Assignment 10

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Outline

Question

Probability, Random Variables and Stochastic Processes Chapter 2, Problem 2-25

A train and a bus arrive at the station at random between 9 A.M. and 10 A.M. The train stops for 10 minutes and the bus for x minutes. Find x so that the probability that the bus and the train will meet equals 0.5

Solution

Let's denote the random variable X_1 map to the set $\{0, 1\}$ where $X_1 = 0$ denote that bus and train don't meet and $X_1 = 1$ denote that they meet.

Let's denote the random variable X_2 map to the set $\{0, 1\}$ where $X_2 = 0$ denote that bus arrives first and $X_2 = 1$ denote that train arrives first.

Graph description

Given, train stops for 10 mins and bus stops for x minutes.
Let's draw a graph with Arrival time of bus in mins on X-axis and Arrival time of train in mins on Y-axis.
For the region in which bus and train meet(in blue color),
$Y < X + x$(train should arrive within x minutes after the bus) and
$X < Y + 10$(bus should arrive within 10 minutes after the train)

Graph

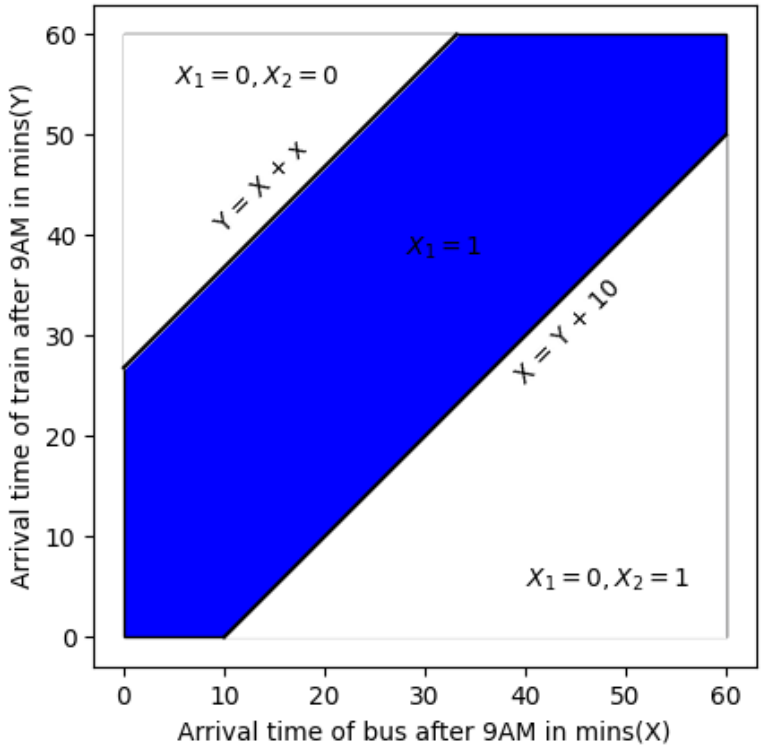

Figure 1: Arrival times of Bus and train

Finding the Value of x

$$\text{Given, } \frac{\Pr(X_1 = 1)}{\sum_{i=0}^{1} \Pr(X_1 = i)} = 0.5 \tag{1}$$

$$\implies \frac{\Pr(X_1 = 0)}{\sum_{i=0}^{1} \Pr(X_1 = i)} = 0.5 \tag{2}$$

Substituting the values from the Figure (1) in equation (2),

$$\frac{\frac{1}{2}\left[(60 - x)^2 + 50 \times 50\right]}{60 \times 60} = 0.5 \tag{3}$$

$$\implies (60 - x)^2 + 50 \times 50 = 60 \times 60 \tag{4}$$

$$\implies (60 - x)^2 = 1100 \tag{5}$$

$$\implies x = 60 - 10\sqrt{11} \approx 26.83 \text{ mins} \tag{6}$$