# Indian Institute of Technology Kanpur

## Aircraft Control Systems (AE322)

Assignment 1

Submission Deadline: 15 February 2025@9.30PM  Max. Marks: 100

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Note: Attach codes wherever MATLAB is used.

1. Consider the following systems where x(t) is the input and y(t) is the output signal. Determine whether they are linear time varying, nonlinear or Linear Time Invariant (LTI) systems:

   (Marks: 15)

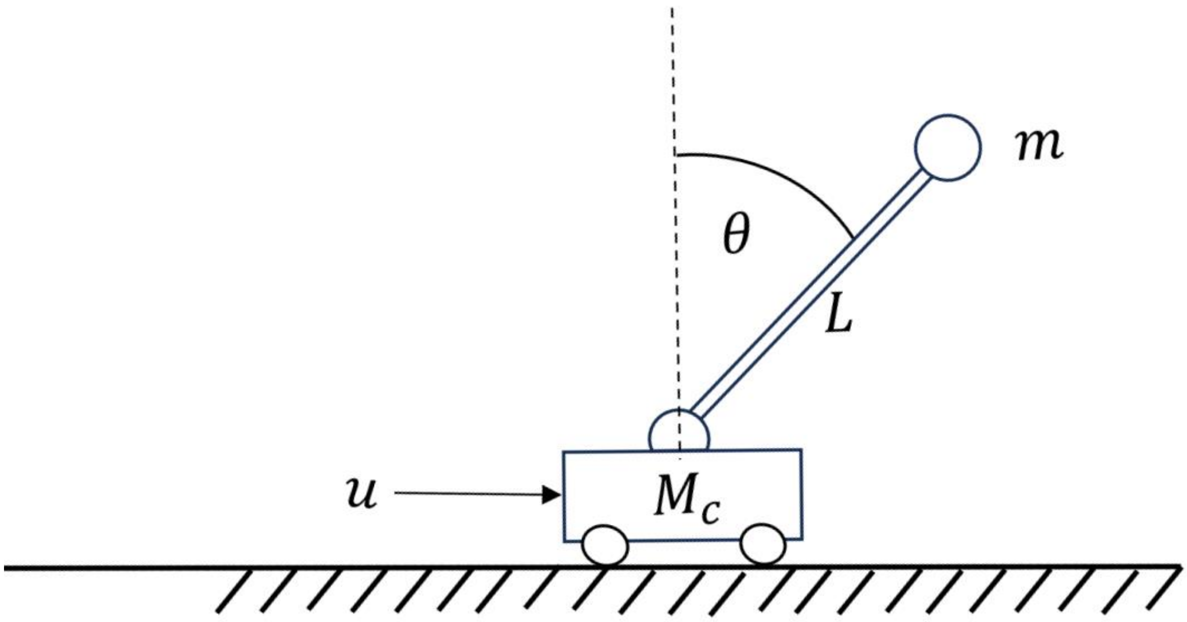
   a. $y(t) = 6\,sin\big(5x(t)\big) + 8\,cos(10x(t))$
   b. $y(t) = tsin(kt) + x(t)$
   c. $y(t) = ax(t) + b$
   d. $y(t) = kx(t - c), \quad c \in \mathbb{R}_+$
   e. $y(t) = t^2 x(t)$

   where, $a, b, k \in \mathbb{R}$

2. Find the transfer functions to the above systems, if possible. If not, give a reason behind it.

   (Marks: 5)

3. Consider the inverted pendulum-cart system. (Marks:80)




Mass of ball(m) = $0.1\ Kg$, mass of Cart ($M_c$) = $1\ Kg$ , length of rod ($L$) $= 0.5m$, $g = 9.81m/s^2$. Equation of motion of the pendulum is given by:

$$\ddot{\theta} = F(\theta, \dot{\theta}) + G(\theta)u$$

Where, $F(\theta, \dot{\theta}) = \frac{\sin\theta}{L(M_c + m\sin^2\theta)}\{-mL\dot{\theta}^2\cos\theta + (M_c + m)g\}$

$$G(\theta) = \frac{-\cos\theta}{L(M_c + m\sin^2\theta)}$$

a) Find all equilibrium points.
b) Linearize the equations of motion about the equilibrium points and write the state space equations.
c) Find the transfer function of $\theta$ with respect to $u$ at the equilibrium points.
d) Identify the stable and unstable equilibrium points using Routh Hurwitz criteria.
e) Write a MATLAB script to plot the output for $0 \le t \le 20$ seconds from-
    i. the time response obtained from the original dynamical equations from a unit step input, with the initial conditions as equilibrium points.
    ii. the unit step response of the linearised systems, with the initial conditions as equilibrium points.

    Compare the responses and draw some conclusions. (Hint: you can use the **step** and **lsim** commands in MATLAB).

f) Simulate the dynamical system for 20 seconds taking the initial conditions as $\theta = 10\ degrees\ \dot{\theta} = 0\ degrees/sec$ and $u(t) \equiv 0$, using
    i. The original dynamical equations.
    ii. The linearized equation of motion, which is obtained from linearization at the nearest equilibrium point of the given initial conditions.

    Compare the responses and draw some conclusions.

g) Design a PD controller to stabilize the system to $(\theta, \dot{\theta}) = (0,0)$, such that the settling time is 1 second and peak overshoot is less than 10 %. Simulate the linearized and original dynamical system by taking this control law. Use initial condition as $\theta = 10\ degrees\ \dot{\theta} = 0\ degrees/sec$ .
h) Now the control requirements are changed. Design another PD controller to stabilise the system to $(\theta, \dot{\theta}) = (0,0)$, such that settling time is 0.5s & peak overshoot is 15%. Simulate the same in MATLAB.
i) Suppose some disturbance force, d(t) is added to the system. Find the steady state error for unit step disturbance. Plot the same in MATLAB to verify.
j) Design a PD controller such that the maximum steady state error is less than 0.5 degrees when the disturbance is unit step.

k) Find the values of kp,kd for marginal stability. Perform simulations to verify oscillations in MATLAB. Plot poles & zeros in MATLAB.