## CS6363: Homework 1

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Ans:  $\Theta(\log_2(n))$   $\Theta(\log_2(n))$ Explanation: If n is odd, then n+3 will be even.  $\therefore 2k+1$  is odd  $\Rightarrow 2(k+1) 2k+1+3 = 2k+4 = 2(k+2)$ So, worst case will be n oscillating between even and odd. When, n becomes even, n will reduce to  $\frac{n}{2}$  resulting in  $\log_2^n$  steps till n becomes constant Recurrence terminates at  $n \le 10$ , and for really large n, number of steps will be order of  $\log_2^n$ .  $\therefore$  Running time will be  $\Theta(\log_2^n)$ .  $\Theta(\log_2^n)$ 

b) An:  $\Theta(n^2)$ Explanation: Function contains 2 for loops and 2
recursive calls.

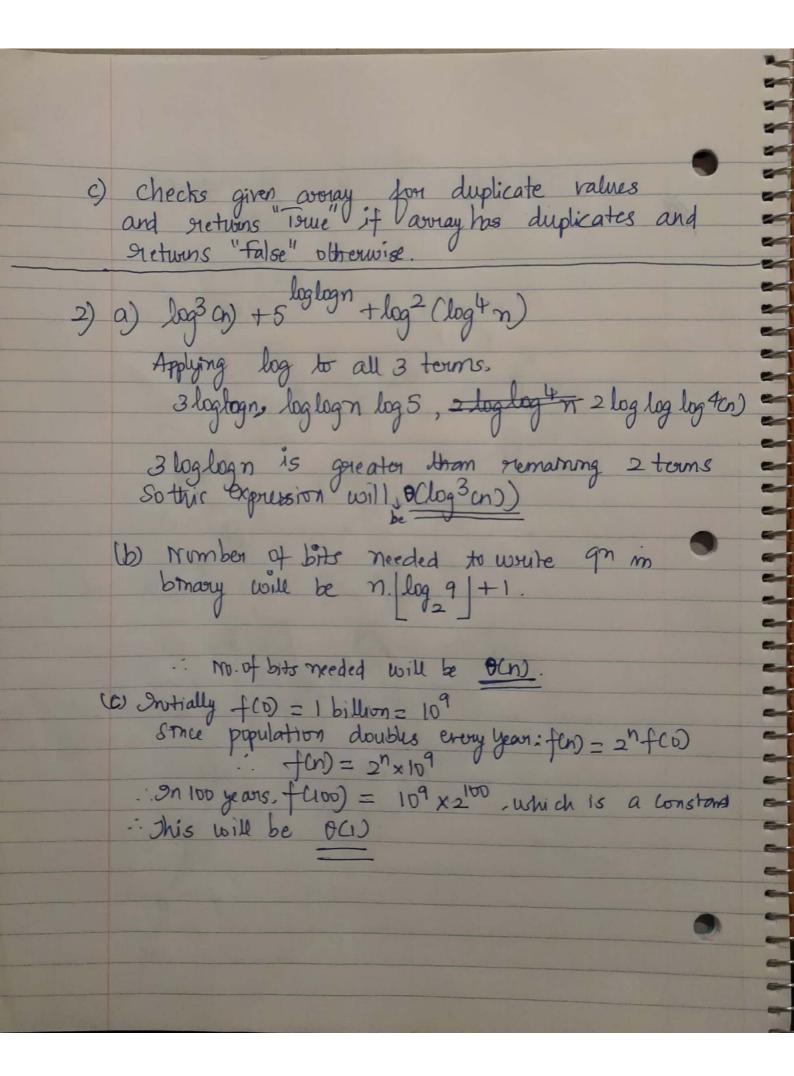
Since one for is inside another for and each

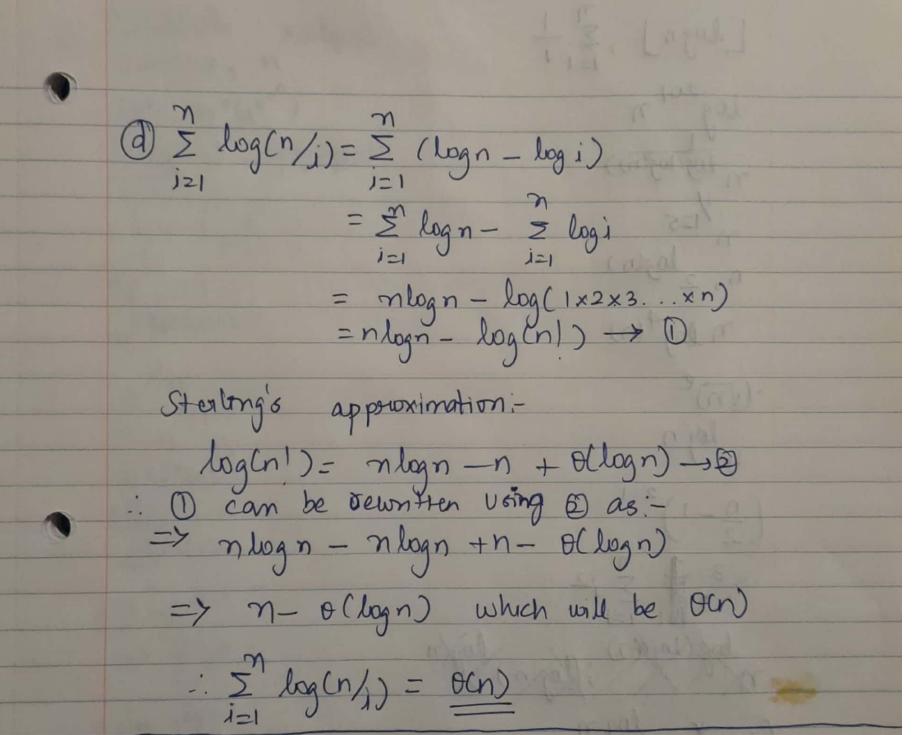
"for is of size n/2, this will be (n/2)(n/2) at
each step.

Recurrence relation will be T(n) = (n/2)(n/2) + T(n/2) + T(n

Solving recurrence  $\rightarrow \frac{n^2/4}{\sqrt{2}} - - \rightarrow \frac{n^2/4}{\sqrt{2}}$   $\frac{(n/2)^2/4}{\sqrt{2}} - - \rightarrow \frac{2 \cdot n^2/4}{16 \cdot 4} = \frac{n^2/6}{16 \cdot 4}$   $\frac{(n/2)^2/4}{\sqrt{2}} - - \rightarrow 4 \cdot \frac{n^2}{16 \cdot 4} = \frac{n^2/6}{16 \cdot 4}$ 

- Ratio of level sums (i+1/i) =  $\frac{1}{2}$  1/2 (<1) ... This, will be  $\Theta(\frac{n}{4}) = \Theta(\frac{n^2}{4})$ 





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Problem 3 3/algn)

Coscn)+2, n

lg*n, lg*(22n)
19
                                                                                                                                                                                                                                                                                     lg ( Vegin ) , 12+ lg ( lg ( n ) )

L lg ( n ) , X = 1;

lg ( lg ( n ) )

1 1/25

2 lg ( n )

1 1/25

2 lg ( n )

1 lg ( n )

                                                                                                                                                                                                                                                                                             (\frac{\gamma_{1}}{2}-1)^{2.1}
                                                                                                                                                                                                                                                                                   (lg*n)lgm

nlg(lg(n)), (lg n)lgn

n
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4) Base case is when there are o' and i' plates. if n=0, return , because there is only one possibility of no plates. if n=1, return 2 because we can place either green plate or red plate. Recurrence relation will be  $T(n) = 2 \cdot T(n-1) + T(n-2)$ (2 cases where last) (ase where last 2 plates)
(plate is either red) (are blue and green on top) umber of buts needed to write of in bainany · T(0)=1 T(0) = 1 Base case T(1) = 2, T(2) = 5T(n) = 2T(n-1) + T(n-2) for 1/2 n>2 Par = noillid = (0) + Heitin = 10 5) a)  $(T(n)) = (n6T(n/3) + n^2)$   $(n^2) + 6$   $(n/3)^2 - -- \rightarrow 6 \cdot (n/3)^2 = (n/3)^2 = (n/3)^2$ polleyel sum = (2/3) in a mortania que s'enitade Ratio of level sums = (2/3) in = 2/3 (a) (Decreasing geometric) : Given recursion is bounded by root.

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Ratio of level sums = 19/20 (<1) (Decreasing geometric)

displayed recursion is bounded by zuroot . (9(n)

c) 
$$T(n) = 7 \cdot T(n/3) + 4n^{3/2}$$

4. 
$$n^{3/2}$$

4.  $(n/3)^{3/2}$ 

Ratio of level sum = 7/127 (>1) (Increasing Geometric)
. Given recursion is bounded by number of leaves.

$$\Rightarrow$$
 Given recursion is  $\Theta(n^{\log_3 7})$ 

T(n) = 8T(n/2) + n3  $\frac{\times 8}{1^3} - \longrightarrow 8 \cdot (\frac{n}{2})^3 = n^3$  $- \longrightarrow 8^2 \left( \frac{\eta}{2^2} \right)^3 = n^3$ : Level -sum =  $8^{i} \left( \frac{\eta}{2i} \right)^{3} = \frac{8^{i}}{8^{i}} \cdot n^{3} = n^{3}$ Rotio of level sums = 19, (21) (Decreasing geometric) -. Ratio of level sums = 1 Given recursion will be bounded by rootx depth. depth= logn, root=n3  $\Theta(n^3, \log_2 n)$ 

P) 
$$T(n) = T(n-1) + n^{1/4} \rightarrow \underline{A}_{M!}$$
:  $\Theta(n^{5/4})$ 
 $n^{1/4}$ 
 $| \text{level sum} = (n-i)^{1/4}$ 
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 $| \text{Ratio of level sum} = (n-i)^{1/4} = (1-\frac{1}{n-i})^{1/4}$ 
 $| \text{T(n)} = n^{1/4} + (n-1)^{1/4} + (n-2)^{1/4} + \cdots + b^{1/4} = (n-i)^{1/4}$ 
 $| \text{As } y = x^{1/4} \text{ is an increasing graph,} = \frac{n^{5/4}}{5^{1/4}}$ 
 $| \text{T(n)} < n^{1/4} + n^{1/4} + \cdots + n \text{ terms} = \frac{n^{5/4}}{5^{1/4}}$ 
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 $| \text{T(n)} < n^{1/4} + n^{1/4} + \cdots + n \text{ terms} =$