Assignment - 2

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(1)

(a) $m(t) = \cos(1000\pi t) \cdot \cos(2000\pi t)$

$$= \frac{2}{2} \times \cos(1000\pi t)\cos(2000\pi t)$$

$$= \frac{1}{2}(2\cos A \cos B) \qquad \because 2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$= \frac{1}{2}\left[\cos(4000\pi t) + \cos(2000\pi t)\right]$$

$$m(t) = \frac{1}{2}\left[\cos(2\pi(2000)t) + \cos(2\pi(1000)t)\right].$$

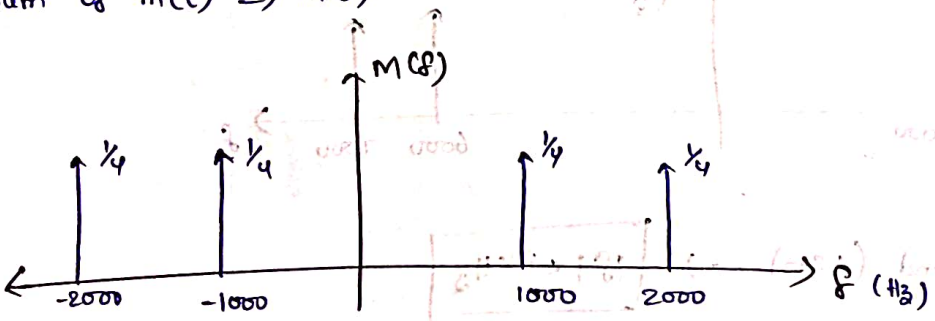
$$\updownarrow$$

$M(f)$ $\qquad m(t) \leftrightarrow M(f)$

$$\therefore M(f) = \frac{1}{2}\left[\frac{\delta(f-2000) + \delta(f+2000)}{2} + \frac{\delta(f-1000) + \delta(f+1000)}{2}\right]$$

$$= \frac{1}{4}\left[\delta(f-2000) + \delta(f+2000) + \delta(f-1000) + \delta(f+1000)\right]$$

Spectrum of $m(t) \Rightarrow M(f)$



(b) We need to sketch spectrum of DSB-SC signal

$$y(t) = m(t)\cos(10{,}000\pi t) = m(t)\cos(2(5000)\pi t)$$

$$y(t) \leftrightarrow Y(f)$$

$$y(t) = m(t) \cdot \cos(2\pi(5000)t)$$

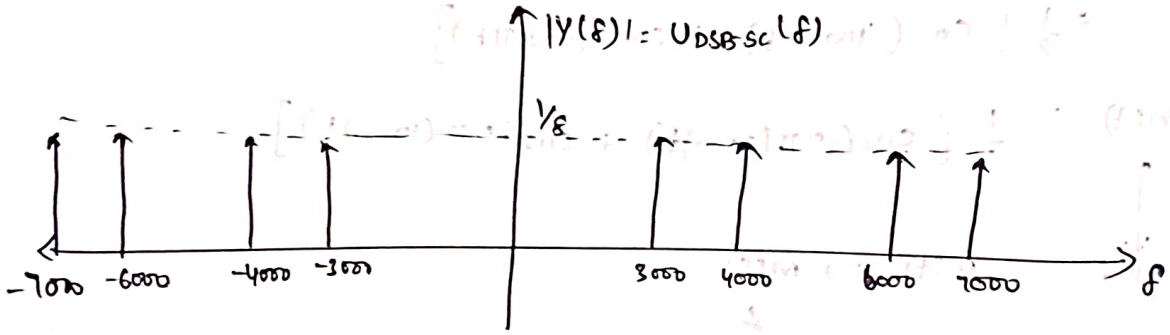
$$\updownarrow$$

$$Y(f) = M(f) * \left[\frac{\delta(f-5000) + \delta(f+5000)}{2}\right]$$

$$Y(f) = M(f) * \left[\frac{\delta(f-5000) + \delta(f+5000)}{2}\right]$$

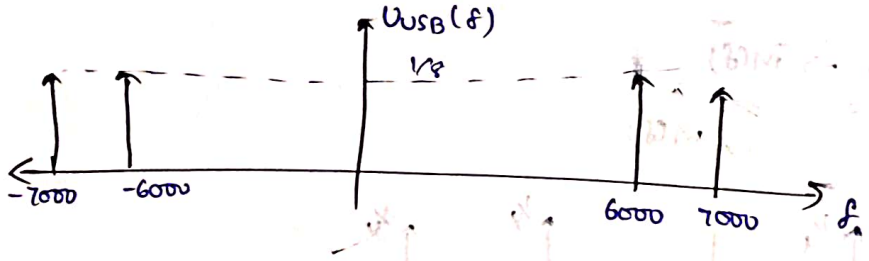
$$Y(f) = \frac{M(f-5000) + M(f+5000)}{2}$$

$$Y(f) = \frac{1}{8}\Big(\delta(f-7000) + \delta(f-6000) + \delta(f-4000) + \delta(f-3000) + \delta(f+3000) + \delta(f+4000) + \delta(f+6000) + \delta(f+7000)\Big)$$

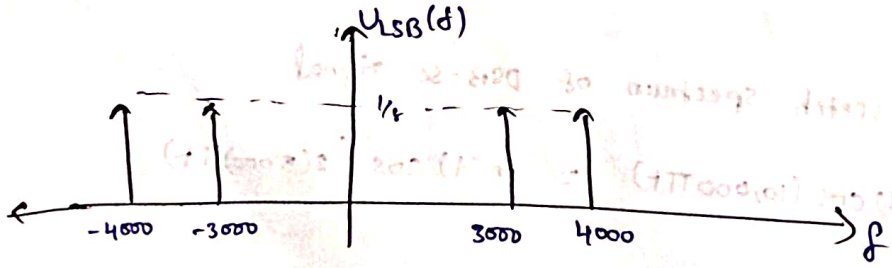


(c) carrier frequency = 5K = 5000 Hz

USB ⇒ upper side Band $|f| > 5K$ $\boxed{|f| > 5K\ Hz}$ ⇒ USB



Lower side Band (LSB) ⇒ $\boxed{|f| < 5KHz}$



(d) Base-band signal $\rightarrow m(t)$ has impulses at $\pm 1K, \pm 2K$

In DSB-SC, the impulses are shifted as:

$1K \longrightarrow 6K$ & $-4K$

$-1K \longrightarrow -6K$ & $4K$

$2K \longrightarrow 7K$ & $-3K$

$-2K \longrightarrow -7K$ & $3K$

Multiplying the carrier at f_c shifts the signal by $\pm f_c$,

here ± 5000

USB-SC :— the impulses are shifted to

$1K \rightarrow 6K$ $\quad$ $-1K \longrightarrow -6K$

$2K \rightarrow 7K$ $\quad$ $-2K \longrightarrow -7K$

only the part of DSB-SC such that it is at $|f| > 5K$ Hz is retained.

In LSB-SC impulses are shifted as.

$1K \rightarrow -4K$ $\quad$ $-1K \longrightarrow 4K$ $\quad$ $\left(\begin{array}{l} 1K \rightarrow 4K \\ -1K \rightarrow 4K \end{array}\right)$

$2K \rightarrow -3K$ $\quad$ $-2K \longrightarrow 3K$

only part of DSB-SC such that $|f| < f_c$ is retained in

LSB-SC

(2) Given message signal

$$m(t) = 2\cos(400t) + 4\sin(500t + \pi/3)$$

& carrier signal:

$$c(t) = A.\cos(8000\pi t)$$

and modulated by DSB (AM) means assumping convential Amplitude modulation (AM)

$$x(t) = U_{DSB}(t) = (A + m(t))\cos(8000\pi t)$$

$$\cancel{M(f) = \delta\left(f - \frac{400}{2\pi}\right) + \delta\left(f + \frac{400}{2\pi}\right)}$$

$$\cancel{M(f) = \delta\left(f - \frac{400}{2\pi}\right) + \delta\left(f + \frac{400}{2\pi}\right) +}$$

$$m(t) = 2\cos(400t) + 4\left(\sin(500t)\frac{1}{2} + \frac{\sqrt{3}}{2}\cos(500t)\right)$$

$$M(t) = 2\cos(400t) + 2(\sin(500t) + \sqrt{3}\cos(500t))$$

$$= 2\cos(400t) + 2\sin(500t) + 2\sqrt{3}\cos(500t)$$

$$M(f) = \cancel{2}.\left[\frac{\delta\left(f - \frac{400}{2\pi}\right) + \delta\left(f + \frac{400}{2\pi}\right)}{\cancel{2}}\right] + \cancel{2}\left[\frac{\delta\left(f - \frac{500}{2\pi}\right) - \delta\left(f + \frac{500}{2\pi}\right)}{\cancel{2}j}\right]$$

$$+ \cancel{2}\sqrt{3}\left[\frac{\delta\left(f - \frac{500}{2\pi}\right) + \delta\left(f + \frac{500}{2\pi}\right)}{\cancel{2}}\right]$$

$$M(f) = \delta\left(f - \frac{400}{2\pi}\right) + \delta\left(f + \frac{400}{2\pi}\right) + \left[\frac{\delta\left(f - \frac{500}{2\pi}\right) - \delta\left(f + \frac{500}{2\pi}\right)}{j}\right]$$

$$+ \sqrt{3}\left[\delta\left(f - \frac{500}{2\pi}\right) + \delta\left(f + \frac{500}{2\pi}\right)\right]$$

$$x(t) = U_{DSB}(t) = (A + m(t))\cos(8000\pi t)$$

$$X(f) = \frac{A}{2}\left[\delta(f - 4000) + \delta(f + 4000)\right] + \frac{M(f)}{2} * \left[\delta(f - 4000) + \delta(f + 4000)\right]$$

$$X(f) = \frac{A}{2}\left(\delta(f-4000) + \delta(f+4000)\right) + \frac{M(f-4000) + M(f+4000)}{2}$$

$$M(f-4000) = \delta(f-4063.66) + \delta(f-3936.3)$$

$$+ \sqrt{3}\left(\delta(f-4079.5) + \delta(f-3920.42)\right)$$

$$+ \frac{1}{j}\left(\delta(f-4079.5) - \delta(f-3920.42)\right)$$

$$M(f+4000) = \delta(f+3936.3) + \delta(f+4063.66)$$

$$+ \sqrt{3}\left(\delta(f+3920.42) + \delta(f+4079.5)\right)$$

$$+ \frac{1}{j}\left(\delta(f+3920.42) - \delta(f+4079.5)\right)$$

$$X(f) = \frac{A}{2}\left(\delta(f-4000) + \delta(f+4000)\right) + \frac{1}{2}\delta(f-4063.66) + \frac{1}{2}\delta(f-3936.3)$$

$$+ \frac{\sqrt{3}}{2}\left(\delta(f-4079.5) + \delta(f-3920.42)\right)$$

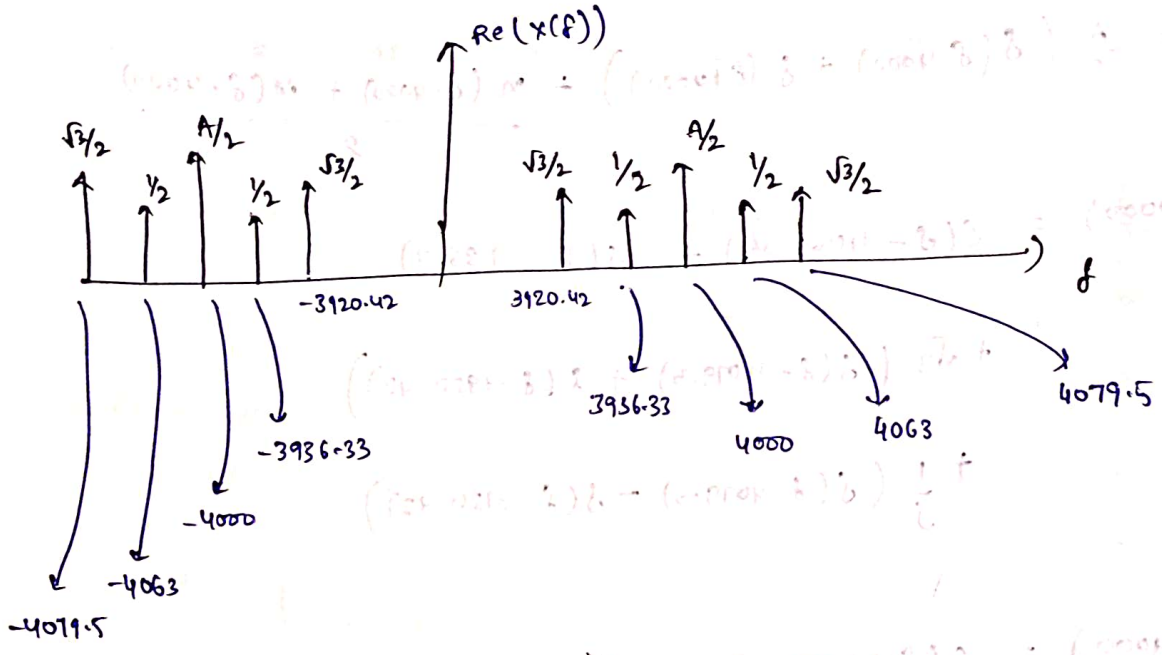
$$+ \frac{1}{2j}\left(\delta(f-4079.5) - \delta(f-3920.42)\right)$$

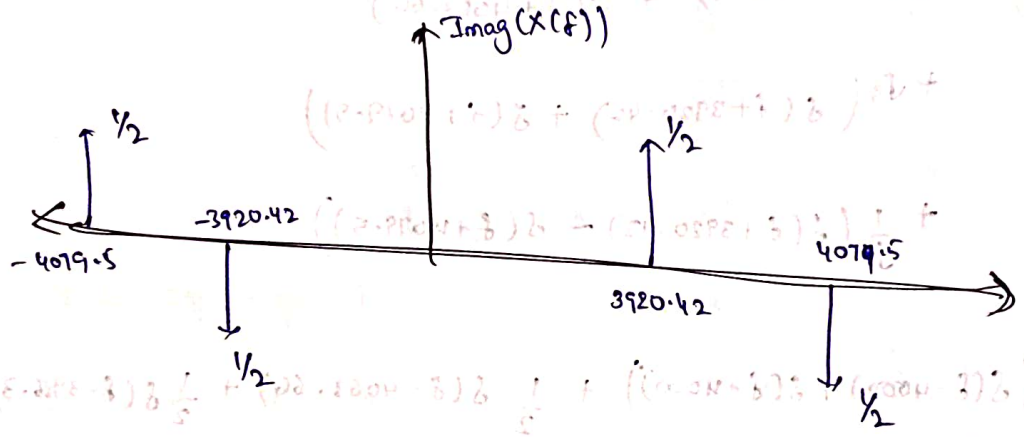
$$\cancel{+ \frac{1}{2}\left(\delta(f-4063.66) + \delta(f+4036.66)\right)}$$

$$+ \frac{1}{2}\left(\delta(f+3936.3) + \delta(f+4063.66)\right)$$

$$+ \frac{\sqrt{3}}{2}\left(\delta(f+3920.42) + \delta(f+4079.5)\right)$$

$$+ \frac{1}{2j}\left(\delta(f+3920.42) - \delta(f+4079.5)\right)$$





Power content:

$$v_{DSB}(t) = (A + m(t)) \cos(8000\pi t)$$

$$= A\cos(8000\pi t) + m(t)\cos(8000\pi t)$$

$$= A\cos(8000\pi t) + 2\cos(400t)\cos(8000\pi t) + 4\sin(500t + \pi/3)\cos(8000\pi t)$$

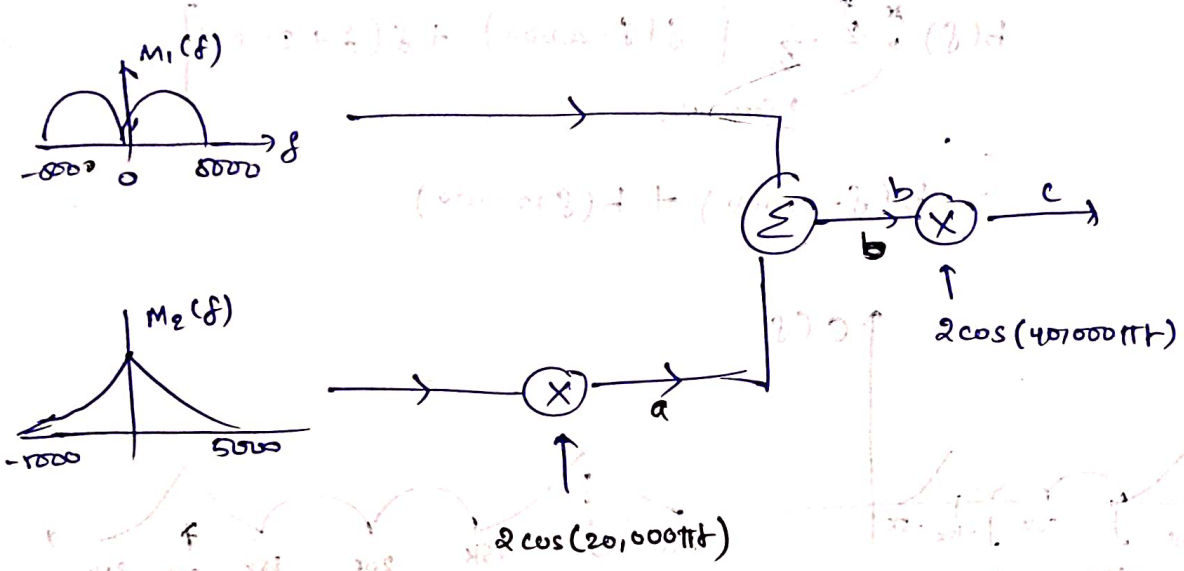
$$= A\cos(8000\pi t) + \cos((8000\pi + 4000)t) + \cos((8000\pi - 400)t) - 2\sin((8000\pi - 500)t - \pi/3) + 2\sin(8000\pi t + 500t + \pi/3)$$

$\Rightarrow$ power $\{\cos\theta(t)\} = 1/2$

$$= \frac{A^2}{2} + 1/2 + 1/2 + \frac{(2)^2}{2} + \frac{(2)^2}{2} \Rightarrow 5 + \frac{A^2}{2}$$

(3) $m_1(t)$ & $m_2(t)$ ⇒ Bandlimited 5 KHz.

Carrier frequency $(f_c) = 20000$ Hz

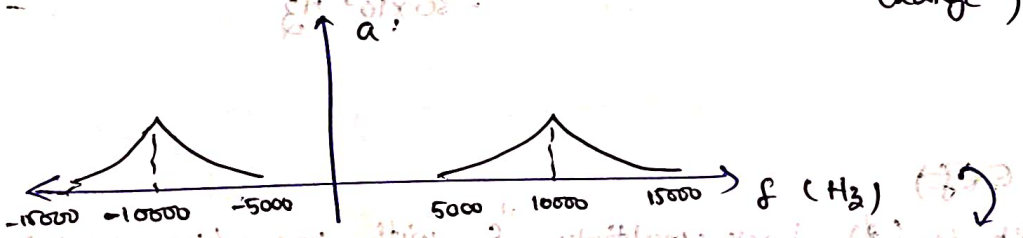


(a)

For (a) ⇒ by multiplying 2cos(20,000πt) the graph $M_2(f)$ will the spectrum will shift by 10000 Hz without changing Amplitude.

(∵ since multiplied "2" → the Amplified won't Change)

a)

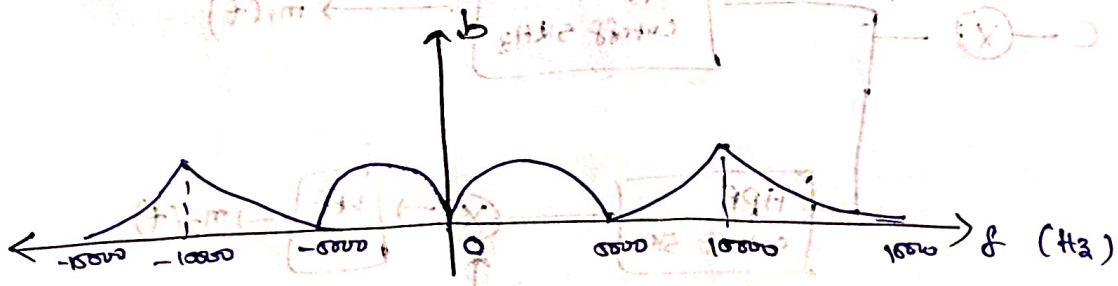


$$a = \left(\frac{M_2(f-10000) + M_2(f+10000)}{2}\right)$$

For (b) ⇒ $b = M_1(f) + a$

$b = M_1(f) + a$ #

$= M_1(f) + M_2(f-1000) + M_2(f+1000)$



For part (c) $\downarrow$

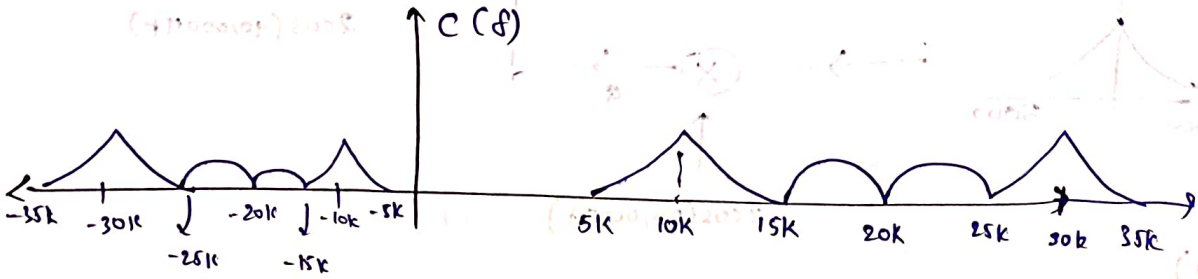
$b(t) \cdot 2\cos(40000\pi t)$

$\Downarrow$

$b(f) * 2 \cdot \frac{1}{2}\left[\delta(f-20000) + \delta(f+20000)\right]$

Convolution

$= b(f-20000) + b(f+20000)$



(b)

Bandwidth of channel $= (35k - 5k)$ Hz

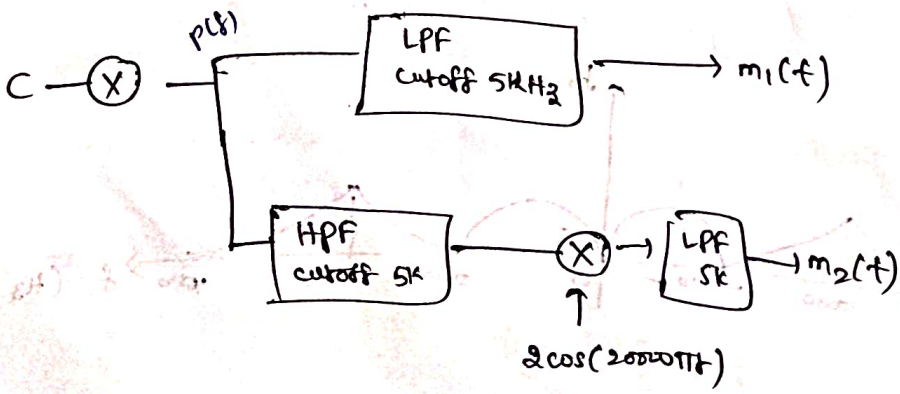
$= 30$ KHz

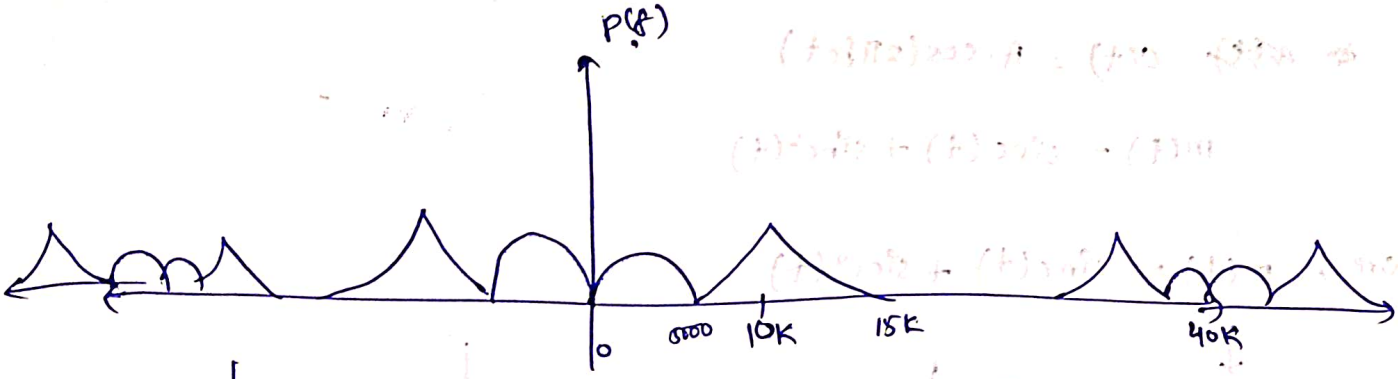
$= 30 \times 10^3$ Hz

(c)

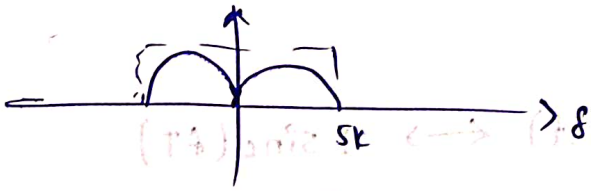
$\Rightarrow$ To get $M_1(t)$ back multiply "c" with $2\cos(40000\pi t)$ and then LPF with cutoff at 5KHz.

$\Rightarrow$ To get $M_2(t)$ back multiply "c" with $2\cos(40000\pi t)$, and then HPF of cutoff 5KHz then multiply with $2\cos(20000\pi t)$ and then LPF at 5KHz.



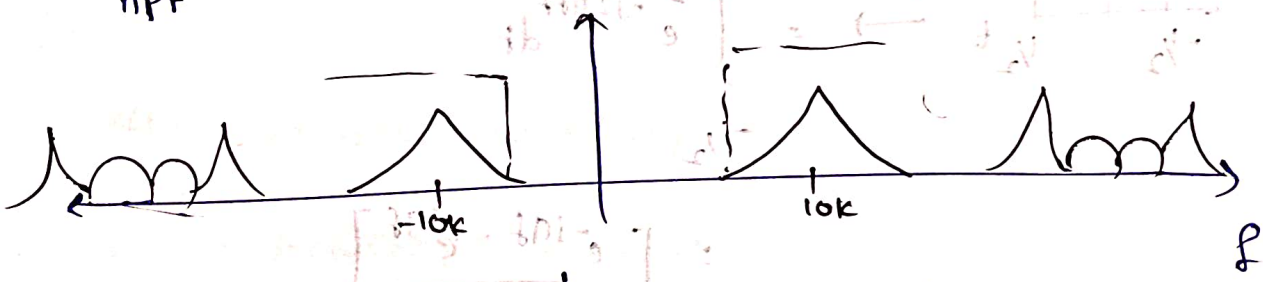


LPF (5kHz)

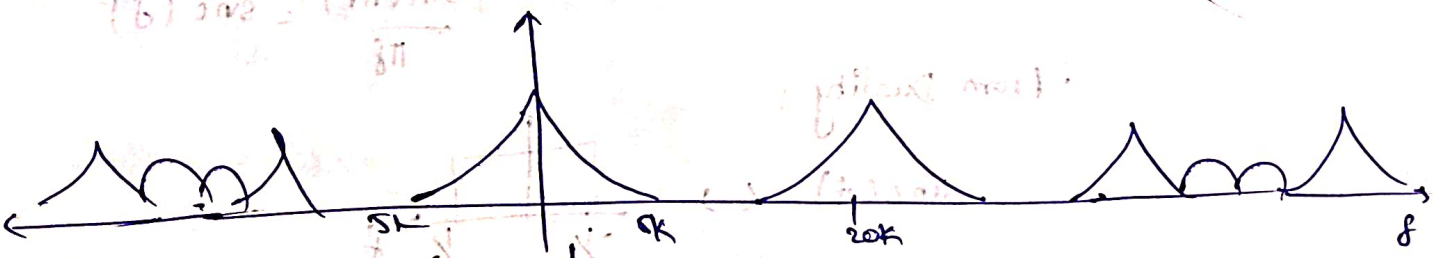


$\Rightarrow m_1(t)$

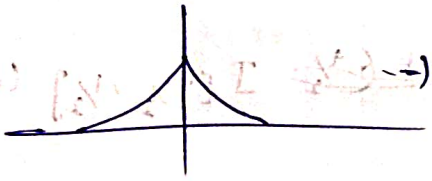
APF



$\otimes\ 2\cos(200000\pi t)$



LPF (5kHz)



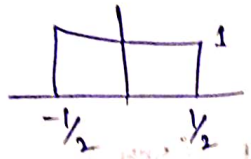
$\Rightarrow m_2(f) \Rightarrow m_2(t)$

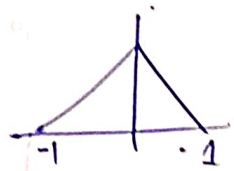
(4) ~~x(t)~~ $c(t) = A \cdot \cos(2\pi f_c t)$

$m(t) = \text{sinc}(t) + \text{sinc}^2(t)$

Here, $m(t) = \text{sinc}(t) + \text{sinc}^2(t)$

$\updownarrow$

$M(f) =$ 

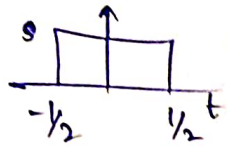
$+$ 

(from sinc prop)

$\text{sinc}(t) \rightarrow$

$I_{[-T/2, T/2]}(t) \longleftrightarrow T\,\text{sinc}(fT)$

↳ property 4

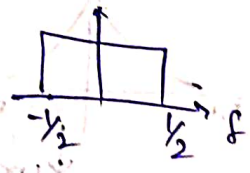
$\therefore$ 

$\xrightarrow{FT} = \int_{-1/2}^{1/2} e^{-j2\pi f t}\,dt$

$= \left[\dfrac{e^{-j\pi f} - e^{j\pi f}}{-j2\pi f}\right]$

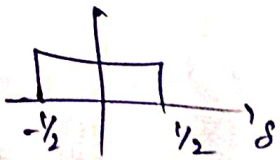
$= \dfrac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$

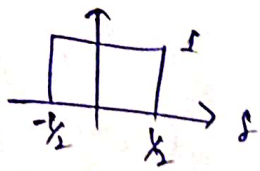
$\therefore$ From Duality:

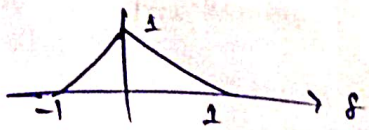
$\text{sinc}(t) \longleftrightarrow$ 

$\text{sinc}^2(t) = \text{sinc}(t) \cdot \text{sinc}(t)$

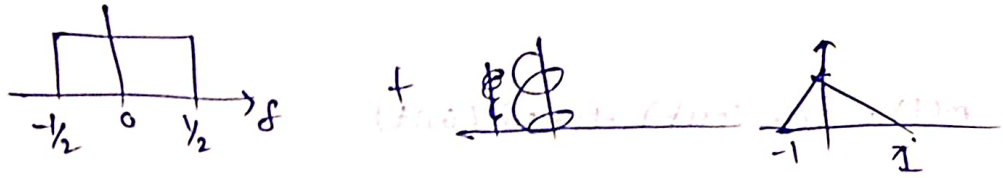
$= \cancel{\text{rect}(t/2)}\ I_{[-1/2, 1/2]}(f) * I_{[-1/2, 1/2]}(f)$

$=$ 

$*$ 

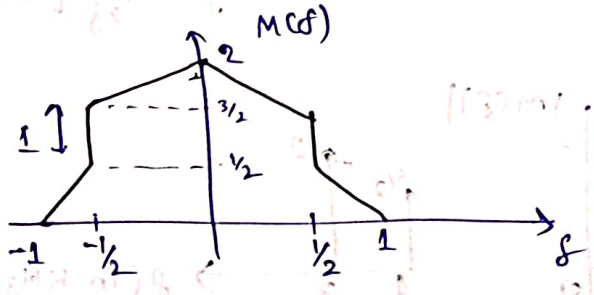
$=$ 

$M(f) =$



$$= I_{[-1/2,\,1/2]}(f) + (1-|f|)\, I_{[-1,1]}(f)$$

$M(f) =$



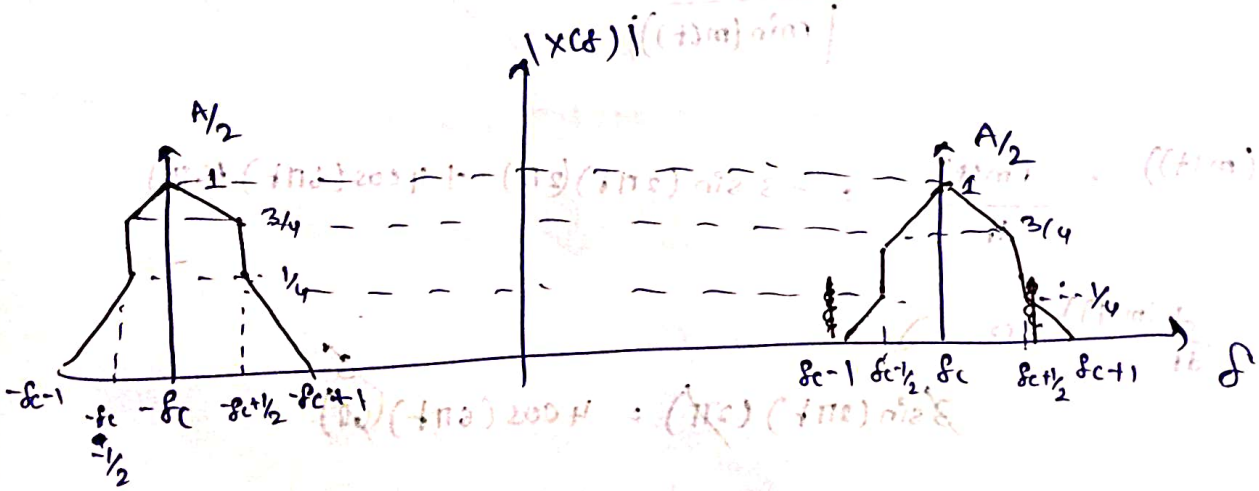
$$x(t) = (A + m(t))\cos(2\pi f_c t)$$

$$x(t) = A\cos(2\pi f_c t) + m(t)\cos(2\pi f_c t)$$

$$\downarrow$$

$$X(f) = \frac{A}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + M(f) * \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

$$X(f) = \frac{A}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{M(f-f_c) + M(f+f_c)}{2}$$



Bandwidth of Modulated signal $\Rightarrow (f_c+1) - (f_c-1) = 2\,Hz$

(5)

$m(t) = 3\cos(2\pi t) + 4\sin(6\pi t)$

Unit ⇒ time in milliseconds

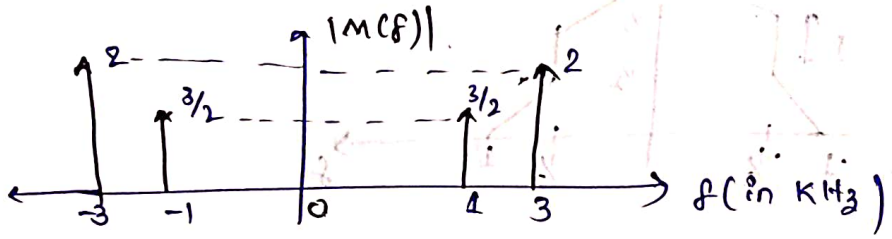
Carrier freq ⇒ 600 KHz

$= 600 \times 10^3$ Hz

$m(t) = 3\cos(2\pi t) + 4\sin(6\pi t)$ ∴ freq in KHz #

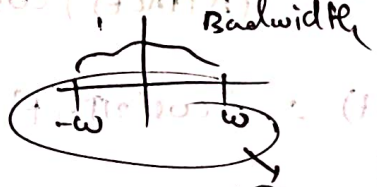
↓

$$M(f) = \frac{3}{2}\left[\delta(f-1000) + \delta(f+1000)\right] + \frac{4}{2j}\left[\delta(f-3000) - \delta(f+3000)\right]$$



Bandwidth of message signal

$= 3$ KHz



ω max freq #

(b) $m_n(t) \rightarrow$ Normalised message

$$m_n(t) = \frac{m(t)}{|\min(m(t))|}$$

$$\min(m(t)) = \frac{dm(t)}{dt} = -3\sin(2\pi t)(2\pi) + 4\cos(6\pi t)(6\pi)$$

$$\frac{d}{dt}(m(t)) = 0$$

$$\cancel{3}\sin(2\pi t)(\cancel{2\pi}) = 4\cos(6\pi t)(\cancel{6\pi})^{3}$$

$$\sin(2\pi t) = 4\cos(6\pi t)$$

$$\sin(2\pi t) = 4\sqrt{1-\sin^2(6\pi t)}$$

$$\Rightarrow \sin^2(2\pi t) = 16\left(1-\sin^2(6\pi t)\right)$$

$$= 16 - 16\sin^2(6\pi t)$$

$$= 16 - 16\left(3\sin(2\pi t) - 4\sin^3(2\pi t)\right)^2$$

let $x = \sin(2\pi t)$

$$x^2 = 16 - 16(3x - 4x^3)^2$$

$$x^2 = 16 - 16(9x^2 + 16x^6 - 24x^4)$$

$$\sin^2(2\pi t) = y(t)$$

$$y = 16 - 144y + 256y^3 - 384y^2$$

$$\Rightarrow 256y^3 - 384y^2 - 144y + 16 = 0$$

$\Rightarrow$ from Desmos we get the minimum value of

$3\cos(z) + 4\sin(3z) \rightarrow$ Desmos

$= -6.627$ at $z = 3.626$

So, $|\min(m(t))| = 6.627$

$$m_n(t) = \frac{m(t)}{|\min(m(t))|} = \frac{3\cos(2\pi t) + 4\sin(6\pi t)}{6.627}$$

(c) $U_{am}(t) = A_c(1 + a_{mod}\, m_n(t))\cos(2\pi f_c t)$

$$= A_c\left(1 + a_{mod}\left(\frac{3\cos(2\pi t) + 4\sin(6\pi t)}{6.627}\right)\right)\cdot\cos(2\pi f_c t)$$

$\Rightarrow$ time is in milliseconds

$$\boxed{a_{mod} = \tfrac{1}{2}}$$

$$= A_c\left(1 + \frac{1}{2}\left(\frac{3\cos(2\pi t) + 4\sin(6\pi t)}{6.627}\right)\right)\cos(2\pi(600k)t)$$

Here time is in Milliseconds

(d) Now, we know the formula power efficiency

$$\eta_{am} = \frac{a^2_{mod}\,\overline{m_n^2}}{1 + a^2_{mod}\,\overline{m_n^2}}$$

$$\overline{m_n^2} = \frac{\left(\frac{3}{6.627}\right)^2}{2} + \frac{\left(\frac{4}{6.627}\right)^2}{2} = 0.2846$$

If $a_{mod} = 0.5 = 1/2$

$$\eta_{am} = \frac{(1/4)(1/2)\left[\left(\frac{3}{6.627}\right)^2 + \left(\frac{4}{6.627}\right)^2\right]}{1 + (1/4)(1/2)\left[\left(\frac{3}{6.627}\right)^2 + \left(\frac{4}{6.627}\right)^2\right]}$$

$$= 0.066$$

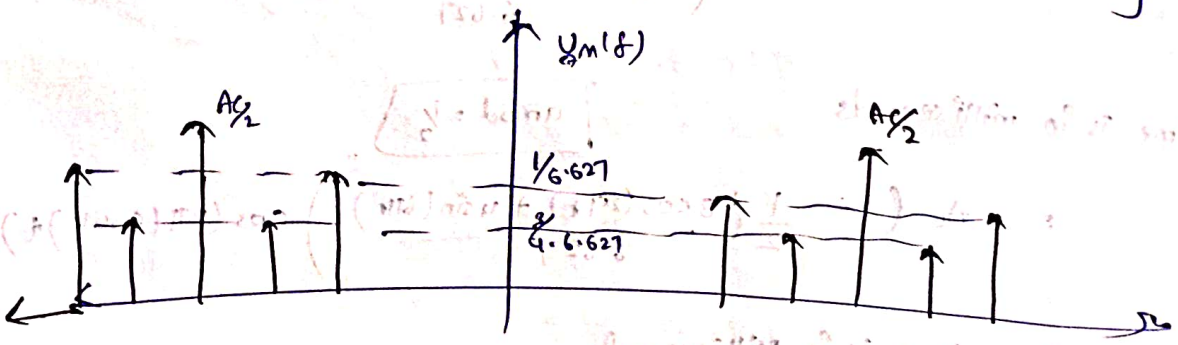
$$\boxed{\therefore \eta_{am} = 0.066}$$

(e)

$$U_{AM}(f) = \frac{A_c}{2}\left[\delta(f-600) + \delta(f+600)\right]$$

$$+ \frac{3}{4 \times 6.627}\left[\delta(f-601) + \delta(f-599) + \delta(f+601) + \delta(f+599)\right]$$

$$+ \frac{1}{6.627 \cdot j}\left[\delta(f-603) - \delta(f-597) - \delta(f+603) + \delta(f+597)\right]$$



(f) The peaks of the carrier appear at a rate of f_c during that the capacitor should not get much discharge.

$$\frac{1}{e^{f_c R_c}} \rightarrow 0 \qquad f_c R_c \gg 1$$

$$RC \gg 1/f_c$$

R_c should be able to detect change in envelope

$$BRC \ll 1$$

$$RC \ll 1/B$$

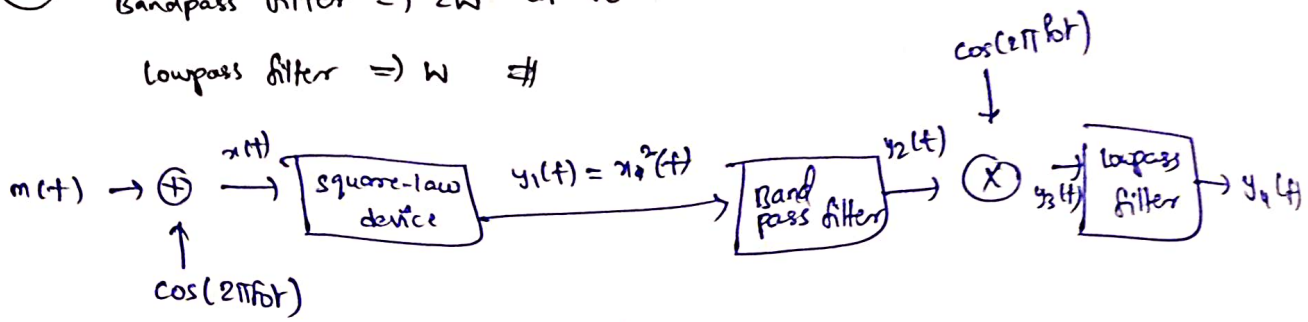
$$C \gg \frac{1}{6\times10^5 \times 50} \quad \& \quad C \ll \frac{1}{3\times10^3 \times 50}$$

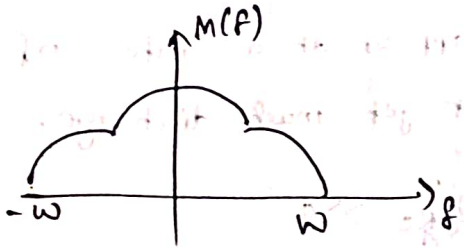
$$C \gg \frac{1}{30}\mu F \qquad C \ll \frac{10^{-4}}{15} F$$

$$\boxed{\frac{1}{30}\mu F \ll C \ll \frac{10^{-4}}{15} F}$$

(6) Bandpass filter $\Rightarrow 2W$ at f_0

Lowpass filter $\Rightarrow W$ #

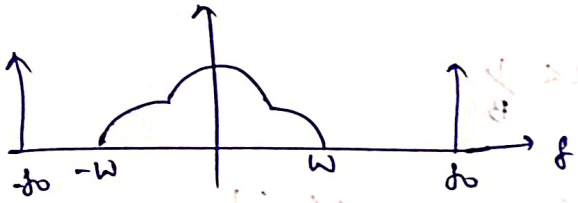




Assuming $f_0 > 3W$

$$x(t) = m(t) + \cos(2\pi f_0 t)$$

$X(f)$



Bandwidth of $x(t) \Rightarrow f_0 \Rightarrow$ (one sided bandwidth)

$$y_1(t) = x^2(t) = m^2(t) + \cos^2(2\pi f_0 t) + 2m(t)\cos(2\pi f_0 t)$$

if $m(t) \longrightarrow (-W \text{ to } W)$

then $m^2(t)$

↳ will be from $-2W$ to $2W$ it is $M(f) * M(f)$

$$\cos^2(2\pi f_0 t) = \frac{1+\cos(4\pi f_0 t)}{2}$$

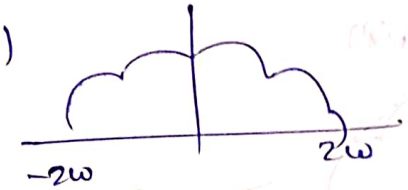
$$= \frac{1+\cos(2(2f_0)\pi t)}{2}$$

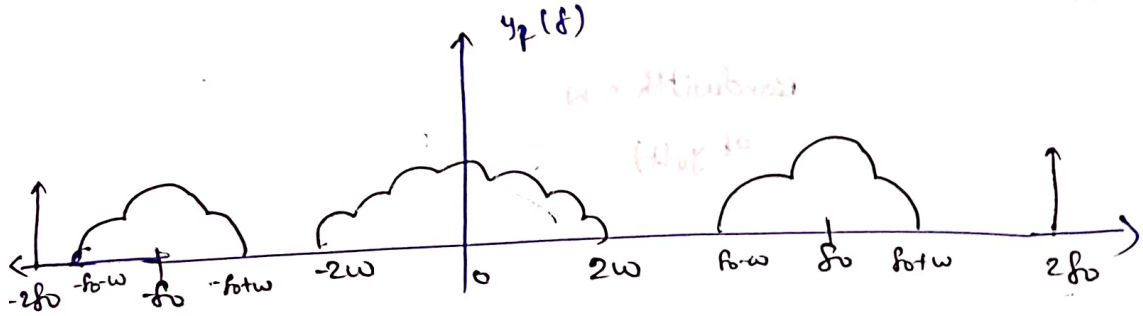
$$= \frac{1}{2} + \cos(2\pi(2f_0)t)$$

$$2m(t)\cos(2\pi f_0 t) \overset{FT}{\Rightarrow} \frac{\cancel{2}\, M(f-f_0) + M(f+f_0)}{\cancel{2}}$$

$$\frac{1}{2} + \cos(2\pi(2f_0)t) \xrightarrow{FT} \frac{\delta(f)}{2} + \frac{\delta(f-2f_0) + \delta(f+2f_0)}{2}$$

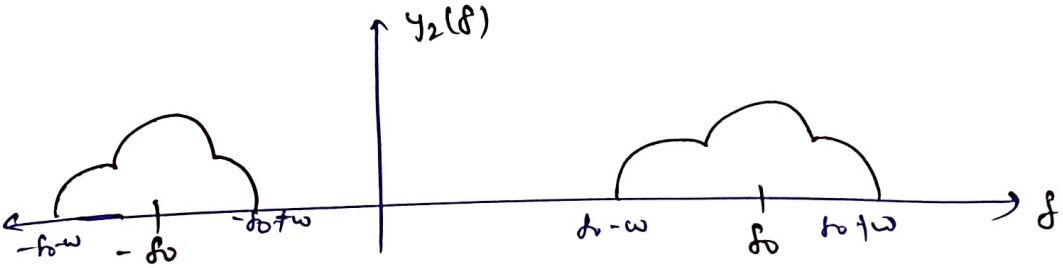
$m^2(t) \Rightarrow m(f) * m(f)$





Bandwidth of $y_1(f) \Rightarrow y_1(t) \Rightarrow 2f_0$

$\downarrow$ Band pass

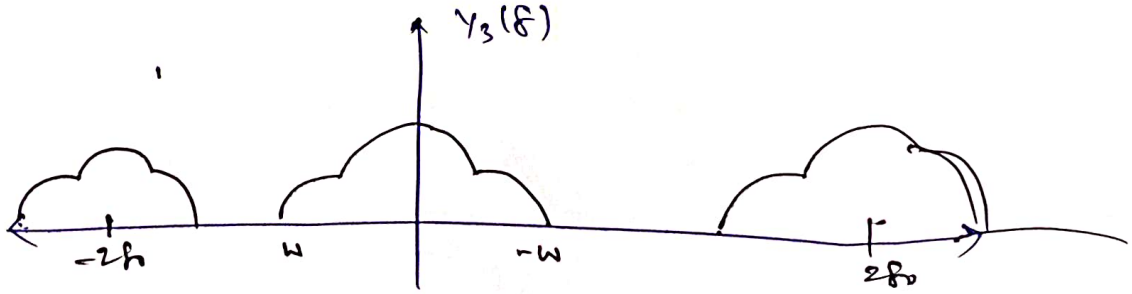


Bandwidth of signal $y_2(t)$ $= (f_0 + w) - (f_0 - w)$

$= 2w$

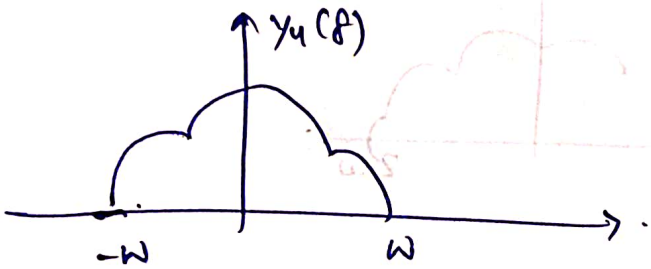
on multiplying the above signal by $\cos(2\pi f_0 t)$.

$$\Rightarrow \frac{Y_2(f-f_0) + Y_2(f+f_0)}{2}$$



on passing $y_3(t)$ through a LPF with bandwidth W

we get the part of $y_3(t)$ only from $-W$ to W



Bandwidth $= W$

of $y_4(t)$

→ THE END —— Theory questions