Capacitated Vehicle Routing Problem with Time Window – Single Depot.

Parameters:-

c_{ij} – Cost to visit node j from node i.

t_{ij} – travel time to visit node j from node i.

d_i – demand at node i (kg)

q_γ – Capacity of vehicle γ

(a_i, b_i) – time window of customer i

t_s – service time at each customer = 20 mins

F_γ – Fixed cost of vehicle γ = $2 \cdot q_\gamma$

C_γ – Variable cost of vehicle γ (per km) $= \left\lceil \dfrac{20000 - q_\gamma}{1000} \right\rceil$

Sets:-

N = orders = set of orders including source & sink as depot.

$C = \{1, 2, \dots K\}$ $\quad N = \{0, \underbrace{1, 2, \dots K}_{\text{orders}}, K+1\}$ $\quad 0, K+1$ – depot

V = vehicle fleet : $\{v_1, v_2 \dots v_\gamma\}$ → vehicle ids

$\{q_1, q_2, \dots q_\gamma\}$ → capacity.

Arc (i, j) ∀ $i \neq j$, $i \neq K+1$, $j \neq 0$.

Decision Variables:-

$$x_{ij\gamma} = \begin{cases} 1, & \text{if vehicle } \gamma \text{ drives from vertex } i \text{ to } j \\ 0, & \text{o/w} \end{cases}$$

$x_{ij\gamma} \in \{0, 1\}$ $\quad \forall\, i, j \in N, \quad \forall \gamma \in V$

$S_{i\gamma}$ = Service time at customer i

i.e., time vehicle γ starts to service customer i.

lb = 480 mins, Cat = Continuous.

$$I_\gamma = \begin{cases} 1, & \text{if veh } \gamma \text{ is used} \\ 0 & \text{o/w} \end{cases}$$

Objective :-

Obj 1 :- Minimize total Cost.

$$\text{Min} \sum_{r \in V} I_r \cdot F_r + \sum_{r \in V} O_r \sum_{i \in N/\text{Sink}} \sum_{j \in N/\text{source}} C_{ij} x_{ijr}$$

Obj 2 : Minimize total travel distance.

$$\text{Min} \sum_{r \in V} \sum_{i \in N/\text{Sink}} \sum_{j \in N/\text{Source}} C_{ij} x_{ijr}$$

Obj 3 : Minimize the number of vehicles used

$$\text{Min} \sum_{r \in V} I_r$$

Constraints :-

Start from depot :-

$$\sum_{j \in N/\text{Source}} x_{\text{Source},j,r} = I_r \quad \forall r \in V$$

End at depot :-

$$\sum_{i \in N/\text{Sink}} x_{i,\text{Sink},r} = I_r \quad \forall r \in V$$

Flow balancing {After a vehicle arrives at a customer it has to leave for another destination}

$$\sum_{i \in N/\text{Sink}} x_{ihr} - \sum_{j \in N/\text{Source}} x_{hjr} = 0 \quad \forall h \in C, \forall r \in V$$

Each Customer is visited Exactly Once :-

$$\sum_{r \in V} \sum_{j \in N/\text{Sink}} x_{ijr} = 1 \quad \forall i \in C$$

Vehicle Capacity Constraint:

$$\sum_{i \in C} d_i \sum_{j \in N} x_{ijr} \leq q_r \cdot I_r \quad \forall r \in V$$

Time Window Constraints :-

$$a_i \leq S_{ir} \leq b_i \quad \forall i \in N, \forall r \in V$$

$$S_{jr} \geq S_{ir} + t_s + t_{ij} - M(1 - x_{ijr})$$

$$\forall r \in V, j \in N / \text{source}, i \in N / \text{sink}.$$

Vehicle Compatibility :-

If r is common $\quad x_{ijr} \leq I_r \quad \forall i, j \in N, \forall r \in V$

else $x_{ijr} = 0$

Linking Constraint :-

$$x_{ijr} \leq I_r \quad \forall i, j \in N, r \in V$$

Note:- Implicitly $i \in N/\text{Sink}$, $j \in N/\text{Source}$.