Assignment 10

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Outline

1. Question
2. Solution

Question

Chapter 13 Example 6 [NCERT 12]

Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.

Solution

Let random variables $X_1, X_2 \in \{0, 1\}$ denote the following events in Table (1)

Variable	Event
$X_1 = 0$	ball is drawn from Bag I
$X_1 = 1$	ball is drawn from Bag II
$X_2 = 0$	ball drawn is red
$X_2 = 1$	ball drawn is black

Table 1

Various Probability values

Various probability values are are given in the Table (2).

Probability	**Value**
$\Pr(X_1 = 0)$	$\frac{1}{2}$
$\Pr(X_1 = 1)$	$\frac{1}{2}$
$\Pr(X_2 = 0 \mid X_1 = 0)$	$\frac{3}{7}$
$\Pr(X_2 = 0 \mid X_1 = 1)$	$\frac{5}{11}$
$\Pr(X_1 = 1 \mid X_2 = 0)$	?

Table 2

Required Probability

By Bayes' theorem,

$$\Pr(X_1 = 1|X_2 = 0) = \frac{\Pr(X_1 = 1)\Pr(X_2 = 0|X_1 = 1)}{\Sigma_{i=0}^{1}\Pr(X_1 = i)\Pr(X_2 = 0|X_1 = i)} \tag{1}$$

$$\implies \Pr(X_1 = 1|X_2 = 0) = \frac{\frac{1}{2} \times \frac{5}{11}}{\frac{1}{2} \times \frac{3}{7} + \frac{1}{2} \times \frac{5}{11}} \tag{2}$$

$$= \frac{35}{68} \tag{3}$$