Assignment 9

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Outline

Question

Probability, Random Variables and Stochastic Processes Chapter 2, Problem 2-22

Show that $2^n - (n+1)$ equations are needed to establish the independence of n events.

Solution

Let's denote the random variable X map to the set $\{0, 1, \ldots, n-1\}$ where each $X = j, j \in \{0, 1, \ldots, n-1\}$ represents an event.
Here it is given that all the events are independent.

Condition for independence

We know that, for independence of n events, $\forall j \in \{0, 1, 2, \ldots, n-1\}$,

$$\Pr(X = i_0, i_1, i_2, \ldots, i_j) = \Pr(X = 0)\Pr(X = 1)\ldots\Pr(X = j) \tag{1}$$

where $i_0, i_1, i_2, \ldots, i_j \subset \{0, 1, 2, \ldots, n-1\}$
We have to find number of equations required to satisfy these conditions.

Required number of equations

for $\Pr(X = i_0, i_1, i_2, \ldots, i_j)$, the required number of equations is same as selecting $j+1$ numbers out of $n = {}^nC_{j+1}$ starting from $j = 1$.

$$\implies \text{number of equations} = {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n \tag{2}$$

We know that

$$ {}^nC_0 + {}^nC_1 + \ldots + {}^nC_n = 2^n \tag{3}$$

$$\implies {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n - {}^nC_0 - {}^nC_1 \tag{4}$$

$$\text{From (2), The number of required equations} = 2^n - (1 + n) \tag{5}$$