Lecture 2

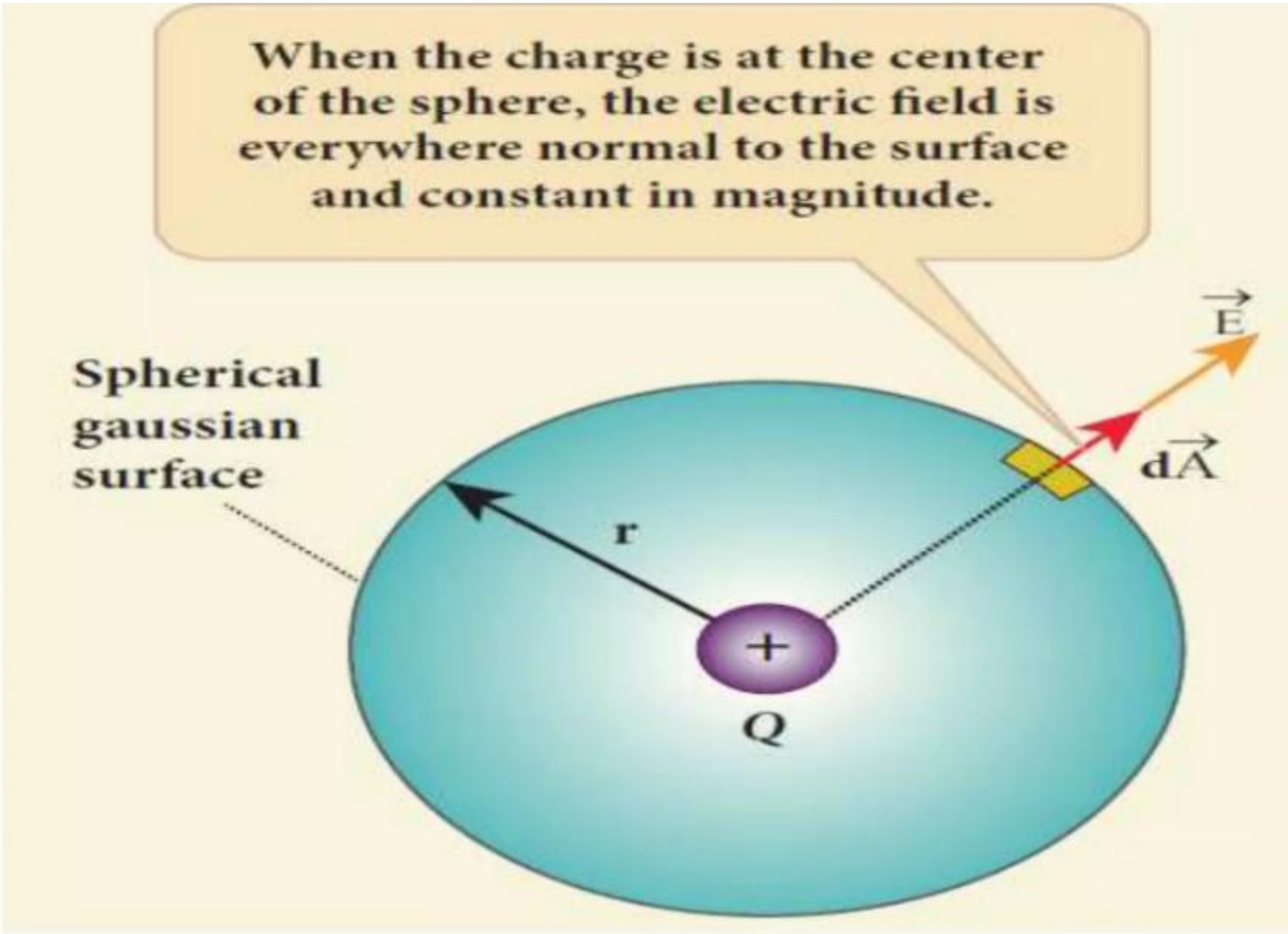
Maxwell Equations

(Origin of Electromagnetic Waves)

Gauss contributed to electromagnetic theory by formulating Gauss's Law for Electricity and Magnetism, these laws are the foundation of Maxwell's equations. which simplifies the calculation of electric and magnetic fields and form

Karl Friedrich Gauss (1777–1855) was a legendary mathematician of the nineteenth century.

Guess's Law


When the charge is at the center of the sphere, the electric field is everywhere normal to the surface and constant in magnitude.
Spherical gaussian surface
r
+
Q
$\vec{E}$
$d\vec{A}$

- A positive point charge Q is surrounded by an imaginary sphere of radius r
- The total electric flux through the closed surface of the sphere using the equation

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E\, dA \cos\theta$$

- The electric field of the point charge is directed radially outward at all points on the surface of the sphere
- Therefore, the direction of the area element dA is along the electric field E and .$\theta = 0^0$

$$\Phi_E = \oint E\,dA \qquad \text{since } \cos 0^0 = 1$$

- *E is uniform on the surface of the sphere*

$$\Phi_E = E\oint dA$$

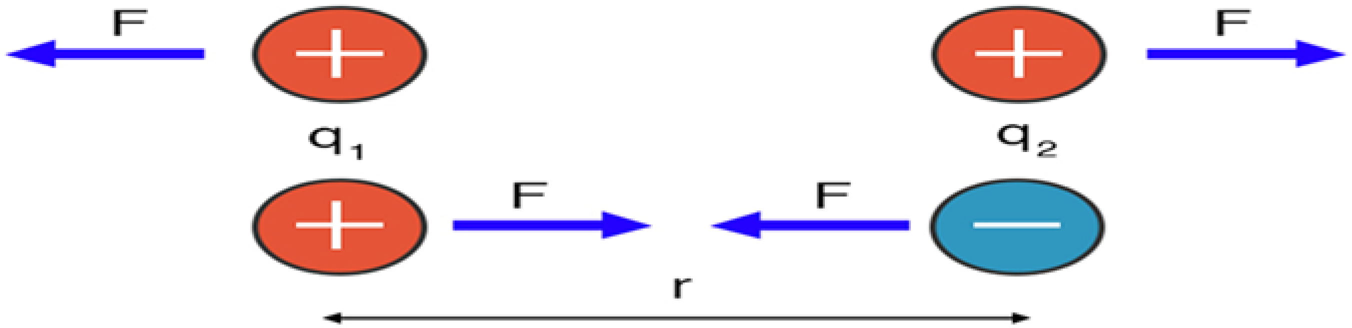
Substituting for $\oint dA = 4\pi r^2$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

$$\Phi_E = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{r^2} \times 4\pi r^2 = 4\pi\frac{1}{4\pi\varepsilon_\circ}Q$$

$$\Phi_E = \frac{Q}{\varepsilon_\circ}$$

- The equation is called as Gauss's law

Coulomb's Law



$$F = k\,\frac{q_1 \cdot q_2}{r^2}$$

* Electrostatic constant K depends on the nature of medium separating the charges and on the system of units.

In SI system, $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2/c^2$

* ϵ_0 electrical permittivity of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \ c^2/Nm^2$$

Limitations of Coulomb’s Law-

- The law is applicable only for the point charges at rest.
- It is difficult to implement Coulomb’s law where charges are in an arbitrary shape because, in such cases, we cannot determine the distance between the charges.

Key Differences:

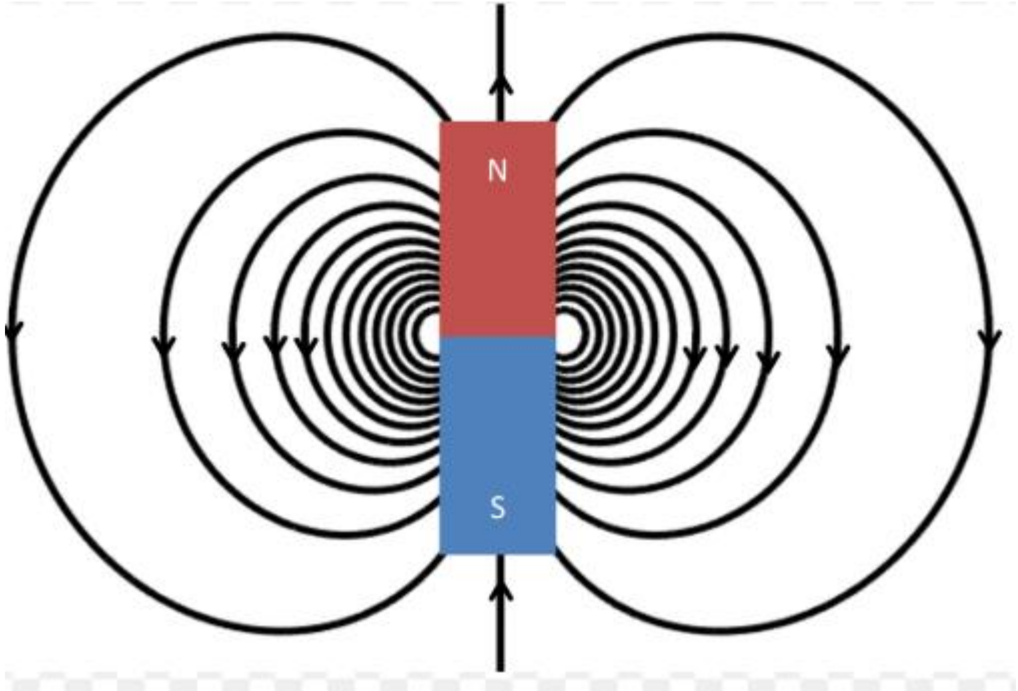
Feature	Coulomb's Law	Gauss's Law
Describes	Force between two charges	Electric flux over a closed surface
Application	Point charges	Continuous charge distributions
Mathematical Form	Inverse square law	Integral form (flux)
Best Used For	Calculating force between two charges	Finding electric fields for symmetrical charge distributions

- **Derive the Coulomb law from the Guess law?**
- **Derive the Guess law from Coulomb Law?**

Guess's Law for Magnetism

- We can write a similar relation for the magnetic case, however, there are *no* isolated magnetic charges (monopoles) exist. Therefore:

$$\boxed{\oint \vec{B} \bullet d\vec{A} = 0}$$



- This is Gauss's Law for Magnetism

- Gauss’s Law for Magnetism tells us that as many lines of magnetic flux enter an enclosed surface, as leave it

- Since magnetic monopoles do not exist, there are no sources or sinks for magnetic field lines, unlike E-Fields

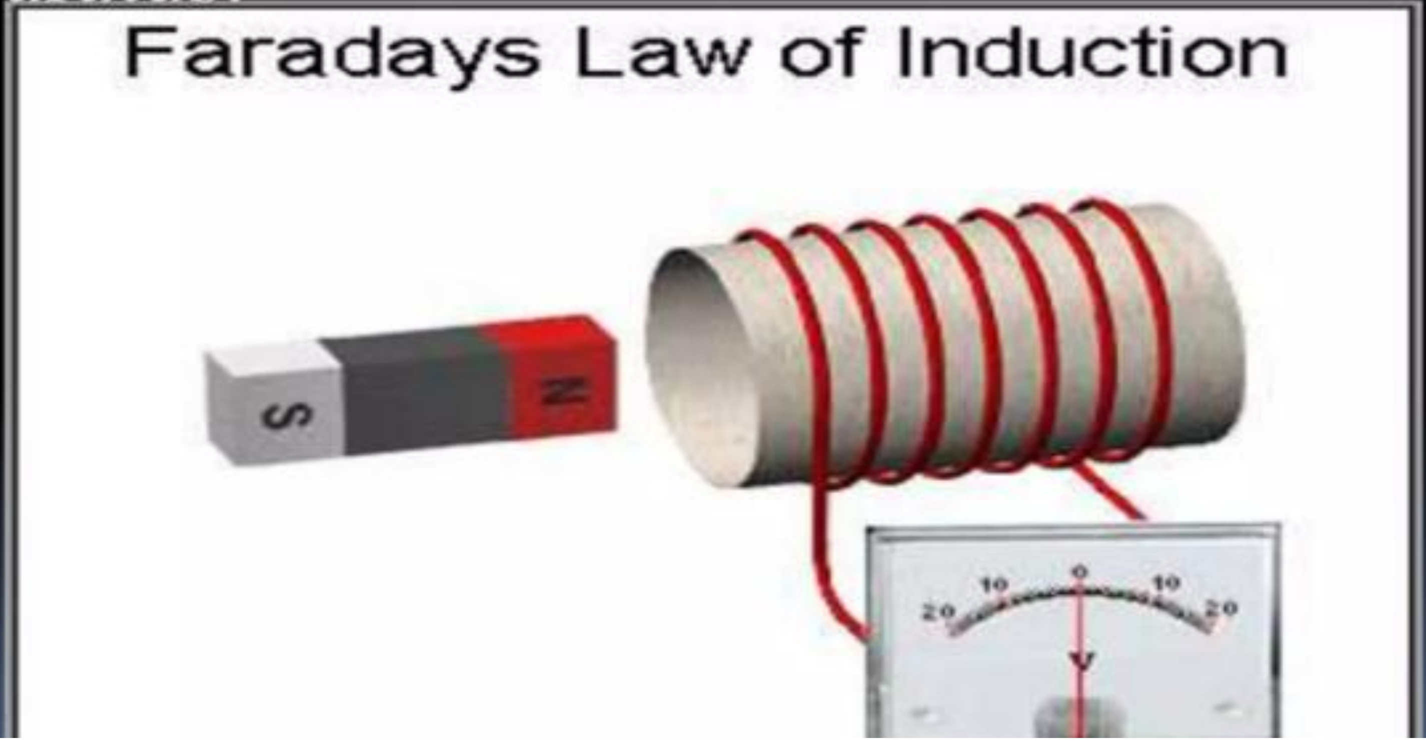
- Therefore, magnetic fields must be continuous (no beginning or end)

Faraday's Law

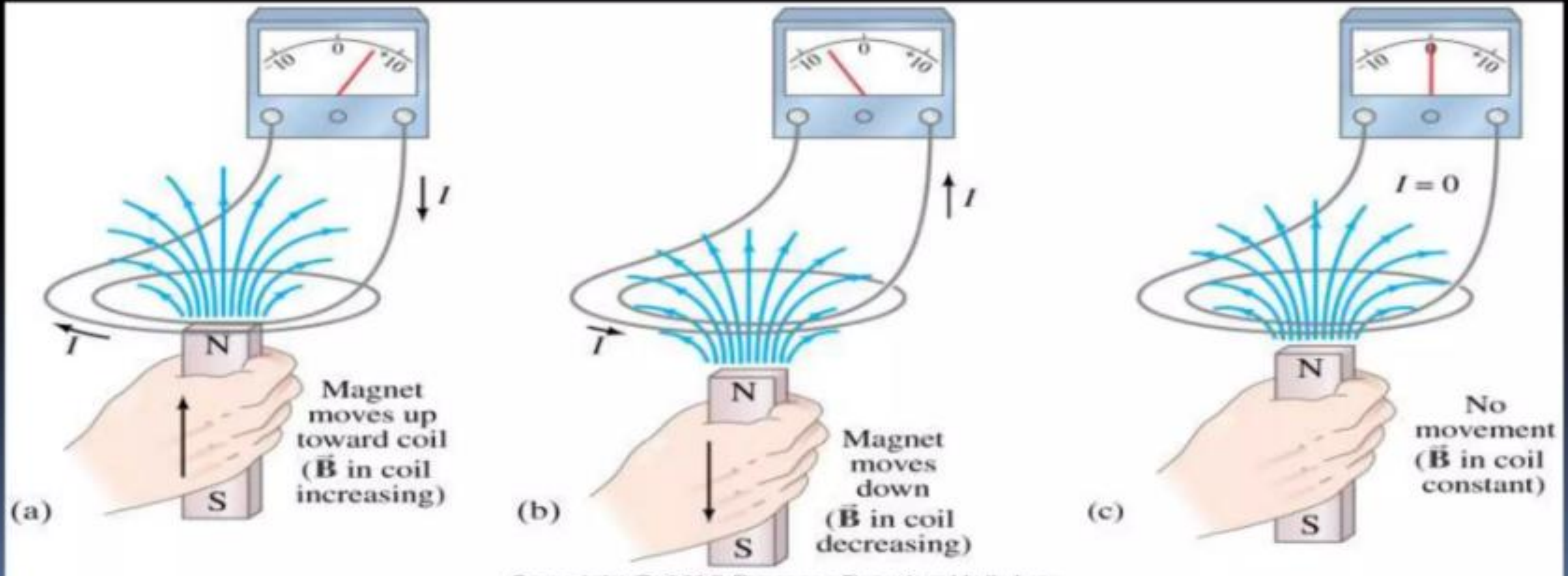
- Great experimental physicist and chemist
- 1791 – 1867
- Contributions to early electricity include:
  - Invention of motor, generator, and transformer
  - Electromagnetic induction
  - Laws of electrolysis: A method of separating bonded elements and compounds by passing an electric current through them

FARADAY'S EXPERIMENT

Almost 200 years ago, around 1831, Faraday looked for evidence that a magnetic field would induce an electric current with this apparatus:



He found no evidence when the magnet was stationary, but did see a deflection in the galvanometer when the magnet was moved away or towards the coil



- Faraday’s law of induction states that “the emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit”
- Mathematically,

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

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- **Lenz's law** : *the induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop*

James Clerk Maxwell (1831-1879)

James Clerk Maxwell, the great physicist and mathematician,was born in Edinburgh,Scotland, on November 13, 1831

most notable achievement was to formulate the classical theory of electromagnetic radiation, bringing together for the first time electricity, magnetism, and light as manifestations of the same phenomenon. Maxwell's equations for electromagnetism have been called the "second great unification in physics" after the first one realised by Isaac Newton.

André-Marie Ampère

His contributions form the basis for modern electromagnetism, including applications in electrical engineering, wireless communication, and power transmission.

Born 20 January 1775 (Lyon, Kingdom of France) Died 10 June 1836 (aged 61)

ACHIEVEMENTS

And here is the result of hard work. By the age of 12, Ampere had independently figured out the basics of higher mathematics - differential calculus, he had learned to integrate, and at the age of 13 he had already submitted his first work on mathematics to the Lyons Academy!

Ampere's Law

"The line integral of the magnetic field B around any closed loop is μ_0 times the net steady current I enclosed by this path"

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I \qquad (1)$$

$$\oint \underline{B}.\underline{dl} = \oint B\, dl \cos 0 = B \oint dl$$

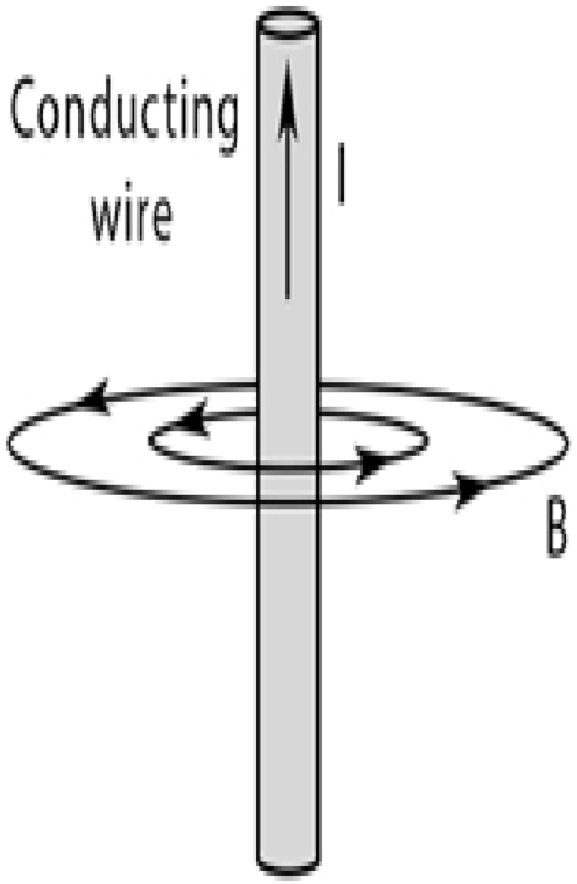
As, $\underline{B}$ is uniform at each point on the field line:

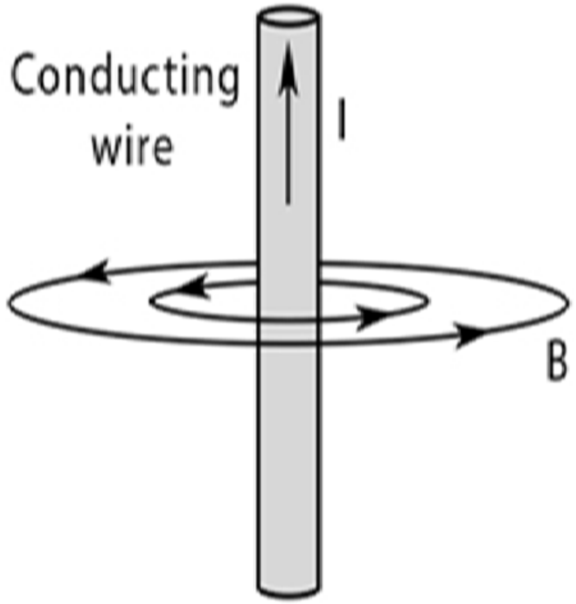
$$\oint \underline{B}.\underline{dl} = B\,(2\pi r) \qquad (2)$$

Equating (1) and (2):

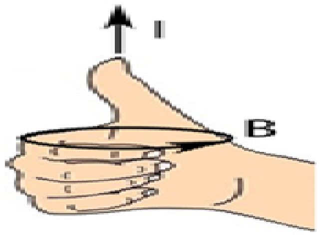
$$B = \frac{\mu_0 I}{2\pi r}$$

i.e. the magnetic field a distance r from a long straight wire carrying a steady current I.





Right hand thumb rule



Thumb points in the direction of the electric current and fingers curl around the current indicating the direction of the magnetic field

- Ampere's law is true for steady currents
- The Magnetic field inside the loop remains constant
- I , arising due to flow of charges, is called the 'conduction current' I_C

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I_c$$

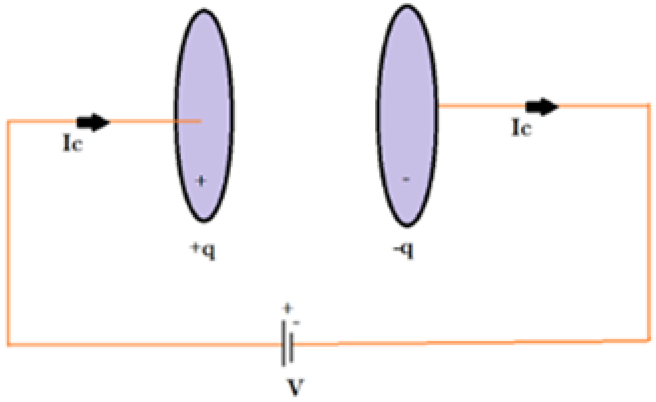
Maxwell's correction to Ampère's law - Displacement current

As the amount of charge on the capacitor increases with time, hence the q charge, changes with time, produces a current:

$$I_d = dq/dt$$

For a capacitor, $q = \varepsilon_0 EA$, and $I_d = \frac{dq}{dt} = \varepsilon_0 \frac{d(\overbrace{EA}^{\Phi_E})}{dt}$

i.e. Current is produced because of changing electric flux w.r.t time OR Electric flux is generated due to the presence of $\underline{E}$ between plates



CONC: Current is not **ZERO** between plates but it is I_d AND

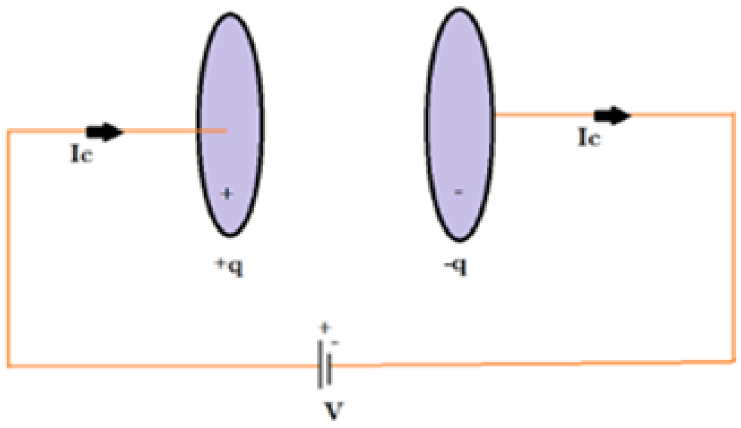
$\underline{B}$ is produced by the time varying $\underline{E}$ or I_d

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 I_d$$

Maxwell correction to Ampere's Law is:

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 (I_C + I_d)$$

$$\Rightarrow \oint \underline{B} \bullet \underline{dl} = \mu_0 I_C + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$



Maxwell correction to Ampere's Law

- There has to be some *I* existing between plates.
- There are no conduction electrons between plates BUT
  There is time-varying $\underline{E}$ between plates, which generates *I*.
- The $\underline{E}$ is directed from +ve plate to –ve plate, because the amount of charge on capacitor increases with time, AND
- The $\underline{E}$ between plates increases with time, as well.
- He called this new current as a **displacement current, I_d** .