**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans) x(mean) = 60, pop mean =45+10=55

Z = (60-55)/8 = 0.625

1-norm.cdf(0.625) = 0.2659

B

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans) mean =38,Sd =6

Z score for 44 = (44-38)/6 = 1 = 84.13%

Z score for 38 = (38-38)/6=0 = 50%

For people above 44 = 100-84.13 = 15.87%

Between 38 and 44 = 84.13-50 = 34.13%

More employees at processing centre older than 44 is False

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans) z score for 30 = (30-38)/6 = -1.33 =9.15% = 36

True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans) According to central limit theorem any large sum of independent idendically distributed random variables is approximately Normal.

The **Normal distribution** is defined by two parameters, the **mean**, , and the **variance**,  and written as 

following the property of multiplication, we get



and following the property of addition,



And the difference between the two is given by



The mean of  and  is same but the var() of   is 2 times more than the variance of .

The difference between the two says that the two given variables are **identically** and **independently** distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans) D

Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z(99.5) = stats.norm.ppf(0.995) = 2.576

a = -(20\*2.576) + 100= 48.5

      b = (20\*2.576)+100= 151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans) 95% of the **probability** **lies**between 1.96 **standard deviations**of the **mean**.

Companys profit= P~N(5+7,3^2+4^2) = N(12,5^2)

Thus **range**is = (12-1.96\*5,12+1.96\*5)

=(Rs99M,1026M)

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans)5th percentile z score = 1.64

p-12/5 = 1.644

p = 3.78M$ or Rs170.1M

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans)loss is when profit is <0

The first division of the company have higher probability of making loss in a given year.

P1 = P(p-5/3) = P(-1.66) = 0.485

P2 = P(p-7/4) = P(-1.75) = 0.401