EC5.203 Communication Theory (3-1-0-4):

Lecture 12:
Digital Modulation

03 March 2025

INTERNATIONAL INSTITUTE OF
INFORMATION TECHNOLOGY

H Y D E R A B A D

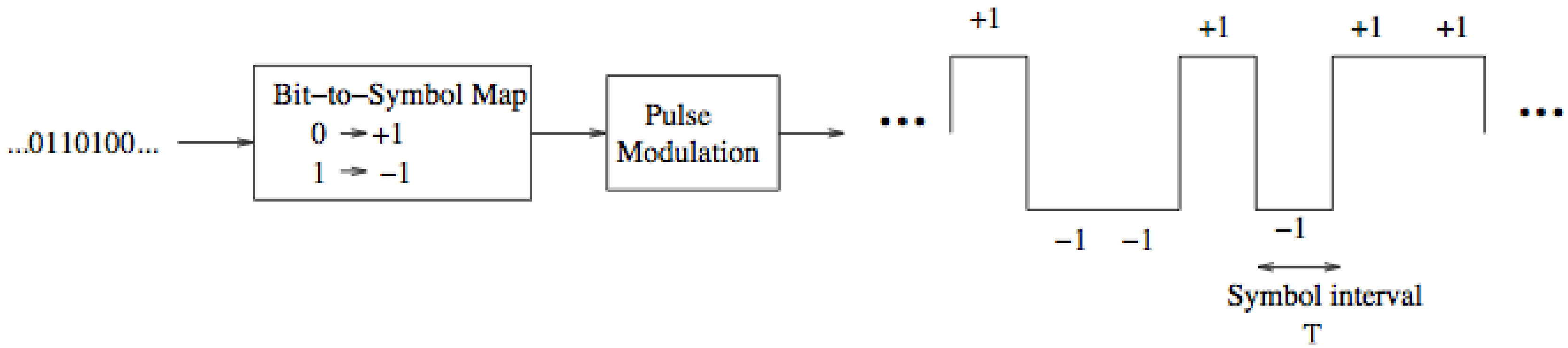
References

- Chap. 4 (Madhow)

Digital modulation

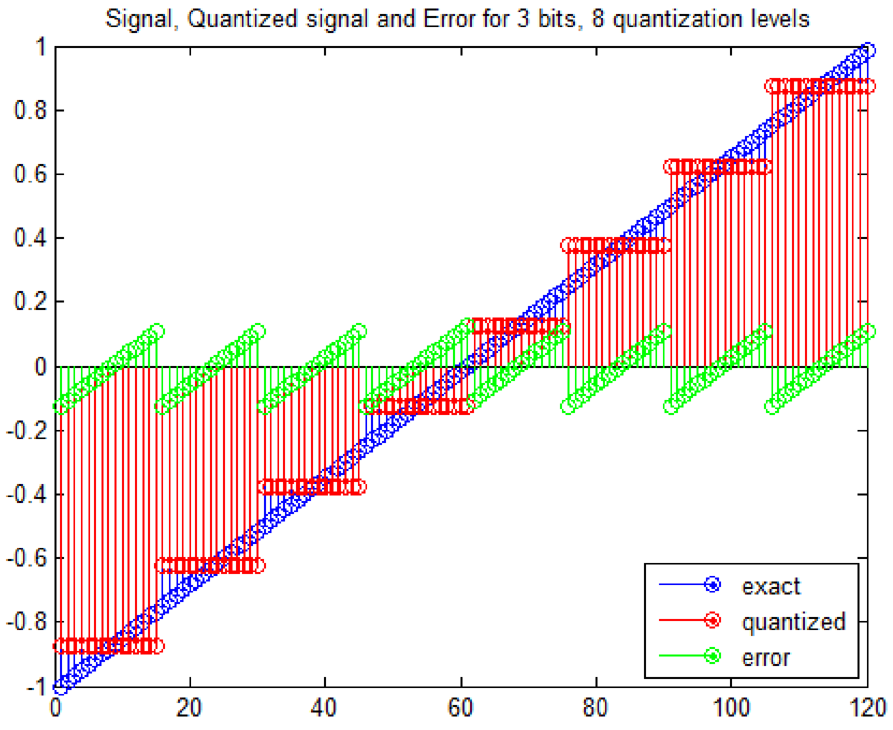
- Digital modulation is the process of translating bits to analog waveforms that can be sent over a physical channel.

- Baseband example: Binary antipodal Signaling

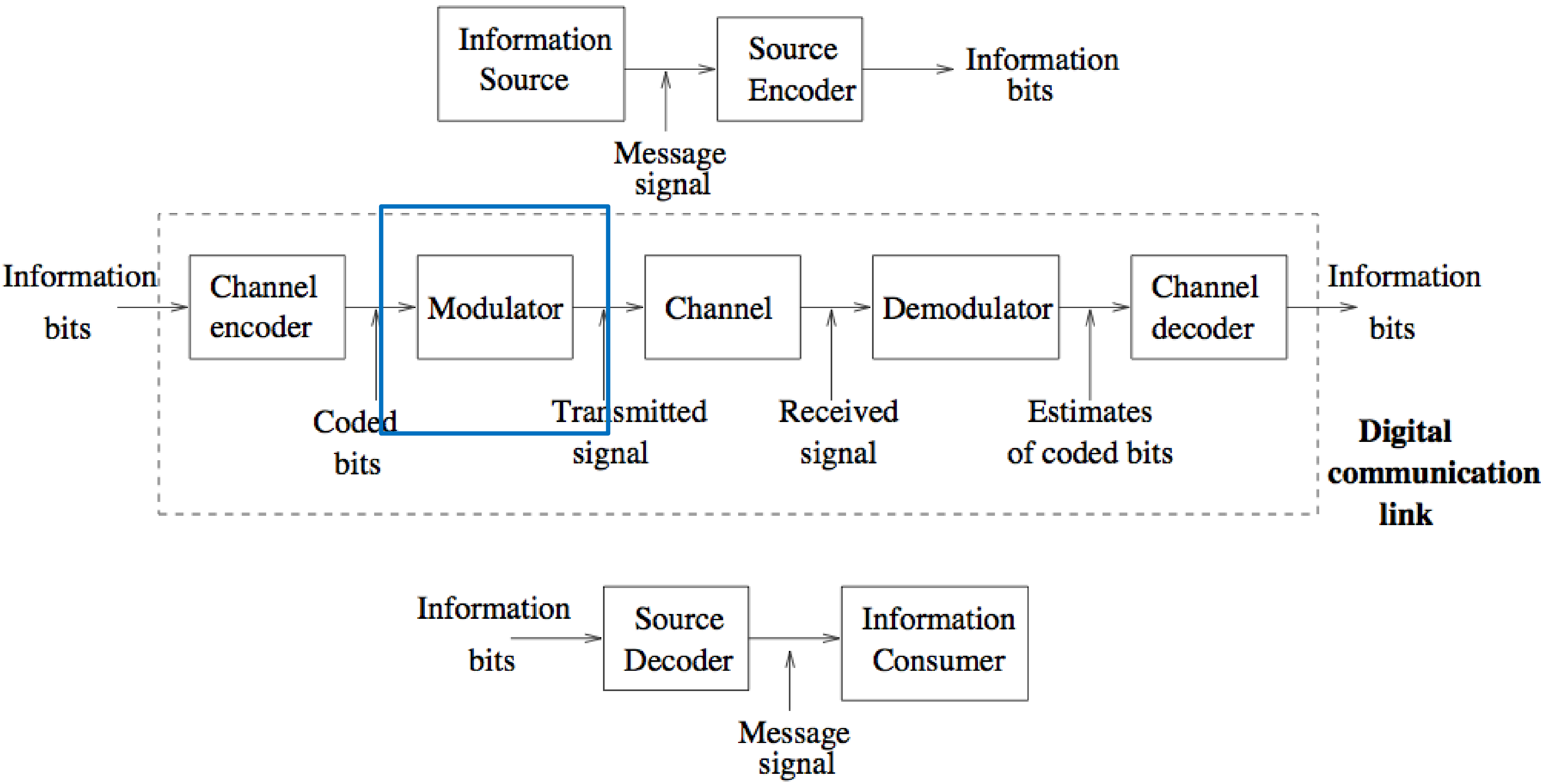


Recap: *Digital Communications*

- Analog signal can be converted to digital signal or sequence by sampling and quantization
  - Songs/Movies stored in CD/DVD, Data in hard-drive

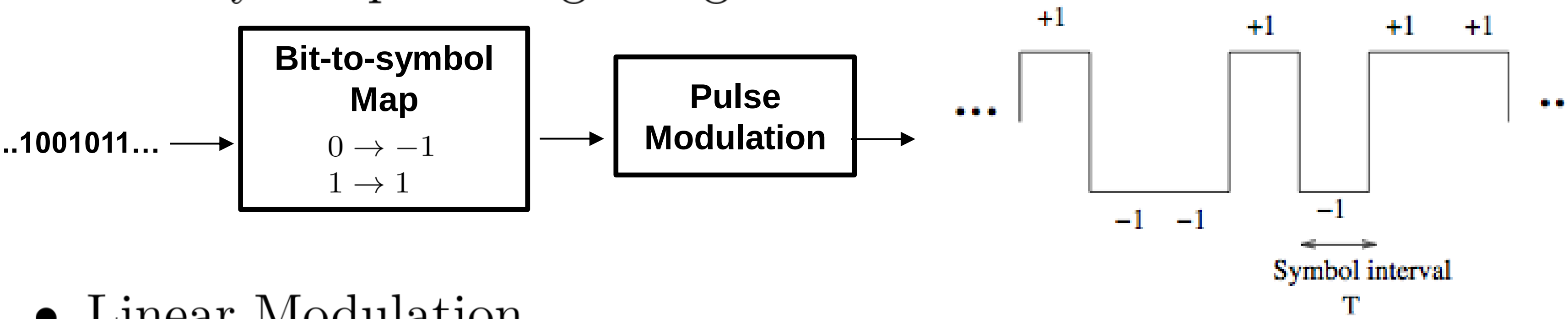


Recap: *Digital Communication Systems*



Digital modulation: baseband example

- Binary antipodal Signaling



- Linear Modulation

$$u(t) = \sum_n b[n]p(t - nT)$$

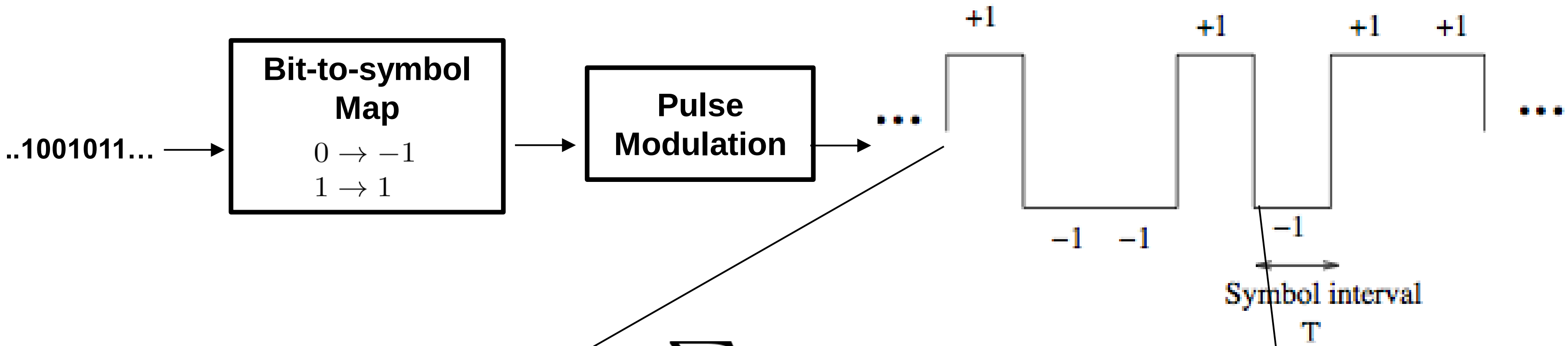
where $\{b[n]\}$ is sequence of symbols and $p(t)$ is modulating pulse for T seconds. For this example

$$p(t) = I_{[0,T]}(t)$$

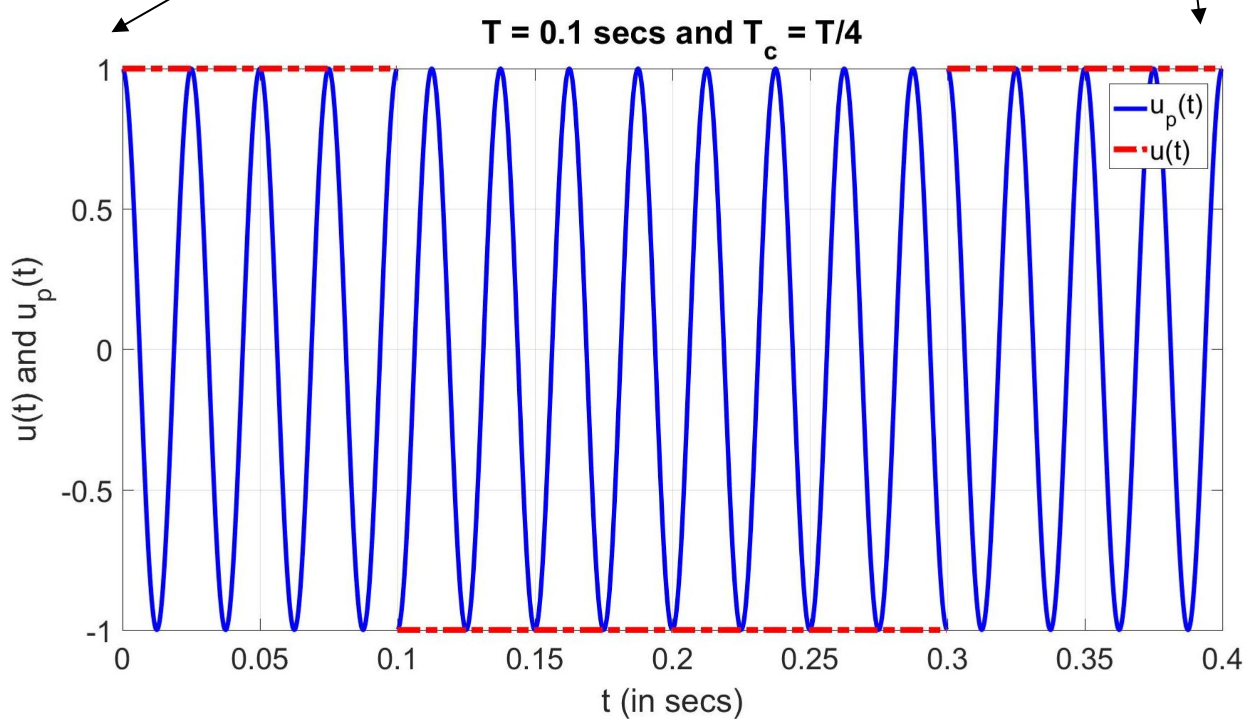
is rectangular window in time domain.

- Baseband signal sent directly over the physical baseband channel.

Digital modulation: passband example



$$u_p(t) = u(t)\cos(2\pi f_c t) = \sum_n b[n]p(t - nT)\cos(2\pi f_c t)$$



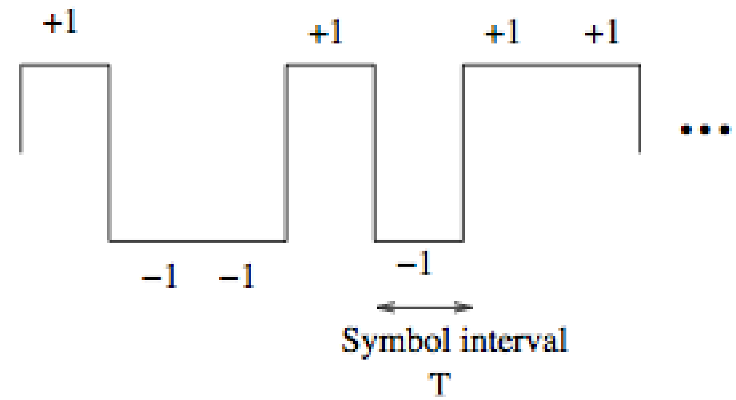
BPSK (also called 2-PAM)

Baseband

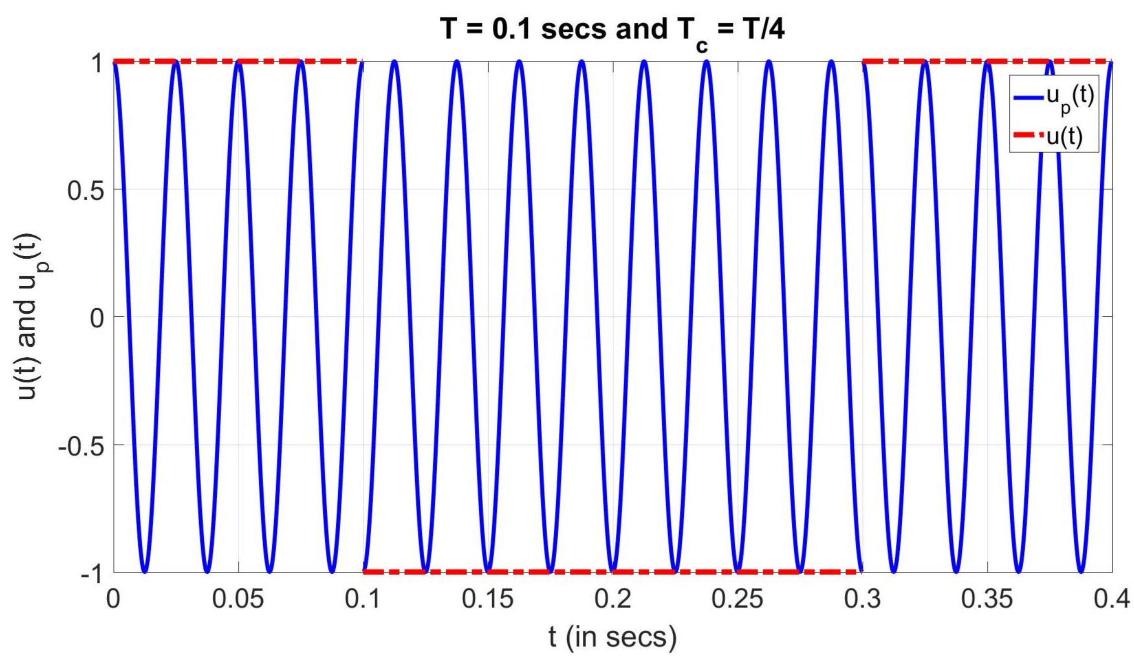
$\{b[n]\}$

1 0 0 1 0 1 1

$$0 \rightarrow -1$$
$$1 \rightarrow 1$$

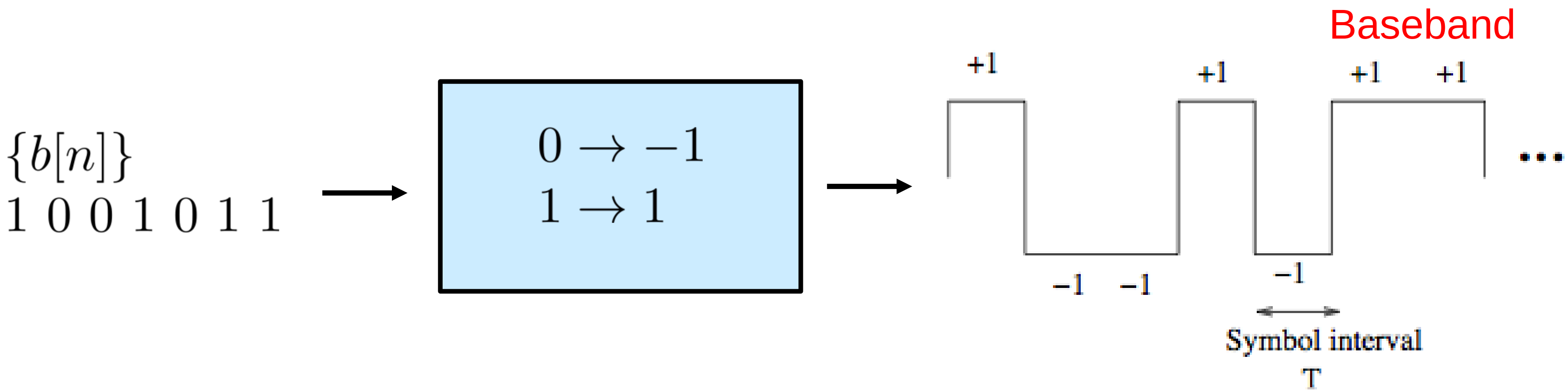


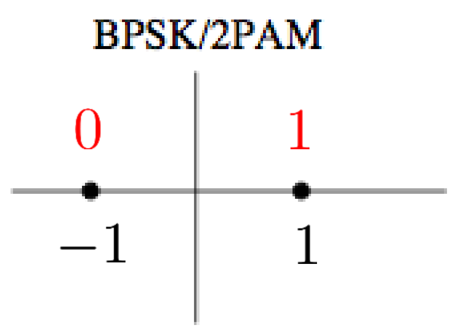
$$u_p(t) = u(t)\cos(2\pi f_c t) = \sum_n b[n]p(t - nT)\cos(2\pi f_c t)$$



Passband

Signal Constellation for BPSK





- *Constellation* or *Alphabet*: The set of values each symbol can take

BPSK (also called 2-PAM)

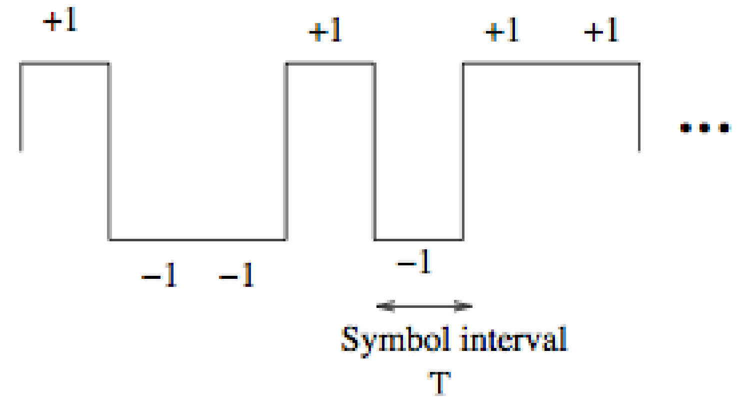
- For BPSK, it does not matter which bits maps to which phase.

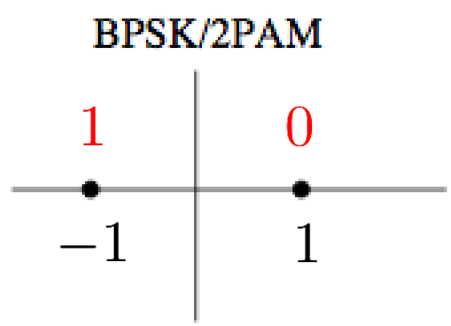
$\{b[n]\}$
0 1 1 0 1 0 0 $\longrightarrow$

$0 \to 1$
$1 \to -1$

$\longrightarrow$

Baseband

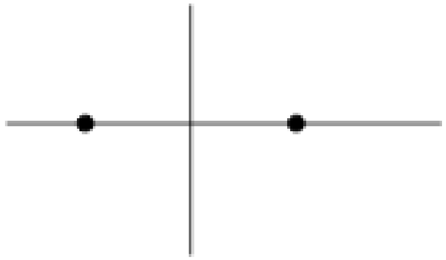




Can we load more bits per symbol?

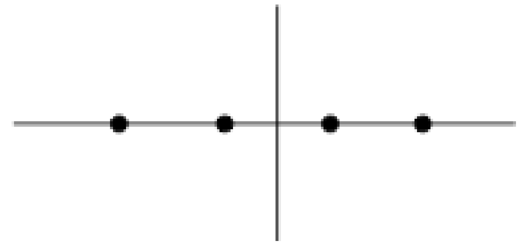
1 bit per symbol of T secs

BPSK/2PAM



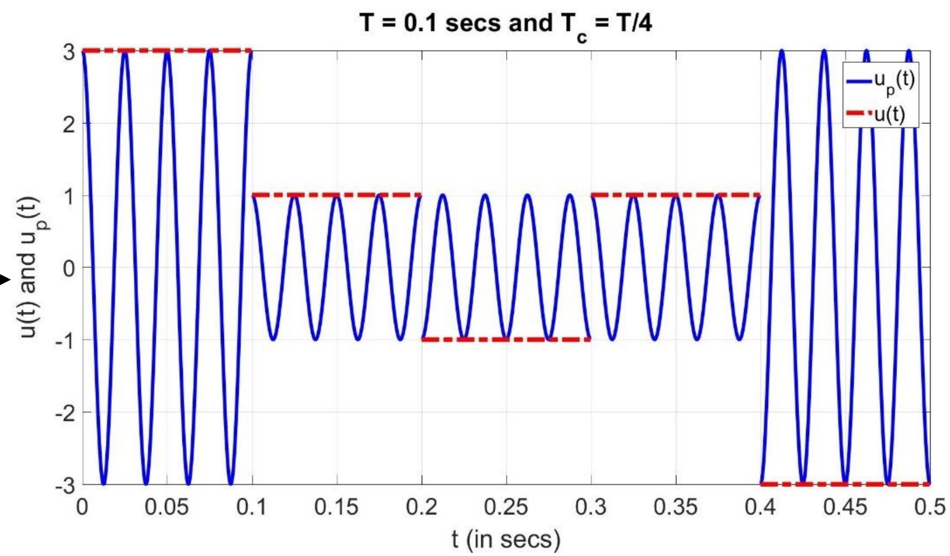
2 bits per symbol

4PAM



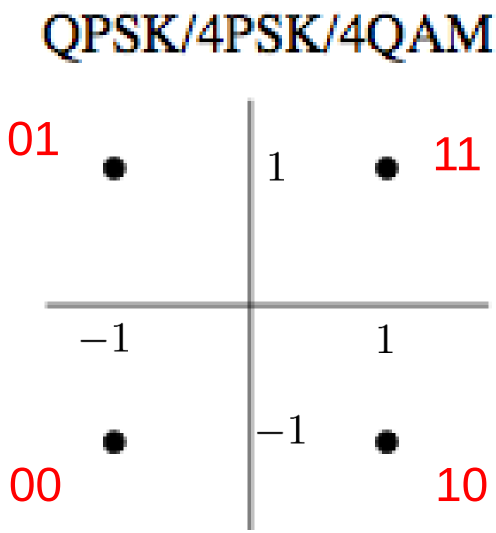
1 1 1 0 0 1 1 0 0 0

$$\begin{aligned} 00 &\rightarrow -3 \\ 01 &\rightarrow -1 \\ 10 &\rightarrow +1 \\ 11 &\rightarrow +3 \end{aligned}$$

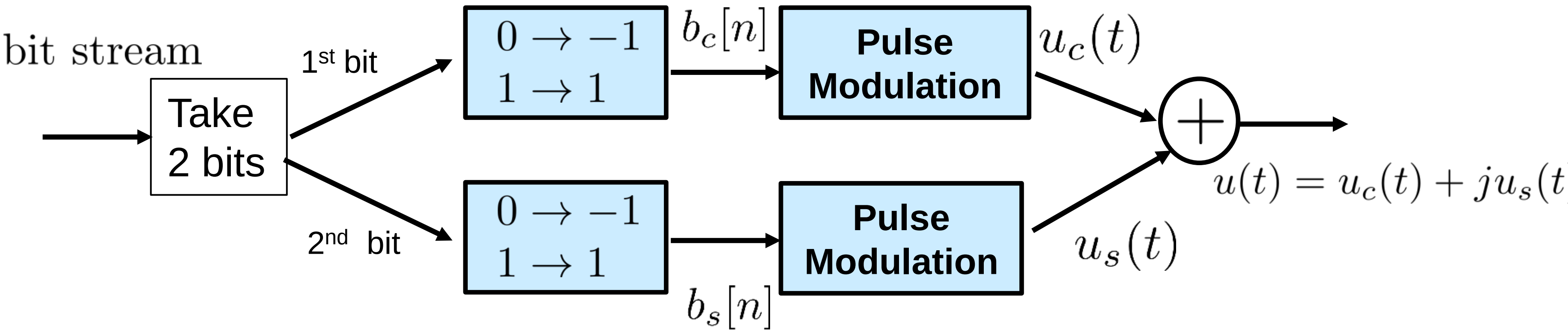


Another way for 2 bits/symbol: QPSK

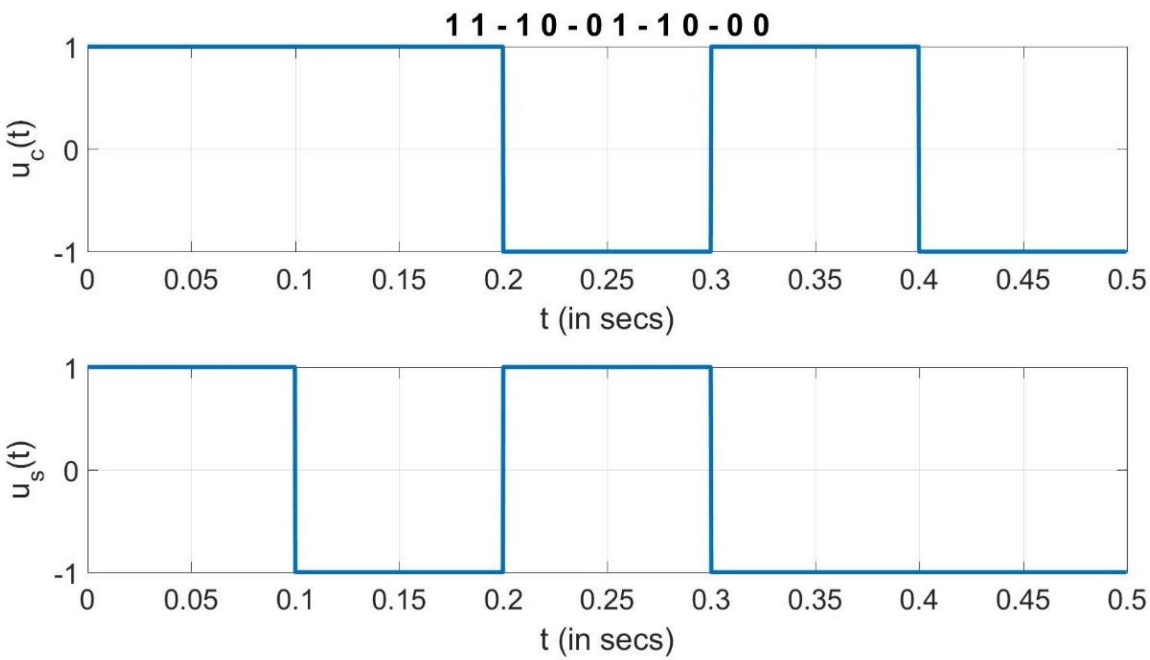
- Load Q component also giving rise to QAM and PSK modulation schemes



QPSK baseband



$$b[n] = b_c[n] + jb_s[n]$$



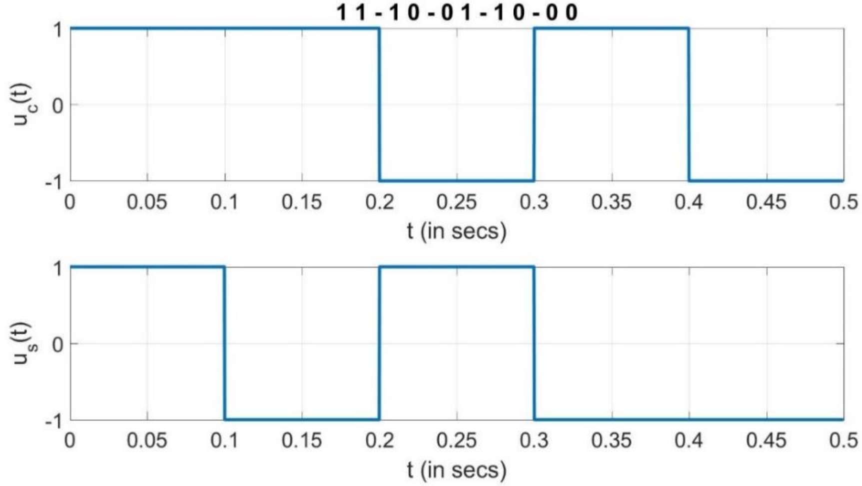
$$u_c(t) = \sum_n b_c[n]p(t - nT)$$

$$u_s(t) = \sum_n b_s[n]p(t - nT)$$

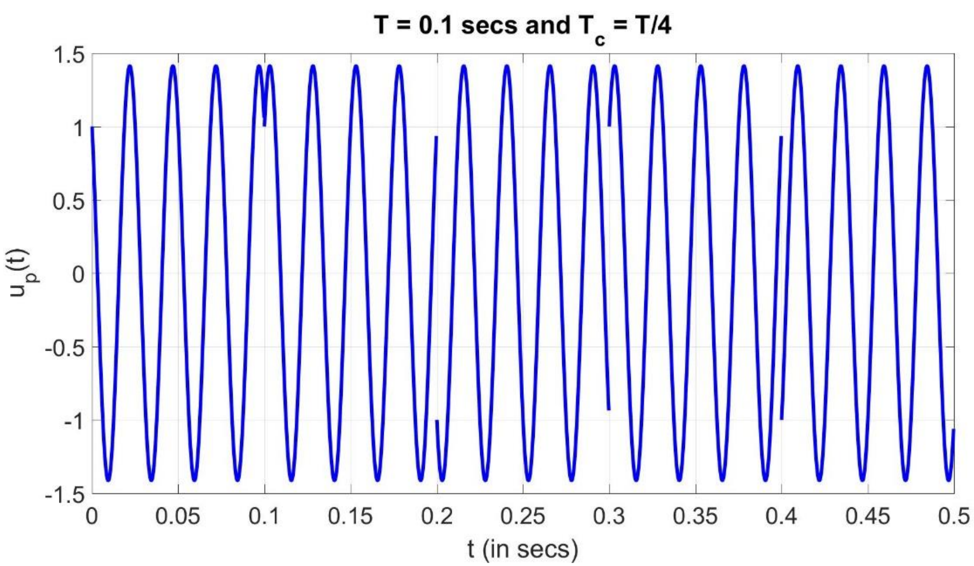
- For single physical baseband channel, QPSK is not possible; simply set $b_s[n] = 0$

QPSK: Passband

1 1 1 0 0 1 1 0 0 0



$$u_p(t) = \Re\{u(t)e^{j2\pi f_c t}\} = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)$$



QPSK: Passband

$$u_p(t) = \Re\{u(t)\cos(2\pi f_c t)\} = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)$$

$$u(t) = u_c(t) + ju_s(t) = \pm 1 \pm j1 \quad nT < t < (n+1)T$$

- When $b_c[n] = 1$ and $b_s[n] = 1$:

$$u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t) = \sqrt{2}\cos(2\pi f_c t + \pi/4)$$

- When $b_c[n] = -1$ and $b_s[n] = 1$:

$$u_p(t) = -u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t) = \sqrt{2}\cos(2\pi f_c t + 3\pi/4)$$

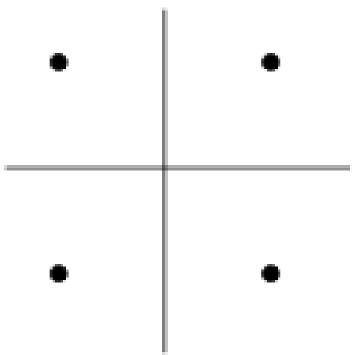
- When $b_c[n] = -1$ and $b_s[n] = -1$:

$$u_p(t) = -u_c(t)\cos(2\pi f_c t) + u_s(t)\sin(2\pi f_c t) = \sqrt{2}\cos(2\pi f_c t - 3\pi/4)$$

- When $b_c[n] = 1$ and $b_s[n] = -1$:

$$u_p(t) = u_c(t)\cos(2\pi f_c t) + u_s(t)\sin(2\pi f_c t) = \sqrt{2}\cos(2\pi f_c t - \pi/4)$$

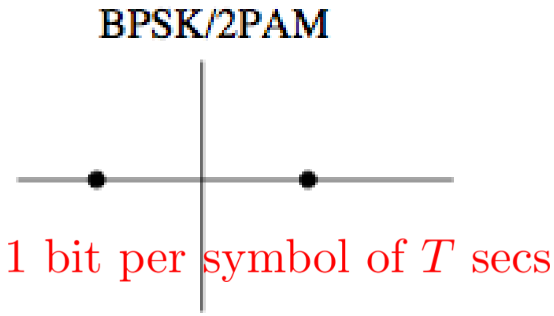
QPSK/4PSK/4QAM

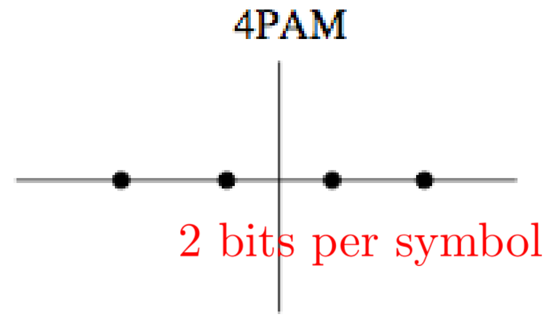


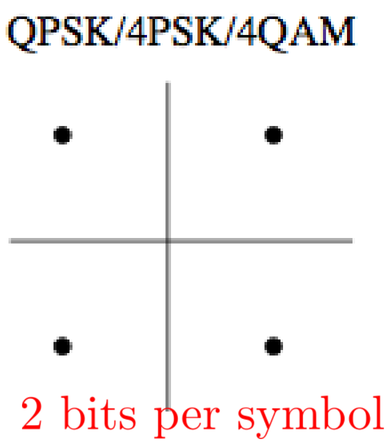
$$b[n] = \sqrt{2}e^{j\theta[n]}$$
$$\theta[n] \in \{\pm\pi/4, \pm 3\pi/4\}$$

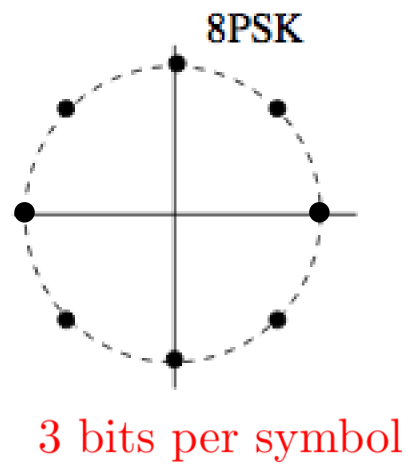
Can we load even more bits?

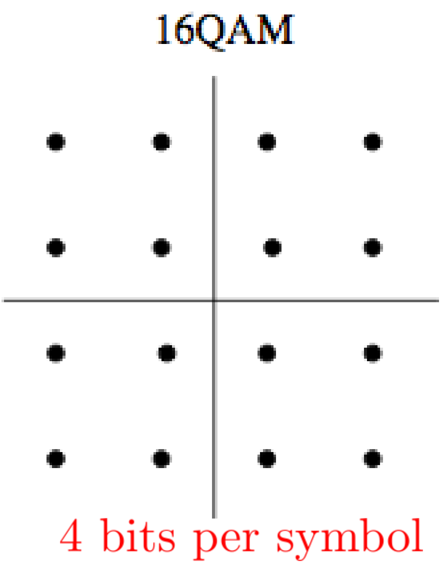
Nomenclature: *M-scheme*









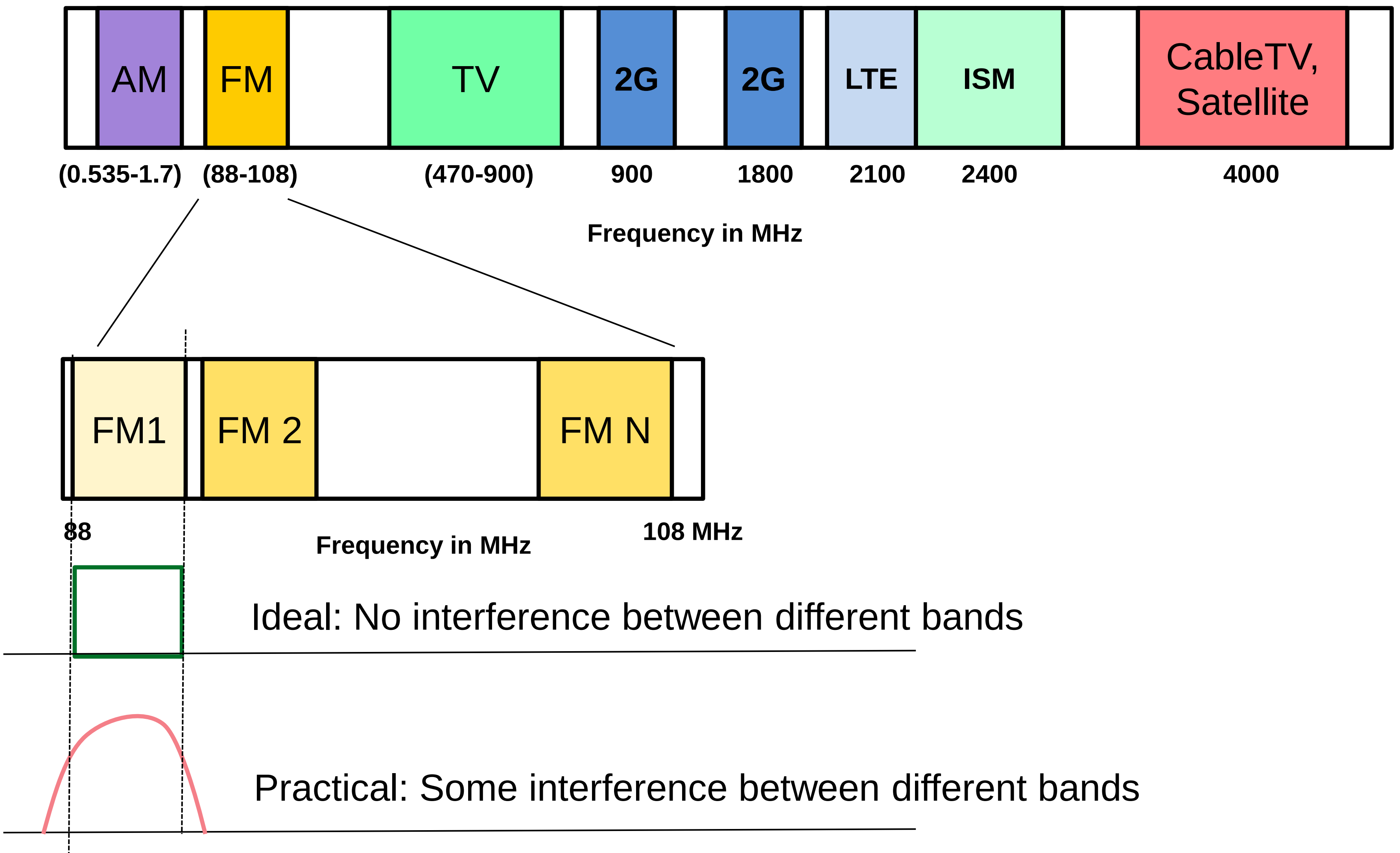


COMMON CONSTELLATIONS

- In general, M-ary modulation scheme can transmit $\log_2 M$ bits per symbol.
- Information rate $= \frac{\log_2 M}{T}$ bits/sec.

Bandwidth Occupancy

Motivation



AM
FM
TV
2G
2G
LTE
ISM
CableTV, Satellite
(0.535-1.7)
(88-108)
(470-900)
900
1800
2100
2400
4000
Frequency in MHz
FM1
FM 2
FM N
88
Frequency in MHz
108 MHz
Ideal: No interference between different bands
Practical: Some interference between different bands

Modeling bandwidth occupancy

- Consider the complex envelope of a linearly modulated signal

$$u(t) = \sum_n b[n]p(t - nT)$$

where $\{b[n]\}$ is sequence of symbols and $p(t)$ is modulating pulse for T seconds.

- $\{b[n]\}$ is modeled as random at the transmitter as well as receiver.

- However for characterizing the bandwidth occupancy of digitally modulated signal u, we define the quantities of interest in terms of average across time.

- We treat $u(t)$ as a finite power signal that can be modeled as a deterministic sequences once $\{b[n]\}$ is fixed.

- Bandwidth is then defined in terms of power spectral density.

Power Spectral Density (PSD)

- Power spectral density $S_x(f)$ for signal $x(t)$ specifies how the power in a signal is distributed in different frequency bands.
- PSD is defined as power per unit frequency.
- Units of watts/hertz or joules (since power/frequency = energy)
- Total power in $x(t)$ is given by

$$P_x = \int_{-\infty}^{\infty} S_x(f)\ df = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{T_0}^{T_0} |x(t)|^2\ dt$$

- PSD is generally used for random or periodic signals.
  - PSD for random process: We are interested in modeling the complete random process instead of a particular realization of it in a given time.

Periodogram-based PSD estimation

- Limit the signal $x(t)$ to a finite observation interval
$$x_{T_0} = x(t) I_{[-T_0/2, T_0/2]}(t)$$
where T_0 is the length of the observation interval. Since T_0 is finite and $x(t)$ has finite power, $x_{T_0}(t)$ has finite energy. So its Fourier transform is given by
$$X_{T_0} = \mathcal{F}(x_{T_0}(t))$$
- The energy spectral density of x_{T_0} is given by $|X_{T_0}(f)|^2$.
- Therefore the estimated PSD is given by
$$\hat{S}_x(f) = \frac{|X_{T_0}(f)|^2}{T_0}$$
- Formally, the PSD is in the limit of large time windows as follows
$$\hat{S}_x(f) = \lim_{T_0 \to \infty} \frac{|X_{T_0}(f)|^2}{T_0}$$

PSD of Linearly Modulated Signal

- Theorem 4.2.1: Consider a linearly modulated signal where the symbol sequence $\{b[n]\}$ is zero mean and uncorrelated with average symbol energy

$$\sigma_b^2 = \overline{|b[n]|^2} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |b[n]|^2,$$

then the PSD is given by

$$S_u(f) = \frac{|P(f)|^2}{T} \sigma_b^2$$

and the power of the modulated signal is

$$P_u = \frac{\sigma_b^2 ||p||^2}{T}$$

where $||p||^2$ denotes the energy of the modulating pulse. Proof.

- Assumptions:
  - The symbols have zero DC value: $\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} b[n] = 0$.
  - The symbols are uncorrelated: $\lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N} b[n] b^*[n-k] = 0 \quad \text{for } k \neq 0$.