Assignment 14

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Outline

Question

Probability, Random Variables and Stochastic Processes Chapter 8, Problem 8-31

A die is tossed 102 times, and the i^{th} face shows k_i = 18, 15, 19, 17, 13, and 20 times. Test the hypothesis that the die is fair with $\alpha = 0.05$ using the chi-square test

Chi Square Test

In this test we a introduce a sum **q** known as *Pearson's test static*.

$$\mathbf{q} = \sum_{i=1}^{m} \frac{(O_i - E_i)^2}{E_i} \tag{1}$$

where,

$$O_i = \text{Observed value of the } i^{th} \text{ element} \tag{2}$$

$$E_i = \text{Expected value of the } i^{th} \text{ element} \tag{3}$$

Here, since $\sum_{i=1}^{m} k_i$ is fixed and expected probability in each case is $\frac{1}{6} \times \sum_{i=1}^{m} k_i$, we have only $n - 1$ degrees of freedom as this has introduced one linear constraint.
If this sum **q** is less than ${\chi^2}_{1-\alpha}\,(m-1)$, where α is significance level, we can accept the hypothesis.

Solution

Let's denote the random variable $X_1 = \{1, 2, 3, 4, 5, 6\}$ where each $X_1 = i$ denote that i appeared on top of the die theoretically.
Let's denote the random variable $X_2 = \{1, 2, 3, 4, 5, 6\}$ where each $X_2 = i$ denote that i appeared on top of the die in the given case.
Here no. of times die was thrown(n) = 102
We know that the sum,

$$\mathbf{q} = \sum_{i=1}^{6} \frac{(n \Pr(X_2 = i) - n \Pr(X_1 = i))^2}{n \Pr(X_1 = i)} \tag{4}$$

$$\text{Here, } \Pr(X_1 = i) = \frac{1}{6}, \forall i \in \{1, 2, 3, 4, 5, 6\} \tag{5}$$

$$\implies \mathbf{q} = \sum_{i=1}^{6} \frac{(6 \times \Pr(X_2 = i) - 17)^2}{17} \tag{6}$$

$$= \frac{1 + 4 + 4 + 0 + 16 + 9}{17} = 2 \tag{7}$$

Solution

If the die is fair,

$$\mathbf{q} < \chi^2_{1-\alpha}(6-1) \tag{8}$$

$$\implies \mathbf{q} < \chi^2_{0.95}(5) \tag{9}$$

$$\text{The value of } \chi^2_{0.95}(5) = 11.07 \tag{10}$$

$$\text{Clearly, } \mathbf{q} < 11.07 \tag{11}$$

Therefore, we can accept that the die is fair.