# CS6363 : Homework 1

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1)

a) Ans: ~~$\Theta(\log_2^n)$~~ $\Theta(\log_2(n))$

Explanation: If $n$ is odd, then $n+3$ will be even.
$\because$ $2k+1$ is odd $\Rightarrow$ ~~$2(k+1)$~~ $2k+1+3 = 2k+4 = 2(k+2)$
So, worst case will be $n$ oscillating between even and odd. When $n$ becomes even, $n$ will reduce to $n/2$ resulting in $\log_2^n$ steps till $n$ becomes constant. Recurrence terminates at $n \le 10$, and for really large $n$, number of steps will be order of $\log_2^{(n)}$
$\therefore$ Running time will be ~~$\Theta(\log_2^{(n)})$~~ $\Theta(\log_2^n)$

b) Ans: $\Theta(n^2)$

Explanation: Function contains 2 "for" loops and 2 recursive calls.
Since one "for" is inside another "for" and each "for" is of size $n/2$, this will be $(n/2)(n/2)$ at each step.
$\therefore$ Recurrence relation will be $T(n) = \left(\frac{n}{2}\right)\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + c$
$\Rightarrow T(n) = 2T(n/2) + n^2/4$

Solving recurrence $\rightarrow$

$$\frac{n^2}{4} \dashrightarrow \frac{n^2}{4}$$
$$\swarrow 2$$
$$\frac{(n/2)^2}{4} \dashrightarrow 2 \cdot \frac{n^2}{16} = \frac{n^2}{8}$$
$$\swarrow 2$$
$$\frac{(n/2^2)^2}{4} \dashrightarrow 4 \cdot \frac{n^2}{16 \cdot 4} = \frac{n^2}{16}$$

$\rightarrow$ Ratio of level sums $(i+1/i)$ = ~~$n^2/4$~~ $1/2\ (<1)$
$\therefore$ This will be $\Theta(n^2/4) = \Theta(n^2)$

c) checks given array for duplicate values and returns "True" if array has duplicates and returns "False" otherwise.

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2) a) $\log^3(n) + 5^{\log\log n} + \log^2(\log^4 n)$

Applying log to all 3 terms.

$3\log\log n$, $\log\log n \log 5$, ~~2 log log⁴ n~~ $2\log\log\log^4(n)$

$3\log\log n$ is greater than remaining 2 terms
So this expression will be $\Theta(\log^3(n))$

(b) Number of bits needed to write $9^n$ in binary will be $n.\lfloor \log_2 9 \rfloor + 1$.

$\therefore$ No. of bits needed will be $\Theta(n)$.

(c) Initially $f(0) = 1$ billion $= 10^9$
Since population doubles every year: $f(n) = 2^n f(0)$
$\therefore f(n) = 2^n \times 10^9$
$\therefore$ In 100 years, $f(100) = 10^9 \times 2^{100}$, which is a constant
$\therefore$ This will be $\Theta(1)$

(d) $\sum_{i=1}^{n} \log(n/i) = \sum_{i=1}^{n} (\log n - \log i)$

$= \sum_{i=1}^{n} \log n - \sum_{i=1}^{n} \log i$

$= n\log n - \log(1 \times 2 \times 3 \ldots \times n)$

$= n\log n - \log(n!) \rightarrow ①$

Sterling's approximation:-

$\log(n!) = n\log n - n + \theta(\log n) \rightarrow ②$

$\therefore$ ① can be rewritten using ② as:-

$\Rightarrow n\log n - n\log n + n - \theta(\log n)$

$\Rightarrow n - \theta(\log n)$ which will be $\theta(n)$

$\therefore \sum_{i=1}^{n} \log(n/i) = \underline{\underline{\theta(n)}}$

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3) Problem 3

$\cos(n)+2$, $n^{3/(2\lg n)}$

$\lg^* n$, $\lg^*(2^{2^{2^n}})$

$\lg(\sqrt{\lg(n)})$, $12+\lg(\lg(n))$

$\sqrt{\lg(n)}$

$\lfloor \lg(n) \rfloor$, $\sum_{i=1}^{n} \frac{1}{i}$

$\lg^{201}(n)$

$n^{\frac{1}{\lg(\lg(n))}}$

$n^{1/125}$

$n$, $2^{\lg(n)}$

$n \lg^4(n)$

$(\sqrt{n})^e$

$4^{\lg(n)}$

$(\frac{n}{2}-1)^{2.1}$

$n^3$, $\sum_{i=1}^{n} i^2$

$(\lg^* n)^{\lg n}$

$n^{\lg(\lg(n))}$, $(\lg n)^{\lg n}$

$n^{\sqrt{n}}$

$(1+\frac{1}{250})^n$

4) Base case is when there are '0' and '1' plates. if $n=0$, return 1 because there is only one possibility of no plates. if $n=1$, return 2 because we can place either green plate or red plate.
Recurrence relation will be

$$T(n) = \underbrace{2 \cdot T(n-1)}_{\text{2 cases where last plate is either red or green}} + \underbrace{T(n-2)}_{\text{Case where last 2 plates are blue and green on top}}$$

$\therefore$ $T(0) = 1$
$T(1) = 2$, $T(2) = 5$ } Base case
$T(n) = 2T(n-1) + T(n-2)$ for $n > 2$

5) a) $T(n) = 6T(n/3) + n^2$

$n^2$

$\times 6$

$(n/3)^2 \dashrightarrow 6 \cdot (n/3)^2 = 2/3\, n^2$

$\times 6$

$(n/3^2)^2 \longrightarrow 6^2 (n/3^2)^2 = (2/3)^2 n^2$

$\vdots$

$(n/3^i)^2 \dashrightarrow 6^i (n/3^i)^2 = (2/3)^i n^2$

Level sum = $(2/3)^i n^2$

Ratio of level sums = $\dfrac{(2/3)^{i+1} \cdot n^2}{(2/3)^i \cdot n^2} = 2/3$ $(<1)$ (Decreasing geometric)

$\therefore$ Given recursion is bounded by root.
$\Rightarrow \Theta(n^2)$

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b) $T(n) = T(n/5) + 3T(n/4) + n$

$$n$$

$$n/5 \qquad n/4 \quad \dashrightarrow \frac{n}{5} + \frac{n}{4} \times 3 = \frac{19n}{20}$$

$$n/5.5 \quad n/5.4 \quad n/5.4 \quad n/4.4 \dashrightarrow \frac{n}{25} + \frac{3n}{20} + \frac{3n}{20} + \frac{9n}{16} = \left(\frac{19}{20}\right)^2 n$$

$\therefore$ level sum $= \left(\frac{19}{20}\right)^i n$

Ratio of level sums $= 19/20$ $(<1)$ (Decreasing geometric)

$\therefore$ Given recurssion is bounded by root. $\Theta(n)$

c) $T(n) = 7 \cdot T(n/3) + 4n^{3/2}$

$$4 \cdot n^{3/2}$$

$$4(n/3)^{3/2} \dashrightarrow 7 \cdot 4 \cdot \frac{n^{3/2}}{\sqrt{27}}$$

$$4(n/3^2)^{3/2} \dashrightarrow 7^2 \cdot 4 \cdot \frac{n^{3/2}}{(\sqrt{27})^2}$$

$\therefore$ level sum $= \left(7/\sqrt{27}\right)^i \cdot 4 \cdot n^{3/2}$

Ratio of level sums $= 7/\sqrt{27}$ $(>1)$ (Increasing Geometric)

$\therefore$ Given recursion is bounded by number of leaves.

$\therefore$

$\#$ leaves $=$ (branch factor)$^{depth}$

$\Rightarrow \#$ leaves $= 7^{\log_3 n} = n^{\log_3 7}$

$\Rightarrow$ Given recursion is $\Theta(n^{\log_3 7})$

d) $T(n) = 8T(n/2) + n^3$

$n^3$

$\times 8$

$(n/2)^3 \longrightarrow 8 \cdot (n/2)^3 = n^3$

$\times 8$

$(n/2^2)^3 \longrightarrow 8^2 \cdot (n/2^2)^3 = n^3$

$\therefore$ Level sum $= 8^i (n/2^i)^3 = \frac{8^i}{8^i} \cdot n^3 = n^3$

$\therefore$ Ratio of level sums $= 1$

$\therefore$ Given recursion will be bounded by root $\times$ depth.

depth $= \log_2 n$, root $= n^3$

$\Rightarrow \Theta(n^3 \cdot \log_2 n)$

f) $T(n) = T(n-1) + n^{1/4} \longrightarrow$ Ans: $\Theta(n^{5/4})$

$n^{1/4}$

$(n-1)^{1/4}$

$(n-i)^{1/4}$

level sum $= (n-i)^{1/4}$

depth $= n$

# leaves $= 1$

Ratio of level sum $= \dfrac{(n-i-1)^{1/4}}{(n-i)^{1/4}} = \left(1 - \dfrac{1}{n-i}\right)^{1/4}$

$T(n) = n^{1/4} + (n-1)^{1/4} + (n-2)^{1/4} + \cdots + 0^{1/4}$

As $y = x^{1/4}$ is an increasing graph,

$T(n) < n^{1/4} + n^{1/4} + \ldots.$ n terms

$T(n) < n \cdot n^{1/4} \Rightarrow T(n) < n^{5/4}$

$\int_0^n n^{1/4} dn = \dfrac{n^{5/4}}{5/4}$

$= 4/5 \cdot n^{5/4}$

$\therefore$ This will be $\Theta(n^{5/4})$

e) $T(n) = T(\sqrt{n}) + 3$

Depth is when $n^{1/2^d} = c$

$\Rightarrow \frac{1}{2^d} \log n = \log c$

$\Rightarrow \log\log n = d \log 2 + \log\log c$

$\Rightarrow d$ is $\Theta(\log\log n)$

3

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Since level sum ratio is "1", Given recursion will be $\Theta(\text{root} \times \text{depth})$

$\Rightarrow \Theta(3 \log\log n) = \Theta(\log\log n)$