EC5.203 Communication Theory (3-1-0-4):

Lecture 15:
Digital Modulation - 4

13 March 2025

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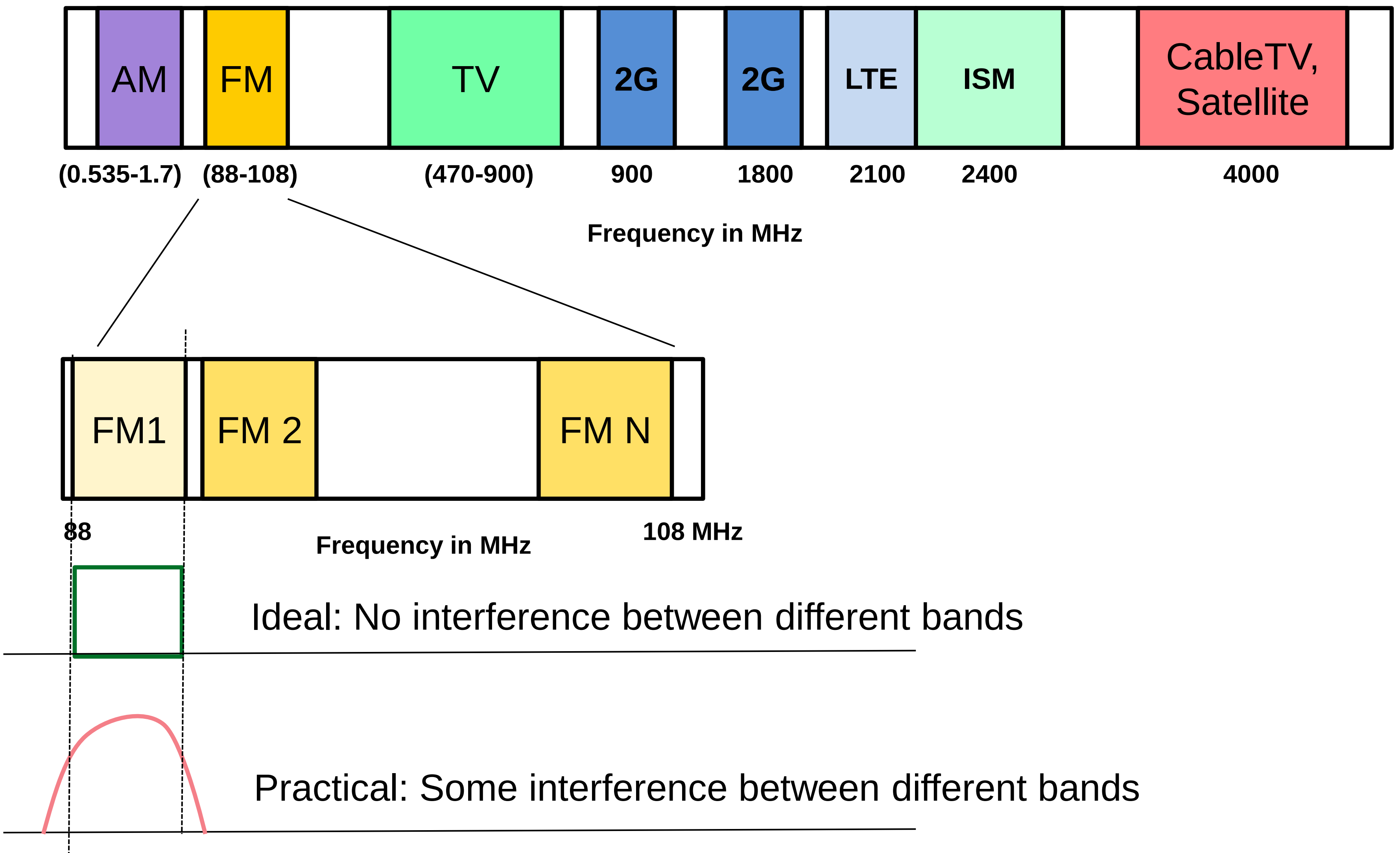
References

- Chap. 4 (Madhow)

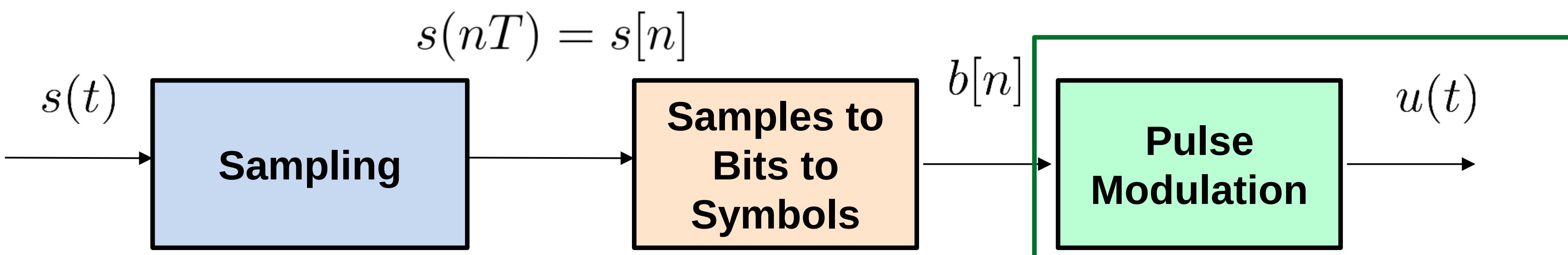
Recap: Design for Bandlimited Channels:

Nyquist Criteria for pulse shaping!

Motivation


AM
FM
TV
2G
2G
LTE
ISM
CableTV, Satellite
(0.535-1.7)
(88-108)
(470-900)
900
1800
2100
2400
4000
Frequency in MHz
FM1
FM 2
FM N
88
108 MHz
Frequency in MHz
Ideal: No interference between different bands
Practical: Some interference between different bands

Design for Bandlimited Channels



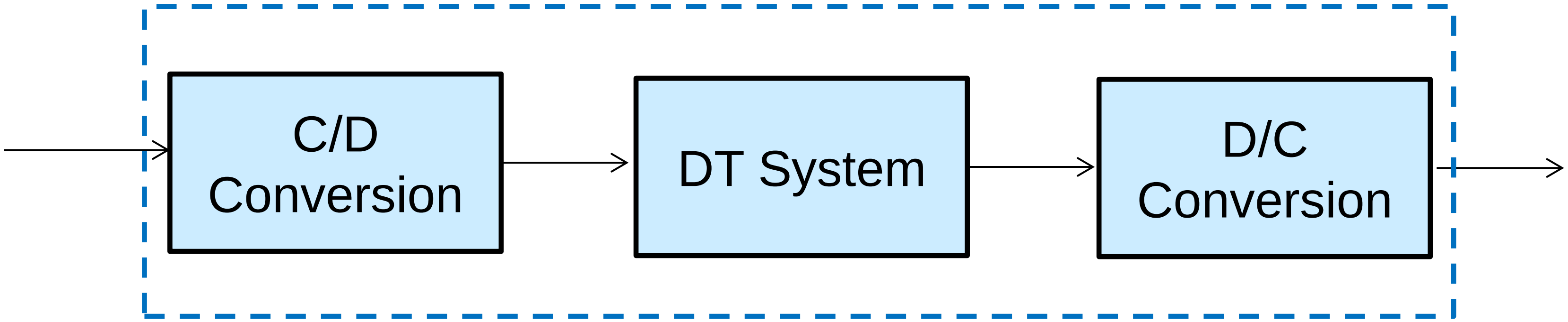
$$s(t) = \sum_{n=-\infty}^{\infty} s\left(\frac{n}{W}\right) g\left(t - \frac{n}{W}\right)$$

$$u(t) = \sum_{n} b[n]p(t - nT)$$

$$S_u(f) = \frac{|P(f)|^2}{T}\sigma_b^2$$

Minimum bandwidth dictated by Sampling
W=1/T

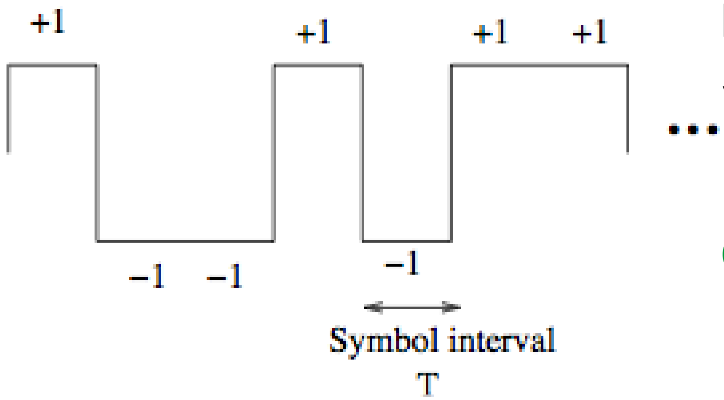
Bandwidth occupancy can be designed based on P(f) and independent of S(f) (FT of s(t)).



What is Inter Symbol Interference (ISI)?

Time Domain *p(t)*

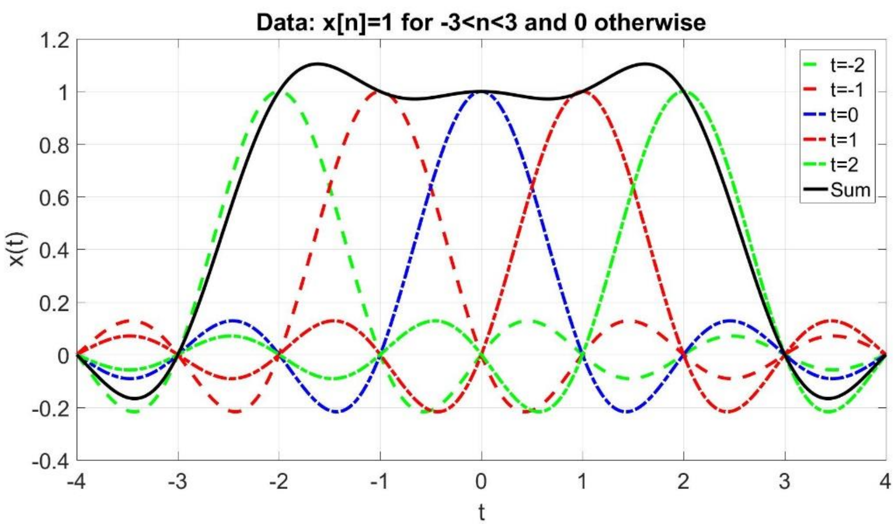
Rectangular Pulse



Freq. Domain *P(f)*
Sinc

No ISI=> SmallTiming offset does not cause issues

Sinc Pulse



Freq. Domain *P(f)*
Rect. Pulse

ISI=> SmallTiming offset does cause issues

No ISI at sampling instances though!

Nyquist Criterion for ISI avoidance

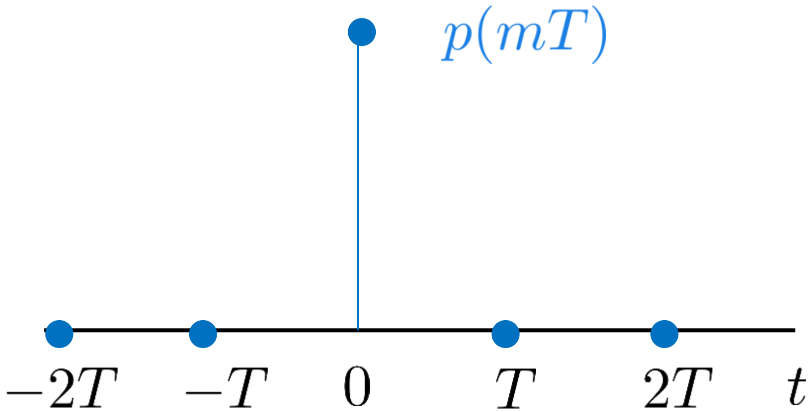
- The pulse $p(t) \leftrightarrow P(f)$ is Nyquist for sampling rate $1/T$ if

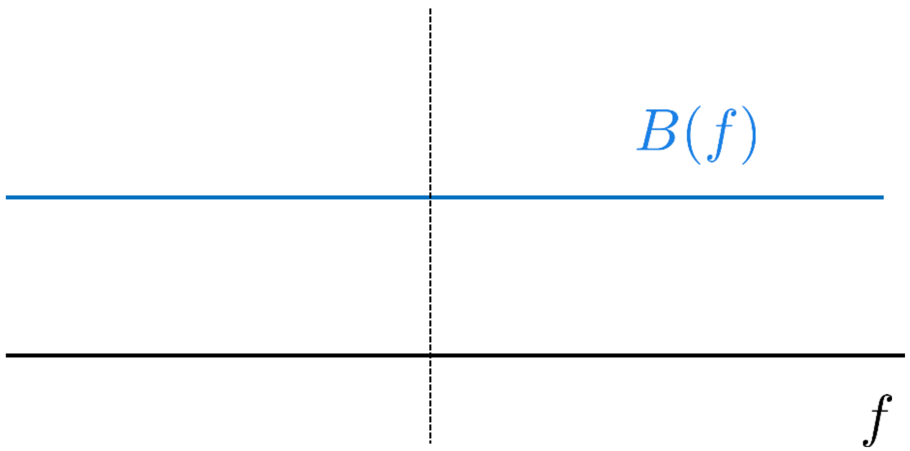
$$p(mT) = \delta_{m0} = \begin{cases} 1, & m = 0 \\ 0, & m \neq 0 \end{cases}$$

or equivalently

DT Fourier Transform Pair

$$B(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} P\left(f + \frac{k}{T}\right) = 1 \quad \forall f$$





Nyquist Criterion for ISI avoidance

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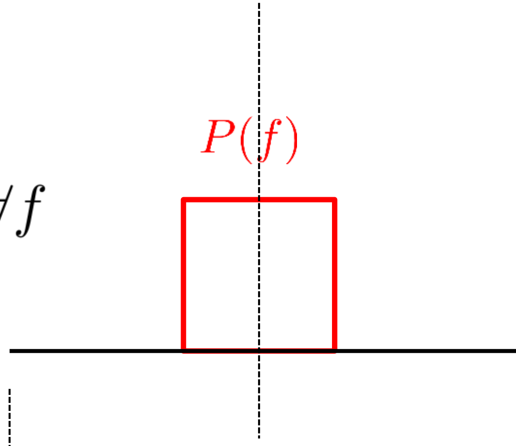
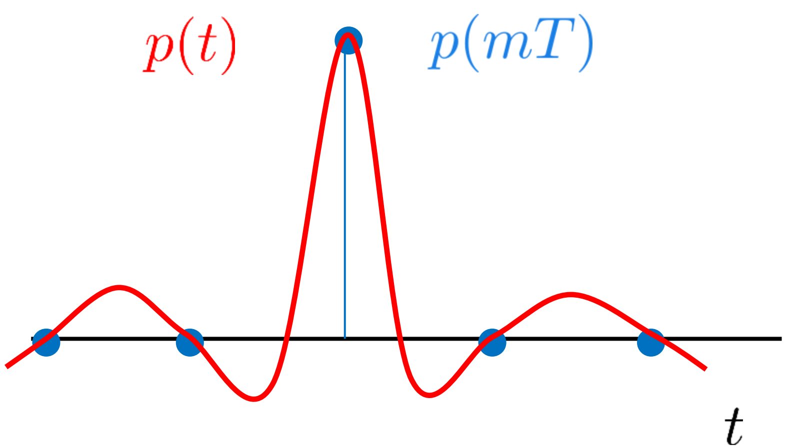
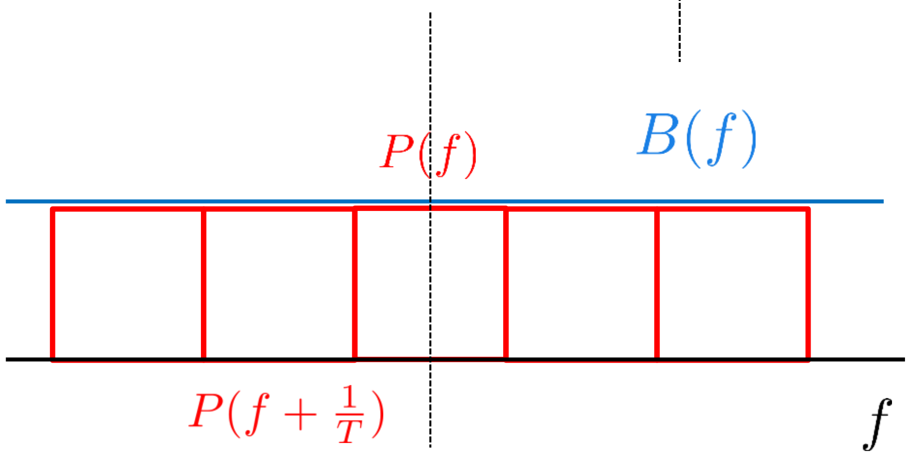






Theorem 4.5.1: Sampling

- Theorem (Sampling): Consider a signal $s(t)$, sampled at rate $1/T_s$. Let $S(f)$ denote the spectrum of $s(t)$, and let

$$B(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} S(f + \frac{k}{T_s})$$

denotes the sum of translates of the spectrum. Then the following observations hold

1. $B(f)$ is periodic with period $1/T_s$.
2. The samples $s(nT_s)$ are Fourier series for $B(f)$, satisfying

$$s(nT_s) = T_s \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} B(f) e^{j2\pi f n T_s} df$$

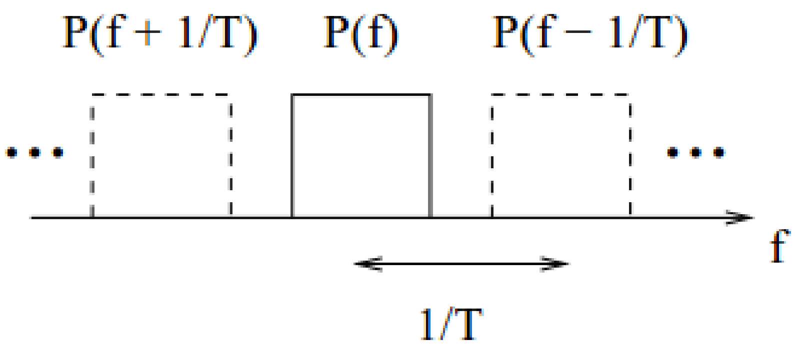
$$B(f) = \sum_{n=-\infty}^{\infty} s(nT_s) e^{-j2\pi f n T_s}$$

Significance of Nyquist Criterion for ISI avoidance

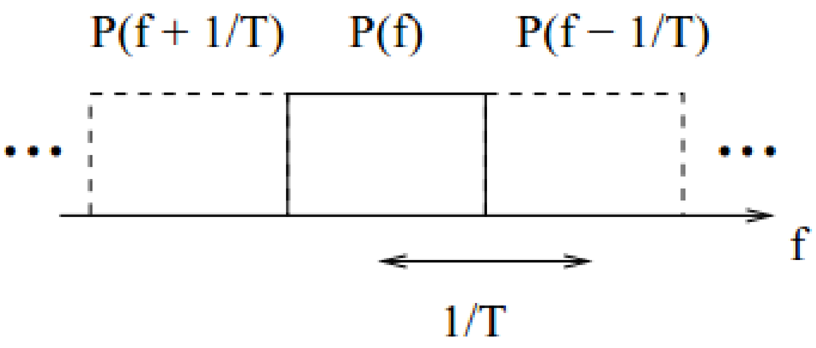
- It provides freedom to expand the modulation in time beyond the symbol duration so that bandwidth containment is better in frequency domain while ensuring that there is no ISI at appropriately chosen sampling intervals despite the significant overlap between successive pulses.

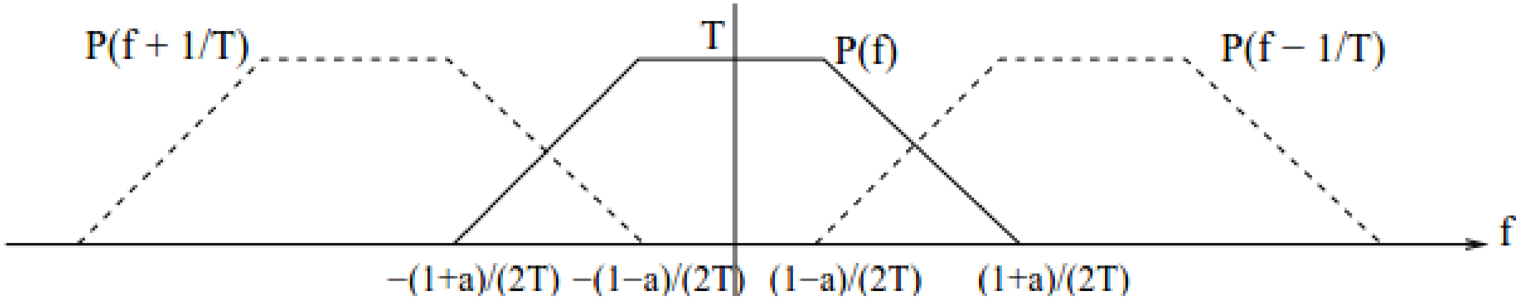
Nyquist Pulses: Excess Bandwidth

Not Nyquist



Nyquist pulse with minimum bandwidth





Nyquist pulse with excess bandwidth

Need of Excess Bandwidth!

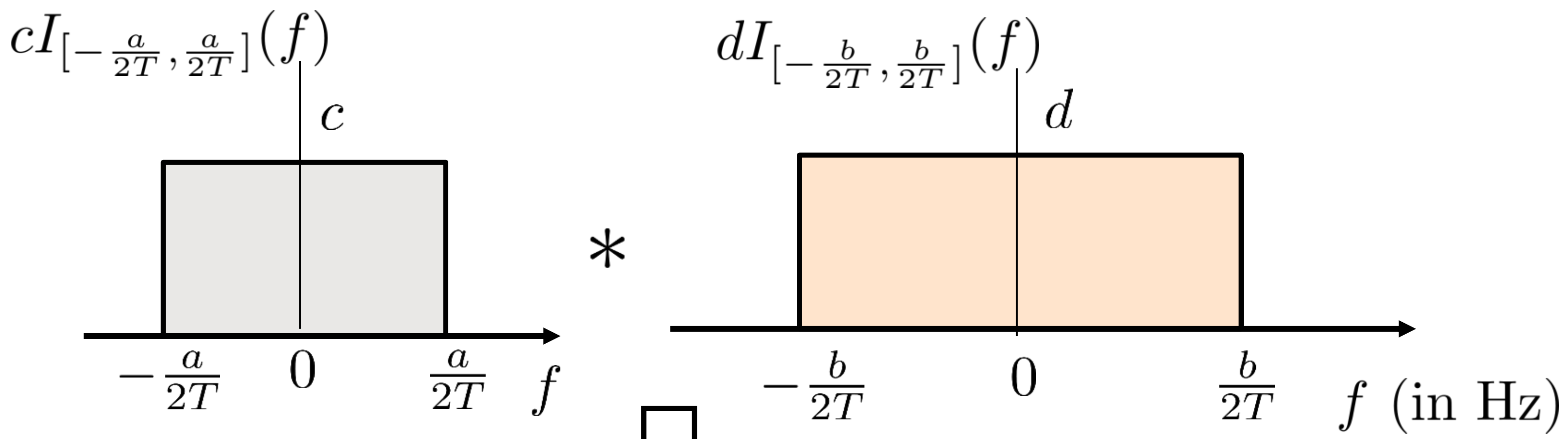
- Sinc pulse decays as $1/|t|$ and the divergence of harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ implies significant interference from distant symbols.

- However a pulse decaying as $1/|t|^b$ with $b > 1$ should work as $\sum_{n=1}^{\infty} \frac{1}{n^b}$ converges for $b > 1$.

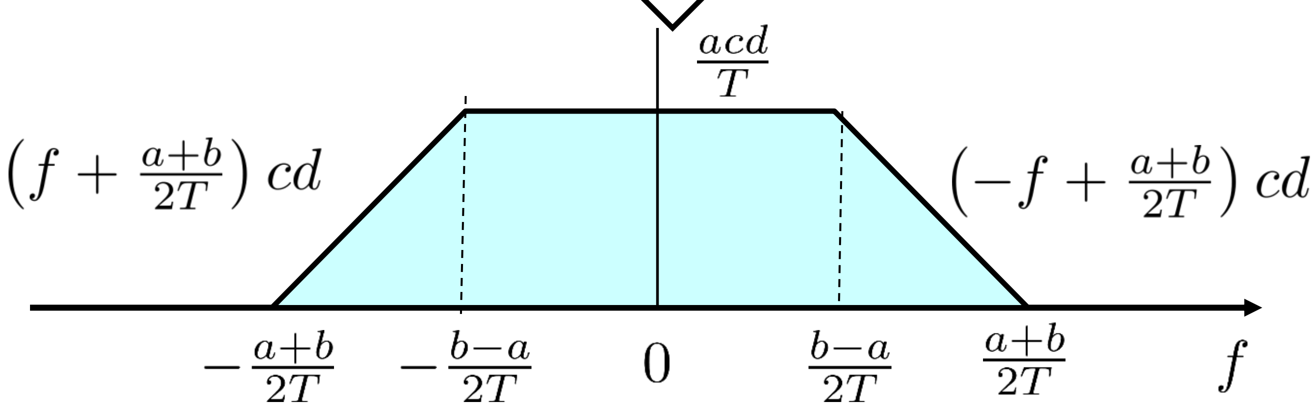
- A faster decay in time requires slower decay in frequency. Thus we need excess bandwidth.

- Excess bandwidth is defined as the fraction of the bandwidth over the minimum required for ISI avoidance at a given symbol rate.

Trapezoidal Pulse: General Expressions with scaling



$$I_{[-\frac{a}{2T},\frac{a}{2T}]}(f) \leftrightarrow \frac{a}{T}\mathrm{sinc}\left(\frac{at}{T}\right)$$



$$c\, I_{[-\frac{a}{2T},\frac{a}{2T}]}(f) * d\, I_{[-\frac{b}{2T},\frac{b}{2T}]}(f) \longleftrightarrow \frac{a\,b\,c\,d}{T^2}\,\mathrm{sinc}\left(\frac{at}{T}\right)\mathrm{sinc}\left(\frac{bt}{T}\right)$$

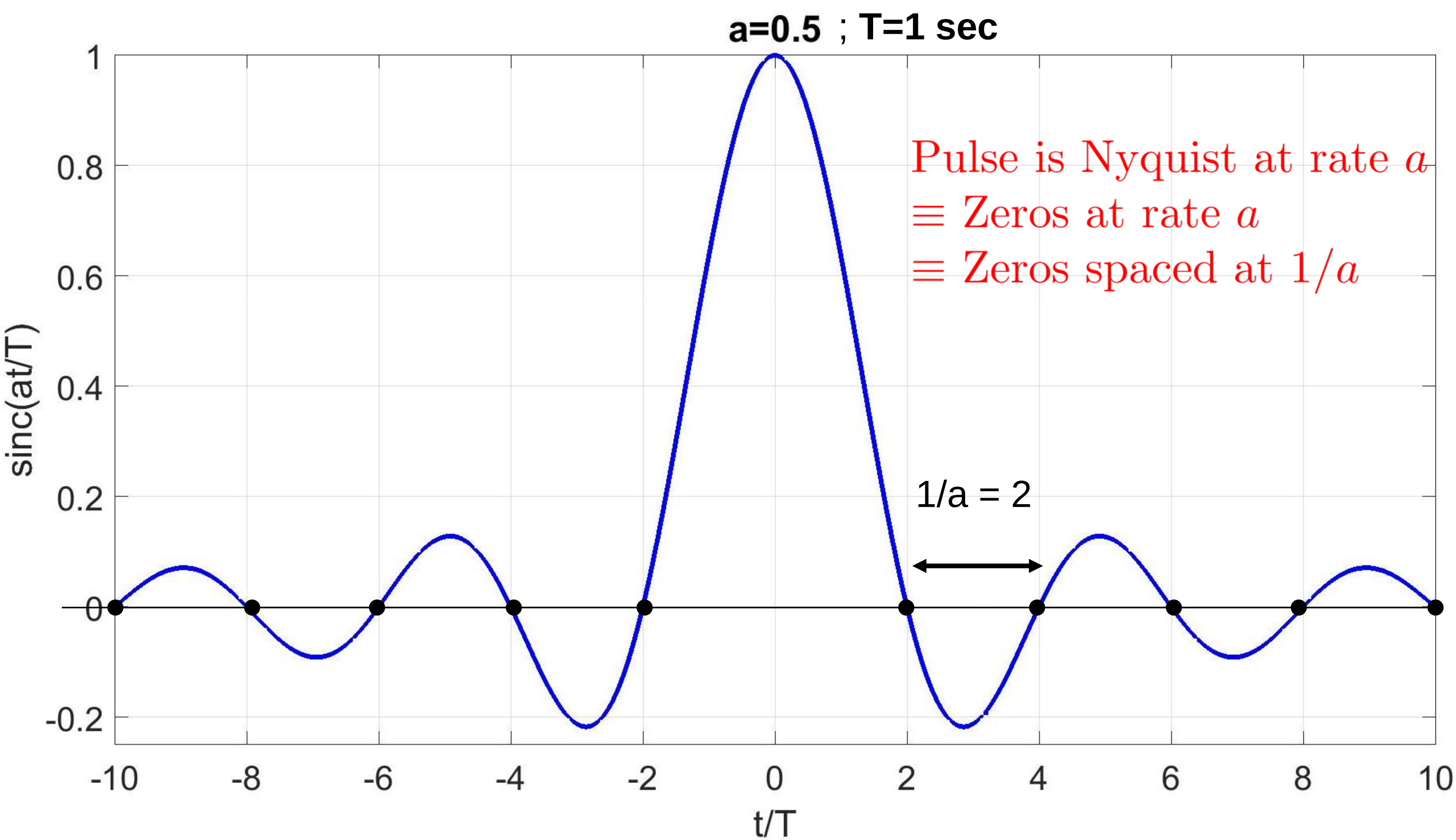
Some Interesting Properties of Nyquist Pulses

- For trapezoidal and the raised-cosine waveforms, the time-domain pulse has a $\text{sinc}(at)$ term that provides zeros at the integer multiples of $1/a$. This means that pulse is Nyquist at rate a. In other words, a time domain factor that provides *zeros at rate a* enables Nyquist signaling at rate a.

- A pulse that is trapedoizal has a time-domain pulse of the form $\text{sinc}(at)\text{sinc}(bt)$, which provides zeros at rate a and b. Thus this is Nyquist at rate a and rate b.

- Once we have zeros at integer multiples of T, we also have zeros at integer multiples of KT where K is any positive integer. In other words, if a pulse is Nyquist at rate $1/T$, then it is also Nyquist at integer submultiples of this rate, i.e., $1/KT$.

Sinc Pulse and Nyquist Rate



Today's Class
Orthogonal Modulation

Orthogonal Modulation

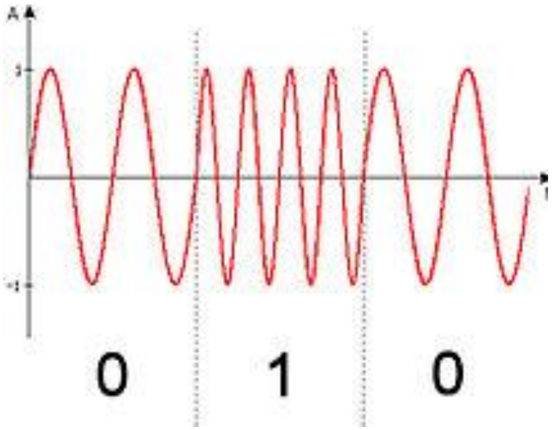
- Power efficient as opposed to bandwidth efficient
- Use of M orthogonal waveforms for sending messages
- Note that $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$ are also orthogonal
- Example: Frequency shift keying (FSK)
- Used in satellite communication

Frequency Shift Keying (FSK)

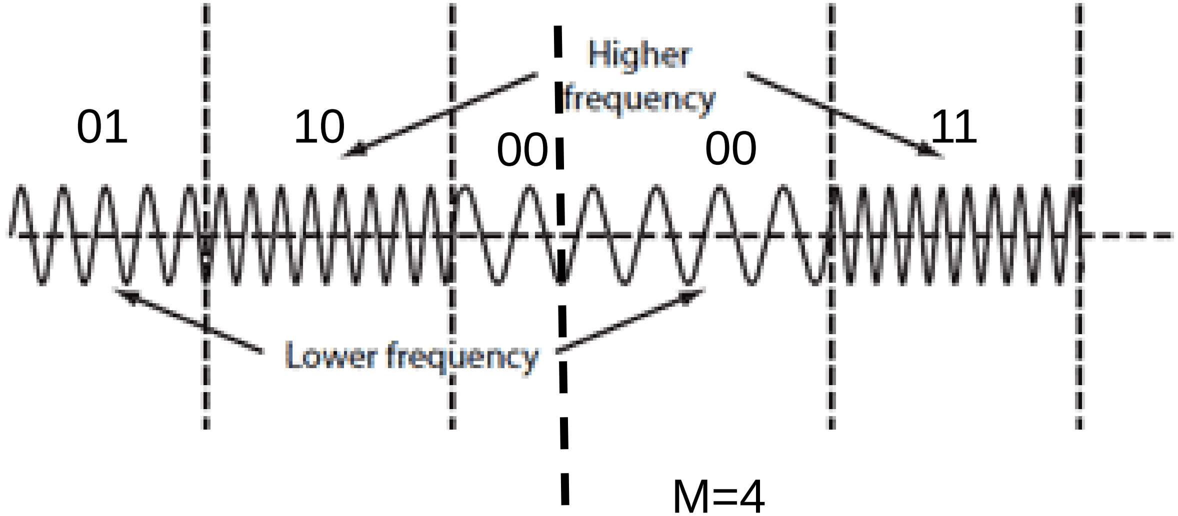
- Use M sinusoidal tones separated by Δf to send M messages (or $\log_2 M$ bits)
- The k^{th} sinusoidal tone (passband signal) is given by

$$u_{p,k} = \cos(2\pi(f_c + k\Delta f)t) \quad 0 \leq t \leq T$$

for $k = 0, 1, \ldots, M-1$.



M=2

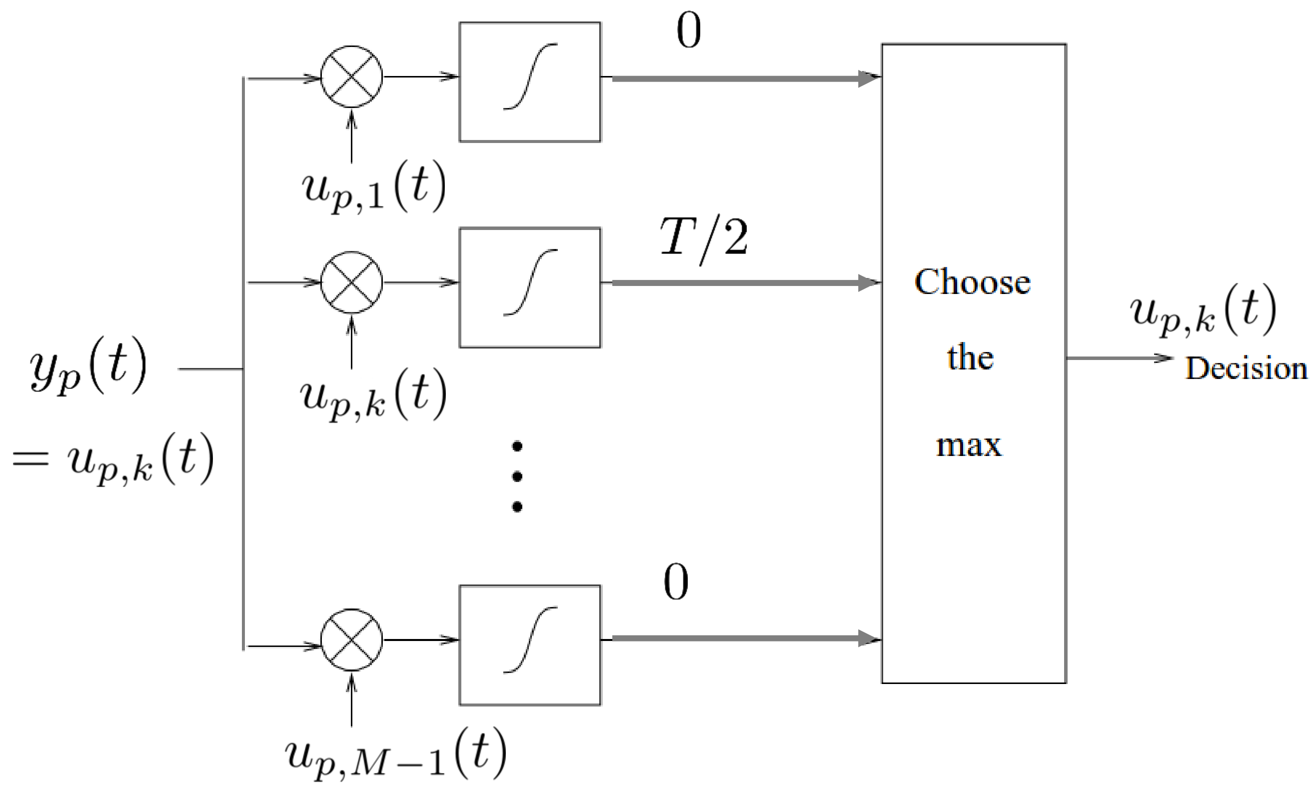


M=4

FSK

- Show that the minimum frequency spacing between two adjacent tones for FSK to be orthogonal is
$$\Delta f = \frac{1}{2T}$$
- Each tone correspond to one dimension so we have M dimensions instead of 2 dimensions as in PAM, PSK and QAM.
- The minimum bandwidth required in this case is $M\Delta f = \frac{M}{2T}$.

FSK: coherent demodulation (passband)



- Correlate the incoming signal with all M reference sinusoidal tones (passband signal) given by

$$u_{p,k} = \cos(2\pi(f_c + k\Delta f)t) \quad 0 \leq t \leq T$$

  for $k = 0, 1, \ldots, M-1$.

- Choose $u_{p,m}$ for which the output is maximum among all the correlated outputs.

FSK: Issue with coherent demodulation

- Consider that $u_{p,m}$ was sent and the corresponding received signal is
$$y_p(t) = \cos(2\pi(f_c + m\Delta f)t + \frac{\pi}{2}) \quad 0 \leq t \leq T$$

- If there is phase offset between the incoming (received) signal $y_p(t)$ and the correct reference signal $u_{p,m}(t)$, then
$$< y_p, u_{p,m} >= \int_0^T y_p(t)u_{p,m}(t)\, dt = 0$$

$$\int_0^T \cos(2\pi(f_c+m\Delta f)t+\pi/2) \cdot \cos(2\pi(f_c+m\Delta f)t)\,dt$$
$$= 0.$$

Noncoherent Reception

- Solution to this problem is to use $|\langle u_k, u_l \rangle|$ instead of $\mathrm{Re}\{\langle u_k, u_l \rangle\}$.
- Show that the design requirement for non-coherent reception is $\Delta f = 1/T$.

Summarizing the concept of Orthogonality

- For a signal set $\{s_k(t)\}$, orthogonality requires that for $k \neq l$, we have

$$\begin{aligned} \mathrm{Re}\{\langle s_k, s_l\rangle\} = 0 \qquad & \text{coherent orthogonality criteria} \\ \langle s_k, s_l\rangle = 0 \qquad & \text{non-coherent orthogonality criteria} \end{aligned}$$

- Bandwidth required for M-ary orthogonal modulation with coherent detection criteria is $\frac{M}{2T}$ while that for orthogonal modulation with non-coherent detection is $\frac{M}{T}$.

- Similarly the bandwidth efficiency for the two schemes is $\eta_B = (\log_2 2M)/M$ for coherent criteria while $\eta_B = (\log_2 M)/M$ for non-coherent criteria.

Biorthogonal Modulation

- If a signal set $\{s_k(t)\}$ for $k = 0, \ldots, M-1$ denote orthogonal modulation, then set $\{\pm s_k(t)\}$ containing $2M$ waveforms represents biorthogonal modulation.
- Bandwidth efficiency of biorthogonal modulation is twice that of orthogonal modulation, i.e., $\eta_B = \frac{\log_2 4M}{M}$.
- Note that this is only applicable to coherent systems.