Assignment 2

Kotikalapudi Karthik (cs21btech11030)

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Question 5(b)

Verify Rolle's theorem for the following function:
$f(x) = e^{-x} \sin x$ on $[0, \pi]$

Solution: Given, $f(x) = e^{-x} \sin x$ on $[0, \pi]$

For the function to satisfy the Rolle's theorem, the following conditions should be satisfied:

1) $f(x)$ should be continuous in $[0, \pi]$
2) $f(0) = f(\pi) = 0$
3) $f(x)$ should be differentiable in $(0, \pi)$

Here, e^{-x} is an exponential and continuous function and $\sin x$ is a trigonometric and continuous function
$\therefore f(x) = e^{-x} \sin x$ is continuous on $[0, \pi]$.
Also,

$$f(0) = e^{0} \sin 0 = 0 \tag{1}$$
$$f(\pi) = e^{\pi} \sin \pi = 0 \tag{2}$$
$$\therefore f(0) = f(\pi) = 0 \tag{3}$$

By differentiating $f(x)$,

$$f'(x) = e^{-x} \left[\cos x - \sin x\right] \tag{4}$$

$\therefore f'(c)$ exists in $(0, \pi)$
$\implies f(x)$ is differentiable in $(0, \pi)$
Thus, all the conditions of the Rolle's theorem are satisfied.
$\therefore \exists$ at least one value of $x = c$ such that $f'(c) = 0$
$\because f'(c) = 0$, substituing $f'(c)$ from equation (4),

$$e^{-c} \left(\cos c - \sin c\right) = 0 \tag{5}$$
$$\implies \cos c - \sin c = 0 \tag{6}$$
$$\implies c = \frac{\pi}{4} \tag{7}$$

Clearly, $\frac{\pi}{4} \in (0, \pi)$
Hence, Rolle's theorem is verified.
Also, we can see from the graph clearly that all the necessary conditions for Rolle's theorem are satisfied and $f'(\frac{\pi}{4}) = 0$

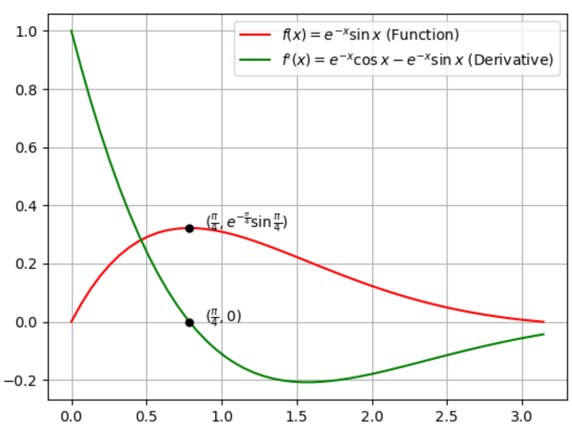

Fig. 1. Graph of $f(x)$ and $f'(x)$ in $[0, \pi]$