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Assignment - 5.

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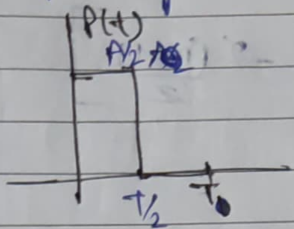
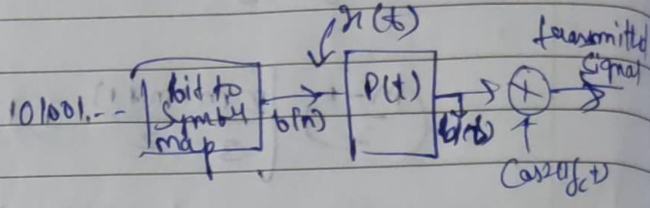
Ques 2

Given Half-width pulse transmission using Polar signalling ->

Therefore





$$x(t) = \sum_{-\infty}^{\infty} b[n]\,\delta(t - nT)$$

Also $b[n] = \pm 1$

Now PSD of $x(t) \Rightarrow$ $S_x(f) = \frac{1}{T}\sum_{n=-\infty}^{\infty} R_n e^{-j2\pi nTf}$

Now, $R_n = \lim_{N\to\infty} \frac{1}{N}\sum_{k=-N/2}^{N/2} b[k]\, b[n+k]$

Now, $R_0 = \lim_{N\to\infty} \frac{1}{N}\sum_{k=-\frac{N}{2}}^{N/2} b^2[k]$

Now considering 0, 1 to be equally likely

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\left[\left(\frac{N}{2}\right)(-1)^2 + \left(\frac{N}{2}\right)(1)^2\right] = 1$$

Also since $b[k]$, $b[n+k]$ are independent.

$$R_1 = \lim_{N\to\infty} \frac{1}{N}\sum_{k=-\frac{N}{2}}^{\frac{N}{2}} b[k]\cdot b[k+1]$$

b_k \ b_{k+1}	-1	1
-1	1	-1
1	-1	1

$$= \lim_{N\to\infty} \frac{1}{N}\left[\left(\frac{N}{4}\right)(-1)^2 + \frac{N}{4}(1)^2 + \frac{N}{4}(1)^2 + \frac{N}{4}(-1)^2\right] = 0$$

Similarly for $n \geq 1$ $R_n = 0$

Hence $S_x(f) = \frac{1}{T_b} = R_b$

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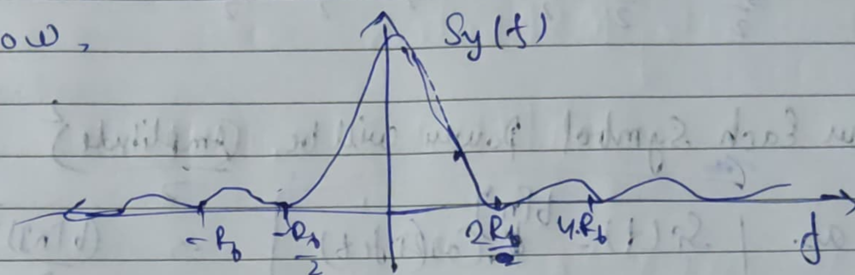
Now; $P(f) = \left(\frac{T}{2}\,\text{sinc}\left(\frac{T}{2}f\right) e^{-j f 2\pi \frac{T}{2}}\right) \cdot \frac{A}{2}$

$$|P(f)|^2 = \left(\frac{T}{2}\right)^2 \left(\frac{A^2}{2}\right) \cdot \text{sinc}^2\left(\frac{T}{2}f\right)$$

(Baseband) $S_y(f) = \frac{T_b}{4} \cdot \frac{A^2}{4} \cdot \text{sinc}^2\left(\frac{T_b}{2}f\right)$, where $T_b = \frac{1}{R_b}$

Now,



Now,

Minimum Bandwidth will ~~be the~~ ~~null~~ ~~as~~ main lobe containment

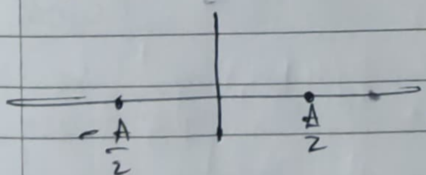
therefore $= 2R_b$

Also, in Passband → (2× Baseband BW) i.e

minimum Transmitted Bandwidth $\boxed{4R_b}$ = Ans

Transmitted Power ⇒ transmitted signal

Since, $S_i(t) = \pm \frac{A}{2} \cos(2\pi f_c t)$



therefore Avg Power:

$$P_{Avg} = \frac{1}{2}\cdot\frac{1}{2}\left(-\frac{A}{2}\right)^2 + \frac{1}{2}\cdot\frac{1}{2}\left(\frac{A}{2}\right)^2 = \frac{1}{2}\cdot\frac{A^2}{4} = \text{Ans } \frac{A^2}{8}$$

classmate

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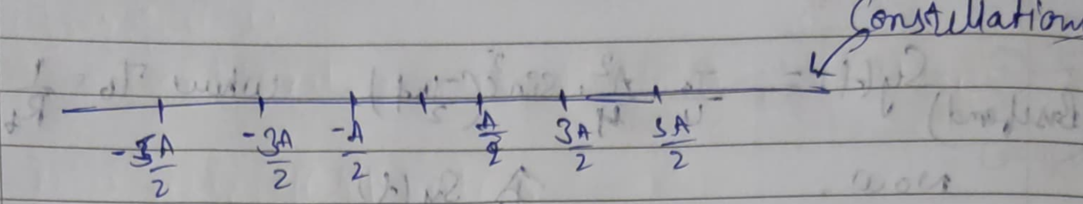
(b) Now, using half-width pulses:

$$\pm\frac{A}{2}, \pm\frac{3A}{2}, \ldots \pm\left[\frac{M-1}{2}\right]A$$



Now for each symbol power will be $\frac{(\text{Amplitude})^2}{2}$

as. $\boxed{s_i(t) = b[n]\cos(2\pi f_c t)} \Rightarrow \frac{(b[n])^2}{2}$

Now, since all are equally probable

$$P_{avg} = \frac{1}{M}\left[\left(\frac{A}{2}\right)^2 + \left(\frac{-A}{2}\right)^2 + \left(\frac{3A}{2}\right)^2 + \left(\frac{-3A}{2}\right)^2 + \cdots \left(A\left(\frac{M-1}{2}\right)\right)^2 + \left(-\frac{(M-1)}{2}A\right)^2\right]$$

for equally probable $P_{avg} = \frac{2\times}{2}\left(\frac{A}{2}\right)^2 \frac{1}{M}\left[1 + 3^2 + \cdots (M-1)^2\right]$

$$P_{Avg} = \frac{A^2}{M\cdot 4}\left[1 + 3^2 + 5^2 + \cdots (M-1)^2\right]$$

$$= \frac{A^2}{M\cdot 4}\sum_{k=1}^{M/2}(2k-1)^2$$

$$= \frac{A^2}{M\cdot 4}\sum_{k=1}^{M/2} 4k^2 - 4k + 1$$

$$= \frac{A^2}{M\cdot 4}\left[\frac{4\times\left(\frac{M}{2}\right)\left(\frac{M}{2}+1\right)(M+1)}{6} - \frac{4\left(\frac{M}{2}\right)\left(\frac{M}{2}+1\right)}{2} + 1\right]$$

$$= \frac{A^2}{M\cdot 4}\cdot\frac{M(M^2-1)}{6} = \frac{A^2\cdot\cancel{M}(M^2-1)}{\cancel{M}\,24}$$

$$P_{Avg\ Symbol} = \frac{A^2(M^2-1)}{24}$$

$$\boxed{\text{Power per bit} = \frac{A^2(M^2-1)}{24\log_2 M}}$$

Ques 3:

(a) Given Bit rate R_b

Now. minimum Theoritical BW ⇒

Baseband $\left(\frac{R_b}{2}\right)$ Passband (R_b)

Now. we are using 16-ary PAM

Now. BW will be

Baseband ⇒ $\frac{R_b}{2(\log_2 16)} = \frac{R_b}{8}$

Passband ⇒ $\frac{R_b}{\log_2 16} = \frac{R_b}{4}$ → Ans

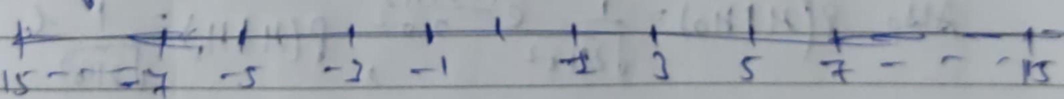
It is reduced by the factor of 4.

(b) Let assume at a rate of R_b 2-PAM was transmitted initially.

$P_{Avg} = \frac{1}{2}\frac{(1)^2}{2} + \frac{1}{2}\frac{(-1)^2}{2}$

⇒ $\frac{1}{2}$

Now for 16-PAM



$P_{Avg} = \frac{2}{2\times 16}\left[(1)^2 + (3)^2 + (5)^2 + \dots + (15)^2\right]$

$P_{Avg} = \frac{680}{16} = 42.5$

Now Increased Power $= \frac{42.5}{0.5} \Rightarrow \boxed{85}$

Power got increased by a factor of 85

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Ques!

Given, Prior Probabilities
$\pi_0 = P(H_0)$
$\pi_1 = P(H_1)$
Observation r.
Conditional Densities $P(r|H_0)$, $P(r|H_1)$

Now using MAP rule →

$$\pi_0 P(r|H_0) \underset{H_1}{\overset{H_0}{\gtrless}} \pi_1 P(r|H_1)$$

$$\frac{P(r|H_0)}{P(r|H_1)} \underset{H_1}{\overset{H_0}{\gtrless}} \frac{\pi_1}{\pi_0} \rightarrow (1)$$

Example : **BPSK**
Let say,
H_0 : Bit 0 was sent → Signal $S_0 = -A$
H_1 : Bit 1 was sent → Signal $S_1 = +A$

Recieved signal
$r = S + N$ Let say $N \sim N(0, \sigma^2)$

Also. $P(r|H_0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(r+A)^2}{2\sigma^2}}$, $P(r|H_1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(r-A)^2}{2\sigma^2}}$

Now, using MAP rules Eq (1).

$$\pi_0 \, e^{-\frac{(r+A)^2}{2\sigma^2}} \underset{H_1}{\overset{H_0}{\gtrless}} \pi_1 \, e^{-\frac{(r-A)^2}{2\sigma^2}}$$

$$e^{\frac{2Ar}{\sigma^2}} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\pi_0}{\pi_1}$$

taking log →

$$\frac{2Ar}{\sigma^2} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\pi_0}{\pi_1}$$

~~Now~~ now, $$r \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\sigma^2}{2A} \ln\left(\frac{\pi_0}{\pi_1}\right)$$

If Prior probabilities are same

$$r \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\sigma^2}{2A}(0) \qquad r \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

Hence This is the matched filter output being compared to zero → Hence the optimal ML detector for BPSK in AWGN.