SYSTEM IDENTIFICATION GENERAL PROCEDURE & APPLICATION TO STANFORD P1 STEER-BY-WIRE VEHICLE

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1. INTRODUCTION

The objective of this document is to give a general overview of frequency-based system identification procedure without any math, apply the techniques to Stanford P1 vehicle in parallel to identify a 3-DOF lateral-directional model of the vehicle.

Classical control methodologies such as linear quadratic regulator (LQR), require a linear, time-invariant (LTI) model that represents the system to be controlled. There are 2 ways in which these models can be built.

- *Physical Modeling*: This model is constructed based on the presumed knowledge of physics that governs the system. E.g. Newton's laws of motion.
- *Data driven modeling*: Measurements of several variables of the process are taken and a model is constructed by identifying a model that matches the dynamics that underlies the measured data. This process of constructing the model of a system from experimental data is called system identification.

But, why system identification?

- It is not always easy to derive the model of a system with what we know about it. For example, exact values of stiffnesses, coefficients of friction & damping are hard to find.
- A model based on the data will better match the dynamics of the actual system.
- System identification is mode time efficient.
- Uncertainty data are readily available for the identified model.

There are two types of models to consider when discussing system identification: nonparametric and parametric models.

Nonparametric models are concerned with characterizing only the measured input-to-output behavior of the aircraft dynamics, not the nature of the aircraft equations of motion. Examples of nonparametric modeling are impulse responses (time) and frequency responses (frequency).

Parametric models such as root locus design and pole placement are completely defined by their coefficients or parameters.

2. FREQUENCY-BASED IDENTIFICATION:

Frequency response-based system identification is very well suited for complex and sensitive systems. The reasons for this include:

1. Bias effects of noise in response measurements and process noise are eliminated from the analysis.
2. Direct and precise identification of time delays caused by linear phase shift with frequency.
3. linearity.
4. Non-parametric models can be identified directly without requiring the parametric model first be identified. This exposes many key aspects of the dynamics.

This methodology has four main steps: frequency response identification, state-space model fitting to the MIMO frequency response database, model structure determination and time domain verification. Each of these steps will be explained in detail during the actual identification process which follows.

3. STANFORD P1 STEER-BY-WIRE VEHICLE

The Stanford "P1" prototype steer-by-wire vehicle is an electric vehicle featuring independent left and right steering mechanisms and independent electric rear-wheel drive. Independent front steering mechanisms provide steering system redundancy, as either front wheel can steer the car independently of the other in the event of a failure. Independent steering also enables research in steering control systems that can tune the relative angles of each wheel separately, maximizing handling performance and minimizing tire wear. The independent electric rear-wheel drive system allows precise computer control of the torque applied by each drive wheel, enabling the option of breaking with one wheel while accelerating with the other. This technique could be used as part of a stability control algorithm or to provide an additional means of steering the car in the case of a failure in the primary steering system.

Fig 1. Stanford P1 vehicle.

3.1 Identification of P1 vehicle.

Two datasets were given, and they were collected on the real vehicle; system identification maneuvers were performed on the runway at NASA Ames research center. The first dataset is *P1_Final_Sweep* which has the results of frequency sweep to control the steering angle collected at the vehicle speed of 33 ft/s.

Data signals provided in the file:
time – in seconds
delta – control steering angle, deg
ay – lateral acceleration, ft/s2.
beta – sideslip angle (at center of gravity), rad

p – roll rate of body, rad/s
r – yaw rate of car, rad/s
phi – roll angle, rad

3.1.1 Collection of Time-History Data

Time hisotry data are collected by specialized flight test maneuvers to excite the dynamics of concern such as the study of dynamics and control (low frequencies of interest) or structural stability and response (higher frequency of interest). Typical excitation for system-identification purpose is *Frequency Sweeps*. Frequency sweep refers to a class of control that has quasi-sinusoidal shape of increasing frequency. Frequency sweeps act as good excitation signals for the frequency-response method for various reasons; 1. It is persistently exciting, which means that all of the modes of a system are excited. 2. The frequency range of excitation can be strictly controlled during the test. 3. Response time histories are roughly symmetric.

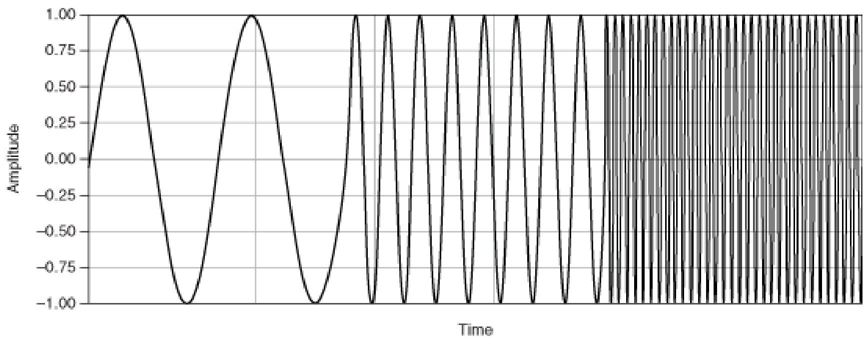


Fig 2. Typical automated frequency sweep input.

The following are the frequency sweep results from P1_Final_Sweep.

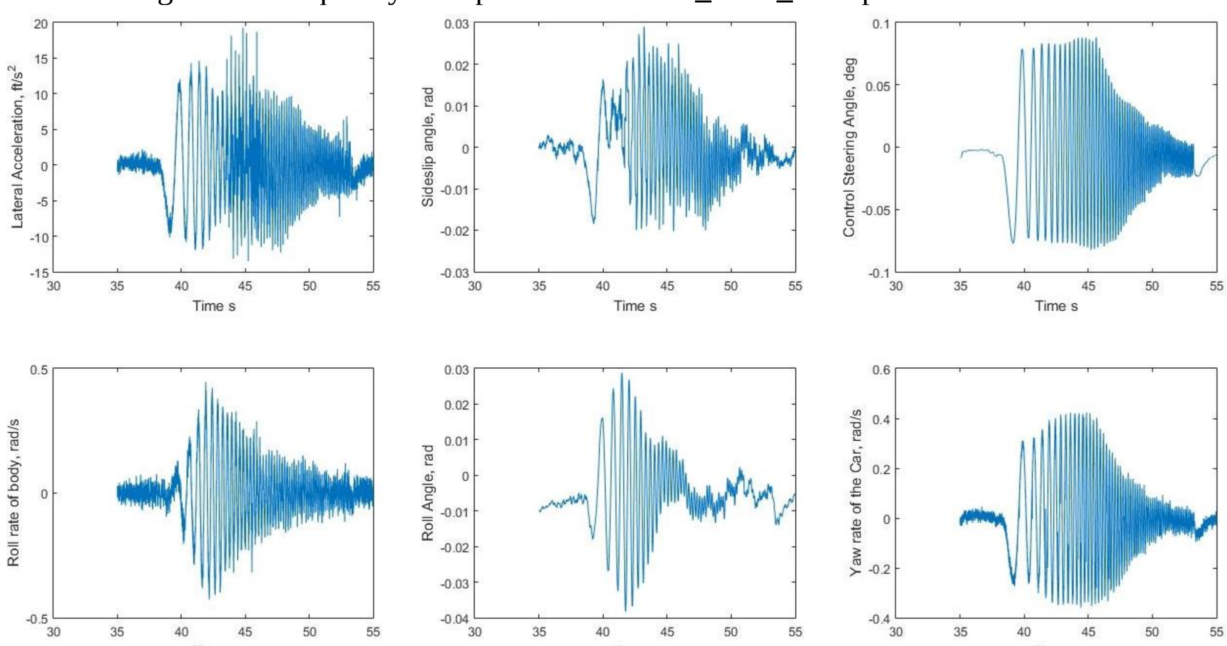


Fig 3. Time history data from frequency sweeps.

Now let us look at the detailed description of the main steps performed in identification methodology.

3.1.2 Frequency response identification from flight data

The frequency response fully characterizes the system's dynamic behavior, in terms of the best linear description of the input-to-output behavior. This doesn't need any prior knowledge of the internal structure of the system at all.

The first step is to perform multivariable spectral analysis (Multi-input-single-output MISO) of the data. The analysis of a signal or input-to-output process as a function of frequency is referred to as spectral analysis. This analysis is a generalization of simple SISO FFT and is required because real data of complex systems involve multiple partially correlated control inputs (Primary and Secondary) during a single excitation maneuver. However, the presence of multiple inputs doesn't necessarily mean that the usual SISO identification is inadequate. The SISO solution is satisfactory for MIMO system if one of the following conditions is true:

1) Inter-axis dynamic coupling is negligible.
2) Secondary inputs are uncorrelated with the primary input.

By repeating the above MISO process for each of the outputs, we get MIMO frequency-response. The MIMO database thus obtained is a nonparametric model as it characterizes the input-output relationship without any model parameters. A byproduct of spectral analysis is the coherence function. It provides a key measure of the frequency-response accuracy as a function of frequency and indicates whether a system has been excited across all the frequency ranges of interest.

There are 2 key features of the spectral analysis; 1. Chirp Z-transform 2. Composite window optimization. Chirp Z transform, also known as zoom transform is a specialized implementation of the Fast Fourier Transform (FFT). It can determine the frequency response with very high accuracy, provides flexibility in the selection of sample rates and window lengths for composite windowing.

Spectral windowing is a process by which the time-history data are segmented, and the frequency response is determined for each segment or window. By using large window sizes, better information content at lower frequencies are captured. But this increases random error as the number of windows can be used also reduces. At higher frequencies, where the signal-to-noise ratio is higher, the oscillation of the frequency curves increases. Smaller windows reduce random error at the expense of diminished information content at the lower frequencies.

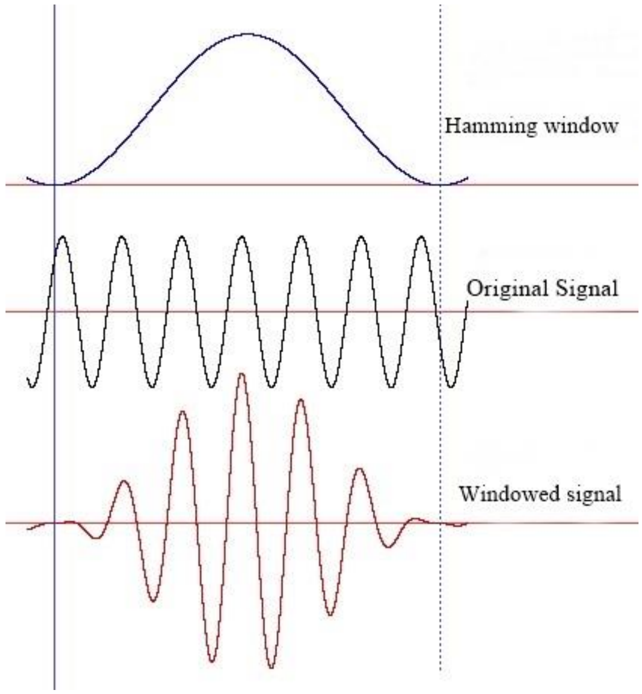
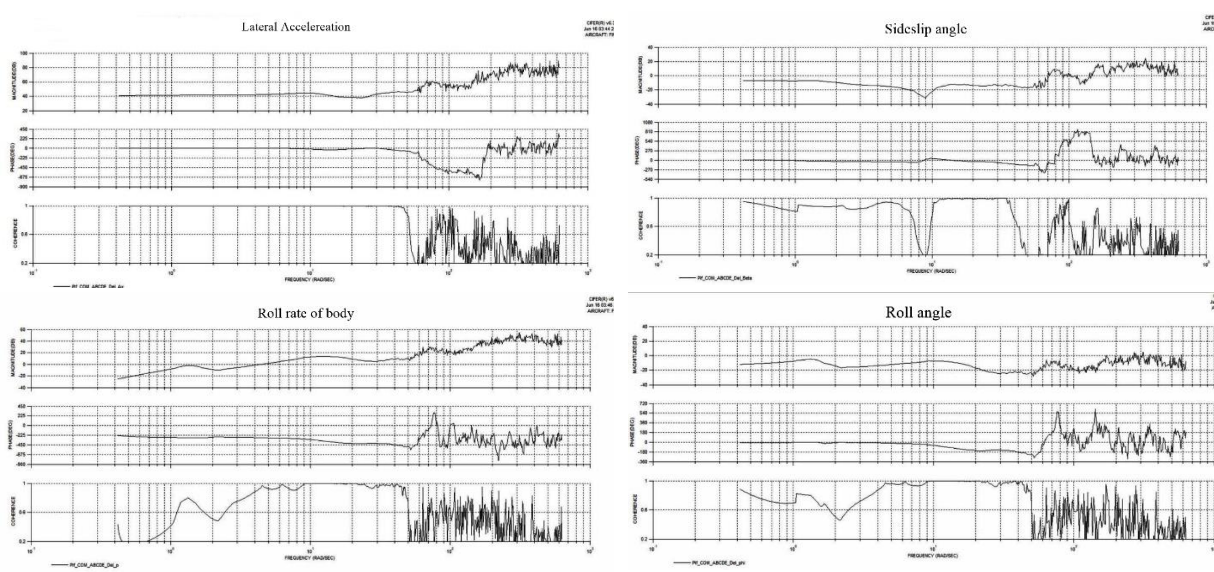


Fig 4. Single hamming window.

Composite windowing is a technique that eliminates the need for repeated, manual optimization of window sizes and produces a single, optimized frequency response that is accurate over the entire frequency range of interest. This is done by combining the weighted average of multiple windows. The weighing function assigns a weight of 1 to the data with minimum error and the windows with higher error or deweighted accordingly. This results in high quality MIMO frequency response database.

Following are the identified frequency responses for Stanford P1 Vehicle.



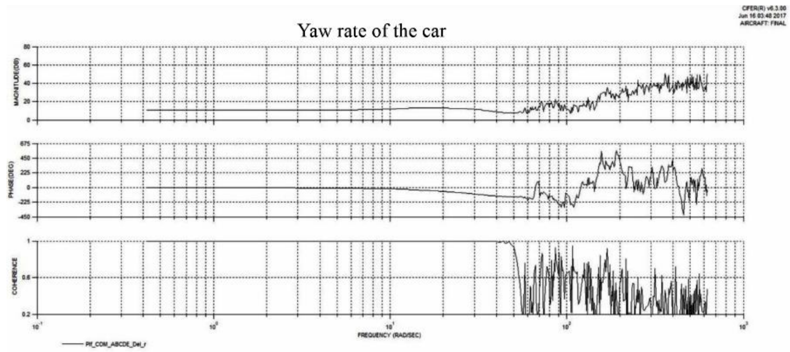


Fig 5. Frequency responses of various signals with respect to steering angle.

RESPONSE	INPUT δ (rad/s)
A_y	0.4189 – 52.15
β	0.4189 – 41.86
p	1.15 – 20
Φ	0.4189 – 20
r	0.4189 – 53.70

Table1 Frequency ranges for model fit based on coherences

3.1.3 State-space model fitting to the MIMO frequency response database

In many applications, the required end-product of system identification is a state-space model expressed in terms of stability and control derivatives or even the physical system parameters. The state-space equations are

$$\boldsymbol{M}\dot{\boldsymbol{x}} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{G}\boldsymbol{u}(t - \boldsymbol{\tau})$$

The matrices M, F, G and the vector τ contain model parameters to be identified. The time delays can be included to accound for unmodeled dynamics. The measurement vector is

$$\boldsymbol{y} = \boldsymbol{H}_0\boldsymbol{x} + \boldsymbol{H}_1\dot{\boldsymbol{x}}$$

The matrices H0 and H1 are composed of known constants.

The solution of the MIMO identification problem involves determining the model matrices M, F, G and τ that produce a frequency-response matrix T(s) that most closely matches the frequency responses obtained from the flight data. An initial state-space model structure is chosen based on the frequency-response plots. The freed state-space model parameters undergo optimization to match the frequency responses identified from the flight data. A coherence weighted quadratic cost function (J) is used to quantify the match between flight data and the state-space model.

An overall average cost function J_{ave} less than or equal to 100 is generally considered as reflecting an acceptable level of accuracy for dynamics modeling. Individual cost functions can reach 150 to 200 without resulting in a noticeable loss of overall predictive accuracy. The uniqueness and validity are tested by calculating the Insensitivities (I) and Cramer-Rao Bounds (CR).

```
PARAM    VALUE
 Yv        -2.723
 Yr        -50.99
 Lv         1.079
 Lr        -27.37
 Lphi      -71.87
 Np         10.01
 Nr        -41.66
 Ydel       314.3
 Ldel       102.6
 Ndel       142.5
 del      0.02871

***  Case: PIf_Final2

Costs after   178 additional iterations (  178  total iterations):
ay  /del  :   50.59577
beta/del  :   67.58646
p   /del  :   102.9061
phi /del  :   86.55997
r   /del  :   17.59249

The average cost function is:              65.0482
```

Table 2 Cost function

3.1.4 Model structure determination

The model structure might need to be redefined. If the confidence of a parameter value is found to be very poor because of a lack of information content or correlation with other parameters, then it is better to eliminate that parameter from the model structure or replace it with a value that is reasonable from physical concepts.

The confidence of a parameter can be found by Cramer-Rao bound and its insensitivity. Parameters that have undesirably large insensitivities and/or Cramer-Rao bounds are systematically removed from the model structure. The model undergoes optimization again and is made to re-converge once a parameter is removed. This ensures that all the parameters are sensitive to cost function, contribute to the model structure and that there are no correlated parameters in the model.

** M-Matrix **

CASE NAME :CASE ID

PIf_Final2 :PIf, Final, Derivid, Iteration, dropped Yp

PIf_Final2

	UBEST	CR Bound	CR Percent	% Insens

No parameters have been defined for this location.

** F-Matrix **

CASE NAME :CASE ID

PIf_Final2 :PIf, Final, Derivid, Iteration, dropped Yp

PIf_Final2

	UBEST	CR Bound	CR Percent	% Insens
Yv	-2.723	0.2816	10.34	2.473
Yp	0.000 +	------	------	------
Yr	-50.99	10.86	21.29	0.3647
g	32.17 !	------	------	------
Lv	1.079	0.1401	12.98	3.407
Lp	0.000 +	------	------	------
Lr	-27.37	4.014	14.66	0.2754
Lphi	-71.87	4.171	5.804	1.184
Nv	0.000 +	------	------	------
Np	10.01	0.7482	7.471	2.047
Nr	-41.66	5.095	12.23	0.1687

\+ Eliminated during model structure determination
! Fixed value in model

No parameter constraints have been defined.

** G-Matrix **

CASE NAME :CASE ID

PIf_Final2 :PIf, Final, Derivid, Iteration, dropped Yp

PIf_Final2

	UBEST	CR Bound	CR Percent	% Insens
Ydel	314.3	35.27	11.22	0.2092
Ldel	102.6	13.30	12.97	0.2689
Ndel	142.5	17.76	12.47	0.1711
del	0.02871	2.455E-03	8.554	2.455

** Working Variables **

CASE NAME :CASE ID

PIf_Final2 :PIf, Final, Derivid, Iteration, dropped Yp

PIf_Final2

	UBEST	CR Bound	CR Percent	% Insens

No parameters have been defined for this location.

** Transfer Function Costs **

CASE NAME :CASE ID

PIf_Final2 :PIf, Final, Derivid, Iteration, dropped Yp

	PIf_Final2
ay /del	50.596
beta/del	67.586
p /del	102.906
phi /del	86.560
r /del	17.592
Average	65.048

Table 3 State-space model structure accuracy results.

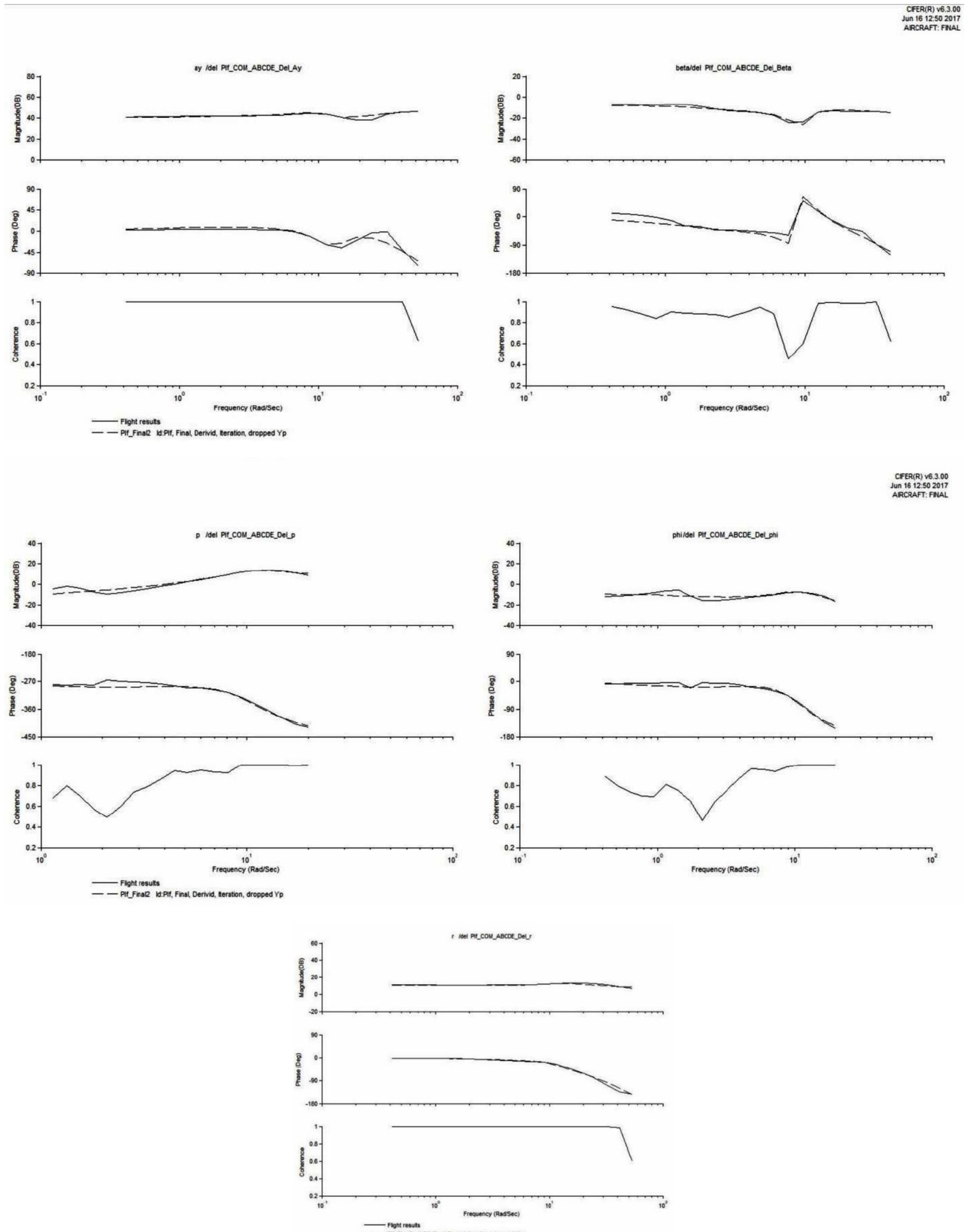


Fig 6. Plots overlaying state-space model and the identified frequency responses

From top left 1) Lateral acceleration, 2) Sideslip angle, 3) Roll rate of body, 4) Roll angle, 5) Yaw rate of the car

3.1.5 Time domain verification

An important assessment of model fidelity, robustness, and the limitations of the linear model is provided by evaluating its predictive capability in the time domain for test inputs, such as steps or doublets, that are dissimilar from those used in the identification. Confidence must be gained that the model is not "overly tuned" to the identification test data and the flight condition at which the data were collected. The identified state-space model is driven with flight data, and the outputs of the model are evaluated against the real flight data. A cost function (J_{rms}) is again used to measure the match between the model and the flight data. J_{rms} must be less than or equal to 1 to 2.

The Theil inequality coefficient (TIC), provides a normalized criterion for assessing the predictive accuracy of the model. A value of TIC = 0 corresponds to a model with perfect predictive accuracy and a value close to TIC = 1 corresponds to no predictive capability at all.

The following are results from doublets/step inputs from P1_verify dataset.

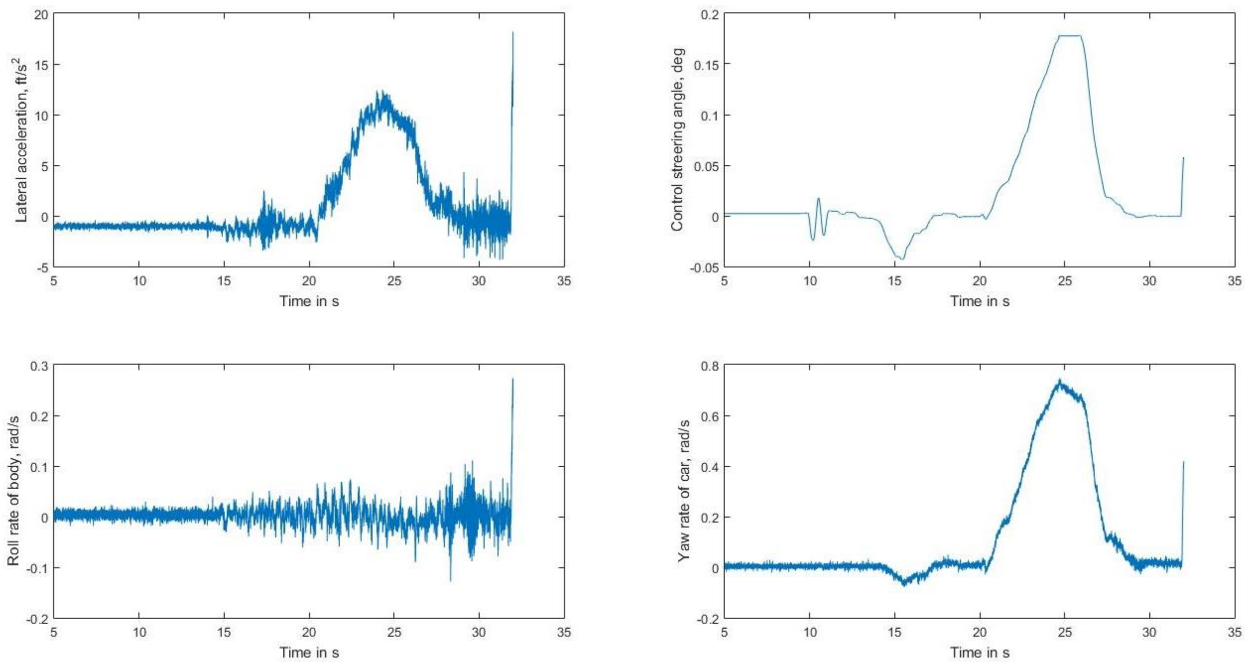


Fig.7 Plots for time domain verification generated by doublets/step inputs.

The following are the results of Time domain verification

```
NTPTS =  11500

  *** Data Decimation Option ***
 New NTPTS =          575
 New total number of output points =        1725
 New DT =   4.0000003E-02
 New sample rate =    25.00000

 Data pre-processing is complete.

 New NTPTS =          575
 New DT    =   4.0000003E-02
 New SAMPLE RATE =    25.00000
 New total number of output points =        1725
 NFIN  =          575
Resetting arrays for TSTAR and TFIN ...    TSTAR =   0.0000000E+00
   NSTAR =           1
   NFIN  =         575
   NTPTS =         575
   NTPTS =         575
   NACT  =         575

     Find initial conditions for biases to state derivatives
     and output biases applied to the state-space model.
     (Output reference value is shifted.)


 Initial cost is :   2.452724
 Initial TIC  is :  0.1434361

Starting bias and reference shift determination ...
(  8 parameters in the least-sq solution of biases.)


 Identified state derivative biases are:
 XD0(           1 ) =    1.502056
 XD0(           2 ) =   0.9000084
 XD0(           3 ) =    1.341514
 XD0(           4 ) =  -2.3880055E-02
 XD0(           5 ) =   0.0000000E+00

 Identified output reference shifts are :
  YREF(           1 ) =   0.4965910
  YREF(           2 ) =   -1.962724
  YREF(           3 ) =   0.5709377

 Cost after least squares solution is    1.802724
 TIC  after least squares solution is   0.1007995
```

Table4 Time domain verification costs and Theil inequality coefficient.

It is seen from the table that, that the cost function is within the Guideline ($J \leq 1–2$) and Theil Inequality Coefficient ($TIC < 0.25$). TIC of 0.1 means that there is only 10% error in the predictive accuracy.

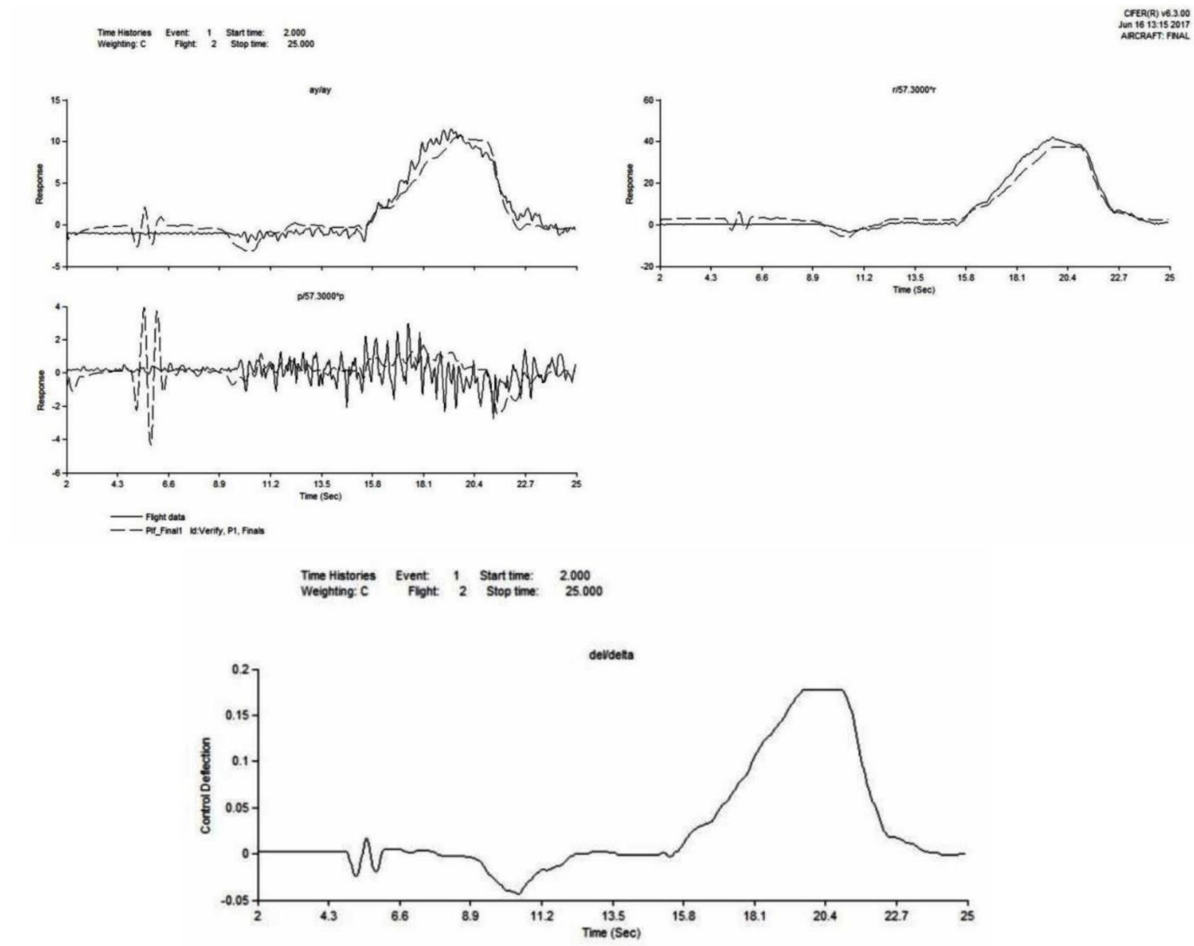


Fig: 8 Plots overlaying the model and the time domain verification data.

The responses figure indicates that the identified model has almost identical response to the data. This means that the model reduction steps process doesn’t degrade the match of the time response to the P1 data, indicating that all the reduction steps were correctly taken.

CONCLUSION

In this brief write-up we explored how frequency-based system identification can be used to model complex dynamical systems and applied the technique to Stanford P1 vehicle to identify and verify a 3-DOF lateral-directional model. This technique has been successfully used to model much more complex systems such as aircrafts and helicopters which have highly unstable response characteristics, as much as 13 degrees of freedom and strong coupling with the dynamics of airflow.

All the system identification steps in this paper were carried out with the help of Student CIFER® software and Matlab.

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