

Assignment 14

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Question

Probability, Random Variables and Stochastic Processes Chapter 8, Problem 8-31

A die is tossed 102 times, and the i^{th} face shows $k_i =$
18, 15, 19, 17, 13, and 20 times. Test the hypothesis that the die is fair with
 $\alpha = 0.05$ using the chi-square test

Solution

Let's denote the random variable $X_1 = \{1, 2, 3, 4, 5, 6\}$ where each $X_1 = i$ denote that i appeared on top of the die theoretically.

Let's denote the random variable $X_2 = \{1, 2, 3, 4, 5, 6\}$ where each $X_2 = i$ denote that i appeared on top of the die in the given case.

Here no. of times die was thrown $(n) = 102$

We know that the sum,

$$q = \sum_{i=1}^6 \frac{(n \Pr(X_2 = i) - n \Pr(X_1 = i))^2}{n \Pr(X_1 = i)} \quad (1)$$

$$\text{Here, } \Pr(X_1 = i) = \frac{1}{6}, \forall i \in \{1, 2, 3, 4, 5, 6\} \quad (2)$$

$$\Rightarrow q = \sum_{i=1}^6 \frac{(\Pr(X_2 = i) - \frac{1}{6})^2}{\frac{1}{6}} \quad (3)$$

$$= \frac{1 + 4 + 4 + 0 + 16 + 9}{17} = 2 \quad (4)$$

Solution

If the die is fair,

$$\mathbf{q} < \chi^2_{1-\alpha}(6-1) \quad (5)$$

$$\implies \mathbf{q} < \chi^2_{0.95}(5) \quad (6)$$

$$\text{The value of } \chi^2_{0.95}(5) = 11.07 \quad (7)$$

$$\text{Clearly, } \mathbf{q} < 11.07 \quad (8)$$

Therefore, we can accept that the die is fair.