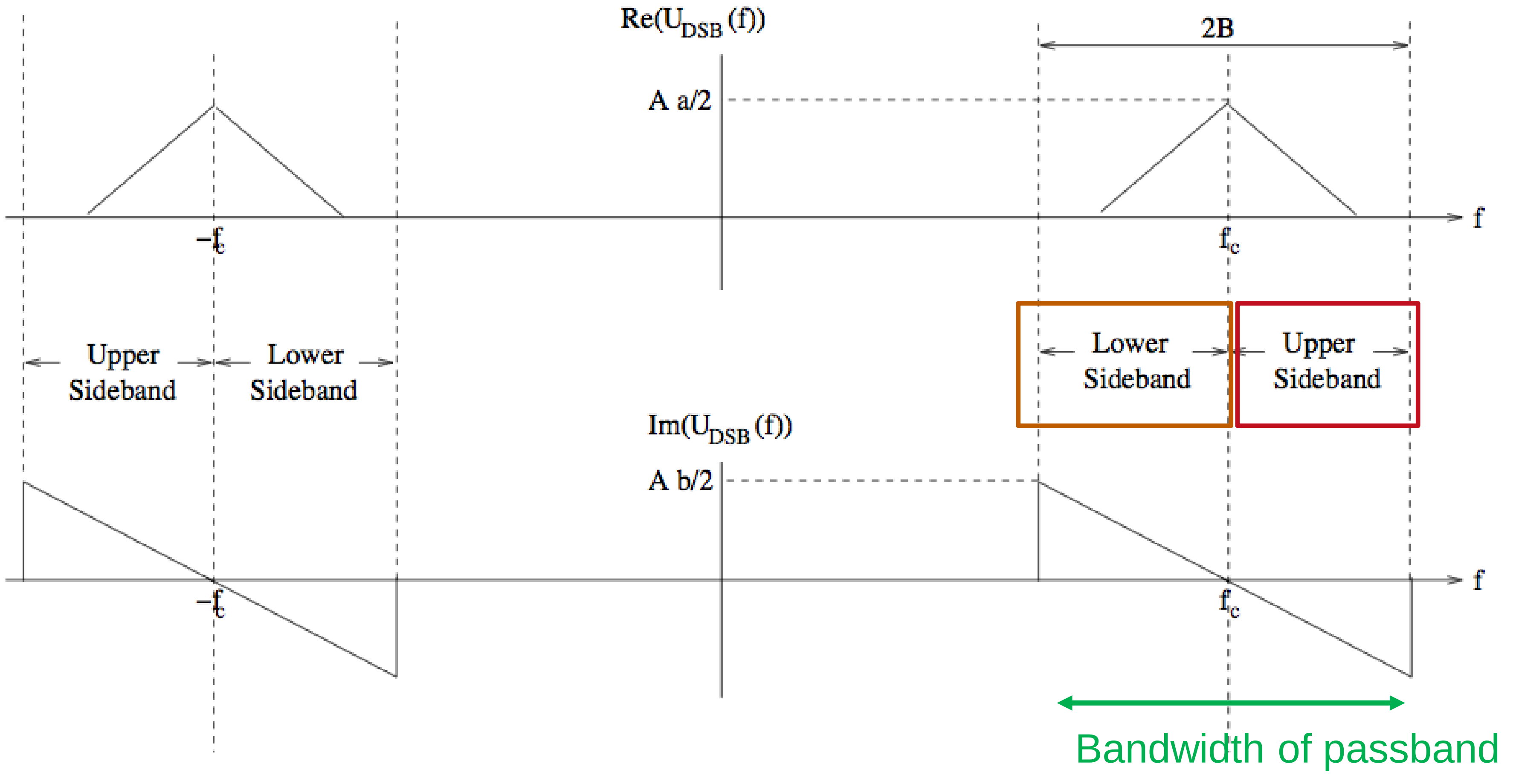
(b) DSB spectrum

Example 2

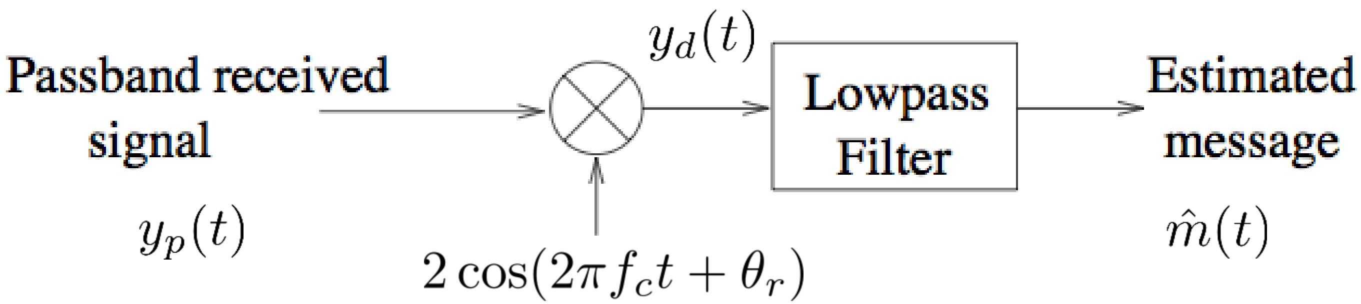
- Consider a message signal $m(t)$ with following frequency response $M(f)$



DSB-SC spectrum for Example 2



Demodulation of DSB-SC



- Standard downconverter to recover I component
- The received signal in the passband is

$$y_p(t) = Am(t)\cos(2\pi f_c t)$$

where θ_r is the phase difference arising from the phase offset with respect to local carrier at Rx.

- The output of multiplier followed by low pass filter is

$$\hat{m}(t) = Am(t)\cos\theta_r$$

Need of Coherent Detection

- The output of multiplier followed by low pass filter is

$$\hat{m}(t) = Am(t)\cos\theta_r$$

- For $\theta_r = 0$, $\hat{m}(t) = Am(t)$
- For $\theta_r = \pi/2$, $\hat{m}(t) = 0$
- For $\theta_r(t) = 2\pi\Delta f t + \phi$, time varying signal degradation in amplitude and unwanted sign changes.
- Need of synchronization of phase of the local oscillator with the phase of incoming signal:
  - Phased locked loop (PLL)
- Switch to other AM techniques which do not need synchronization
  - Conventional AM or DSB (with carrier)

Conventional AM

- Add a large carrier component to a DSB-SC signal so that the passband has the following form

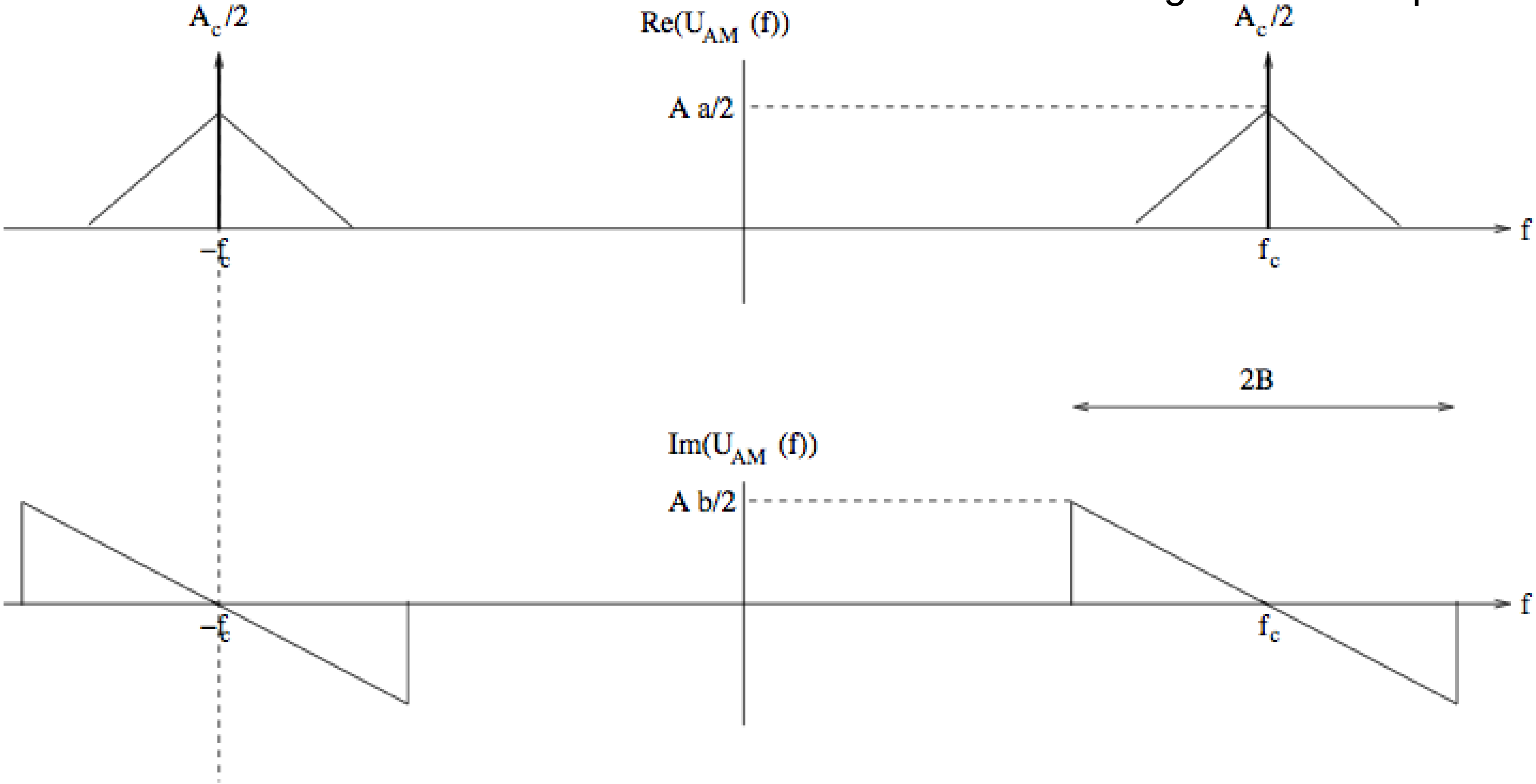
$$\begin{aligned} u_{\mathrm{AM}}(t) &= (Am(t) + A_c)\cos(2\pi f_c t) \\ &= Am(t)\cos(2\pi f_c t) + A_c\cos(2\pi f_c t) \end{aligned}$$

- Taking Fourier transform

$$U_{\mathrm{AM}}(f) = \frac{A}{2}(M(f-f_c)+M(f+f_c))+\frac{A_c}{2}(\delta(f-f_c)+\delta(f+f_c))$$

Conventional AM: spectrum

Strong carrier component



Sidestepping sync requirement

- The envelope (or magnitude of complex envelope) does not depend on carrier phase
- Suppose we can extract the envelope of a passband signal (will soon see a simple circuit for this purpose)
  - Does not require carrier sync
- Can we recover the message?

Modulation Index

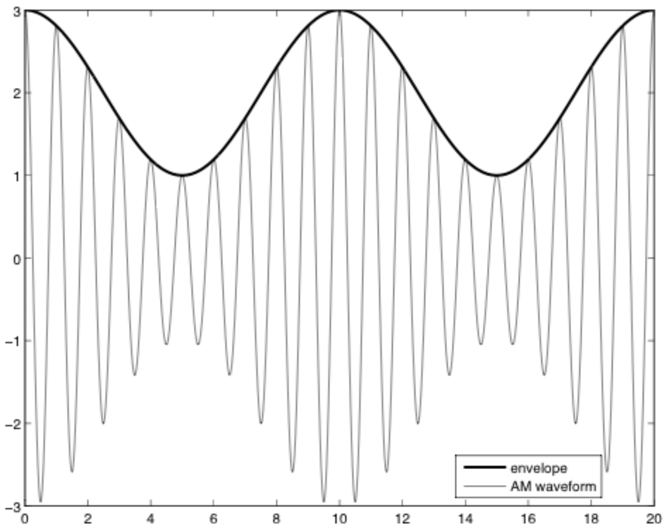
- Condition needed for envelope to preserve message info

$$A\ m(t) + A_c > 0 \quad \forall t$$
$$A \min_t m(t) + A_c > 0$$

- Can be expressed in terms of modulation index

$$a_{\text{mod}} = \frac{AM_0}{A_c} = \frac{A|\min_t m(t)|}{A_c}$$

- For signal to be recoverable, $a_{\text{mod}} \leq 1$.



AM signal in terms of modulation index

- Convenient to normalize message so that the largest negative swing is -1

$$m_n(t) = \frac{m(t)}{M_0} = \frac{m(t)}{|\min_t m(t)|}$$

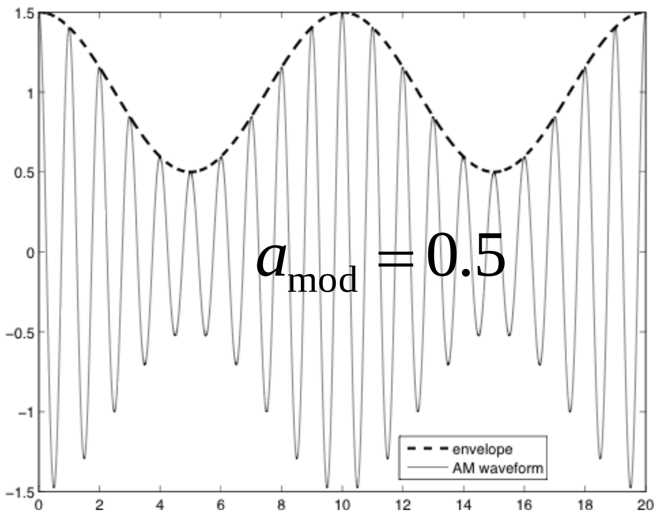
$$\min_t m_n(t) = \frac{\min_t m(t)}{M_0} = -1$$

- AM signal in terms of modulation index and normalized message

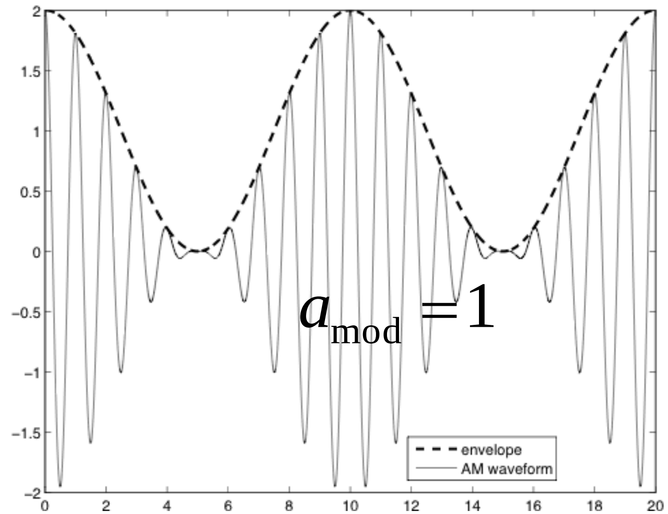
$$y_p(t) = B(1 + a_{\mathrm{mod}} m_n(t)) \cos(2\pi f_c t + \theta_r)$$

Effect of modulation index

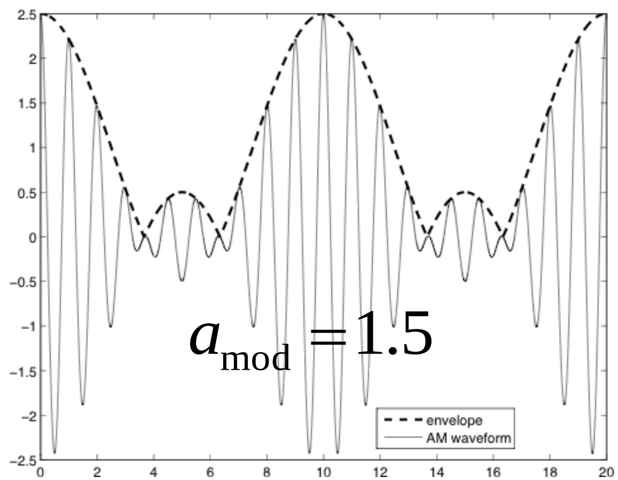
Example of sinusoidal message



Envelope = message + DC

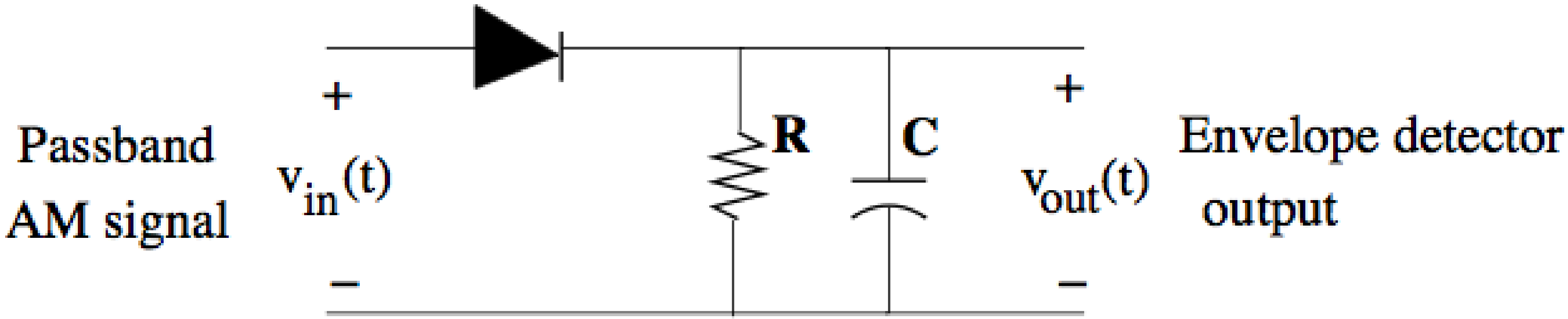


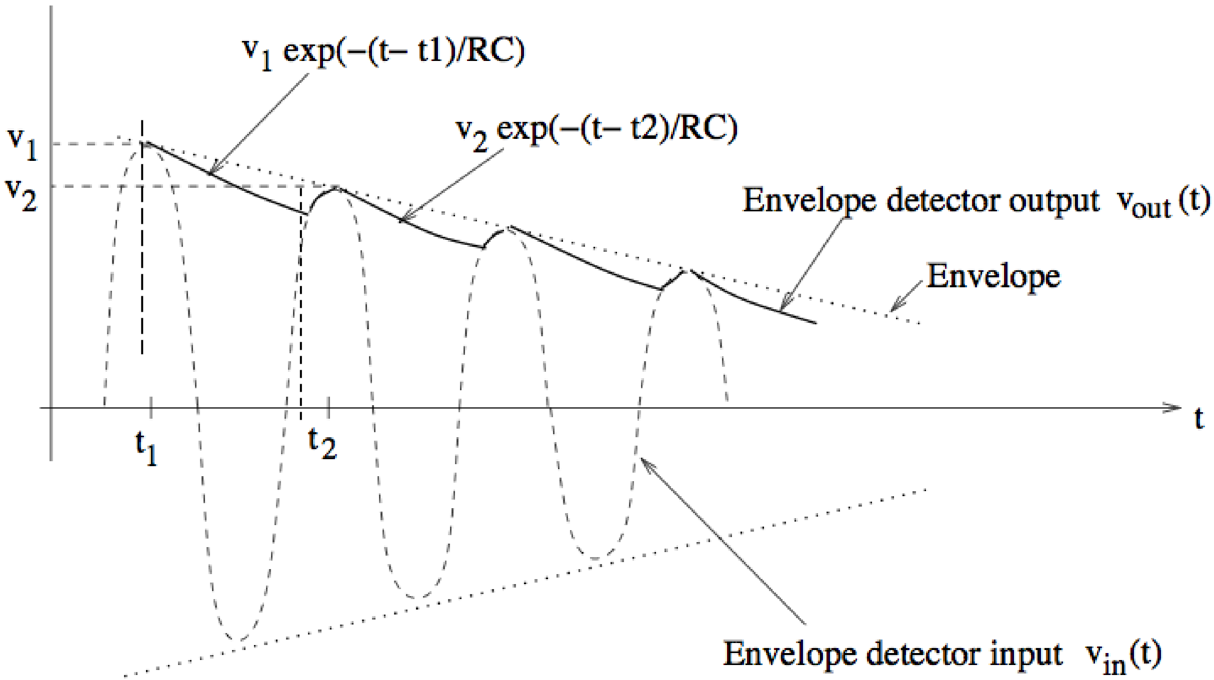
Envelope = message



Message info not preserved in envelope

Envelope Detectors





- At $t = t_1$, diode reverse biases and capacitor starts discharging.
- When $t = t_2'$, diode forward biases and capacitor starts charging.

Positive carrier cycle ➔ capacitor charges up (reaches value of envelope)
Negative carrier cycle ➔ capacitor discharges with RC time constant

Conventional AM modulation

- Use of multiplier
  - Several ways: Analog multiplier such as Sheingold, Variable gain amplifier, etc
  - It is rather difficult to maintain linearity in this kind of amplifier
  - They are expensive
- Few of other simple yet practical methods
  - Non-linear modulators
  - Switching modulators

B.P.Lathi pages: 155-159

Power efficiency of conventional AM

- DSB expression

$$u_{\mathrm{AM}}(t) = Am(t)\cos(2\pi f_c t) + \boxed{A_c \cos(2\pi f_c t)}$$

Extra Non-information carrying component

- Power efficiency is given by

$$\eta = \frac{\text{Power in information carrying signal}}{\text{Power in total signal}}$$

- Prove that power efficiency for conventional AM is given by

$$\eta_{\mathrm{AM}} = \frac{a_{\mathrm{mod}}^2 \overline{m_n^2}}{1 + a_{\mathrm{mod}}^2 \overline{m_n^2}}$$

- Further prove that

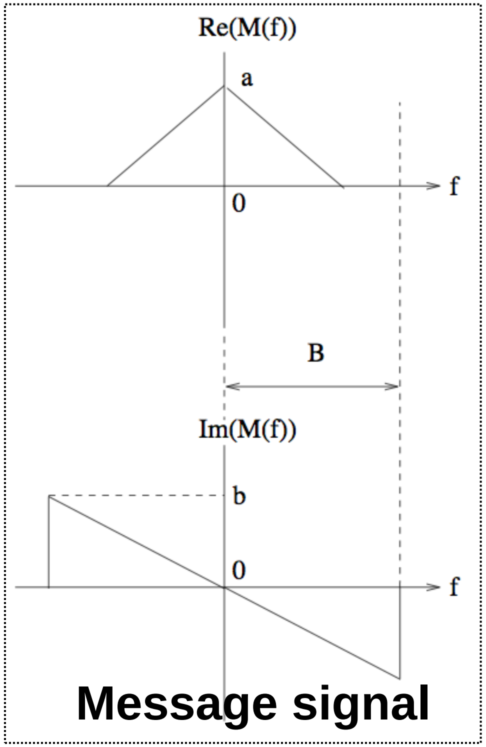
$$\eta_{\mathrm{AM}} \leq 50\%$$

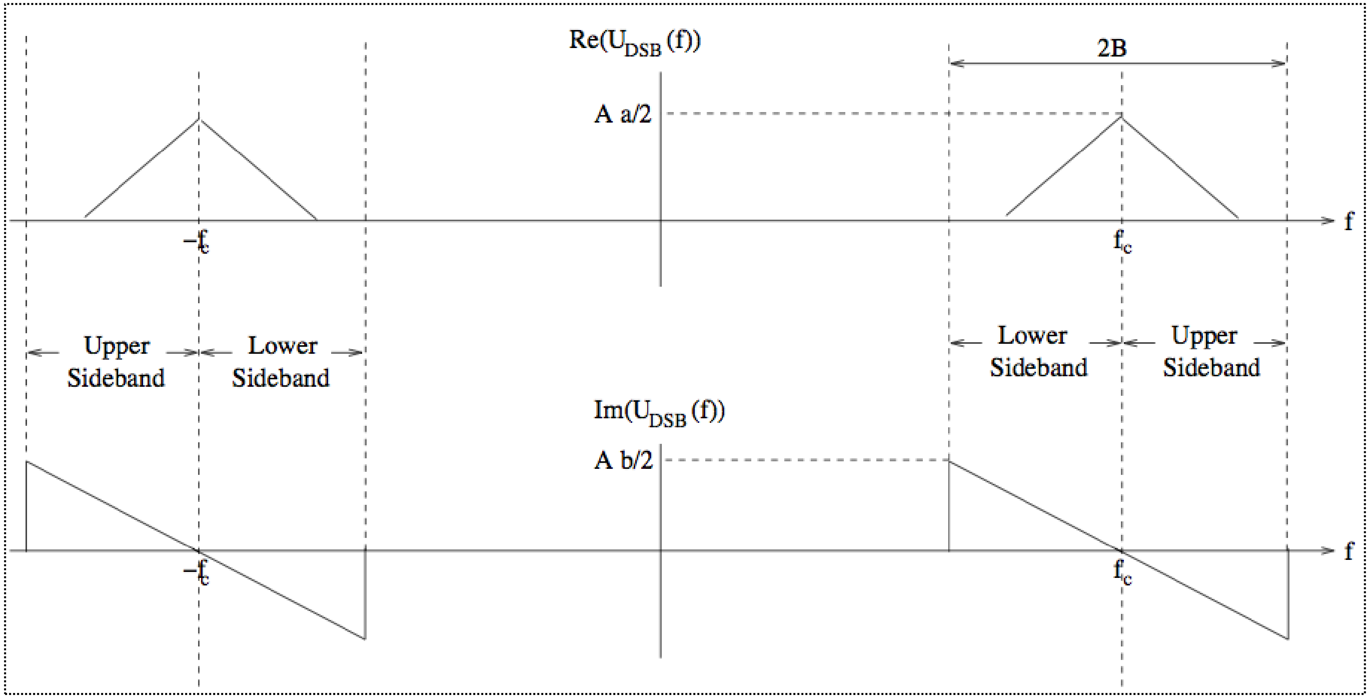
- Solve: Find η_{AM} for sinusoidal message signal $m(t) = A_m \cos(2\pi f_m t)$

Todays' Class

Amplitude Modulation:
Single Side Band

SSB: Motivation



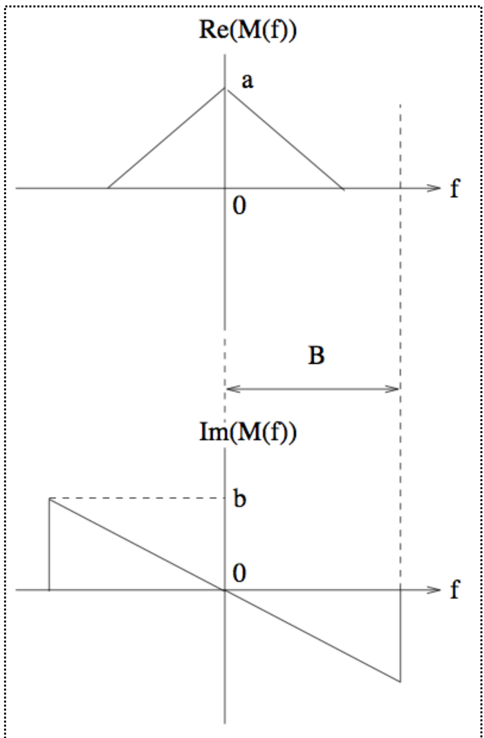


- Each sideband has enough information to extract the original message.

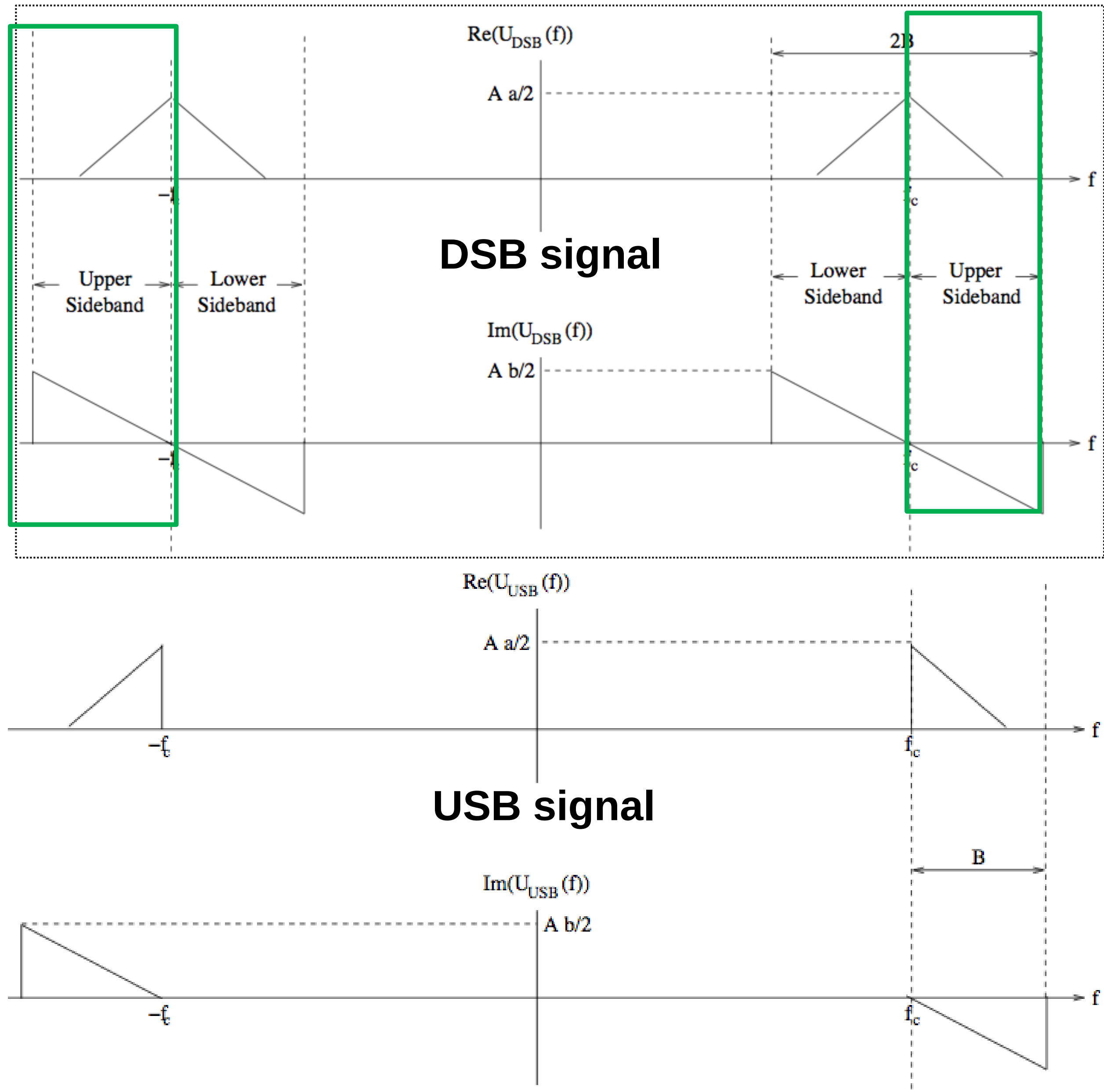
m(t) is complex envelope of DSB

- Message $m(t)$ is the I component of an DSB signal.
- Sending only one sideband reduces our bandwidth requirement by 50%.

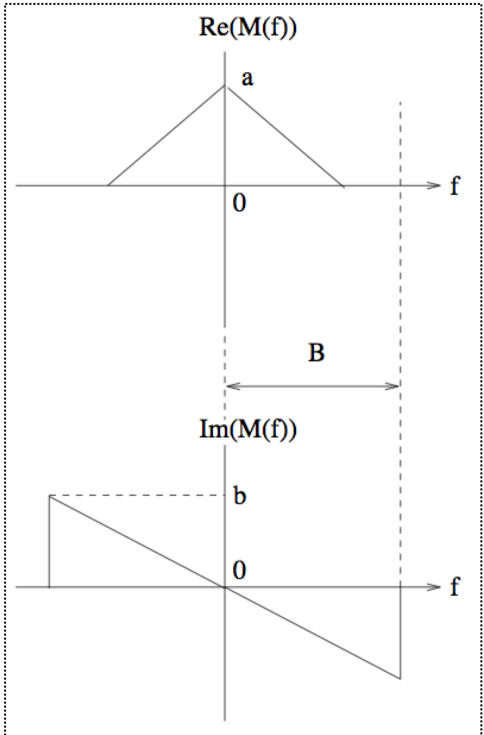
DSB ➔ USB (SSB)



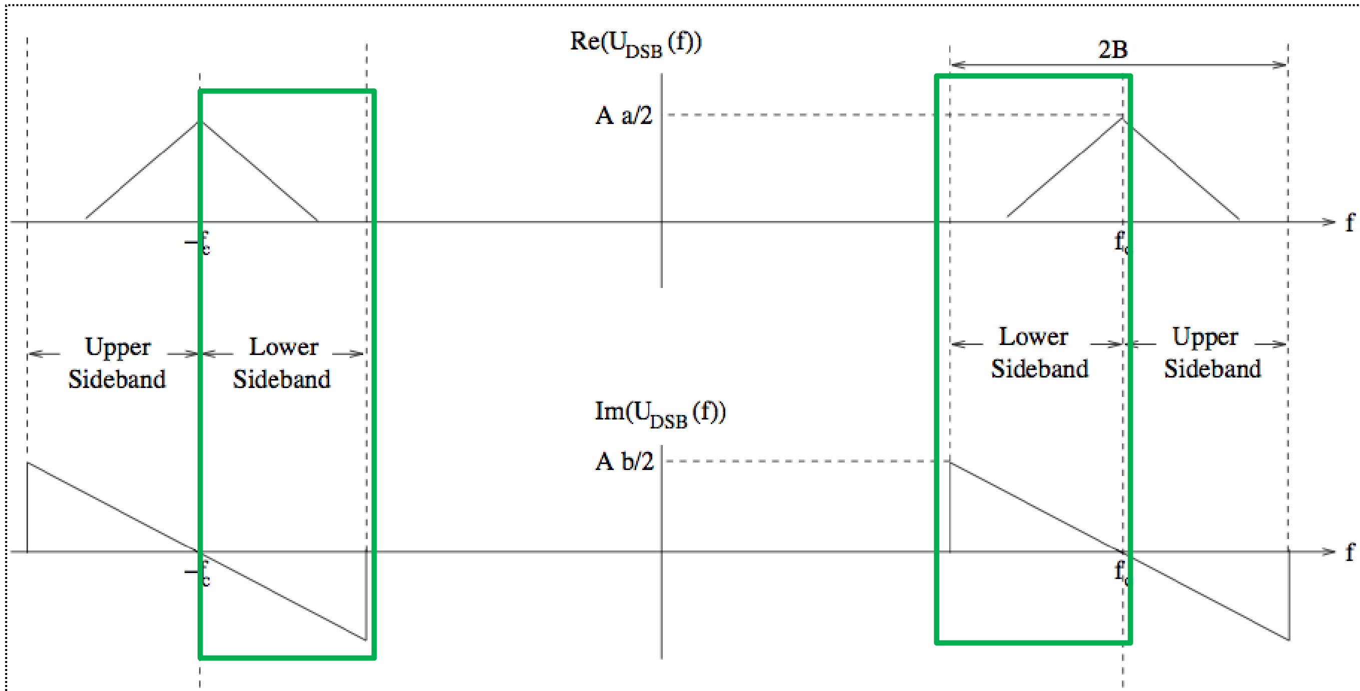
Message signal



DSB → LSB (SSB)

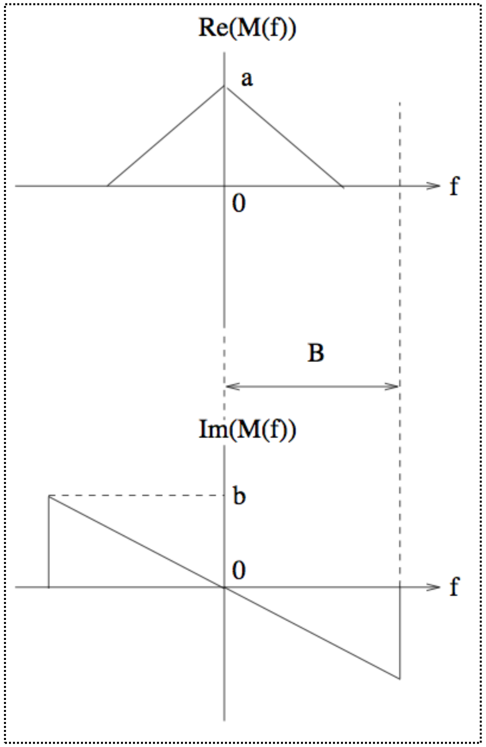


Message signal

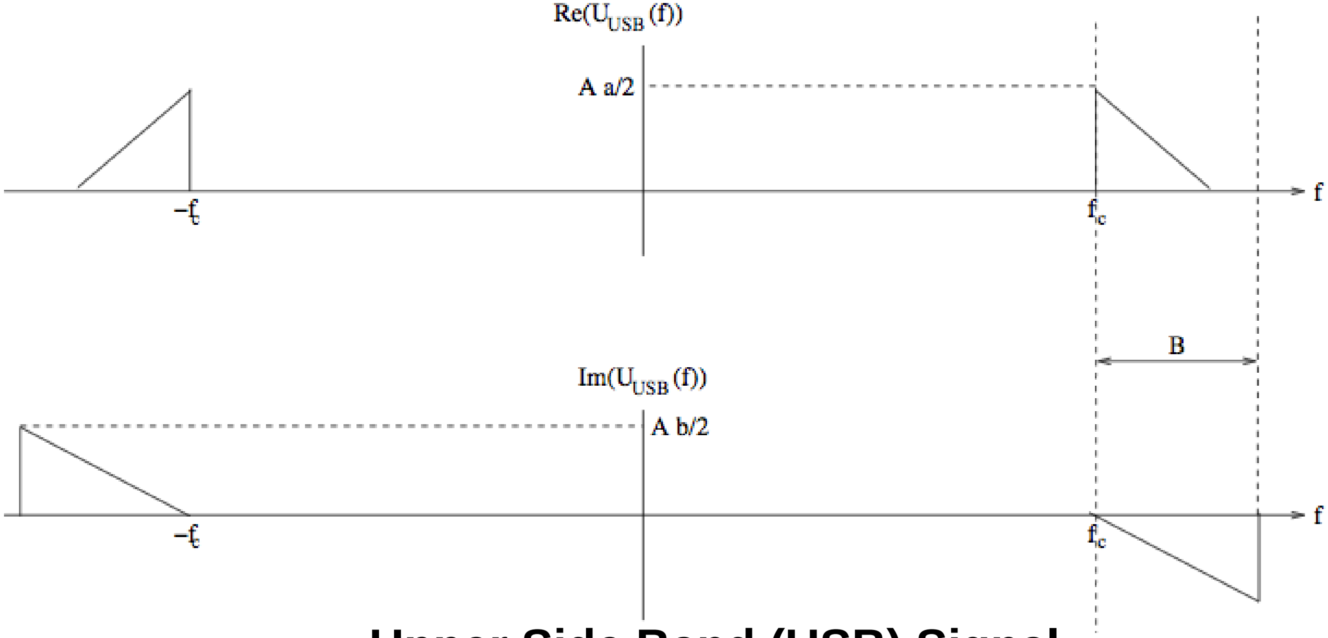


DSB signal

Recover Signal from USB Signal? Solve!

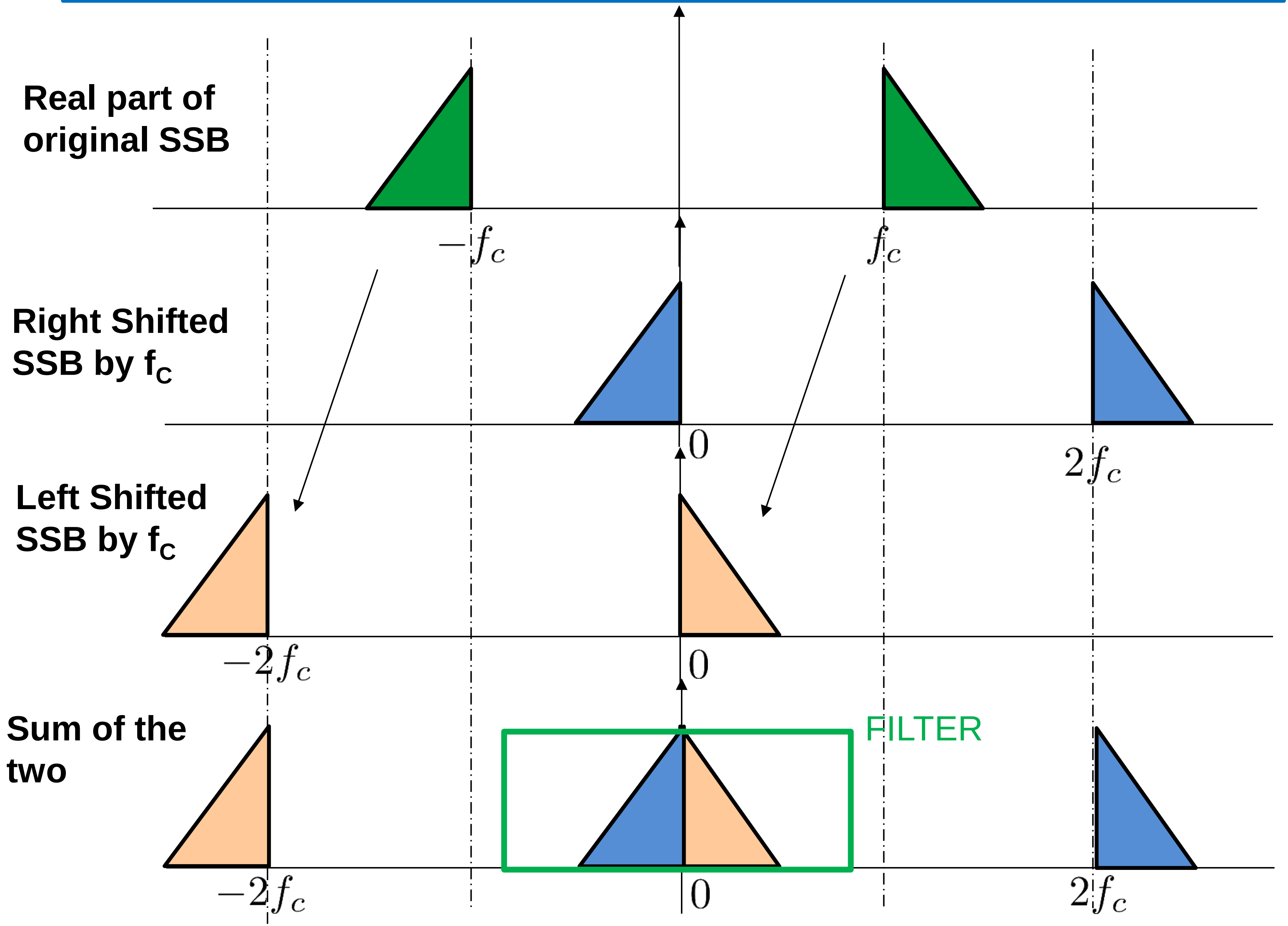


Message signal

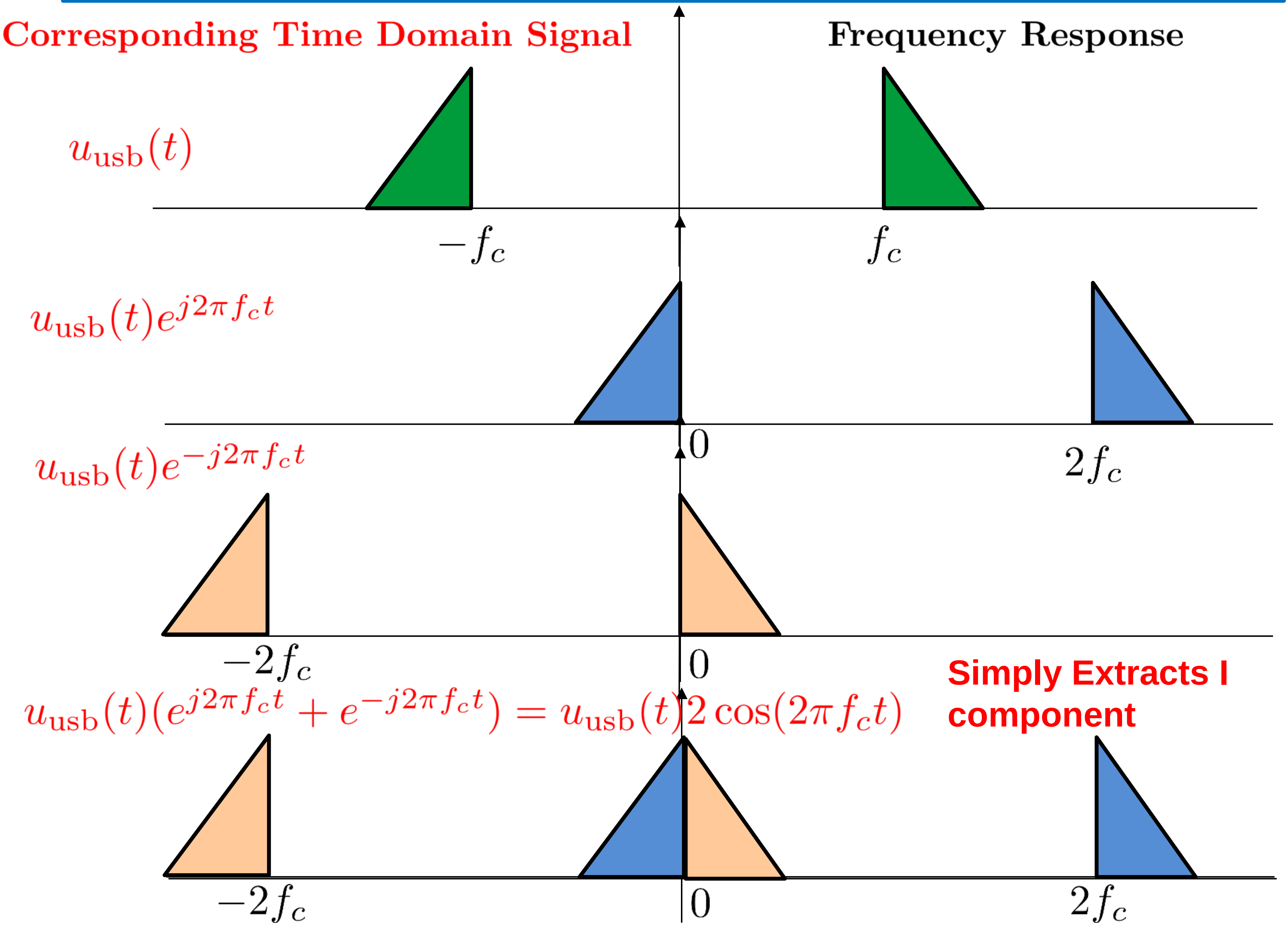


Upper Side Band (USB) Signal

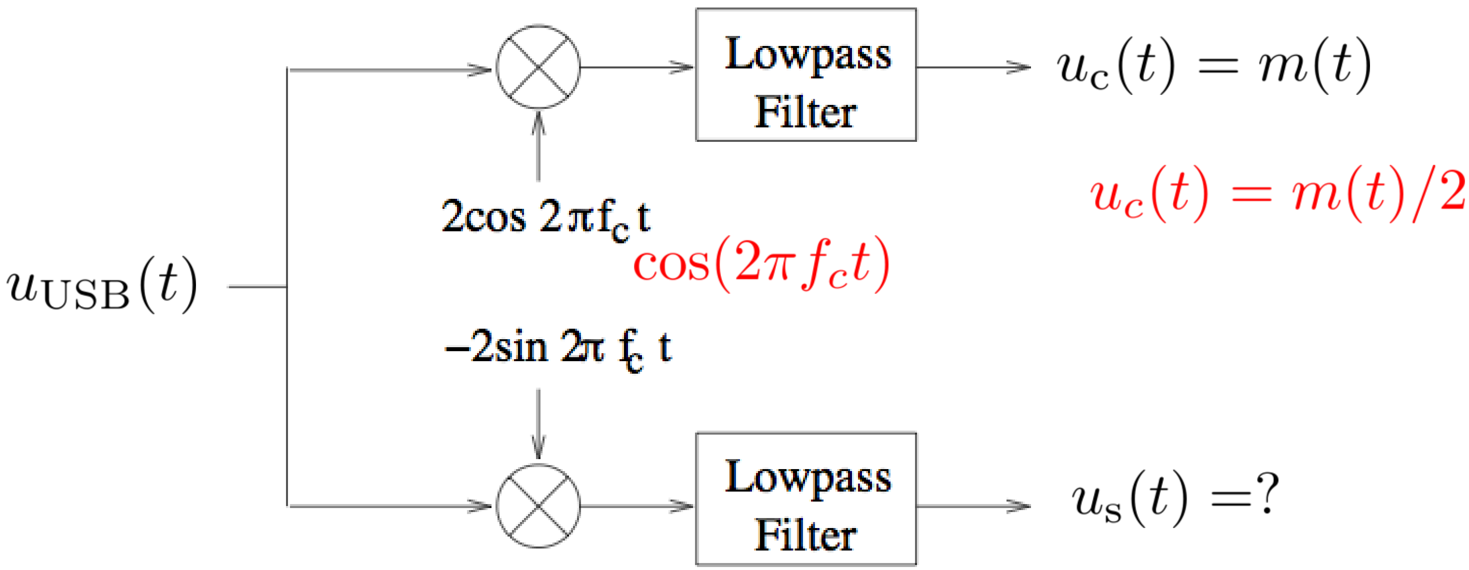
Solution!


Real part of original SSB
$-f_c$
f_c
Right Shifted SSB by f_C
0
$2f_c$
Left Shifted SSB by f_C
$-2f_c$
0
Sum of the two
FILTER
$-2f_c$
0
$2f_c$

Corresponding Time Domain Equations

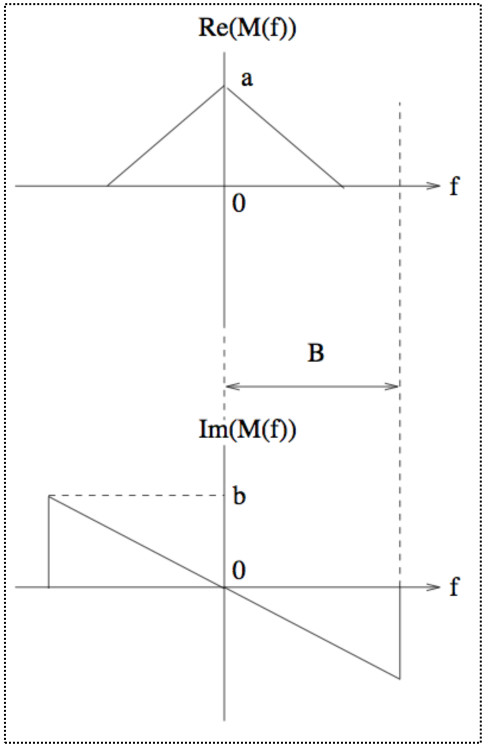


Message signal is I component of filter o/p

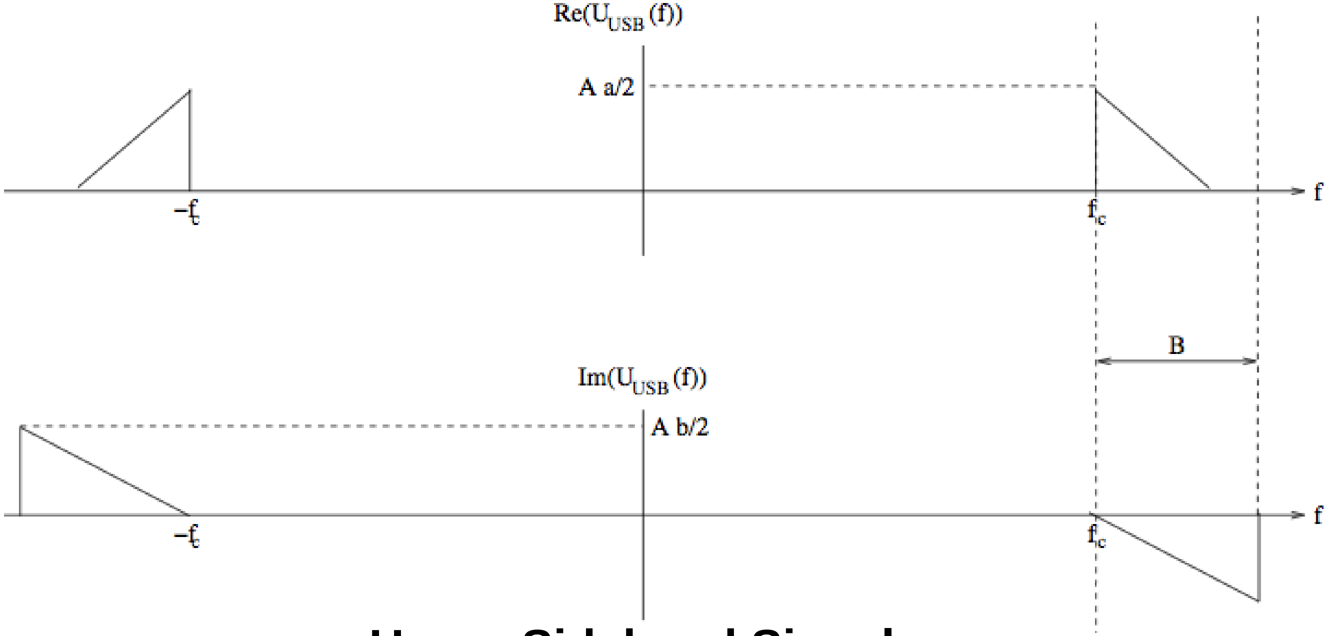


Downconversion
(passband to baseband)

Recover Signal from USB Signal!



Message signal

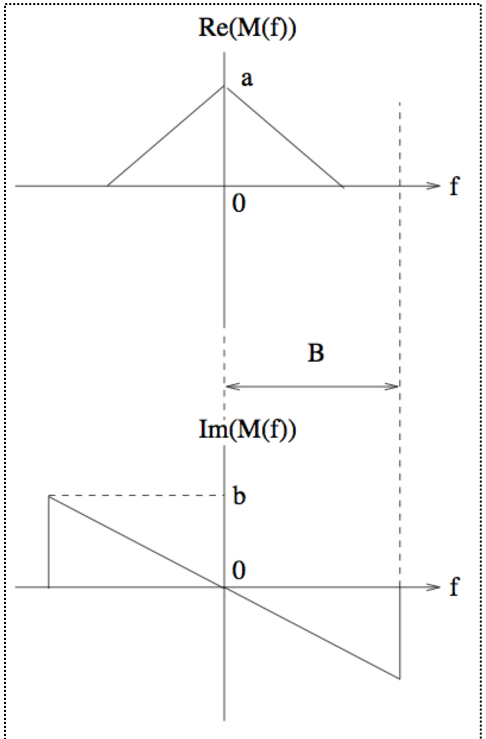


Upper Sideband Signal

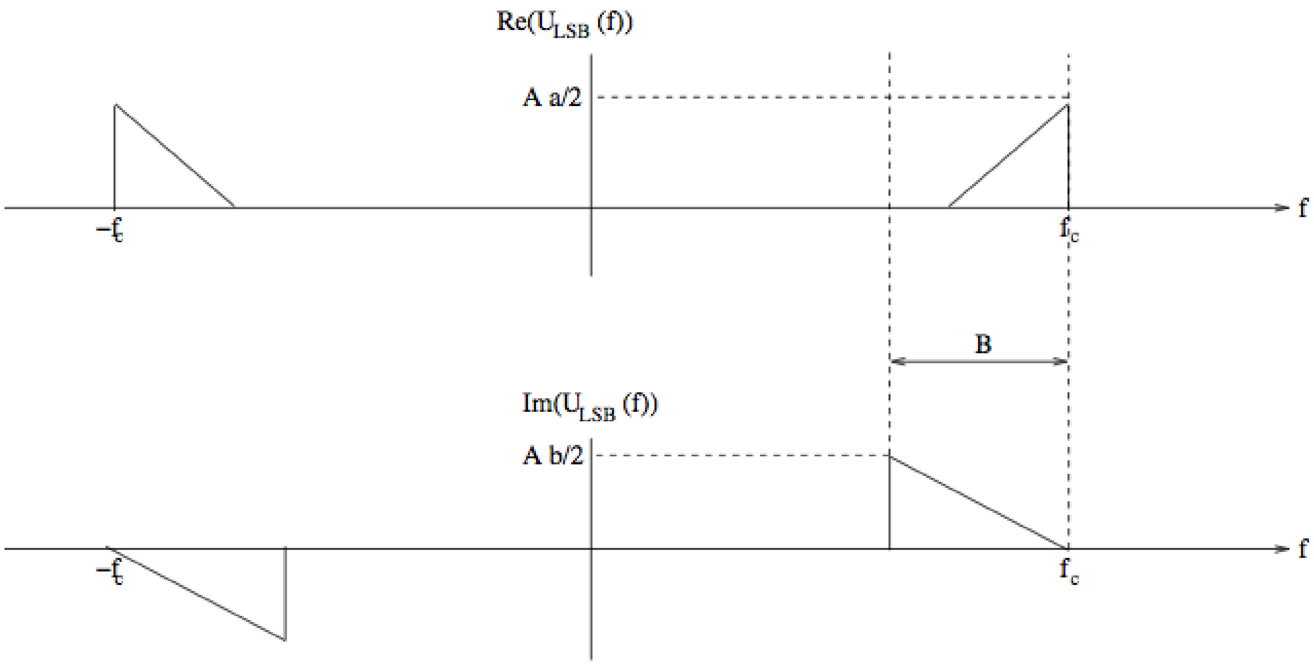
The original signal can be obtained from the USB signal by

1. Shifting the USB signal to right by f_c
2. Shifting the USB signal to left by f_c
3. Add the two signals
4. Pass the resultant signal through low pass filter to filter $2f_c$ component

Recover Signal from LSB Signal?



Message signal



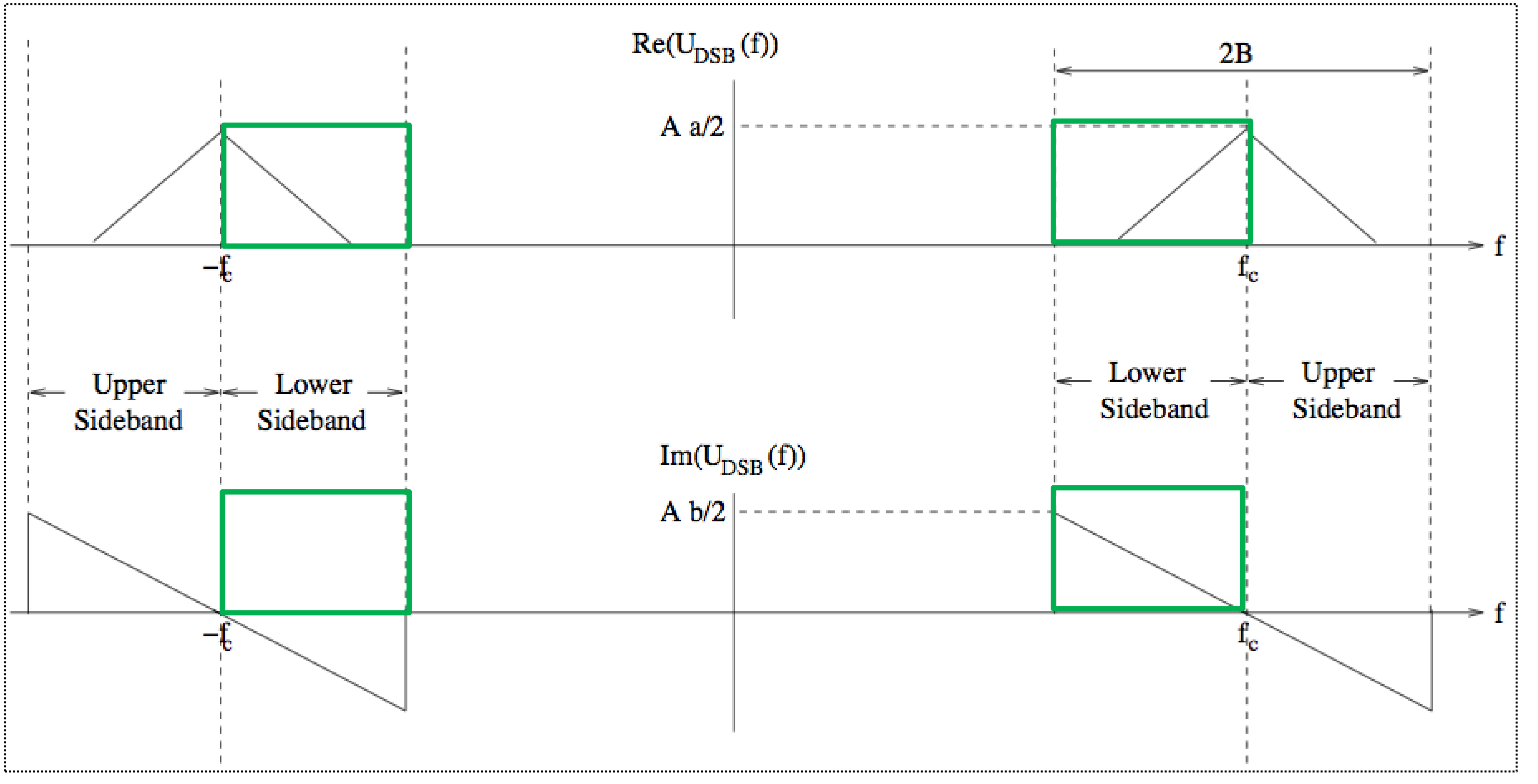
Lower Sideband (LSB) Signal

The original signal can be obtained from the USB signal by

1. Shifting the LSB signal to right by f_c
2. Shifting the LSB signal to left by f_c
3. Add the two signals
4. Pass the resultant signal through low pass filter to filter $2f_c$ component

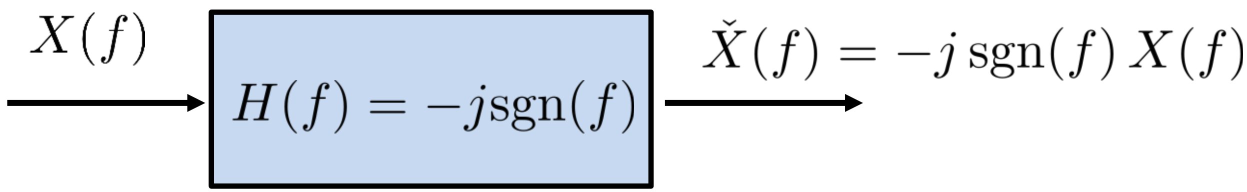
Use of Hilbert Transform for SSB Generation

Motivation: *Requirement of ideal filters*

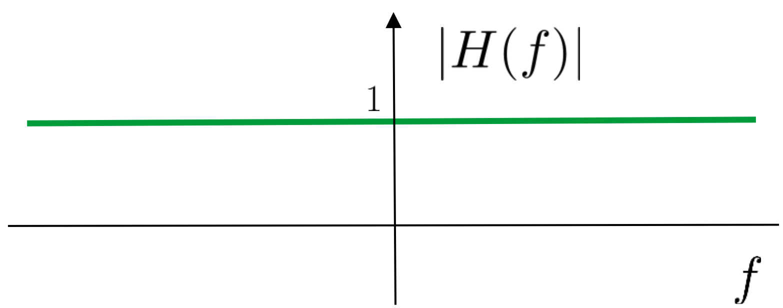


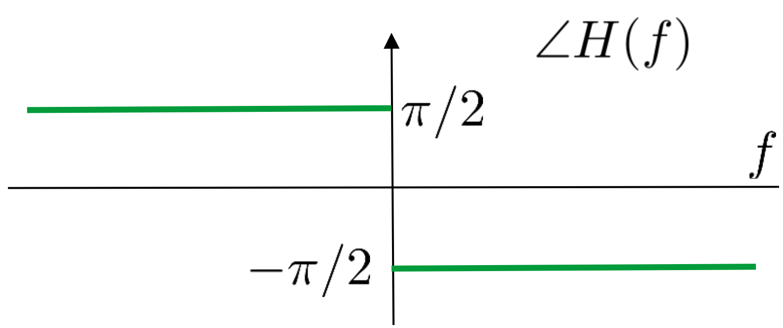
- Logical approach: Filtering one of the sideband requires rectangular filters with sharp cut-off!
- Practically infeasible!!!
- Implementing Hilbert transform in baseband avoids need for sharp filtering at passband!

Hilbert transform

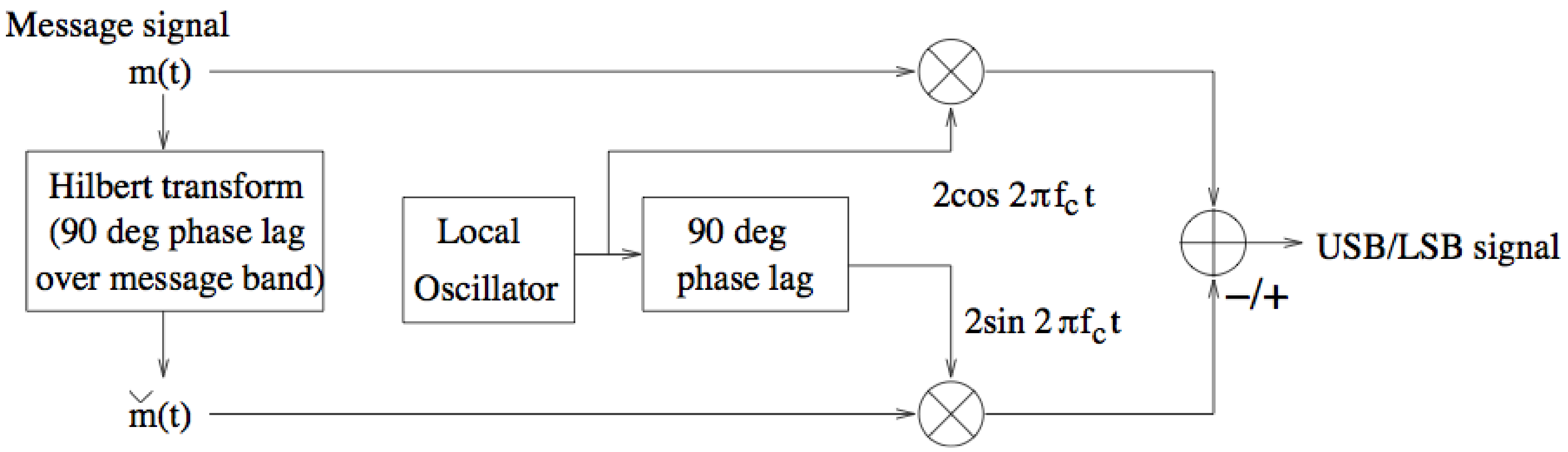


$$H(f) = -j\,\mathrm{sgn}(f) \longleftrightarrow h(t) = \frac{1}{\pi t}$$



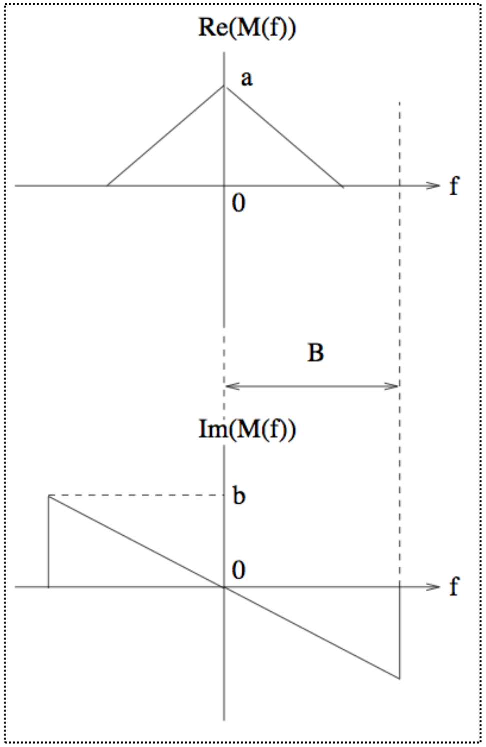


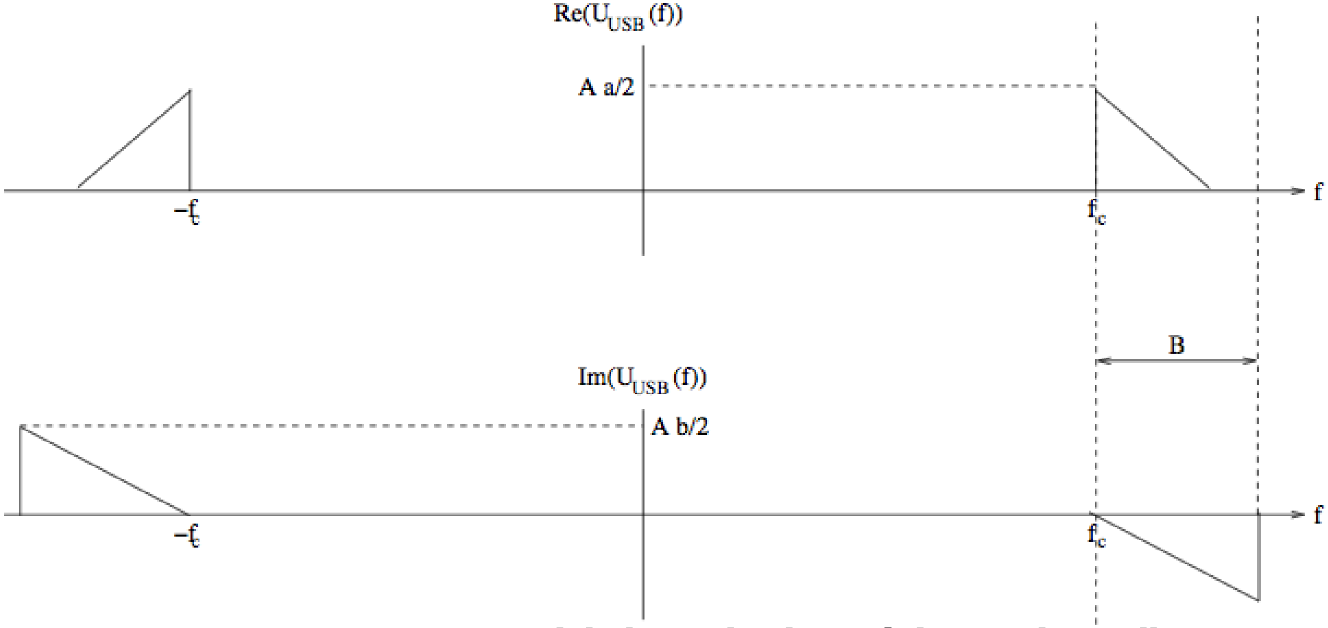
SSB in baseband using Hilbert Transform



- Implementing Hilbert transform in baseband avoids need for sharp filtering at passband!
- In next few slides, we will see why it works!

USB passband signal is real!





Upper Sideband Signal (Passband)

Baseband Message signal

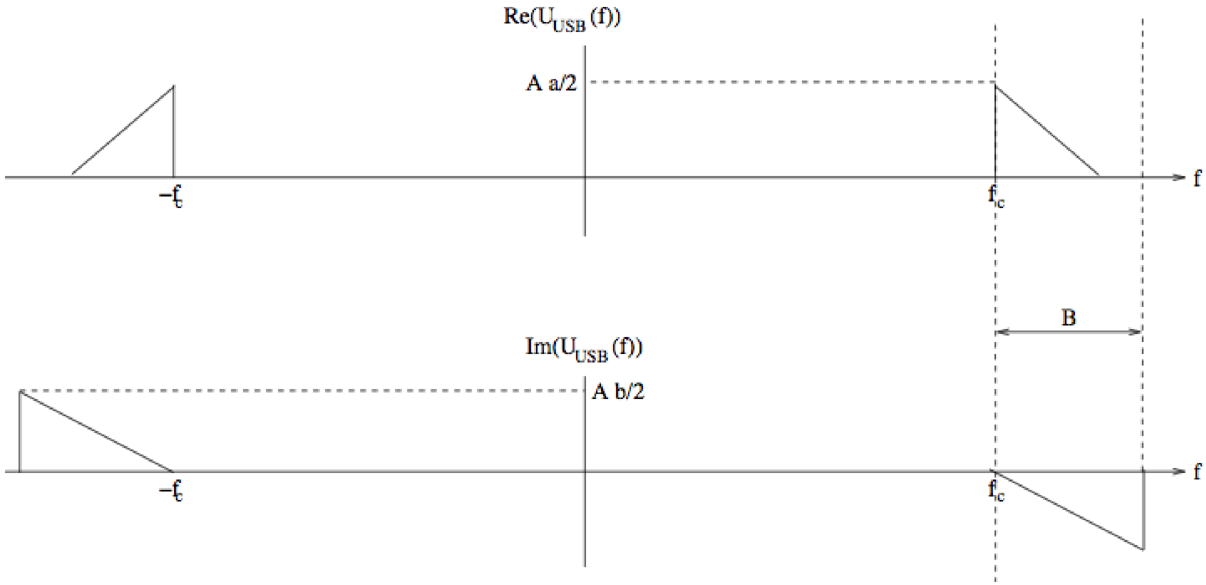
For a real signal $x(t)$

- $X(f) = X^*(-f)$ Even symmetry for magnitude spectrum
- $\mathrm{Re}\{X(f)\} = \mathrm{Re}\{X^*(-f)\}$, i.e., Even Symmetry
- $\mathrm{Im}\{X(f)\} = -\mathrm{Im}\{X^*(-f)\}$ Odd Symmetry

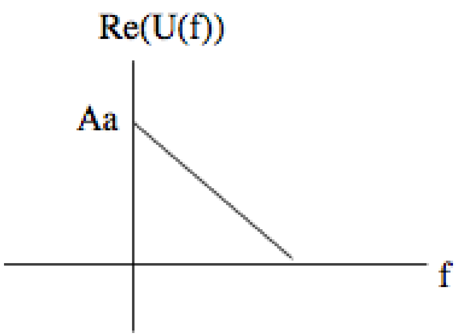
m(t) will be real!

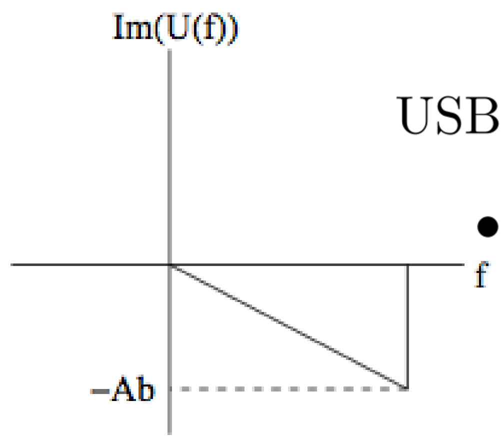
m(t) is complex envelope of DSB
In SSB discussion, u(t) will be complex envelope of USB

Complex envelope for USB signal



USB passband



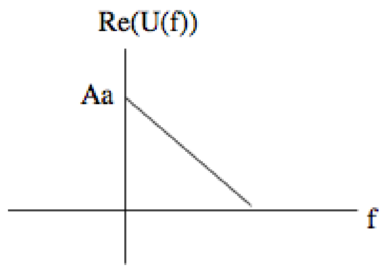


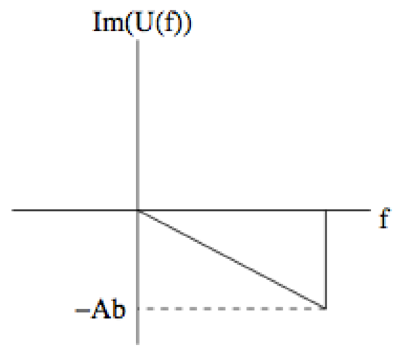
USB baseband signal $u(t)$ is complex

- No odd or even symmetry!

In SSB discussion, u(t) will be complex envelope of USB

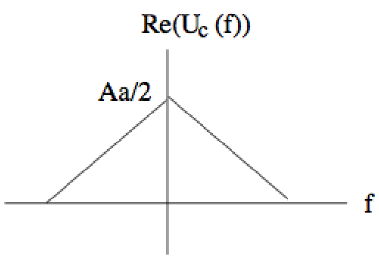
I and Q components for SSB

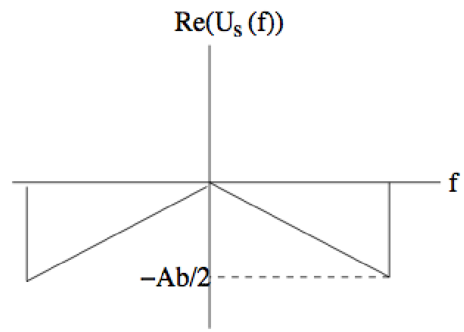


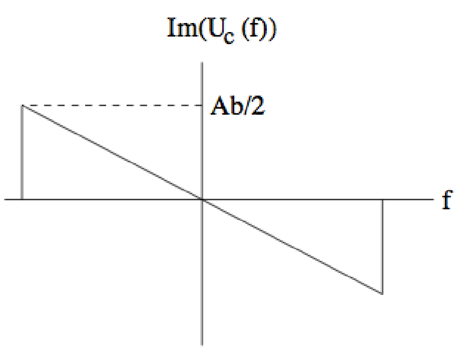


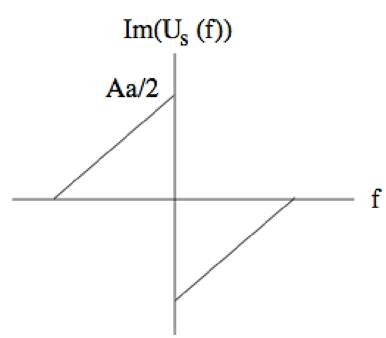
I component

Q component









USB Complex envelope $U(f)$

USB I and Q components

$$U_c(f) = \frac{U(f) + U^*(-f)}{2}$$

$$U_s(f) = \frac{U(f) - U^*(-f)}{2j}$$

Prove

$$U_c(f) = A\,M(f)/2$$

$$U_s(f) = -j\,\mathrm{sgn}(f)\,U_c(f)$$

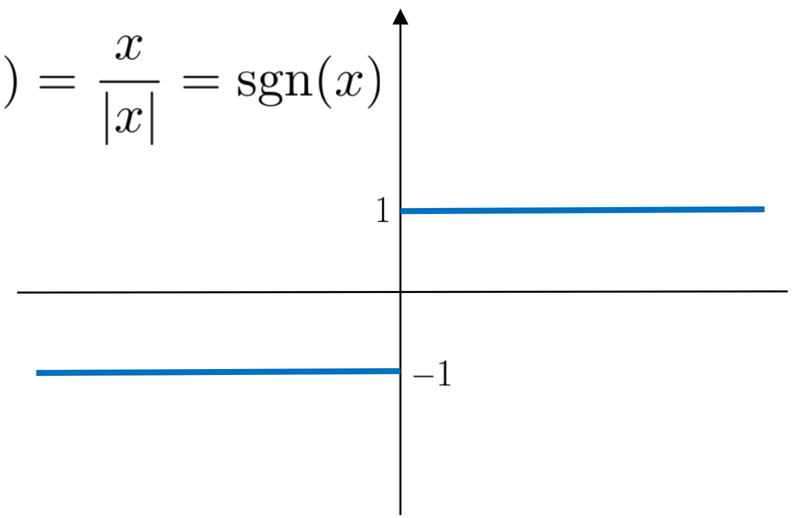
Sign Function

$$U_s(f) = -jU_c(f) \quad f > 0$$

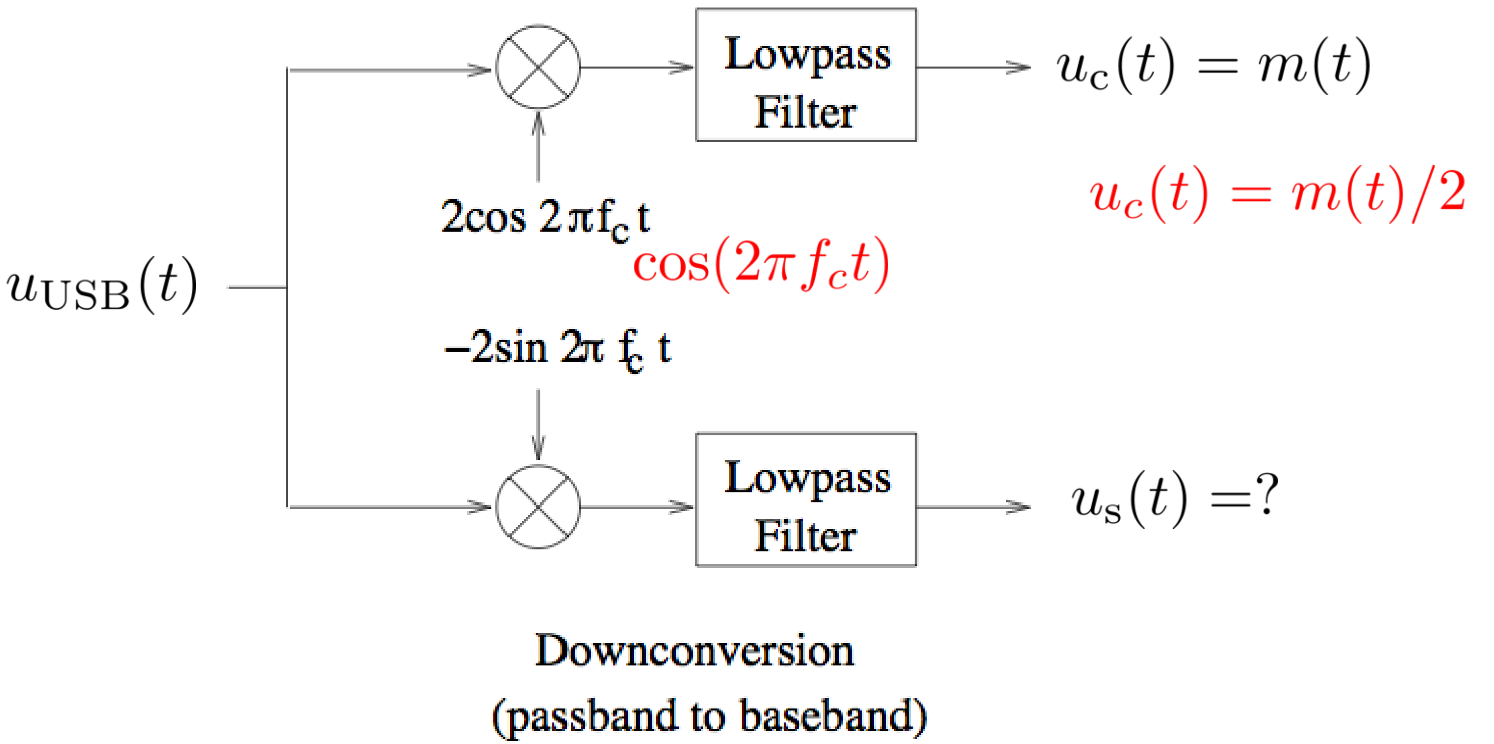
$$U_s(f) = jU_c(f) \quad f < 0$$

$$U_s(f) = -j\,\mathrm{sgn}(f)U_c(f)$$

$$y(x) = \frac{x}{|x|} = \mathrm{sgn}(x)$$

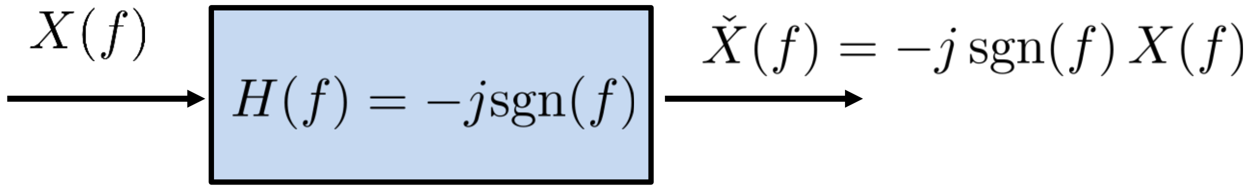


Message signal is I component of filter o/p

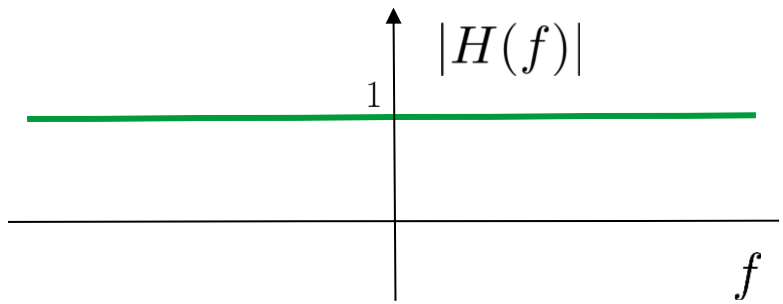


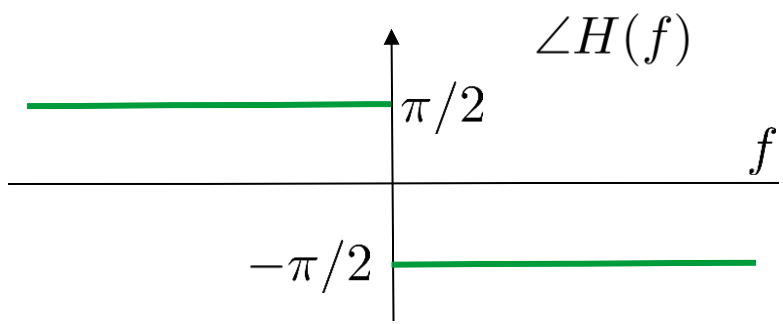
- The key to efficient SSB generation lies in this figure!

Hilbert transform

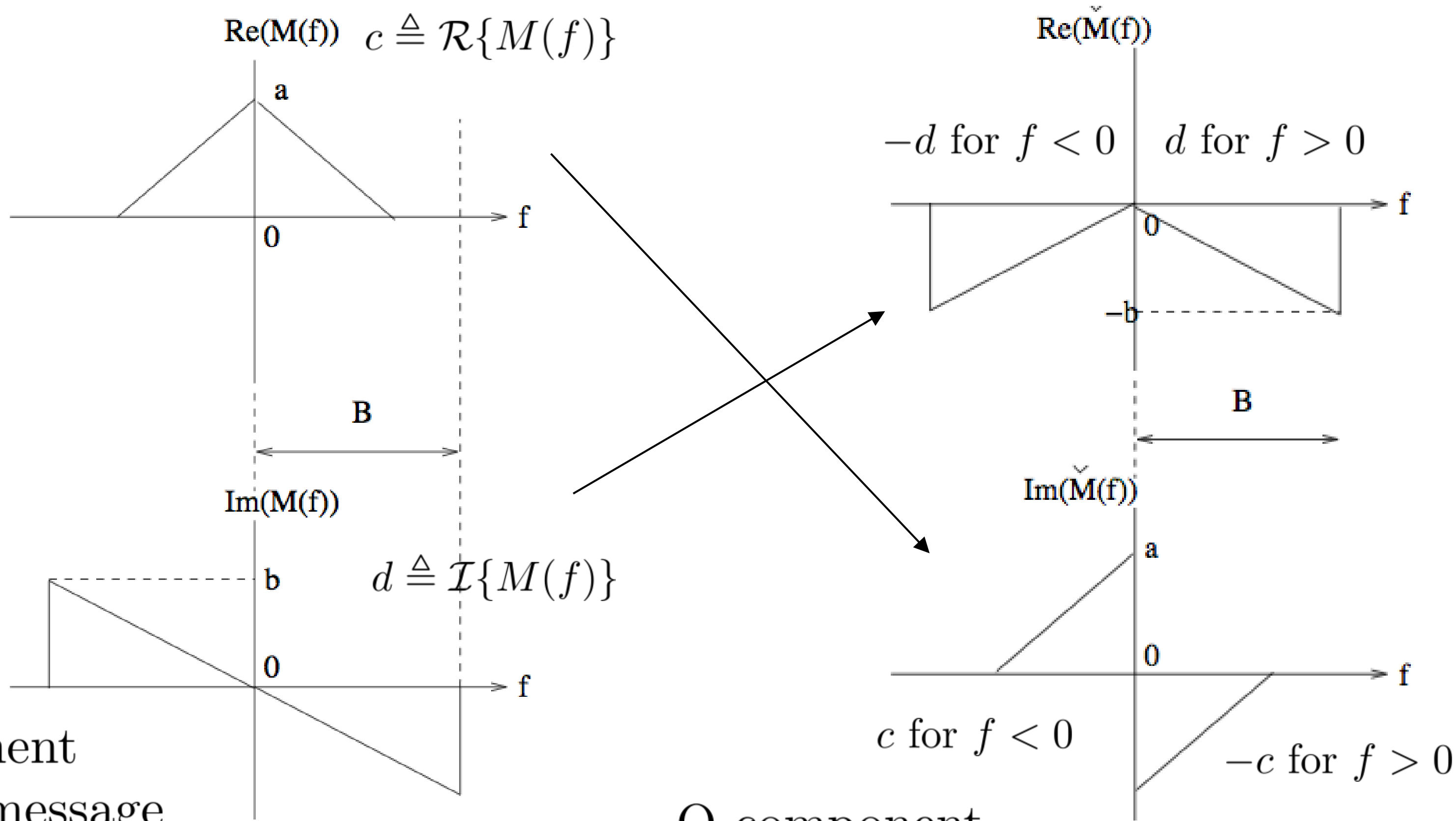


$$H(f) = -j\,\text{sgn}(f) \longleftrightarrow h(t) = \frac{1}{\pi t}$$





SSB and Hilbert Transform

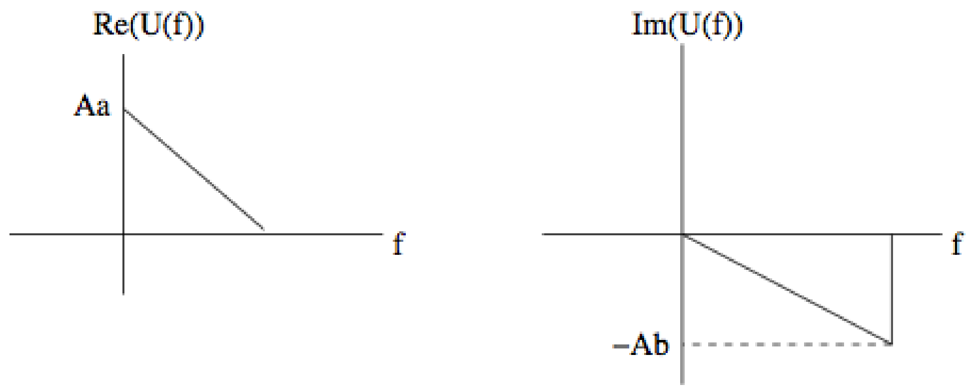


I component
original message

Q component
Hilbert transform of original message

- let $M(f) \triangleq c + jd$, then
  for $f > 0$, $\check{M}(f) = -jM(f) = -jc + d \quad \rightarrow \quad \mathcal{R}\{\check{M}(f)\} = d$ and $\mathcal{I}\{\check{M}(f)\} = -c$
  for $f < 0$, $\check{M}(f) = jM(f) = jc - d \quad \rightarrow \quad \mathcal{R}\{\check{M}(f)\} = -d$ and $\mathcal{I}\{\check{M}(f)\} = c$

Complex envelope for SSB signal



- USB baseband complex envelope in terms of message is given as

$$U(f) = U_c(f) + jU_s(f)$$
$$= M(f) + j\check{M}(f)$$

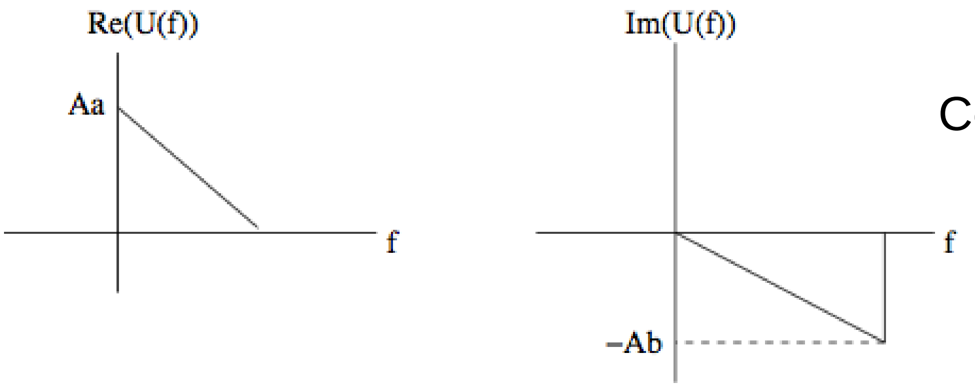
Taking inverse FT, we get

$$u(t) = m(t) + j\check{m}(t)$$

where $\check{m}(t) = m(t) * \frac{1}{\pi t}$.

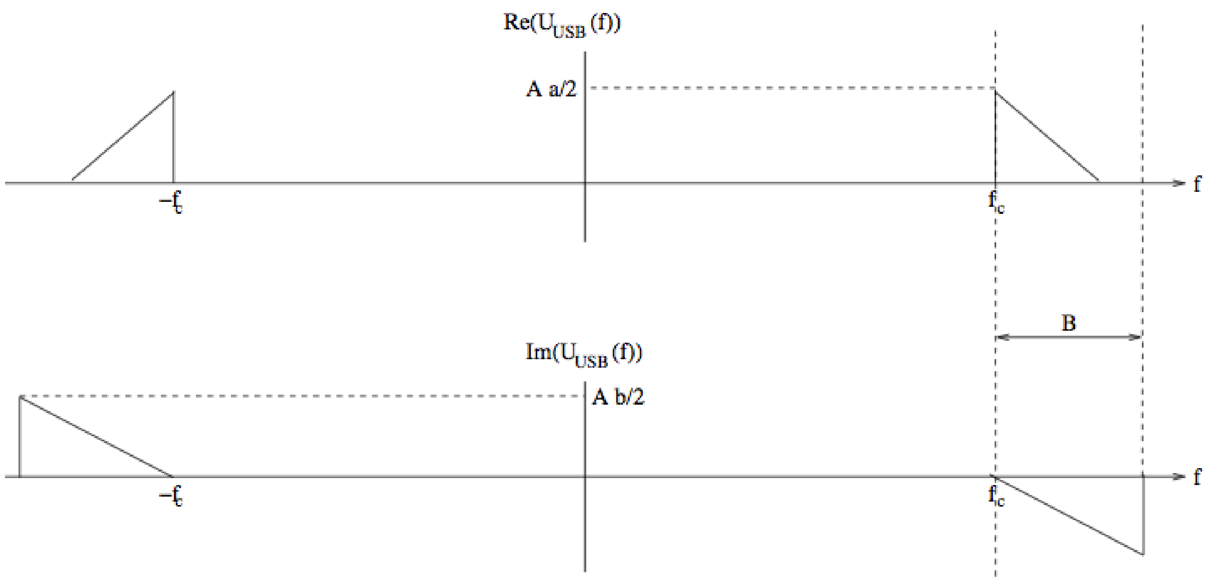
- Thus for USB, $u_c(t) = m(t)$ and $u_s(t) = \check{m}(t)$.

Complex envelope for SSB signal



Complex baseband

$$
\begin{aligned}
u_{\mathrm{USB}}(t) &= \mathrm{Re}\{u(t)e^{j2\pi f_c t}\} \\
&= m(t)\cos(2\pi f_c t) - \check{m}(t)\sin(2\pi f_c t)
\end{aligned}
$$

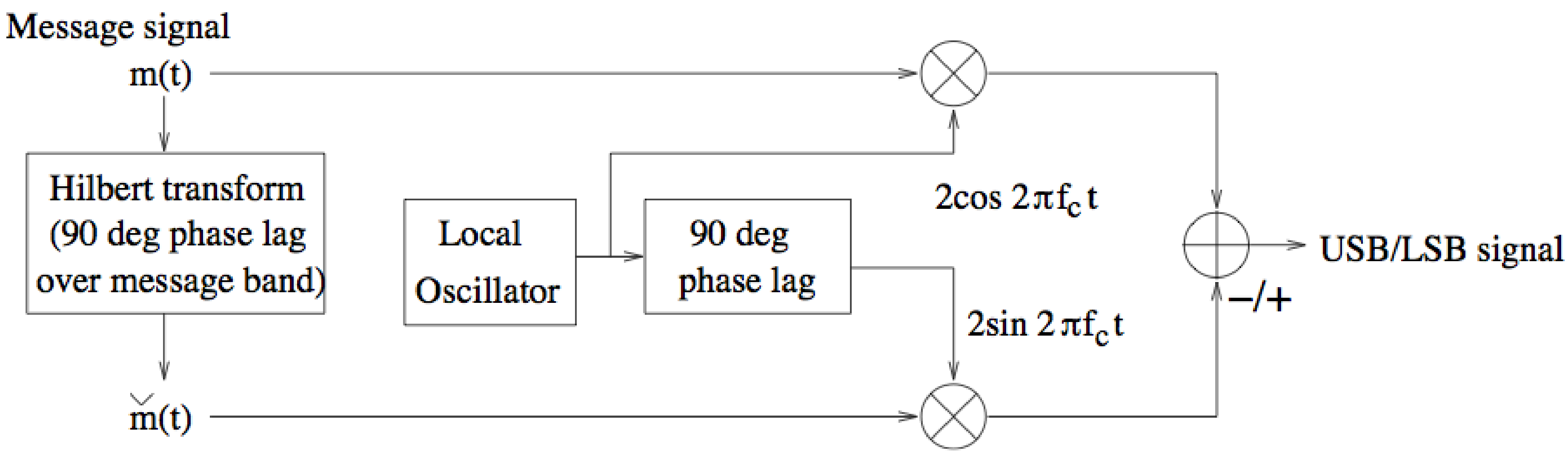


Real USB passband

Real in time
Even and Odd
Symmetric in frequency domain

Implementing SSB in baseband

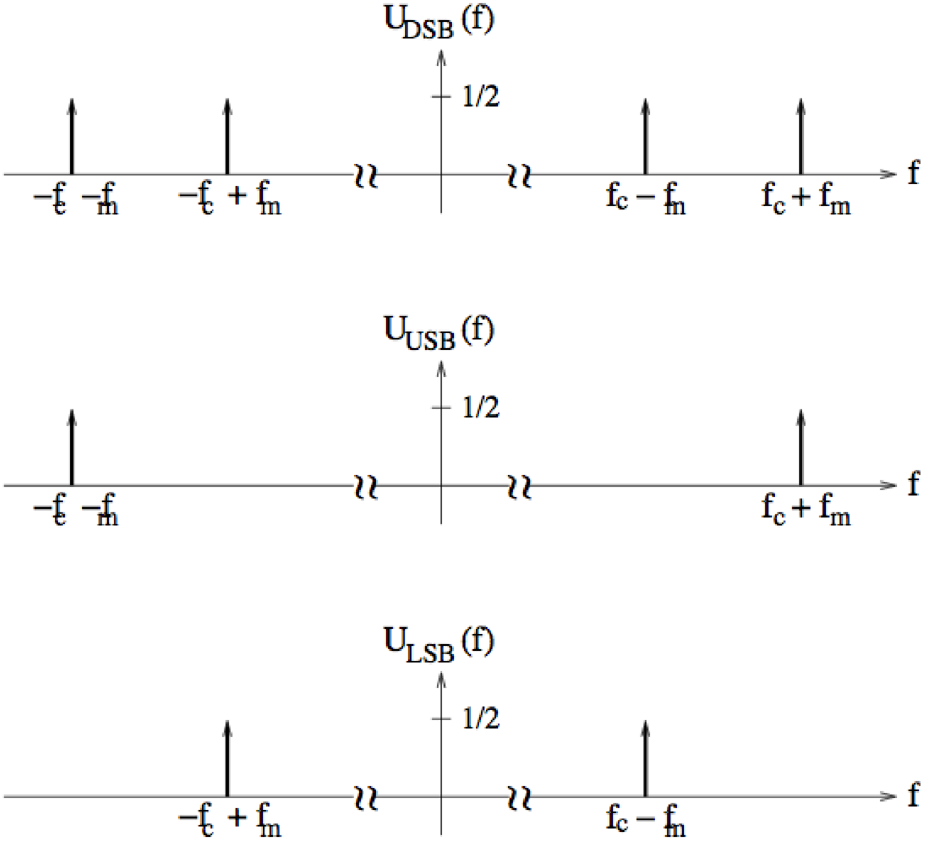
$$u_{\mathrm{USB}}(t) = \mathrm{Re}\{u(t)e^{j2\pi f_c t}\}$$
$$= m(t)\cos(2\pi f_c t) - \check{m}(t)\sin(2\pi f_c t)$$
$$u_{\mathrm{LSB}}(t) = \mathrm{Re}\{l(t)e^{j2\pi f_c t}\}$$
$$= m(t)\cos(2\pi f_c t) + \check{m}(t)\sin(2\pi f_c t)$$



Implementing Hilbert transform in baseband avoids need for sharp filtering at passband

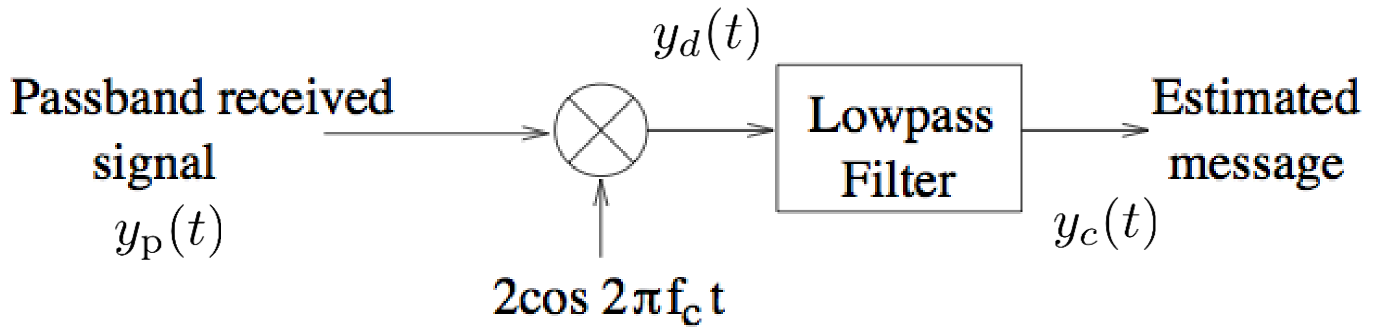
SSB for sinusoidal message

- Consider message $m(t) = \cos(2\pi f_m t)$.
  - Find $\check{m}(t)$.
  - Find $u_{\text{DSB}}(t)$, $u_{\text{USB}}(t)$, $u_{\text{LSB}}(t)$ assuming $\overline{u^2_{\text{DSB}}} = 1$.
  - Plot spectrum for DSB, USB, and LSB.



SSB demodulation: Coherent

- Synchronous demodulation to extract I component



- Prove that for SSB signal,

$$y_c(t) = m(t)\overset{\text{Attenuation}}{\boxed{\cos\theta_r}} - \overset{\text{Interference}}{\boxed{\check{m}\sin\theta_r}}$$

where θ_r is the phase difference between the received signal and local oscillator. Assignment!

- Vulnerable to carrier phase offset!!

SSB demodulation: Noncoherent

- Add strong carrier component and employ envelope detection
- The expression for the received signal with strong carrier component

$$y_p(t) = (A + m(t)) \cos 2\pi f_c t + \theta_r \pm \check{m}(t) \sin(2\pi f_c t + \theta_r)$$

- Message info preserved in envelope

$$e(t) = \sqrt{(A + m(t))^2 + \check{m}^2(t)} \approx A + m(t)$$

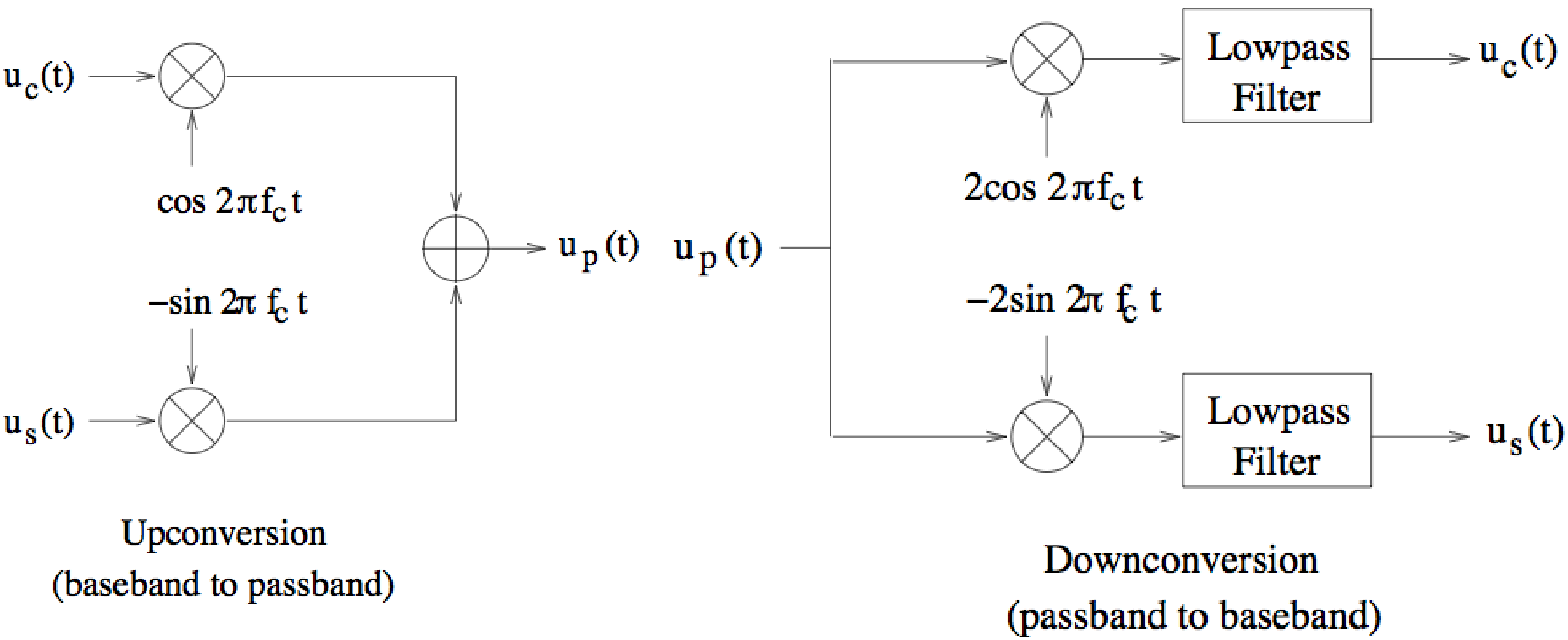
as long as $|(A + m(t))| \gg |\check{m}(t)|$.

Questions?

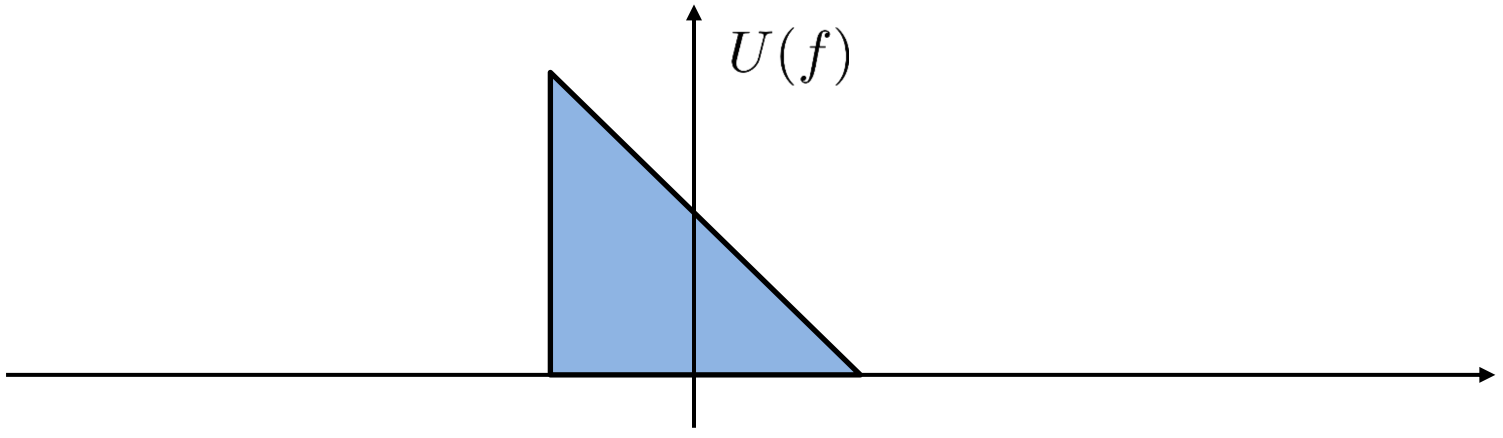
Quadrature Amplitude Modulation

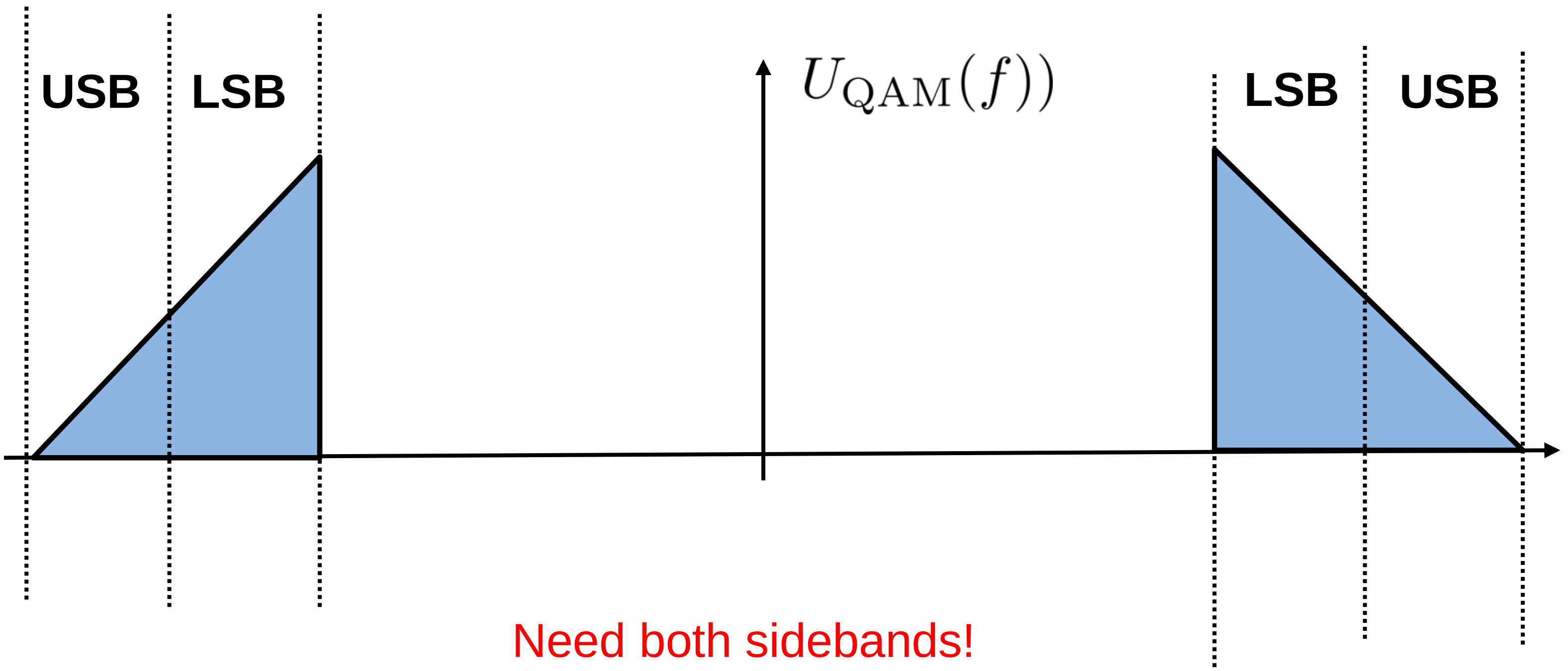
QAM

$$
\begin{aligned}
u(t) &= u_c(t) + j u_s(t) \\
u_{\text{QAM}}(t) &= \text{Re}\{u(t) e^{j2\pi f_c t}\} \\
&= u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)
\end{aligned}
$$



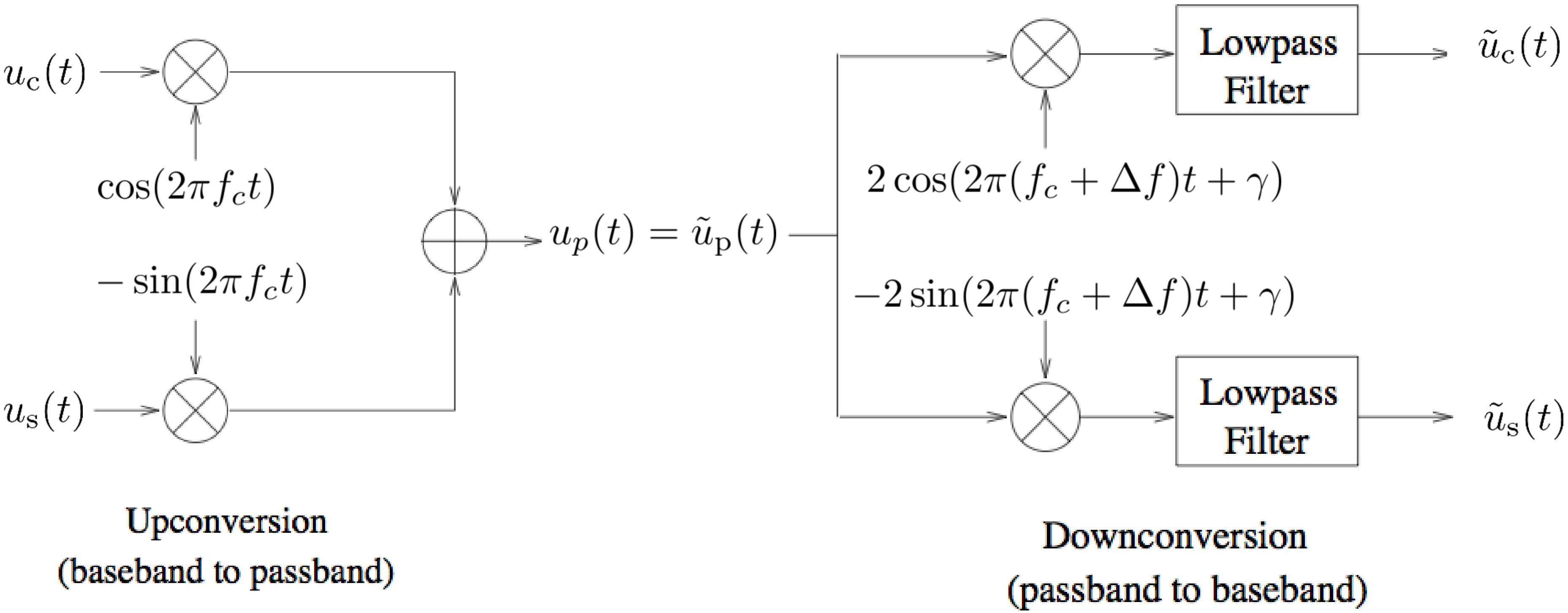
QAM





Need both sidebands!

Effect of Frequency and Phase Offset



- We have already seen this in Ch. 2: In this case

$$\tilde{u}_c(t) = u_c(t)\cos\phi(t) + u_s(t)\sin\phi(t)$$
$$\tilde{u}_s(t) = -u_c(t)\sin\phi(t) + u_s(t)\cos\phi(t)$$

where $\phi(t) = 2\pi\Delta f t + \gamma$ is the phase offset resulting from frequency offset Δf and the phase offset γ.

Coherent Detection: Synchronization

- Frequency offset and phase offset cause cross-interference between I and Q components

$$\tilde{u}_\mathrm{c}(t) = u_c(t)\cos\phi(t) + u_s(t)\sin\phi(t)$$
$$\tilde{u}_\mathrm{s}(t) = -u_c(t)\sin\phi(t) + u_s(t)\cos\phi(t)$$

  where $\phi(t) = 2\pi\Delta t + \gamma$ is the phase offset resulting from frequency offset Δf and the phase offset γ.

- Either have tight synchronization, i.e., $\Delta f \approx 0$ and $\gamma \approx 0$.

- Compensate for the offset $u(t) = \tilde{u}(t)e^{j\phi}$.

Questions?