Assignment 6

Kotikalapudi Karthik (cs21btech11030)

***Abstract*—This document contains the solution to Example 6 of Chapter 13 (Probability) in the NCERT Class 12.**

NCERT class 12

Chapter 13(Probability) Example 6

A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?

Solution:

Let

(i) A = Event that sum of the numbers on the die is 6

(ii) B = Event that at least one of the number on the die is 4.

Let's denote the random variables X_i map to the set $\{0, 1\}$ where $X_1 = 1$ denote that the event A occurs and $X_2 = 1$ denote that the event B occurs. The sample space for die thrown twice is given by

$$\mathcal{S} = \{(x, y) : x, y \in \{1, 2 \dots 6\}\} \tag{1}$$

$$\therefore \left|\mathcal{S}\right| = 6 \times 6 = 36 \tag{2}$$

The event that $X_1 = 1$ is given by

$$\mathcal{S}_A = \{(x, y) : x + y = 6 \ \ \& \ \ x, y \in \{1, 2 \dots 6\}\} \tag{3}$$

$$\left|\mathcal{S}_A\right| = 5 \tag{4}$$

The event that $X_2 = 1$ is given by

$$\mathcal{S}_B = \{(4, x) \cup (x, 4) : x \in \{1, 2 \dots 6\}\} \tag{5}$$

$$\left|\mathcal{S}_B\right| = 11 \tag{6}$$

The event that $X_1 = 1 \ \ \& \ \ X_2 = 1$ is given by

$$\mathcal{S}_{A \cap B} = \{(2, 4), (4, 2)\} \tag{7}$$

$$\left|\mathcal{S}_{A \cap B}\right| = 2 \tag{8}$$

The probabilities for different values of X_i are given in Table I

Probability	Value
$\Pr(X_1 = 1)$	$\frac{5}{36}$
$\Pr(X_2 = 1)$	$\frac{11}{36}$
$\Pr(X_1 = 1, X_2 = 1)$	$\frac{2}{36}$
$\Pr(X_1 = 1 \vert X_2 = 1)$	?

TABLE I

$$\therefore \Pr(X_2 = 1 | X_1 = 1) = \frac{\Pr(X_2 = 1, X_1 = 1)}{\Pr(X_1 = 1)} \tag{9}$$

$$\implies \Pr(X_2 = 1 | X_1 = 1) = \frac{\frac{2}{36}}{\frac{5}{36}} \tag{10}$$

$$\implies \Pr(X_2 = 1 | X_1 = 1) = \frac{2}{5} \tag{11}$$