Assignment 13

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Outline

1. Question
2. Solution

Question

Probability, Random Variables and Stochastic Processes Chapter 2, Problem 7-11

The number of daily accidents is a Poisson random variable **n** with parameter a. The probability that a single accident is fatal equals p. Show that the number **m** of fatal accidents in one day is a Poisson random variable with parameter ap.

Known equations

Given, **n** is a Poisson random variable with parameter a. We know that,

$$\Pr\left(\mathbf{n} = n\right) = \frac{e^{-a}a^n}{n!} \tag{1}$$

$$\implies E\left\{z^{-\mathbf{n}}\right\} = \sum_{n=0}^{\infty} \frac{e^{-a}a^n}{n!} z^{-n} \tag{2}$$

$$= e^{-a}e^{az^{-1}} \tag{3}$$

$$= e^{-a\left(1-z^{-1}\right)} \tag{4}$$

If n accidents happen in a day, the probability that m will be fatal,

$$\Pr\left(\mathbf{m} = m | \mathbf{n} = n\right) = {}^nC_m p^m q^{n-m} \tag{5}$$

Solution

$$E\left\{z^{-\mathbf{m}}|\mathbf{n}=n\right\} = \sum_{m=0}^{\infty} {}^{n}C_{m} p^{m} q^{n-m} z^{-n} \tag{6}$$

$$= \left(pz^{-1}+q\right)^{n} \tag{7}$$

We know that,

$$E\left\{z^{-\mathbf{m}}\right\} = E\left\{E\left\{z^{-\mathbf{m}}|\mathbf{n}\right\}\right\} \tag{8}$$

$$\implies E\left\{z^{-\mathbf{m}}\right\} = E\left\{\left(pz^{-1}+q\right)^{n}\right\} \tag{9}$$

$$= \sum_{n=0}^{\infty} \left(pz^{-1}+q\right)^{n} \frac{e^{-a}a^{n}}{n!} \tag{10}$$

$$= e^{a\left(pz^{-1}+q\right)} e^{-a} \tag{11}$$

Solution

We know that,

$$q = 1 - p \tag{12}$$

By substituting this in equation (11),

$$E\left\{z^{-\mathbf{m}}\right\} = e^{-ap\left(1-z^{-1}\right)} \tag{13}$$

Comparing this with equation (4), we get that the **m** is a Poisson random variable with parameter *ap*.