Assignment 9

Kotikalapudi Karthik (cs21btech11030)

***Abstract*—This document contains the solution to Problem 2-22 of Chapter 2 of the book Probability, Random Variables and Stochastic Processes.**

Probability, Random Variables and Stochastic Processes Chapter 2

Problem 2-22 Show that $2^n - (n+1)$ equations are needed to establish the independence of n events.

Solution:

Let's denote the random variable X map to the set $\{0, 1, \ldots, n-1\}$ where each $X = j$, $j \in \{0, 1, \ldots, n-1\}$ represents an event.

Here it is given that all the events are independent. We know that, for independence of n events, $\forall j \in \{0, 1, 2, \ldots, n-1\}$,

$$\Pr(X = i_0, i_1, i_2, \ldots, i_j) = \Pr(X = 0)\Pr(X = 1)\ldots\Pr(X = j) \quad (1)$$

where $i_0, i_1, i_2, \ldots, i_j \subset \{0, 1, 2, \ldots, n-1\}$

We have to find number of equations required to satisfy these conditions. for $\Pr(X = i_0, i_1, i_2, \ldots, i_j)$, the required number of equations is same as selecting $j+1$ numbers out of $n = {}^nC_{j+1}$ starting from $j = 1$.

$$\implies \text{number of equations} = {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n \quad (2)$$

We know that

$$ {}^nC_0 + {}^nC_1 + \ldots + {}^nC_n = 2^n \quad (3)$$

$$\implies {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n - {}^nC_0 - {}^nC_1 \quad (4)$$

From (2), number of equations $= 2^n - (1 + n)$ (5)