Q → component

$y_p(t) \xrightarrow{c.e} y(t)$

$$y(t) = y_c(t) + j y_s(t)$$

$$\boxed{Im\{y(t)\} = y_s(t)}$$

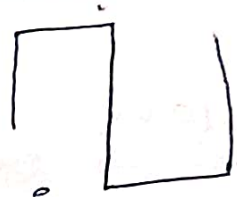
$$\theta(t) = \theta(0) + 2\pi k f_0 \int_0^t m(t)\,dt$$

$\beta \rightarrow$ FM, PM. $\quad a_{mod} = \frac{A_{m_0}}{A_c}$

$a_{mod} \rightarrow A_m \quad m_n = \frac{m(t)}{m_0}$

Digital modulation:

$0 \to -1 \qquad 1 \to +1$ $\to$ Pulse modulation

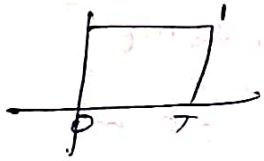


This signal is Baseband.

BPSK:

Tx	Rx	we can swap the maping
$0 \to$	-1	$0 \to +1$
$1 \to$	$+1$	$1 \to -1$

$$\boxed{p(t) = I_{[0,T]}(t)}$$



Linear modulation

$x(t) = b[0]p \qquad x(t_0) = b[t_0]\delta(t-t_0)$

$$x(t) \longrightarrow \boxed{p(t)} \longrightarrow y(t) = x(t) * p(t)$$

$$= b[t_0]\delta(t-t_0) * p(t)$$

$b[n] \to +1/-1$

$$\boxed{y(t) = b[t_0]\, p(t-t_0)}$$

$y_p(t) = y(t)\cos(2\pi f_c t)$

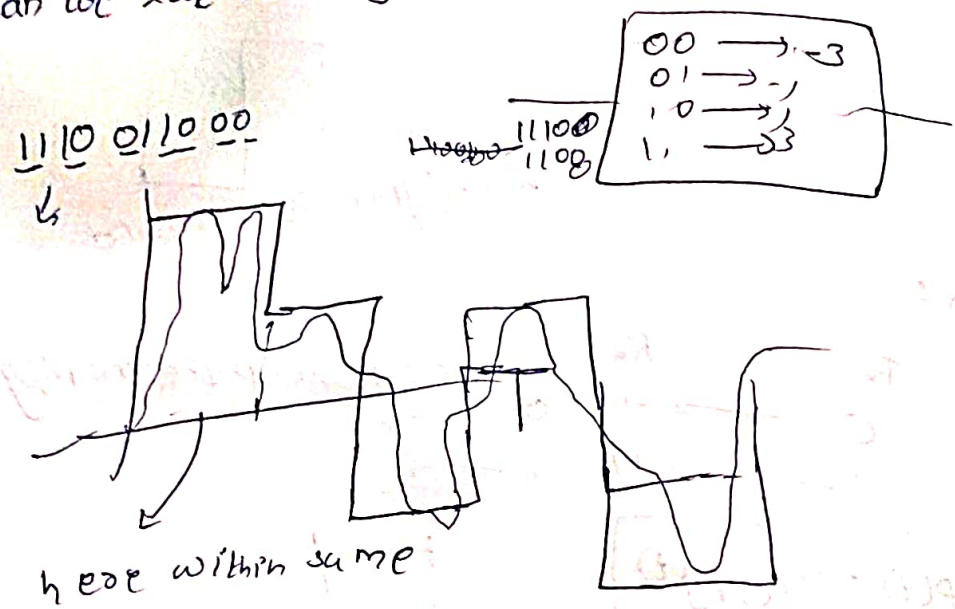
$$= \sum b[n]\, p(t-nT)\cos(2\pi f_c t)$$

0	1
-1	1
$-\cos(2\pi f_c t)$	$\cos(2\pi f_c t)$

BPSK (Binary phase shift key) $\longrightarrow$ $+1, -1$

$\cos(2\pi f t) - \cos(2\pi f t)$

Can we send more bit symbols?



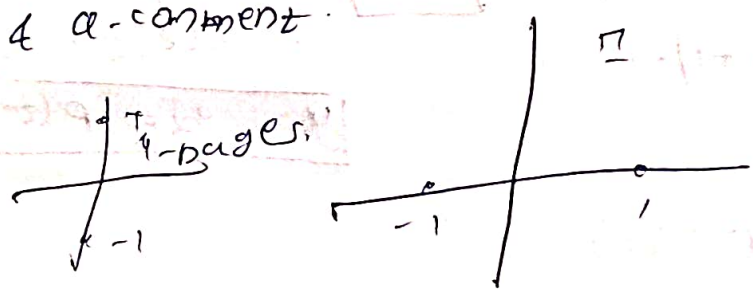
here within same

period 2 bits:

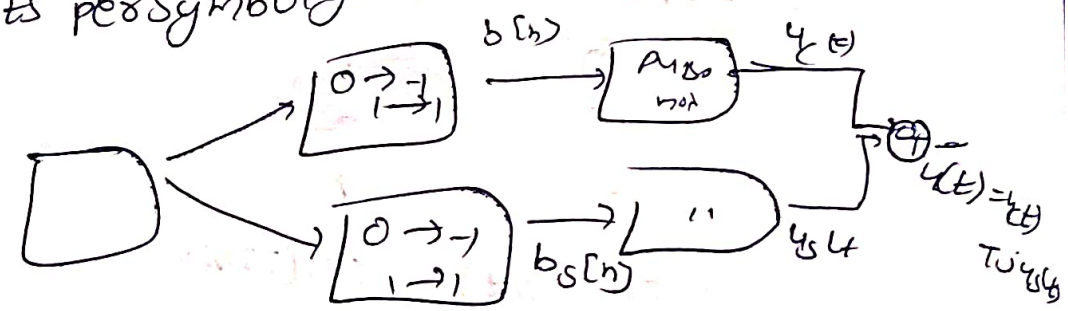
$\log_2 M \longrightarrow \frac{\log_2 M}{T} \rightarrow \log_2 4 = 2$

Here If the voltages ↑ then power also increase.

Both I & Q component.



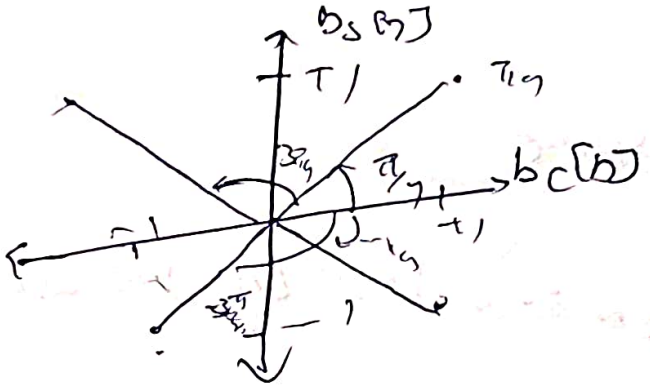
2 bits per symbol.



$b(n) = b_c[n] + j b_s[n]$

we can use I & Q andhen coming.

$$u_p(t) = y_c(t)\cos(2\pi f_c t) - y_s(t)\sin(2\pi f_c t)$$



→ In passband we are sending lots. bits.

people have send 10-12 bits It becomes dificult to measure

In 6th chapter we will show noise effects & how is recovered.

$n \rightarrow$ no. of symbols (constellation size - 16)

$$\boxed{\text{Information rate} - \frac{\log_2 n}{T}}$$

Bandwidth:

Interference happence we will send time limited signal so Bandwidth $\rightarrow \infty$

$b[n] \rightarrow$ is random in transmission & recieved.

Because what signal is transmitting we dont known

Ergodicity

$$E[x] = \int x\, p(x)\, dx \approx \frac{1}{N} \Sigma$$

Assumption 1:

~~DC zero~~ DC value

Assumption 2:

$$\frac{1}{2N} \sum_{n=-N}^{N} x[n] b^*[n-k] = 0 \text{ for } k \neq 0$$

we need to find PSD of

digital noise

$$u(t) = \sum_n b[n]\, p(t-nT)$$

consider the observation inverted

consider the observation inverted.

$[-NT, NT]$

$$u_{T_0}(t) = \sum_{n=-N}^{N} b[n]\, p[t-nT]$$

$\downarrow$ Taking FT

$$u_{T_0}(f) = \sum_{n=-N}^{N} b[n]\, P(f)\, e^{-j2\pi f n T}$$

Energy spectral density.

$$|u_{T_0}(f)|^2 = u_{T_0}(f)\, u_{T_0}^*(f)$$

$$= \left(\sum_n b[n] P(f)\, e^{-j2\pi f n T}\right)$$

$$\left(\sum_m b[m]\, P^*(f)\, e^{j2\pi f m T}\right)$$

$$= |P(f)|^2 \sum_n \sum_m b[n]\, b^*[m]$$

$$e^{-j2\pi(m-n)fT}$$

PSD is given by $\hat{S}_u(f) = \frac{|u_{T_0}(f)|^2}{T_0} = \frac{|u_{T_0}(f)|^2}{2NT}$

$$= \frac{|P(f)|^2}{T} \sum_k \frac{e^{-j2\pi k f T}}{2N} \sum_n b[n]\, b^*[n-k]$$

using $\frac{1}{2} \sum_{n=-N}^{N} b[n]\, b^*[n-k] = 0$ for $k \neq 0$

$$\hat{S}_u(f) = \frac{|P(f)|^2}{T} \frac{1}{2N} \sum_{n=-N}^{N} |b[n]|^2$$

$$S_u(f) = \lim_{T_0 \to \infty} \hat{S}_u(f) = \lim_{N \to \infty} \frac{|P(f)|^2}{T} \frac{2N+1}{2N} \frac{1}{2N+1} \sum_n |b[n]|^2$$

$$= \frac{|P(f)|^2}{T} \sigma_b^2$$

$$P_u = \int_{-\infty}^{\infty} S_u(f)\, df = \int_{-\infty}^{\infty} \frac{|P(f)|^2}{T} \sigma_b^2\, df$$

$$\boxed{P_u = \frac{\sigma_b^2}{T} \int_{-\infty}^{\infty} |P(f)|^2\, df}$$

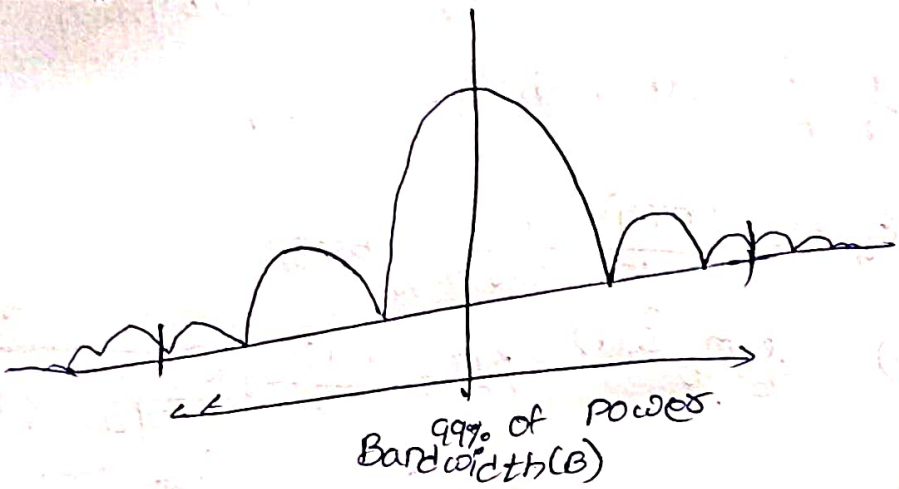
$$P_u = \frac{\sigma_b^2}{T} \int_{-\infty}^{\infty} p^2(t)\, dt = \frac{\sigma_b^2}{T} \|p\|^2$$

$$\boxed{P_u = \frac{\sigma_b^2}{T} \|p\|^2}$$

6th March 2025:

Bandwidth on PSD

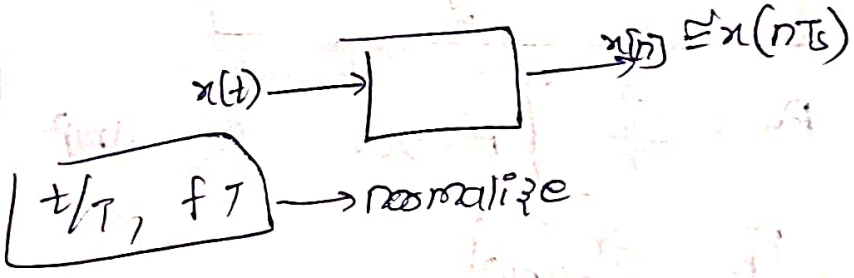
$$\int_{-B/2}^{B/2} S_u(f)\,df = \gamma P_u = \gamma \int_{-\infty}^{\infty} S_u(f)\,df.$$



$\tilde{u}(t) \approx u(t)$ 99%

If 90% $\tilde{u}(t) \longleftrightarrow u(t)$ then error will be more.

Fraction Power containment Bandwidth:



$$S_u(f) = \frac{|\sigma_b^2|\,\|P\|}{T}.$$

$$\int_{-B/2}^{B/2} S_u(f)\,df = \gamma P_u = \gamma \int_{-\infty}^{\infty} S_u(f)\,df.$$

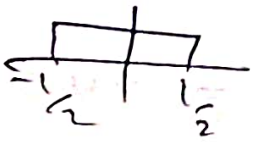
$$= \int_{-B/2}^{B/2} \frac{\|P(f)\|^2 \sigma_b^2}{T}\,df = \gamma \int_{-\infty}^{\infty} \frac{\|P(f)\|^2 \sigma_b^2}{T}\,df$$

everything do P(f)

$\gamma = 0.99$

$$S_{y_1}(f)\,df = \int_{-B_1/2}^{B_1/2} \sin c^2(f)\,df = \gamma \int_{-\infty}^{\infty} \sin c^2(f)\,df.$$

$$= 2\gamma \int_{0}^{1/2} t\,dt.$$



$$= 2\gamma \frac{t^2}{2} = \gamma$$

$$\boxed{\int_{0}^{B_1/2} S_{y_1}(f)\,df = \gamma/2}$$

"for normalised signal"

$\gamma = 0.99$, $B_1 = 10.2$ and for $\gamma = 0.9$ $B_1 = 0.85$

Bit rate = no. of bits per symbol × no. of symbols per sec

$$\text{symbol rate} = \frac{1}{T} = \frac{\text{bit rate}}{\text{no. of bits per symbol}}$$

$$= \frac{20\,\text{mbps}}{4\,\text{bits/symbol}} = \frac{5\,\text{M symbols}}{\text{sec}}$$

$$99\% = B_1 \times \frac{1}{T} = 10.2 \times \frac{1}{T} = 51\,\text{MHz}$$

$$= 10.2 \times 5 = 51\,\text{MHz}.$$

$\frac{\text{symbol}}{\text{bit rate}}$ — bits is → symbols symbol =

For $\gamma = 0.9$, $B_1 = 0.85$ $B = 0.85 \times 5 = 4.25$ MHz.

QPSK → 2 bits per symbol

$T =$ → symbols

$\frac{}{}$ = bit rate → is → 2 bits to

$$99\% \text{ symbol rate} = \frac{20\,\text{M}}{2\,\text{bits/symb}}$$

$$= 10\,\text{M}\ \cancel{\text{symbol}}\ \text{bps}.$$

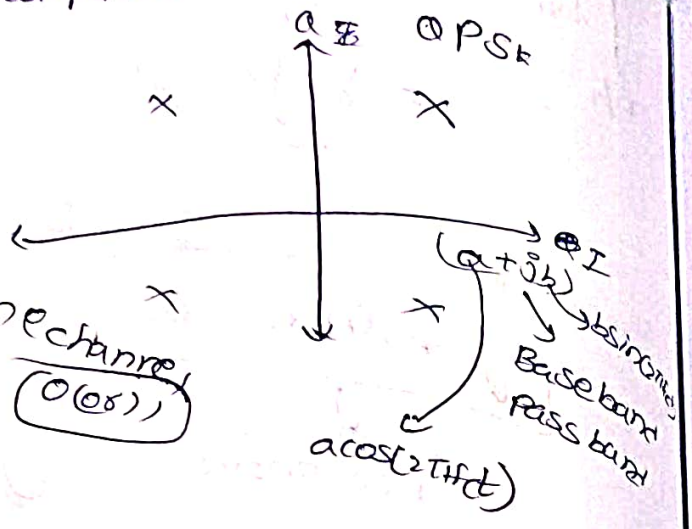
$$99\% = 10.2 \times 10 = 102\,\text{MHz}.$$

$$90\% = 0.85 \times 10 = 8.5\,\text{MHz}$$

(c) ✓

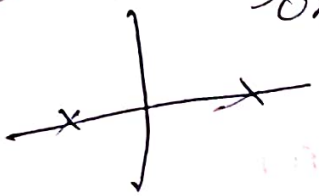
Passband

Base band → I and Q component



QPSK → 4

(2-PSK) → BPSK → one channel (0(or))



[BPSK (or) 2-PAM]

4-PAM and 4-BSK are same.

PSK → only change in phase

QAM → Amplitude & ~~Qua~~ phase Both change.

If T → is more

then more robust

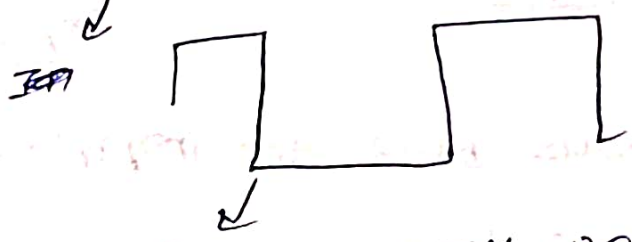
But less Bits per rate.

Sampling Theorem:

$\boxed{\omega_s > 2\omega_m}$

Design of Band limited channels:

$g(t) = \text{sinc}(Wt)$



In frequency we are having '∞' BW

Lecture-14 10th march 2024.5

$s(nT) = s[n]$

$$s(t) = \sum_{n=-\infty}^{\infty} s\left(\frac{n}{W}\right) g\left(t - \frac{n}{W}\right)$$

can we use sinc pulse → Advantage
↓
Bandwidth limited 'But '∞' time which is not feasible.

ISI → "Inter symbol Interference"

Trying to design modulated pulse "s-t" in freq domain

there is no interference.

"complex symbols → QPSK, QAM"

For sampling we are using train of impulses.

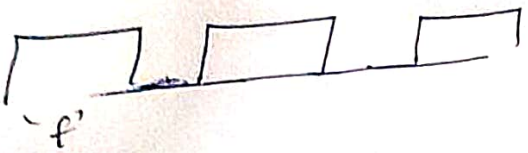
$u(t)$ → is transmitted and then in Reciever we will sample at 1, 2, 3....

$$\boxed{u(t) = \sum_{n=} b[n]\, p(t - nT)}$$

$$\boxed{B(f) \xleftrightarrow{DTFT} p(nT)}$$

$$\boxed{P(f) \xrightarrow{FT} p(t)}$$

All Summation of $P(f)$ is constant.



$\longrightarrow$ we wont have impulse

It should be cotiguous b.coz foo impulse FT is a constant But If it is not cotiguous then it wont be a constant in frequency domain.

transmit $b(n)$

imp

$$s(nT_s) = T_s \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} B(f)\, e^{j2\pi f n T_s} df.$$

$$\boxed{B(f) = \sum_{n=-\infty}^{\infty} s(nT_s)\, e^{-j2\pi f n T_s}}$$

a) $$B\left(f + \frac{1}{T_s}\right) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} s\left(f + kT_s + \frac{1}{T_s}\right)$$

use $m = k+1$

$$= \frac{1}{T_s} \sum_{m=-\infty}^{\infty} s\left(f + \frac{m}{T_s}\right) = B(f)$$

b) Inverse FT of $s(f)$

$$s(t) = \int_{-\infty}^{\infty} S(f)\, e^{j2\pi f t} df.$$

At $t = nT_s$

$$s(nT_s) = \int_{-\infty}^{\infty} S(f)\, e^{j2\pi n T_s t} df.$$

$$= \sum_{k=-\infty}^{\infty} \underbrace{\int_{(k-1/2)/T_s}^{(k+1/2)/T_s} s(f)\, e^{j2\pi f n T_s} df}_{I_k}$$

use sub $v = f - \frac{k}{T_s}$.

$$I_k = \int_{-\frac{1}{2}T_s}^{\frac{1}{2}T_s} S\left(v+\frac{k}{T_s}\right) \underbrace{e^{j2\pi\left(v+\frac{k}{T_s}\right)nT_s}}_{e^{j2\pi kn}} dv$$

$$f = \frac{k+\frac{1}{2}}{T_s}$$

$$v = \frac{1}{2T_s}.$$

$$= \int_{-\frac{1}{2}T_s}^{\frac{1}{2}T_s} S\left(v+\frac{k}{T_s}\right) e^{j2\pi v nT_s} dv$$

sub I_k in (1)

$$s(nT_s) = \sum_k \int_{-\frac{1}{2}T_s}^{\frac{1}{2}T_s} S\left(v+\frac{k}{T_s}\right) e^{j2\pi v nT_s} dv$$

$$= \int_{-\frac{1}{2}T_s}^{\frac{1}{2}T_s} \underbrace{\boxed{\sum_k S\left(v+\frac{k}{T_s}\right)}}_{T_s B(f)} e^{j2\pi v nT_s} dv.$$

$$\boxed{s(nT_s) = T_s \int_{-\frac{1}{2}T_s}^{\frac{1}{2}T_s} B(f)\, e^{j2\pi v nT_s} dv}$$

This is just formulae for the ~~coeff~~ formulae for F.S coefficients.

$$B(f) = \sum_{n=-\infty}^{\infty} s(nT_s) e^{-j2\pi f nT_s}.$$

$$u(t) = \sum_n u_n e^{j2\pi n f_0 t}$$

"Dimensionless FSK"

→ Doubt??

$$u_k = \frac{1}{T_0} \int_{\langle T_0 \rangle} u(t) e^{-j2\pi k f_0 t} dt.$$

$$\boxed{u(-t) \longleftrightarrow u_{-k}}$$

Sinc pulse → decays slowly "1/(t)"

$\sum_n \frac{1}{n}$ → divergent

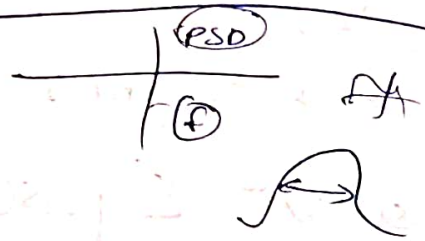
If 1000 bits are transmitting with offset.

If we have

1/(t)² ~~fast~~ If B↑ then interference will be low b/c3.

To get trapezoid in f domain

$$\boxed{ab\sin(at)\sin(bt) \longleftrightarrow \Pi\left[-\frac{a}{2},\frac{a}{2}\right] * \Pi\left[-\frac{b}{2},\frac{b}{2}\right]}$$



Lecture-15:

13th March 2025:

Chapter-4:

2-PAM | BPSK — for 2 bits

4-PSK | QAM — " 4 "

PSK → circle

orthogonal modulation → m-dimension

means M-orthogonal signals

$$\boxed{\Delta f = \frac{1}{2T}}$$

Consider Mary FSK

$$u_{PSK}(t) = \cos(2\pi(f_c + k\Delta f)t) \longrightarrow (1) \quad 0 \le t \le T$$

$k = 0, 1, 2 \ldots M-1$

where $f_c \gg \frac{1}{T}$

corresponding complex envolope are

$$u_k(t) = \exp(j2\pi k\Delta f t) \longrightarrow (2)$$

Note that

$$u_{p,k}(t) = Re\{u_k e^{j2\pi f_c t}\}.$$

we need

$$\langle u_{p,k}, u_{p,l} \rangle = 0$$

$p \rightarrow$ passband $\quad k \rightarrow k^{th}$ sinusoid

we know;

$$\langle u_{p,k}, u_{p,l} \rangle = \frac{1}{2} \mathrm{Re} \langle u_k, u_l \rangle \longrightarrow (4)$$

$$\langle u_k, u_l \rangle = \int_0^T u_k(t)\, u_l^*(t)\, dt = \int_0^T \exp(j 2\pi k \Delta f t) \exp(j 2\pi l \Delta f t)\, dt$$

$$= T \quad \text{for } k = l. \longrightarrow (5)$$

for $k \neq l$

$$\langle u_k, u_l \rangle = \frac{\exp(j 2\pi (k-l) \Delta f t)}{j 2\pi (k-l) \Delta f} \Bigg|_0^T = \frac{e^{j 2\pi (k-l) \Delta f t} - 1}{j 2\pi (k-l) \Delta f}$$

$$= \frac{e^{j\pi (k-l) \Delta f T}}{\pi (k-l) \Delta f} \left[\frac{e^{j\pi (k-l) \Delta f T} - e^{-j\pi (k-l) \Delta f T}}{j 2} \right]$$

$$= \frac{\sin(\pi (k-l) \Delta f T)}{\pi (k-l) \Delta f} e^{j\pi (k-l) \Delta f T} \longrightarrow (6)$$

putting (6) & (4)

$$\langle u_{pk}, u_{p,l} \rangle = \frac{1}{2} \cdot \frac{2 \sin(\pi (k-l) \Delta f T)}{2\pi (k-l) \Delta f} \cos(\pi (k-l) \Delta f T)$$

$$= \frac{1}{2} \frac{\sin(2\pi (k-l) \Delta f T)}{2\pi (k-l) \Delta f}$$

for $l = k+1$

$$\langle u_{p,k}, u_{p,l} \rangle = \frac{1}{2} \frac{\sin(2\pi \Delta f T)}{2\pi \Delta f} \longrightarrow (8)$$

This goes to zero when $2\pi \Delta f T = I\pi$

$2\pi \Delta f T = I 2\pi$

$$\boxed{\therefore \Delta f = \frac{1}{2T}}$$

For non coherent

~~[illegible]~~ have $\boxed{\Delta f = \frac{1}{T}}$ $\qquad \Delta f = \frac{n}{2T}$

From (6), we have

$$\langle u_k, u_l \rangle = \frac{\sin \pi (k-l) \Delta f T}{\pi (k-l) \Delta f} \exp(j 2\pi (k-l) \Delta f T)$$

$$|\langle u_k, u_l \rangle| = \frac{|\sin(\pi (k-l) \Delta f T)|}{\pi (k-l) \Delta f} \quad \because |e^{j2\pi(k-l)\Delta f T}| = 1$$

For $l = k+1$,

$$|\langle u_k, u_{k+1} \rangle| = \frac{|\sin(\pi \Delta f T)|}{\pi \Delta f}.$$

This will be zero for $\pi \Delta f T = \pm \pi, \pm 2\pi$.

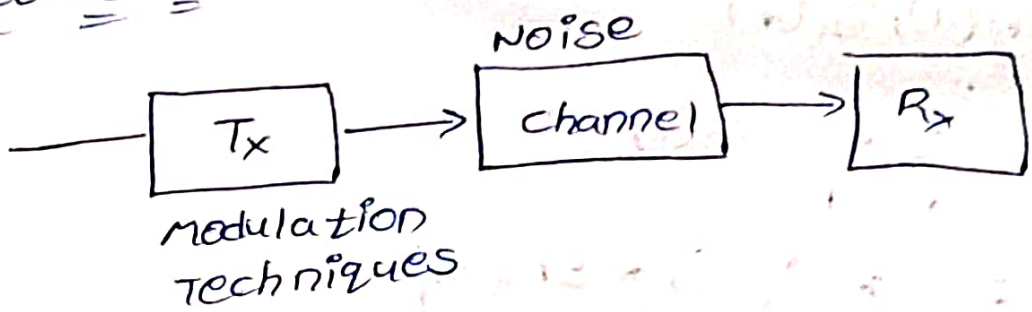
So min of Δf for which adjacent tone is orthogonal $\boxed{\Delta f = \frac{1}{T}}$

coherent $\qquad T \rightarrow$

$$\frac{\frac{\log_2 M \text{ bits}}{T}}{M/2T} = \frac{2 \log_2 M}{M} \rightarrow \text{coherent.}$$

$\bar{\vartheta}_n = 4TKBRB$ $\qquad \Delta f = \frac{1}{2T} \frac{M}{2T}$

Lecture-16 :



what is noise?

anything which is undesired

$$\boxed{\overline{v_n^2} = 4RK_BTB}$$

σ will have more variations

If the B.W is more then we will have more noise.

$$\overline{P_n^2} = \frac{\left(\overline{V_n/2}\right)^2}{R} = K_BTB = N_0B$$

$n \longrightarrow$ const

$R_x(n_1, n_2) = f(n_1 - n_2)$

PSD is given by

Noise created by reciever is dominant then the " " " environment.

collection of joint PDF's $\longrightarrow$ random process.

$$S_y(f) = S_x(f)\,|G(f)|^2$$

$$y(t) = x(t) * g(t)$$

$$Y(f) = X(f)\,G(f)$$

$$S_y(f) = \lim_{T_0 \to \infty} \frac{|Y_{T_0}(f)|^2}{T_0}$$

$$= \lim_{T_0 \to \infty} \frac{|X_{T_0}(f)\,G(f)|^2}{T_0}$$

$$= |G(f)|^2 \lim_{T_0 \to \infty} \frac{|X_{T_0}(f)|^2}{T_0}$$

$$= |G(f)|^2 S_x(f)$$

$$= G(f)\, G^*(f) S_x(f)$$

$$\Big\updownarrow \text{IFT}$$

$$\boxed{R_y(\tau) = g(\tau) * g^*(-\tau) * SR_x(\tau)}$$

$$x(t) \longleftrightarrow X(f)$$

$$x^*(t) \longleftrightarrow X^*(-f)$$

$$x^*(-t) \longleftrightarrow X^*(f)$$

$y(t) = x(t) * g(t)$ [∵ as $x(t)$ is only random]

$$E[y(t)] = \cancel{g(t)}\ E[x(t)] * g(t)$$

$$= m_x(t) * g(t)$$

$$= \int \underbrace{m_x(t-u)}_{\text{const.}} g(u)\, du$$

$$= m_x \int g(u)\, du$$

$$\boxed{E[y(t)] = m_x\ G(0) = \text{const}}$$

$R_{yx}(t_1, t_2)$, $R_y(t_1, t_2)$

$$R_{yx}(t+\tau, t) = E[y(t+\tau)\, x^*(t)]$$

$$= E[(x(t+\tau) * g(t+\tau))\, x^*(t)]$$

$$= E\left[\left(\int x(t+\tau-u)\, g(u)\, du\ x^*(t)\right)\right]$$

$$= \int E[x(t+\tau-u)\, x^*(t)]\, g(u)\, du.$$

$$= \int R_x(\tau-u)g(u)du = R_x(\tau) * g(\tau)$$

$\therefore$ $R_{yx}(t+\tau, t)$ depends only of difference τ. $\rightarrow$ ①

The autocorrelation function of Y is given by.

$$R_y(t+\tau, t) = E[y(t+\tau)\, y^*(t)]$$

$$= E\left[y(t+\tau)\left(\int x(t-u)g(u)du\right)^* \right]$$

$$= \int E[Y(t+\tau)\, x^*(t-u)]\, g^*(u)\, du$$

$t_1 = t+\tau,\ t_2 = t-u. \Rightarrow t_1 - t_2 = \tau + u.$

$$= \int R_{yx}(\tau+u)\, g^*(u)\, du$$

Let $g_{mp}(u) \triangleq g^*(-u)$

$$R_y(t+\tau, t) = \int R_{yx}(\tau+u)\, g_{mp}(-u)\, du$$

$$= R_{yx}(\tau) * \underbrace{g_{mp}(\tau)}_{\text{matched Filter}}$$

$R_y(\tau)$ is also depends only on timedifference.

$$R_y(\tau) = R_x(\tau) * g(\tau) * \cancel{g^*(\tau)}$$

$$R_y(\tau) \rightarrow S_y(f) \qquad g(\tau) \xrightarrow{g_{mp}(\tau)} G(f)$$

$$R_x(\tau) \longrightarrow S_x(f) \qquad g^*(-\tau) \longrightarrow G^*(f)$$

20th march 2025

Additive white Gaussian noise.

AWGN

CLT → Gaussian

"white → PSD is flat"

$$\boxed{IFT\{PSD\} = \delta_x}$$

$$R_x(\tau) = 0 \quad \tau \neq 0$$

$$\boxed{R_x(\tau) = \sigma^2 \delta(\tau)}$$

$$|S(f)|^2$$

$$S_n(f) = \frac{N_0}{2}$$

(a) PSD of $y(t)$

$$S_y(f) = S_x(f)\,|G(f)|^2$$

$$= \frac{N_0}{2}|G(f)|^2 \longrightarrow (1)$$

(b) Autocorrelation:

$$R_y(\tau) = R_x(\tau) * g(\tau) * g^*(-\tau)$$

$$= \frac{N_0}{2}\delta(\tau) * g(\tau) * g^*(-\tau)$$

$$= \frac{N_0}{2}[g(\tau) * g^*(-\tau)]$$

$$= \frac{N_0}{2}\int g(\tau - s)\, g^*(-s)\, ds$$

$$\downarrow s = -s$$

$$= \frac{N_0}{2}\int g^*(s)\, g(s+\tau)\, ds$$

$$\downarrow s + \tau = \ell.$$

$$R_y(\tau) = \frac{N_0}{2}\int g^*(\ell - \tau)\, g(\ell)\, d\ell.$$

$R_y \longleftrightarrow S_y \quad \int_{-\infty}^{\infty} S_y = \text{power}$

$$\boxed{R_y(\tau) = \int S_y(f)\, e^{j2\pi f \tau} df}$$

$E[x^2(n)]$

$R_y(0) = E(y(n)y(n+\tau))$

(a) power of o/p

$$P_y = R_y(0) = \frac{N_0}{2}\int \underbrace{g^*(\ell)g(\ell)}_{|g(\ell)|^2} d\ell = \frac{N_0}{2}\|g\|^2 \rightarrow \text{energy}$$

$$P_y = \int_{-\infty}^{\infty} S_y(f)\, df$$

from (1)

$$= \frac{N_0}{2}\int_{-\infty}^{\infty} |G(f)|^2 df = \frac{N_0}{2}\|g\|^2$$

$$\boxed{SNR = \frac{|\langle s, g\rangle|^2}{\frac{N_0}{2}\|g\|^2}}$$

$\langle s, g\rangle \rightarrow$ LTI system.

$\frac{N_0}{2}\|g\|^2 \rightarrow$ noise.

$z = \langle y, g\rangle = \int y(t)\, g(t)\, dt$ ∴ real signal

$$= \int (s(t) + n(t))\, g(t)\, dt$$

$$= \underline{\int s(t)\, g(t)\, dt} + \underline{\int n(t)\, g(t)\, dt}$$

SNR is defined as.

$$SNR = \frac{|\langle s, g\rangle|^2}{E[|\langle n, g\rangle|^2]}$$

$$SNR = \frac{\text{signal power}}{\text{noise power}}$$

$$= \frac{|\langle s, g\rangle|^2}{E\left[|\langle n, g\rangle|^2\right]}$$

$\iint E[n(t)n(s)]g(t)g(s)\,dt\,ds$

$$E\left[|\langle n,g\rangle|^2\right] = E\left[\left|\int n(t)g(t)\,dt\right|^2\right]$$

$$= E\left[\left(\int n(t)g(t)\,dt\right)\left(\int n(s)g(s)\,ds\right)\right]$$

$$= E\left[\iint n(t)n(s)g(t)g(s)\,dt\,ds\right]$$

$$= \iint E[n(t)n(s)]g(t)g(s)\,dt\,ds.$$

$$\overset{\tau = t-s}{=} \iint R_n(\tau)\, g(t)g(s)\,dt\,ds.$$

$$= \iint \frac{N_0}{2}\delta(t-s)\, g(t)g(s)\,dt\,ds.$$

$$= \frac{N_0}{2}\int g(s)g(s)\,ds = \frac{N_0}{2}\|g\|^2.$$

$$SNR = \frac{|\langle s, g\rangle|^2}{\frac{N_0}{2}\|g\|^2} = \frac{\left|\int s(t)g(t)\,dt\right|^2}{\frac{N_0}{2}\int g^2(t)\,dt}.$$

Cauchy Schwarz Inequality.

$$\left[\int g_1(t)g_2(t)\,dt\right]^2 \le \left[\int g_1^2(t)\,dt \int g_2^2(t)\,dt\right].$$

$$\left[\int s(t)g(t)\,dt\right]^2 \le \left(\int g^2(t)\,dt\right)\left(\int s^2(t)\,dt\right).$$

equality when $g_1(t) = c g_2(t)$

Equality achieved when

$$g(t) = c s(t)$$

$$\int s(t) g(t) dt = c \int s^2(t) dt.$$

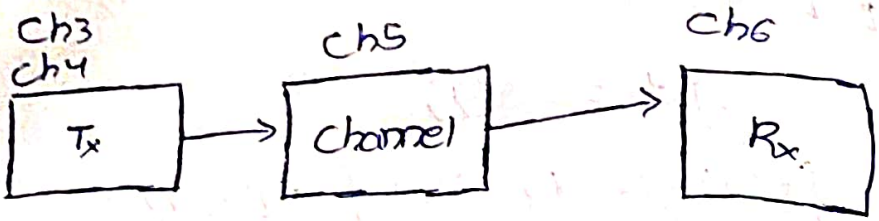
$$\text{max SNR} = \frac{c\left(\int s^2(t) dt\right)^2}{\frac{N_0}{2} c^2 \int s^2(t) dt} = \frac{\int s^2(t) dt}{N_0/2}.$$

$$= \frac{2 \|s\|^2}{N_0}.$$

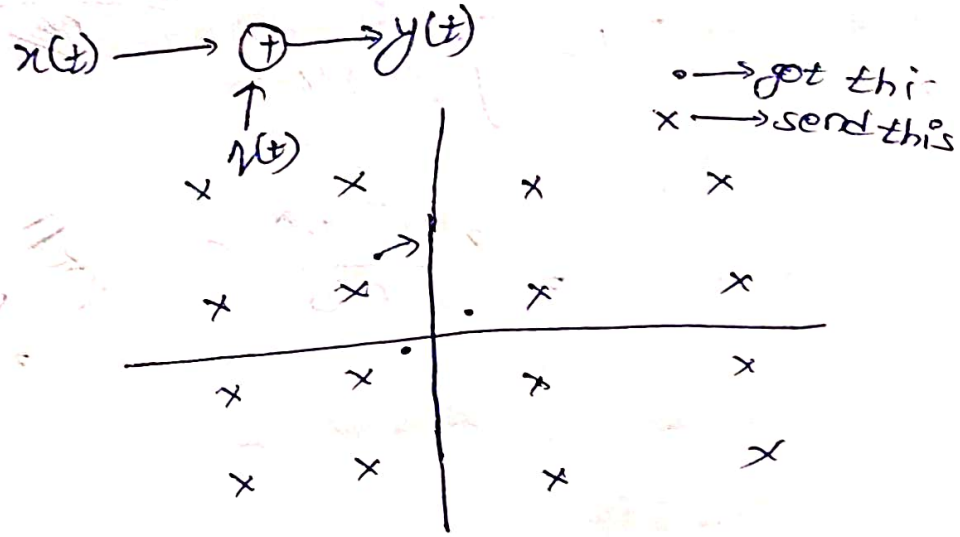
$\|m\| = \int m(t) dt$ // $s(t) dt$

Lecture - 18

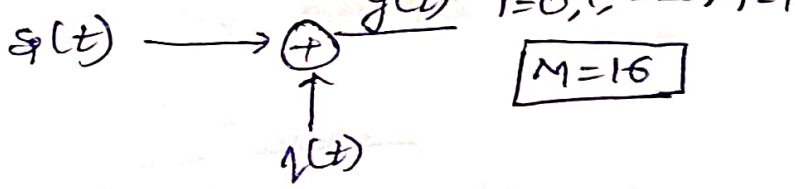
24th march 2025 :



This demodulation is done for 8 Transmitters in Ch3.



For 4-bits we sent QAM signal



$i = 0, 1, \ldots M-1$

$\boxed{M = 16}$

which of these 16 symbols were sent

SNR = 20 N = 1000 symbols → 4000 bits

$\boxed{SNR = 10\log_{10}\left(\frac{S}{N}\right) = 10}$ $S = 10$,

$\boxed{\sigma^2 = \langle \quad \rangle}$

$$\boxed{\frac{1}{n_2 - n_1 + 1} \sum_{n_1}^{n_2} |x[n]|^2}$$

H_1 : 1 is sent

H_0 : 0 is sent

$\pi_0 = P(0 \text{ is sent})$

$\pi_1 = P(1 \text{ is sent})$

$\boxed{\pi_0 + \pi_1 = 1}$ $\boxed{\pi_0 = \pi_1 = \frac{1}{2}}$

Assuming that both '0' & '1' are more likely,

a) $\pi_0 = \pi_1 = 0.5$

$P_{avg} = \pi_0 P_0 + \pi_1 P_1$

$= \pi_0 (0) + \frac{1}{2}(m^2)$ $\boxed{E[m^2] = m^2}$

$\boxed{P_{avg} = \frac{m^2}{2}}$

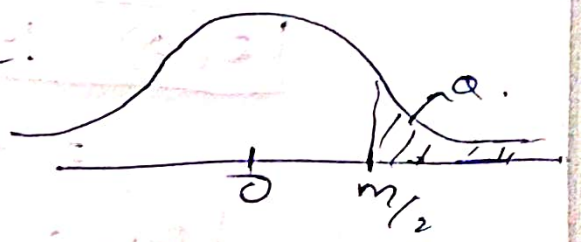
b) If '0' is sent we have recieved '1' ~~why~~ $P(\epsilon r = 0)$

Given '0' is sent, $Y = n$ $\therefore Y \sim N(0, \sigma^2)$

The conditional probability of error $= P(\text{decide } 1 / 0 \text{ is sent})$

$= P(Y > m/2 \mid 0 \text{ sent})$

$\frac{P(Y > m/2 \cap 0 \text{ is sent})}{\frac{P(0 \text{ is sent})}{1/2}} = \int_{m/2}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx.$

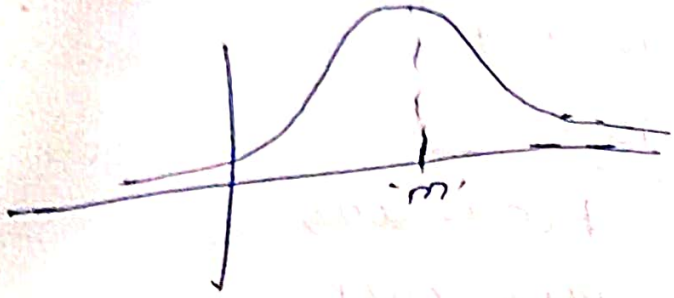


$= Q\left(\frac{m}{2\sigma}\right)$

$$\boxed{Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx.}$$

$Y \le \frac{m}{2}$

$Y = m + n$



$Y \le \frac{m}{2}$

$$\int_{-\infty}^{\frac{m}{2}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{(x-m)^2}{2\sigma^2}\right)} dx.$$

$$1 - \int_{m/2}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{(x-m)^2}{2\sigma^2}\right)} dx. = 1 - Q($$

$Y \sim N(m, \sigma^2)$ "$Y = m + n$"

$P_{e|1} = P(\text{decide } 0 \mid 1 \text{ is sent})$

$= P(Y \le \frac{m}{2} \mid 1 \text{ is sent})$

$$= \int_{-\infty}^{\frac{m}{2}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}} dx$$

$$= 1 - Q\left(\frac{-m}{2\sigma}\right) \qquad Q\left(\frac{\overset{m/2}{2} - \overset{m}{\cancel{1}}}{\underset{\sigma}{\cancel{2}}}\right)$$

$$\boxed{Q(-x) = 1 - Q(+x)}$$

$\log_{10} = 1.3$

$(10^{0.3})$

$(10) \cdot 10^{0.3}$

(20)

$$= \cancel{1} - \cancel{1} + Q\left(\frac{m}{2\sigma}\right)$$

$$\boxed{P_{e|1} = Q\left(\frac{m}{2\sigma}\right)}$$

(iv) Avg probability

$$P_e = \pi_0 P_{e|0} + \pi_1 P_{e|1}$$

$$= \frac{1}{2}\left[Q\left(\frac{m}{2\sigma}\right)\right]$$

$$\boxed{P_e = Q\left(\frac{m}{2\sigma}\right)}$$

$$SNR = \frac{m^2/2}{\sigma^2}$$

variance $= E[x^2] - (E[x])^2$

$\boxed{E[x] = 0}$

variance $= E[x^2] \rightarrow$ power.

$$SNR = \frac{m^2}{2\sigma^2}$$

$$\frac{m}{\sqrt{2}\,\sigma} = \sqrt{SNR} \Rightarrow \boxed{\frac{m}{2\sigma} = \sqrt{\frac{SNR}{2}}}$$

Not in dB scale it is in normal scale.

$$SNR = 10^{SNR_{dB}/10}$$

$$= 10^{13/10} = 10^{3} \approx 20.$$

$$P_e = Q\left(\sqrt{\frac{20}{2}}\right) = Q(\sqrt{10})$$

$$= 7.8 \times 10^{-4}.$$

25th march 2025

optimal demodulation:

$$y(t) = s_i(t) + n(t)$$

we want to minimize the prob. of wrong guess.

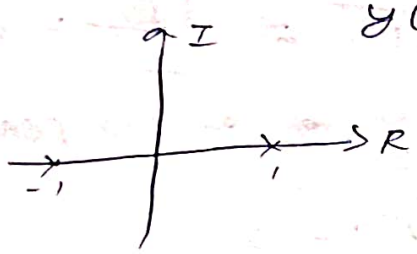
$P(y|i) = P(y|H_i)$ $H_0, H_1, \ldots H_{M-1}$

↳ stands for hypot

Prior prob $\pi_i \longrightarrow$ what is prob that 'i' is sent.

$\delta \longrightarrow$ not impulse

Decision rule



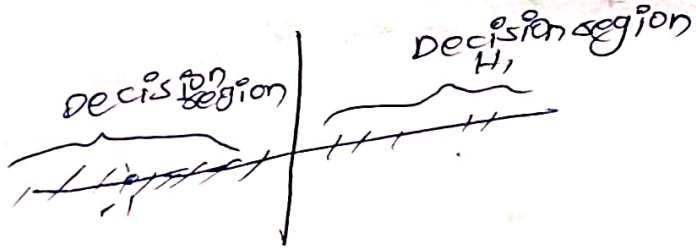
$y(t) = s(t) + n(t)$

↓ Demodulate

$Re\{y(t)\}$.

FSK $\longrightarrow \mathbb{R}^2$

BPSK $\longrightarrow$ 1D	QAM $\longrightarrow$ 2D
FSK $\longrightarrow$ 'mD' $\mathbb{R}^m$	



Start with observation space $\longrightarrow$ Get the decision rule.

16-QAM $\rightarrow H_0, \ldots H_{15}$

$\boxed{H_0, H_1, H_2 \ldots H_{15}}$ send H_2 but get any thing from this then it is correct.

$$P_{e|i} = \sum_{j \neq i} P(Y \in \Gamma_j / H_i) = 1 - P(Y \in \Gamma_i / H_i)$$

$$\boxed{P_{c|i} = 1 - P_{e|i}}$$

$$P_e = \sum_{i=1}^{M} \pi_i P_{e|i} \qquad P_c = \sum_{i=1}^{n} \pi_i P_{c|i}$$

for → for → for

All decision regions are mutually exclusive "

"MAP Rule"

$$\delta_{MAP}(y) = \arg\max_i \{P(H_i / Y = y)\}$$

$$P(H_1/y) \uparrow \quad P(H_0/y) \downarrow$$

$$P(A/B) = \frac{P(B/A)\,P(A)}{P(B)}$$

$$\delta_{MAP}(y) = \arg\max_i \frac{P(Y=y/H_i)P(H_i)}{P(Y=y)}$$

$$= \arg\max_i P(y/i)\,P(i)$$

$$= \left[\log \pi_i + \log P(y/i)\right] \rightarrow \text{argmax}.$$

$$T = f(Y) > \eta \Rightarrow H_1$$

$$< \eta \Rightarrow H_0$$

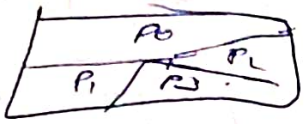
, minimizing the average prob. of error is equivalent to maximizing probability of correct detection as $P_c = 1 - P_e$.

consider an arbitrary decision rule δ, with corresponding regions $\{\Gamma_i\}$. The conditional probabilities of correct decision is

$$P_{c|i} = P(y \in \Gamma_i \mid H_i)$$

$$= \int_{\Gamma_i} p(y|i)\,dy \quad i = 0, 1, \ldots M-1$$

$$= \int_{\Gamma_0} p(y|H_0)$$



Avg prob of correct decision

$$= \sum_i \pi_i P(c|i)$$

$$= \sum_i \pi_i \int p(y|i)\,dy$$

Any point $y \in \Gamma$ can belong in exactly one of the M decision regions. If we decide to put it in Γ_i, then point contribute term $\pi_i p(y|i)$ to the integrand.

Since we ~~wi~~ wish to maximize P_c

maximize each integrand

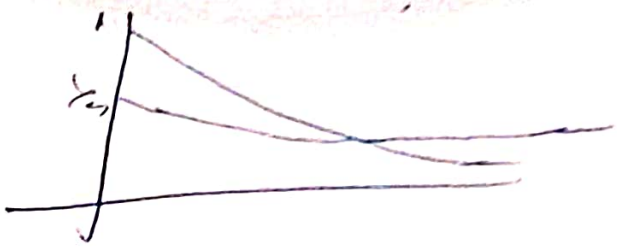
put y in the decision region which makes the largest contribution to integrand.

Put y in Γ_i such that $\pi_i P(y|i)$ is maximized

which is MAP rule

ML (maximum likelihood)

$p(y|i)$



$p(y|0) = e^{-y} I_{[y \geq 0]}$

$p(y|1) = \frac{e^{-y/4}}{4} I_{[y \geq 0]}(y)$

$$p(y|1) \underset{H_0}{\overset{H_1}{\gtrless}} p(y|0)$$

$$\ln \frac{1}{4} - \frac{y}{4} \underset{H_1}{\overset{H_0}{\gtrless}} -y$$

$$y \underset{H_0}{\overset{H_1}{\gtrless}} \frac{4}{3} \ln 4 = 1.8484$$

$$y \underset{H_0}{\overset{H_1}{\gtrless}} 1.8484$$

$P_{e|0} = P(\text{decide } H_1 | H_0 \text{ is true})$

$= P(Y > 1.8484 | H_0)$

$= 1 - P(Y \leq 1.8484 | H_0)$

$= 1 - CDF_0(1.8484)$

$= e^{-1.8484}$.

MAP crite

$\pi_1 \, p(y|1) \underset{H_0}{\overset{H_1}{\gtrless}} \pi_0 p(y|0)$

$y \underset{H_0}{\overset{H_1}{\gtrless}} 3.6968$

$P_{e|0} = 0.029$ $\quad \pi_0 = 4/5 \quad \pi_1 = 1/5.$

$P_{e|1} = 0.6$

Avg Prob $= 0.1405$