Communication Theory
Spring-2025

Assignment 3 **Deadline: March 11, 5PM**

Instructions:

- All questions are compulsory.
- Clearly state the assumptions *(if any)* made that are not specified in the questions.
- Submission format: `Rollnumber.pdf`

Cautions:

(a) One late homework assignment is allowed without penalty.

(b) 2 marks will be deducted on other late assignments.

Questions

1. Let $p(t) = I_{[-\frac{1}{2},\frac{1}{2}]}(t)$ denote a rectangular pulse of unit duration. Construct the signal

$$m(t) = \sum_{n=-\infty}^{\infty} (-1)^n p(t-n)$$

The signal $m(t)$ is input to an FM modulator, whose output is given by

$$u(t) = 20\cos(2\pi f_c t + \varphi(t))$$

where

$$\varphi(t) = 20\pi \int_{-\infty}^{t} m(\tau)\, d\tau + a$$

and a is chosen such that $\varphi(0) = 0$.

   (a) Carefully sketch both $m(t)$ and $\varphi(t)$ as a function of time.

   (b) Approximating the bandwidth of $m(t)$ as $W \approx 2$, estimate the bandwidth of $u(t)$ using Carson's formula.

   (c) Suppose that a very narrow ideal BPF (with bandwidth less than 0.1) is placed at $f_c + \alpha$. For which (if any) of the following choices of α will you get nonzero power at the output of the BPF: (i). $\alpha = 0.5$, (ii). $\alpha = 0.75$, (iii). $\alpha = 1$.

2. Figure 1 shows, as a function of time, the phase deviation of a bandpass FM signal modulated by a sinusoidal message.

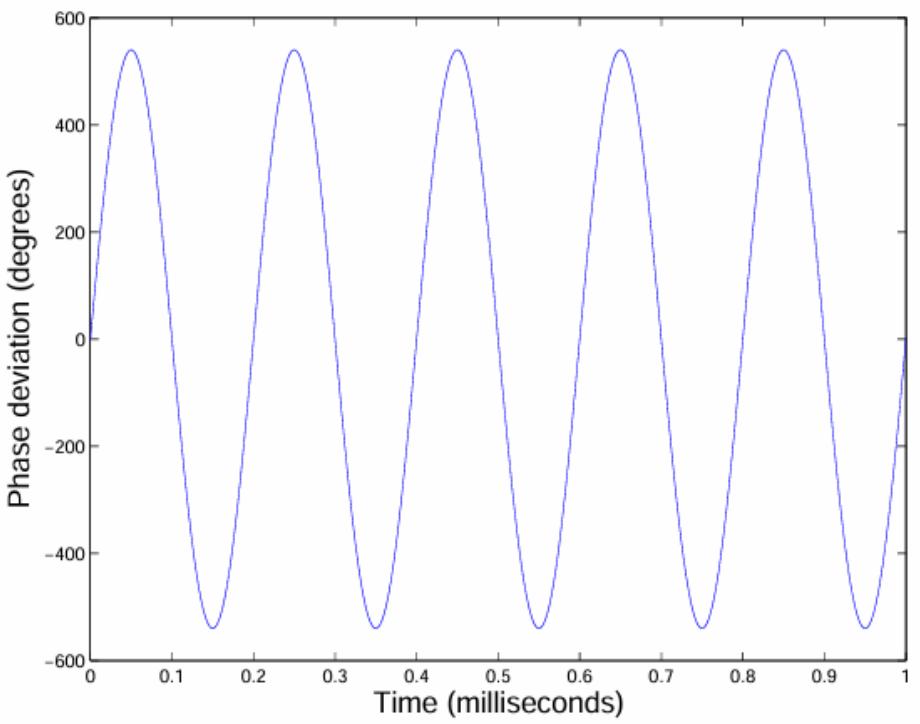

Figure 1: Phase deviation of FM signal for problem 2

(a) Find the modulation index (assume that it is an integer multiple of π for your estimate).

(b) Find the message bandwidth.

(c) Estimate the bandwidth of the FM signal using Carson's formula.

3. An angle-modulated signal with carrier frequency $f_c = 2\pi * 10^5$ is described by the equation $u_p(t) = 10\cos(f_c + 5\sin 3000t + 10\sin 2000\pi t)$

(a) Find the power of the modulated signal

(b) Find the frequency deviation

(c) Find the modulation index β

(d) Estimate the bandwidth of $u_p(t)$

4. The input $m(t)$ to an FM modulator with $k_f = 1$ has Fourier transform:

$$M(f) = \begin{cases} j2\pi f, & |f| < 1 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

The output of the FM modulator is given by:

$$\phi_{FM}(t) = A\cos(2\pi f_c t + \phi(t)) \tag{2}$$

where f_c is the carrier frequency.

(a) Find an explicit time domain expression for $\phi(t)$ and carefully sketch $\phi(t)$ as a function of time.

(b) Determine $\phi_{FM}(t)$, the frequency modulated signal, and plot it as a function of time.

(c) Find the magnitude of the instantaneous frequency deviation from the carrier at time $t = \frac{1}{4}$.

(d) Using the result from (b) as an approximation for the maximum frequency deviation, estimate the bandwidth of $u(t)$.

Note: Here $\phi(t)$ is the phase term, while $\phi_{FM}(t)$ is the FM signal. Do not get confused.

5. Provide a MATLAB code for the following questions, do not use inbuilt function to perform the modulation

(a) Generate an FM signal using a sinusoidal message signal with a carrier frequency of 1 kHz, frequency deviation of 50 Hz, and a modulation index of 5, with appropriate f_m.

(b) Plot the time-domain waveform of both the message and FM signal.

(c) Compute and plot the frequency spectrum of the FM signal.

(d) Implement a demodulation technique (have to select a demodulator discussed in the class) to recover the message signal.