

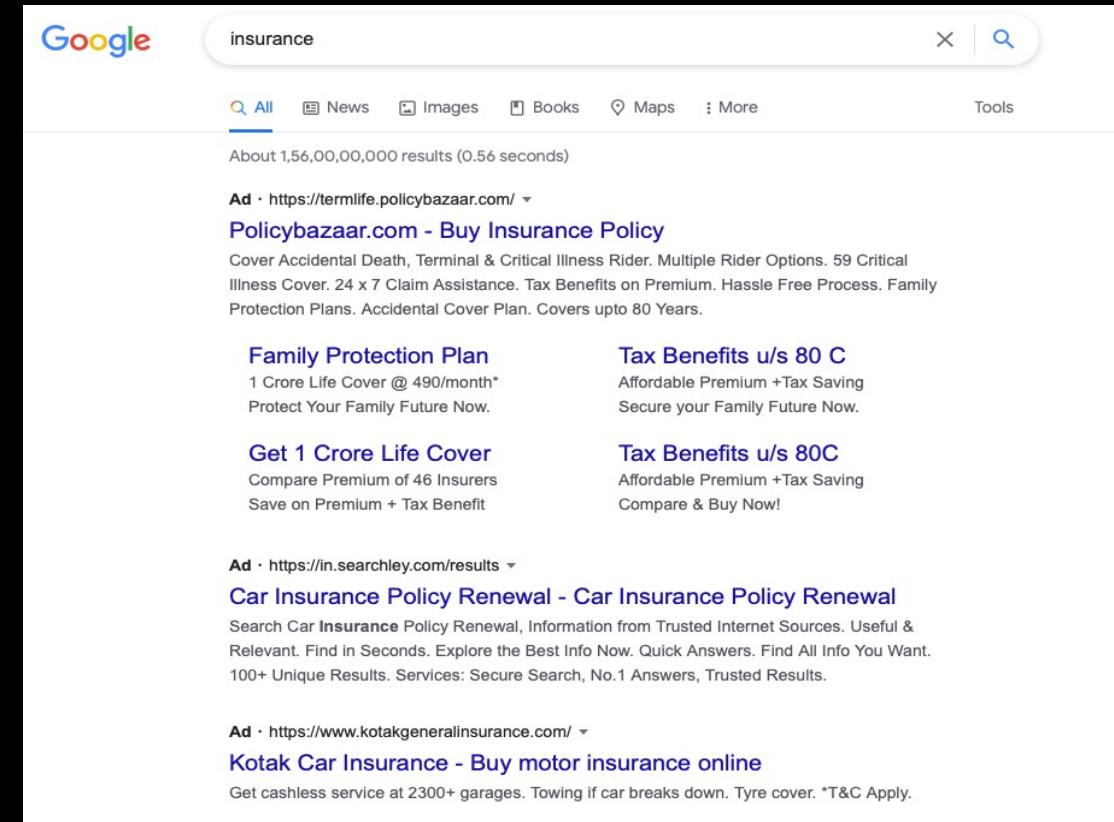
Attribution Models

Applied AI Course

"Half the money I spend on advertising  
is wasted; the trouble is I don't know  
which half" - John Wanamaker

# Digital marketing

ad-click vs ad-impr



# Digital marketing

## search vs display ads

The screenshot shows the homepage of The Times of India. At the top, there's a large banner for 'A SAFE SPACE FOR CHILDREN, ESPECIALLY GIRLS CAN HELP THEM REWRITE THEIR FUTURE' with a 'SUPPORT NOW' button. Below it, there are several news headlines: 'Blip in 2nd Covid wave? Maha caseload in first 10 days of July over 79,500', 'Live: India far better prepared to tackle 3rd Covid wave', 'When education went online, 80% schools didn't have internet', 'India pulls officials from Kandahar as Taliban widen control', 'UP: BJP claims win on 648 out of 825 block pramukh seats', and 'Gr Noida: Doctor falls to death' from 14th-floor flat. On the right side, there's an advertisement for 'NATIONAL RAIL & TRANSPORTATION INSTITUTE (NRTI)' with details about admissions and programs like BBA, BSC, PGDM, and PGC. There are also smaller ads for 'Save the Children'.

This screenshot shows a Google search results page for the query 'insurance'. The search bar at the top has 'insurance' typed in. Below the search bar, there are tabs for All, News, Images, Books, Maps, More, and Tools. The main content area shows search results starting with an ad for Policybazaar.com: 'Policybazaar.com - Buy Insurance Policy'. It highlights features like Accidental Death, Terminal & Critical Illness Rider, and Family Protection Plans. Below the ad, there are two more ads: 'Family Protection Plan' and 'Tax Benefits u/s 80 C', both from Policybazaar.com. Further down, another ad for 'Car Insurance Policy Renewal' from 'in.searchley.com/results' is shown, followed by an ad for 'Kotak Car Insurance - Buy motor insurance online' from 'www.kotakgeneralinsurance.com/'. The page indicates about 1,56,00,00,000 results found in 0.56 seconds.

# Traditional marketing :

TV → prime-time  
→ TV-Series  
::

Print → Front-Page  
→ Page 3  
→ Sports page

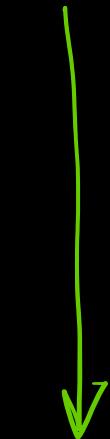
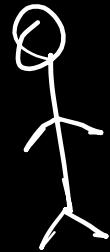
Outdoor → hoardings  
→ Buses  
;

## (Digital) Channels:

- email
- G-Search
- Youtube Video
- Youtube Search
- FB feed
- FB display ads
- Instagram Feed
- Twitter (promoted) Feed
- Amazon Search
- Amazon Display
- Google Display
- :
- :

# Marketing & Advertising:

Amazon



Time

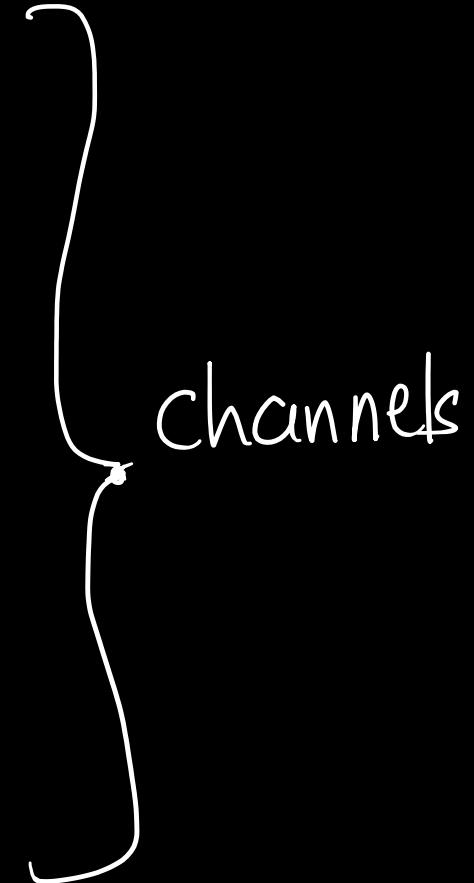
Facebook Ad Impression

Google Search Ad Impression + Click

Instagram Ad Impression

Youtube Search Ad Impression

Buys on Amazon for \$100



# Marketing & Advertising:

Amazon



Facebook Ad Impression

Google Search Ad Imp<sup>r</sup> + click R

Instagram Ad Imp<sup>r</sup>

Youtube Search Ad Imp<sup>r</sup>

Time

Buy's on Amazon

\$100

How  
much  
Contribution?

Big Ques:

How much did each ad (mp) | click  
contribute to the final purchase?

Big players:

- Google Ads & Analytics
- FB
- Amazon
- ⋮

Rule-based:

- gut-feeling driven
- clicks more important than impressions
- Search ads are more influential than display ads.



In the **Last Interaction** attribution model, the last touchpoint—in this case, the *Direct channel*—would receive 100% of the credit for the sale.



In the **Last Non-Direct Click** attribution model, all direct traffic is ignored, and 100% of the credit for the sale goes to the last channel that the customer clicked through from before converting—in this case, the *Email* channel.



In the **Last Google Ads Click** attribution model, the last Google Ads click—in this case, the first and only click to the *Paid Search* channel—would receive 100% of the credit for the sale.



In the **First Interaction** attribution model, the first touchpoint—in this case, the *Paid Search* channel—would receive 100% of the credit for the sale.



In the **Linear** attribution model, each touchpoint in the conversion path—in this case the *Paid Search, Social Network, Email, and Direct* channels—would share equal credit (25% each) for the sale.



In the **Time Decay** attribution model, the touchpoints closest in time to the sale or conversion get most of the credit. In this particular sale, the *Direct* and *Email* channels would receive the most credit because the customer interacted with them within a few hours of conversion. The *Social Network* channel would receive less credit than either the *Direct* or *Email* channels. Since the *Paid Search* interaction occurred one week earlier, this channel would receive significantly less credit.



In the **Position Based** attribution model, 40% credit is assigned to each the first and last interaction, and the remaining 20% credit is distributed evenly to the middle interactions. In this example, the *Paid Search* and *Direct* channels would each receive 40% credit, while the *Social Network* and *Email* channels would each receive 10% credit.

Rule-based

→ Why?

## Data - driven models

- can we decipher from raw - data ?
- many methods

Other - names for Attribution - modeling:

Multi - channel attribution

Marketing mix modeling

## Granularity of attribution:

- For each sale/conversion
- Across many sales/conversions in a day/week

Regression-based MMM:

e.g.: Spend on TV, radio, newspaper ] every week  
revenue / sales

$$x_i = \langle TV_i, Radio_i, Newspaper_i \rangle$$

$$y_i = Sales_i$$

Linear regression model:

$$Sales_i = w_0 + w_1 TV_i + w_2 Radio_i + w_3 Newspaper_i$$

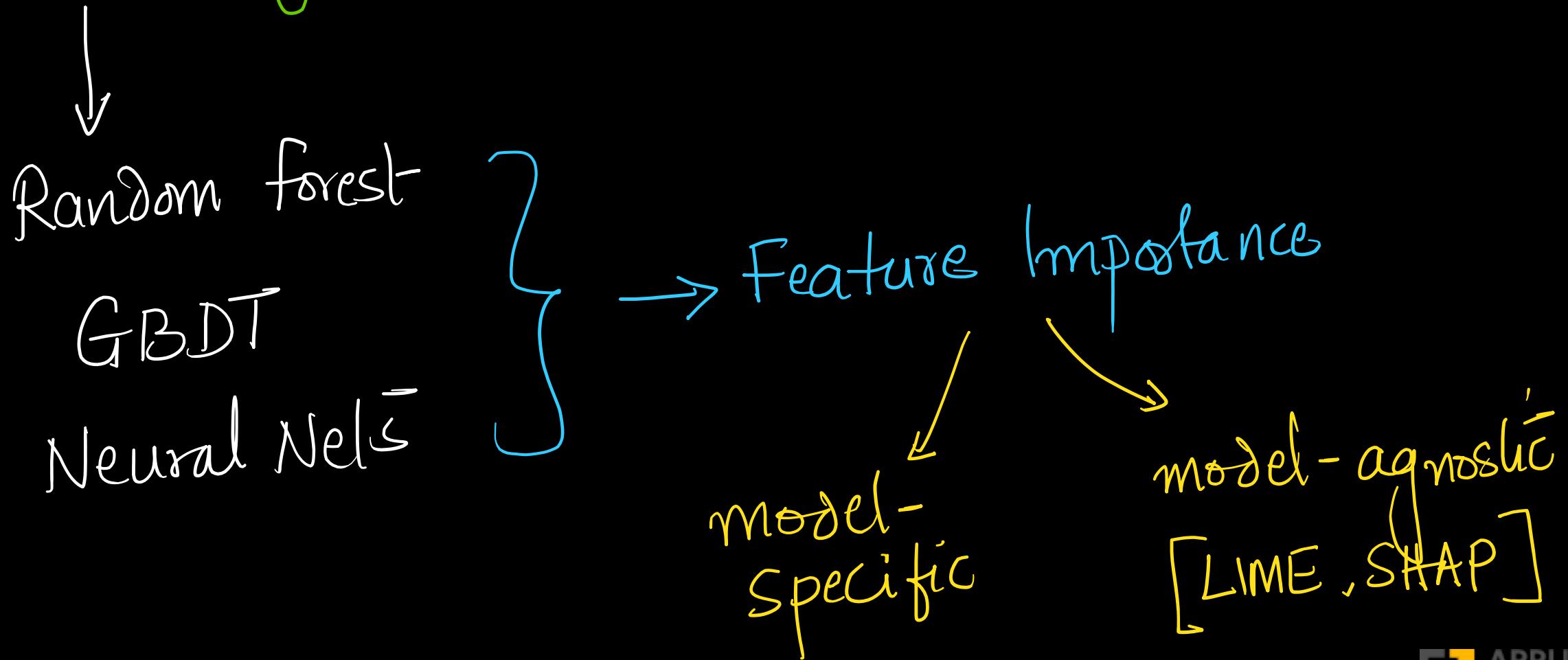
weights for attribution:

$$\text{Sales}_i = 100k + 2 \cdot TV_i + 1 \cdot Radio_i + 0.5 \cdot \text{newspaper}_i$$



a \$1 Spent on TV gives 4 times  
more sales than \$1 on newspaper

Other regression models also work



Problems:

Feature importances are not perfect

but are very useful

Be careful:

- time -varying sales (due to festival(s))

aka seasonality



add seasonality features

day of week, Week of Year, isHolidays ...

- Carry-over effects



time-gap between ads and sales

e.g.: Home, Car, smartphone, chocolate-bar

Code - example:

<https://towardsdatascience.com/building-a-simple-marketing-mix-model-with-ols-571ac3d5b64f>

# Game-Theoretic

- Shapley values [2012 Nobel prize in Economics]
- (\*) - Mathematical guarantees & very rigorous foundations
- "The Beautiful mind" movie

## → Co-operative Games

↳ economics: Team of 10 (players) build & sell a truck. How much of the value is to be attributed to each?

each channel is a player



n - players (let)

$$N = \{1, 2, 3, \dots, n\}$$

[Terminology]

A = coalition  $\subseteq N$

Characteristic Function:

$$v : 2^N \rightarrow \mathbb{R}$$

e.g.:  $A = \{1, 3, 6\}$  ,  $v(A) = \$100$

Payoff / Value generated by  
coalition A .

$v(A)$  in Attribution modeling

- Total number of conversions/sales (Simpler generated)
- $P(\text{conversion} | A)$  using an ML-model

Shapley value  
of  $i^{\text{th}}$  channel/player = How much credit  
to give to channel  $i$

given  $v(A)$  &  $N = \{1, 2, \dots, n\}$

$$\phi_i(v)$$

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|! (n - |S| - 1)!}{n!} [v(S \cup \{i\}) - v(S)]$$

↓                            ↓                                      ↓

sum                        weighted                         incremental value  
across all    due to player<sub>i</sub>  
subsets

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|! (n - |S| - 1)!}{n!} [v(S \cup \{i\}) - v(S)]$$

$$\frac{\text{---}}{n_C |S|}$$

Problem:

- Exponential time - complexity

- lots of data needed

Code Sample: <https://medium.com/data-from-the-trenches/marketing-attribution-e7fa7ae9e919>

Affibution Models

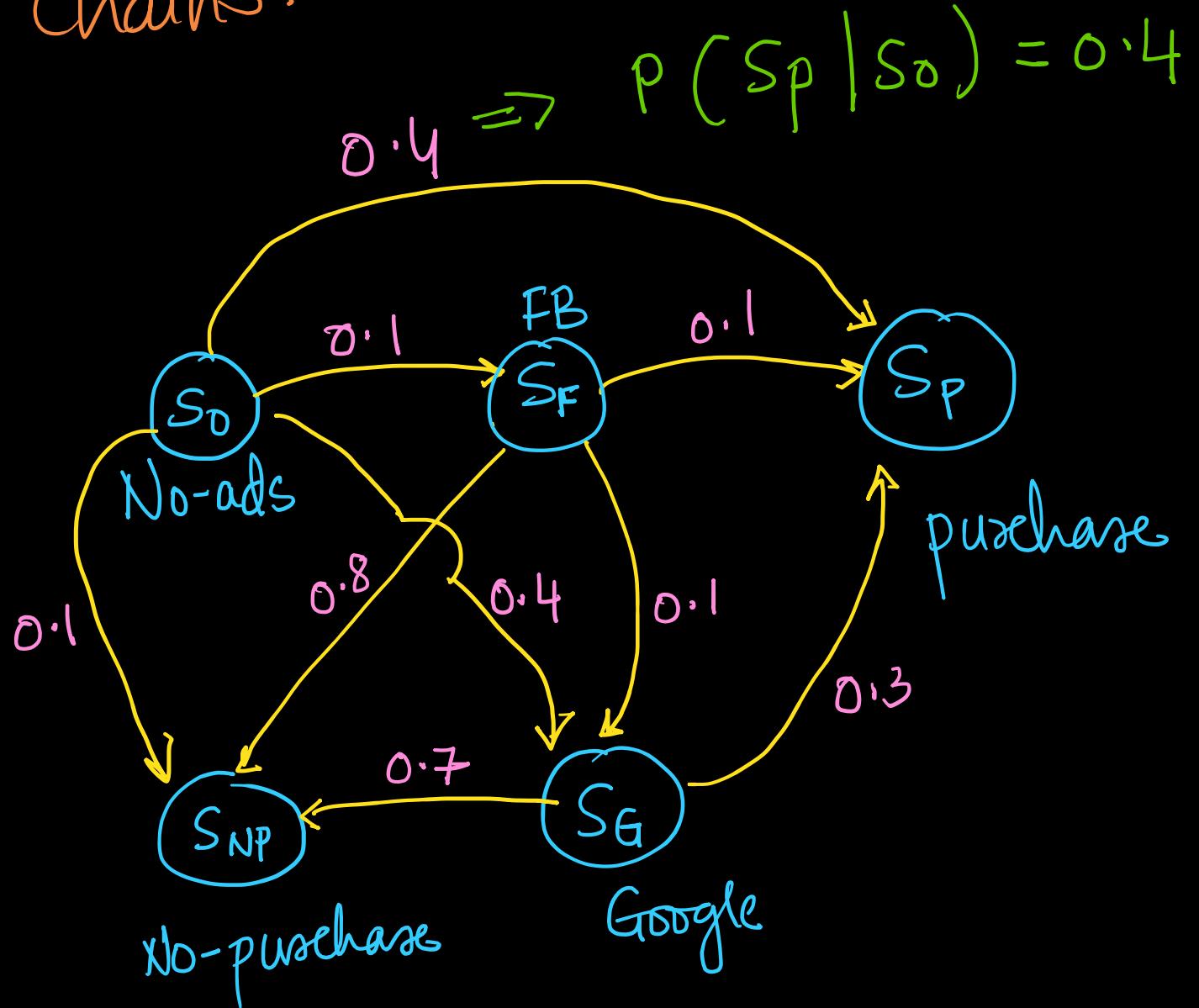
Game theoretic (SOTA)

ML-based (popular)

Markov-chains (Classical)

Survival Analysis  
(Classical)

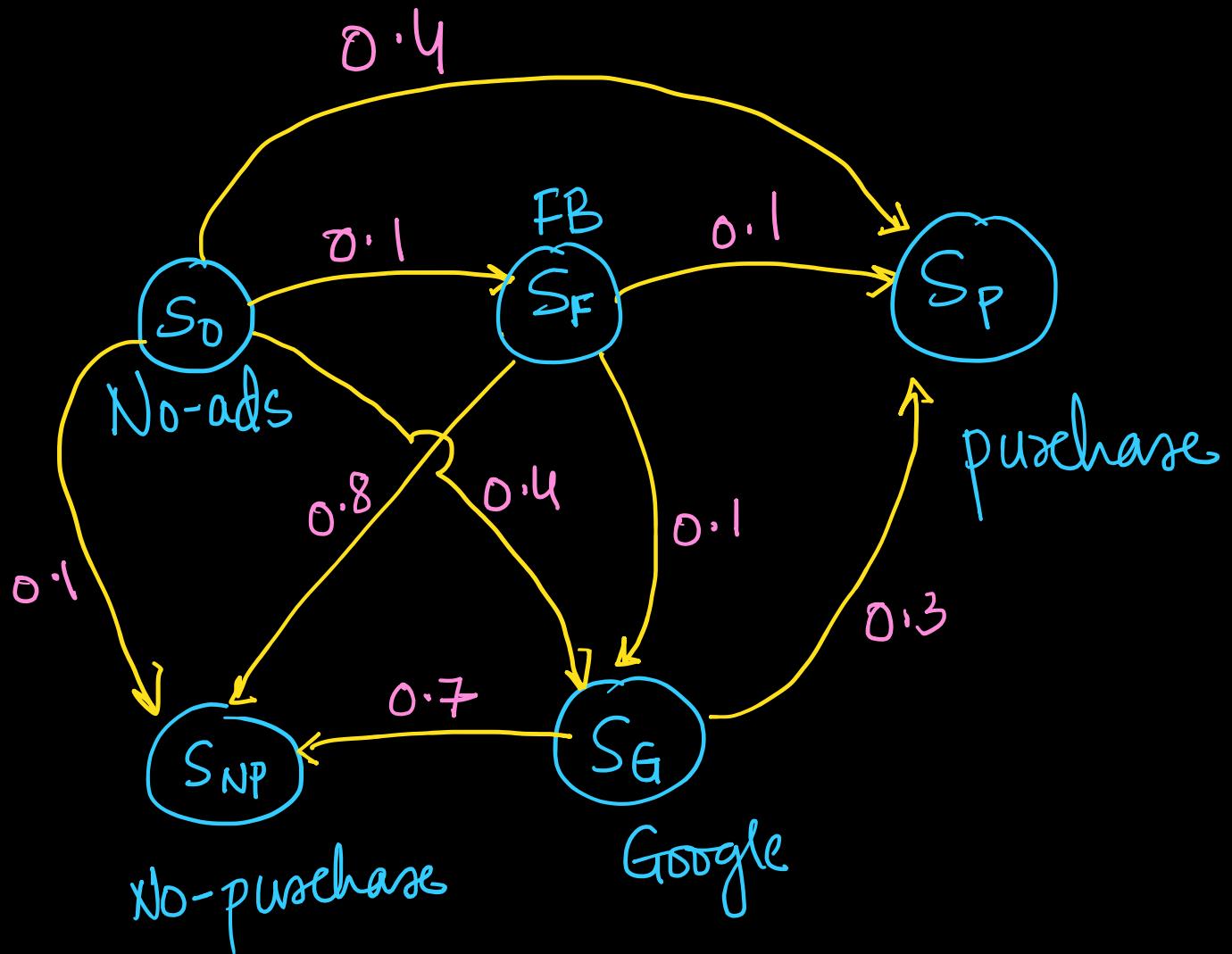
Markov chains:



Markov Property: 'memorless'

$$P(S_j | S_{j-1}, S_{j-2}, \dots, S_0) = P(S_j | S_{j-1})$$

history does not matter



$P(\text{conversion or Purchase})$

$$= 0.4 +$$

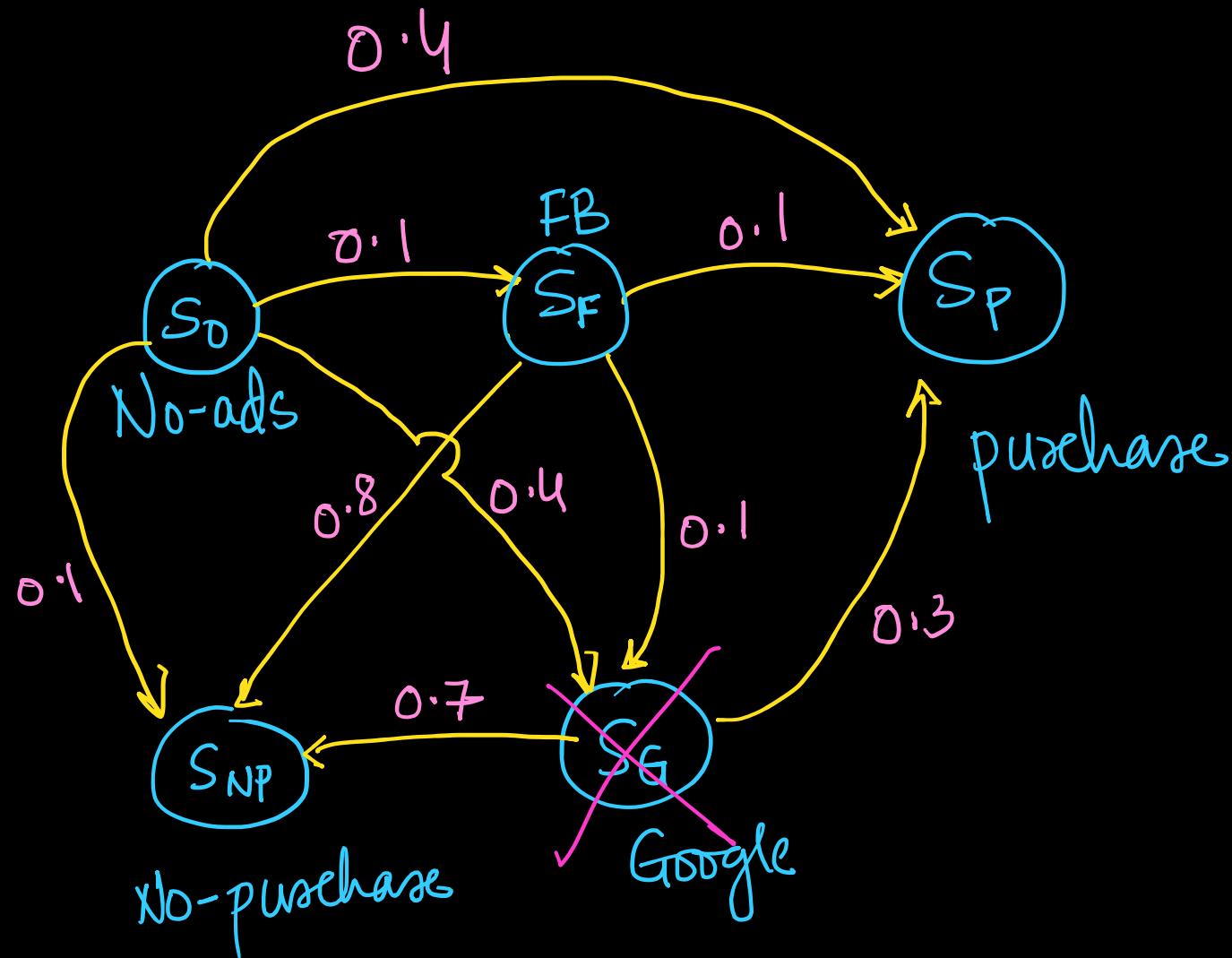
$$0.1 \times 0.1 +$$

$$0.1 \times 0.1 \times 0.3 +$$

$$0.4 \times 0.3$$

$$= 0.533$$

Removal effect of SG



$P(\text{purchase})$

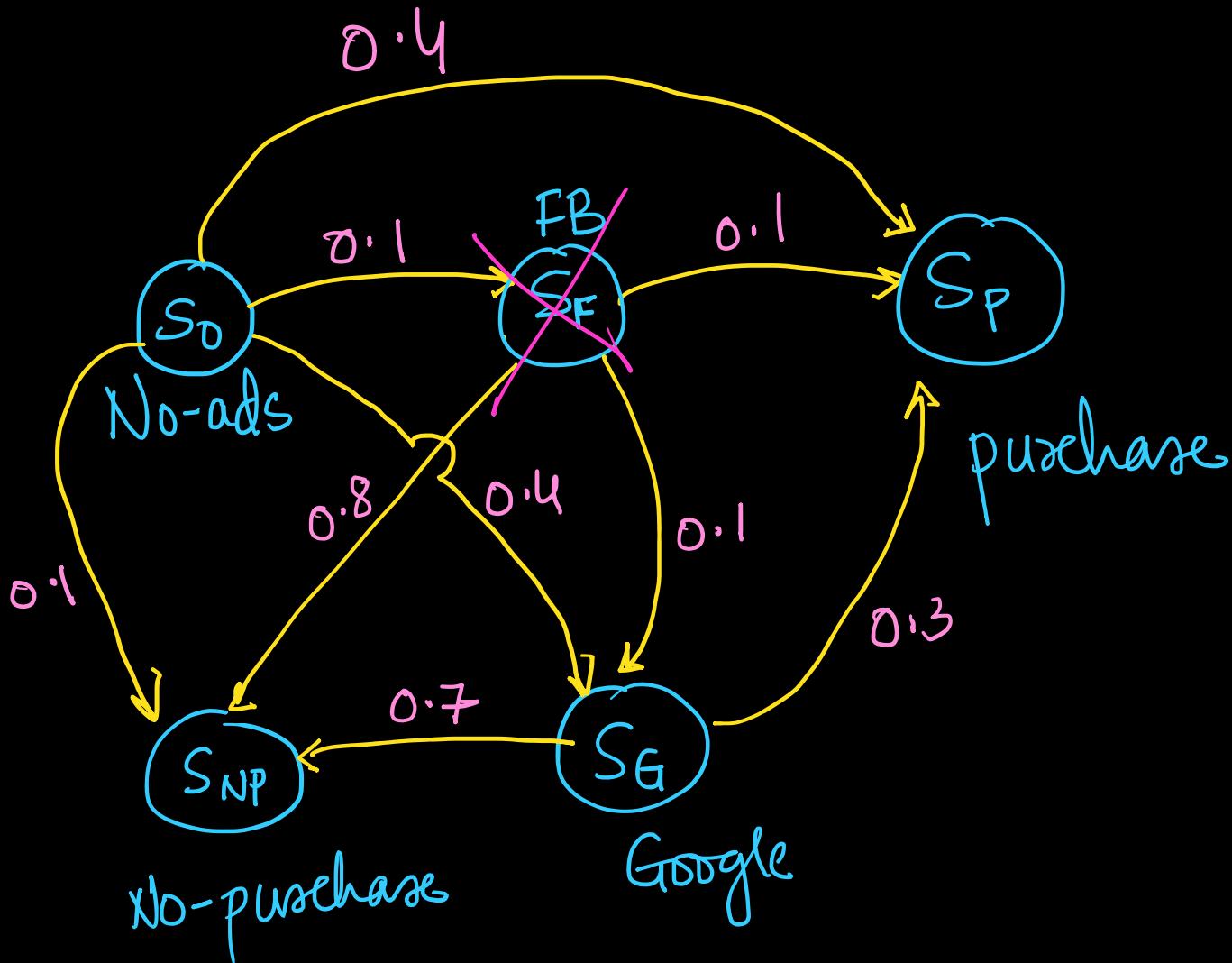
$$\begin{aligned} &= 0.4 + \\ &\quad 0.1 \times 0.1 \\ &= 0.41 \end{aligned}$$

Effect of removing  $S_G = 1 - \left( \frac{0.41}{0.533} \right)$

What %age of conversions  
would be lost if we  
donot use Google-ads

$$= 0.2307$$

Removal effect of  $S_F$



$P(\text{purchase})$

$$= 0.4 +$$

$$0.4 \times 0.3$$

$$= 0.52$$

Effect of removing  $S_F = 1 - \left( \frac{0.52}{0.533} \right)$

What %age of conversions  
would be lost if we  
don't use FB-ads

$= 0.024$

$$\Rightarrow \text{Direct} \rightarrow 100 - (23.01 + 2.4) = 74.53\%$$

Google Ads  $\rightarrow$  23.01%

FB ads  $\rightarrow$  2.4%

What's the drawback of Hasse chains?

$$P(s_j | s_{j-1}, s_{j-2}, \dots, s_0) = P(s_j | s_{j-1})$$

(

ignoring all these

Advantage of Markov chains

Computationally easy → Paths in  
a Graphs

remove &  
recompute  
probabilities

Trade off between

computation & rigour

Markov chains

Shapley values

Dataset + Code

<https://towardsdatascience.com/marketing-channel-attribution-with-markov-chains-in-python-part-2-the-complete-walkthrough-733c65b23323>

# Survival Analysis for Attribution.

↳ healthcare analytics

→ survival of a patient

→ In marketing : Conversion

'Death' in Health  
case  
↑  
event of interest

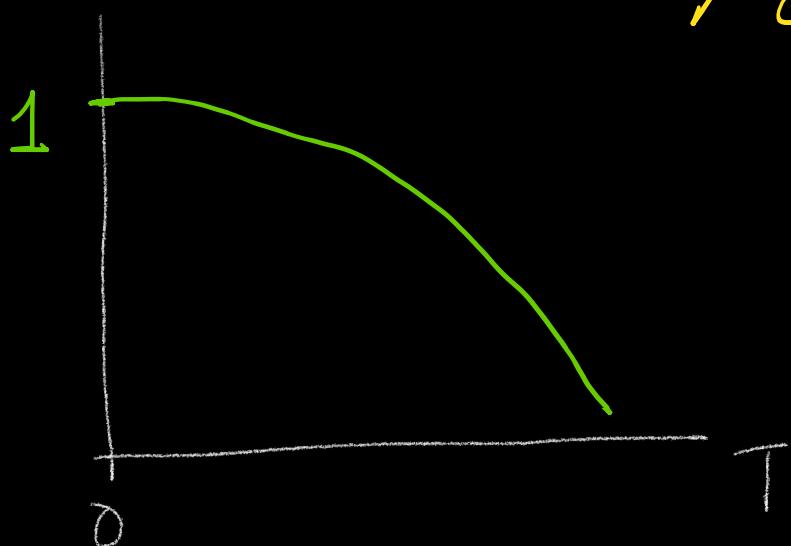
Survival Function:

$$S(t) = P(T > t) \rightarrow \text{Probability of Survival @ time } t$$

Note  $T \geq 0$

$$S(t) = P(T > t) = 1 - F(t)$$

$$\Rightarrow S(t) = 1 - P(T \leq t)$$



$$S'(t) = -f(t)$$

↳ PDF

Hazard Funktion:

$$\lambda(t) = \lim_{dt \rightarrow 0} \frac{P(t \leq T \leq t+dt \mid T > t)}{dt}$$

$$\begin{aligned}
 \lambda(t) &= \lim_{dt \rightarrow 0} \frac{Pr(t \leq T \leq t + dt | T > t)}{dt} \\
 &= \lim_{dt \rightarrow 0} \frac{Pr(t \leq T \leq t + dt) / Pr(T > t)}{dt} \\
 &= \lim_{dt \rightarrow 0} \frac{(F(t + dt) - F(t)) / S(t)}{dt} \\
 &= \frac{f(t)}{S(t)} \\
 &= -\frac{S'(t)}{S(t)}.
 \end{aligned}$$

PDF →  $f(t)$   
 CDF →  $F(t + dt) - F(t)$

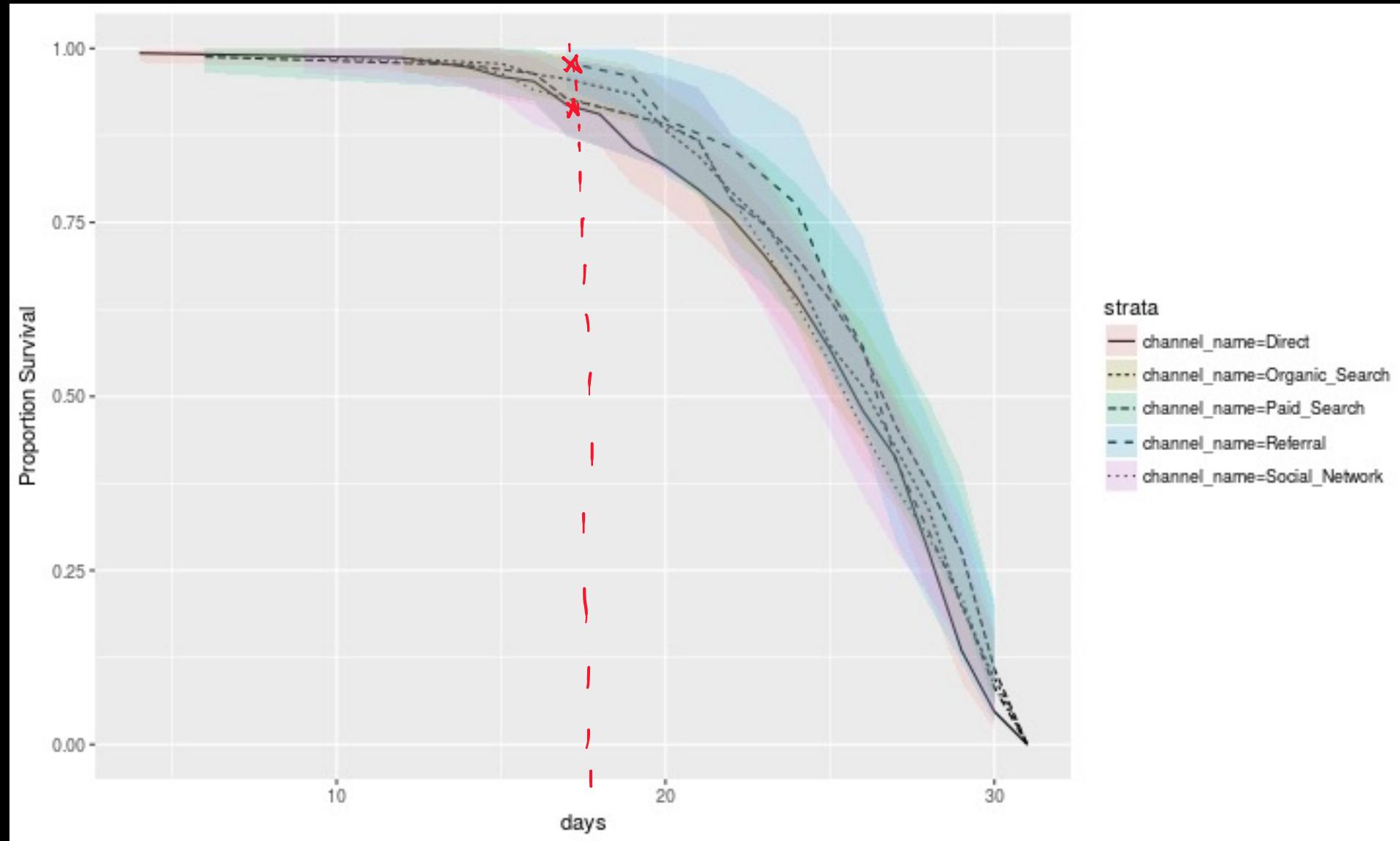
$$\frac{d \log(g(t))}{dt} = \frac{g'(t)}{g(t)}$$

So,  $\lambda(t) = -\frac{d}{dt} \log(s(t))$

$$\Rightarrow s(t) = \exp\left(-\int_0^t \lambda(x) dx\right)$$

if we know  $x(t)$ , we can compute  
 $s(t)$

Univariate



# Cox - Proportional Hazards Regression Model:

$$\lambda(t|x) = b_0(t) \cdot \exp \left\{ \sum_{i=1}^d b_i x_i \right\}$$

↓

Co-variables  
or feature-vec

baseline  
Hazard  
with  $x = \vec{0}$

↓

partial  
Hazards

$$\lambda(t|x) = b_0(t) \cdot \exp \left\{ \sum_{i=1}^d b_i x_i \right\}$$



multiplicative linear-model

$b_i$ 's: importance of each channel

Regression + feature importance  
in the Survival analysis

Framework.







