Project
on

Automotive Suspension System

Course code: EE650A
Basics of Modern Control Systems

Submitted by

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Submitted To,

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Contents:

Introduction

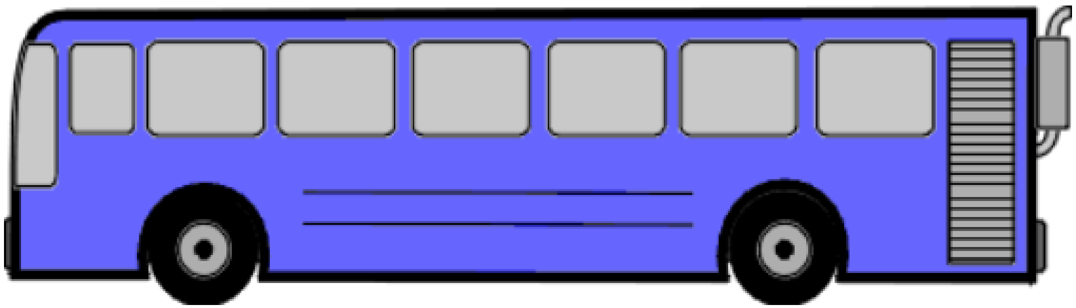
- An Automotive Suspension system Allows the vehicle to absorb shocks and maintain contact with the road, even on rough surfaces.

- Consist of springs, shock absorbers, and various linkages that help to manage the movement of the wheels relative to the body of the vehicle.

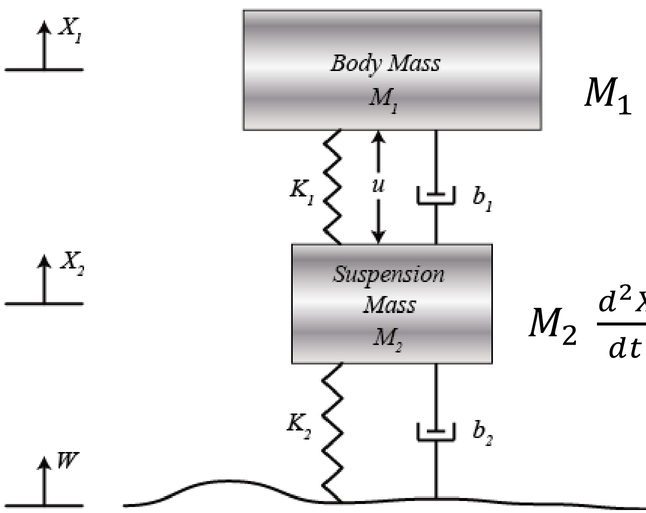
- Springs absorb vertical energy from the wheels, when a vehicle goes over bumps, helping maintain a smooth ride.

- Shock Absorbers (Dampers) control the motion of the springs, preventing excessive bounce and keeping the vehicle stable.

Mathematical Model



U=Control input force



$$M_1 \frac{d^2X_1}{dt^2} = -b1\left(\frac{dX_1}{dt} - \frac{dX_2}{dt}\right) - k1(X_1 - X_2) + U$$

$$M_2 \frac{d^2X_2}{dt^2} = b1\left(\frac{dX_1}{dt} - \frac{dX_2}{dt}\right) + k1(X_1 - X_2) + b2\left(\frac{dW}{dt} - \frac{dX_2}{dt}\right) + k2(W - X_2) - U$$

Schematic diagram of an automotive suspension system

Transfer functions are calculated as:

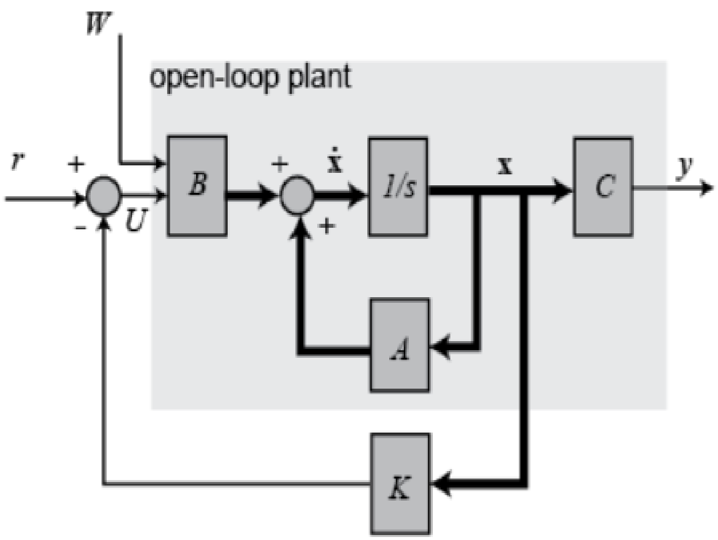
$$G_1(s)=\frac{X_1(s)-X_2(s)}{U(s)}=\frac{(M_1+M_2)s^2+b_2s+K_2}{\Delta}$$

When, U is present, W= 0

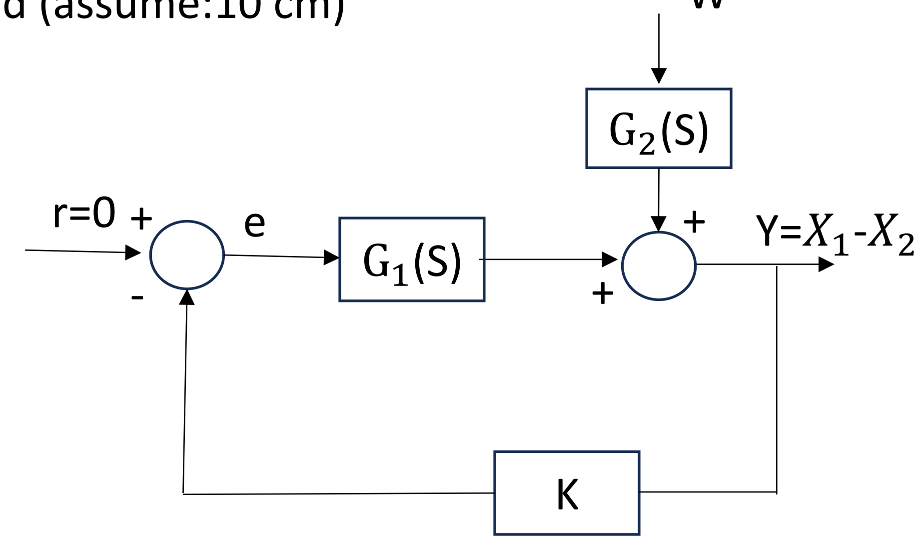
$$G_2(s)=\frac{X_1(s)-X_2(s)}{W(s)}=\frac{-M_1b_2\,s^3-M_1K_2s^2}{\Delta}$$

When, W is present, U= 0

W=deep pothole on the road (assume:10 cm)



Controller



Controller

State Space Model of Automotive Suspension System

State Space Model

The state space representation of automotive suspension system:

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

Where, $X = \begin{bmatrix} X1 \\ X2 \\ \dot{X}1 \\ \dot{X}2 \end{bmatrix}$

System Matrix, input & output matrices are computed as :

$$[A] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K1/M1 & K1/M1 & -b1/M1 & b1/M1 \\ K1/M2 & -(K1+K2)/M2 & b1/M2 & -(b1+b2)/M2 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 \\ 0 \\ 1/M1 \\ -1/M2 \end{bmatrix}$$

$$[C] = \begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix}$$

$$[D] = \begin{bmatrix} 0 \end{bmatrix}$$

State Space Model

The state space representation of system:

$$\dot{X}=AX+BU$$

$$Y=CX+DU$$

Vehicle ¼ mass: M1 = 3000 Kg;
Suspension mass: M2 = 350 Kg;
Spring constant: K1 = 80000 N/m;
Spring constant: K2 = 500000 N/m;
Damper constant: b1 = 350 N-S/m;
Damper constant: b2 = 15020 N-S/m;

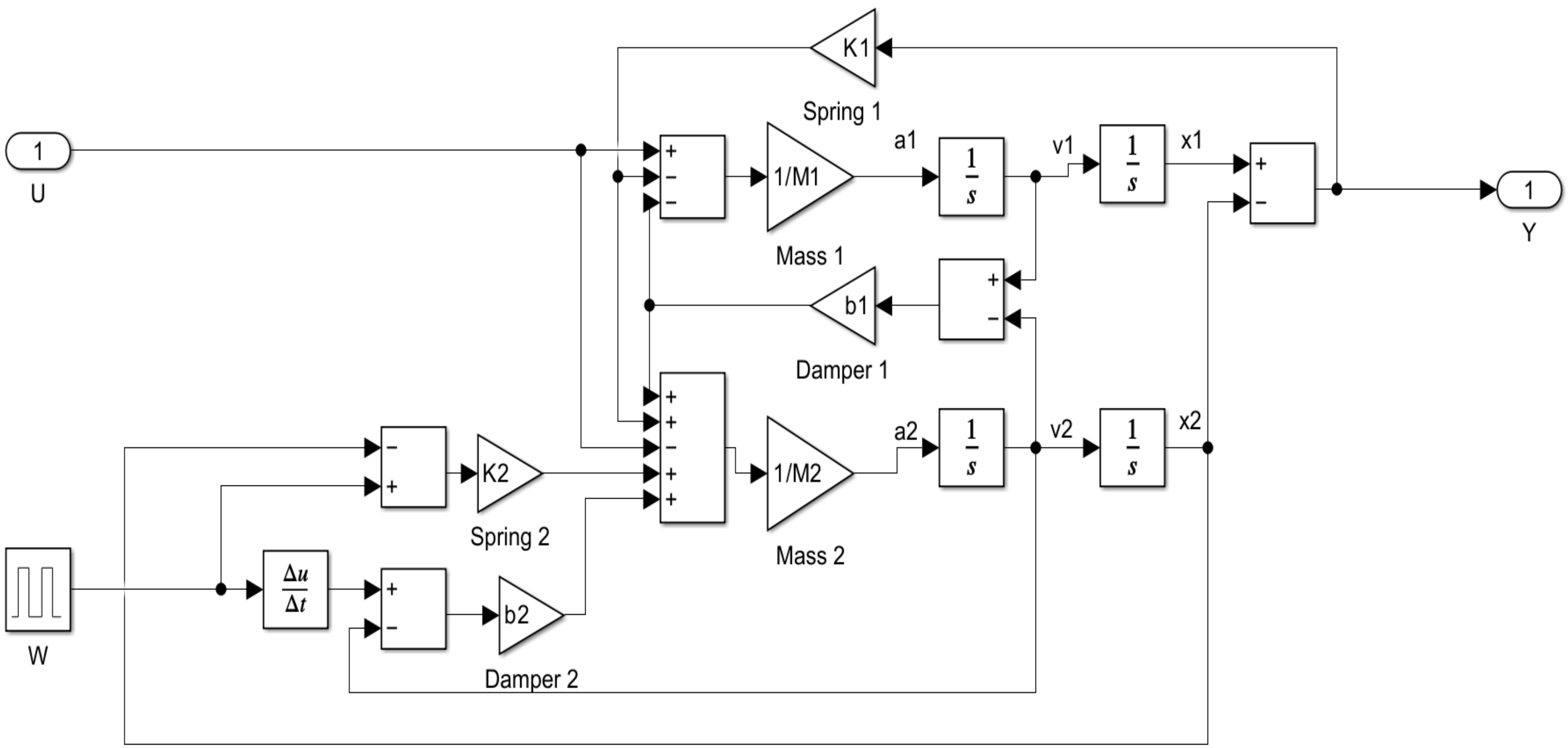
Matrix [A]= $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -26.7 & 26.7 & -0.1 & 0.1 \\ 200 & -1450 & 0.9 & -38.4 \end{bmatrix}$

Matrix [B]= $\begin{bmatrix} 0 \\ 0 \\ 0.0003 \\ -0.0025 \end{bmatrix}$

Matrix [C]= $[1 \ \ -1 \ \ 0 \ \ 0]$

Matrix [D]= $[\ 0\ ]$

MATLAB Simulink Model of Plant



Suspension system model of Plant

Controllability & Observability Tests

Controllability Matrix:

C=[B : AB : A^2B :………… … . : A^{n-1}B]

For our system:

C=[B : AB : A^2B : A^3 B]

[C] is a 4x4 matrix

Rank of [C]=4

Hence , All states (X1, X2, $\dot{X}$1, $\dot{X}$2) are controllable.

We can design State feedback controller.

Observability Matrix:

O=[C^T : $A^T C^T$: $(A^T^2)C^T$: … … … … … … : $(A^T$^n-1$)C^T$]

For our system:

O=[C^T : $A^T C^T$: $(A^T^2)C^T$: $(A^T^3)C^T$]

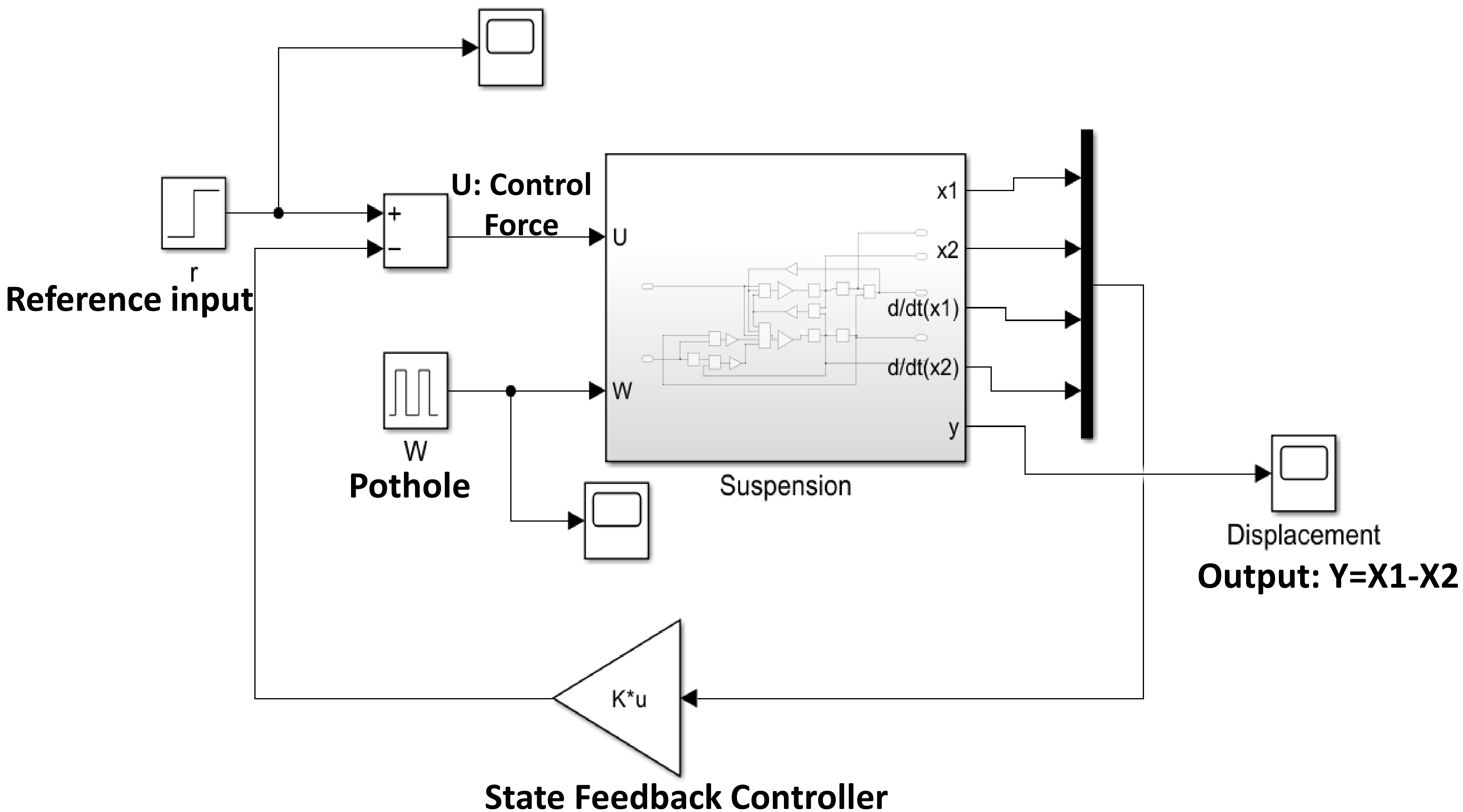
[O] is a 4x4 matrix

Rank of [O]= =4

Hence , All states (X1, X2, $\dot{X}$1, $\dot{X}$2) are observable.

We can design Leunberger observer.

MATLAB Simulink Model of Closed Loop System



State Feedback Controller

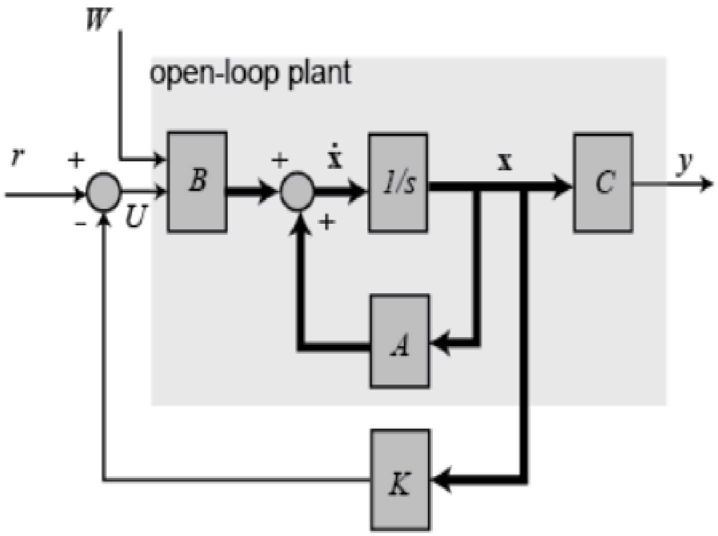
Design a feedback controller so that the output, Y= (X1-X2) has following requirements:

- Overshoot (OS)<10%
- Settling time <10 Sec.

The state space representation of system:

$$\dot{x} = Ax + Bu, \quad U = -Kx$$

$$\dot{x} = (A - Bk)x$$



Controller

Desired Pole Placement:

S1, S2= -24.05± 35.50 j

S3, S4= -0.61± 4.92 j

The desired characteristic equation:

(S-S1)(S-S2)(S-S3)(S-S4)=0

$$S^4 + 49S^3 + 1922S^2 + 3429S + 45342 = 0$$

The K matrix obtained using Pole Placement Method:

$$K = [28820 \ \ -14950 \ \ 4610 \ \ -3700]$$

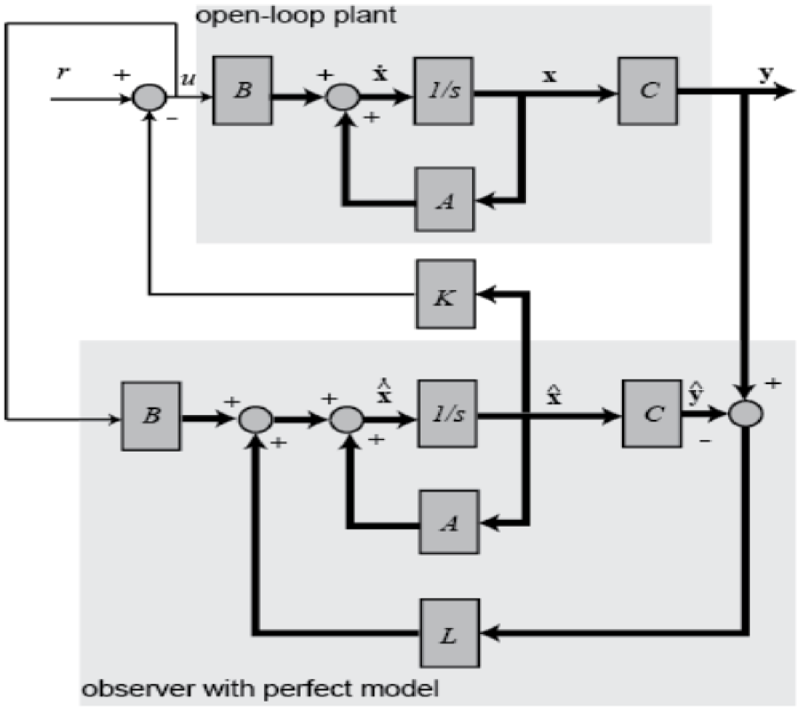
Leunberger Observer

Design a Leunberger observer so that the output, Y= (X1-X2) has following requirements:

- Overshoot (OS)<10%
- Settling time <10 Sec.

The state space representation of system:

$$\dot{\hat{x}}=A\hat{x}+Bu+L(y-\hat{y}) \text{ ,}$$
$$\hat{y}=C\hat{x}$$
$$\dot{e}=\dot{x}-\dot{\hat{x}}=(A-LC)e$$



Observer design

Desired Observer Poles:

S1, S2,S3, S4= -30,-33,-38,-40

The desired characteristic equation:

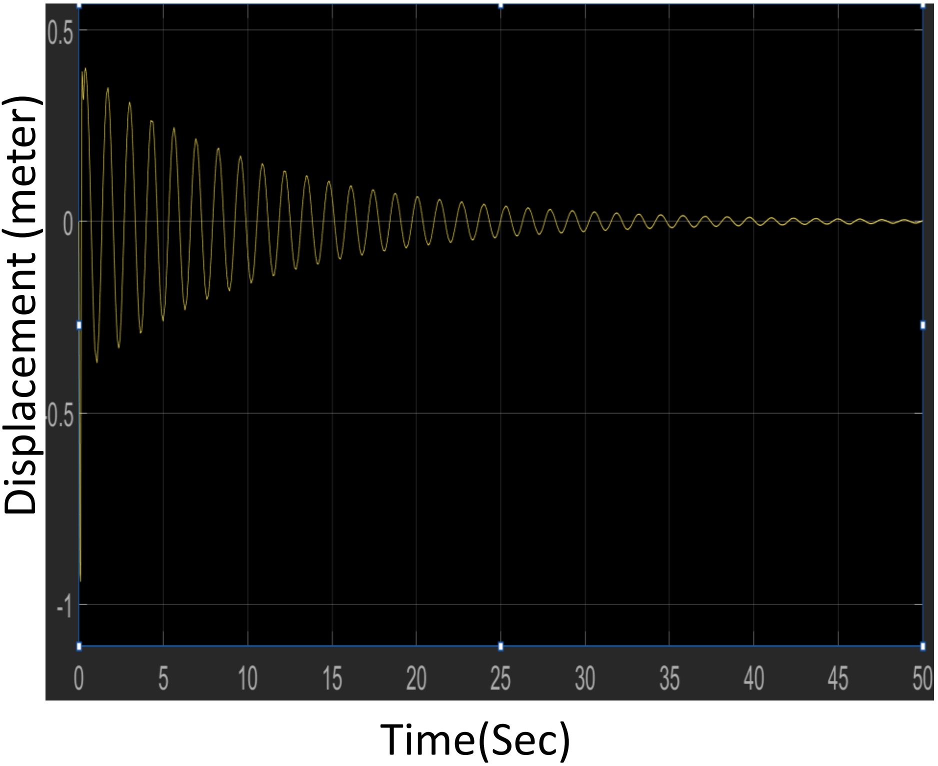
(S-S1)(S-S2)(S-S3)(S-S4)=0

$$S^4+141S^3+7424S^2+172980S+1504800=0$$

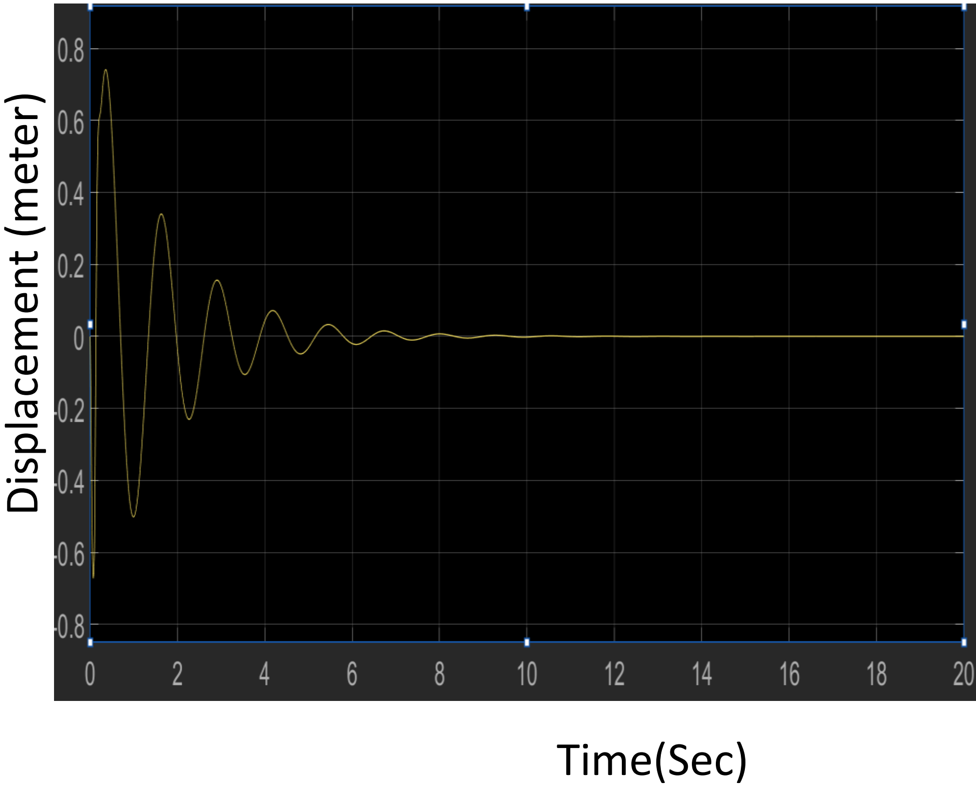
The L matrix obtained using Pole Placement Method:

$$L=\begin{bmatrix} 102.1 \\ -0.4 \\ 1165.2 \\ -828.8 \end{bmatrix}$$

Simulation Results



Without State Feedback Controller

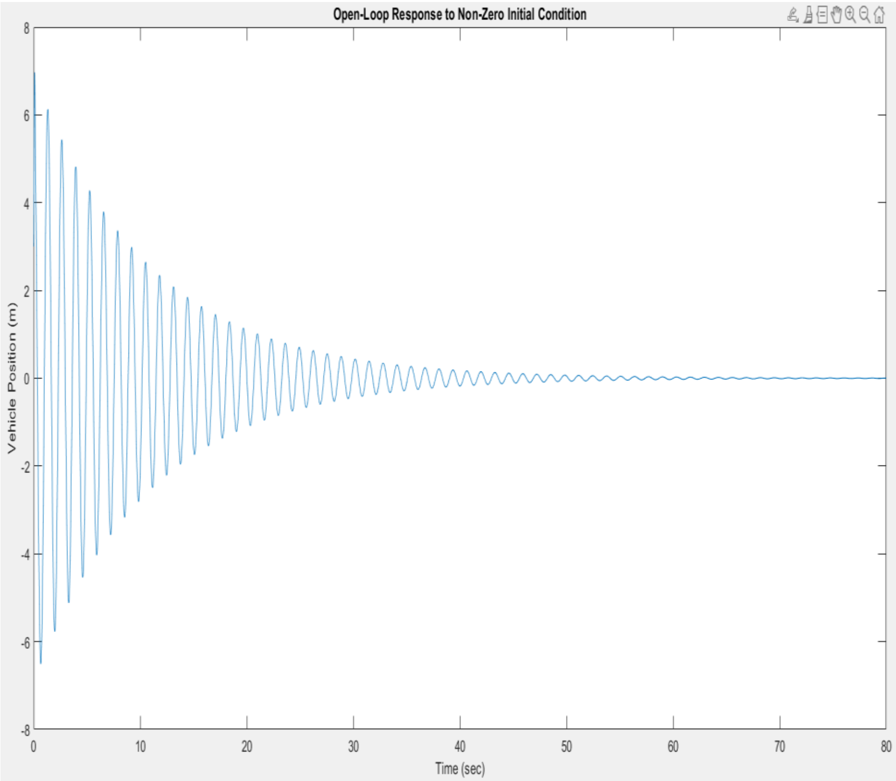


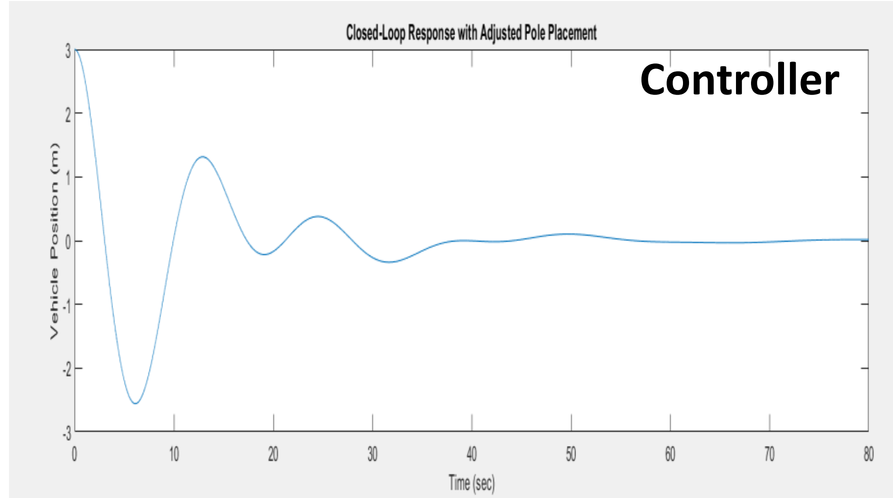
With State Feedback Controller

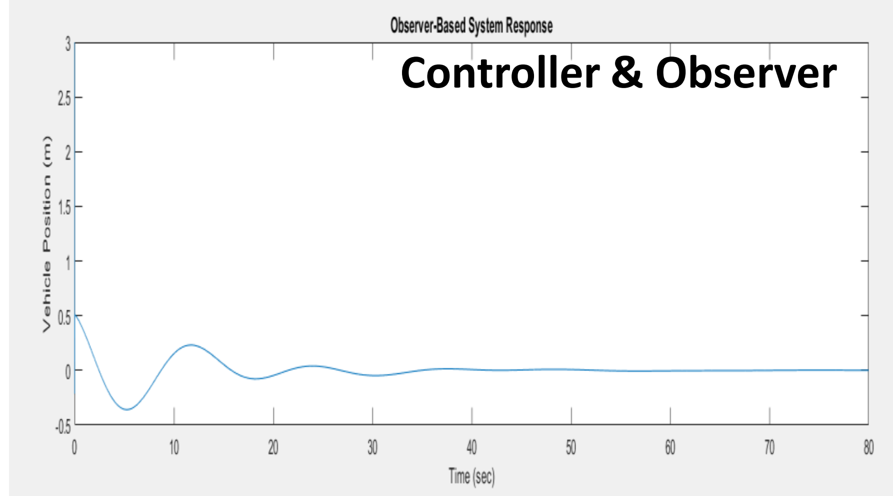
Assuming deep pothole, W=10 cm

Closed loop response:

Open loop response:







Controller Poles = [-0.1 + 0.5i, -0.1 - 0.5i, -0.05 + 0.2i, -0.05 - 0.2i]
Observer Poles = [-300 + 50i, -300 - 50i, -400 + 30i, -400 - 30i]

Conclusions

- Design of Automotive suspension system carried out in MATLAB Simulink environment.
- All states are found to be controllable & observable.
- State feedback controller is used to get desired response within specifications.
- Design of state feedback controller, so that when the road disturbance (W) is simulated by a pulse input, the output Y=(X1-X2) has a settling time less than 10 seconds and an overshoot less than 10%.
- When the bus runs onto a 10 cm high step, the bus body will oscillate within a range of +/- 10 mm and will stop oscillating within 10 seconds.
- Leunberger observer has designed for estimation of systems unmeasurable states.
- System is also simulated with controller and observer.

References:

- Linear System Theory and Design by Chi-Tsong Chen.
- https://ctms.engin.umich.edu/CTMS/?example=Suspension&section=Control State Space.

Thank you