Assignment 15

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Outline

1. Question
2. Solution

Probability, Random Variables and Stochastic Processes Chapter 12, Problem 12-12

Show that if we use as estimate of the power spectrum $S(\omega)$ of a discrete-time process $x[n]$ the function

$$S_w(\omega) = \sum_{m=-N}^{N} w_m R[m]\, e^{-jm\omega T}$$

then

$$S_w(\omega) = \frac{1}{2\sigma} \int_{-\sigma}^{\sigma} S(y)\, W(\omega - y)\, dy$$

$$W(\omega) = \sum_{-N}^{N} w_n e^{-jn\omega T}$$

Solution

Given,

$$S_w(\omega) = \sum_{m=-N}^{N} w_m R[m] e^{-jm\omega T} \tag{1}$$

We know that

$$S(y) = \mathcal{F}[R[m]] \tag{2}$$

$$\implies R[m] = \mathcal{F}^{-1}[S(y)] \tag{3}$$

$$\implies R[m] = \frac{1}{2\sigma}\int_{-\sigma}^{\sigma} S(y) e^{jmyT} dy \tag{4}$$

Here y is a variable in the frequency domain.
If $T = 1$sec, the limits will be $-\pi$ to π.
As we don't know the value of T, let the limits be $-\sigma$ to σ.

Solution

Substituting equation (4) in equation (1),

$$S_w(\omega) = \sum_{m=-N}^{N} w_m \left(\frac{1}{2\sigma} \int_{-\sigma}^{\sigma} S(y)\, e^{jmyT} dy \right) e^{-jm\omega T} \tag{5}$$

$$\implies S_w(\omega) = \frac{1}{2\sigma} \left(\int_{-\sigma}^{\sigma} \left(S(y) \sum_{m=-N}^{N} w_m e^{jmyT} \right) dy \right) e^{-jm\omega T} \tag{6}$$

$$\implies S_w(\omega) = \frac{1}{2\sigma} \int_{-\sigma}^{\sigma} \left(S(y) \sum_{m=-N}^{N} w_m e^{-jm(\omega-y)T} \right) dy \tag{7}$$

Solution

$W(w)$ is the Discrete-time Fourier Transform(DTFT) of w_n

$$W(\omega) = \sum_{n=-N}^{N} w_n e^{-jm\omega T} \tag{8}$$

$$\implies W(\omega - y) = \sum_{n=-N}^{N} w_n e^{-jm(\omega-y)T} \tag{9}$$

Substituting equation (9) in equation (7),

$$S_w(\omega) = \frac{1}{2\sigma}\int_{-\sigma}^{\sigma} (S(y)\, W(\omega - y))\, dy \tag{10}$$