Statistical Tests - 1

Plan:

concept; Terminology:

- Quick recap (coin-toss)

Classical Stat-Tests

- one-sided vs 2-sided tests
- z-test
- T-test
- Z-proportions test
- χ^2-test
- ANOVA

Assumption

↓

hyp-testing

↓

Permutation testing (advanced)

Task: determine if coin is fair or not

$\boxed{\text{COIN-toss}}$

Expt: toss 100 times

$\boxed{T_{obs} = 65}$ let

Test-statistic (T) = # heads in 100 tosses

sensible

$H_0: T \sim 50$

$H_a: T >> 50$

H_0: coin is fair

H_a: coin is biased towards heads

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$$\text{p-val} = P\left(\text{observing } \underset{(T_{obs})}{65} \text{ heads or more in } 100 \text{ tosses} \,\middle|\, H_0\right)$$

H_0: coin is fair $\Downarrow$ $P(H) = 1/2$

$$= {}^{100}C_{65} \left(\tfrac{1}{2}\right)^{65} \left(\tfrac{1}{2}\right)^{35} + {}^{100}C_{66} \left(\tfrac{1}{2}\right)^{66} \left(\tfrac{1}{2}\right)^{34} + \dots + {}^{100}C_{100} \left(\tfrac{1}{2}\right)^{100} \left(\tfrac{1}{2}\right)^{0}$$

$$= \sum_{i=65}^{100} {}^{100}C_i \left(\tfrac{1}{2}\right)^{100}$$

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T Test-statistic = observing how many heads we get in 100 tosses

bernouli trail/expt

coin-toss: Bernouli r.v → binary → 0 success p
1 failure q=1-p

Binomial r.v → counts the number of successes in n bernouli trails

n = (100)

$$T \sim \text{Binomial}\left(n = 100;\ p = \tfrac{1}{2}\right)$$

Test-statistic disb under H_0

param { coin-toss → $T \sim \text{Binomial}(n=100; p=1/2)$ under H_0

non-param { KS-test → $T = \sup(\text{gap b/w CDFs}) \sim$ Kolgomorov distb

non-param { Permutation test
– donot have the theoritical distb of test-statistic

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$$\checkmark \left\{ T \sim \mathrm{Bin}(n = 100;\ p = 1/2) \right.$$

$$\text{p-val} = P(\overbrace{T \geq 65}) = 1 - P(\overbrace{T \leq 64})$$

$$= 1 - \mathrm{CDF}(T = 64) = 0.17\%$$

$$P(\text{obs} \geq 65 \text{ heads in } 100 \text{ tosses} \mid \text{coin is fair}) = 0.17\%$$

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$\alpha = 5\%$ (default)

$p\text{-val} = 0.17\% < \alpha$

$\Downarrow$

reject null hyp $\Rightarrow$ accept H_a: coin is biased towards heads

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Visually :

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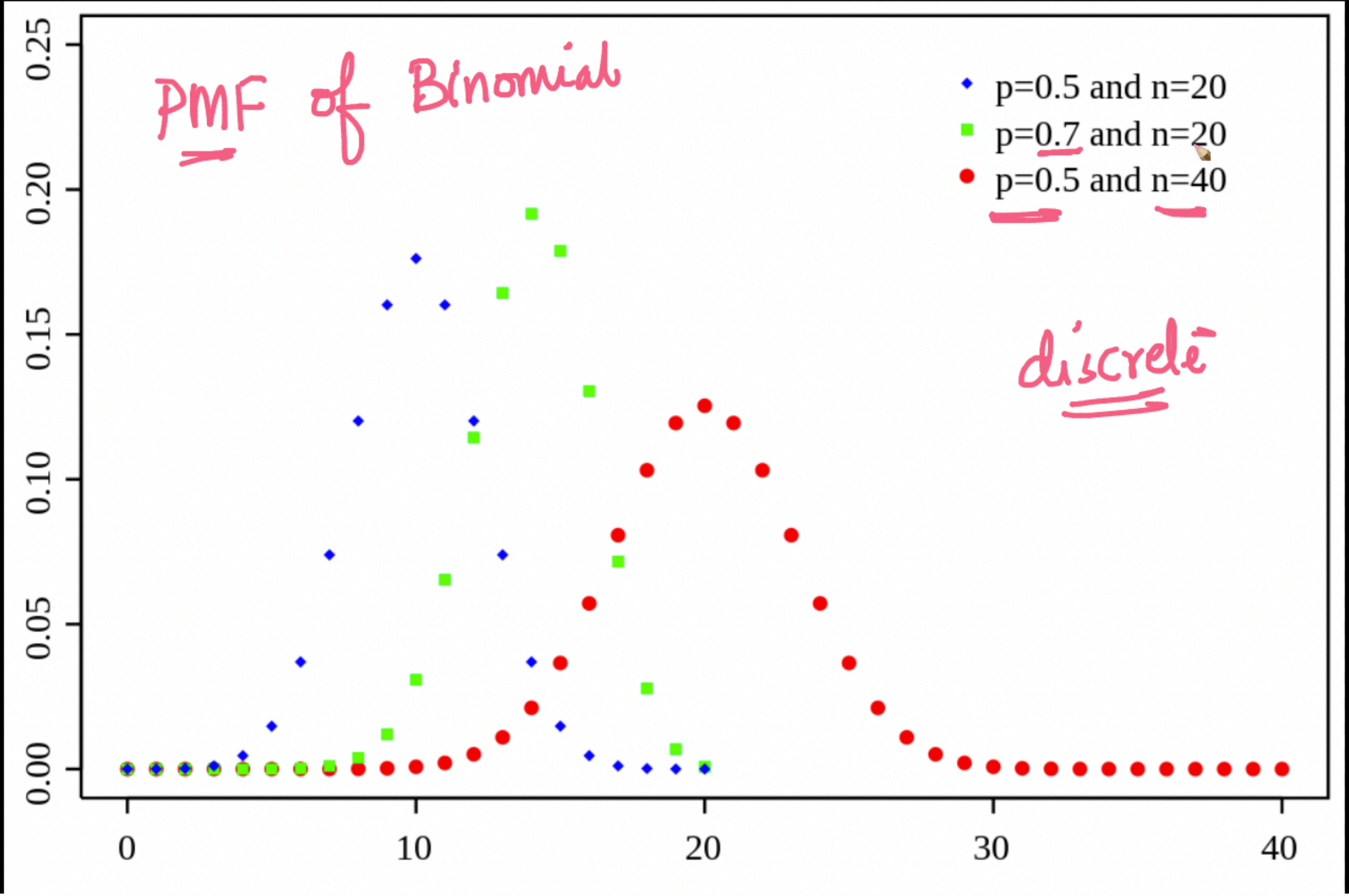


Probability mass function for the binomial distribution

More details

Tayste - Own work

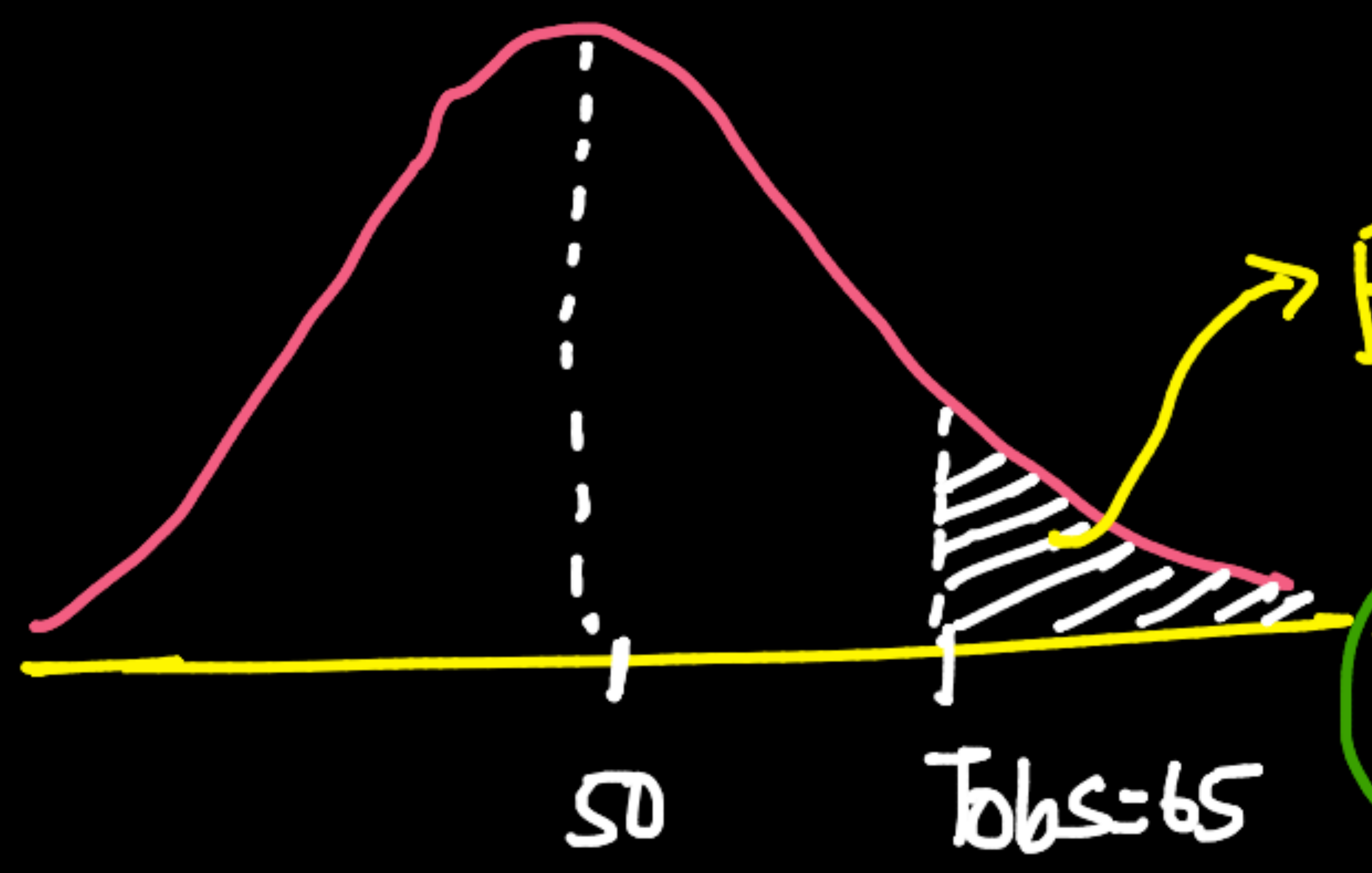
Public Domain

one-side

Not-Gaussian

Ha: coin is biased towards heads

PMF



p-value

$T \sim \text{Bin}(n=100, p=0.5)$ under H_0

p-val < α
Reject H_0
else
Reject H_a

$$P(T \geq T_{obs} \mid H_0) = \text{p-value}$$

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One-side (Right) → compute p-val

→ Ha

→ T

One-sided tests

Right-sided tests

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Alt:

✓ H_0: coin is fair

Expt: 100 tosses

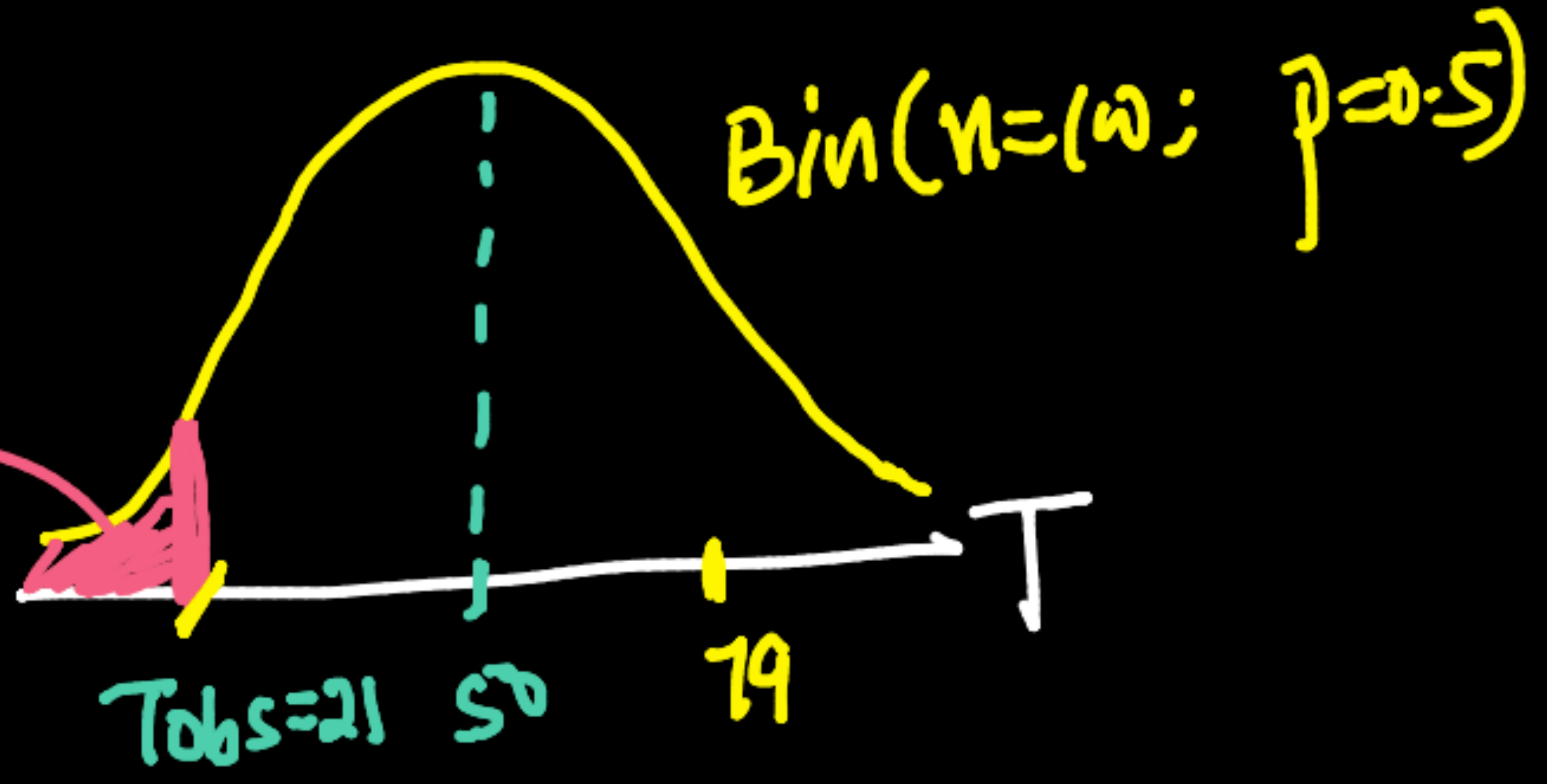
changed: H_a: coin is biased towards tails

✓ T: # heads in 100 tosses

$T_{obs} = 21$ heads (let)

$T \sim \text{Binomial}(n=100;\ p=0.5)$ under H_0

p-val



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H_a

$$\text{p-val} = P(\text{observing } 21 \text{ or fewer heads in 100 tosses} \mid H_0)$$

$$P(\text{observing an extreme val Toss} \mid H_0)$$

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H_a & T

p-val $\Rightarrow$ left-sided test

p-val $< \alpha$

reject H_0

else reject H_a ✓

→ revisiting & rewatching

~3

Upload ←

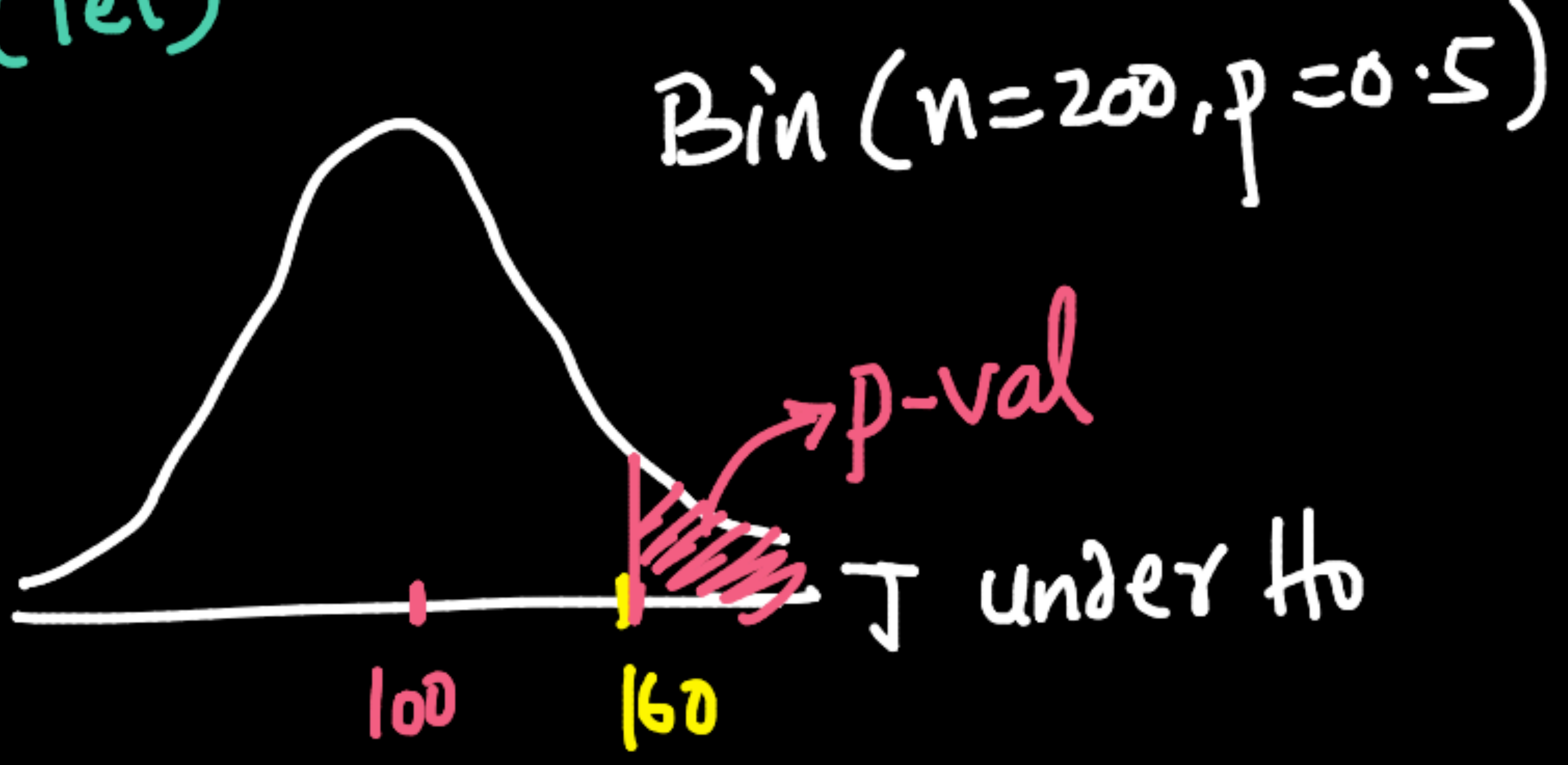
Expt: 200 coin tosses

H_0: coin is fair

H_a: coin is biased towards tails

T: # tails in 200 tosses

$T_{obs} = 160$ (let)



$$\text{p-val} = P(T \geq 160)$$

Right sided test

H_0: coin is fair

H_a: coin is unfair $\rightarrow H$, $\rightarrow T$

Expt: 100 coin tosses

$T_{obs} = 30$

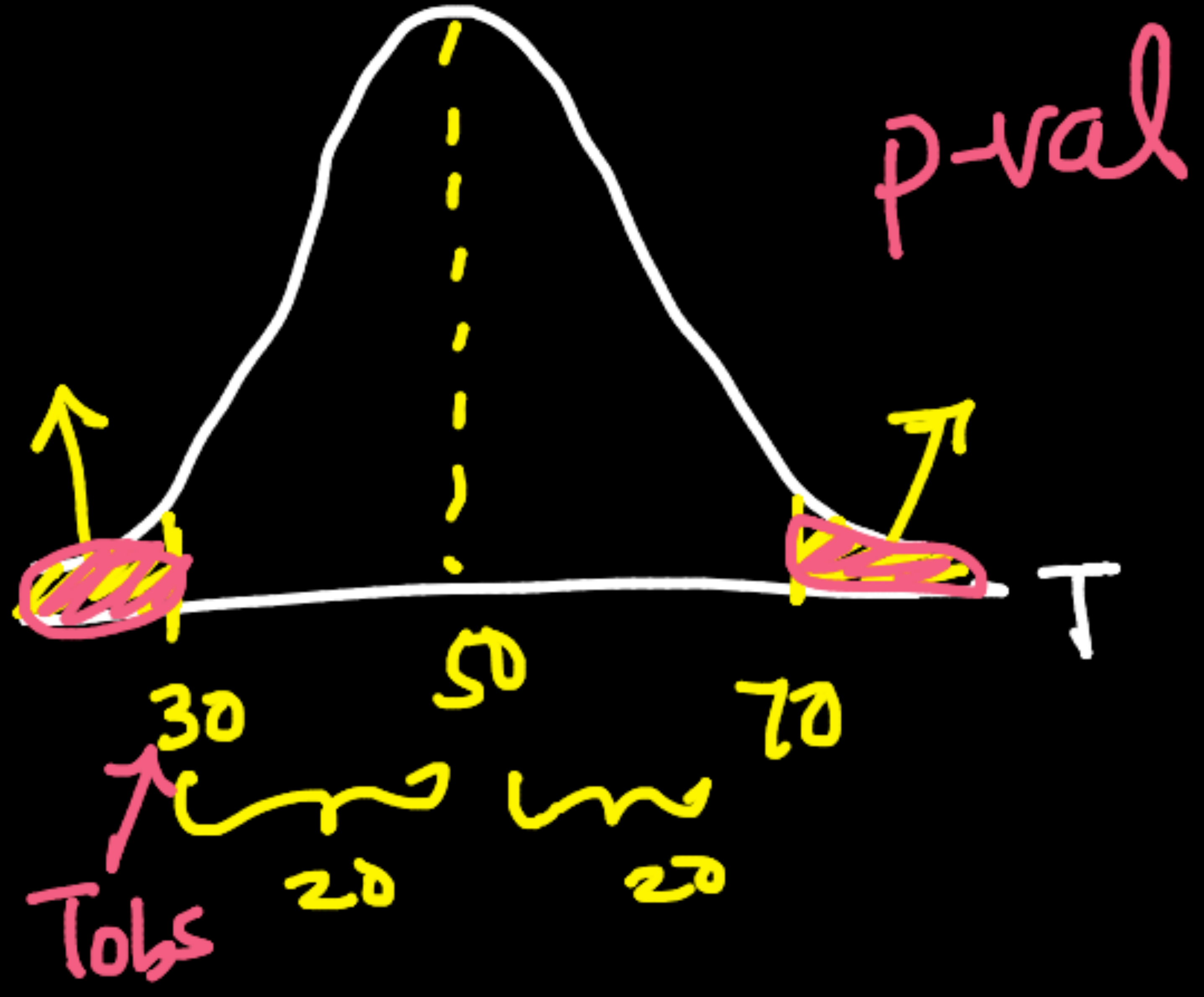
2-sided

T = # heads in 100 tosses

↳ $\sim \text{Bin}(n=100;\ p=0.5)$ under H_0

p-val = P(obs a value as extreme as 30 | H_0)

$= P(T \geq 70 \text{ or } T \leq 30 \mid H_0)$



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Futile

H_0: coin is unfair

$p(H) \neq 0.5$

$\hookrightarrow 0.1, 0.12, 0.13,$
$0.96, 0.53 \ldots$

(∞-many)

T = # heads in 100 tosses

$\hookrightarrow \text{Bin}(n=100;\ \cancel{p=?})$

$P(T \geq T_{obs} \mid H_0)$

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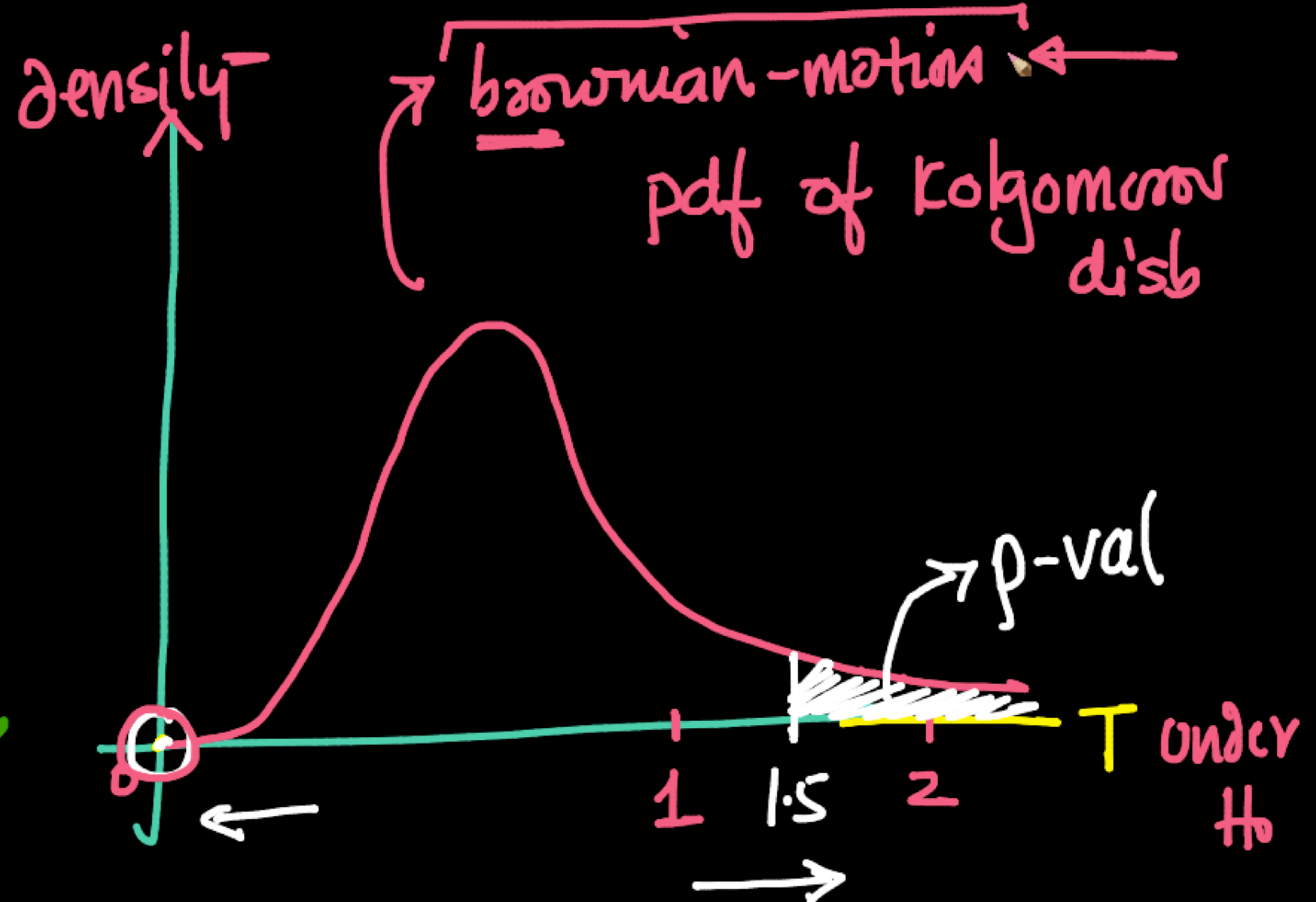
KS-test

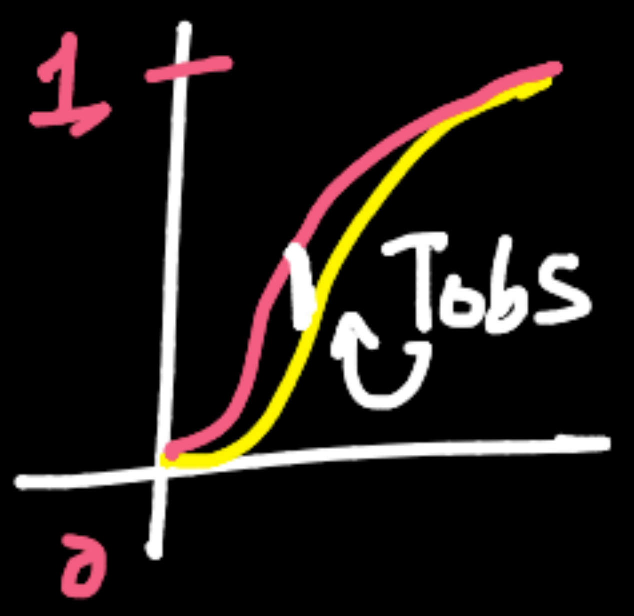
✓ $T = \sup(|\text{gap b/w CDFs}|)$

Ho: X & Y have same disb

Ha: X & Y donot same disb



right-sided



$$p\text{-val} = P(T \geq T_{obs})$$

How to choose H_0:

H_0: $\longrightarrow$ simulate using permutation testing

$\longrightarrow$ dist^n T under H_0

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pfizer: covid

GBDT ~~X~~

↓

classification

AB Tests

100 M1 → T: #days to recovery

(dummy) M0 → placebo

100

age, gen ...

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Coin-toss: $T_{obs} = 65$ in 100 tosses

$$\text{p-val} = p(\text{observing an extreme value of } T \mid H_0)$$

T_{obs}

$\alpha = 5\%$

H_0

coin is fair

$H_a \Rightarrow p(H) = 0.51, 0.52 \ldots$

coin is biased towards heads

65 heads in 100 tosses $\Rightarrow 0.17\%$

1st → breadth & basics

then deeper (later)

live-sessions (optional)

hyp-testing

A/B Tests

AB Testing platform

2-sided:

H_0: coin is fair

H_a: coin is biased towards heads → $

→ Right-sided

T = #heads in 100 tosses

H_a: unfair
H
T

2-sided

p-val = 6% > 5%

↓

accept H_0

p-val: 3%

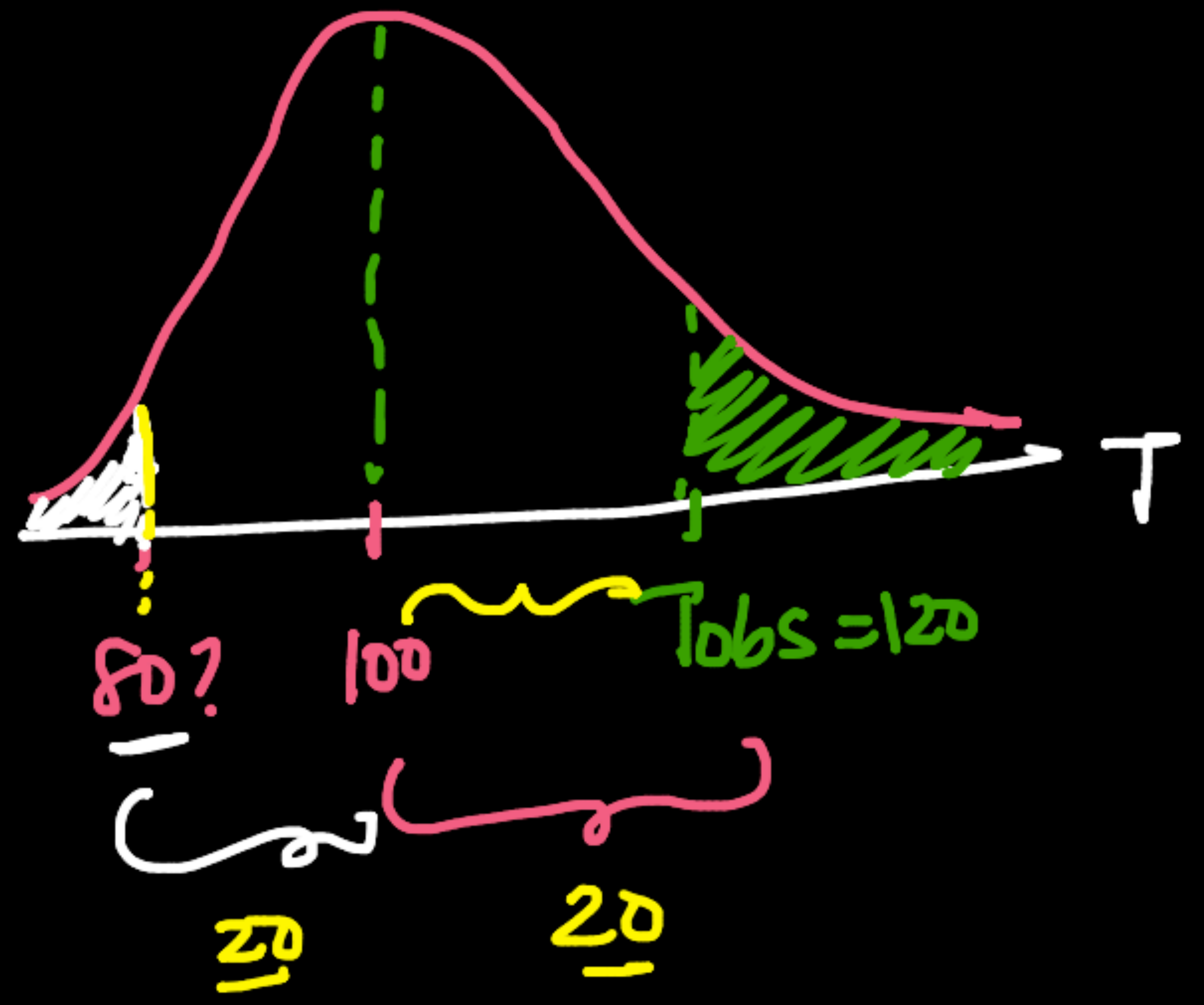
$\alpha = 5\%$

accept H_a

T

non-symmetric

2-sided



Linear-regression

Stats

DL

Bayesian DL }

z test
T test

Height

{disb of the population

sample means are Gaussian

[disb of Test statistic

$$T = \frac{M_1 - \mu}{\sqrt{\frac{S_1^2}{n_1} + \frac{\sigma_1^2}{n_2}}}$$

μ, σ

$H_0: M_1 = \mu$

$H_a: M_1 \neq \mu$

H_0

$T \rightarrow$ Simulate H_0

$\rightarrow$ distb of T under H_0

H_0

H_a

p-val = 0.17%

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① T-test

n is small

sample-sizes

n = 25

②

Permutation testing

n is large

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$$p\text{-value} < \alpha$$

$$0.17\% < 5\%$$

$$p(\text{obs} \mid H_0)$$

H_0

0.17%

H_a ✓