GATE

Be sure with Vidyalankar Since 1960

Pearl Centre, S.B. Marg, Dadar (W), Mumbai – 400 028. Tel. 4232 4232

CS : Computer Science and Information Technology

Module 2 : Discrete Mathematics

INDEX

Chapter - 1 : Propositional Logic

1.1 Definition

A **statement** or **proposition** is a declarative sentence that is either true or false, but not both.

Solved Example 1 :

Which of the following declarative sentences are propositions?

1. Washington, D.C., is the capital of the United States of America.
2. Toronto is the capital of Canada.
3. $1 + 1 = 2$
4. $2 + 2 = 3$

Solution :

All the sentences are propositions.

Propositions 1 and 3 are true, whereas 2 and 4 are false.

Solved Example 2 :

Consider the following sentences.

1. What time is it?
2. Read this carefully.
3. $x + 1 = 2$
4. $x + y = z$

Solution :

Sentence 1 is a question and sentence 2 is a command so they are not propositions. Sentence 3 and 4 are not propositions even though they are declarative sentences the values of x, y and z are unknown these sentences are neither true nor false. Note each of sentences 3 and 4 can be turned into a proposition if we assign values to the variables x, y and z.

Small letters p, q, r, s… are used to denote **propositional variables**, that is, variables that represent propositions, just as letters x, y, z,… are used to denote numerical variables in mathematics.

<u>For Example</u> : **p:** The sun is shining today. **q:** It is cold.

The **truth value** of a proposition is **true**, denoted by **T**, if it is a true proposition, and the **truth value** of a proposition is **false**, denoted by **F**, if it is a false proposition.

1.2 Logical Operators

Propositional statements or variables can be combined using logical connectives to form new propositions called **compound propositions.** The truth value of a compound statement depends only on the truth value of the statements being combined and on the types of connectives being used.

Negation

Let p be a proposition. The **negation of p**, denoted by $\neg p$ (also denoted by ~p or $\overline{p}$), is the statement **"It is not the case that p."** The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$ is true if p is false, and $\neg p$ is false if p is true. Table 1 displays the truth table for $\neg p$.

TABLE 1 : The Truth Table for the **Negation** of a Proposition.

p	**¬p**
T	F
F	T

Solved Example 3 :

Find the negation of the proposition and express this in simple English.

"Vandana's smartphone has at least 32GB of memory"

Solution :

The negation is

"It is not the case that Vandana's smartphone has at least 32GB of memory."

This negation can also be expressed as

"Vandana's smartphone does not have at least 32GB of memory"

or even more simply as

"Vandana's smartphone has less than 32GB of memory."

Conjunction

Let p and q be propositions. The **conjunction of p and q**, denoted by $p \wedge q$, is the proposition **"p and q."** The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise. Table 2 displays the truth table of $p \wedge q$.

TABLE 2 : The Truth Table for the **Conjunction** of Two Propositions.		
p	**q**	**$p \wedge q$**
T	T	T
T	F	F
F	T	F
F	F	F

Solved Example 4 :

Find the conjunction of the propositions p and q.

p: Rebecca's PC has more than 16 GB free hard disk space

q: The processor in Rebecca's PC runs faster than 1 GHz

Solution:

The conjunction of these propositions, $p \wedge q$, is the proposition

"Rebecca's PC has more than 16 GB free hard disk space, and the processor in Rebecca's PC runs faster than 1 GHz."

This conjunction can be expressed more simply as

"Rebecca's PC has more than 16 GB free hard disk space, and its processor runs faster than 1 GHz."

Disjunction

Let p and q be propositions. The **disjunction of p and q**, denoted by $p \vee q$, is the proposition **"p or q."** The disjunction $p \vee q$ is false when both p and q are false and is true otherwise. Table 3 displays the truth table for $p \vee q$.

TABLE 3 : The Truth Table for the **Disjunction** of Two Propositions.		
p	**q**	**$p \vee q$**
T	T	T
T	F	T
F	T	T
F	F	F

Solved Example 5 :

Find the disjunction of the propositions p and q.

p: Students who have taken calculus can take this class

q: Students who have taken computer science can take this class

Solution :

The disjunction of these propositions, $p \vee q$, is the proposition

"Students who have taken calculus can take this class or Students who have taken computer science can take this class"

This disjunction can be expressed more simply as

"Students who have taken calculus or computer science can take this class."

Exclusive Or

Let p and q be propositions. The exclusive or of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise. Table 4 displays the truth table for $p \oplus q$.

TABLE 4 : The Truth Table for the **Exclusive or** of Two Propositions.

p	**q**	**$p \oplus q$**
T	T	F
T	F	T
F	T	T
F	F	F

Solved Example 6 :

Find the Exclusive or of the propositions *p* and *q*.

p: Students who have taken calculus can take this class

q: Students who have taken computer science can take this class

Solution :

The Exclusive or of these propositions, $p \oplus q$, is the proposition

"Students, who have taken calculus or computer science, but not both, can take this class."

Conditional Operator

Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition **"if p, then q."** The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise.

TABLE 5 : The Truth Table for the **Conditional** Statement $p \rightarrow q$.

p	**q**	$\mathbf{p \rightarrow q}$
T	T	T
T	F	F
F	T	T
F	F	T

Table 5 displays the truth table $p \rightarrow q$. In conditional statement $p \rightarrow q$, p is called the hypothesis or antecedent or premise and q is called the conclusion or consequence. A conditional statement is also called an **implication.** Following are some of the ways to express conditional statements:

"if p, then q"
"p implies q"
"if p, q"
"p only if q"
"p is sufficient for q"
"a sufficient condition for q is p"
"q if p"
"q whenever p"
"q when p"
"q is necessary for p"
"a necessary condition for p is q"
"q follows from p"
"q unless $\neg$p"

Solved Example 7 :

Find the implication $p \rightarrow q$ for the propositions p and q.

p: Maria learns discrete mathematics.

q: Maria will find a good job.

Solution :

The conditional statement $p \rightarrow q$ is the proposition

"If Maria learns discrete mathematics, then she will find a good job."

There are many other ways to express this conditional statement in English.

"Maria will find a good job when she learns discrete mathematics."

"For Maria to get a good job, it is sufficient for her to learn discrete mathematics."

"Maria will find a good job unless she does not learn discrete mathematics."

Converse : The proposition $q \rightarrow p$ is called the converse of $p \rightarrow q$.

Contrapositive : The proposition $\neg q \rightarrow \neg p$ is called the contrapositive of $p \rightarrow q$.

Inverse : The proposition $\neg p \rightarrow \neg q$ is called the inverse of $p \rightarrow q$.

TABLE 6 : The Truth Table for the **Converse, Contrapositive,** and **Inverse** of $p \rightarrow q$

p	**q**	$p \rightarrow q$	$q \rightarrow p$	$\neg q \rightarrow \neg p$	$\neg p \rightarrow \neg q$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

It is observed from the Table 6 that the **converse is equivalent to the inverse** of the proposition and **contrapositive is equivalent to the original proposition** $p \rightarrow q$.

Solved Example 8 :

Find the converse, contrapositive, and inverse of the conditional statement

“The home team wins whenever it is raining”

Solution :

Because “q whenever p” is one of the ways to express the conditional statement $p \rightarrow q$, the original statement can be rewritten as “If it is raining, then the home team wins.”

The converse is “If the home team wins, then it is raining.”

The contrapositive of this conditional statement is “If the home team does not win, then it is not raining.”

The inverse is “If it is not raining, then the home team does not win.”

Biconditional Operator

Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition **“p if and only if q.”** The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.

TABLE 7 : The Truth Table for the **Biconditional** Statement p ↔ q.		
p	**q**	**p ↔ q**
T	T	T
T	F	F
F	T	T
F	F	T

The truth table for p ↔ q is shown in Table 6. The statement p ↔ q is true when both the conditional statements p ↔ q and q ↔ p are true and is false otherwise. Biconditional statements are also called bi-implications or equivalence. Following are some other common ways to express p ↔ q:

"p is necessary and sufficient for q"
"if p then q, and conversely"
"p iff q."

Solved Example 9 :

Find the bi-implication p ↔ q for the propositions p and q.

p: You can take the flight

q: You buy a ticket

Solution :

Then p ↔ q is the statement

"You can take the flight if and only if you buy a ticket."

Truth Tables of Compound Propositions

Logical connectives are used to build complicated compound propositions involving any number of propositional variables. The truth tables could be used to determine the truth values of these compound propositions.

The truth table has a row for each possible truth values of the proposition variables. If there is a single variable it can take up to 2 possible values hence there would be 2 rows in the table. If there are 2 propositional variables there are $2 \times 2 = 4$ rows in the truth table. If there are 3 propositional variables there are $2 \times 2 \times 2 = 8$ rows in the truth table. Thus if a compound statement contains n propositional variables there would be $2 \times 2 \times \ldots \times 2 = 2^n$ rows in the truth table.

A truth table may be systematically constructed in the following way:

1. The first n columns of the table are labeled by the component propositional variables. Further columns are included for all intermediate combinations of the variables, culminating in a column for the full compound statement.
2. Under each of the first n headings, we list the 2^n possible n-tuples of truth values for the n propositional variables.
3. For each of the remaining columns compute in sequence the remaining truth values.

Solved Example 10 :

Make a truth table for the statement $(p \vee \sim q) \rightarrow (p \wedge q)$.

Solution :

The compound statement has 2 propositional variables hence there will be 2^2 rows in the truth table.

TABLE 8 : The Truth Table of $(p \vee \sim q) \rightarrow (p \wedge q)$

p	**q**	**~q**	**$(p \vee \sim q)$**	**$(p \wedge q)$**	**$(p \vee \sim q) \rightarrow (p \wedge q)$**
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Tautology: A compound proposition that is **always true** for all possible truth values of the propositional variables that occur in it, is called a tautology.

Contradiction: A compound proposition that is **always false** for all possible truth values of the propositional variables that occur in it, is called a contradiction.

Contingency: A compound proposition that is **neither a tautology nor a contradiction** is called a contingency.

Solved Example 11 :

Show that $p \wedge q \rightarrow p$ is a tautology

Solution :

TABLE 9 : The Truth Table for $p \wedge q \rightarrow p$

p	**q**	**$p \wedge q$**	**$p \wedge q \rightarrow p$**
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

It is observed from Table 9 $p \wedge q \rightarrow p$ is always true for all possible truth values of variables p and q. Hence it is a tautology.

Solved Example 12 :

Show that $\sim(p \wedge q) \leftrightarrow (q \wedge p)$ is a contradiction

Solution :

TABLE 10 : The Truth Table for $\sim(p \wedge q) \leftrightarrow (q \wedge p)$					
p	**q**	$\boldsymbol{\sim(p \wedge q)}$	$\boldsymbol{\sim(p \wedge q)}$	$\boldsymbol{q \wedge p}$	$\boldsymbol{\sim(p \wedge q) \leftrightarrow (q \wedge p)}$
T	T	T	F	T	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	T	F	F

From table 10 it is observed that $\sim(p \wedge q) \leftrightarrow (q \wedge p)$ is always false hence it is a contradiction.

Solved Example 13 :

Show that $(p \vee r) \rightarrow (p \wedge q)$ is a contingent.

Solution :

Since there are 3 proposition variables in the given compound proposition there will be 8 rows in the truth table. From table 11 it is observed that $(p \vee r) \rightarrow (p \wedge q)$ has both true and false truth values hence it a contingency.

TABLE 11 : The Truth Table for $(p \vee r) \rightarrow (p \wedge q)$					
p	**q**	**r**	$\boldsymbol{p \vee r}$	$\boldsymbol{p \wedge q}$	$\boldsymbol{(p \vee r) \rightarrow (p \wedge q)}$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	F	F
F	T	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F
F	F	F	F	F	T

Precedence of Logical Operators

The parentheses have the highest precedence. Table 9 shows the order of precedence of the logical operators.

TABLE 12 : Precedence of Logical Operators.	
Precedence	**Operator**
1	$\sim$
2	$\wedge$
3	$\vee$
4	$\rightarrow$
5	$\leftrightarrow$

For Example :

The statement $\sim p \wedge q$ is the conjunction of $\sim p$ and q, namely, $(\sim p) \wedge q$, and not the negation of the conjunction of *p* and *q*, namely $\sim(p \wedge q)$.

For Example :

$p \wedge q \vee r$ means $(p \wedge q) \vee r$ rather than $p \wedge (q \vee r)$.

For Example :

$p \vee q \rightarrow r$ is the same as $(p \vee q) \rightarrow r$.

Logical Equivalence

Two Compound propositions are logically equivalent if and only if they have the same truth values in all possible cases.

The compound propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

Solved Example 14 :

Show that $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent.

Solution :

TABLE 13 : Logical Equivalence between $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \vee (q \rightarrow r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

From table 13 it is observed that the columns $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ have same truth values for all possible truth values of propositional variables p, q and r. Hence, the given compound propositions are equivalent.

1.3 Laws of Logic

Table 14 contains some important equivalences. In these equivalences, **T** denotes the compound proposition that is always true and **F** denotes the compound proposition that is always false.

TABLE 14 : Logical Equivalences.

Equivalence	**Name**
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg (p \wedge q) \equiv \neg p \vee \neg q$ $\neg (p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation laws

Tables 15 and 16 display some useful equivalences for compound propositions involving conditional and biconditional operators, respectively

TABLE 15 : Logical Equivalences Involving Conditional Operator
$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg (p \to \neg q)$
$\neg (p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 16 : Logical Equivalences Involving Biconditional Statements.
$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Solved Example 15 :

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent

Solution :

$\neg (p \vee (\neg p \wedge q))$

$\equiv \neg p \wedge \neg(\neg p \wedge q)$

…by the second De Morgan law

$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$

…by the first De Morgan law

$\equiv \neg p \wedge (p \vee \neg q)$

…by the double negation law

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

…by the second distributive law

$\equiv F \vee (\neg p \wedge \neg q)$ …by Negation law

$\equiv (\neg p \wedge \neg q) \vee F$

…by the commutative law for disjunction

$\equiv \neg p \wedge \neg q$ …by the identity law for F

Consequently $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

Solved Example 16 :

Show that $(p \wedge q) \to (p \vee q)$ is a tautology

Solution:

$(p \wedge q) \to (p \vee q)$

$\equiv \neg (p \wedge q) \vee (p \vee q)$ …by Conditional law

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$

…by the first De Morgan law

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$

…by the associative and commutative laws for disjunction

$\equiv T \vee T$

…by Negation law and the commutative law for disjunction

$\equiv T$ …by the domination law

Satisfiable

A compound proposition is satisfiable if **there is an assignment** of truth values to its variables that makes it **true**.

Unsatisfiable

A compound proposition is unsatisfiable if for **all assignments** of truth values to its variables, the compound proposition **remains false**.

1.4 Argument

An argument is an assertion that given a set of propositions P_1, P_2,……P_n, called premises, yields (has a consequence) another proposition Q, called the conclusion. Such an argument is denoted by

$$P_1, P_2, \ldots\ldots, P_n \vdash Q$$

Valid argument

An argument $P_1, P_2, P_3, \ldots., P_n \vdash Q$ is said to be valid if Q is true whenever all the premises $P_1, P_2, \ldots., P_n$ are true.

The argument $P_1, P_2, P_3\ldots. P_n \vdash Q$ is valid if and only if the proposition

$(P_1, \wedge P_{2\ \ldots.}\wedge P_n) \rightarrow Q$ is a tautology.

Fallacy argument

An argument which is not valid is called a fallacy.

Solved Example 17 :

Show the following argument is valid :

$p, p \rightarrow q \vdash q$

Solution :

This is clear from the table given below.

TABLE 17 : The Truth Table for $p, (p \rightarrow q) \vdash q$

p	**q**	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Here, p and $p \rightarrow q$ are true simultaneously only in row1, and in this case q is also true.

Hence $p, p \rightarrow q \vdash Q$ is valid.

On the other hand, the argument $p \rightarrow q, q \vdash p$ is a fallacy.

TABLE 18 : The Truth Table for $p \rightarrow q \vdash p$				
p	**q**	$\mathbf{p \rightarrow q}$	$\mathbf{(p \rightarrow q) \wedge q}$	$\mathbf{((p \rightarrow q) \wedge q) \rightarrow p}$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Law of Syllogism

A fundamental principle of logical reasoning states : "If p implies q and q implies r, then p implies r."

That is, the following argument is valid, $p\rightarrow q, q\rightarrow r \vdash p\rightarrow r$

For Example :

The truth table below shows that the following proposition is a tautology.

Q : $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

TABLE 19 : The Truth Table for $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$							
p	**q**	**r**	$\mathbf{p\rightarrow q}$	$\mathbf{q\rightarrow r}$	$\mathbf{(p\rightarrow q) \wedge (q \rightarrow r)}$	$\mathbf{p\rightarrow r}$	**Q**
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Logical Implication

A proposition P(p, q, …) is said to logically imply a proposition Q(p, q, …) written as

P(p, q, ……) $\Rightarrow$ Q(p, q,…) of Q(p, q, …) is true whenever P(p, q, …) is true.

For Example :

Consider the following truth table :

Table 20 shows $p \vee q$ is true whenever p is true. Hence, p logically implies $p \vee q$.

TABLE 20 : The Truth Table for p logically implies $p \vee q$

p	**q**	**$p \vee q$**
T	T	T
T	F	T
F	T	T
F	F	F

Now if Q(p, q, …) is true whenever P(p, q, …..) is true, then the argument P(p, q, …) $\vdash$ Q(p, q, ….) is valid.

Furthermore, the argument P $\vdash$ Q is valid if and only if the conditional statement $P \rightarrow Q$ is always true, i.e. a tautology. Hence we can conclude the following concept.

For any propositions P(p, q, …) and Q(p, q, …), the following three statements are equivalent :

- P(p, q, …) logically implies Q(p, q, …..).
- The argument P(p, q, …) $\vdash$ Q(p, q, ….) is valid.
- The proposition P(p, q, …) $\rightarrow$ Q(p, q, …) is a tautology.

❑ ❑ ❑ ❑ ❑

Assignment – 1

Duration : 45 Min. **Max. Marks : 30**

Q.1 to Q.8 carry one mark each

1. Which of the following well–formed formulas are equivalent ? **[1988]**

(A) $P \to Q$ (B) $\sim P \to Q$

(C) $\sim P \vee Q$ (D) $\sim Q \to P$

2. Which of the following is/are tautology?

(A) $a \vee b \to b \wedge c$ **[1992]**

(B) $a \wedge b \to b \vee c$

(C) $a \vee b \to (b \to c)$

(D) $a \to b \to (b \to c)$

3. The proposition $p \wedge (\sim p \vee q)$ is

(A) a tautology **[1993]**

(B) logically equivalent to $p \wedge q$

(C) logically equivalent to $p \vee q$

(D) a contradiction

(E) None of the above

4. Which of the following propositions is a tautology ? **[1997]**

(A) $(p \vee q) \to p$ (B) $p \vee (q \to p)$

(C) $p \vee (p \to q)$ (D) $p \to (p \to q)$

5. What is the converse of the following assertion ? **[1998]**

I stay only if you go

(A) I stay if you go

(B) If I stay then you go

(C) If you do not go then I do not stay

(D) If I do not stay then you go

6. Consider two well–formed formulas in propositional logic

F1 : $P \Rightarrow \neg P$

F2 : $(P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

Which of the following statements is correct? **[2001]**

(A) F1 is satisfiable, F2 is valid

(B) F1 unsatisfiable, F2 is satisfiable

(C) F1 is unsatisfiable, F2 is valid

(D) F1 and F2 are both satisfiable

7. The statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statements below? **[2017]**

I. $p \Rightarrow q$ II. $q \Rightarrow p$

III. $(\neg q) \vee p$ IV. $(\neg p) \vee q$

(A) I only (B) I and IV only

(C) II only (D) II and III only

8. Let p, q, r denote the statements "It is raining", "It is cold", and "It is pleasant", respectively. Then the statement "It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold" is represented by **[2017]**

(A) $(\neg p \wedge r) \wedge (\neg r \to (p \wedge q))$

(B) $(\neg p \wedge r) \wedge ((p \wedge q) \to \neg r)$

(C) $(\neg p \wedge r) \vee ((p \wedge q) \to \neg r)$

(D) $(\neg p \wedge r) \vee (r \to (p \wedge q))$

Q.9 to Q.19 carry two marks each

9. Indicate which of the following well-formed formula are valid:
[1990]
(A) $((P \to Q) \wedge (Q \to R)) \to (P \to R)$
(B) $(P \to Q) \to (\neg P \to \neg Q)$
(C) $(P \wedge (\neg P \vee \neg Q)) \to Q$
(D) $((P \to R) \vee (Q \to R)) \to ((P \vee Q) \to R)$

10. Which one of the following is false? Read $\wedge$ as AND, $\vee$ as OR, $\sim$ as NOT, $\to$ as one way implication and $\leftrightarrow$ as two way implication. **[1996]**
(A) $((x \to y) \wedge x) \to y$
(B) $((\sim x \to y) \wedge (\sim x \to \sim y)) \to x$
(C) $(x \to (x \vee y))$
(D) $((x \vee y) \leftrightarrow (\sim x \to \sim y))$

11. If F_1, F_2 and F_3 are propositional formulae such that $F_1 \wedge F_2 \to F_3$ and $F_1 \wedge F_2 \to \sim F_3$ are both tautologies, then which of the following is true :
[1991]
(A) Both F_1 and F_2 are tautologies
(B) The conjunction $F_1 \wedge F_2$ is not satisfiable
(C) Neither is tautology
(D) Neither is satisfiable
(E) None of the above

12. If the proposition $\sim p \Rightarrow q$ is true, then the truth value of the proposition $\sim p \vee (p \Rightarrow q)$, where $\sim$ is negation, '$\vee$' is inclusive or and $\Rightarrow$ is implication, is
(A) true **[1995]**
(B) multiple–valued
(C) false
(D) cannot be determined

13. Let a, b, c, d be propositions, assume that the equivalences $a \leftrightarrow (b \vee -b)$ and $b \leftrightarrow c$ hold. Then the truth value of the formula $(a \wedge b) \to ((a \wedge c) \vee d)$ is always **[2000]**
(A) True
(B) False
(C) Same as the truth value of b
(D) Same as the truth value of d

14. The following resolution rule is used in logic programming :
Derive clause $(P \vee Q)$ from clauses $(P \vee R)$, $(Q \vee \neg R)$
Which of the following statements related to this rule is FALSE? **[2003]**
(A) $((P \vee R) \wedge (Q \vee \neg R)) \Rightarrow (P \vee Q)$ is logically valid
(B) $(P \vee Q) \Rightarrow ((P \vee R) \wedge (Q \vee \neg R))$ IS logically valid
(C) $(P \vee Q)$ is satisfiable if and only if $(P \vee R) \wedge (Q \vee \neg R)$ is satisfiable
(D) $(P \vee R) \Rightarrow$ FALSE if and only if both P and Q are unsatisfiable.

15. The following propositional statement is

$(P \rightarrow (Q \vee R)) \rightarrow ((P \wedge Q) \rightarrow R)$

(A) satisfiable but not valid **[2004]**

(B) valid

(C) a contradiction

(D) None of the above

16. Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $\left[(\neg p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)\right] \rightarrow (\neg s \rightarrow q)$

Q: $\left\lfloor(\neg p \vee q) \wedge (q \rightarrow (p \rightarrow r))\right\rfloor \rightarrow \neg r$

R: $\left[(q \vee r) \rightarrow p\right] \wedge (\neg q \vee p) \rightarrow r$

S: $\left[p \wedge (p \rightarrow r) \wedge (q \vee \neg r)\right] \rightarrow q$

Which of the above arguments are valid ? **[2004]**

(A) P and Q only (B) P and R only

(C) P and S only (D) P, Q, R and S

17. Let P, Q and R be three atomic prepositional assertions.

Let X denote $(P \vee Q) \rightarrow R$ and Y denote $(P \rightarrow R) \vee (Q \rightarrow R)$. Which one of the following is a tautology?

[2005]

(A) $X \equiv Y$ (B) $X \rightarrow Y$

(C) $Y \rightarrow X$ (D) $-Y \rightarrow X$

18. Consider the following propositional statements: **[2006]**

P1: $((A \wedge B) \rightarrow C)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$

P2: $((A \vee B) \rightarrow C)) \equiv ((A \rightarrow C) \vee (B \rightarrow C))$

(A) P1 is tautology, but not P2

(B) P2 is a tautology, but not P1

(C) P1 and P2 are both tautologies

(D) Both P1 and P2 are not tautologies

19. A logical binary relation ⊙, is defined as follows.

A	B	A ⊙ B
True	True	True
True	False	True
False	True	False
False	False	True

Let ~ be the unary negation (NOT) operator, with higher precedence then ⊙. Which one of the following is equivalent to $A \wedge B$? **[2006]**

(A) (~A ⊙ B) (B) ~(A ⊙ ~B)

(C) ~(~A ⊙ ~B) (D) ~(~A ⊙ B)

❑ ❑ ❑ ❑ ❑

Assignment – 2

Duration : 45 Min. **Max. Marks : 30**

Q.1 to Q. 8 carry one mark each

1. "If X then Y unless Z" is represented by which of the following formulas in propositional logic ? ("$\neg$" is negation, "$\wedge$" is conjunction, and "$\rightarrow$" is implication) **[2002]**

(A) $(X \wedge \neg Z) \rightarrow Y$ (B) $(X \wedge Y) \rightarrow \neg Z$

(C) $X \rightarrow (Y \wedge \neg Z)$ (D) $(X \rightarrow Y) \wedge \neg Z$

2. Let p and q be two propositions. Consider the following two formulae in propositional logic.

$S_1 : (\neg p \wedge (p \vee q)) \rightarrow q$

$S_2 : q \rightarrow (\neg p \wedge (p \vee q))$

Which one of the following options is correct? **[2021]**

(A) Both S_1 and S_2 are tautologies.

(B) S_1 is a tautology but S_2 is not a tautology.

(C) S_1 is not a tautology but S_2 is a tautology.

(D) Neither S_1 and S_2 is a tautology.

3. Consider the following statements:

P: Good mobile phones are not cheap

Q: Cheap mobile phones are not good

L: P implies Q

M: Q implies P

N: P is equivalent to Q

Which one of the following about L, M, and N is CORRECT? **[2014]**

(A) Only L is TRUE.

(B) Only M is TRUE.

(C) Only N is TRUE.

(D) L, M and N are TRUE.

4. Which one of the following is NOT equivalent to $p \leftrightarrow q$? **[2015]**

(A) $(\neg p \vee q) \wedge (p \vee \neg q)$

(B) $(\neg p \vee q) \wedge (q \rightarrow p)$

(C) $(\neg p \wedge q) \vee (p \wedge \neg q)$

(D) $(\neg p \wedge \neg q) \vee (p \wedge q)$

5. Consider the following two statements.

S1: If a candidate is known to be corrupt, then he will not be elected.

S2: If a candidate is kind, he will be elected.

Which one of the following statements follows from S1 and S2 as per sound inference rules of logic? **[2015]**

(A) If a person is known to be corrupt, he is kind

(B) If a person is not known to be corrupt, he is not kind

(C) If a person is kind, he is not known to be corrupt

(D) If a person is not kind, he is not known to be corrupt

6. In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in the room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking, the person replies the following

"The result of the toss is head if and only if I am telling the truth."

Which of the following options is correct? **[2015]**

(A) The result is head

(B) The result is tail

(C) If the person is of Type 2, then the result is tail

(D) If the person is of Type 1, then the result is tail

7. Let p, q, r, s represent the following propositions.

p: $x \in \{8, 9, 10, 11, 12\}$

q: x is a composite number

r: x is a perfect square

s: x is a prime number

The integer $x \geq 2$ which satisfies $\neg\left((p \Rightarrow q) \wedge (\neg r \vee \neg s)\right)$ is___. **[2016]**

8. Consider the following expressions:

(i) false (ii) Q

(iii) true (iv) $P \vee Q$

(v) $\neg Q \vee P$

The number of expressions given above that are logically implied by $P \wedge (P \Rightarrow Q)$ is________. **[2016]**

Q.9 to Q.19 carry two marks each

9. P and Q are two propositions. Which of the following logical expressions are equivalent ? **[2008]**

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(A) Only I and II

(B) Only I, II and III

(C) Only I, II and IV

(D) All of I, II, III and IV

10 Which one of the following propositional logic formulas is TRUE when exactly two of p, q, and r are TRUE? **[2014]**

(A) $((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(B) $(\sim(p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(C) $((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(D) $(\sim(p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

11. Which one of the following Boolean expressions is NOT a tautology?**[2014]**

(A) $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$

(B) $(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$

(C) $(a \wedge b \wedge c) - (c \vee a)$

(D) $a \rightarrow (b \rightarrow a)$

12. Let p, q, and r be propositions and the expression $(p \rightarrow q) \rightarrow r$ be a contradiction. Then, the expression $(r \rightarrow p) \rightarrow q$ is **[2017]**

(A) a tautology

(B) a contradiction

(C) always TRUE when p is FALSE

(D) always TRUE when q is TRUE

13. An island has three kinds of people: knights who always tell the truth, knaves who always lie, and spies who can either lie or tell the truth. You encounter three people, A, B, and C. You know one of these people is a knight, one is a knave, and one is a spy. Each of the three people knows the type of person each of other two is. If possible determine who the knave, knight, and spy are. A says "I am the knight," B says "I am the knight," and C says "I am the knight."

(A) A is the spy, B is the knight, C is the knave.

(B) A is the knave, B is the spy, C is the knight.

(C) A is the knight, B is the spy, C is the knave.

(D) Any of the three can be the knight, any can be the spy, any can be the knave.

14. Steve would like to determine the relative salaries of three coworkers using two facts. First, he knows that if Fred is not the highest paid of the three, then Janice is. Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most. Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least?

(A) In order of decreasing salary: Fred, Janice, Maggie

(B) In order of decreasing salary: Fred, Maggie, Janice

(C) In order of decreasing salary: Janice, Fred, Maggie

(D) In order of decreasing salary: Janice, Maggie, Fred

15. A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the gardener and the handyman are not both lying; and if the handyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning.

(A) Butler and Cook are lying and Gardner and Handyman are telling the truth
(B) Butler and Cook are telling the truth and Gardner and Handyman are lying
(C) Butler and Cook are lying and cannot be determined whether Gardner and Handyman are telling the truth
(D) Butler, Gardner and Handyman are telling the truth and Cook is lying

16. Suppose there are signs on the doors to two rooms. The sign on the first door reads "In this room there is a lady, and in the other one there is a tiger"; and the sign on the second door reads "In one of these rooms, there is a lady, and in one of them there is a tiger." Suppose that you know that one of these signs is true and the other is false. Behind which door is the lady?

(A) First door has lady and second door has tiger
(B) First door has tiger and second door has tiger
(C) First door has tiger and second door has lady
(D) Cannot be determined

17. If p, q, r have the same truth value then the proposition

$(p \vee \sim q) \wedge (q \vee \sim r) \wedge (r \vee \sim p)$ is

(A) True
(B) Multiple – Valued
(C) False
(D) Cannot be determined

18. The proposition $\sim (p \vee (\sim p \wedge q))$ is

(A) Tautology
(B) Logically equivalent to $\sim p \wedge \sim q$
(C) Logically equivalent to $\sim p \vee \sim q$
(D) Contradiction

19. The proposition $[p \wedge (p \rightarrow q)] \rightarrow q$ is

(A) Tautology
(B) Contradiction
(C) Logically equivalent to $p \wedge q$
(D) Contingency

❑ ❑ ❑ ❑ ❑