Analysis of 3rd and 4th order Butterworth and Chebyshev Bandpass Filters with Audio Amplifier application

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***Abstract*—This project focuses on comparing third-order and fourth-order Butterworth and Chebyshev filters for use in audio amplifier systems, specifically targeting signals in the 15Hz to 50kHz frequency range. The goal is to understand how each filter performs in terms of total harmonic distortion (THD), power consumption, and temperature characteristics. We designed and implemented each filter using analog components and tested them within a real audio amplification setup. By analyzing their frequency responses and behavior under different signal conditions, we observed clear trade-offs between the filter types and orders. The findings offer practical guidance for choosing the right filter design when building efficient and high-quality audio systems.**

Keywords— Filters ,High pass , Low pass , Bandwidth , Bode Plots, Audio amplifier, pre-amplification, gain stage, power amplification , distortion, slew rate.

I. Introduction

A. What are Active and Passive Filters?

A filter is a circuit that is designed to pass a specified band of frequencies while attenuating all signals outside this band. Filter networks may be either active or passive. Passive f ilter networks contain only resistors, inductors, and capacitors. Active filters, which are the only type covered in this text, employ operational amplifiers (op-amps) as well as resistors and capacitors.

Filters built from resistors (R), inductors (L) and capacitors (C) are known as RLC or passive filters and are the dominant type of filter for high frequency applications. The performance of these filters is typically limited by the inductors which are large, expensive and far from ideal in an electrical sense. By using amplifiers and feedback, the same filter characteristics can be achieved without the use of inductors, these filters are known as “active filters”. An active filter will perform well only to the extent that the amplifiers in it behave in an ideal sense, so traditionally active filters have been limited to applications at frequencies below 1 MHz.

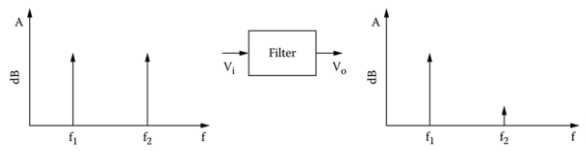

Fig. 1. Using a filter to reduce the effect of an undesired signal.

B. Active Bandpass Filters?

Band-pass filters pass frequencies within a specified band and reject components outside the band. Band-pass filters are geometrically symmetrical, i.e., symmetrical around a center frequency when plotted on linear-log graph paper with frequency on the logarithmic axis. The center can be computed by:

$$f_0 = \sqrt{f_1 f_2}$$

where f_1 is the lower cutoff and f_2 is the upper cutoff frequency.

Fig. 2. Center Frequency

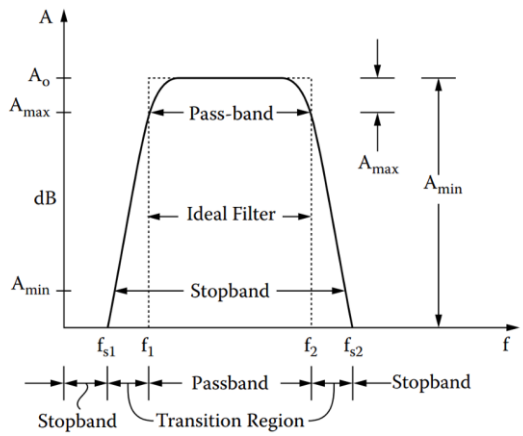

Fig. 3. Band-pass frequency response.

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II. SPECIFICATIONS

If an ideal filter existed, it would eliminate signals above the cutoff frequency and perfectly pass signals below the cutoff frequency. In real filters, various trade-offs are made to get optimum performance for a given application.

Filter Types:

Butterworth Response: The Butterworth response maximizes the flatness in the pass band of the filter. The response is very flat near DC then begins to slowly roll off so that it is at-3 dB at the cutoff frequency, and approaches an ultimate rolloff rate of-20ndB/decade where n is the order of the filter. Butterworth filters are used it is very important to maintain gain flatness, especially at low frequencies.

Chebyshev Response: In some applications, the most important factor is how fast the filter cuts off the unwanted signal. Faster rolloff than what is seen by the Butterworth f ilter can be achieved if you are willing to accept some ripple in the passband.

For Audio AMPLIFIER the specifications are:

- Supply Voltage: 0-5
- Input Signal Voltage: 10–40mV (peak-to-peak)
- Voltage Gain: 500 (achieved through pre-amplification and gain stages)
- Frequency Response: 20 Hz to 20 kHz (audible frequency range)
- Output Power: 1.5W
- Load Resistance: 10
- Total Harmonic Distortion (THD): Minimal

III. WHY ACTIVE FILTERS ?

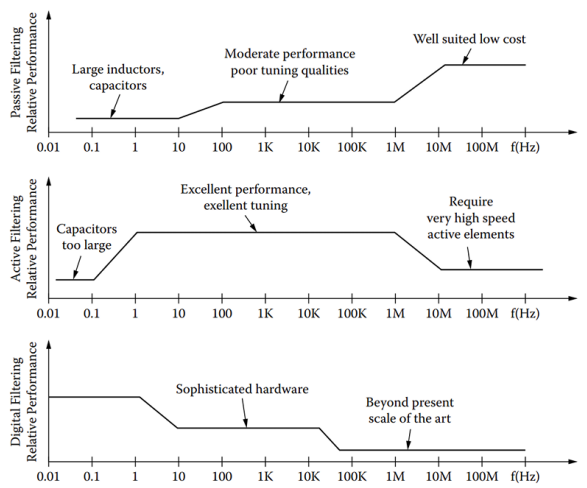


Fig. 4. Comparing filter techniques

An active filter is a network of passive R, C elements, and one or more active elements. Its function is to simulate the action of the usual passive RLC filters. The active element is usually one or more op-amps. The single system parameter that dictates the filter technology is frequency. The below figure illustrates the advantages of active filtering compared to passive techniques as a function of frequency. Active filters offer accuracy, stable tuning, and high immunity to electromagnetic interference. The high input and low output impedance found in active filters allow combinations of two or more stages without the interaction found in passive cascades.

Active filters function similar to simple, frequency-selective control systems; as such, any desired filter characteristic can be generated from the interconnection of integrators, inverter, summing amplifier, and lossy integrators. Efficient and lowcost active filter design, therefore, depends on the realization of a desired transfer function into a circuit that uses the fewest components while maintaining all performance requirements.

IV. WHY SALLEN-KEY TOPOLOGY?

There are many ways of constructing active filters. One general-purpose circuit that is widely used is that of Sallen and Key. We refer to the Sallen and Key circuit as a VCVS because it uses an op-amp and two resistors connected so as to constitute a voltage-controlled voltage source (VCVS). Such a configuration offers good stability, requires a minimum number of elements, and has low impedance, which is important for cascading filters with four or more poles.

Table 1-2. Summary of Architecture Trade-Offs

Architecture	Advantages	Disadvantages
Sallen-Key	Not sensitive to component variation at unity gain	High-frequency response limited by the frequency response of the amplifier
MFB	Less sensitive to component variations and excellent high-frequency response	Less simplifications available to ease design

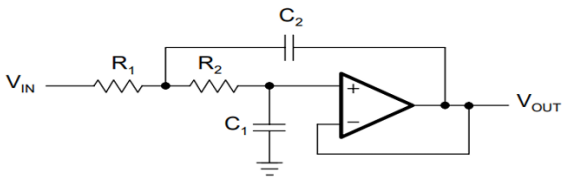


Fig. 1. schematic for a 2nd order unity gain LPF.

- unity gain 2nd order high-pass filter

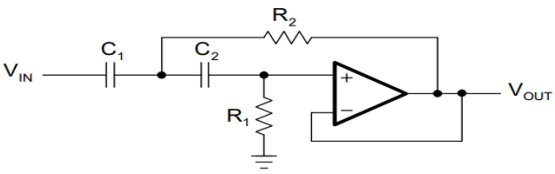


Fig. 2. schematic for a 2nd order unity gain HPF.

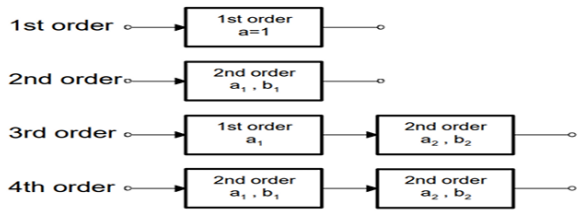


Fig. 3. combinations to make higher order filter.

V. BUTTERWORTH FILTERS

The first, and probably best-known, filter is the Butterworth

or maximally flat response. It exhibits a nearly flat passband. The rolloff is 20 dB/decade or 6 dB/octave for every pole. The general equation for a Butterworth filter's amplitude response is-

$$|H(j\omega)| = \frac{K}{\left[1+\left(\frac{s}{\omega_1}\right)^{2n}\right]^{1/2}}$$

where n is the order of the filter and can be a positive whole number, w1 is the -3 dB frequency of the filter, and K is the gain of the filter. We see that and is monotonically decreasing with w. In addition, the 0.707 or -3 dB point is at w = 1 for all n; that is,

$$|H(j\omega)| = \frac{K}{\sqrt{2}} \quad \text{for all } n$$

The cutoff frequency is thus seen to be w = 1. The parameter n controls the closeness of approximation in both the band and the stopband. The amplitude approximation of Equation (1.12) is called Butterworth or maximally flat response. The reason for the term maximally flat is that when we expand in a power series about w = 0, we have:

$$|H(j\omega)| = K\left(1-\frac{1}{2}\omega^{2n}+\frac{3}{8}\omega^{4n}-\frac{5}{16}\omega^{6n}+\frac{35}{128}\omega^{8n}+\cdots\right)$$

We see that the first 2n-1 derivatives of are equal to zero at w = 0. For w >> 1, the amplitude response of a Butterworth function can be written as (with K = 1)

$$|H(j\omega)| \cong \frac{1}{\omega^n} \quad \omega >> 1$$

We observe that asymptotically, H(jw) falls off as w-n for Butterworth response. In terms of dB, the asymptotic slope is obtained as

$$A = 20\log|H(j\omega)| = -20n\log\omega$$

Consequently, the amplitude response falls asymptotically at a rate of -20n dB/ decade or -6n dB/octave.

Figure 1.6 shows the amplitude response curves of Butterworth low-pass filters of various orders. The frequency scale is normalized to f/f1 so that all of the curves show 3-dB attenuation for f/f1 and K = 1.

Figure 1.7 shows the step response of Butterworth low-pass filters of various orders. Note that the amplitude and duration of the ringing increase as n increases.

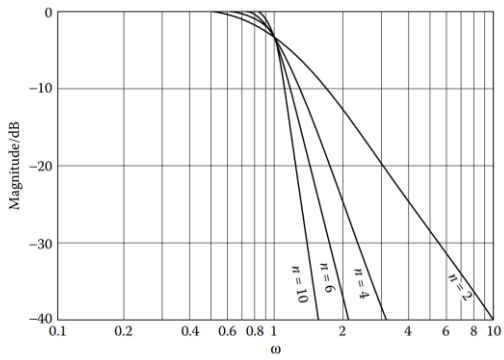


FIGURE 1.6 Amplitude response curves for Butterworth filters of various orders ($K = 1$).

TABLE 1.1
Butterworth filters: $s^n + b_{n-1}s^{n-1} \ldots b_1 s + b_0$

n	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7
1	1.000							
2	1.000	1.414						
3	1.000	2.000	2.000					
4	1.000	2.613	3.414	2.613				
5	1.000	3.236	5.236	5.236	3.236			
6	1.000	3.863	7.464	9.142	7.464	3.864		
7	1.000	4.494	10.098	14.592	14.592	10.098	4.494	
8	1.000	5.126	13.137	21.846	25.688	21.846	13.137	5.126

VI. Chebyshev Bandpass Filters

Another important approximation to the ideal filter is the Chebyshev or equal ripple response. As the latter name implies, this sort of filter will have a ripple in the passband amplitude response. The amount of passband ripple is one of the parameters used in specifying a Chebyshev filter. The Chebyshev characteristic has a steeper rolloff near the -3 dB frequency when compared to the Butterworth, but at the expense of less "flatness" in the passband and poorer transient response. The general equation for a Chebyshev filter's amplitude response is

$$|H(j\omega)| = \frac{K}{\left[1+\varepsilon^2 C_n^2(\omega)\right]^{1/2}}$$

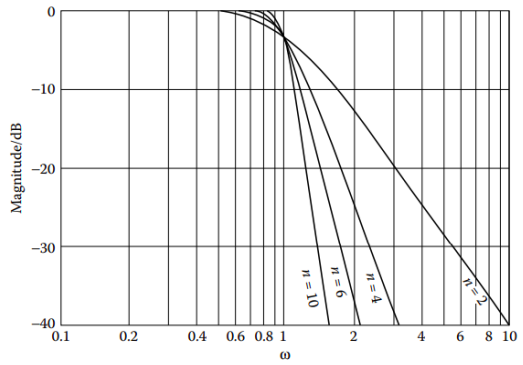


FIGURE 1.6 Amplitude response curves for Butterworth filters of various orders ($K = 1$).

where

$$\cos(n\cos^{-1}\omega) \quad |\omega| \le 1 \tag{1.18}$$

$$C_n(\omega) = \cosh(n\cosh^{-1}\omega) \quad |\omega| > 1 \tag{1.19}$$

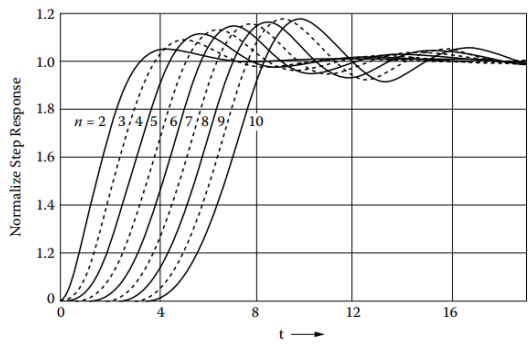


FIGURE 1.7 Step response for Butterworth low-pass filters; $w_i = 1$ rad/s, and step amplitude is 1.

Chebyshev 3 dB

n	
1	$(s+1.002)$
2	$(s^2+0.645s+0.708)$
3	$(s+0.299)(s^2+0.299s+0.839)$
4	$(s^2+0.411s+0.196)(s^2+0.170s+0.903)$
5	$(s+0.178)(s^2+0.287s+0.377)(s^2+0.110s+0.936)$
6	$(s^2+0.285s+0.089)(s^2+0.209s+0.522)(s^2+0.07s+0.955)$
7	$(s+0.126)(s^2+0.228s+0.204)(s^2+0.158s+0.627)(s^2+0.056s+0.966)$
8	$(s^2+0.217s+0.050)(s^2+0.184s+0.321)(s^2+0.123s+0.704)(s^2+0.043s+0.974)$

For $n = 0$ we have:

$$C_0(\omega) = 1 \tag{1.20}$$

and for $n = 1$ we have:

$$C_1(\omega) = \omega \tag{1.21}$$

Higher-order Chebyshev polynomials are obtained through the recursive formula

$$C_n(\omega) = 2\omega C_{n-1}(\omega) - C_{n-2}(\omega) \tag{1.22}$$

Thus, for $n = 2$, we obtain $C_2(\omega)$ as

$$C_2(\omega) = 2\omega(\omega) - 1 = 2\omega^2 - 1 \tag{1.23}$$

In Table 1.8 are given Chebyshev polynomials of order up to $n = 8$.

Within the interval $|\omega| \le 1$, $|H(j\omega)|$ oscillates about unity such that the maximum value is 1 and the minimum is $1/(1+\varepsilon^2)$. Outside this interval, $C_n^2(\omega)$ becomes very large, so that as increases, a point will be reached where $\varepsilon^2 C_n^2(\omega) >> 1$ and $|H(j\omega)|$ approaches zero very rapidly with further increase in w. Thus, we see that $|H(j\omega)|$ in Equation (1.17) is indeed a suitable approximation for the ideal low-pass characteristics.

Figure 1.8 shows a Chebyshev approximation to the ideal low-pass filter. We see that within the passband $0 \le \omega \le 1$, $|H(j\omega)|$ ripples between the value 1 and $1/(1+\varepsilon^2)$. The **ripple height**, or distance between maximum and minimum in the passband, is given as

$$Ripple = 1 - \frac{1}{(1+\varepsilon^2)^{1/2}} \tag{1.24}$$

TABLE 1.8
Chebyshev polynomials $C_n(\omega) = \cos(n\cos^{-1}\omega)$

n	
0	1
1	w
2	$2w^2 - 1$
3	$4w^3 - 3w$
4	$8w^4 - 8w^2 + 1$
5	$16w^5 - 20w^3 + 5w$
6	$32w^6 - 48w^4 + 18w^2 - 1$
7	$64w^7 - 112w^5 + 56w^3 - 7w$
8	$128w^8 - 256w^6 + 160w^4 - 32w^2 + 1$

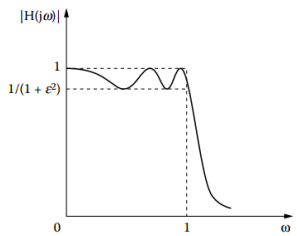


FIGURE 1.8 Chebyshev approximation to low-pass filter.

At w = 1, we have:

$$|H(j1)| = \frac{1}{(1+\varepsilon^2)^{1/2}} \tag{1.25}$$

because $C_n^2(1) = 1$.

In the stopband, that is, for $|\omega \ge 1|$, as ω increases, we reach a point ω_p where $\varepsilon^2 C_n^2(\omega) >> 1$ so that

$$|H(j\omega)| \cong \frac{1}{\varepsilon C_n(\omega)} \quad \omega > \omega_p \tag{1.26}$$

The transfer function in dB is given as

$$\begin{aligned} A &= 20\log|H(j\omega)| \\ &\cong -\left[20\log\varepsilon + 20\log C_n(\omega)\right] \end{aligned} \tag{1.27}$$

For large ω, $C_n(\omega)$ can be approximated by its leading term $2^{n-1}\omega^n$, so that

$$\begin{aligned} A &= -\left[20\log\varepsilon + 20\log 2^{n-1}\omega^n\right] \\ &= -\left[20\log\varepsilon + 6(n-1) + 20n\log\omega\right] \end{aligned} \tag{1.28}$$

A few different Chebyshev filter responses are shown in Figure 1.9 for various values of n. Note that a filter of order n will have n-1 peaks or dips in the

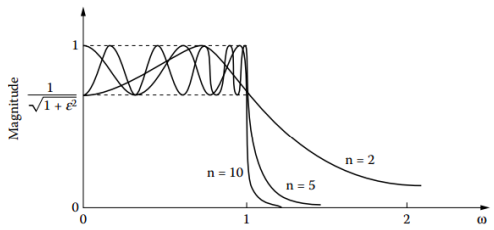


FIGURE 1.9 Examples of Chebyshev amplitude response: (a) 3-dB ripple, (b) expanded view of passband region showing form of responses below cutoff frequency.

passband response. Note also that the nominal gain of the filter ($K = 1$) is equal to the filter's maximum passband gain. An odd-order Chebyshev will have a dc gain to the nominal gain, with "dips" in the amplitude response curve to the ripple value. An even-order Chebyshev will have its dc gain equal to the nominal filter minus the ripple values; the ripple in this case increases the gain to the nominal value.

The addition of a passband ripple as a parameter makes the specification process for a Chebyshev filter more complicated than for a Butterworth filter, but also increases flexibility, because passband ripple can be treated for cutoff slope.

Figure 1.10 shows the step response of 3-dB ripple Chebyshev filters of various orders. As with the Butterworth filters, the higher-order filters ring more.

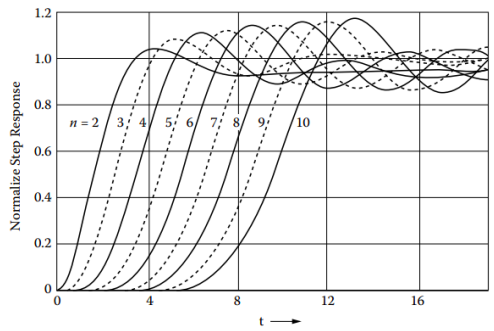


FIGURE 1.10 Step response for Chebyshev low-pass filters; $\omega_1 = 1$ rad/s and step amplitude is 1.

To determine which basic filter is most suitable for a given application, it is useful to have a side-by-side comparison of their amplitude, phase, and delay characteristics. These characteristics are determined by the location, in the s plane, of the n poles of H(s). Thus, the poles of a Butterworth filter lie on a semicircle in the left-half s plane, those of a Chebyshev filter on an ellipse that becomes narrower with increasing ripple, and those of a Bessel filter on a curve outside the Butterworth semicircle. This is shown in Figure 1.13.

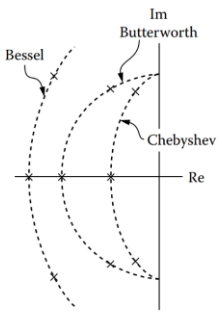


FIGURE 1.13 Comparison of pole location of Butterworth, Chebyshev, and Bessel filters.

Table 1-1. Summary of Filter Type Trade-Offs

Filter Type	Advantages	Disadvantages
Butterworth	Maximum pass-band flatness	Slight overshoot in response to pulse input and moderate rate of attenuation above f_c
Bessel	Constant group delay – no overshoot with pulse input	Slow rate of attenuation above f_c
3-dB Chebyshev	Fast rate of attenuation above f_c	Large overshoot and ringing in response to pulse input

VII. Ripples in Chebyshev

Ripple in passband

Advantage	Description	Disadvantage	Description
1. Sharper Cutoff	Faster transition to stopband than Butterworth.	**1. Amplitude Distortion**	Gain variations in passband can distort signals.
2. Lower Order	Achieves specs with fewer components.	**2. Phase Distortion**	Non-linear phase affects time-sensitive signals.
3. Better Stopband Attenuation	More attenuation with same filter order.	**3. Not Ideal for Audio/Precision**	Ripples are undesirable in flat-response systems.
4. High Selectivity	Good for tight frequency control (e.g., RF).	**4. Sensitive to Tolerances**	High Q values increase sensitivity to component variations.
5. Adjustable Ripple	Control ripple level for trade-off in performance.	**5. Complex Design**	Requires careful tuning for stability and specs.
6. Efficient Implementation	Fewer components = simpler and smaller circuits.	**6. Adds Noise in Low-SNR**	Ripples may amplify unwanted noise.

VIII. Comparison

Characteristic	Butterworth Filter	Chebyshev Filter
Passband Response	Maximally flat (no ripple)	Contains ripple (Type I: passband ripple, Type II: stopband ripple)
Stopband Response	Monotonically decreasing	Steeper roll-off than Butterworth
Phase Response	More linear phase response	Less linear phase response (higher phase distortion)
Group Delay	More constant	Less constant (varies more with frequency)
Step Response	Less overshoot	More overshoot and ringing
Pole Locations	Equally spaced on circle in s-plane	Elliptical pattern in s-plane
Filter Order	Higher order needed for sharp transitions	Lower order achieves steeper roll-off
Transition Width	Wider	Narrower for same order
Applications	Audio processing, biomedical signals	Communications, spectrum analysis

Stability	More robust stability	More sensitive to component variations
Transient Behavior	Better transient response	Poorer transient response

IX. Derivations

FREQUENCY RESPONSE NORMALIZATION-

Several parameters are used to characterize a filter's performance. The most commonly specified parameter is frequency response. When given a frequency-response specification, the designer must select a filter design that meets these requirements. This is accomplished by transforming the required response to a normalized lowpass specification having a cutoff of 1 rad/s. This normalized response is compared with curves of normalized low-pass filters that also have a 1 rad/s cutoff. After a satisfactory low-pass filter is determined from the curves, the tabulated normalized element values of the chosen filter are transformed or denormalized to the final design. The basic for normalization of filters is the fact that a given filter's response can be scaled or shifted to a different frequency range by dividing the reactive elements by a frequency-scaling factor (FSF). The FSF is the ratio of the desired cutoff frequency of the active filter to the normalized cutoff frequency, i.e.:

The FSF must be a dimensionless number. So, both the numerator and denominator of Equation (2.1) must be expressed in the same units, usually rad/s. Frequency-scaling a filter has the effect of multiplying all points on the frequency axis of the response curve by the FSF. Therefore, a normalized response curve can be directly used to predict the attenuation of the denormalized filter. Any linear active or passive network maintains its transfer function if all resistors are multiplied by an impedance-scaling factor (ISF) and all capacitors are divided by the same factor ISF. This occurs because the ISFs cancel one another out in the transfer function. Impedance scaling can be mathematically expressed as

$$R = ISF \times R_n \tag{2.2}$$

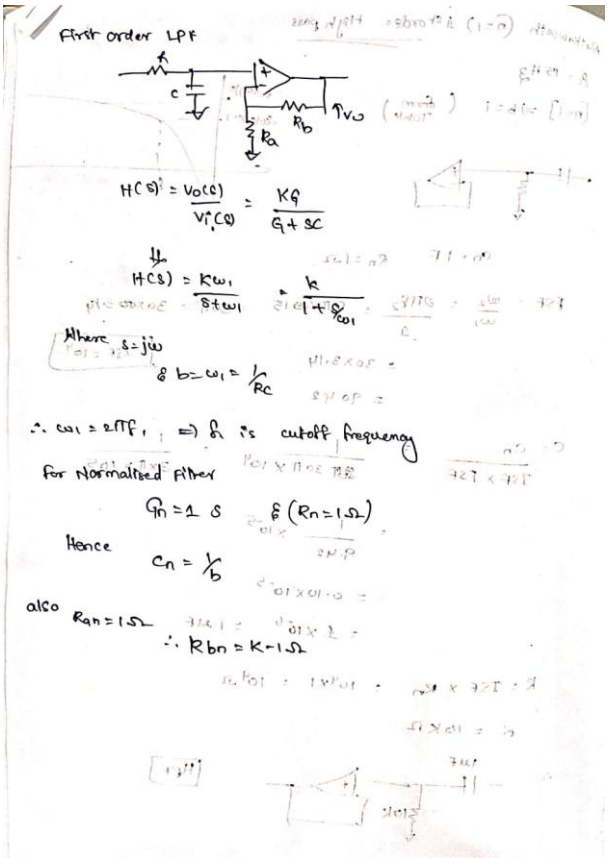
$$C = \frac{C_n}{ISF} \tag{2.3}$$

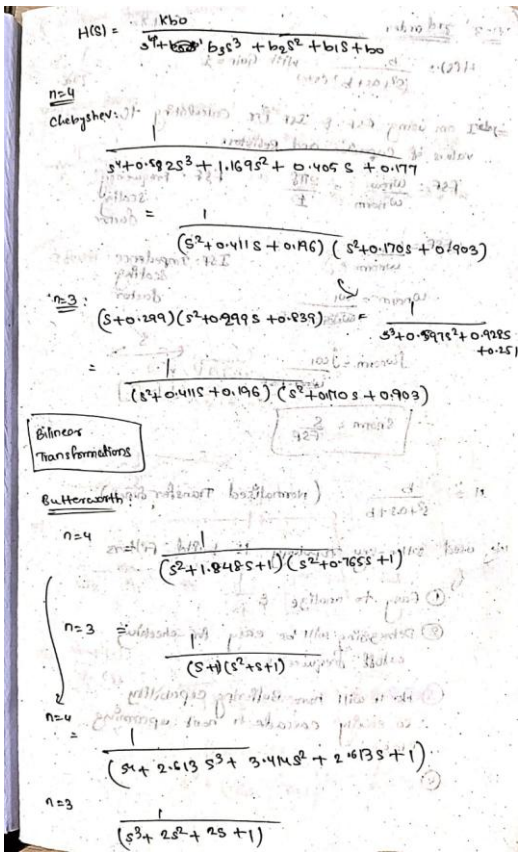
where R_n and C_n are the normalized values.

Frequency and impedance scaling are normally combined into one step rather than performed sequentially. The denormalized values are then given by

$$R = ISF \times R_n \tag{2.4}$$

$$C = \frac{C_n}{ISF \times FSF} \tag{2.5}$$





$n=3$ 3rd order

$\cosh x =$

$$H(s) = \frac{b}{(s^2+as+b)(s+1)} \quad \text{with Gain} = 1$$

$\frac{1}{\pi R_1 C_1}$

⇒ I am using FSF & ISF for calculating the values of Capacitor and Resistors.

$$FSF = \frac{\omega_{new}}{\omega_{norm}} = \frac{2\pi f}{1}$$

FSF: Frequency Scaling factor

$$FSF = \frac{\omega_1}{\omega_{norm}}$$

ISF: Impedence Scaling factor

$$\omega_{norm} = \frac{\omega_1}{FSF}$$

$$j\omega_{norm} = \frac{j\omega_1}{FSF}$$

$$\therefore \boxed{s_{norm} = \frac{s}{FSF}}$$

$$H = \frac{b}{s^2+as+b}$$ (Normalised Transfer funcⁿ)

We used Sallen-key Topology to build filters

① Easy to analize &

② Debugging will be easy for checking cutoff frequencies

③ It will have Buffering capability ∴ so easily cascade to next upcoming stages

If we have Narrow Bandwidth → Multifeedback will be useful

Low pass filters:

$$H(s) = \frac{\omega_0^2}{s^2 + as + \omega_0^2} \qquad a = \frac{\omega_0}{Q}$$

$$a = \frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 2\zeta\omega_0$$

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} = 2\pi f_0$$

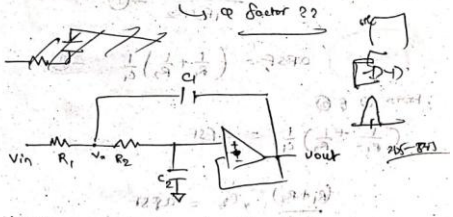
Q factor $= \frac{1}{2\zeta}$

$$\therefore \frac{\omega_0}{a} = Q = \frac{\omega_0}{2\zeta\omega_0} = \frac{1}{2\zeta}$$

$$\therefore Q = \frac{\sqrt{R_1R_2C_1C_2}}{C_2(R_1+R_2)}$$

⇒ → Q factor ??

⇒



$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1 + C_2(R_1+R_2)s + C_1C_2R_1R_2s^2}$$

$$= \frac{1/R_1R_2C_1C_2}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1}\right)s + \frac{1}{R_1C_1R_2C_2}}$$

$H(s) \Rightarrow$ for $\frac{b}{s^2+as+b}$ for Normalized frequency

Now $H(s)$ will be $\boxed{s \to \frac{s}{FSF}}$

$$\therefore = \frac{b}{\frac{s^2}{FSF^2} + \frac{as}{FSF} + b}$$

$$= \frac{bFSF^2}{s^2 + (aFSF)s + bFSF^2}$$

2nd order (called Key LPF)

Our

$$H(s) = \frac{\frac{1}{R_1R_2C_1C_2}}{s^2 + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_1}s + \frac{1}{R_1R_2C_1C_2}}$$

$$\therefore bFSF^2 = \frac{1}{R_1R_2C_1C_2} \quad \text{—①}$$

$$aFSF = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_1} \quad \text{—②}$$

∴ From ① & ②

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_1} = aFSF$$

$$\frac{(R_1+R_2)}{R_1R_2C_1} \times C_2 = aFSF$$

$$= (R_1+R_2)\cdot C_2 \cdot (bFSF^2) = aFSF$$

For low pass

$$\therefore \boxed{(R_1+R_2)C_2 = \frac{a}{bFSF}} \quad **$$

For High pass higher order filters ($n=2$)

↳ We need to find R_1, C_1, R_2, C_2 for particular f_c as with derivation of LPF (Low pass)

Then the values for HPF are

$$C_1, \text{highpass} = \frac{1}{R_{1LPF}}$$

$$C_{2HPF} = \frac{1}{R_{2LPF}}$$

$$R_{1HPF} = \frac{1}{C_{1LPF}}$$

$$R_{2HPF} = \frac{1}{C_{2LPF}}$$

X. Calculations

The following are the calculations for the 1st and 2nd order high pass and ow pass filters which have been cascaded for the higher orders.

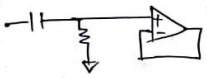
1st order-

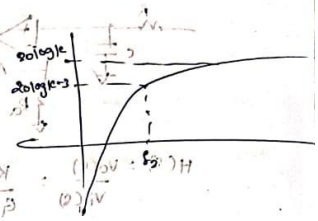
High pass-

Butterworth (n=1) 1st order High pass

$f_c = 15\ Hz$

$[n=1] \Rightarrow b=1$ (from Table)





$C_n = 1F \quad R_n = 1\Omega$

$FSF = \frac{\omega_2}{\omega_1} = \frac{2\pi f_2}{1} = 2\pi \times 15 = 30\pi = 30 \times 3.14$

$= 30 \times 3.14$

$= 90.42$

$\boxed{ISF = 10^4}$

$C = \frac{C_n}{ISF \times FSF} = \frac{1}{30\pi \times 10^4} = \frac{1}{3 \times \pi \times 10^5}$

$= \frac{1}{9.42} \times 10^{-5}$

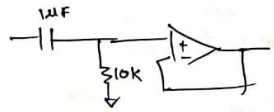
$= 0.10 \times 10^{-5}$

$= 1 \times 10^{-6} = 1\mu F$

$R = ISF \times R_n = 10^4 \times 1 = 10^4\ \Omega$

$R = 10K\Omega$

$\therefore$



$\boxed{HPF}$

The following are the calculations for the low pass-

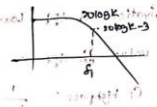
Low pass-

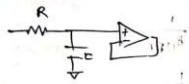
First order LPF $\Rightarrow$ Gain=1

cutoff frequency = 50KHz

= 501000 Hz

For Butterworth filter, [n=1]





$C_n = 1F, \quad R_n = 1\Omega$

I am using $ISF = 10^4$

and $FSF = \frac{\omega_2}{\omega_1} = \frac{2\pi f_1}{1} = 2\pi \times 50 \times 10^3$

$= 100\pi \times 10^3$

$= \pi \times 10^5$

$R = ISF \times R = 10^4 R\Omega$

$= 10000\ \Omega$

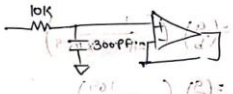
$C = \frac{C_n}{ISF \times FSF} = \frac{1}{\pi \times 10^5 \times 10^4} = \frac{1}{\pi} \times 10^{-9}$

$= 0.3 \times 10^{-9}\ nF$

$= 3 \times 10^{-10} \times \frac{100}{100}$

$= 300 \times 10^{-12} = 300pF$

$\therefore$



2nd order-

Butterworth-

Low pass-

We fix $C_2 = \inf$ $\Big\}$ eqn reduced to

$(R_1 + R_2)(1 \times 10^{-9}) = \frac{a}{b\,FSF}$ $\Big\}$ $FSF = 2\pi f_c = \omega_c$ cutoff freq

$f_c = 50K$

$(R_1 + R_2) = \frac{a}{b \times 2 \times \pi \times 50 \times 10^3 \times 1 \times 10^{-9}}$

$= \frac{a}{b}\left(\frac{1}{100 \times \pi \times 10^3 \times 10^{-9}}\right)$

$= \frac{a}{b}\left(\frac{1}{\pi 10^{-4}}\right)$

$= \frac{a}{b} \times \frac{10000}{\pi}$

$= 3186.7 \times \frac{a}{b}$

$\boxed{R_1 + R_2 = 3186.7\left(\frac{a}{b}\right)}$

for n=2 Butterworth

$a = 1.414, \quad b = 1$

$C_2 = \inf$

$\therefore R_1 + R_2 = 3186.7\left(\frac{1.414}{1}\right)$

$R_1 + R_2 = 1.414 \times 3186.7$

$= 4505.9$

$\simeq 4506$

$\boxed{\therefore R_1 + R_2 = 4506}$

lab

let $R_1 = 2.2K$

then $R_2 = 4506 - 2200$

taken values are to avail of lab

$R_2 = 2306$

$\boxed{R_2 = 2.3K}$

$R_1 = 2.2K$

$\therefore f_c = \dfrac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$

$f_c^2 = \dfrac{1}{4\pi^2 R_1R_2C_1C_2}$

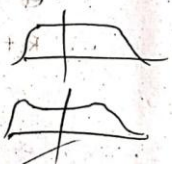
$C_1 = \dfrac{1}{4\pi^2 R_1R_2C_2 f_c^2}$

$= \dfrac{1000}{498,895,760,000}$

$= 2.004\ nf$

$\boxed{\therefore C_1 = 2nf}$

$\therefore \boxed{\begin{array}{l} f_c = 50K, \quad R_1 = 2.2K \\ R_2 = 2.3K \\ C_1 = 2nf \\ C_2 = 1nf \end{array}}$



High pass-

$\Rightarrow$ Now,

$(R_1+R_2)C_2 = \dfrac{a}{bfsf}$

Let $C_2 = 1nF$

$(R_1+R_2)1\times10^{-9} = \dfrac{a}{bfsf}$

$f_c = 15Hz$

$(R_1+R_2) = \dfrac{a}{b\times2\times\pi\times15\times10^{-9}}$

$= \left(\dfrac{a}{b}\right)\left(\dfrac{1}{2\pi\times15\times10^{-9}}\right)$

$= \left(\dfrac{a}{b}\right)\left(\dfrac{10^9}{30\pi}\right)$

$= \left(\dfrac{a}{b}\right)\left(\dfrac{1}{94.2}\right)$

$= \left(\dfrac{a}{b}\right)\left(\dfrac{1}{94.2}\right)\times10^9$

$(R_1+R_2) = \left(\dfrac{a}{b}\right)(0.001)\times10^9$

For butterworth $(n=2)$

$a = 1.414$, $b = 1$

$(R_1+R_2) = (1.414)(0.001)\times10^9$

$= 1.41\times0.001\times10^9$

$= 1.414\times10^{-3}\times10^9$

$= 1.414\times10^6$

$\boxed{R_1+R_2 = 1.414\times10^6}$

$= 1.4\ M\Omega$

Let $R_1 = 750K\Omega$

$\therefore R_2 = 1.4M - 0.75M$

$= 1400K - 750K\Omega$

$\boxed{R_2 = 650K\Omega}$

$\therefore f_c = \dfrac{1}{\sqrt{R_1R_2C_1C_2}\cdot2\pi}$

$C_1 = \dfrac{1}{4\pi^2R_1R_2C_2f_c^2}$

$= \dfrac{1}{4\times(3.14)^2\times(750)\times10^3\times(650)\times10^3\times(10^{-9})\times(15)^2}$

$$= \frac{1}{4325899500 \times 10^{-3}}$$

$$= 0.00000000023 \times 10^{3}$$

$$= 0.23\,\mu$$

$$= 0.23\,\mu = 0.23 \times 10^{6} \times \frac{1000}{1000}$$

$$= 0.23 \times 1000 \times 10^{-9}\,F$$

$$\boxed{C_1 = 230\,nF}$$

C_1 LPF = 230nF

C_2 LPF = 1nF

R_1 LPF = 750kΩ

R_2 LPF = 650 kΩ

∴ For Now for High pass

$$C_{1,HPF} = \frac{1}{R_{1\,LPF}}$$

$$= \frac{1}{750 \times 10^{3}} = \frac{1}{750} \times 10^{-3}$$

$$= 0.0013 \times 10^{-3}$$

$$= 13 \times 10^{-4} \times 10^{-3}$$

$$= 13 \times 10^{-7}$$

$$= 13 \times \frac{10}{10} \times 10^{-7} = 1.3\,\mu$$

$$\boxed{C_{1,HPF} = 1.3\,\mu}$$

$$C_{2,HPF} = \frac{1}{R_{2\,LPF}}$$

$$= \frac{1}{650 \times 10^{3}} = 0.0015 \times 10^{-3}$$

$$= 1.5 \times 10^{-6} = 1.5\,\mu$$

$$\boxed{C_{2,HPF} = 1.5\,\mu}$$

$$R_{1,HPF} = \frac{1}{C_{1\,LPF}}$$

$$= \frac{1}{230 \times 10^{-9}} = 0.0043 \times 10^{9}$$

$$= 43 \times 10^{5}$$

$$= 4.3\,M\Omega$$

$$\boxed{\therefore R_{1,HPF} = 4.3\,MHz}$$

$$R_{2\,HPF} = \frac{1}{C_{2\,LPF}} = \frac{1}{1nF} \times 10^{9}\,\Omega$$

$$= 1000\,M\Omega$$

We are doing available values from lab

$$\frac{4.3M}{100} = 4.3 \times 10^{4} = 43K \qquad \frac{1000\,M}{100} = 100M$$

$$\boxed{\therefore R_1 = 43k\,\Omega \quad R_2 = 10\,M\Omega}$$

Chebyshev-
Low pass-

n=2 chebyshev filter (2nd order)

$\alpha = 0.708$

a = 0.645 b = 0.708

We fix $C_2 = 1nF$.

equation reduced to

$$(R_1 + R_2)(1 \times 10^{-9}) = \frac{a}{bFSF}$$

our $f_c = 50K$

$$\therefore (R_1 + R_2) = \frac{a}{b \times 2\pi \times 50 \times 10^{3} \times 10^{-9}}$$

$$= \frac{a}{b}\left(\frac{1}{100 \times \pi \times 10^{3} \times 10^{-9}}\right)$$

$$= \frac{a}{b}\left(\frac{1}{\pi \times 10^{-4}}\right)$$

$$= \frac{a}{b}\left(\frac{10000}{\pi}\right)$$

$$= \frac{a}{b} \times 3186.7$$

$$\boxed{\therefore R_1 + R_2 = 3186.7\left(\frac{a}{b}\right)}$$

We know from 3-dB Ripple chebyshev,

a = 0.645 b = 0.708

$$R_1 + R_2 = 3186.7\left(\frac{0.645}{0.708}\right)$$

$$= 3186.7 \times 0.911$$

$$\boxed{R_1 + R_2 = 2903.13}$$

Taken R_1 & R_2 according to lab available values

let R_1 = 1K

then R_2 = 2903.13 − 1000 Ω

$R_2 = 1903.13\ \Omega$

$\approx R_2 = 1903.13\ \Omega$

$\boxed{R_2 = 1900\ \Omega}$

We know,

$$f_c = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}}$$

$$f_c^2 = \frac{1}{4\pi^2 (R_1 C_1 R_2 C_2)}$$

$$C_1 = \frac{1}{4\pi^2 R_1 R_2 C_2 f_c^2}$$

$$= \frac{1}{4\times\pi^2\times 10^3\times 1.9\times10^3\times 1\times10^{-9}\times(50)^2\times10^6}$$

$$= \frac{1}{187{,}332.4\times10^9}$$

$$= 0.0000053\times10^{-3}$$

$$C_1 = 5.3\ nF$$

$\therefore$ For $f_c = 50\,kHz$ (Chebyshev)

$R_1 = 1000\ \Omega$

$R_2 = 1900\ \Omega$

$C_1 = 5.3\ nF$

$C_2 = 1\ nF$

High pass-

For Chebyshev High pass 2nd order

$a = 0.645$, $b = 0.708$

fix $C_2 = 10\,\mu F$

$$(R_1+R_2)(10\times10^{-6}) = \frac{a}{b}\,FSF$$

$f_c = 15\,Hz$

$$(R_1+R_2) = \frac{a}{b}\left(\frac{1}{2\times\pi\times 15\times10^{-5}}\right)$$

$$= \frac{a}{b}\left(\frac{10^5}{30\pi}\right)$$

$$= \frac{a}{b}(0.01)\times10^5$$

$a = 0.645$, $b = 0.708$ (from table)

$$(R_1+R_2) = \left(\frac{0.645}{0.708}\right)(0.01)\times10^5$$

$$= 0.911\times0.01\times10^5$$

$$= 0.00911\times10^5$$

$\boxed{R_1+R_2 = 911\ \Omega}$

let $R_1 = 500\ \Omega$

then $R_2 = 411\ \Omega$

$$f_c = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$$

$$C_1 = \frac{1}{4\times\pi^2\times R_1\times R_2\times C_2\times f_c^2}$$

$$= \frac{1}{4\times(3.14)^2\times500\times411\times10^5\times15\times15}$$

$$= \frac{1}{1823533020}\times10^5$$

$$= 0.0000000005483\times10^5$$

$$= 5.483\times10^{-10}\times10^5$$

$$= 5.48\times10^{-5} = 54.8\,\mu F$$

$\boxed{C_1 = 54.8\,\mu F}$

HPF

$$R_1\ HPF = \frac{1}{C_1\ LPF} = \frac{1}{54.8\times10^{-6}} = 10^6\times\frac{1}{54.8}$$

$$= 0.018\times10^6$$

$\boxed{R_1, HPF = 18\ k\Omega}$

$$R_2\ HPF = \frac{1}{C_2, HPF} = \frac{1}{10\mu} = 10^5\ \Omega$$

$\boxed{R_2, HPF = 100\ k\Omega}$

$$C_1, HPF = \frac{1}{R_1\ LPF} = \frac{1}{5000} = 0.002$$

$\boxed{C_1, HPF = 20\,\mu F}$

$$C_2, HPF = \frac{1}{R_2, LPF} = \frac{1}{4110} = 0.00024$$

$\boxed{C_2, HPF = 24\,\mu F}$

XI. Results

Below are the results for all the filters. We have provided both hardware and ltspice and tried our best to analyse most of the parameters.

For temperature analysis-

Butterworth Filters (3rd & 4th Order): Show stable, flat passband response across temperatures with minimal gain variation and slight center frequency drift. Higher-order filters are slightly more sensitive but maintain smooth response.

Chebyshev Filters (3rd & 4th Order): Exhibit sharper roll-off but are more affected by temperature. Passband ripple may vary slightly, and center frequency shift is more noticeable, especially in 4th order due to tighter component tolerances.

For THD-

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} V_{n_RMS}^2}}{V_{f_RMS}}$$

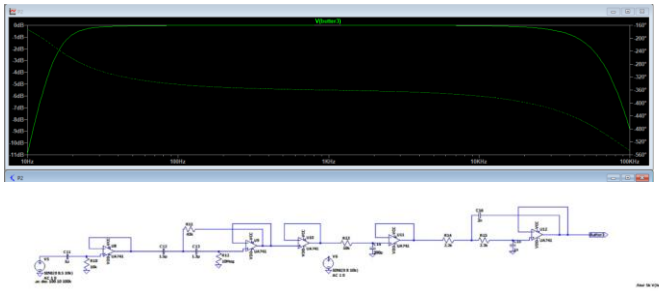
- THD is the total harmonic distortion present in the signal
- V_{n_RMS} is the RMS voltage of the nth harmonic
- V_{f_RMS} is the RMS voltage of the fundamental frequency

3RD ORDER BUTTERWORTH BANDPASS

It is a type of linear filter characterized by a smooth and maximally flat frequency response within the passband, meaning it has no ripples and maintains a consistent gain until the cutoff edges. It is a cascade of the 1st order and 2nd order bandpass filters.

For the plots below, both in hardware and LTSpice we can see that the bode plot has a flat response for the frequency range along with the monotonically decreasing transitions in the stopband.

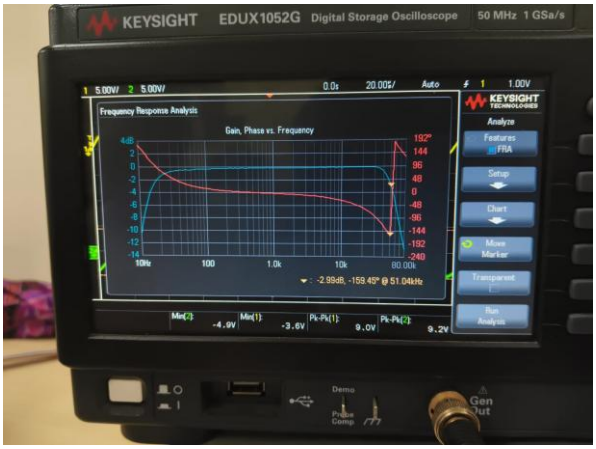
A. LTSpice simulation



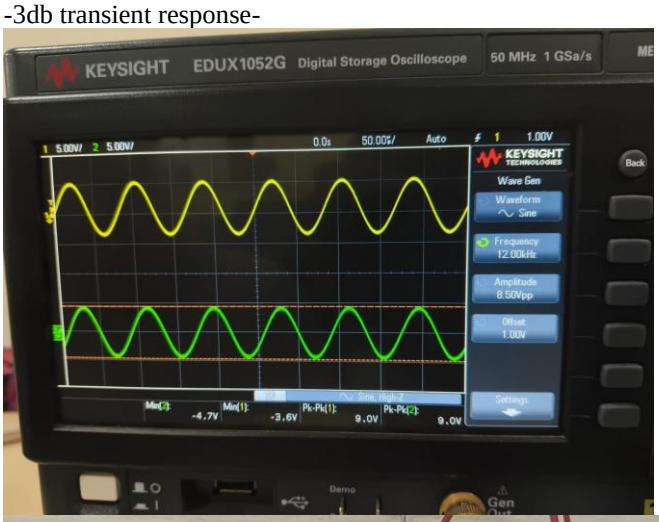
B. Hardware circuit

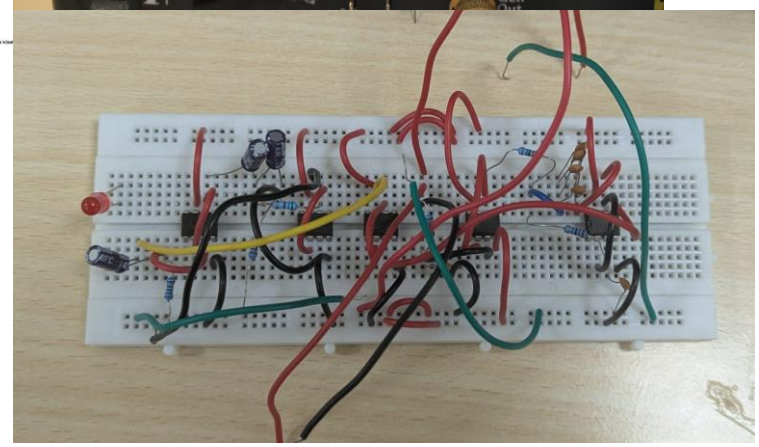
-3db fl

-3db fh



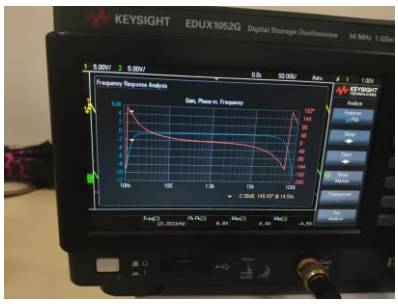
-3db transient response-

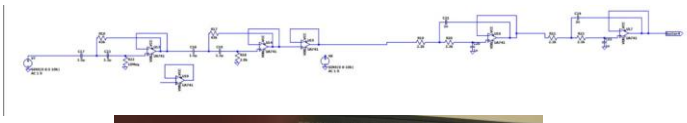


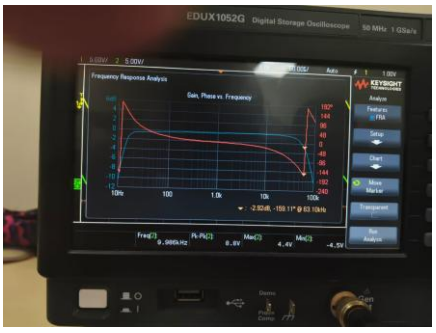


XII. 4TH ORDER BUTTERWORTH BANDPASS

A. Hardware circuit-

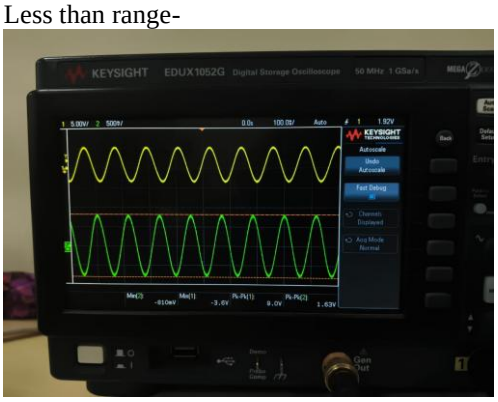




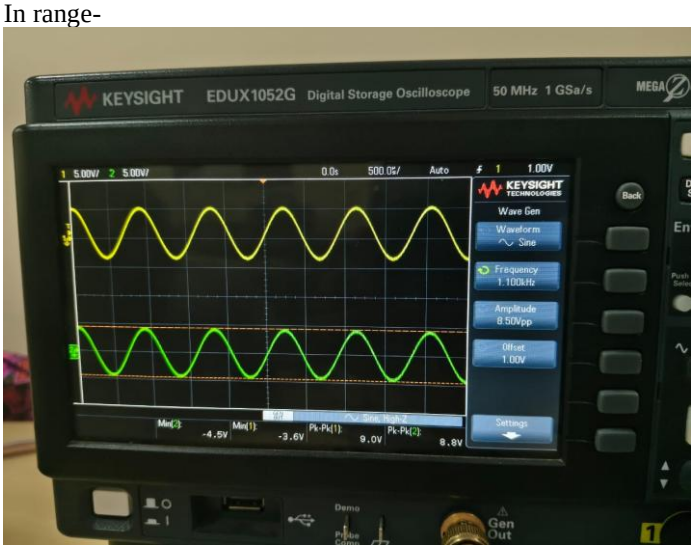


Transient response-

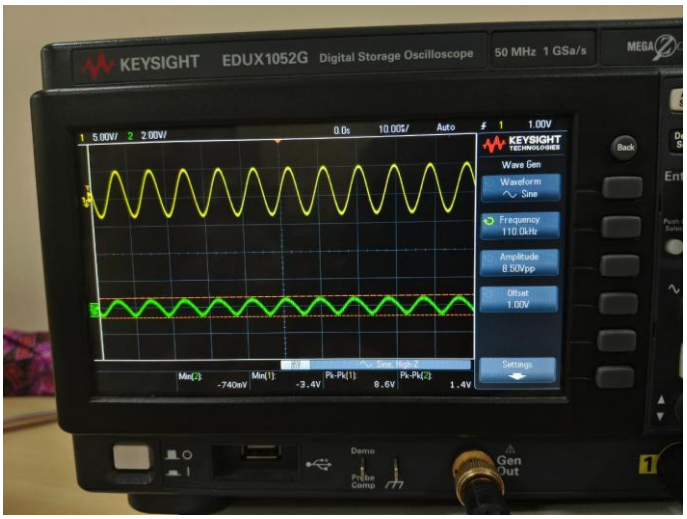
Less than range-



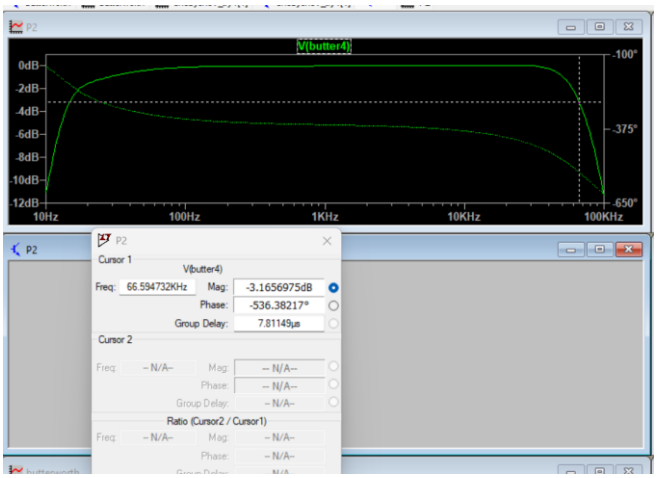
In range-



Out of range-

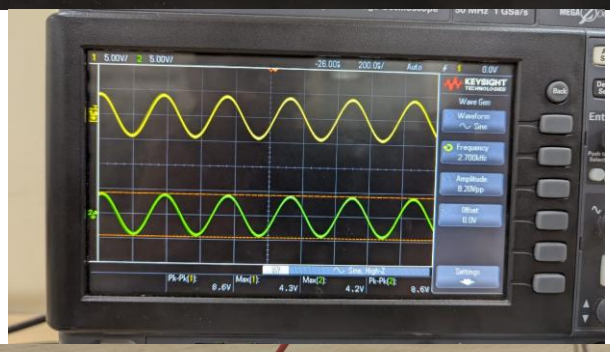


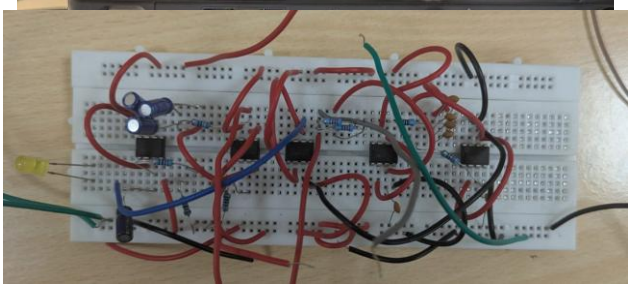
B. Ltspice-



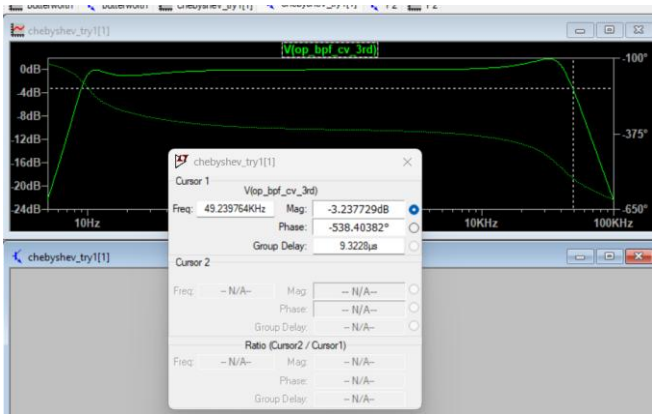
XIII. 3RD ORDER CHEBYSHEV BANDPASS

A. Hardware circuit



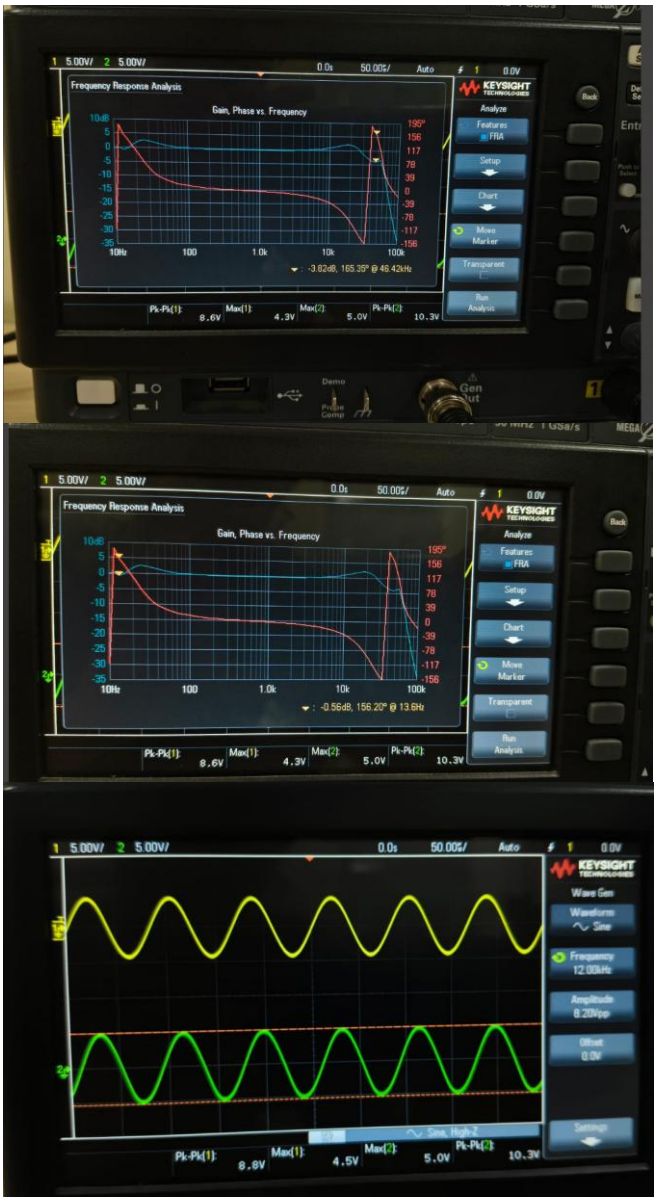


B. *Ltspice-*

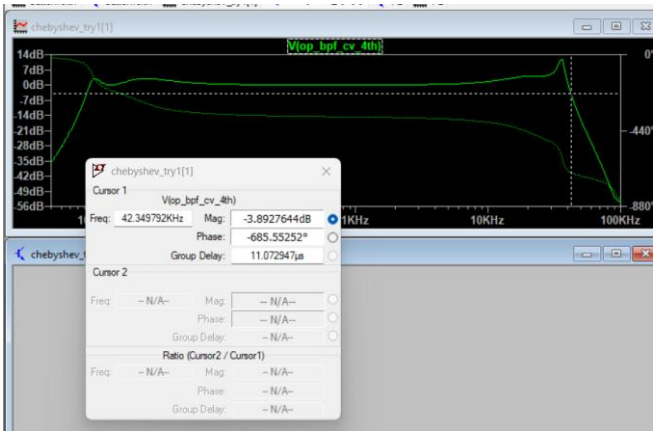


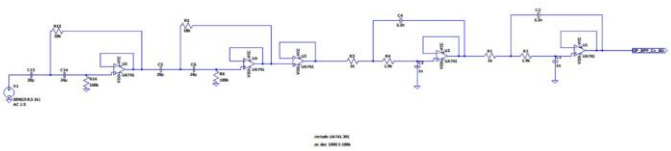
XIV. 4TH ORDER CHEBYSHEV BANDPASS

A. *Hardware circuit*



B. *Ltspice-*





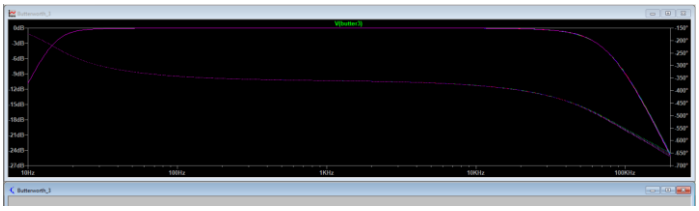
XV. More Observations

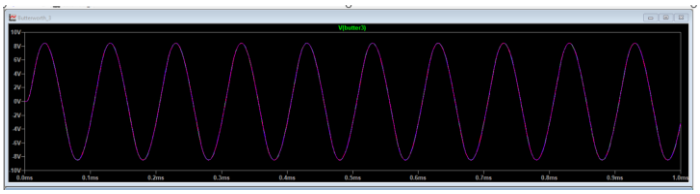
It is a type of linear filter characterized by a smooth and maximally flat frequency response within the passband, meaning it has no ripples and maintains a consistent gain until the cutoff edges. It is a cascade of the 1st order and 2nd order bandpass filters.

For the plots below, both in hardware and LTSpice we can see that the bode plot has a flat response for the frequency range along with the monotonically decreasing transitions in the stopband.

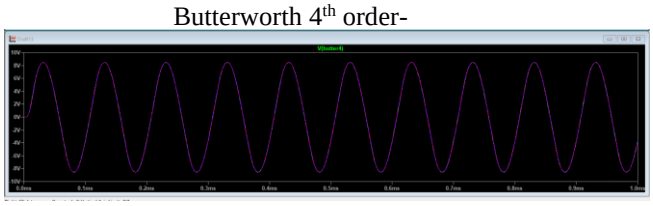
A. *Temperature Analysis*

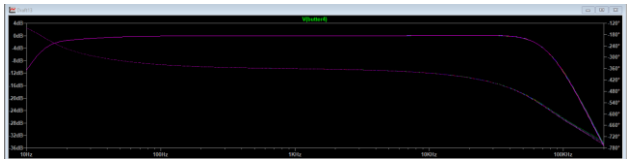
Butterworth 3rd order-



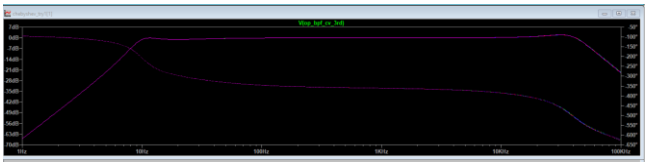


Butterworth 4th order-

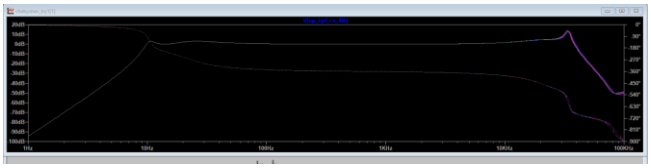




Chebyshev 3rd order-



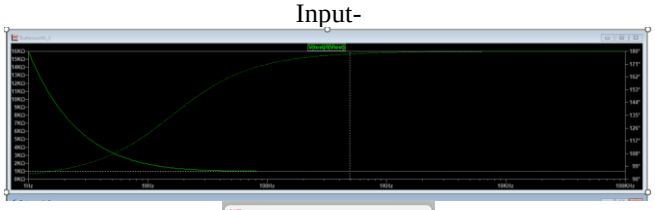
Chebyshev 4th order-

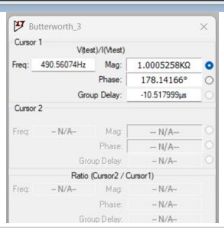


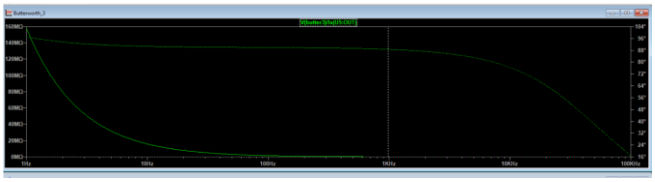
B. *Input and output impedances*

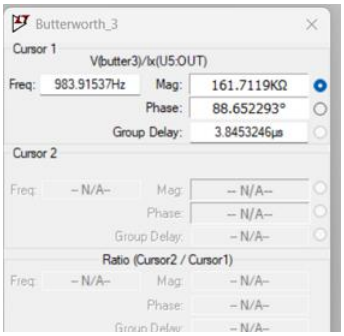
3rd order Butterworth-

Input-



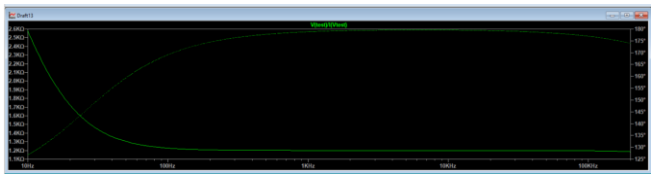




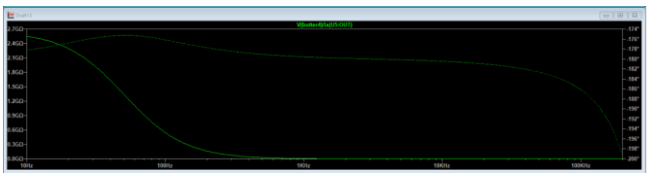


4th order Butterworth-

Input-

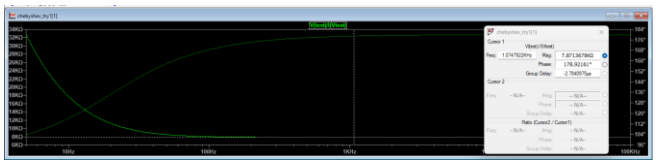


Output-

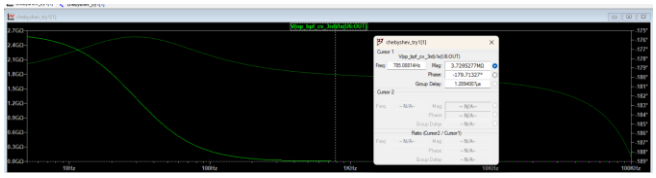


3rd order Chebyshev-

Input-

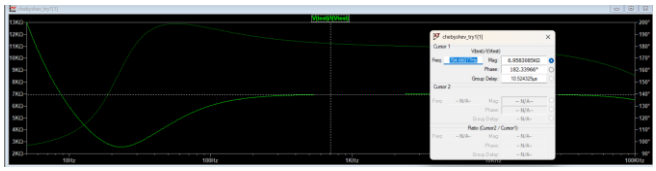


Output-

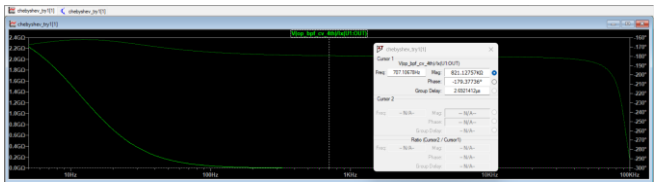


4th order Chebyshev-

Input-



Output-



C. THD-

Butterworth Filter (Order 3 & 4)- Low distortion in passband due to smooth and flat magnitude response. Order 4 offers better stopband attenuation and narrower transition band than order 3. THD remains low, especially at mid-band frequencies due to minimal gain peaking. Phase nonlinearity increases with filter order but is still smoother compared to Chebyshev.

Chebyshev Filter (Order 3 & 4)- Higher THD near the ripple peaks in the passband. As filter order increases, Ripple density increases, potentially introducing more harmonic distortion. Steeper roll-off leads to better selectivity but at the cost of nonlinear phase and higher thd. Order 4 Chebyshev can have significantly higher harmonic distortion than its Butterworth counterpart.

3rd order Butterworth-

```
N-Period=1
Fourier components of V(butter3)
DC component:-0.0249331

Harmonic     Frequency     Fourier     Normalized    Phase       Normalized
 Number        [Hz]       Component     Component   [degree]     Phase [deg]
    1        1.000e+4     8.431e+0     1.000e+0     110.11°        0.00°
    2        2.000e+4     1.742e-2     2.066e-3     143.18°       33.06°
    3        3.000e+4     1.175e-1     1.393e-2      -97.26°     -207.37°
    4        4.000e+4     1.119e-2     1.328e-3     -141.22°     -251.34°
    5        5.000e+4     4.049e-2     4.802e-3       -3.72°     -113.83°
    6        6.000e+4     6.084e-3     7.216e-4      -37.08°     -147.20°
    7        7.000e+4     1.441e-2     1.710e-3      105.65°       -4.47°
    8        8.000e+4     3.109e-3     3.688e-4       76.35°      -33.76°
    9        9.000e+4     5.597e-3     6.639e-4     -142.30°     -252.42°
Partial Harmonic Distortion: 1.507357%
Total Harmonic Distortion:   1.508264%
```

4th order Butterworth-

```
N-Period=1
Fourier components of V(butter4)
DC component:-0.0294449

Harmonic     Frequency     Fourier     Normalized    Phase       Normalized
 Number        [Hz]       Component     Component   [degree]     Phase [deg]
    1        1.000e+4     8.501e+0     1.000e+0     114.28°        0.00°
    2        2.000e+4     1.689e-2     1.987e-3     145.68°       31.40°
    3        3.000e+4     1.126e-1     1.325e-2      -91.48°     -205.76°
    4        4.000e+4     1.137e-2     1.337e-3     -129.61°     -243.89°
    5        5.000e+4     4.058e-2     4.773e-3        8.68°     -105.60°
    6        6.000e+4     6.153e-3     7.238e-4      -20.59°     -134.87°
    7        7.000e+4     1.409e-2     1.657e-3      128.66°       14.38°
    8        8.000e+4     3.141e-3     3.695e-4      114.52°        0.24°
    9        9.000e+4     5.656e-3     6.653e-4      -93.78°     -208.06°
Partial Harmonic Distortion: 1.441926%
Total Harmonic Distortion:   1.442387%
```

3rd order Chebyshev-

```
N-Period=1
Fourier components of V(op_bpf_cv_3rd)
DC component:-0.0103663

Harmonic     Frequency     Fourier     Normalized    Phase       Normalized
 Number        [Hz]       Component     Component   [degree]     Phase [deg]
    1        1.000e+4     8.690e+0     1.000e+0     116.03°        0.00°
    2        2.000e+4     1.878e-2     2.162e-3     142.61°       26.58°
    3        3.000e+4     1.392e-1     1.601e-2      -86.20°     -202.23°
    4        4.000e+4     1.543e-2     1.775e-3      -93.26°     -209.28°
    5        5.000e+4     4.902e-2     5.640e-3       50.63°      -65.39°
    6        6.000e+4     4.952e-3     5.698e-4       88.24°      -27.79°
    7        7.000e+4     1.327e-2     1.527e-3     -148.13°     -264.16°
    8        8.000e+4     1.734e-3     1.995e-4      -52.91°     -168.94°
    9        9.000e+4     1.584e-2     1.822e-3       73.95°      -42.08°
Partial Harmonic Distortion: 1.737996%
Total Harmonic Distortion:   1.747314%
```

4th order Chebyshev-

```
N-Period=1
Fourier components of V(op_bpf_cv_4th)
DC component:-0.0394006

Harmonic     Frequency     Fourier     Normalized    Phase       Normalized
 Number        [Hz]       Component     Component   [degree]     Phase [deg]
    1        1.000e+4     6.741e+0     1.000e+0      -30.56°        0.00°
    2        2.000e+4     5.995e-2     8.893e-3      -64.62°      -34.06°
    3        3.000e+4     1.266e+0     1.878e-1      -60.61°      -30.06°
    4        4.000e+4     1.038e-2     1.540e-3       -4.16°       26.40°
    5        5.000e+4     3.706e-1     5.498e-2      -76.75°      -46.19°
    6        6.000e+4     2.024e-2     3.002e-3      -60.30°      -29.74°
    7        7.000e+4     1.392e-1     2.064e-2     -140.86°     -110.30°
    8        8.000e+4     1.939e-2     2.877e-3      -99.98°      -69.42°
    9        9.000e+4     1.380e-1     2.048e-2      176.89°      207.45°
Partial Harmonic Distortion: 19.804787%
Total Harmonic Distortion:   19.866010%
```

XVI. Applications

Butterworth-

Butterworth bandpass filters are widely used in applications where a maximally flat frequency response in the passband is essential. This characteristic ensures minimal distortion within the desired frequency range, making them ideal for audio signal processing tasks such as equalizers, tone controls, and audio crossover networks. In biomedical signal analysis, these filters help clean ECG and EEG signals by removing noise while preserving the integrity of the signal shape. Communication systems benefit from Butterworth filters through consistent channel amplification, which is

crucial for uniform signal reception. Control systems use these filters to process sensor signals without introducing ripple that could affect system stability. Additionally, in seismology, Butterworth filters are employed in instruments that isolate seismic waves at specific frequencies.

Chebyshev-

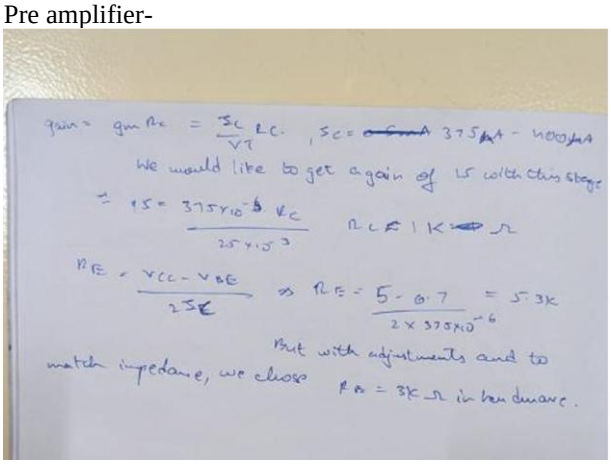
Chebyshev bandpass filters, on the other hand, are known for their steeper roll-off and the presence of ripples in the passband, which make them suitable for applications demanding high selectivity. In RF and microwave engineering, these filters enable sharp frequency selectivity for antenna and receiver design. Radar systems use them for precise frequency filtering where roll-off is prioritized over a flat passband. In speech processing, Chebyshev filters are useful for noise reduction and formant filtering when tighter band separation is required. They also play a role in digital communication systems for data demodulation and efficient signal extraction in scenarios that require minimal transition width. Finally, in spectral analysis, these filters are applied where high selectivity is crucial and the presence of passband ripple is acceptable.

For the given frequency range for us (15 Hz to 50kHz), the best application is the audio amplifier as it allows the human audibe frequncy range, which has been extended for oor project. Since many applications use small frequency ranges (around 20- 300Hz or till 5k in general), our frequency range does work for all applications but inorder to show specifically for our range, we chose audio amplifier.

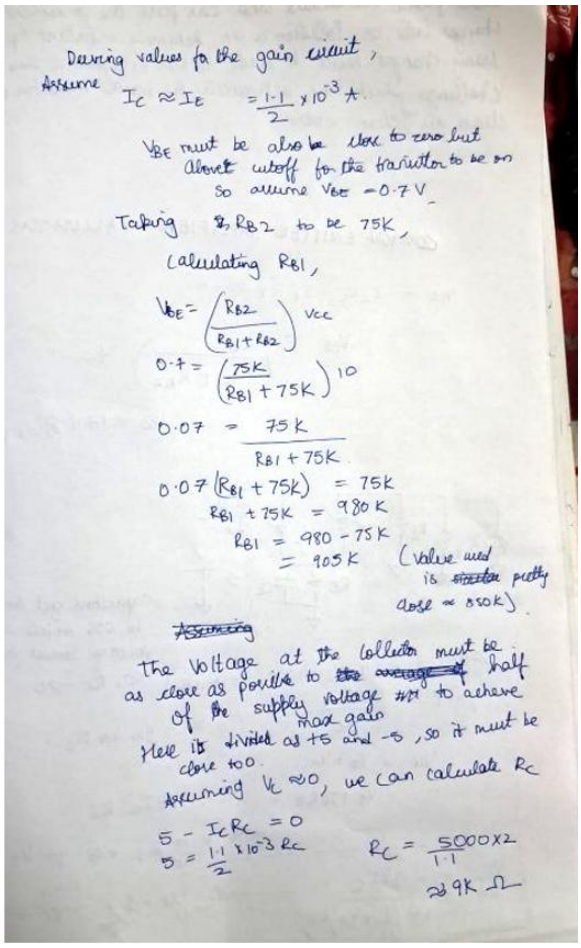
XVII. AUDIO AMPLIFIER

In order to ensure the impedance matching due to the insertion of our filters, we have done the following calculations again by adjusting the values of resistors and capacitors in terms of assumptions that we generally take. So the below calculations are the process followed-

Pre amplifier-

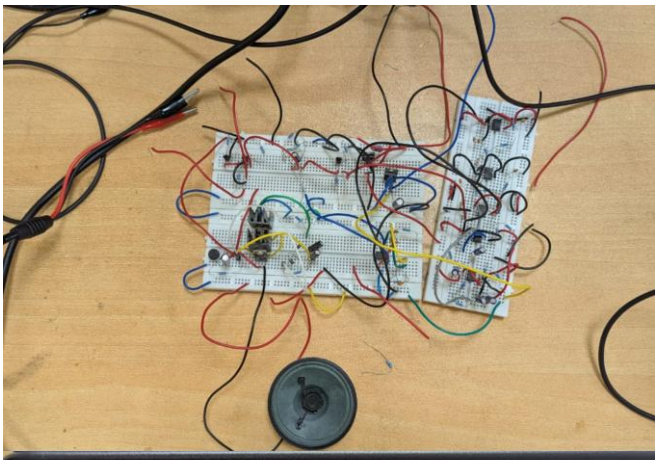


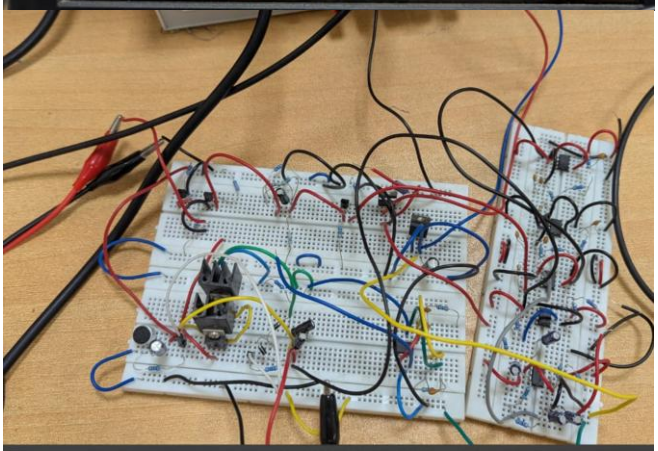
Gain stage-

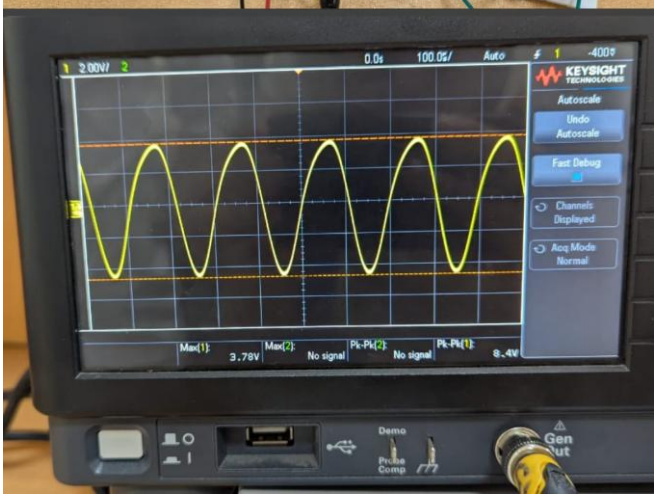


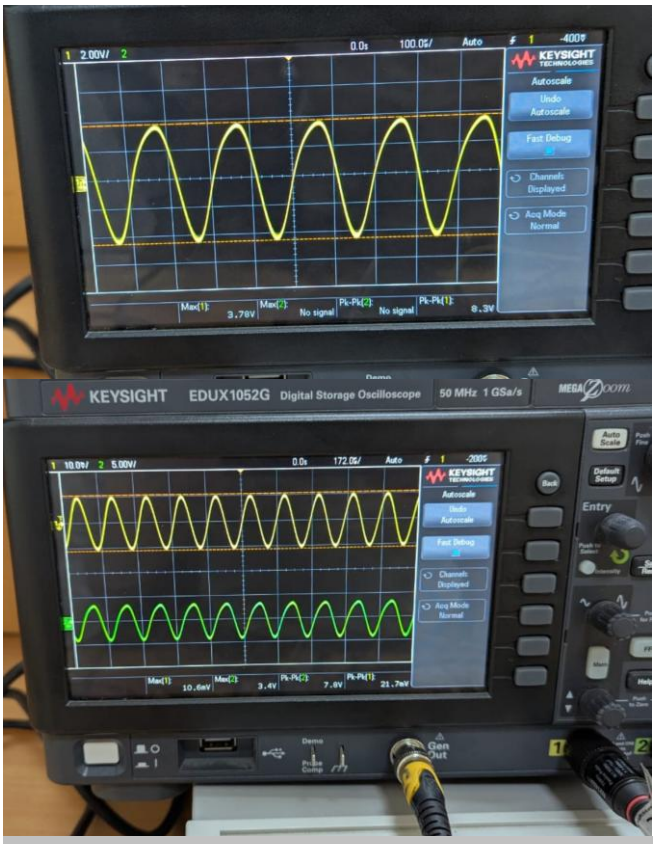
Power amplier calculations remained the same with very slight negligible changes.

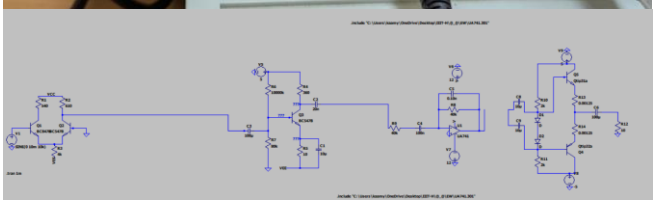
The results were similarly ideal for other cases too.











XVIII. Slew Rate

Slew rate (S.R) is the maximum rate at which the output voltage of an amplifier can change with time.

S.R.=max(dVout / dt)

A higher slew rate means the amplifier can handle rapid voltage changes efficiently, which is important for high-frequency signals.

If the slew rate is too low, the output cannot keep up with fast input transitions, leading to slew induced distortion—where the output waveform appears stretched and lags the input. • Ways to increase slew rate include increasing the maximum operating voltage, reduce capacitive impedance in subcircuits and allowing operation at higher frequencies.

$S=2\pi fmVm$

XIX. Conclusions

The analysis of 3rd and 4th order Butterworth and Chebyshev bandpass filters for audio amplifier applications (15Hz-50kHz range) revealed distinct performance characteristics across several critical parameters:

Frequency Response: Butterworth filters provided maximally flat response in the passband with smooth transitions. Chebyshev filters demonstrated

steeper roll-off but with characteristic ripples in the passband

Temperature Stability: Butterworth filters showed superior stability across temperature variations with minimal gain fluctuation. Chebyshev filters exhibited more sensitivity to temperature changes, especially the 4th order variant

Total Harmonic Distortion (THD): Butterworth filters maintained lower THD throughout the passband due to their flat response. Chebyshev filters showed increased distortion near ripple peaks, with 4th order configurations exhibiting significantly higher THD than their Butterworth counterparts

Implementation Complexity: 4th order filters provided better stopband attenuation but required more precise component selection. 3rd order implementations offered a reasonable compromise between performance and complexity.

1) *Final specifications-*

Proposed Frequency Range- 15 Hz to 50kHz

Frequency range achieved for-

a) *Butterworth 3rd order bandpass- 15.7Hz to 51kHz*

b) *Butterworth 4th order bandpass- 14.5Hz to 62kHz*

c) *Chebyshev 3rd order bandpass- 21Hz to 47.8kHz*

d) *Chebyshev 4th order bandpass- 13Hz to 45.5kHz*

Hence, we achieved very close values to the required range.

2) *Gain- 0 for all filters*

3) *Phase distortion is less in Butterworth because of the location of the poles on the circle in s plane on converting from bilinear transformation. Whereas for Chebyshev, they lie on the epplise which gives less linear phase response compared to Butterworth.*

4) *The values of the input and output impedances have been highlighted in the ltspice plots itself above.*

5) *The THD has also been compared to show the low distortion but high values at the ripples in Chebyshev.*

6) *The numebr of ripples is equal to the half of the order of the filter.*

To conclude, we use Butterworth filters when we want to get maximally flat passband response and allow the entire frequency range properly. But if we want sharp cutoff for no interference with other channels and sharp transitions, we prefer Chebyshev filters. In general, higher orders are better than lower ones.

https://drive.google.com/drive/folders/1oTU_SLZKmBAeYsH4jzWtmSvMO99PpXsy?usp=sharing

(Link for the video)