# CT Assignment-4



Bhargav

2023112014

1)

a) we have to choose 'a' and 'b' such that p(t) is Nyquist with 50% excess BW of data rate 40Mbps usin 16 QAM.

$$\boxed{R_b = 40\,Mbps}$$

$$R_s = \frac{R_b}{\log_2 M}$$

As we known

$$\text{sinc}(\omega t) \xleftrightarrow{FT} \frac{1}{\omega} I\left[-\frac{\omega}{2}, \frac{\omega}{2}\right](f)$$

Given $R_b = 40\,mbps$, $M = 16$ and excess Bandwidth $(\beta) = 0.5$

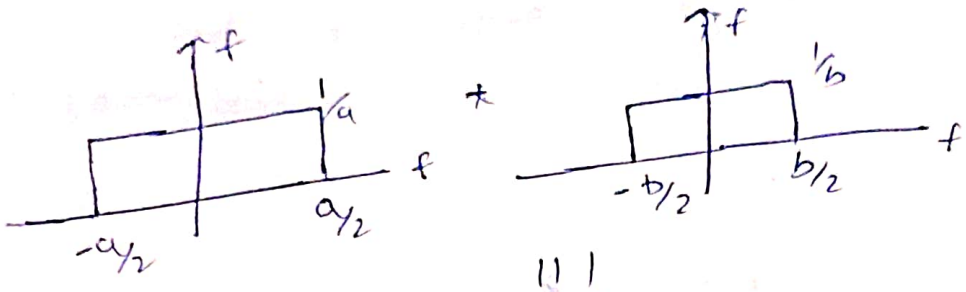
$$\underbrace{R_s}_{\text{Symbol rate}} = \frac{\overbrace{R_b}^{\text{Bit rate}}}{\log_2 M}$$

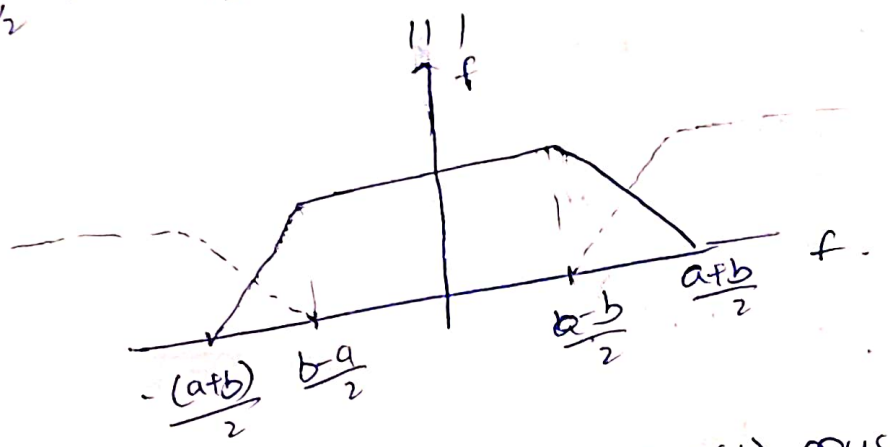
$$= \frac{40M}{\log_2 16} = \frac{\overset{10}{\cancel{40}}M}{\cancel{4}}$$

$R_s = 10M$ symbols/sec

$$\boxed{R_s = 10MHz}$$

$$P(f) = \frac{1}{a} I\left[-\frac{a}{2}, \frac{a}{2}\right] * \frac{1}{b} I\left[-\frac{b}{2}, \frac{b}{2}\right]$$




To get $P(t)$ to be nyquist $P(t)$ must be zero

either at $\sin c(aT)=0$ (or) $\sin c(bT)=0$

let $\frac{a}{2}=\frac{1}{2T}$

$$\boxed{a=\frac{1}{T}}$$

$a=\frac{1}{T}=R_s=10MHz$

$$\boxed{a=10MHz}$$

$B=(1+\beta)B_{min}$

$2B_{min}=2R_s$

$$\boxed{2R_s\leq 2B}$$

$B_{min}=\frac{R_s}{2}$

$=\frac{10M}{2}$

$$\boxed{B_{min}=5MHz}$$

3) Given a 16QAM

$\boxed{M = 16}$

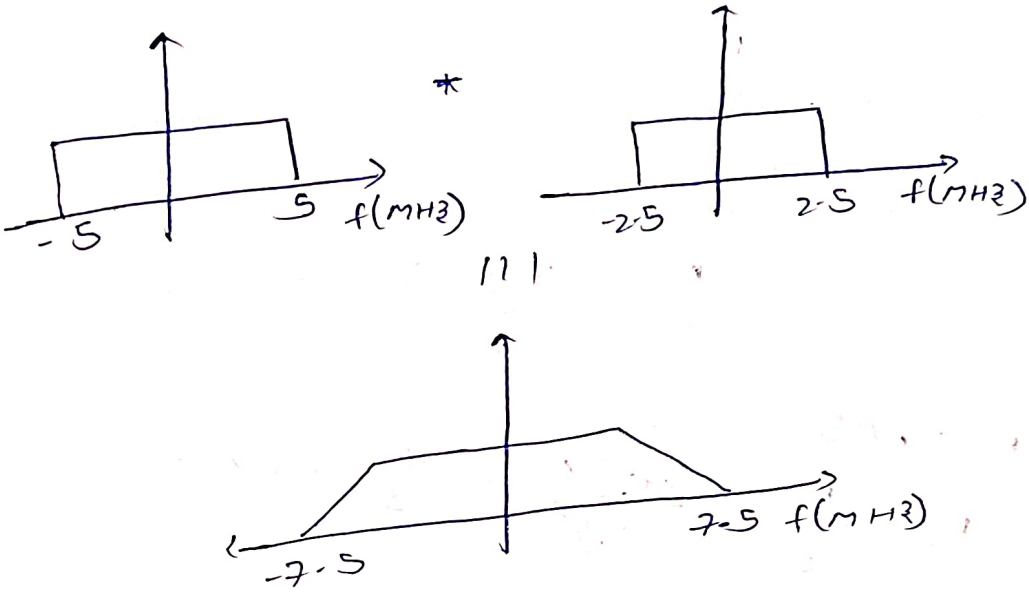
$R_b = 50\ Mbps$ $\quad \beta = 50\% = 0.5$

Total BW => $B = (1+\beta)B_{min}$

$B = (1+0.5)5$

$= (1.5)5$

$\boxed{B = 7.5\ MHz}$




as we known $a = 10\ MHz$.

$\frac{a+b}{2} = (1+\beta)\left(\frac{1}{2T}\right)$

$= (1+0.5)(5)$

$\frac{a+b}{2} = 7.5$

$a+b = 15$

$\boxed{b = 5\ MHz}$

so B.W occupied $= \frac{a+b}{2} = 7.5\ MHz$.

#3

b) we have to choose 'a' and 'b' such that P(t) can be used for Nyquist sampling both for a 16QAM system with a 40Mbps data and for an 8PSK system with an 18Mbps data rate

16 QAM $\Rightarrow M=16$

$$R_S = \frac{R_b}{\log_2 M}$$

$$= \frac{40M}{\log_2 16} = \frac{40M}{4} = 10MHz = \frac{1}{T_1}$$

8PSK $\Rightarrow M=8$

$$R_S = \frac{R_b}{\log_2 M} = \frac{18M}{\log_2 8} = \frac{18M}{3} = 6MHz = \frac{1}{T_2}$$

$\text{sinc}(at)\,\text{sinc}(bt) = 0$

$\text{sinc}(at) = 0$ (or) $\text{sinc}(bt) = 0$

$a = \frac{1}{T_1} \Rightarrow a = 10MHz$

$b = \frac{1}{T_2} \Rightarrow b = 6MHz$

$R_{S_1} = \frac{1}{T_1}$

$B_{min_1} = \frac{R_{S_1}}{2} = \frac{10M}{2} = 5MHz$

$R_{S_2} = \frac{1}{T_2} \Rightarrow B_{min_2} = \frac{1}{2T_2} = \frac{6M}{2} = 3MHz$

$B = B_{min_1}(1+\alpha)$

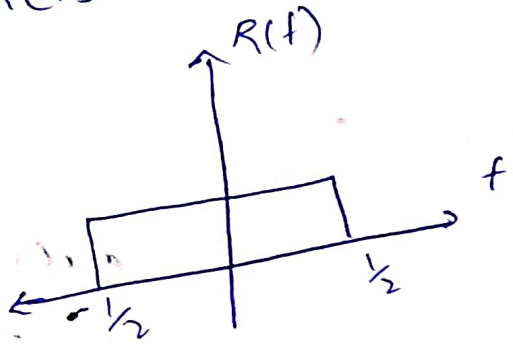
BW occupied $= \frac{10+6}{2} = 8MHz$

$8 = 5(1+\alpha) \Rightarrow \alpha = \frac{8}{5} - 1 = \frac{3}{5} = 0.6$ $\boxed{\alpha = 60\%}$

2) Given $R(f) = I_{[-\frac{1}{2},\frac{1}{2}]}(f)$

$$c(f) = \frac{\pi}{2a}\cos\left(\frac{\pi f}{a}\right) I_{[-\frac{a}{2},\frac{a}{2}]}(f)$$

a) plot for $R(f)$




plot for $c(f)$

at $f = -\frac{a}{2}$

$$\frac{\pi}{2a}\cos\left(\frac{\pi}{a}\left(-\frac{a}{2}\right)\right) \Rightarrow \frac{\pi}{2a}\cos\left(\frac{\pi}{2}\right) = 0$$

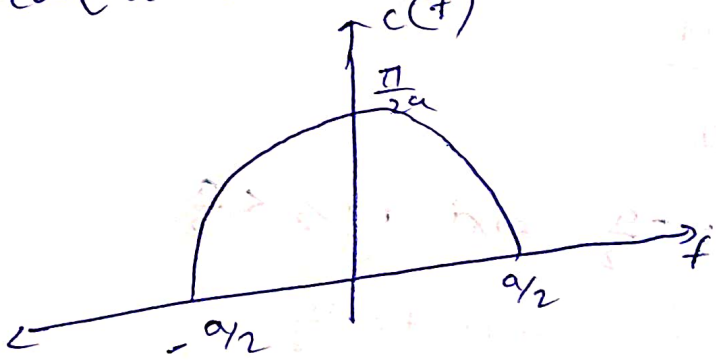
$$\frac{\pi}{2a}\cos\left(-\frac{\pi}{2}\right)$$

at $f = \frac{a}{2}$

$$\frac{\pi}{2a}\cos\left(\frac{\pi}{a}\frac{a}{2}\right) = 0$$

at $f = 0$

$$\frac{\pi}{2a}\cos\left(\frac{\pi}{a}(0)\right) = \frac{\pi}{2a}$$

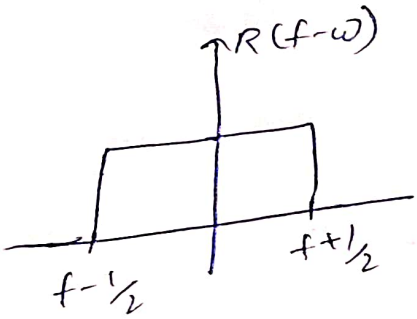


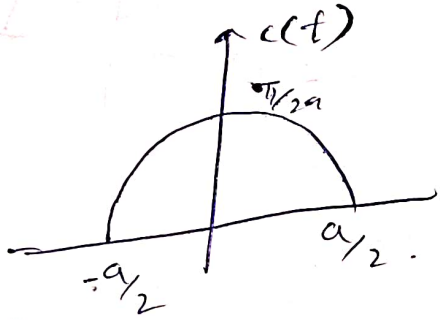
b) we have to show that the frequency domain raised cosine pulse can be written as

$$S(f) = R * C(f)$$

Proof:

$$S(f) = \int_{\omega=-\infty}^{\infty} R(f-\omega)\, C(\omega)\, d\omega$$




given $0<a<1$

for $f+\frac{1}{2} \leq \frac{-a}{2}$

$f < \frac{-(a+1)}{2}$

$$S(f) = \int_{-\infty}^{\infty} (0)(0)\, d\omega = 0$$

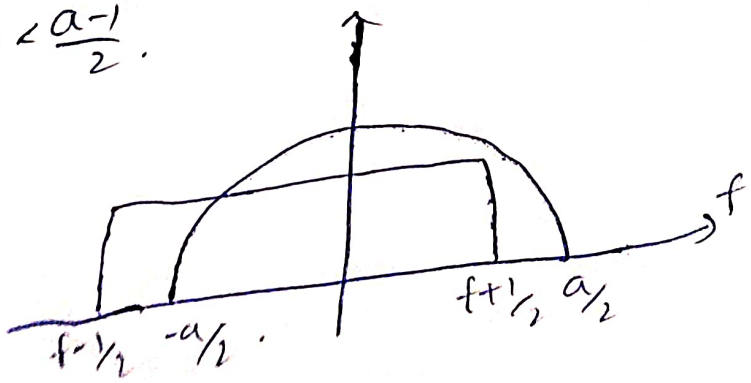
$\boxed{S(f) = 0}$ $\therefore$ As there is no common area.

Case-II:

$f+\frac{1}{2} \geq \frac{-a}{2}$ and $f+\frac{1}{2} < \frac{a}{2}$.

$f > \frac{-a-1}{2}$ and $f < \frac{a-1}{2}$.

$\frac{-a-1}{2} < f < \frac{a-1}{2}$.



$$s(f) = \int_{-a/2}^{f+1/2} (1) \frac{\pi}{2a} \cos\left(\frac{\pi\omega}{a}\right) d\omega$$

$$= \frac{\pi}{2a} \left[ \frac{\sin\left(\frac{\pi\omega}{a}\right)}{\pi/a} \right]_{-a/2}^{f+1/2}$$

$$= \frac{1}{2} \left[ \sin\left(\frac{\pi(f+1/2)}{a}\right) - \sin\left(\frac{\pi}{a}\left(\frac{-a}{2}\right)\right) \right]$$

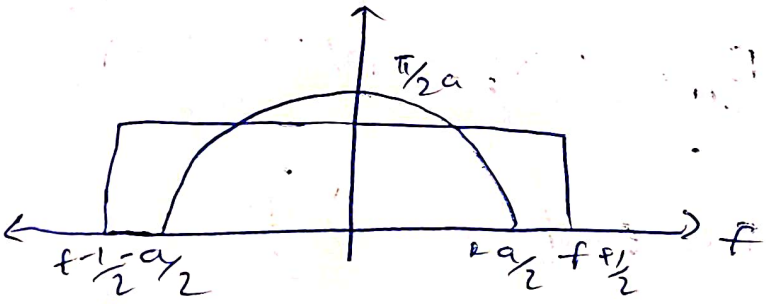
$$\boxed{s(f) = \frac{1}{2} \left[ \sin\left(\frac{\pi(f+1/2)}{a}\right) + 1 \right]}$$

case- III

$f - 1/2 < \frac{-a}{2}$ and $f + 1/2 > \frac{a}{2}$.

$f < \frac{-a+1}{2}$ and $f + \frac{1}{2} > \frac{a-1}{2}$

$$\frac{a-1}{2} < f < \frac{-a+1}{2}.$$




$$s(f) = \int_{-a/2}^{a/2} (1) \frac{\pi}{2a} \cos\left(\frac{\pi\omega}{a}\right) d\omega$$

$$= \frac{\pi}{2a} \left[ \frac{\sin\left(\frac{\pi\omega}{a}\right)}{\frac{\pi}{a}} \right]_{-a/2}^{a/2}.$$

$$= \frac{\pi}{2a} \times \frac{a}{\pi} \left[ \sin\left(\frac{\pi a}{2a}\right) - \sin\left(\frac{-\pi a}{2a}\right) \right]$$

$$= \frac{1}{2} [1+1] = 1$$

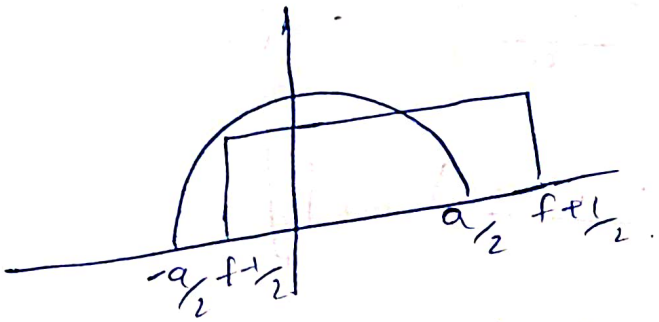
$$\boxed{s(f) = 1}$$

Case - IV

$f-\frac{1}{2} > \frac{-a}{2}$ and $f+\frac{1}{2} > \frac{a}{2}$ and $f-\frac{1}{2} < \frac{a}{2}$

$f > \frac{-a+1}{2}$ and $f > \frac{a-1}{2}$ $\qquad f < \frac{a+1}{2}$

$\frac{-a+1}{2} < f < \frac{+a+1}{2}$




$$S(f) = \int_{f-1/2}^{a/2} (1)\ \frac{\pi}{2a}\cos\left(\frac{\pi\omega}{a}\right) d\omega$$

$$= \frac{\pi}{2a}\left[\frac{\sin\left(\frac{\pi\omega}{a}\right)}{\frac{\pi}{a}}\right]_{\omega=f-1/2}^{a/2}$$

$$= \frac{1}{2}\left[\sin\left(\frac{\pi}{a}\left(\frac{a}{2}\right)\right) - \sin\left(\frac{\pi(f-1/2)}{a}\right)\right]$$

$$= \frac{1}{2}\left[1 - \sin\left(\frac{\pi}{a}(f-1/2)\right)\right]$$

To see that is indeed a raised cosine shape set $\hat{f}$ take $\sin\theta \approx \theta$.

$$= \cancel{\frac{1}{2}}\left[\cancel{1} - \frac{\pi}{a}f + \cancel{\frac{1}{2}}\right]$$

$$\tilde{f} = \frac{\pi}{a}\left(f - \frac{(1-a)}{2}\right)$$

so

$$\frac{\pi}{a}\left(f - \frac{1}{2}\right) = \tilde{f} - \frac{\pi}{2}$$

$$S(\tilde{f}) = \frac{1}{2}\left[1 - \sin\left(\tilde{f} - \frac{\pi}{2}\right)\right]$$

$$s(\tilde{f}) = \frac{1}{2}[1+\cos\tilde{f}]$$

Case-V

$f > \frac{1+a}{2} \leftarrow f - \frac{1}{2} > \frac{a}{2}$

There is no common a or a so

$$\boxed{s(f) = 0}$$

$$s(f) = \begin{cases} 0 & f < -\frac{(a+1)}{2} \\ \frac{1}{2}\left[1+\sin\left(\frac{\pi(f+1/2)}{a}\right)\right] & -\frac{a-1}{2} < f < \frac{a-1}{2} \\ 1 & \frac{a-1}{2} < f < -\frac{a+1}{2} \\ \frac{1}{2}\left[1-\sin\left(\frac{\pi}{a}(f-1/2)\right)\right] & -\frac{a+1}{2} < f < \frac{a+1}{2} \\ 0 & f > \frac{a+1}{2} \end{cases}$$

c) we have to find the time domain pulse $s(t) = r(t)\,c(t)$ and identify wether the zeros of $s(t)$ are and conclude that $s(t/T)$ is a Nyquist pulse with duration T.

$$s(t) = r(t)\,c(t)$$

$$R(f) = I_{[-\frac{1}{2}, \frac{1}{2}]}(f) \longleftrightarrow r(t) = \operatorname{sinc}(t)$$

$$C(f) = \frac{\pi}{2a}\cos\left(\frac{\pi f}{a}\right) I_{[-\frac{a}{2}, \frac{a}{2}]}(f)$$

$$c(t) = \int_{-\infty}^{\infty} C(f)\, e^{j2\pi f t}\, df$$

$$= \frac{\pi}{2a}\int_{-a/2}^{a/2} \frac{1}{2}\left[e^{j\pi f/a} + e^{-j\pi f/a}\right] e^{j2\pi f t}\, df$$

$$= \left(\frac{\pi}{2a}\right)\left(\frac{1}{2}\right)\int_{-a/2}^{a/2} e^{j\pi f/a[1+2at]} + e^{-j\frac{\pi f}{a}[1-2at]}\, df.$$

$$= \frac{\pi}{4a}\left[\frac{e^{j\frac{\pi f}{a}[1+4\pi^2 t]}}{j\frac{\pi}{a}[1+4\pi^2 t]} + \frac{e^{-j\frac{\pi f}{a}[1-4\pi^2 t]}}{-j\frac{\pi}{a}[1-4\pi^2 t]}\right]_{-a/2}^{a/2}$$

$$= \frac{\pi}{4a}\left[\frac{e^{j(\frac{\pi}{a}+2\pi t)f}}{j(\frac{\pi}{a}+2\pi t)} + \frac{e^{j(-\frac{\pi}{a}+2\pi t)f}}{j(-\frac{\pi}{a}+2\pi t)}\right]_{-a/2}^{a/2}$$

$$= \frac{\pi}{4a}\left[\frac{e^{j(\frac{\pi}{2}+\pi a t)} - e^{-j(\frac{\pi}{2}+\pi a t)}}{j(\frac{\pi}{a}+2\pi t)} + \frac{e^{j(-\frac{\pi}{2}+\pi a t)} - e^{j(\frac{\pi}{2}-\pi a t)}}{j(-\frac{\pi}{a}+2\pi t)}\right]$$

$$= \frac{\pi}{4a}\left\{\frac{2j\sin(\pi a t+\frac{\pi}{2})}{j(\frac{\pi}{a}+2\pi t)} - \frac{2j\sin(\frac{\pi}{2}-\pi a t)}{j(-\frac{\pi}{a}+2\pi t)}\right\}$$

$$= \frac{\pi}{2a}\left[2\pi t\left[\cancel{\sin(\frac{\pi}{2}+\pi a t)}\ \cos(\pi a t)\right.\right.$$

$$= \frac{\pi}{2a}\left[\frac{\cos(\pi a t)[-\frac{\pi}{a}+2\pi t] - \cos(\pi a t)[\frac{\pi}{a}+2\pi t]}{4\pi^2 t^2 - \frac{\pi^2}{a^2}}\right]$$

$$= \frac{-\pi}{2a}\left[\frac{\cos(\pi a t)(\frac{2\pi}{a})}{\pi^2[4t^2-\frac{1}{a^2}]}\right] \qquad \frac{\pi}{2a}\left[\frac{(-\frac{2\pi}{a})\cos(\pi a t)}{4\pi^2 t^2-\frac{\pi^2}{a^2}}\right]$$

$$= -\frac{1}{a^2}\left[\frac{\cos(\pi a t)}{[4t^2-\frac{1}{a^2}]}\right]$$

$$\boxed{c(t) = \frac{\cos(\pi a t)}{1-4a^2t^2}}$$

Thus time domain-pulse is given by

$$\boxed{s(t) = \text{sinc}(t)\,\frac{\cos(\pi a t)}{1-4a^2t^2}}$$

$s(t) = 0$ for all $t = \pm 1, \pm 2 \ldots$ (all non-zero integers)

That means $s(t/T)$ is a nyquist pulse i.e. zero at all multiples of $T$ (except $T=0$) so when sampled at multiples of $T$ the pulse doesnt interetere with neighbouring symbols.

d) we have to sketch an argument that shows that if the pulse $s(t/T)$ is used for BPSK signalling at rate $1/T$, then magnitude of transmitted waveform is always finite

$$s(t/T) = \frac{\text{sinc}(t/T)\cos\left(\frac{\pi a}{T}\right)}{1-4a^2\left(\frac{t}{T}\right)^2}$$

let $g(t) = s(t/T)$

$$x(t) = \sum b[n]\,g(t-nT)$$

$$= \sum b[n]\,s((t-nT)/T)$$

$$= \sum_n b[n]\,s\left(\frac{t}{T}-n\right)$$

$s\left(\frac{t}{T}-n\right)$ decays as $1/n^3$ //

$\sum_n \frac{1}{n^3}$ converges to '0' finite value for any $t$.

The modified cosine pulse $s(t)$ decays as $1/t^3$

$g(t) = s(t^{1/4})$ considered a linearly modulated

wave form

$x(t) = \sum_n a[n] s(t^{1/4} - n)$

finally $s(t^{1/4} - n)$ decays roughly as $1/n^3$ the

convergence to $\sum_n \frac{1}{n^3}$ implies both the sum

and the sum converges to a finite value for

$a[n]$

The raised cosine pulse $s(t)$ decays at $1/t^3$

$g(t) = s(t/T)$ consider a linearly modulated wave form.

$$x(t) = \sum_n b[n]\, s(t/T - n)$$

Fixing $s(t/T - n)$ decays roughly as $1/n^3$ the convergence $\sum_n \frac{1}{n^3}$ implies that the sum in the eqn converges to a finite value for any $t$.

3) Given a 16QAM

$\boxed{M = 16}$

$R_b = 50\,Mbps$ $\quad B = 50\% = 0.5$

$$R_s = \frac{R_b}{\log_2 M}$$

$R_b$ → bit rate

$R_s$ → symbol rate

$$= \frac{50\,M}{\log_2 16}$$

$$= \frac{50}{4}$$

$$\boxed{R_s = 12.5\ M\text{symbols per sec}}$$

minimum Bandwidth $= \frac{1}{T}$

Total Bandwidth $= 12.5(1+B)$

$= 12.5(1+\frac{1}{2})$

$= 12.5(\frac{3}{2})$

$= 18.75\ MHz$.

for carrier freq of 5.2GHz.

passband signal takes $[f_c - \frac{B}{2}, f_c + \frac{B}{2}]$.

$B = 18.75\,MHz$.

range $\in [f_c - \frac{18.75M}{2}, f_c + \frac{18.75M}{2}]$

$\in [5.2GHz - 9.375\,MHz, 5.2\,GHz + 9.375\,MHz]$

$\in [5.190625, 5.209375]\,GHz$.

b) $R_{sa} = 12.5$ MHsymbols /sec.

$R_{bb} = \log_2 M \times R_{sa}$

as for QPSK $M=4$

$= \log_2 4 \times 12.5$

$= 2 \times 12.5$ MH

$\boxed{R_{bb} = 25 \text{ Mbps}}$

So the bitrate of Part (b) is 25Mbps which half of part (a)

c) For same Reliability

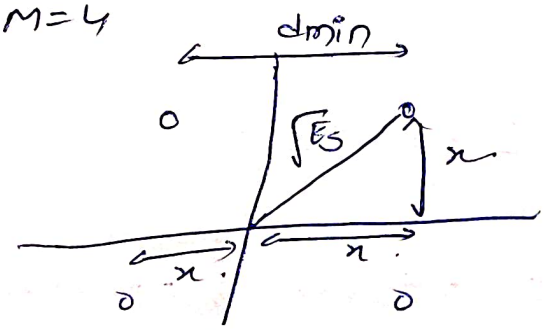
$$\left(\frac{\eta_p R_b}{N_0}\right)_{16QAM} = \left(\frac{\eta_p R_b}{N_0}\right)_{QPSK}.$$

$$\eta_p = \frac{d_{min}^2}{E_b}$$

$$E_b = \frac{E_s}{\log_2 M}.$$

$\eta_p$ for QPSK.

$M=4$




$\boxed{d_{min} = 2x}$

From pythagorous theorm

$$x^2 + x^2 = E_s$$

$$2x^2 = E_s$$

$$\eta_p = \frac{d_{min}^2}{E_b}$$

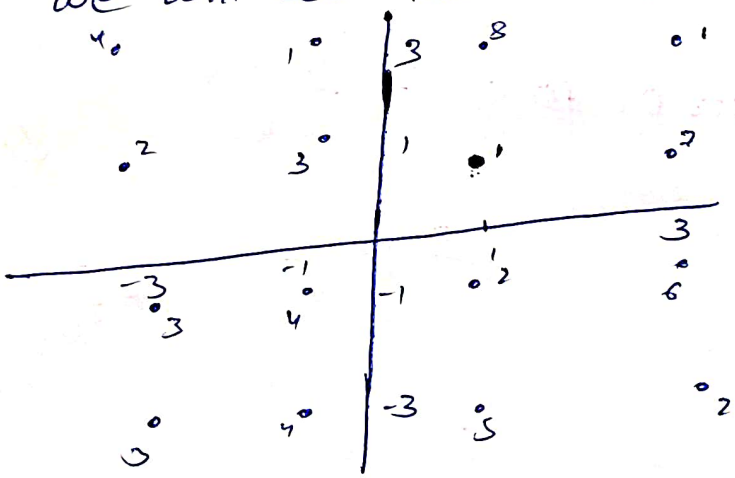
$$E_b = \frac{E_s}{\log_2 M}$$

$$= \frac{2x^2}{\log_2 4} = \frac{2x^2}{2} = x^2.$$

$$\eta_p = \frac{4x^2}{x^2} = 4$$

$$\boxed{(\eta_p)_{qpsk} = 4}$$

Now we will see for 16 QAM.



There are 8 $(|x|, |y|) = (1,3)$ or $(3,1)$

and 4 $(|x|, |y|) = (1,1)$

4 $(|x|, |y|) = (3,3)$

$$E_s = \frac{4}{16}[2] + \frac{4}{16}[3^2+3^2] + \frac{8}{16}[1^2+3^2]$$

$$= \frac{4}{16}[2+9+9] + \frac{8}{16}[9+1]$$

$$= \frac{4}{16}[20+20] = \frac{4(40)}{16} = \frac{160}{16} = 10$$

$$E_b = \frac{E_s}{\log_2 m}$$

as $M=16$ for 16-QAM

$$= \frac{10}{\log_2 16} = \frac{10}{4}$$

$$E_b = \frac{10}{4}$$

$$\boxed{d_{min} = 2}$$

$$(\eta_P)_{16\text{-}QAM} = \frac{4}{\frac{10}{4}}$$

$$= \frac{\cancel{16}\,8}{\cancel{10}\,5}$$

$$\boxed{(\eta_P)_{16\text{-}QAM} = \frac{8}{5}}$$

as reliab

$$\text{scale factor of power} = \frac{(\eta)_{16\text{-}QAM}}{(\eta)_{QPSK}} \times \frac{(R_b)_{QPSK}}{(R_b)_{16\text{-}QAM}}$$

$$= \left(\frac{\cancel{8}\,2}{5}\right) \times \frac{1}{\cancel{4}} \times \frac{\cancel{2}\,5}{\cancel{5}\,2}$$

$$= \left(\frac{1}{2}\right)\left(\frac{2}{5}\right)$$

$$\frac{\text{QPSK power}}{\text{16-QAM power}} = \frac{1}{5}$$

$$\text{QPSK power} = \frac{1}{5} \times (\text{16-QAM power})$$

$$= \frac{1}{5} \times 100\,mW$$

$$\boxed{\text{QPSK power} = 20\,mW}$$