B.M.S. COLLEGE OF ENGINEERING, BENGALURU-560019.
DEPARTMENT OF MATHEMATICS
FIRST TEST - III SEMESTER B.E.(CSE/ISE)
STATISTICS AND DISCRETE MATHEMATICS - 19MA3BSSDM

Q.No.	**SOLUTIONS & SCHEME OF VALUATION**	Marks
	PART-A	

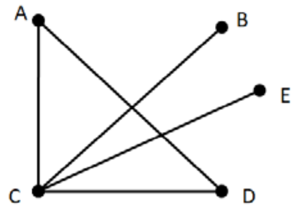
1 Suppose that in a group of 5 people: A, B, C, D, and E, the following pairs of people are acquainted with each other. — **5M**

A and C
A and D
B and C
C and D
C and E

a) Draw a graph G to represent this situation.
b) Identity the degree of each vertex.

Soln

a) Graph G — **3M**



b) — **2M**

$vertex$	A	B	C	D	E
$degree$	2	1	4	2	1

PART-B

2(a) Obtain an expression for the number of edges in a complete graph G(V,E) and hence find the fewest vertices needed to construct a complete graph with at least 1000 edges. — **5M**

Soln

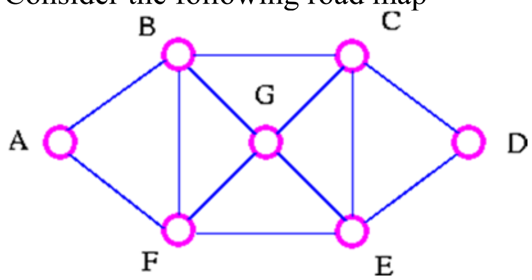
In a complete graph , there exists exactly one edge between every pair of vertices. The number of edges in a complete graph is equal to the number of pairs of vertices. If the number of vertices is n, then the number of pairs of vertices is $nC_2 = \frac{n(n-1)}{2}$. — **2M**

Given the number of edges(m) is at least 1000.

$$\frac{n(n-1)}{2} \geq 1000 \Rightarrow n^2 - n - 2000 \geq 0 \Rightarrow n = 45.22, \; -44.22$$

Thus, the fewest number of vertices required to construct complete graph with at least 1000 edges is 46. — **3M**

2(b) Consider the following road map — **5M**



An explorer wants to explore all the routes between a number of cities. Can a tour be found which traverses each route only once? If yes, justify.
Particularly, find a tour which starts at A, goes along each road exactly once,

and ends back at A.

Soln Yes, tour can be found which traverses each route only once because the given graph consist an Euler circuit/ degree of every vertex is even. **3M**

A tour that starts at A, traverse each road exactly once is

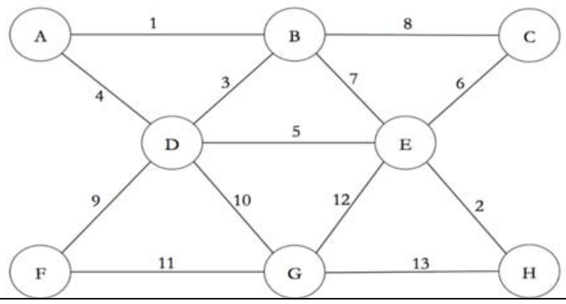
$A \to B \to C \to D \to E \to G \to F \to A$ **2M**

2(c) If G is a simple graph with n vertices in which the degree of every vertex is at least $\left(n-1\right)/2$, prove that G is connected. **5M**

Sol. Take any two vertices u and v of G. Then they are adjacent or not adjacent. If they are adjacent, then G is connected. Otherwise, each has at least (n-1)/2 neighbours, because the degree of every vertex is at least (n-1)/2. Therefore, u and v tahen together have at least n-1 neighbours. But, since G has a total of n vertices, the total number of neighburs which u and v together can have is only n-2. Therefore, at least one vertex, say x is a neighbor of both u and v. Hence, there is a edge between u and x and there is an edge between x and v. Thus, there is a path between u and v . G must be connected. **5M**

PART-C

3(a) Using Kruskal's algorithm, find a minimal spanning tree for the following weighted graph. **6M**

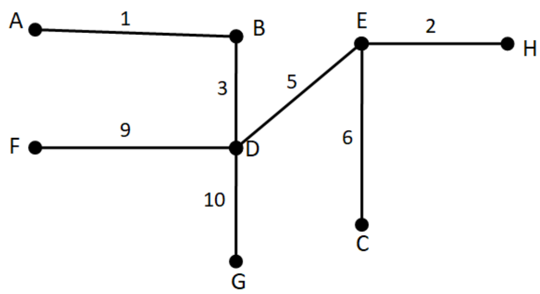


Sol. The given graph has 8 vertices and therefore a spanning tree will have 7 edges.

Edge	*AB*	*EH*	*BD*	*AD*	*DE*	*EC*	*BE*	*BC*	*DF*	*DG*	*FG*	*GE*	*GH*
Weight	1	2	3	4	5	6	7	8	9	10	11	12	13
Select	*Y*	*Y*	*Y*	*N*	*Y*	*Y*	*N*	*N*	*Y*	*Y*	*N*	*N*	*N*

3M

Spanning tree T



2M

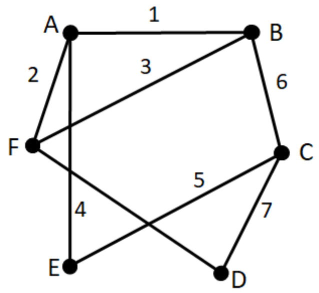
Wt(T)=43 **1M**

OR

3(b) For the given adjacency matrix, draw the graph and hence write its incidence matrix. 6M

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Sol. Graph G for the given adjacency matrix.



3M

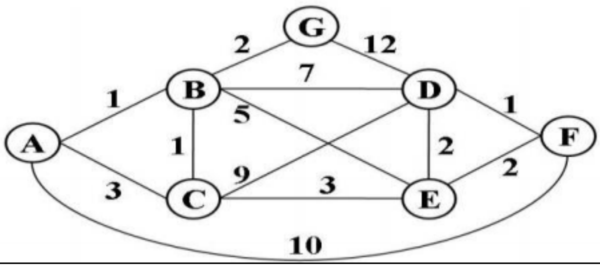
Incidence matrix A(G)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

3M

OR

4(a) The diagram below shows roads connecting villages near to city A. The numbers on each arc represent the distance, in miles, along each road. Leon lives in city A and works in city F. Use Dijkstra's algorithm to find the minimum distance for Leon's journey to work. 7M



Sol.

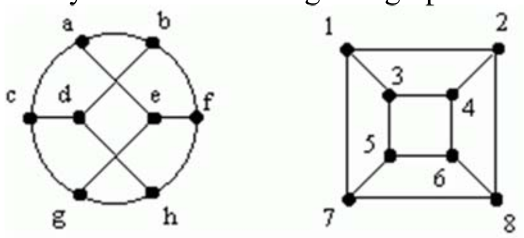
marked	A	B	C	D	E	G	F
A	$\boxed{0}$	∞	∞	∞	∞	∞	∞
B	$\boxed{0}$	$\boxed{1}$	3	∞	∞	∞	10
C	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$	8	6	3	10
G	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$	8	5	$\boxed{3}$	10
D	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$	8	$\boxed{5}$	$\boxed{3}$	10
E	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$	$\boxed{7}$	$\boxed{5}$	$\boxed{3}$	7
F	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$	7	$\boxed{5}$	$\boxed{3}$	$\boxed{7}$

4M

3M

OR

4(b) Verify that the following two graphs are isomorphic. 7M



Sol. For proving vertex mapping and edge mapping 6M
conclusion 1M

OR

5(a) A question paper contains 10 questions of which 7 are to be answered. In how many ways a student can select the 7 questions 7M

(i) If he can choose any seven?
(ii) If he should select three questions from the first five and four questions from the last five?
(iii) If he should select at least three from the first five?

Soln

(i) Number of ways to select 7 from 10 questions in $10C_7 = 120$ *ways*. 2M
(ii) Number of ways to select three questions from the first five and four questions from the last five in $5C_3 \times 5C_4 = 50$ *ways*. 2M
(iii) Number of ways to select at least three from the first five in $5C_3 \times 5C_4 + 5C_4 \times 5C_3 + 5C_5 \times 5C_1 = 110$ *ways*. 3M

OR

5(b) Find the number of ways of arranging the letters of the word ASSOCIATIONS. In how many of them (i) all the three S's come together ii) The two A's do not come together. 7M

Soln	The given word has 12 letters of which 2 are A, 3 are S, 2 are O, 2 are I and 1 each C, T, N. Number of arrangements of these letters is $\frac{12!}{2!2!3!2!} = 9979200$	1M
	In a arrangement, all the three S's come together Number of arrangements of these letters is $\frac{10!}{2!2!2!} = 453600$	2M
	Since 2 A's do not come together, first arrange the remaining 10 letters in $\frac{10!}{2!2!3!} = 151200$ ways.	2M
	Now, we have 11 locations for two A's to be placed so that no A's do not come together. These locations can be chosen in $11C_2 = 55\, ways$	
	Number of arrangement which have no A's come together is $151200 \times 55 = 8316000$	2M

Note: Suitable marks to be awarded for alternate methods.

P.GOMATHI