

Matrix Project

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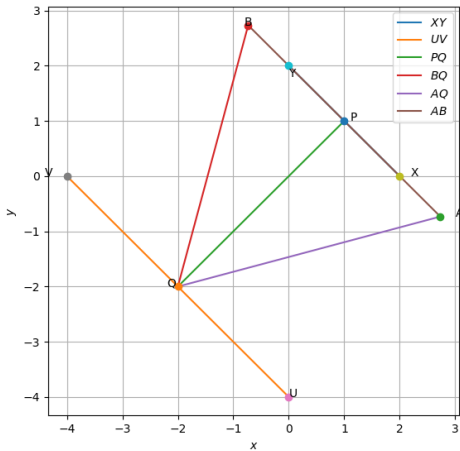
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Problem

If an equilateral triangle, having centroid at the origin, has a side along the line $x + y = 2$ then find the area of the triangle.



- Let A,B,C be the vertices of the triangle
- A,B lie on the given line $(1 \ 1)\mathbf{X} = 2$
 $(1 \ 1)\mathbf{A} = 2$
 $(1 \ 1)\mathbf{B} = 2$
- Given centroid is ORIGIN i.e $\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$
 $\mathbf{C} = -(\mathbf{A} + \mathbf{B})$
 From above equations $(1 \ 1)\mathbf{C} = -4$
- Third vertex C lies on the line with equation $(1 \ 1)\mathbf{X} = -4$

As the given triangle is equilateral, so its altitude must pass through its centroid, which is origin. And it should be perpendicular to the given line $(1 \ 1)\mathbf{X} = 1$. So the equation of altitude is $(1 \ -1)\mathbf{X} = 0$.

- The point of intersection of altitude and line $(1 \ 1)\mathbf{X} = -4$ is the third vertex C.
- The point of intersection of altitude and line $(1 \ 1)\mathbf{X} = 1$ is the mid-point of the line AB. The point be P.

Solving for \mathbf{P}

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\mathbf{x} = -\frac{1}{2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Solving for Q

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} -4 \\ 0 \end{bmatrix}$$

$$\mathbf{x} = -\frac{1}{2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ 0 \end{bmatrix}$$

$$\mathbf{c} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

Finding A and B

P is the mid-point of the side **AB**. So, we can find out **A** and **B**.

$$\mathbf{A} = \mathbf{P} + m\mathbf{N}$$

$$\mathbf{B} = \mathbf{P} - m\mathbf{N}$$

where $\mathbf{N} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and m is $\frac{a}{2\sqrt{2}}$, where a is the side of the triangle.

Finding the height and length of the side

$$\text{Height} = \|\mathbf{P} - \mathbf{C}\| = 4.242 \text{ units}$$

$$h = \frac{\sqrt{3}}{2}a$$

$$a = 4.898 \text{ units}$$

Vertices of the triangle

$$\mathbf{A} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$$

Area of a triangle

Now we have all the vertices of the triangle. So area of the triangle is

$$\text{area} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

The End