Assignment 14

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Outline

Question

Probability, Random Variables and Stochastic Processes Chapter 8, Problem 8-31

A die is tossed 102 times, and the i^{th} face shows $k_i = 18, 15, 19, 17, 13, \text{and } 20$ times. Test the hypothesis that the die is fair with $\alpha = 0.05$ using the chi-square test

Chi Square Test

Let the total number of trails be n. In this test we a introduce a sum **q** known as *Pearson's test static*.

$$\mathbf{q} = \sum_{i=1}^{m} \frac{(k_i - np_{0i})^2}{np_{0i}} \tag{1}$$

where,

$$k_i = \text{Observed value for the event } i \tag{2}$$

$$p_{0i} = \text{Expected probability of the event } i \tag{3}$$

If this sum **q** is less than $\chi^2{}_{1-\alpha}\,(m-1)$, where α is significance level, we can accept the hypothesis.

Chi Square Test

let p_i denote observed probability of the event i.
We know that,
The ratio, $\frac{k_i}{n} \to p_{0i}$ as $n \to \infty$
From this, $|k_i - np_{0i}|$ is small if $p_i = p_{0i}$ and increases as $|p_i - p_{0i}|$ increases.
This justifies the use of random variable **q** and the set $q > c$ as critical region of the test.
For large value of n, **q** has χ^2 distribution.
Because of the constraint $\sum p_{0i} = 1$, the distribution of **q** has only $m - 1$ degrees of freedom.
$\implies$ **q** has $\chi^2(m-1)$ distribution.
If X is a random variable having χ^2 distribution with n degrees of freedom, then ${\chi^2}_{1-\alpha}(n)$ can be calculated by $\Pr\left(X \geq {\chi^2}_{1-\alpha}(n)\right) = \alpha$
Therefore, if this sum **q** is less than ${\chi^2}_{1-\alpha}(m-1)$, we can accept the hypothesis.

Solution

Let's denote the random variable $X_1 = \{1, 2, 3, 4, 5, 6\}$ where each $X_1 = i$ denote that i appeared on top of the die theoretically.
Let's denote the random variable $X_2 = \{1, 2, 3, 4, 5, 6\}$ where each $X_2 = i$ denote that i appeared on top of the die in the given case.
Here no. of times die was thrown(n) = 102
We know that the sum,

$$\mathbf{q} = \sum_{i=1}^{6} \frac{\left(n \Pr\left(X_2 = i\right) - n \Pr\left(X_1 = i\right)\right)^2}{n \Pr\left(X_1 = i\right)} \tag{4}$$

$$\text{Here, } \Pr\left(X_1 = i\right) = \frac{1}{6}, \forall i \in \{1, 2, 3, 4, 5, 6\} \tag{5}$$

$$\implies \mathbf{q} = \sum_{i=1}^{6} \frac{\left(6 \times \Pr\left(X_2 = i\right) - 17\right)^2}{17} \tag{6}$$

$$= \frac{1 + 4 + 4 + 0 + 16 + 9}{17} = 2 \tag{7}$$

Solution

If the die is fair,

$$\mathbf{q} < \chi^2_{1-\alpha}(6-1) \tag{8}$$

$$\implies \mathbf{q} < \chi^2_{0.95}(5) \tag{9}$$

$$\text{The value of } \chi^2_{0.95}(5) = 11.07 \tag{10}$$

$$\text{Clearly, } \mathbf{q} < 11.07 \tag{11}$$

Therefore, we can accept that the die is fair.