8E and 8F: Finding the Probability P(Y==1|X)

8E: Implementing Decision Function of SVM RBF Kernel

After we train a kernel SVM model, we will be getting support vectors and their corresponsing coefficients α_i

Check the documentation for better understanding of these attributes:

https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html

As a part of this assignment you will be implementing the `decision_function()` of kernel SVM, here decision_function() means based on the value return by `decision_function()` model will classify the data point either as positive or negative

Ex 1: In logistic regression After traning the models with the optimal weights w we get, we will find the value $\frac{1}{1+\exp(-(wx+b))}$, if this value comes out to be < 0.5 we will mark it as negative class, else its positive class

Ex 2: In Linear SVM After traning the models with the optimal weights w we get, we will find the value of $sign(wx+b)$, if this value comes out to be -ve we will mark it as negative class, else its positive class.

Similarly in Kernel SVM After traning the models with the coefficients α_i we get, we will find the value of $sign(\sum_{i=1}^{n}(y_i\alpha_i K(x_i,x_q)) + intercept)$, here $K(x_i,x_q)$ is the RBF kernel. If this value comes out to be -ve we will mark x_q as negative class, else its positive class.

RBF kernel is defined as: $K(x_i,x_q) = exp(-\gamma ||x_i - x_q||^2)$

For better understanding check this link: https://scikit-learn.org/stable/modules/svm.html#svm-mathematical-formulation

Task E

> 1. Split the data into X_{train}(60), X_{cv}(20), X_{test}(20)
> 2. Train $SVC(gamma=0.001, C=100.)$ on the (X_{train}, y_{train})
> 3. Get the decision boundry values f_{cv} on the X_{cv} data i.e. f_{cv} `= decision_function(` X_{cv} `)` you need to implement this decision_function()

In [59]:
```
import numpy as np
import pandas as pd
from sklearn.datasets import make_classification
import numpy as np
from sklearn.svm import SVC
import math
from sklearn.model_selection import train_test_split
```

In [60]:
```
X, y = make_classification(n_samples=5000, n_features=5, n_redundant=2,
                           n_classes=2, weights=[0.7], class_sep=0.7, random_state=15)
```

Pseudo code

clf = SVC(gamma=0.001, C=100.)
clf.fit(Xtrain, ytrain)

def decision_function(Xcv, ...): #use appropriate parameters
    for a data point x_q in Xcv:
        #write code to implement $(\sum_{i=1}^{\text{all the support vectors}}(y_i\alpha_i K(x_i,x_q)) + intercept)$, here the values y_i, α_i, and $intercept$ can be obtained from the trained model
return # the decision_function output for all the data points in the Xcv

fcv = decision_function(Xcv, ...) # *based on your requirement you can pass any other parameters*

Note: Make sure the values you get as fcv, should be equal to outputs of clf.decision_function(Xcv)

In [61]:
```
#train test split
X_train,X_test,y_train,y_test = train_test_split(X,y,test_size = 0.2,random_state = 0)
X_train_final,X_cv,y_train_final,y_cv = train_test_split(X_train,y_train,test_size = 0.25,random_state = 0)
#train the algorithm
print("Shape of the X_train:{0} and Y_train:{1}".format(X_train_final.shape,y_train_final.shape))
print("Shape of the X_test : {0} and y_test : {1}".format(X_test.shape,y_test.shape))
print("Shape of the Validation :{0} and Y_cv : {1}".format(X_cv.shape,y_cv.shape))
support_vector_classifier = SVC(gamma = 0.001,C = 100)
support_vector_classifier.fit(X_train_final,y_train_final)
```

```
Shape of the X_train:(3000, 5) and Y_train:(3000,)
Shape of the X_test : (1000, 5) and y_test : (1000,)
Shape of the Validation :(1000, 5) and Y_cv : (1000,)
```

Out[61]: `SVC(C=100, gamma=0.001)`

In [62]:
```
class SupportVectorClassifier:

    def __init__(self,gamma = 0.001,C = 100):

        self.gamma = gamma
        self.C = C

    def fit(self,X_train,y_train):

        model = SVC(gamma = self.gamma,C = self.C).fit(X_train,y_train)
        return model

    def compute_similarity_matrix(self,xi,xq,gamma):

        norm = np.sum(( xi - xq) ** 2 ,axis = -1)

        similarity_function  = np.exp(- (gamma) * norm)

        return similarity_function

    def kernal_gram_matrix(self,Xi,Xj,gamma):
        kernal_matrix_similarity = np.zeros((Xi.shape[0],Xj.shape[0]))
        for idx,row in enumerate(Xi):
            for idx_col,column in enumerate(Xj):
                rbf_kernal = self.compute_similarity_matrix(row,column,self.gamma)
                kernal_matrix_similarity[idx][idx_col] = rbf_kernal
        return kernal_matrix_similarity

    def decision_function(self,XCV,support_vectors,dual_coeff,intercept):

        #compute the kernal similarity Matrix
        kernal_gram_matixes= self.kernal_gram_matrix(XCV, support_vectors,self.gamma)

        #yi* alpha i * kernal(xi,xq)
        decision_fun = dual_coeff * kernal_gram_matixes
        decision_custom = np.sum(decision_fun,axis = -1) + intercept

        return decision_custom.reshape(-1,1)
```

In [63]:
```
svc = SupportVectorClassifier(gamma = 0.001,C = 100)
support_vector_classifier = svc.fit(X_train_final,y_train_final)
#intercept b
intercept = support_vector_classifier.intercept_

#dual_coefficient yi*alphai

svm_dual_coeff_ = support_vector_classifier.dual_coef_

#get the support_vector
support_vector = support_vector_classifier.support_vectors_

f_cv = svc.decision_function(X_cv,support_vector,svm_dual_coeff_,intercept)
```

In [64]:
```
sk_df = support_vector_classifier.decision_function(X_cv)
sk_df = sk_df.reshape(-1,1)
```

In [65]:
```
#check the difference between the custom predction and actual predictiom
difference = f_cv - sk_df
differece_error = [i  for i in range(len(difference)) if difference[i] > 1e-10]
print(differece_error)
```

```
[]
```

8F: Implementing Platt Scaling to find P(Y==1|X)

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TASK F

> 1. Apply SGD algorithm with (f_{cv}, y_{cv}) and find the weight W intercept b `Note: here our data is of one dimensional so we will have a one dimensional weight vector i.e W.shape (1,)`
>
> Note1: Don't forget to change the values of y_{cv} as mentioned in the above image. you will calculate y+, y- based on data points in train data
>
> Note2: the Sklearn's SGD algorithm doesn't support the real valued outputs, you need to use the code that was done in the `'Logistic Regression with SGD and L2'` Assignment after modifying loss function, and use same parameters that used in that assignment. if Y[i] is 1, it will be replaced with y+ value else it will replaced with y- value
>
> 1. For a given data point from X_{test}, $P(Y=1|X) = \frac{1}{1+exp(-(W*f_{test}+b))}$ where f_{test} `= decision_function(` X_{test} `)`, W and b will be learned as metioned in the above step

Note: in the above algorithm, the steps 2, 4 might need hyper parameter tuning, To reduce the complexity of the assignment we are excluding the hyerparameter tuning part, but intrested students can try that

In [66]:
```
#implemented Calibrated CV

#input actual_y -> function(x)-> ycv
#construct a data set

def label_count(Y):
    positive_label,negative_label = 0,0
    for label in range(len(Y)):
        if Y[label] == 1:
            positive_label += 1
        else:
            negative_label +=1
    assert((positive_label +negative_label) == len(Y) )
    return positive_label,negative_label
no_of_positive_point,no_of_negative_point = label_count(y_train_final)
print("Positive Label : {0} and Negative Label : {1}".format(no_of_positive_point,no_of_negative_point))
y_positive,y_negative = (no_of_positive_point + 1) / (no_of_positive_point + 2), 1 / (no_of_negative_point + 2)
print("Calibrated positive point:{0} and Calibrated Negative Point : {1}".format(y_positive,y_negative))
```

```
Positive Label : 906 and Negative Label : 2094
Calibrated positive point:0.998898678414097 and Calibrated Negative Point : 0.00047709923664122136
```

In [67]:
```
#In platt Scaling We build a new validation dataset- create a bucket and computing the averagefrom our case have no of points in low we directly apply y_avg
def average_y(Y,y_positive,y_negative):

    y_bucket = []
    for label in range(len(Y)):
        if Y[label] == 1:
            y_bucket.append(y_positive)
        else:
            y_bucket.append(y_negative)
    return y_bucket
y_cv_modified = average_y(y_cv,y_positive,y_negative)
```

In [68]:
```
class SGD:
    def __init__(self,learning_rate,epochs):

        self.learning_rate = learning_rate
        self.epochs = epochs

    def sigmoid(self,z):
        sigmoid_of_Z= 1 / (1 + np.exp(-z))
        return sigmoid_of_Z

    def gradient(self,x,y,w,b,alpha,N):
        dw = ((x * ((y - self.sigmoid(np.dot(w,x) + b))) - ((alpha / N) * w)))
        db = y - self.sigmoid(np.dot(w,x) + b)
        return dw,db
    def logloss(self,y_true,y_pred):

        loss = 0
        n = len(y_true)
        if(len(y_true) == len(y_pred)):
            for l in range(len(y_true)):

                loss += (y_true[l] * math.log10(y_pred[l])) + ((1-y_true[l]) * math.log10(1-y_pred[l]))

            loss = (-1 / n) * loss

        return loss

    def fit(self,X,Y,alpha):

        loss_list = []
        wait = 0
        #initialize weight
        w = 0
        b = 0
        print(self.learning_rate)
        for epoch in range(1,epochs+1):

            for data in range(len(X)):
                #compute gradient
                dw,db= self.gradient(X[data],Y[data],w,b,alpha,len(X))
                #update weight
                w = w +  self.learning_rate * dw
                #update bias
                b = b +  self.learning_rate * db
            y_pred  = [self.sigmoid(np.dot(w,X[data]) + b) for data in range(len(X))]
            loss = self.logloss(Y,np.array(y_pred))

            loss_list.append(loss)
            if epoch >= 2:
                if loss_list[epoch-2]  == loss_list[epoch-1]:
                    print("No improvement in training")
                    break

            print('Epoch--{0}'.format(epoch) )

            print("Training Loss  : {0} ".format(loss[0]))

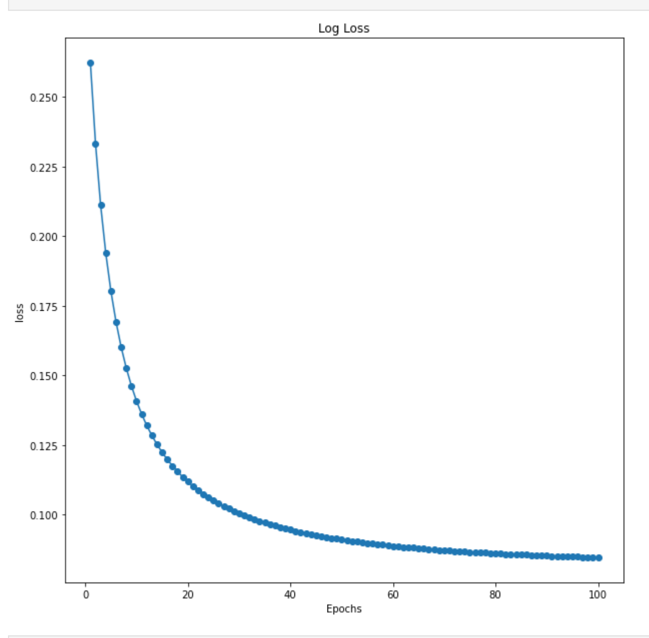
        return loss_list,w,b
    def predict(self,x,w,b):
        z = np.dot(w,x) + b
        return self.sigmoid(z)
```

In [69]:
```
learning_rate = 1e-4
epochs = 100
alpha=0.0001
y_cv_modified = np.array(y_cv_modified).reshape(-1,1)
sgd = SGD(learning_rate,epochs)
```

In [70]:
```
loss_list,w,b= sgd.fit(f_cv,y_cv_modified,alpha)
print("Optimized W:{0} and b:{1}".format(w,b))
```

```
0.0001
Epoch--1
Training Loss  : 0.26234390399314991
Epoch--2
Training Loss  : 0.23328949504531726
Epoch--3
Training Loss  : 0.21112989001987448
Epoch--4
Training Loss  : 0.19390095342750988
Epoch--5
Training Loss  : 0.1802395179581706
Epoch--6
Training Loss  : 0.16920443188047013
Epoch--7
Training Loss  : 0.16014004963550305
Epoch--8
Training Loss  : 0.15258251814728516
Epoch--9
Training Loss  : 0.14619776722588454
Epoch--10
Training Loss  : 0.14074077868139112
Epoch--11
Training Loss  : 0.13602866186090992
Epoch--12
Training Loss  : 0.1319226272846691
Epoch--13
Training Loss  : 0.12831572881244926
Epoch--14
Training Loss  : 0.12512439016558863
Epoch--15
Training Loss  : 0.12228245064105836
Epoch--16
Training Loss  : 0.11973691421734095
Epoch--17
Training Loss  : 0.11744486869036001
Epoch--18
Training Loss  : 0.11537122090892515
Epoch--19
Training Loss  : 0.1134870096789125
Epoch--20
Training Loss  : 0.11176813331551005
Epoch--21
Training Loss  : 0.11019437877586269
Epoch--22
Training Loss  : 0.10874867287090426
Epoch--23
Training Loss  : 0.10741649892603101
Epoch--24
Training Loss  : 0.1061854380530169
Epoch--25
Training Loss  : 0.10504480524062751
Epoch--26
Training Loss  : 0.10398535829020995
Epoch--27
Training Loss  : 0.10299906322149323
Epoch--28
Training Loss  : 0.10207890382703495
Epoch--29
Training Loss  : 0.10121872601838272
Epoch--30
Training Loss  : 0.10041310979663653
Epoch--31
Training Loss  : 0.09965726331235482
Epoch--32
Training Loss  : 0.09894693470715851
Epoch--33
Training Loss  : 0.09827833836006895
Epoch--34
Training Loss  : 0.0976480928728578
Epoch--35
Training Loss  : 0.09705316867628976
Epoch--36
Training Loss  : 0.09649084356375484
Epoch--37
Training Loss  : 0.09595866479027237
Epoch--38
Training Loss  : 0.09545441663524254
Epoch--39
Training Loss  : 0.09497609253320682
Epoch--40
Training Loss  : 0.09452187104051173
Epoch--41
Training Loss  : 0.09409009503663503
Epoch--42
Training Loss  : 0.0936792536640946
Epoch--43
Training Loss  : 0.09328796659580965
Epoch--44
Training Loss  : 0.09291497028774075
Epoch--45
Training Loss  : 0.09255910593087173
Epoch--46
Training Loss  : 0.09221930886265292
Epoch--47
Training Loss  : 0.091894599235923
Epoch--48
Training Loss  : 0.09158407377461211
Epoch--49
Training Loss  : 0.09128689847149252
Epoch--50
Training Loss  : 0.09100230210482986
Epoch--51
Training Loss  : 0.09072957046884425
Epoch--52
Training Loss  : 0.09046804122800774
Epoch--53
Training Loss  : 0.09021709931792915
Epoch--54
Training Loss  : 0.08997617282631465
Epoch--55
Training Loss  : 0.08974472929657472
Epoch--56
Training Loss  : 0.08952227240436805
Epoch--57
Training Loss  : 0.08930833896393585
Epoch--58
Training Loss  : 0.08910249622669368
Epoch--59
Training Loss  : 0.08890433943934649
Epoch--60
Training Loss  : 0.08871348963291924
Epoch--61
Training Loss  : 0.08852959161763156
Epoch--62
Training Loss  : 0.08835231216162523
Epoch--63
Training Loss  : 0.08818133833417724
Epoch--64
Training Loss  : 0.08801637599634181
Epoch--65
Training Loss  : 0.0878571484239491
Epoch--66
Training Loss  : 0.08770339504962435
Epoch--67
Training Loss  : 0.0875548703120069
Epoch--68
Training Loss  : 0.08741134260166905
Epoch--69
Training Loss  : 0.08727259329439284
Epoch--70
Training Loss  : 0.0871384158634822
Epoch--71
Training Loss  : 0.0870086150636789
Epoch--72
Training Loss  : 0.08688300618004377
Epoch--73
Training Loss  : 0.08676141433584818
Epoch--74
Training Loss  : 0.08664367385414927
Epoch--75
Training Loss  : 0.08652962766825541
Epoch--76
Training Loss  : 0.08641912677677396
Epoch--77
Training Loss  : 0.086312029739364
Epoch--78
Training Loss  : 0.08620820220969132
Epoch--79
Training Loss  : 0.08610751650243058
Epoch--80
Training Loss  : 0.08600985119145745
Epoch--81
Training Loss  : 0.0859150907366405
Epoch--82
Training Loss  : 0.08582312513689555
Epoch--83
Training Loss  : 0.08573384960736886
Epoch--84
Training Loss  : 0.08564716427882373
Epoch--85
Training Loss  : 0.08556297391746323
Epoch--86
Training Loss  : 0.08548118766360062
Epoch--87
Training Loss  : 0.08540171878770683
Epoch--88
Training Loss  : 0.08532448446251112
Epoch--89
Training Loss  : 0.08524940554993635
Epoch--90
Training Loss  : 0.08517640640175544
Epoch--91
Training Loss  : 0.08510541467295472
Epoch--92
Training Loss  : 0.08503636114686645
Epoch--93
Training Loss  : 0.08496917957122135
Epoch--94
Training Loss  : 0.0849038065043314
Epoch--95
Training Loss  : 0.08484018117068305
Epoch--96
Training Loss  : 0.08477824532527921
Epoch--97
Training Loss  : 0.08471794312611652
Epoch--98
Training Loss  : 0.0846592210142411
Epoch--99
Training Loss  : 0.0846020276008571
Epoch--100
Training Loss  : 0.08454631356101536
Optimized W:[1.44961872] and b:[-0.18626539]
```

In [71]:
```
import matplotlib.pyplot as plt
plt.figure(figsize = (10,10))
plt.plot(range(1,epochs + 1),loss_list)
plt.scatter(range(1,epochs+1),loss_list)
plt.title("Log Loss")
plt.xlabel("Epochs")
plt.ylabel("loss")
plt.show()
```



In [72]:
```
#test Data
ftest = svc.decision_function(X_test,support_vector,svm_dual_coeff_,intercept)
test_prediction  = [sgd.predict(ftest[data],w[0],b[0])[0] for data in range(len(ftest))]
```

In [73]:
```
print("test Prediction : {0}".format(test_prediction))
```

```
test Prediction : [0.004969085753400267, 0.47172988567687335, 0.06292162797181014, 0.4095663384280869, 0.01140033712095906, 0.9259763715110881, 0.123656385165
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If any one wants to try other calibration algorithm istonic regression also please check these tutorials

1. http://fa.bianp.net/blog/tag/scikit-learn.html#fn:1
2. https://drive.google.com/open?id=1MzmA7QaP58RDzocB0RBmRiWfl7Co_VJ7
3. https://drive.google.com/open?id=133odBinMOIVb_rh_GQxxsyMRyW-Zts7a
4. https://stat.fandom.com/wiki/Isotonic_regression#Pool_Adjacent_Violators_Algorithm