Lecture 7

EM Waves &Radio Frequency-Based Sensors

- Doubts?

- How does the polarization vector affect capacitance in the dielectric medium?
- Frequency of System and Resonance Frequency.

How polarization vector effect capacitance in dielectric medium

1. Polarization Vector ($\vec{P}$):

- The polarization vector represents the electric dipole moment per unit volume of a dielectric material:

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

  where:

  - χ_e = electric susceptibility of the material
  - ϵ_0 = permittivity of free space
  - $\vec{E}$ = electric field

- Polarization occurs when the dielectric material in a capacitor becomes polarized in response to the applied electric field.
- The positive and negative charges within the dielectric material align slightly, creating **induced dipoles**.

2. Polarization and Electric Displacement Field ($\vec{D}$):

The electric displacement field ($\vec{D}$) is given by:

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

or

$$\vec{D} = \epsilon \vec{E}$$

where:

- $\epsilon = \epsilon_0 (1 + \chi_e) = \epsilon_0 \epsilon_r$
- ϵ_r = relative permittivity (dielectric constant)

3. Capacitance with Dielectric:

The capacitance of a capacitor without a dielectric is:

$$C_0 = \frac{\epsilon_0 A}{d}$$

When a dielectric is introduced, the capacitance becomes:

$$C = \frac{\epsilon A}{d} = \frac{\epsilon_0 \epsilon_r A}{d}$$

Thus:

$$C = \epsilon_r \times C_0$$

This shows that **introducing a dielectric increases the capacitance by a factor of the dielectric constant (ϵ_r)**.

4. How Polarization Affects Capacitance:

- The polarization vector ($\vec{P}$) increases the **effective charge storage capacity** of the capacitor by reducing the effective electric field between the plates.
- This happens because the induced electric field from the polarization opposes the external field, leading to a **reduced net field** inside the dielectric.
- As a result, **more charge** can be stored for the same applied voltage, effectively increasing the capacitance.
- The higher the polarization ($\vec{P}$), the greater the relative permittivity (ϵ_r), and therefore, the **higher the capacitance**.

What is a Bound Charge?

A **bound charge** is a charge that is associated with the **molecules or atoms** of a dielectric material and cannot move freely. These charges arise due to the **polarization** of the dielectric when an external electric field is applied.

There are two types of bound charges:

1. **Volume Bound Charge Density** (ρ_b):

$$\rho_b = -\nabla \cdot \mathbf{P}$$

   It represents the charge density distributed throughout the volume of a polarized material.

2. **Surface Bound Charge Density** (σ_b):

$$\sigma_b = \mathbf{P} \cdot \mathbf{n}$$

   It represents the bound charge accumulated at the surface of the dielectric.

EM wave frequency and resonance frequency:

Aspect	Frequency of EM Wave	Resonance Frequency
Definition	The number of oscillations of electric and magnetic fields per second.	The natural frequency at which a system oscillates with maximum amplitude.
Origin	Determined by the **source of the wave** (e.g., antenna, light source).	Determined by the **system's properties** (e.g., LC components, mass).
Nature	**Intrinsic** to the wave itself.	**Inherent** to the system; changes with system characteristics.
Change with Medium	Remains **constant** when changing medium.	Can **change** if the system's properties change (like inductance).
Context	Applies to **light, radio waves, microwaves, etc.**	Applies to **oscillating systems** (like circuits or vibrating objects).
Effect on Amplitude	Amplitude does not inherently increase or decrease.	Amplitude **maximizes** when excited at resonance frequency.
Application	Communication, optics, microwave heating.	Radio tuning, electrical circuits, mechanical resonance.

Key Points-

- The **EM wave frequency** generated by the radio station is **fixed** and determined by the station.
- The **resonance frequency** of the radio circuit must be **adjusted** (by changing the capacitance) to **match** the EM wave frequency.
- When matched, **resonance occurs**, resulting in **maximum signal reception**.

How the Frequencies Interact:

- When the **EM wave frequency (100 MHz)** matches the **resonance frequency (100 MHz)** of the LC circuit, the circuit **resonates**.
- This resonance maximizes the **current and voltage** in the radio circuit, allowing for **efficient signal reception**.
- If the radio is tuned to a different frequency (e.g., **101 MHz**), the circuit does not resonate, and the signal is weak or absent.

What Changes: Wavelength and Speed

The **speed (v)** and **wavelength** (λ) of the wave change according to the medium's properties:

Speed of EM Wave in a Medium:

$$v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

Where:

- c = Speed of light in a vacuum ($3 \times 10^8 \ \mathrm{m/s}$)
- ϵ_r = Relative permittivity of the medium
- μ_r = Relative permeability of the medium

Relationship between Speed, Wavelength, and Frequency:

$$v = f\lambda$$

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Since **frequency (f) is constant**, the change in **speed (v)** directly affects the **wavelength** (λ):

$$\lambda = \frac{v}{f}$$

Change in Wavelength:

When an EM wave moves from one medium to another:

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} = \frac{n_1}{n_2}$$

Where n is the **refractive index**:

$$n = \frac{c}{v}$$