Terrain analysis – Trend Surface Analysis

ESSE 4640: LAB 3

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Introduction

This lab focuses on terrain surface modeling using a global interpolation function. The function following the general math model:

$$Z(X,Y) = \sum_{i=0}^{k}\sum_{j=0}^{k-i} X^i Y^j + e(x,y),$$

is a general linear used to find the best fit of a trend surface over a set of points whose X, Y, and Z coordinates are known. $Z(X,Y)$ are the measured elevation at the sampled points. X and Y are the planimetric coordinates the sampled points. $e(x,y)$ is the residual. The tasks in this lab include estimating the coefficients of the second and third order surfaces and their residuals. The quality of the trend surface was evaluated and then the best trend surface was selected. The elevation of given points were found by applying the selected trend surface equation. Plots were also generated.

Methodology

Read in File

The given data file "DTM_Lab3_XYZ_F2021.txt" was input into an excel sheet which was read in by column using the "xlsread" function to create variables for X, Y, and Z coordinates. The were 92 irregularly spaced coordinates given in the file.

Generating Trend Surfaces

The following equation was used for the second order trend surface is as follows:

$$Z(X,Y) = a_0 + a_1X + a_2Y + a_3X^2 + a_4XY + a_5Y^2 + e_{x,y}$$

The following equation was used for the third order trend surface is as follows:

$$Z(X,Y) = a_0 + a_1X + a_2Y + a_3X^2 + a_4XY + a_5Y^2 \ + a_6X^3 + a_7X^2Y + a_8XY^2 + a_9Y^3 + \ e_{x,y}$$

The X, Y, Z values of the given set of coordinates were input into the equations to solve for the coefficients. The centroid and normalised coordinates were also found. The set of given coordinates we normalised as follows:

$$X_N = \frac{X - X_{min}}{X_{max} - X_{min}}$$

$$Y_N = \frac{Y - Y_{min}}{Y_{max} - Y_{min}}$$

$$Z_N = \frac{Z - Z_{min}}{Z_{max} - Z_{min}}$$

$$X_{Normal} = X_N(X_{max} - X_{min}) + X_{min}$$

$$Y_{Normal} = Y_N(Y_{max} - Y_{min}) + Y_{min}$$

$$Z_{Normal} = Z_N(Z_{max} - Z_{min}) + Z_{min}$$

The coordinates were shifted to the centroid of the area of interest by, first, finding the value to shift the coordinates by using the following formulas where n is the number of points:

$$X_C = \frac{\Sigma X}{n}$$

$$Y_C = \frac{\Sigma \mathrm{Y}}{n}$$

$$Z_C = \frac{\Sigma Z}{n}$$

The difference between these values and the given set is the centroid coordinates:

$$X_{cent} = X - X_C$$

$$Y_{cent} = Y - Y_C$$

$$Z_{cent} = Z - Z_C$$

Using Least Squares Adjustment, the coefficients were found by defining the unknowns as the coefficients and observable of the elevations of the given points. The polynomial trend surface equations were used as the math model. The design matrix was created by differentiating the math model by the coefficients: $A = \frac{\delta Z}{\delta x}$. The unknowns were found as using the following formulas given that the weight matrix, P, is equal to the identity matrix.

$$\hat{x} = \; N^{-1}A^T l$$

Accuracy of the Estimated Coefficients

3) Estimate the accuracy of the estimated coefficients and comment on the numerical stability of the coefficient matrix of the normal equations. (10 marks)

The aposteriori variance factor was found to assess the accuracy of the unknowns:

$$\hat{\sigma}_0^2 = \frac{r^T r}{dof}$$

Estimate Elevations and Residuals

The adjusted coefficients were used to calculate the adjusted observables:

$$\hat{l} = A\,\hat{x}$$

The residuals were then found as the difference between the observables and the adjusted observables.

$$r = \; e(x, y) = l - \; \hat{l}$$

The mean, standard deviation, minimum, maximum and root mean square error were found using the built in functions.

Evaluate Goodness of the fit

5) For all the estimated trend surfaces assess their "worthiness" by evaluating the goodness-of-fit values R2 and by performing the analysis of variance test by evaluating the F-statistic (level of significance a=0.05). (15 marks)

Total sum of squares

$$SS_T = \Sigma Z^2 - \frac{(\Sigma Z)^2}{2} = \Sigma\left(Z_i - \hat{Z}\right)^2$$

Trend Sum if Surfaces

$$SS_T = \Sigma Z\hat{Z}^2 - \frac{\Sigma\hat{Z}}{n} = \Sigma\left(\hat{Z}_i - \bar{Z}\right)^2$$

Residual sum of squares

$$SS_D = SS_T - SS_R = \Sigma\left(Z_i - \hat{Z}\right)^2$$

Percentage of goodness of fit

$$R^2\% = \frac{SS_R}{SS_T}$$

The total variance, the trend variance, and the residual variance were calculated for the ANOVA test and calculated as:

$$Total\ Variance\ MS_T = \frac{SS_T}{n-1}$$

$$Trend\ Variance\ MS_T = \frac{SS_R}{m-1}$$

$$Residual\ Variance\ MS_P = \frac{SS_P}{n-m}$$

These values were used for the F-test statistic

$$\frac{MS_R}{MS_P} \sim F_{(m-1,n-1)}$$

Here m is the number of coefficients and n is the number of given points. The F-test statistic was compared to the critical value found on F test evaluation tables. The critical value for the second order trend surface is $F_{(5,91)}$. The critical value for the second order trend surface is $F_{(9,91)}$.

Elevation at Specific Points

The coordinates were substituted into the trend surface polynomial.

Graphs

Graphs of the given elevation points, the original and generated trend surfaces, the contours for the original were generated.

Results and Discussion

Second Order Coefficients

a_0	194.9606
a_1	0.0118
a_2	0.0047
a_3	-9.4185×10^{-7}
a_4	2.01666×10^{-7}
a_5	-6.3498×10^{-7}

Third Order Coefficients

a_0	303.0618
a_1	-0.0315
a_2	-0.0119
a_3	4.7623×10^{-6}
a_4	3.2718×10^{-6}
a_5	1.4766×10^{-6}
a_6	-2.4567×10^{-10}
a_7	-1.7130×10^{-10}
a_8	-1.1710×10^{-10}
a_9	-1.1788×10^{-10}

Accuracy of the Coefficients

Second Order Coefficients

$$\hat{\sigma}_0^2 = 79.7394$$

Third Order Coefficients

$$\hat{\sigma}_0^2 = 82.2067$$

Since the a posteriori variance factors are quite large, the coefficients that were calculated are not accurate. The a posteriori variance factor should ideally be much closer to zero. Between the second and third orders, the coefficients of the third order are less accurate judging solely on the a posteriori variance factor.

Residual Statistics

	Second Order	Third Order
Mean	3.9809×10^{-12}	2.4209×10^{-9}
Standard Deviation	8.6809	8.6068
Minimum	-27.2911	-27.5121

Maximum	25.9480	25.6089
Root Mean Square Error	8.6336	8.5599

Goodness of the Fit

	Second Order	Third Order
Total Sum of Squares	$8.825440359168235e+03$	$8.825440359168235e+03$
Trend Sum of Squares	$1.978533265316117e+03$	$2.103196070518944e+03$
Residual Sum of Squares	$6.811853310383596e+03$	$6.717574190376617e+03$
Percentage Goodness of the fit	0.2242	0.2383

	Second Order	Third Order
Total Variance	98.0604	98.0604
Trend Variance	329.7555	210.3196
Residual Variance	81.0935	83.8697
F-test Statistic	4.0664	2.5047

The critical value for the second order trend surface is $F_{(5,91)} = 2.315$. The critical value for the second order trend surface is $F_{(9,91)} = 1.984$. Both critical values are smaller than the F Test Statistic and the null hypothesis are rejected. Therefore, neither trend surfaces are not a good fit.

Selection of the Trend Surface

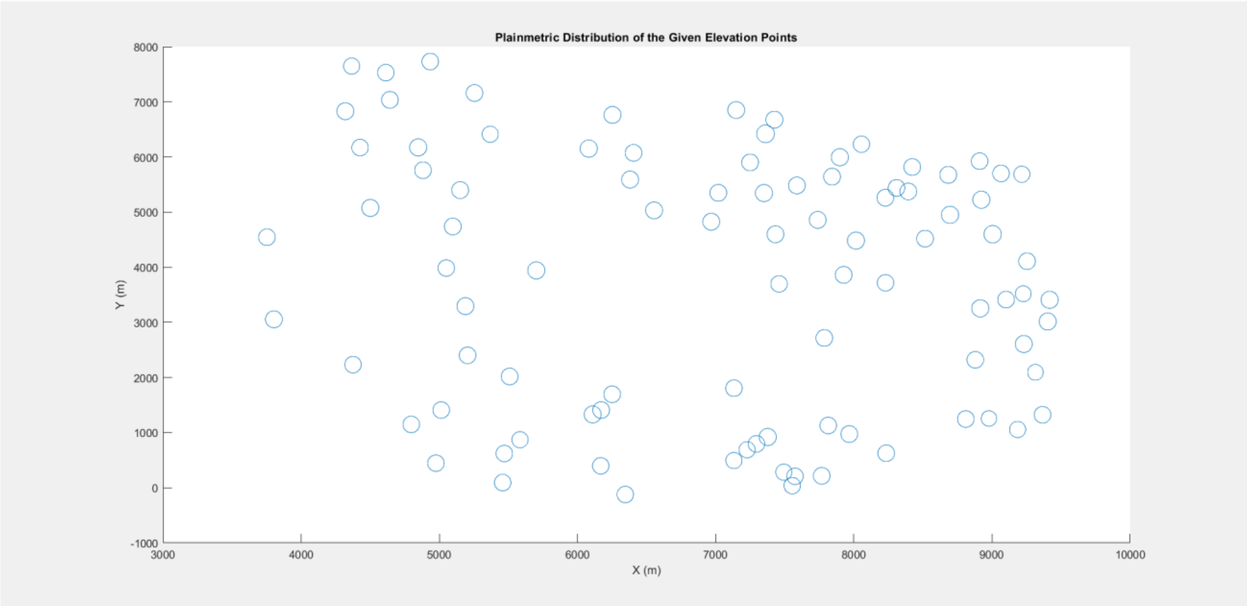
Both trend surfaces are not at all accurate and neither should be picked but of the two options the second order trend surface was chosen. For both trend sources, the first coefficient, a_0, is very high and the rest are closer to zero. Neither are a good fits according to the F- test. Although the a posteriori variance factors are very high, the second order trend surface is relatively smaller than the third order trend surface. For both trend surfaces, the means of the residuals are small but the second order trend surface is relatively smaller. For these reasons, the second order trend surface would be selected.

Elevation at three locations

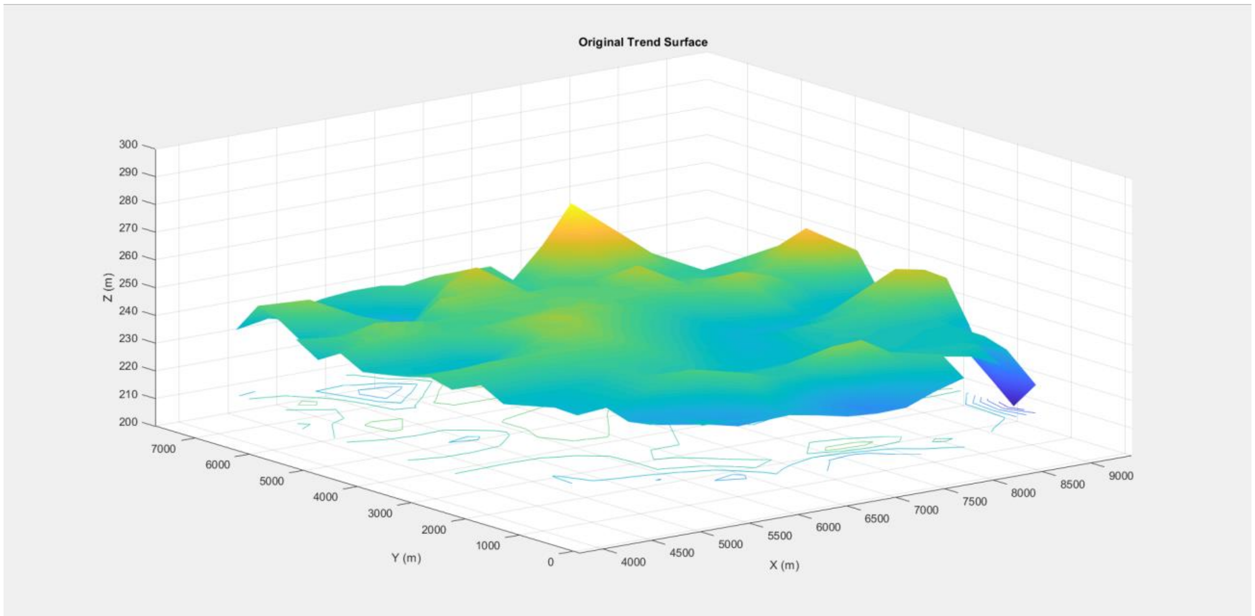
Point	X	Y	Z from Second Order	Z from Third Order
A	$7000m$	$3500m$	1.3010×10^{16}	5.0526×10^{25}
B	$6350m$	$5500m$	2.0029×10^{16}	1.0253×10^{26}
C	$8800m$	$2400m$	1.4273×10^{16}	4.7901×10^{25}

Graphs

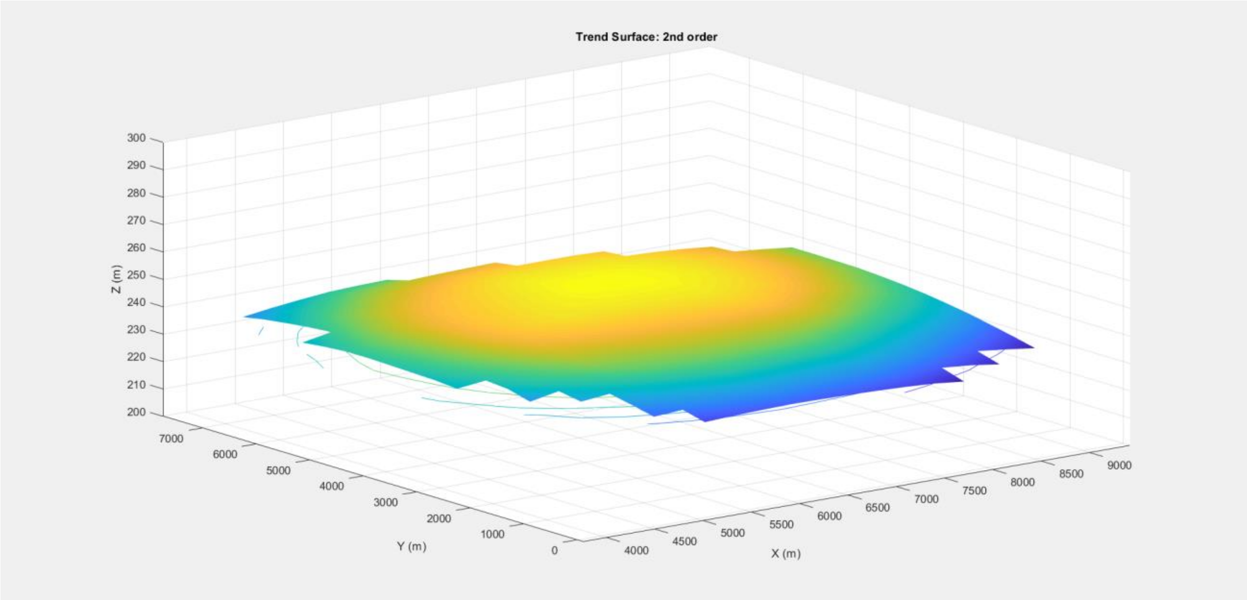
The following graph shows the distribution of the given points.



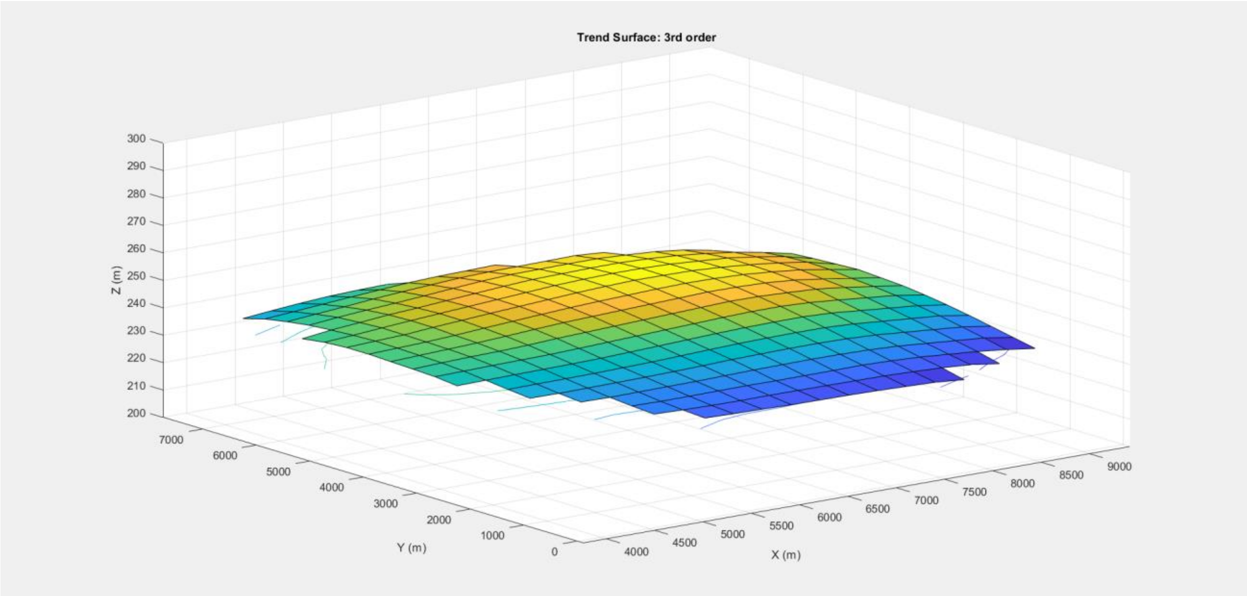
The following graph is a generated surface of the given points.



The following graph is the second order trend surface of the given points.



The following graph is the third order trend surface of the given points.



Conclusion

This lab applied knowledge learned from the lectures. The results could be improved.

References

2021. *ESSE 4640: Lab 3 Manual.*

Jadidi, Mojgan. 2021. "ESSE 4640 Digital Terrain Modelling: Lecture 5." Lecture Slides.

Statology. https://www.statology.org/wp-content/uploads/2018/09/f1.png

Appendix

```
%% ESSE4640 DTM Lab 3

clc

clear all

%% Read Data

% read file

data = xlsread('lab3esse4640');

%%Initial (Given) Coordinates

xi = data(:,1);

yi = data(:,2);

zi = data(:,3);



%% Generate 2nd and 3rd order trend surfaces

% Initial coeffiecnt values

n = 92;

x = sum(xi);

y = sum(yi);

z = sum(zi);

% Estimating the corresponding polynomial coefficient

X = xi;

Y = yi;

Z = zi;

xy = xi.* yi;

xz = xi.* zi;

yz = yi.* zi;

for i = 2 : 8

   X(:,i) = x.^i;

   Y(:,i) = y.^i;
```

```
end
for i = 1 : 8
   SumX(i) = sum(X(:,i));
   SumY(i) = sum(Y(:,i));
end
xySum = sum(xy);
xzSum = sum(xz);
yzSum = sum(yz);

% Estimate using the normalized coordinates
Xmx = max(xi); Xmn = min(xi);
Ymx = max(yi); Ymn = min(yi);
Zmx = max(zi); Zmn = min(zi);

Xn = (xi-Xmn)/(Xmx-Xmn);
Yn = (yi-Xmn)/(Ymx-Ymn);
SumZn(:,1) = (zi-Xmn)/(Zmx-Zmn);
xyn = Xn.* Yn;
xzn = Xn.* SumZn(:,1);
yzn = Yn.* SumZn(:,1);
for i = 2 : 8
   Xn(:,i) = Xn(i,1).^i;
   Yn(:,i) = Yn(i,1).^i;
end
for i = 1 : 8
   SumXn(i) = sum(Xn(:,i));
   SumYn(i) = sum(Yn(:,i));
end
xynSum = sum(xyn);
```

```
xznSum = sum(xzn);

yznSum = sum(yzn);

SumZn = sum(SumZn(:,1));


% Estimate using the centroid coordinates

Xcent = xi/n;

Ycent = yi/n;

Zcent = zi/n;

SumXcent = sum(Xcent);

SumYcent = sum(Ycent);


for i = 2 : 8

   Xcent(:,i) = x.^i;

   Ycent(:,i) = y.^i;

end


xycent = xi.* yi;

xzcent = xi.* zi;

yzcent = yi.* zi;


for i = 1 : 8

   SumXcent(i) = sum(Xcent(:,i));

   SumYcent(i) = sum(Ycent(:,i));

end

xycentSum = sum(xycent);

xzcentSum = sum(xzcent);

yzcentSum = sum(yzcent);

SumZcent = sum(zi);
```

```
%% Normalized First Order Polynomial Equation
% Design Matrix
An = [n   SumXn(:,1)     SumYn(:,1);
    SumXn(:,1) SumXn(2) xynSum;
    SumYn(:,1)  xynSum  SumYn(2)];
Zon = [SumZn xznSum yznSum];
con = An/Zon
condn = abs (max(con)/min(con))

%% Centroid First Order Polynomial Equation
% Design Matrix
Acent = [n     SumXcent(:,1)      SumYcent(:,1);
       SumXcent(:,1)  SumXcent(2) xycentSum;
       SumYcent(:,1)  xycentSum  SumYcent(2)];
Zocent = [SumZcent xzcentSum yzcentSum];
cocent = Acent/Zocent
condcent = abs (max(cocent/min(cocent)));

%% Second Order Polynomial Equation
% Design Matrix
A2 = [n       x        y        SumX(2)       xySum         SumY(2);
     x       SumX(2)    xySum      SumX(3)       SumX(2)*y      SumY(2)*x;
     y       xySum      SumY(2)    SumX(2)*y      SumY(2)*x      SumY(3);
   SumX(2)     SumX(3)    SumX(2)*y   SumX(4)       SumX(3)*y      SumX(2)*SumY(2);
   xySum       SumX(2)*y   SumY(2)*x   SumX(3)*y      SumX(2)*SumY(2) SumY(3)*x;
   SumY(2)     SumY(2)*x   SumY(3)     SumX(2)*SumY(2) SumY(3)*x      SumY(4)];

Zo2 = [z z*x z*y SumX(2)*z  xySum*z SumY(2)*z];
co2 = A2/Zo2;
```

cond2 = abs (max(co2)/min(co2));

%% Normalized Second Order Polynomial Equation

% Design Matrix

An2 = [n SumXn(:,1) SumYn(:,1) SumXn(2) xynSum SumYn(2);

SumXn(:,1) SumXn(2) xynSum SumXn(3) SumXn(2)*SumYn(:,1) SumY(2)*SumXn(:,1);

SumYn(:,1) xynSum SumYn(2) SumXn(2)*SumYn(:,1) SumY(2)*SumXn(:,1) SumYn(3);

SumXn(2) SumXn(3) SumXn(2)*SumYn(:,1) SumXn(4) SumXn(3)*SumYn(:,1) SumXn(2)*SumYn(2);

xynSum SumXn(2)*SumYn(:,1) SumYn(2)*SumXn(:,1) SumXn(3)*SumYn(:,1) SumXn(2)*SumYn(2) SumYn(3)*SumXn(:,1);

SumYn(2) SumYn(2)*SumXn(:,1) SumYn(3) SumXn(2)*SumYn(2) SumYn(3)*SumXn(:,1) SumYn(4)];

Zon2 = [SumZn(:,1) SumZn(:,1)*SumXn(:,1) SumZn(:,1)*SumYn(:,1) SumXn(2)*SumZn(:,1) xynSum*SumZn(:,1) SumYn(2)*SumZn(:,1)];

con2 = An2/Zon2;

condn2 = abs (max(con2)/min(con2));

%% Centroid Second Order Polynomial Equation

% Design Matrix

Acent2 = [n SumXcent(:,1) SumYcent(:,1) SumXcent(2) xycentSum SumYcent(2);

SumXcent(:,1) SumXcent(2) xycentSum SumXcent(3) SumXcent(2)*SumYcent(:,1) SumYcent(2)*SumXn(:,1);

SumYcent(:,1) xycentSum SumYcent(2) SumXcent(2)*SumYcent(:,1) SumYcent(2)*SumXcent(:,1) SumYcent(3);

SumXcent(2) SumXcent(3) SumXcent(2)*SumYcent(:,1) SumXcent(4) SumXcent(3)*SumYcent(:,1) SumXcent(2)*SumYcent(2);

xycentSum SumXcent(2)*SumYcent(:,1) SumYcent(2)*SumXcent(:,1) SumXcent(3)*SumYcent(:,1) SumXcent(2)*SumYcent(2) SumYcent(3)*SumXcent(:,1);

SumYcent(2) SumYcent(2)*SumXcent(:,1) SumYcent(3) SumXcent(2)*SumYcent(2) SumYcent(3)*SumXcent(:,1) SumYcent(4)];

```
Zocent2 = [SumZcent(:,1) SumZcent(:,1)*SumXcent(:,1) SumZcent(:,1)*SumYcent(:,1)
SumXcent(2)*SumZcent(:,1)  xycentSum*SumZcent(:,1) SumYcent(2)*SumZcent(:,1)];

cocent2 = Acent2/Zocent2

condcent2 = abs (max(cocent2/min(cocent2)));



%% Third Order Polynomial Equation

%Design Matrix

A3 = [ n        x          y          SumX(2)         xySum       SumY(2)       SumX(3)       SumX(2)*y
x*SumY(2)      SumY(3);

      x        SumX(2)       xySum       SumX(3)          SumX(2)*y      x*SumY(2)      SumX(4)
SumX(3)*y      SumX(2)*SumY(2)   x*SumY(3) ;

      y        xySum       SumY(2)       SumX(2)*y         x*SumY(2)      SumY(3)       SumX(3)*y
SumX(2)*SumY(2)   x*SumY(3)      SumY(4) ;

      SumX(2)     SumX(3)       SumX(2)*y      SumX(4)          SumX(3)*y      SumX(2)*SumY(2)   SumX(5)
SumX(4)*y      SumX(3)*SumY(2)   SumX(2)*SumY(3) ;

      xySum     SumX(2)*y      x*SumY(2)      SumX(3)*y         SumX(2)*SumY(2)   x*SumY(3)
SumX(4)*y      SumX(3)*SumY(2)   SumX(2)*SumY(3)   x*SumY(4);

      SumY(2)     x*SumY(2)      SumY(3)       SumX(2)*SumY(2)      x*SumY(3)      SumY(4)
SumX(3)*SumY(2)   SumX(2)*SumY(3)   x*SumY(4)      SumY(5);

      SumX(3)     SumX(4)       SumX(3)*y      SumX(5)          SumX(4)*y      SumX(3)*SumY(2)   SumX(6)
SumX(5)*y      SumX(4)*SumY(2)   SumX(3)*SumY(3);

      SumX(2)*y   SumX(3)*SumY(2)   SumX(2)*SumY(2)   SumX(4)*y         SumX(3)*SumY(2)
SumX(2)*SumY(3)   SumX(5)*y      SumX(4)*SumY(2)   SumX(3)*SumY(3)   SumX(2)*SumY(4);

      x*SumY(2)   SumX(2)*SumY(2)   x*SumY(3)      SumX(3)*SumY(2)      SumX(2)*SumY(3)   x*SumY(4)
SumX(4)*SumY(2)   SumX(3)*SumY(3)   SumX(2)*SumY(4)   x*SumY(5);

      SumY(3)     x*SumY(3)      SumY(4)       SumX(2)*SumY(3)      x*SumY(4)      SumY(5)
SumX(3)*SumY(3)   SumX(2)*SumY(4)   x*SumY(5)      SumY(6)];

Zo3 = [z z*x y*z SumX(2)*z xySum*z SumY(2)*z SumX(3)*z SumX(2)*y*z x*SumY(2)*z SumY(3)*z];

co3 = A3/Zo3;

cond3 = abs(max(co3)/min(co3));


%% Normalized Third Order Polynomial Equation
```

```
%Design Matrix

An3 = [ n      SumXn(:,1)      SumYn(:,1)      SumXn(2)      xynSum      SumYn(2)
SumXn(3)      SumXn(2)*SumYn(:,1)      SumXn(:,1)*SumYn(2)      SumYn(3);

      SumXn(:,1)      SumXn(2)      xynSum      SumXn(3)      SumXn(2)*SumYn(:,1)
SumXn(:,1)*SumYn(2)      SumXn(4)      SumXn(3)*SumYn(:,1)      SumXn(2)*SumYn(2)
SumXn(:,1)*SumYn(3) ;

      SumYn(:,1)      xynSum      SumYn(2)      SumXn(2)*SumYn(:,1)      SumXn(:,1)*SumYn(2)
SumYn(3)      SumXn(3)*SumYn(:,1)      SumXn(2)*SumYn(2)   SumXn(:,1)*SumYn(3)      SumYn(4) ;

      SumXn(2)    SumXn(3)      SumXn(2)*SumYn(:,1)      SumXn(4)      SumXn(3)*SumYn(:,1)
SumXn(2)*SumYn(2)   SumXn(5)      SumXn(4)*SumYn(:,1)      SumXn(3)*SumYn(2)
SumXn(2)*SumYn(3) ;

      xynSum    SumXn(2)*SumYn(:,1)      SumXn(:,1)*SumYn(2)      SumXn(3)*SumYn(:,1)
SumXn(2)*SumYn(2)   SumXn(:,1)*SumYn(3)      SumXn(4)*SumYn(:,1)      SumXn(3)*SumYn(2)
SumXn(2)*SumYn(3)   SumXn(:,1)*SumYn(4);

      SumYn(2)    SumXn(:,1)*SumYn(2)      SumYn(3)      SumXn(2)*SumYn(2)      SumXn(:,1)*SumYn(3)
SumYn(4)      SumXn(3)*SumYn(2)   SumXn(2)*SumYn(3)   SumXn(:,1)*SumYn(4)      SumYn(5);

      SumXn(3)    SumXn(4)      SumXn(3)*SumYn(:,1)      SumXn(5)      SumXn(4)*SumYn(:,1)
SumXn(3)*SumYn(2)   SumXn(6)      SumXn(5)*SumYn(:,1)      SumXn(4)*SumYn(2)
SumXn(3)*SumYn(3);

      SumXn(2)*SumYn(:,1)   SumXn(3)*SumYn(2)   SumXn(2)*SumYn(2)   SumXn(4)*SumYn(:,1)
SumXn(3)*SumYn(2)   SumXn(2)*SumYn(3)   SumXn(5)*SumYn(:,1)      SumXn(4)*SumYn(2)
SumXn(3)*SumYn(3)   SumXn(2)*SumYn(4);

      SumXn(:,1)*SumYn(2)   SumXn(2)*SumYn(2)   SumXn(:,1)*SumYn(3)      SumXn(3)*SumYn(2)
SumXn(2)*SumYn(3)   SumXn(:,1)*SumYn(4)      SumXn(4)*SumYn(2)   SumXn(3)*SumYn(3)
SumXn(2)*SumYn(4)   SumXn(:,1)*SumYn(5);

      SumYn(3)    SumXn(:,1)*SumYn(3)      SumYn(4)      SumXn(2)*SumYn(3)      SumXn(:,1)*SumYn(4)
SumYn(5)      SumXn(3)*SumYn(3)   SumXn(2)*SumYn(4)   SumXn(:,1)*SumYn(5)      SumYn(6)];

Zon3 = [SumZn(:,1) SumZn(:,1)*SumXn(:,1) SumYn(:,1)*SumZn(:,1) SumXn(2)*SumZn(:,1)
xynSum*SumZn(:,1) SumYn(2)*SumZn(:,1) SumXn(3)*SumZn(:,1) SumXn(2)*SumYn(:,1)*SumZn(:,1)
SumXn(:,1)*SumYn(2)*SumZn(:,1) SumYn(3)*SumZn(:,1)];

con3 = An3/Zon3;

condn3 = abs(max(con3)/min(con3));


%% Centroid Third Order Polynomial Equation

%Design Matrix
```

```
Acent3 = [ n         SumXcent(:,1)           SumYcent(:,1)           SumXcent(2)          xycentSum
SumYcent(2)       SumXcent(3)       SumXcent(2)*SumYcent(:,1)      SumXcent(:,1)*SumYcent(2)
SumYcent(3);

      SumXcent(:,1)        SumXcent(2)        xycentSum        SumXcent(3)
SumXcent(2)*SumYcent(:,1)      SumXcent(:,1)*SumYcent(2)      SumXcent(4)
SumXcent(3)*SumYcent(:,1)      SumXcent(2)*SumYcent(2)   SumXcent(:,1)*SumYcent(3) ;

      SumYcent(:,1)        xycentSum        SumYcent(2)        SumXcent(2)*SumYcent(:,1)
SumXcent(:,1)*SumYcent(2)      SumYcent(3)        SumXcent(3)*SumYcent(:,1)
SumXcent(2)*SumYcent(2)   SumXcent(:,1)*SumYcent(3)      SumYcent(4) ;

      SumXcent(2)     SumXcent(3)        SumXcent(2)*SumYcent(:,1)      SumXcent(4)
SumXcent(3)*SumYcent(:,1)      SumXcent(2)*SumYcent(2)   SumXcent(5)
SumXcent(4)*SumYcent(:,1)      SumXcent(3)*SumYcent(2)   SumXcent(2)*SumYcent(3) ;

      xycentSum     SumXcent(2)*SumYcent(:,1)      SumXcent(:,1)*SumYcent(2)
SumXcent(3)*SumYcent(:,1)         SumXcent(2)*SumYcent(2)   SumXcent(:,1)*SumYcent(3)
SumXcent(4)*SumYcent(:,1)      SumXcent(3)*SumYcent(2)   SumXcent(2)*SumYcent(3)
SumXcent(:,1)*SumYcent(4);

      SumYcent(2)     SumXcent(:,1)*SumYcent(2)      SumYcent(3)        SumXcent(2)*SumYcent(2)
SumXcent(:,1)*SumYcent(3)      SumYcent(4)        SumXcent(3)*SumYcent(2)
SumXcent(2)*SumYcent(3)   SumXcent(:,1)*SumYcent(4)      SumYcent(5);

      SumXcent(3)     SumXcent(4)        SumXcent(3)*SumYcent(:,1)      SumXcent(5)
SumXcent(4)*SumYcent(:,1)      SumXcent(3)*SumYcent(2)   SumXcent(6)
SumXcent(5)*SumYcent(:,1)      SumXcent(4)*SumYcent(2)   SumXcent(3)*SumYcent(3);

      SumXcent(2)*SumYcent(:,1)   SumXcent(3)*SumYcent(2)   SumXcent(2)*SumYcent(2)
SumXcent(4)*SumYcent(:,1)         SumXcent(3)*SumYcent(2)   SumXcent(2)*SumYcent(3)
SumXcent(5)*SumYcent(:,1)      SumXcent(4)*SumYcent(2)   SumXcent(3)*SumYcent(3)
SumXcent(2)*SumYcent(4);

      SumXcent(:,1)*SumYcent(2)   SumXcent(2)*SumYcent(2)   SumXcent(:,1)*SumYcent(3)
SumXcent(3)*SumYcent(2)      SumXcent(2)*SumYcent(3)   SumXcent(:,1)*SumYcent(4)
SumXcent(4)*SumYcent(2)   SumXcent(3)*SumYcent(3)   SumXcent(2)*SumYcent(4)
SumXcent(:,1)*SumYcent(5);

      SumYcent(3)     SumXcent(:,1)*SumYcent(3)      SumYcent(4)        SumXcent(2)*SumYcent(3)
SumXcent(:,1)*SumYcent(4)      SumYcent(5)        SumXcent(3)*SumYcent(3)
SumXcent(2)*SumYcent(4)   SumXcent(:,1)*SumYcent(5)      SumYcent(6)];

Zocent3 = [SumZcent(:,1) SumZcent(:,1)*SumXcent(:,1) SumYcent(:,1)*SumZcent(:,1)
SumXcent(2)*SumZcent(:,1) xycentSum*SumZcent(:,1) SumYcent(2)*SumZcent(:,1)
SumXcent(3)*SumZcent(:,1) SumXcent(2)*SumYcent(:,1)*SumZcent(:,1)
SumXcent(:,1)*SumYcent(2)*SumZcent(:,1) SumYcent(3)*SumZcent(:,1)];

cocent3 = Acent3/Zocent3;
```

```
condcent3 = abs(max(cocent3)/min(cocent3));


%Second order Design Matrix
for i=1:1:92
   A2(i,1) = (1);
   A2(i,2) = xi(i);
   A2(i,3) = yi(i);
   A2(i,4)= xi(i)^2;
   A2(i,5) = xi(i)*yi(i);
   A2(i,6)= yi(i)^2;
end
[X2, L2, R2, apost2] = LnrPrmtrcLSA(zi,A2)

%Third Order Design Matrix
for i=1:1:92
   A3(i,1) = (1);
   A3(i,2) = xi(i);
   A3(i,3) = yi(i);
   A3(i,4)= xi(i)^2;
   A3(i,5) = xi(i)*yi(i);
   A3(i,6)= yi(i)^2;
   A3(i,7)= xi(i)^3;
   A3(i,8)= xi(i)^2*yi(i);
   A3(i,9)= yi(i)^2*xi(i);
   A3(i,10)= yi(i)^3;
end
[X3, L3, R3, apost3] = LnrPrmtrcLSA(zi,A3)
%% Estimate Accuracy
```

```
apost2; apost3;

%% Elevations and Residuals


%%Second order
for i=1:1:92
   A2(i,1) = (1);
   A2(i,2) = xi(i);
   A2(i,3) = yi(i);
   A2(i,4)= xi(i)^2;
   A2(i,5) = xi(i)*yi(i);
   A2(i,6)= yi(i)^2;
end
p1 = inv(transpose(A2)*A2);
p2 = transpose(A2);
Xhat2 = A2*p1*p2*zi;


%Second Order
Mean_R2 = mean(R2)
Max_R2 = max(R2)
Min_R2 = min(R2)
STD_R2 = std (R2)
RMSE_R2 = sqrt(mean(((zi-Xhat2).^2)))

%%Third Order
for i=1:1:92
   A3(i,1) = (1);
```

```
    A3(i,2) = xi(i);
    A3(i,3) = yi(i);
    A3(i,4)= xi(i)^2;
    A3(i,5) = xi(i)*yi(i);
    A3(i,6)= yi(i)^2;
    A3(i,7)= xi(i)^3;
    A3(i,8)= xi(i)^2*yi(i);
    A3(i,9)= yi(i)^2*xi(i);
    A3(i,10)= yi(i)^3;
end
Xhat3 = A3*inv(transpose(A3)*A3)*transpose(A3)*zi;

%Third Order
Mean_R3 = mean(R3)
Max_R3 = max(R3)
Min_R3 = min(R3)
STD_R3 = std (R3)
RMSE_R3 = sqrt(mean(((zi-Xhat3).^2)))

% Worthiness
%%Second Order
avg_Z = mean(zi);
for i=1:1:92
SST_or2(i) = sum((zi(i)-avg_Z)^2);
SSR_or2(i) = sum ((Xhat2(i)-avg_Z)^2);
SSD_or2(i) = sum((zi(i)-Xhat2(i))^2);
end
SST_o2 = sum(SST_or2)
SSR_o2 = sum(SSR_or2)
```

```
SSD_o2 = sum(SSD_or2)

%Goodness-of-Fit

R2 = SSR_o2/SST_o2

nn = 92

m = 6

MST_o2 = SST_o2 / nn

MSR_o2 = SSR_o2 / m

MSD_o2 = SSD_o2 / (n-m)

% Fstat

F2_test = MSR_o2/MSD_o2

%%Third Order

for i=1:1:92

SST_or3(i) = sum((zi(i)-avg_Z)^2);

SSR_or3(i) = sum ((Xhat3(i)-avg_Z)^2);

SSD_or3(i) = sum((zi(i)-Xhat3(i))^2);

end

SST_o3 = sum(SST_or3)

SSR_o3 = sum(SSR_or3)

SSD_o3 = sum(SSD_or3)

% of Goodness-of-Fit

R3 = SSR_o3/SST_o3

n_3 = 92

m_3 = 10

MST_o3 = SST_o3 / n_3

MSR_o3 = SSR_o3 / m_3

MSD_o3 = SSD_o3 / (n_3-m_3)

% Fstat

F3_test = MSR_o3/MSD_o3
```

```
%% Elevation

point = ['A','B','C'];

x_elevLoc = [7000, 6350, 8800];

y_elevLoc = [3500, 5500, 2400];

for i = 1:3

    point(i)

    elev_2(i) = cocent2(1)+(cocent2(2)*x_elevLoc(i)) + (cocent2(3)*y_elevLoc(i)) +
(cocent2(4)*(x_elevLoc(i)^2)) + (cocent2(5)*x_elevLoc(i)*y_elevLoc(i)) + (cocent2(6)*(y_elevLoc(i)^2))

    elev_3(i) = cocent3(1)+(cocent3(2)*x_elevLoc(i)) +(cocent3(3)*y_elevLoc(i)) +
(cocent3(4)*(x_elevLoc(i)^2)) +(cocent3(5)*x_elevLoc(i)*y_elevLoc(i)) +
(cocent3(6)*(y_elevLoc(i)^2))+(cocent3(7)*(x_elevLoc(i)^3))+(cocent3(8)*(x_elevLoc(i)^2)*y_elevLoc(i))
+ (cocent3(9)*x_elevLoc(i)*(y_elevLoc(i)^2))+(cocent3(10)*(y_elevLoc(i)^3))

end



%% Visualize 1

figure(1)

PlanimetricPlot = scatter(xi,yi,zi);

hold on

title('Plainmetric Distribution of the Given Elevation Points')

xlabel('X (m)')

ylabel('Y (m)')


%% Visualize 2

figure;

stem3(xi, yi, zi)

grid on

xv = linspace(min(xi), max(xi), 20);

yv = linspace(min(yi), max(yi), 20);

[X,Y] = meshgrid(xv, yv);
```

```
Z = griddata(xi,yi,zi,X,Y);

%figure(2)

sc = surfc(X, Y, Z);

grid on

set(gca, 'ZLim',[0 100])

shading interp

zlim([200,300])

title('Original Trend Surface')

xlabel('X (m)')

ylabel('Y (m)')

zlabel('Z (m)')

%% Visualize 3

figure;

stem3(xi, yi, Xhat2)

grid on

xv = linspace(min(xi), max(xi), 20);

yv = linspace(min(yi), max(yi), 20);

[X,Y] = meshgrid(xv, yv);

Z = griddata(xi,yi,Xhat2,X,Y);

sc = surfc(X, Y, Z);

grid on

set(gca, 'ZLim',[0 100])

shading interp

zlim([200,300])

title('Trend Surface: 2nd order')

xlabel('X (m)')

ylabel('Y (m)')

zlabel('Z (m)')

%% Visualize 4
```

```
figure;
stem3(xi, yi, Xhat3)
grid on
xv = linspace(min(xi), max(xi), 20);
yv = linspace(min(yi), max(yi), 20);
[X,Y] = meshgrid(xv, yv);
Z = griddata(xi,yi,Xhat3,X,Y);
sc = surfc(X, Y, Z);
grid on
set(gca, 'ZLim',[0 100])
zlim([200,300])
title('Trend Surface: 3rd order')
xlabel('X (m)')
ylabel('Y (m)')
zlabel('Z (m)')

%% Adjustment
function [X, L, R,apost] = LnrPrmtrcLSA(l,A)
   P = diag(ones (length(l),1));
   X =  inv(A'*P*A)*A'*inv(P)*l;
   L =  A*X;
   R =  l - L;
   dof =  length(L) - length(X);
   apost = (R'*P*R)/dof;
end
```