Communication Theory

Spring-2025

Assignment 5 **Deadline: 11:59 pm April 18th**

Instructions:

- All questions are compulsory.
- Clearly state the assumptions *(if any)* made that are not specified in the questions.
- Submission format: `Rollnumber.pdf`

Cautions:

- One late homework assignment is allowed without penalty.
- 2 marks will be deducted on other late assignments.

1. In a binary hypothesis testing problem with hypotheses H_0 and H_1, having prior probabilities π_0 and π_1, and observation model given by conditional densities $p(r \mid H_0)$ and $p(r \mid H_1)$, what decision rule should be used to make the most informed choice between H_0 and H_1? Justify your answer, and support your reasoning with a relevant example from a digital communication system.

2. Consider a case of binary transmission via polar signaling that uses half-width rectangular pulses of amplitudes $A/2$ and $-A/2$. The data rate is R_b bit/s.

   (a) What is the minimum transmission bandwidth and the transmitted power.

   (b) This data is to be transmitted by M-ary rectangular half-width pulses of amplitudes

   $$\pm\frac{A}{2}, \pm\frac{3A}{2}, \pm\frac{5A}{2}, \dots, \pm\left[\frac{(M-1)}{2}\right]A$$

   Note that to maintain about the same noise immunity, the minimum pulse amplitude separation is A. If each of the M-ary pulses is equally likely to occur, show that the transmitted power is

   $$P = \frac{(M^2-1)A^2}{24\log_2 M}$$

   Also determine the transmission bandwidth.

3. Binary data is transmitted over a certain channel at a rate R_b bit/s. To reduce the transmission bandwidth, it is decided to use 16-ary PAM signaling to transmit this data.

   (a) By what factor is the bandwidth reduced?

   (b) By what factor is the transmitted power increased, assuming minimum separation between pulse amplitudes to be the same in both cases?

4. MATLAB - Simulate a binary communication system over an AWGN channel where two deterministic signals $s_0 = -A$ and $s_1 = +A$ are transmitted with prior probabilities π_0 and π_1, respectively. The received signal is given by

$$r = s_i + n, \quad \text{where } n \sim \mathcal{N}(0, \sigma^2).$$

   (a) Implement both the **MAP** and **ML** decision rules for a fixed SNR.

   (b) Simulate the transmission of 10^6 bits and compute the empirical probability of error under both rules.

   Note: You may assume $A = 1$ and $\sigma^2 = 1$ (i.e., SNR $= 0$ dB) for simplicity.

5. MATLAB - The effect of noise on the performance of a binary communication system can be observed from the received signal plus noise at the input to the detector. For example, let us consider binary orthogonal signals, for which the input to the detector consists of the pair of random variables (r_0, r_1), where either

$$(r_0, r_1) = \left(\sqrt{E} + n_0, n_1\right)$$

or

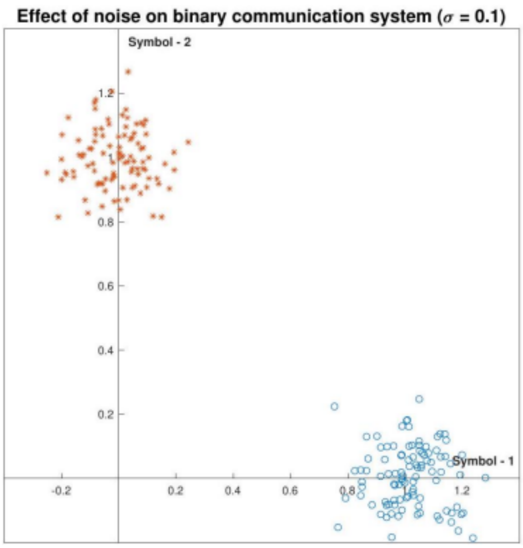
$$(r_0, r_1) = \left(n_0, \sqrt{E} + n_1\right)$$

The noise random variables n_0 and n_1 are zero-mean, independent Gaussian random variables with variance σ^2.

Use Monte Carlo simulation to generate 100 samples of (r_0, r_1) for each value of $\sigma = 0.1, \sigma = 0.3$, and $\sigma = 0.5$. Plot these 100 samples for each σ on different two-dimensional plots. The energy E of the signal may be normalised to unity.

(a) What do you think, what kind of detector can be used for this kind of binary communication?

(b) What will be the effect of increasing noise variance (decreasing SNR) on the detector?



Hint: Fig (a) shows a sample plot for $\sigma = 0.1$.

6. (a) MATLAB - 4-PAM Mapping Function

Write a function `fourpammap` that maps two 0/1 bits to a symbol in the 4-PAM constellation. The 4-PAM constellation consists of the symbols $\{\pm 1, \pm 3\}$.

- Input: Two 0/1 bits.
- Output: A symbol taking one of the following values: $\{\pm 1, \pm 3\}$.

(b) Bit Error Probability for 4-PAM

Using the nearest neighbors approximation, the ideal bit error probability for Gray-coded 4-PAM is given by:

$$P_e = Q\left(\frac{\sqrt{4E_b}}{5\sqrt{N_0}}\right)$$

where $Q(x)$ is the Q-function.

- Plot this error probability on a log scale as a function of E_b/N_0 (in dB) over the range $0 \le E_b/N_0 \le 10$ dB.
- Find the value of E_b/N_0 (in dB) corresponding to a bit error probability of 10^{-2}.

(c) Decision Statistics and Error Probability for 4-PAM

For 6000 transmitted 4-PAM symbols, perform the following tasks:

- Generate decision statistics at the receiver after passing through the receive filter.
- Plot the real versus imaginary parts of the decision statistics.
- Use an appropriate decision rule to estimate the two parallel bit streams of 6000 bits from the 6000 complex decision statistics.
- Measure the bit error probability for the 4-PAM symbols.