Matrix Project

P Karthik (EP17BTECH11014)
K Abhiram (MS17BTECH11009)

EE1390: Intro to AI and ML

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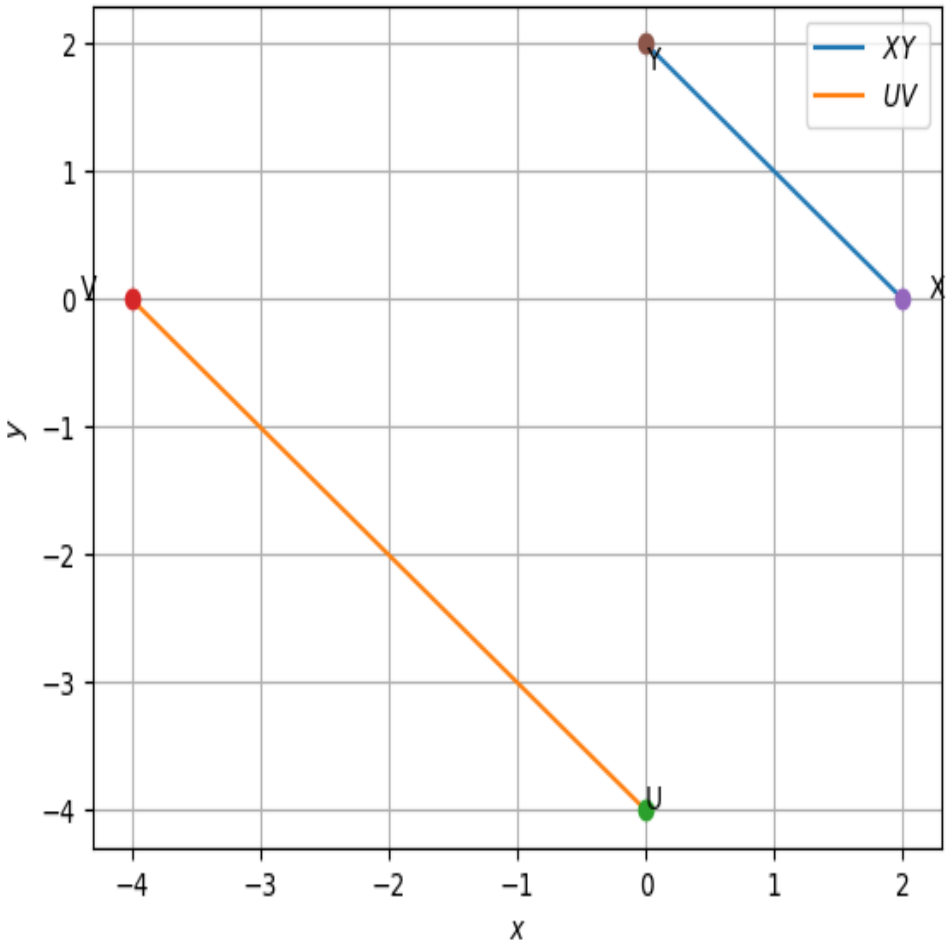
Contents in the Presentation

Problem

If an equilateral triangle, having centroid at the origin, has a side along the line $(1\ 1)\ \mathbf{X} = 2$
then find the area of the triangle.

- Let A,B,C be the vertices of the triangle
- A,B lie one the given line $(1\ 1)\ X = 2$
  $(1\ \ 1)\ A = 2$
  $(1\ \ 1)\ B = 2$
- Given centroid is ORIGIN i.e $A + B + C = 0$
  $C = -(A + B)$
  From above equations $(1\ \ 1)\ C = -4$
- Third vertex C lies on the line with equation $(1\ 1)\ X = -4$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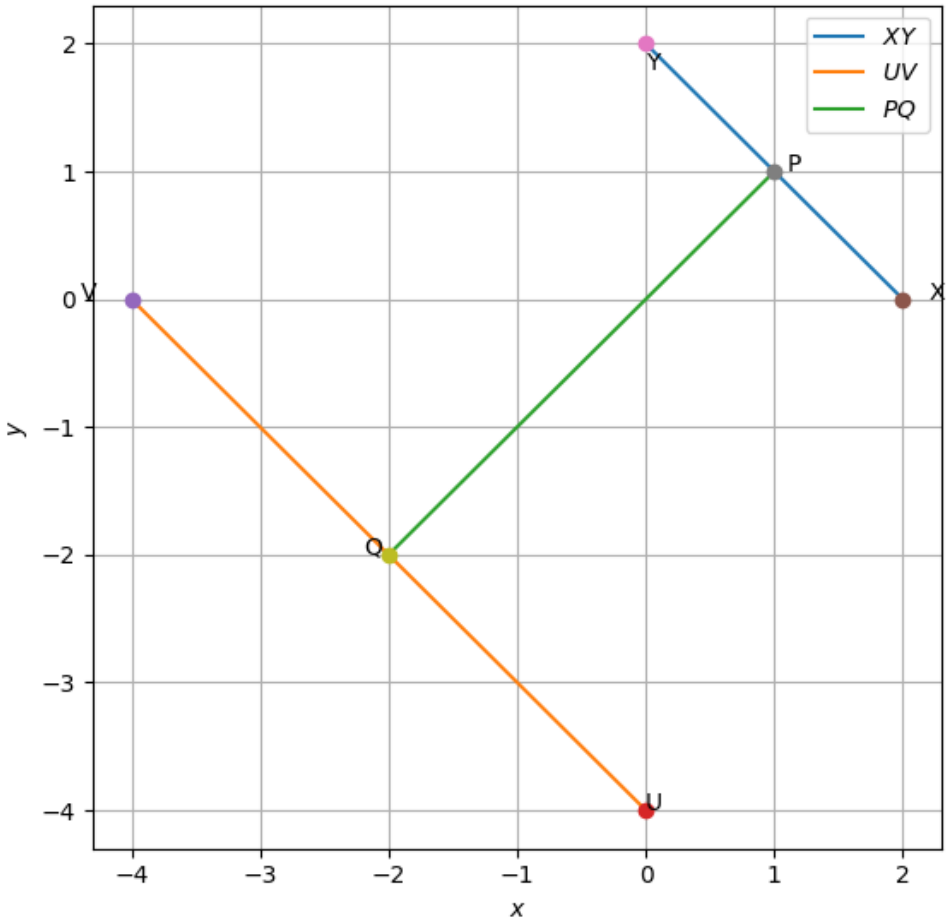
x
y

As the given triangle is equilateral, so it's altitude must pass through its centroid, which is origin. And it should be perpendicular to the given line (1 1) X $= 1$. So the equation of altitude is (1 -1) X $= 0$.

- The point of intersection of altitude and line (1 1) X = -4 is the third vertex C. The point be Q.
- The point of intersection of altitude and line (1 1) X = 1 is the mid-point of the line AB. The point be P.
- Distance between P and Q is the height of the given triangle.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x

Solving for P

Solving equations 1 and 3, the point of intersection P is (x y)

Solving for Q

Solving equations 2 and 3, the point of intersection Q is (x y)

Finding the height

Distance between P and Q gives the height of the triangle, which is 4.242 units

Solving for P

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} X = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad X = -\frac{1}{2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad P = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Solving for Q

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} X = \begin{bmatrix} -4 \\ 0 \end{bmatrix} \qquad X = -\frac{1}{2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ 0 \end{bmatrix} \qquad Q = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

For an Equilateral triangle $h = \frac{\sqrt{3}}{2}a$

h is the height and a is the side of the triangle. $a = 4.898$ units

Solution

Area of triangle

The area of triangle is $\frac{1}{2} \times a \times h$

Area of given triangle

The area of given triangle is 10.392 units^2

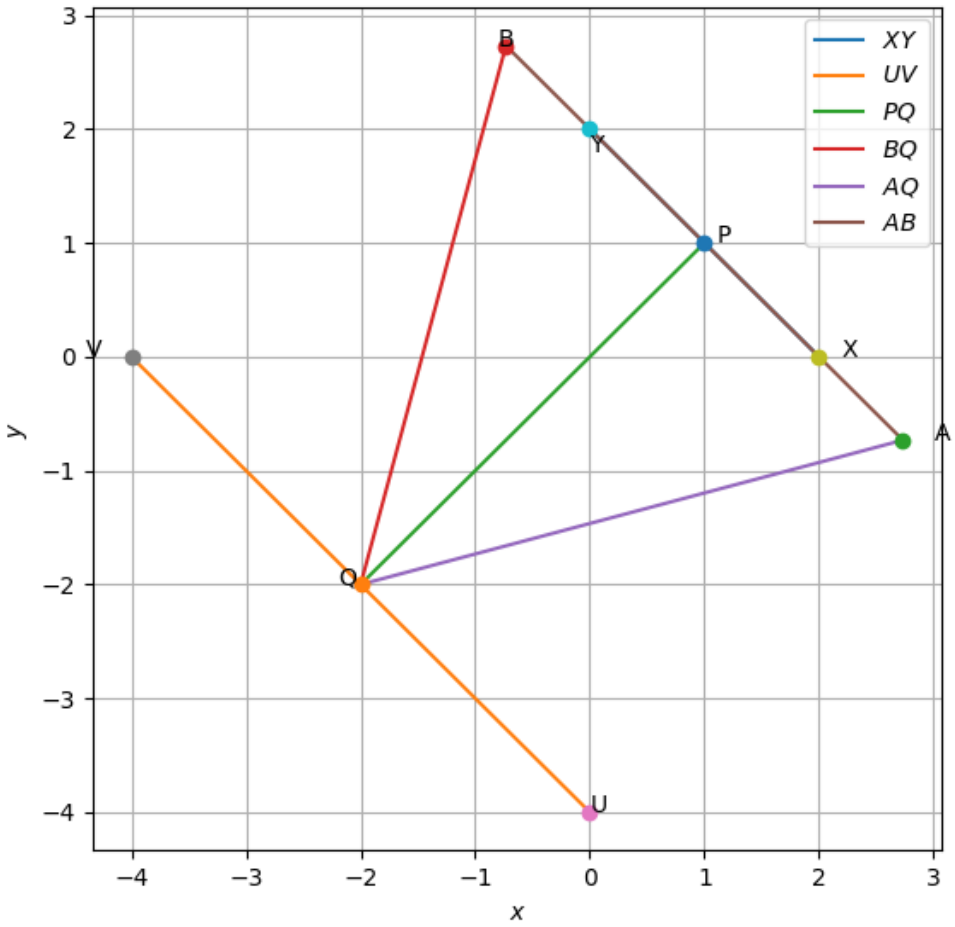
Another way to solve

P is the mid-point of the side AB. So, we can find out A and B.

A = P + M*N

B = P - M*N

where N is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and M is $\frac{a}{2\sqrt{2}}$, where a is the side of the triangle.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x
y

Area of a triangle

Now we have all the vertices of the triangle. So area of the triangle is

$$\text{area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Solution

Area of the triangle

Area of the triangle is 10.392 units^2

The End