①

Reason for arising error

Due to different reason error will arise. There are 3 types of error arise.

(I) Inherent error
(II) Round-off error
(III) Truncation error.

(I) Inherent error :-

This type of error is already present in the statement of problem before its solution.

This is also due to the physical measurement of the parametres of the problem.

(2)

9 **Ex** In mathematical problem

10 we use π, but we put

11 the value of $\pi = \frac{22}{7} = 3.14$

12 which is not exact value.

1 In that problem error will

2 arrise. That error is called

3 inherent error.

4

5 **Ex** Similarly when we solve

6 mathematical problem

7 we use $e = 2.718$

which is not exact value.

In that problem inherent

error will arise.

(3)

(II) Round-off error :-

This type of error will arise due to the rounding-off numbers to certain places.

This rounding is obtained either by chopping or by rounding.

Ex Consider a number

$x = 0.2357$

We have to make 3 digit standard form.

$\therefore\ x = 0.235$ (chopping)

$x = 0.236$ (rounding)

In this case when we put the value of x in any problem round-off error will arrise.

(iii) Truncation error :-

This type of error will arrise due to the truncation of mathematical expression.

Ex we have

$$e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

which is a infinite series.

If we take to solve our problem

$$\boxed{e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!}}$$

Then error will arrise.

That error is called Truncation error.

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Ex We know

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

infinite series.

If we take

$$\sin x = x - \frac{x^3}{3!}$$ for our calculation of a problem then error will arise. That error is called **Truncation error.**

Q. Define significant digit.

Significant digits:-

The number of digits including first non-zero digit and the digits to the right of a decimal number is called significant digit of that number.

Ex 1, .01, .001, .00003

These are all one-significant digit.

Ex 12, 0.012, 0.010, 0.000022

These are all two-significant digit.

Ex 133, 0.0133, 0.330

0.0320, 0.00145

0.205, 0.0205

These are all 3-significant digit.

Remark :-

Any number can be expressed as in two different ways

(I) fixed point form

(II) floating point form

(7)

(1) Fixed point form :-

In this form, we write the number as the sum of the product of the power of the base and the numbers at that place.

Ex

$$(X)_\beta = \alpha_1\beta^{-1} + \alpha_2\beta^{-2} + \alpha_3\beta^{-3} + \cdots + \alpha_n\beta^{-n}$$

$$= (.\alpha_1\alpha_2\cdots\alpha_n)_\beta$$

Here $0 \le \alpha_1, \alpha_2, \alpha_3 \ldots \alpha_n \le 9$

$\beta \rightarrow$ base.

Ex

$$(.1234)_{10} = 1\times10^{-1} + 2\times10^{-2} + 3\times10^{-3} + 4\times10^{-4}$$

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(11) Floating point form :-

In this form, a number can be expressed as

$$\boxed{.d_1 d_2 d_3 \cdots d_t \times \beta^e}$$

where

$$0 \leq d_i < \beta$$

for $i = 1, 2, 3 \cdots t$

Ex $46.5186 = 0.465186 \times 10^2$

Ex $3.147 = 0.3147 \times 10^1$

Ex $216.78 = 0.21678 \times 10^3$

Intermediate value Theorem

If $f(x)$ is continuous on $[a,b]$. and $f(a) \cdot f(b) < 0$, then the eqn $f(x)=0$ has at least one real root or odd number of roots in the interval (a,b).

Pf Consider an eqn $f(x)=0$.

$f(x)$ is continuous $[a,b]$.

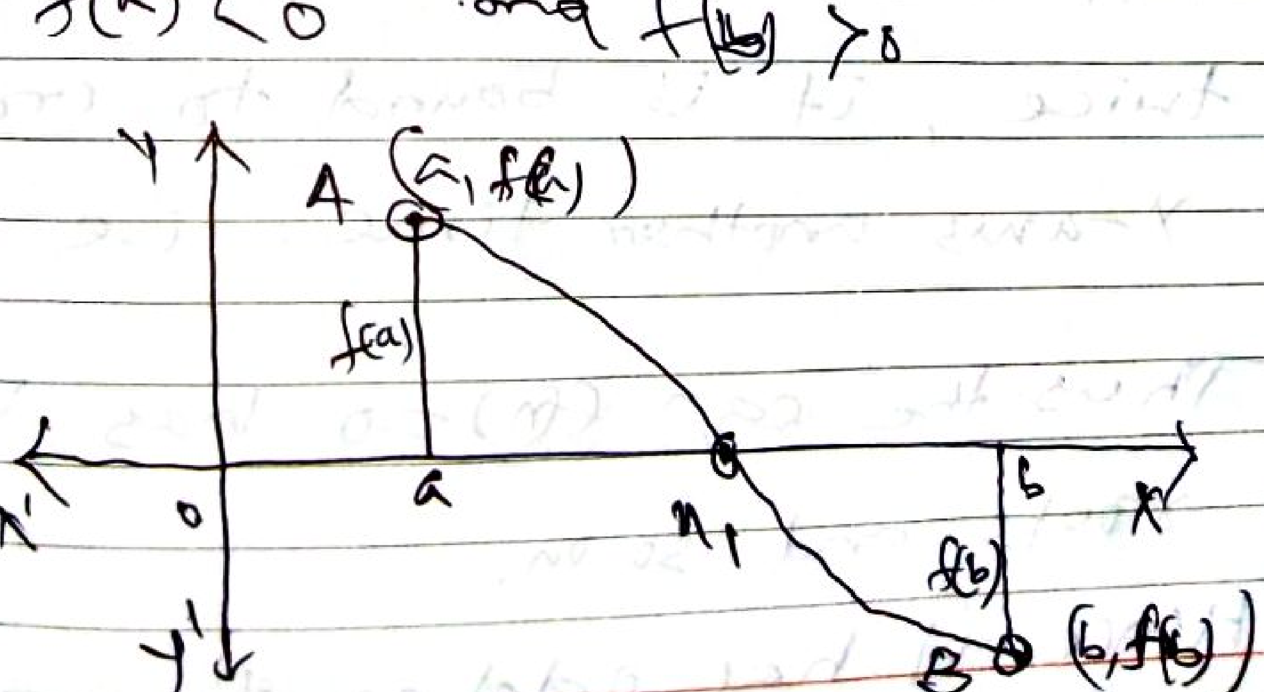
Also $\boxed{f(a) \cdot f(b) < 0}$

$\therefore$ $f(a)$ and $f(b)$ are opposite sign.

Hence we have two cases

(I) $f(a) > 0$ and $f(b) < 0$

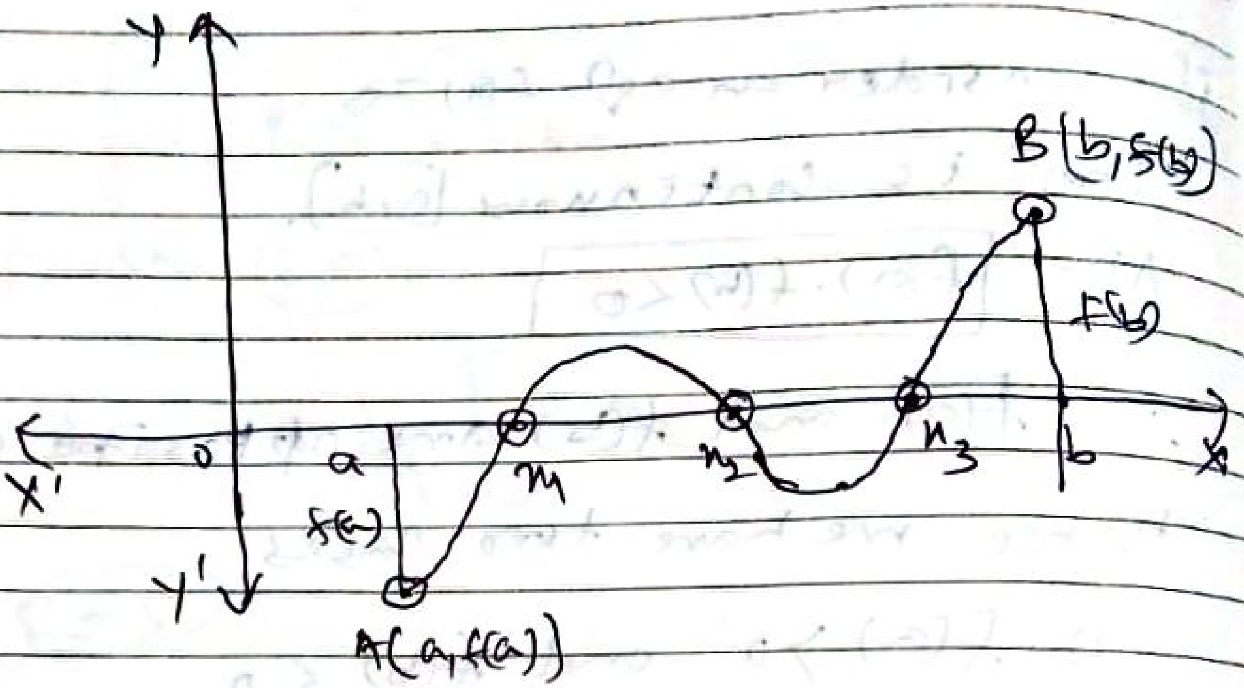
(II) $f(a) < 0$ and $f(b) > 0$



(2)

While moving from A to B, the curve will meet x-axis at some where.

That point is called root of the eqn $f(x)=0$.



Suppose the curve crosses the x-axis twice, it is bound to cross the x-axis another time. i.e. thrice.

Thus the eqn $f(x)=0$ has three roots and so on.

 Hence it has odd no. of roots.

③

Bisection Method

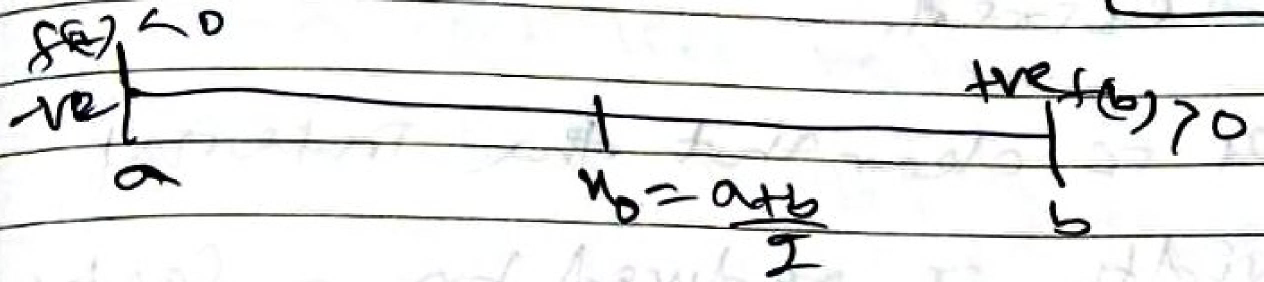
This method is based on the repeated application of intermediate value theorem.

Let $f(x)$ is continuous on the $[a,b]$ and $f(a) \cdot f(b) < 0$. Then there exists atleast one root between a and b.

Let $f(a) < 0$ and $f(b) > 0$

Then root lies between a and b.

its approximate value is $\boxed{x_0 = \frac{a+b}{2}}$



If $f(x_0) = 0$, we conclude that x_0 is a root of the eqn $f(x) = 0$

otherwise the root lies either between "a" and "x_0" or "x_0" and "b".

depending on $f(x_0)$ ~~negative~~ positive or negative.

④

we take the new interval $[a_1, b_1]$ whose length is $\frac{b-a}{2}$.

As before, this is bisected at x_1. and the new interval will be exactly half the length of previous one.

Repeat this process until the latest interval is as small as desired.

It is clear that the interval width is reduced by a factor of one half at each step and at the end of nth step. Then the new interval $[a_n, b_n]$ it's length is $\frac{|b-a|}{2^n}$.

Working Rule (5)

Step-(I) :- choose two real numbers a and b such that

$$\boxed{f(a) \cdot f(b) < 0}$$

Step-(II) :-

put $\boxed{x_r = \frac{a+b}{2}}$

for $r = 0, 1, 2, 3 \ldots$

Step-(III) :- Find $f(x_r)$. If $f(a) \cdot f(x_r) < 0$

then root lies between a and x_r

Then set $x_r = b$. go to step-(II)

Step-(IV) :- If $f(a) \cdot f(x_r) > 0$

then root lies between x_r and b then set $x_r = a$

go to step-(II).

Step-(V) :- Repeat this procedure until getting required decimal places.

(6)

If the two successive roots are equal.
or error is negligible.

Q Using BSM find a root of the eqn $x^4-x-10=0$ which is correct upto 2 decimal places.

Soln Given that

$$x^4-x-10=0 \quad \text{(1)}$$

which is on the form of

$$f(x)=0$$

Here $f(x)=x^4-x-10$

Taking $x=0$

$$f(0)=0^4-0-10=-10<0 \quad (-ve)$$

Taking $x=1$

$$f(1)=1^4-1-10=-10<0 \quad (-ve)$$

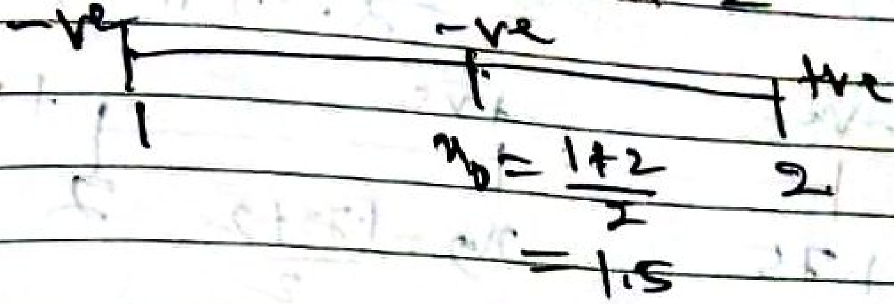
∴ root does not lie between 0 and 1

taking $x=2$

$$f(2) = 2^4 - 2 - 10 = 4 > 0 \text{ +ve}$$

As $\boxed{f(1)\cdot f(2) < 0}$

$\therefore$ root lies between 1 and 2



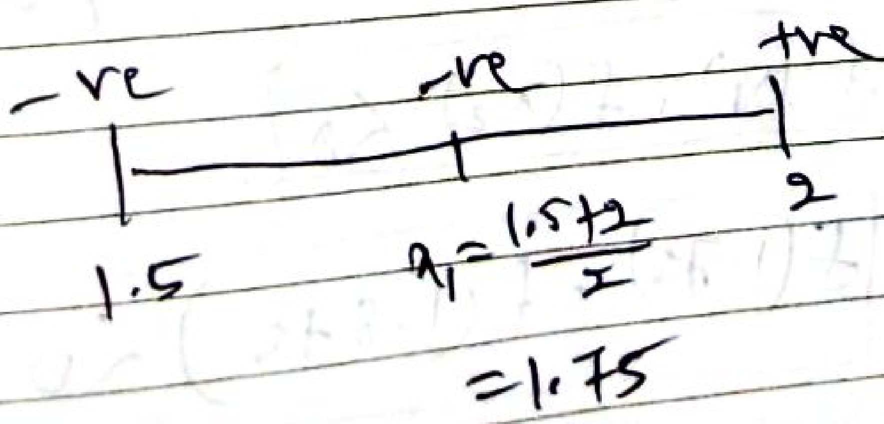
$$\therefore x_0 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$$

$$f(x_0) = f(1.5) = (1.5)^4 - 1.5 - 10$$
$$= 5.0625 - 11.5$$
$$= -6.4375 < 0$$

Now $\boxed{f(x_0)\cdot f(2) < 0}$

i.e. $f(1.5)\cdot f(2) < 0$

$\therefore$ root lies between 1.5 and 2



$\therefore x_1 = \frac{1.5+2}{2} = 1.75$

$f(x_1) = f(1.75) = (1.75)^4 - 1.75 - 10$

$= 9.3789 - 11.75$

$= -2.3711 < 0$

-ve +ve +ve

1.75 $x_2 = \frac{1.75+2}{2}$ 2

$= 1.875$

$\therefore x_2 = \frac{1.75+2}{2} = 1.875$

Now $f(x_2) = f(1.875)$

$= (1.875)^4 - 1.875 - 10$

$= 12.3596 - 11.875$

$= 0.4846 > 0$

$\therefore \boxed{f(x_1) \cdot f(x_2) < 0}$

i.e. $\boxed{f(1.75) \cdot f(1.875) < 0}$

$\therefore$ root lies between x_1 and x_2

(9)

-ve -ve +ve

$x_1 = 1.75$ $x_3 = 1.8125$ $x_2 = 1.875$

$$\therefore x_3 = \frac{x_1+x_2}{2} = \frac{1.75+1.875}{2}$$

$$= \frac{3.625}{2} = 1.8125$$

$$f(x_3) = (1.8125)^4 - 1.8125 - 10$$

$$= 10.7923 - 11.8125$$

$$= -1.0202 < 0$$

$$\therefore \boxed{f(x_3) . f(x_2) < 0}$$

ie

$$f(1.8125) . f(1.875) < 0$$

root lies between x_3 and x_2

-ve +ve

$x_3 = 1.8125$ $x_4 = 1.8438$ $x_2 = 1.875$

$$\therefore x_4 = \frac{1.8125 + 1.875}{2} = \frac{3.6875}{2} = 1.8438$$

(18)

$\therefore |x_4 - x_3| = |1.8438 - 1.8125|$

$= 0.0$

$\therefore$ root $= 1.8$.

Hw

Q-1 Using BSM find a real root of $x^3 - 4x - 9 = 0$ which is correct up to two significant figure

Q-2 Using BSM find a real root of $xe^x - 1 = 0$ which is correct upto 3 decimal places

Q-3 Using BSM find a root of $\cos x - xe^x = 0$ which is correct up to 2 decimal places.

①

Newton's Iterative formula

Q-① Obtain Newton's Iterative formula for finding $\sqrt{N}$, where N is positive real numbers. Hence find the value of $\sqrt{142}$.

Soln Let $x = \sqrt{N}$

squaring both sides

$$x^2 = N$$

$$\Rightarrow x^2 - N = 0 \quad \text{--- (1)}$$

which is in the form of

$$f(x) = 0$$

Here $f(x) = x^2 - N$ --- (2)

$f'(x) = 2x$ --- (3)

We know N-R method

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}} \quad \text{--- (A)}$$

for $k = 0, 1, 2 \ldots$

$$f(x_k) = x_k^2 - N$$

$f'(x_k) = 2x_k$

Then from relation (A) we get

$$x_{k+1} = x_k - \left(\frac{x_k^2 - N}{2x_k}\right)$$

$$= x_k - \left(\frac{x_k^2}{2x_k} - \frac{N}{2x_k}\right)$$

$$= x_k - \frac{x_k}{2} + \frac{N}{2x_k}$$

$$= \frac{2x_k - x_k}{2} + \frac{N}{2x_k}$$

$$= \frac{x_k}{2} + \frac{N}{2x_k}$$

$$\boxed{x_{k+1} = \frac{1}{2}\left[x_k + \frac{N}{x_k}\right]}$$

SUNDAY

for $k = 0, 1, 2$

which is called Newton's iterative formula for finding $\sqrt{N}$.

we have to find $\sqrt{142}$

i.e. $\sqrt{142} = \sqrt{N}$

Here $N = 142$

Let $x_0 = 12$

∴ First approximation is

K=0

$$x_1 = \frac{1}{2}\left[x_0 + \frac{N}{x_0}\right]$$

$$\Rightarrow x_1 = \frac{1}{2}\left[12 + \frac{142}{12}\right]$$

$$\Rightarrow \boxed{x_1 = 11.9167}$$

Taking K=1

The 2nd approximation

$$x_2 = \frac{1}{2}\left[x_1 + \frac{N}{x_1}\right]$$

$$= \frac{1}{2}\left[11.9167 + \frac{142}{11.9167}\right]$$

$$\boxed{x_2 = 11.9164}$$

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WK 38 (260-105)

(4)

Taking $\underline{K=2}$

The 3rd approximation

$$x_3 = \frac{1}{2}\left[x_2 + \frac{N}{x_2}\right]$$

$$= \frac{1}{2}\left[11.9164 + \frac{142}{11.9164}\right]$$

$$\boxed{x_3 = 11.9164}$$

Now $|x_3 - x_2| = 0.0000$

$\therefore \sqrt{142} = 11.9164$. (Ans)

Q. Obtain N.I.F and find the value $\sqrt{20}$ which is correct upto two decimal places

Q. Obtain N.I.F ~~and~~ and find the value $\sqrt{29}$ which is correct upto five decimal places

(5)

Q. Obtain the N.I.F for finding the value of $\frac{1}{N}$ where N is a non-zero real number and calculate the value of $\frac{1}{26}$ which is correct upto four decimal places.

Ans Let $x = \frac{1}{N}$

$\Rightarrow N = \frac{1}{x}$

$\Rightarrow \frac{1}{x} - N = 0$ —— (1)

which is in the form of

$f(x) = 0$

Here $f(x) = \frac{1}{x} - N$ —— (2)

$f'(x) = -\frac{1}{x^2}$ —— (3)

we know N-R method

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}} \quad \text{—— (A)}$$

for $k = 0, 1, 2 \ldots$

(6)

$$\therefore f(x_k) = \frac{1}{x_k} - N$$

$$f'(x_k) = -\frac{1}{x_k^2}$$

From relation (A) we get

$$x_{k+1} = x_k - \left[\frac{\frac{1}{x_k} - N}{-\frac{1}{x_k^2}}\right]$$

$$= x_k + x_k^2\left[\frac{1}{x_k} - N\right]$$

$$= x_k + x_k - Nx_k^2$$

$$= 2x_k - Nx_k^2$$

$$\boxed{x_{k+1} = x_k\left[2 - Nx_k\right]}$$

for $k = 0, 1, 2 \ldots$

which is called Newton's iterative formula. for finding the value of $\frac{1}{N}$.

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Now we have to find $\frac{1}{26}$

Here $N = 26$

Let $x_0 = 0.04$

we have

$$\boxed{x_{k+1} = x_k [2 - N x_k]}$$

$k = 0, 1, 2 \ldots$

Putting $k = 0$

1st Approximation

$$x_1 = x_0 [2 - N x_0$$

$$x_1 = 0.04 [2 - 26 (0.04)]$$

$$\boxed{x_1 = 0.0384}$$

2nd approximation

$$x_2 = x_1 [2 - N x_1]$$

$$= 0.0384 [2 - 26 (0.0384)]$$

$$\boxed{x_2 = 0.0385}$$

$K = 2$

3rd approximation

$$x_3 = x_2(2 - N x_2)$$

$$= 0.0385\,[2 - 26(0.0385)]$$

$$\boxed{x_3 = 0.0385}$$

$$\therefore |x_3 - x_2| = 0.0000$$

$$\therefore \boxed{\frac{1}{26} = 0.0385}$$

Which is correct up to four decimal places.

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Q/ Obtain N.I.F for finding the value of $\frac{1}{N}$ and calculate the value of $\frac{1}{31}$ which is correct up to four decimal places.

3rd sem (1)

Q. Obtain Newton's iterative formula for finding $\frac{1}{\sqrt{N}}$ where N is a positive real number and find the value of $\frac{1}{\sqrt{14}}$ which is correct up to four decimal places.

Ans Let $x = \frac{1}{\sqrt{N}}$

$\Rightarrow$ Squaring both sides

$$\Rightarrow x^2 = \frac{1}{N}$$

$$\Rightarrow x^2 - \frac{1}{N} = 0 \quad \text{——— (1)}$$

Which is in the form of

$$f(x) = 0$$

Here $f(x) = x^2 - \frac{1}{N}$ — (2)

$$f'(x) = 2x \quad \text{——— (3)}$$

We know N-R method

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}} \text{ for } k = 0, 1, 2 \quad \text{——— (A)}$$

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(5)

Now $f(x_k) = x_k^2 - \frac{1}{N}$

$f'(x_k) = 2x_k$

from relation (A) we get

$$x_{k+1} = x_k - \frac{x_k^2 - \frac{1}{N}}{2x_k}$$

$$= \frac{2x_k^2 - x_k^2 + \frac{1}{N}}{2x_k}$$

$$= \frac{x_k^2 + \frac{1}{N}}{2x_k}$$

$$= \frac{1}{2}\left[\frac{x_k^2}{x_k} + \frac{\frac{1}{N}}{x_k}\right]$$

$$= \frac{1}{2}\left[x_k + \frac{1}{N x_k}\right]$$

$$\therefore \boxed{x_{k+1} = \frac{1}{2}\left[x_k + \frac{1}{N x_k}\right]}$$

for $k = 0, 1, 2 \ldots$

which is called Newton's iterative formula.

(3)

Now we have to find the value of $\frac{1}{\sqrt{14}}$

Here $\frac{1}{\sqrt{14}} = \frac{1}{\sqrt{N}}$

Now $\boxed{N=14}$

Let $x_0 = 0.25$ $\qquad \frac{1}{\sqrt{16}} = \frac{1}{4} = 0.25$

we have N.I.F

$$\boxed{x_{k+1} = \frac{1}{2}\left[x_k + \frac{1}{Nx_k}\right]}$$

for $k=0,1,2\ldots$

taking $k=0$

$\therefore$ 1st approximation

$$x_1 = \frac{1}{2}\left[x_0 + \frac{1}{Nx_0}\right]$$

$$= \frac{1}{2}\left[0.25 + \frac{1}{14(0.25)}\right]$$

$$\boxed{x_1 = 0.26785}$$

(4)

Taking $k=1$

The 2nd approximation is

$$x_2 = \frac{1}{2}\left(x_1 + \frac{1}{Nx_1}\right)$$

$$= \frac{1}{2}\left[0.26785 + \frac{1}{14.(0.26785)}\right]$$

$$\boxed{x_2 = 0.26726}$$

Taking $k=2$

The 3rd approximation is

$$x_3 = \frac{1}{2}\left[x_2 + \frac{1}{Nx_2}\right]$$

$$= \frac{1}{2}\left[0.26726 + \frac{1}{14(0.26726)}\right]$$

$$\boxed{x_3 = 0.26726}$$

$$\therefore |x_3 - x_2| = 0.00000$$

$$\therefore \frac{1}{\sqrt{14}} = 0.26726 = 0.2673$$

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(5)

Obtain Newton's iterative formula for finding the value of $\sqrt[3]{N}$ where N is the real number and find the value of $\sqrt[3]{24}$ which is correct upto four decimal place.

Ans Let $x = \sqrt[3]{N}$

Raising power '3' both sides

$\Rightarrow x^3 = N$

$\Rightarrow x^3 - N = 0$ —— (1)

Which is in the form of

$f(x) = 0$

$\therefore f(x) = x^3 - N$ —— (2)

$f'(x) = 3x^2$ —— (3)

we know N-R method

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}} \quad \text{—— (A)}$$

for $k = 0, 1, 2 \ldots$

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Now $f(x_k) = x_k^3 - N$

$$f'(x_k) = 3x_k^2$$

from relation (A) we get

$$x_{k+1} = x_k - \frac{x_k^3 - N}{3x_k^2}$$

$$= \frac{3x_k^3 - x_k^3 + N}{3x_k^2}$$

$$= \frac{2x_k^3 + N}{3x_k^2}$$

$$= \frac{1}{3}\left[\frac{2x_k^3 + N}{x_k^2}\right]$$

$$= \frac{1}{3}\left[\frac{2x_k^3}{x_k^2} + \frac{N}{x_k^2}\right]$$

$$\boxed{x_{k+1} = \frac{1}{3}\left[2x_k + \frac{N}{x_k^2}\right]}$$

for $k = 0, 1, 2 \cdots$

 which is Newton's iterative formula

Now we have to find $\sqrt[3]{24}$

now $\sqrt[3]{24} = \sqrt[3]{N}$

Here $N = 24$

Let $x_0 = 3$ As $(27)^{1/3} = 3$

we have N.I.F.

$$\boxed{x_{k+1} = \frac{1}{3}\left[2x_k + \frac{N}{x_k^2}\right]}$$

for $k = 0, 1, 2 \ldots$

Taking $k=0$

The 1st approximation

$$x_1 = \frac{1}{3}\left[2x_0 + \frac{N}{x_0^2}\right]$$

$$= \frac{1}{3}\left[2(3) + \frac{24}{(3)^2}\right] = \frac{8.66667}{3}$$

$$\boxed{x_1 = 2.88889}$$

Taking k=1

The 2nd approximation is

$$x_2 = \frac{1}{3}\left[2x_1 + \frac{N}{x_1^2}\right]$$

$$= \frac{1}{3}\left[2(2.88889) + \frac{24}{(2.88889)^2}\right]$$

$$\boxed{x_2 = 2.88451}$$

Taking k=2

3rd approximation is

$$x_3 = \frac{1}{3}\left[2x_2 + \frac{N}{x_2^2}\right]$$

$$= \frac{1}{3}\left[2(2.88451) + \frac{24}{(2.88451)^2}\right]$$

$$\boxed{x_3 = 2.8845}$$

$\therefore \quad |x_3 - x_2| = 0.0000$

$\therefore \quad \sqrt[3]{24} = 2.8845.$

Q. Obtain N.I.F for finding $\sqrt[3]{N}$ by using N-R method where N is +ve real number and $\sqrt[3]{10}$ which is correct up to 4 decimal places. $\sqrt[3]{15} = 2.449$. $x_0 = 2$

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Note: To find $\sqrt[R]{N}$ by using N-Raphson method we get

$$x_{K+1} = \frac{1}{R}\left[(R-1)x_K + \frac{N}{x_K^{(R-1)}}\right]$$

$R = 3, 4, 5, 6, \ldots$

Suppose $R=4$

$$x_{K+1} = \frac{1}{4}\left[3x_K + \frac{N}{x_K^3}\right]$$

$R=5$

$$x_{K+1} = \frac{1}{5}\left[4x_K + \frac{N}{x_K^4}\right]$$

and so on.

07 sept 20

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Note:- The rate of convergence of secant method is $\underline{1.618}$ / $\underline{1.62}$

Q Using secant method find a real root of the eqn $x^2-3=0$ which is correct upto 3 decimal places.

Soln Given that

$$x^2-3=0 \quad \text{—— (1)}$$

which is in the form of

$$f(x)=0$$

Here $f(x)=x^2-3$

putting $x=0$

$$f(0)=-3<0$$

Again $x=1$

$$f(1)=-2<0$$

Taking $x=2$ $f(2)=1>0$

(2)

$\therefore$ root lies between 1 and 2...

Let us choose $x_0 = 1$, $x_1 = 2$.

which are initial approximation.

We know that secant method

$$\boxed{x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} \cdot f(x_k)}$$

for $k = 1, 2, 3 - - -$

Taking $k = 1$

$$\boxed{x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_1)}$$

Now $f(x_0) = f(1) = 1^2 - 3 = -2$

$f(x_1) = f(2) = 2^2 - 3 = 1$

$\therefore x_2 = 2 - \frac{2-1}{1-(-2)} \cdot 1$

$= 2 - \frac{1}{3} = \frac{5}{3} = 1.6667$

M	T	W	T	F	S	S	M	T	W	T	F
			1	2	3	4	5	6	7	8	9
12	13	14	15	16	17	18	19	20	21	22	23
26	27	28	29	30	31						

③

$$\therefore \boxed{x_2 = 1.6667}$$

Taking $K=2$

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} \cdot f(x_2)$$

$$= 1.6667 - \frac{1.6667 - 2}{f(1.6667) - f(2)} \cdot f(1.667)$$

Now $f(x_2) = f(1.6667) = (1.6667)^2 - 3$

$$= 2.7779 - 3$$

$$= -0.2221$$

$$\therefore x_3 = 1.6667 - \frac{-0.3333}{-0.2221 - 1} \cdot (-0.2221)$$

$$= 1.6667 - \frac{0.3333}{1.2221} \cdot (-0.2221)$$

$$= 1.6667 + \frac{0.3333}{1.2221} (0.2221)$$

$$= 1.6667 + (0.2727)(0.2221)$$

(4)

$= 1.6667 + 0.0606 = 1.7273$

$\therefore \boxed{x_3 = 1.7273}$

Taking $\underline{k=3}$

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} \cdot f(x_3)$$

$$= 1.7273 - \frac{1.7273 - 1.6667}{f(1.7273) - f(1.6667)} \cdot f(1.7273)$$

Here $f(1.7273) = (1.7273)^2 - 3$

$= 2.9836 - 3$

$= -0.0164$

$$\therefore x_4 = 1.7273 - \frac{0.0606}{-0.0164 - (-0.2221)} (-0.0164)$$

$$= 1.7273 - \frac{0.0606}{-0.0164 + 0.2221} \cdot (-0.0164)$$

$$= 1.7273 - \frac{0.0606}{0.2057} (-0.0164)$$

M T W T F S S M T W T F
1 2 3 4 5 6 7 8 9
12 13 14 15 16 17 18 19 20 21 22 23
26 27 28 29 30 31

5

9 $= 1.7273 + (0.2946)\cdot(0.0164)$

10 $= 1.7273 + 0.0048$

11 $= 1.7321$

12 $\therefore \boxed{x_4 = 1.7321}$

1 Taking $\underline{K=4}$

2

3 $$x_5 = x_4 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} f(x_4)$$

4

$$= 1.7321 - \frac{1.7321 - 1.7273}{f(1.7321) - f(1.7273)} \cdot f(1.7321)$$

5

6

$$= 1.7321 - \frac{0.0048}{f(1.7321) - (-0.0164)} \cdot f(1.7321)$$

7

Now $f(1.7321) = (1.7321)^2 - 3$

$= 3.0002 - 3$

$= 0.0002$

(6)

$\therefore x_5 = 1.7321 - \frac{0.0048}{0.0002 + 0.0164} \cdot (0.0002)$

$= 1.7321 - \frac{0.0048}{0.0166} (0.0002)$

$= 1.7321 - (0.2892)(0.0002)$

$= 1.7321 - 0.0001$

$= 1.732$

$\therefore \boxed{x_5 = 1.732}$

$\therefore |x_5 - x_4| = 0.0001.$

$\therefore$ root $= 1.732.$

which is correct up to required decimal places.

H.W
Q-1 Using Secant method find a real root of $x^3 - 2x - 5 = 0$ which is correct up to three significant figures.

M T W T F S S M T W
1 2 3 4 5 6 7 8
12 13 14 15 16 17 18 19 20 21 22
26 27 28 29 30 31

(7)

Q-(2) Using secant method find the value of r_2 starting with $x_0 = 1$ and $x_1 = 2$, which is correct upto three-significant figures.

Q Using secant method find the root of an eqn $x - \cos x = 0$ where $x_0 = 0.5$ and $x_1 = 1$.

Soln Given that

$$x - \cos x = 0 \quad \text{——} \quad (1)$$

which is in the form of

$$f(x) = 0$$

Here $f(x) = x - \cos x$

$$f(x_0) = f(0.5) = 0.5 - \cos(0.5)$$

$$= 0.5 - 0.8776$$

$$= -0.3776$$

(8)

$f(1) = 1 - \cos 1 = 1 - 0.5403$
$= 0.4597$

We know secant method

$$\boxed{x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} \cdot f(x_k)}$$

for $k = 1, 2, 3 \ldots$

Taking $\underline{k=1}$

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1)$$

$$= 1 - \frac{1 - 0.5}{0.4597 - (-0.3776)} \cdot (0.4597)$$

$$= 1 - \frac{0.5}{0.4597 + 0.3776} \cdot 0.4597$$

$$= 1 - \frac{0.5}{0.8373}(0.4597)$$

$$= 1 - (0.5972)(0.4597)$$

$$= 1 - 0.2745 = 0.7255$$

9

$$\therefore \boxed{x_2 = 0.7255}$$

$$f(x_2) = f(0.7255) = 0.7255 - \cos(0.7255)$$
$$= 0.7255 - 0.7482$$
$$= -0.0227$$

Taking $k=2$

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$$

$$= 0.7255 - \frac{0.7255 - 1}{-0.0227 - 0.4597}(-0.0227)$$

$$= 0.7255 - \frac{-0.2745}{-(0.0227 + 0.4597)}(-0.0227)$$

$$= 0.7255 - \frac{0.2745}{0.4824}(-0.0227)$$

$$= 0.7255 + (0.5690)(0.0227)$$

$$= 0.7255 + 0.0129$$

$$\boxed{x_3 = 0.7384}$$

Taking $k=3$

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} \cdot f(x_3)$$

10

$$x_4 = 0.7384 - \frac{0.7384 - 0.7255}{f(0.7384) - f(0.7255)} \cdot f(0.7384)$$

Now

$$f(0.7384) = 0.7384 - \cos(0.7384)$$
$$= 0.7384 - 0.7395$$
$$= -0.0011$$

$$x_4 = 0.7384 - \frac{0.0129}{-0.0011 - (-0.0227)} \cdot (-0.0011)$$

$$= 0.7384 - \frac{0.0129}{0.0216} \cdot (-0.0011)$$

$$= 0.7384 + \frac{(0.0129)}{0.0216} \cdot (0.0011)$$

$$= 0.7384 + (0.5972)(0.0011)$$

$$= 0.7384 + 0.00067$$

$$\boxed{x_4 = 0.7391}$$

$$\therefore \ |x_4 - x_3| = |0.7391 - 0.7384|$$
$$= 0.0807$$

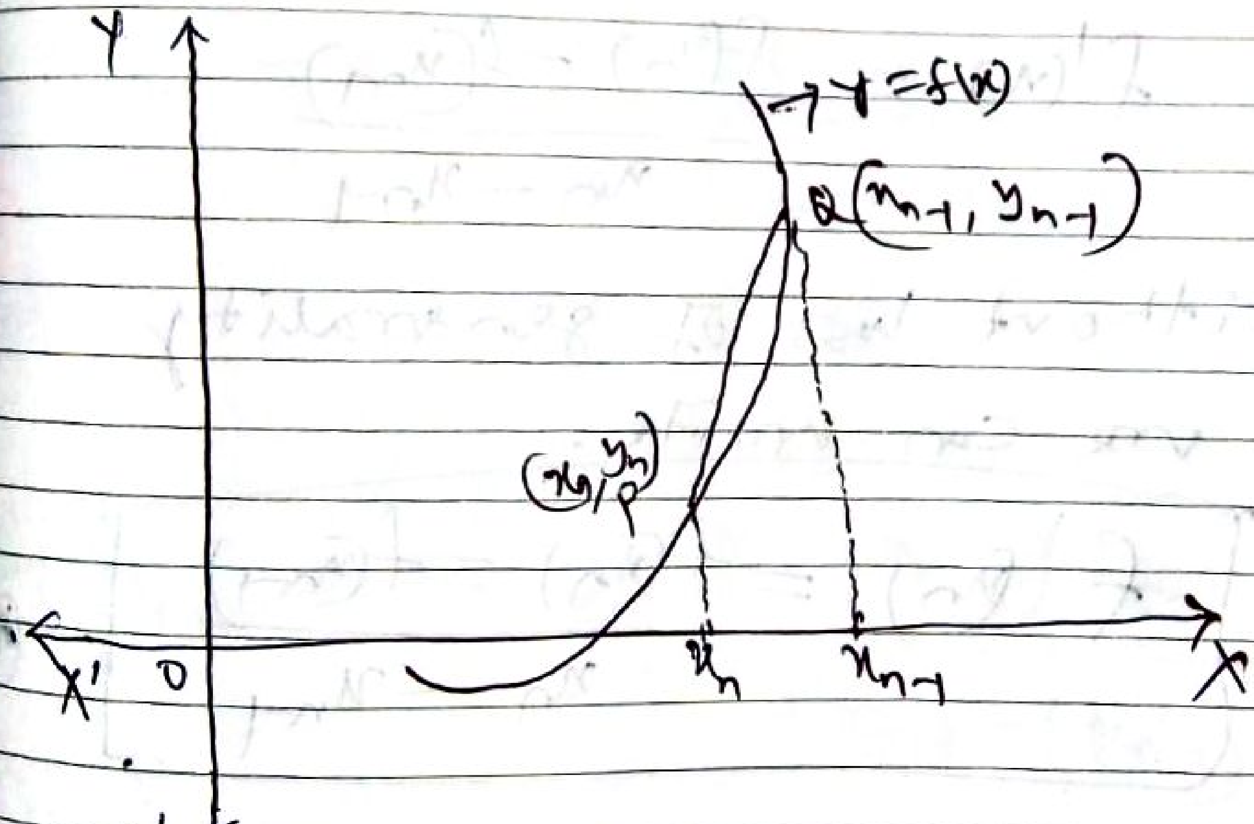
$$\therefore \ \boxed{\text{root} = 0.738}$$

which is correct upto 3 decimal places.

Q. Derive Secant Method.

Ans. Let us consider a curve $y=f(x)$ which cuts X-axis at a point. Let P and Q are two points on the curve $y=f(x)$.

The Co-ordinate of P and Q are (x_n, y_n) and (x_{n-1}, y_{n-1}) respectively.



$$\text{Slope of } PQ = \frac{y_n - y_{n-1}}{x_n - x_{n-1}}$$

(2)

since $y = f(x)$

$$\therefore \; y_n = f(x_n)$$

$$y_{n-1} = f(x_{n-1})$$

$$\text{slope} = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

i.e. $$\frac{dy}{dx} = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

$$f'(x) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

without loss of generality we can write

$$\boxed{f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}} \quad \text{— (A)}$$

(3)

We know N-R method

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

for $n=0,1,2,3\ldots$

putting the value of $f'(x_n)$ in the above formula we get

$$x_{n+1} = x_n - \frac{f(x_n)}{\frac{f(x_n)-f(x_{n-1})}{x_n - x_{n-1}}}$$

$$\Rightarrow \boxed{x_{n+1} = x_n - \frac{(x_n - x_{n-1})\, f(x_n)}{f(x_n) - f(x_{n-1})}}$$

for $n=1,2,3\ldots$

Which is known secant method

(4)

It is also called **chord method**

Note :- It is also expressed as

$$\boxed{x_{k+1} = x_k - \frac{(x_k - x_{k-1}) f(x_k)}{f(x_k) - f(x_{k-1})}}$$

for $k = 1, 2, 3, \ldots$

(*) **Working Rule** :-

To find the roots of an eqn $f(x) = 0$ by secant method we use the following steps.

Step-(1) :- Take two initial approximation x_0 and x_1

Step-(II) :- find $f(x_0)$ and $f(x_1)$

Step-(III) Apply the formula

$$\boxed{x_{n+1} = x_n - \frac{(x_n - x_{n-1}) f(x_n)}{f(x_n) - f(x_{n-1})}}$$

for $n=1, 2, 3 \ldots$

Here taking $n=1$ we get
the ~~a~~ first approximation x_2.

Step-(IV) :- Repeat the procedure
until getting desired
decimal place accuracy.

OR

error tolerance is
negligible.

(1)

9 ## Fixed point iteration Method

10 Let us consider an eqn $f(x) = 0$ — (1)

11 Let 'α' be the exact root of eqn (1)

12 $\therefore \quad f(\alpha) = 0.$

1 The eqn ~~eqn~~ (1) can be expressed

2 as $\boxed{x = \varphi(x)}$ ← (2)

3 Let I be the interval containing the root 'α' i.e. $\alpha \in I$.

If $|\varphi'(x)| < 1 \quad \forall\, x \in I$, then the sequence of approximation $x_0, x_1, x_2, \dots x_n$ will converge to α. When the initial approximation 'x_0' is choosen in the interval I.

2

Working Rule

Step-I :- First express the given eqn (1) in the form of

$$\boxed{x = \varphi(x)} \quad \text{--- (2)}$$

where $|\varphi'(x)| < 1$

Step-II :- Choosing x_0 be the initial approximation to the eqn (2).

Step-III :- We get

$$\boxed{x_1 = \varphi(x_0)}$$

which is 1st approximation

Step-IV :- The 2nd approximation

$$x_2 = \varphi(x_1)$$

and 3rd approximation

$$x_3 = \varphi(x_2)$$

(3)

Similarly proceeding in this way we get

$$\boxed{x_n = \varphi(x_{n-1})}$$

Step-(iv) :- The sequence of approximation $x_1, x_2, x_3 \ldots x_n$ which will converge to α.

or. $|x_n - x_{n-1}| \leq \epsilon$.

or error tolerance is negligible.

Note :- (I) The condition for convergence of fixed point iteration method is $|\varphi'(x)| < 1$. which is the sufficient condition.

(II) The rate of convergence of fixed point iteration method is '1' i.e linearly convergent.

(4)

Q Using fixed pt iteration method find a real root of the eqn $\cos x = 3x - 1$ which is correct upto 4 decimal places.

Soln Given that

$$\cos x = 3x - 1$$

$$\Rightarrow \cos x - 3x + 1 = 0 \quad \text{---} (1)$$

which is in the form of

$$f(x) = 0$$

Here $f(x) = \cos x - 3x + 1$.

Now eqn (1) can be written as

$$3x = 1 + \cos x$$

$$\boxed{x = \frac{1 + \cos x}{3}} \quad \text{---} (2)$$

which is in the form of

$$x = \phi(x)$$

Here $\phi(x) = \frac{1 + \cos x}{3}$

Now $\varphi'(x) = -\frac{1}{3}\sin x$

$$\Rightarrow |\varphi'(x)| = \left|-\frac{1}{3}\sin x\right| = \left|-\frac{1}{3}\right||\sin x|$$

$$\Rightarrow \boxed{|\varphi'(x)| \le \frac{1}{3}} < 1$$

$(\because |\sin x| \le 1)$

$\therefore |\varphi'(x)| < 1$

which is condition of convergent.

Here the eqn

$$\boxed{x = \frac{1}{3}(1+\cos x)}$$

Let us choose $x_0 = \frac{1}{2}$

$\therefore$ 1st approximation

$$x_1 = \frac{1}{3}[1+\cos x_0]$$

$$= \frac{1}{3}\left[1+\cos\left(\frac{1}{2}\right)\right]$$

(6)

$$x_1 = \frac{1}{3}(1.8776) = 0.6258$$

$$\boxed{x_1 = 0.6258}$$

which is 1st approximation.

The 2nd approximation

$$x_2 = \phi(x_1) = \frac{1}{3}[1+\cos x_1]$$

$$= \frac{1}{3}[1+\cos(0.6258)]$$

$$= \frac{1}{3}[1+0.8105]$$

$$= 0.6035$$

$$\boxed{x_2 = 0.6035}$$

3rd approximation

$$x_3 = \phi(x_2) = \frac{1}{3}[1+\cos x_2]$$

$$= \frac{1}{3}[1+\cos(0.6035)]$$

$$= \frac{1}{3}[1+0.8234] = 0.6078$$

(7)

$$\boxed{x_3 = 0.6078}$$

4th approximation

$$x_4 = \varphi(x_3) = \frac{1}{3}\left[1+\cos x_3\right]$$

$$= \frac{1}{3}\left[1+\cos(0.6078)\right]$$

$$= \frac{1}{3}\left[1.8209\right]$$

$$= \cancel{0.6079}\ 0.6070$$

5th approximation

$$x_5 = \varphi(x_4) = \frac{1}{3}\left[1+\cos x_4\right]$$

$$= \frac{1}{3}\left[1+\cos(0.6070)\right]$$

$$= \frac{1}{3}\left[1+0.8214\right]$$

$$\boxed{x_5 = 0.6071}$$

6th. approximation

$$x_6 = \varphi(x_5)$$

8

$$x_6 = \frac{1}{3}\left[1 + \cos x_5\right]$$

$$= \frac{1}{3}\left[1 + \cos(0.6071)\right]$$

$$= \frac{1}{3}\left[1 + 0.8213\right]$$

$$= 0.6071$$

$$\boxed{x_6 = 0.6071}$$

$\therefore \left|x_6 - x_5\right| = 0.0000$

$\therefore$ The root $= 0.6071$ which is correct upto 4 decimal place.

Using fixed pt iteration method find the root of the eqn $x^3 - 2x - 1 = 0$ which is correct upto 3 significant figures

(1)

System of linear eqns:—
[Gauss-Seidel Method]

Let us consider a system of linear eqns.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \quad \text{—— (1)}$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \quad \text{—— (2)}$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \quad \text{—— (3)}$$

The above system of linear eqn can be written in matrix form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times 1}$$

c R

$$\Rightarrow \boxed{A \cdot X = B}$$

where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$

(2)

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} \text{ and } B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times 1}$$

Now to solve the system of linear eqn by Gauss-Siedel method we use the following steps.

i.e. we have to find the value of x_1, x_2, x_3.

<u>Working Rule</u> :-

<u>Step-I</u> :- check the given system of linear eqn satisfy convergent condition.

i.e. diagonally dominant.

$\therefore |a_{11}| \geq |a_{12}| + |a_{13}|$

$|a_{22}| \geq |a_{21}| + |a_{23}|$

$|a_{33}| \geq |a_{31}| + |a_{32}|$

3

Step-II :- If the given eq^ns are not satisfying convergent condition, then rearrange the eq^ns such that they satisfy convergent condition.

Step-III :- Applying initial approximation

$$x_1 = \frac{1}{a_{11}}\left[b_1 - a_{12}x_2 - a_{13}x_3\right]$$

$$x_2 = \frac{1}{a_{22}}\left[b_2 - a_{21}x_1 - a_{23}x_3\right]$$

$$x_3 = \frac{1}{a_{33}}\left[b_3 - a_{31}x_1 - a_{32}x_2\right]$$

Step-IV :- Taking the initial approximat $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$

Step-V :- Repeat the procedure until getting two approximation are equal (nearly equal).

Q Using Gauss-Seidel method solve the following system of linear eqn.

$8x_1 + 2x_2 - 2x_3 = 8$

$x_1 - 8x_2 + 3x_3 = -4$

$2x_1 + x_2 + 9x_3 = 12$

Sol^n Given that

$8x_1 + 2x_2 - 2x_3 = 8$ —— (1)

$x_1 - 8x_2 + 3x_3 = -4$ —— (2)

$2x_1 + x_2 + 9x_3 = 12$ —— (3)

check it is diagonally dominant

$\boxed{|a_{11}| \geq |a_{12}| + |a_{13}|}$

$\therefore |8| > |2| + |-2|$

i.e. $8 > 2+2$

$\boxed{|a_{22}| \geq |a_{21}| + |a_{23}|}$

$\therefore |-8| > |1| + |3|$

$8 > 1+3$

$\boxed{|a_{33}| \geq |a_{31}| + |a_{32}|}$

$\therefore |9| > |2| + |1|$ i.e. $9 > 2+1$

(5)

W.K.T

$$x_1 = \frac{1}{a_{11}}\left[b_1 - a_{12}x_2 - a_{13}x_3\right]$$

$$x_2 = \frac{1}{a_{22}}\left[b_2 - a_{21}x_1 - a_{23}x_3\right]$$

$$x_3 = \frac{1}{a_{33}}\left[b_3 - a_{31}x_1 - a_{32}x_2\right]$$

$$\therefore \quad x_1 = \frac{1}{8}\left[8 - 2x_2 + 2x_3\right] \quad \text{— (a)}$$

$$x_2 = \frac{1}{-8}\left[-4 - x_1 - 3x_3\right] \quad \text{— (b)}$$

$$x_3 = \frac{1}{9}\left[12 - 2x_1 - x_2\right] \quad \text{— (c)}$$

Taking initial approximation

$$x_1^{(0)} = 0, \quad x_2^{(0)} = 0, \quad x_3^{(0)} = 0$$

From (a), (b) (c) we get the

Ist approximation

$$x_1^{(1)} = \frac{1}{8}\left[8 - 2(0) + 2(0)\right] = 1$$

$$\boxed{x_1^{(1)} = 1}$$

01 (6)

$$x_2^{(1)} = \frac{1}{-8}\left[-4-10-3(0)\right] = \frac{-5}{-8}$$

$$\boxed{x_2^{(1)} = 0.625}$$

$$x_3^{(1)} = \frac{1}{9}\left[12-2(1)-\frac{5}{8}\right] = \frac{1}{9}\left[10-\frac{5}{8}\right]$$

$$= \frac{75}{72} = 1.04$$

$$\boxed{x_3^{(1)} = 1.04}$$

∴ The 1st approximation are

$$x_1^{(1)} = 1, \quad x_2^{(1)} = 0.625, \quad x_3^{(1)} = 1.04$$

The 2nd approximation are

$$x_1^{(2)} = \frac{1}{8}\left[8 - 2x_2^{(1)} + 2x_3^{(1)}\right]$$

$$= \frac{1}{8}\left[8 - 2(0.625) + 2(1.04)\right]$$

$$\boxed{x_1^{(2)} = 1.104}$$

$$x_2^{(2)} = -\frac{1}{8}\left[-4 - x_1^{(2)} - 3x_3^{(1)}\right]$$

7

9 $= -\frac{1}{8}\left[-4 - 1.104 - 3(1.04)\right]$

10 $\boxed{x_2^{(2)} = 1.027}$

11

$x_3^{(2)} = \frac{1}{9}\left[12 - 2x_1^{(2)} - x_2^{(2)}\right]$

12

1 $= \frac{1}{9}\left[12 - 2(1.104) - 1.027\right]$

2

$x_3^{(2)} = 0.986$

3

Now 3rd approximations are

4

$x_1^{(3)} = \frac{1}{8}\left[8 - 2x_2^{(2)} + 2x_3^{(2)}\right]$

5

6 $= \frac{1}{8}\left[8 - 2(1.028) + 2(0.987)\right]$

7 $\boxed{x_1^{(3)} = 0.986}$

03 SUNDAY

$x_2^{(3)} = \frac{1}{\ }\left[-4 \ldots\right.$

7

$$= -\frac{1}{8}\left[-4 - 1.1024 - 3(1.04)\right] = \frac{8.22}{8}$$

$$\boxed{x_2^{(2)} = 1.028}$$

$$x_3^{(2)} = \frac{1}{9}\left[12 - 2x_1^{(2)} - x_2^{(2)}\right]$$

$$= \frac{1}{9}\left[12 - 2(1.1024) - 1.028\right]$$

$$\boxed{x_3^{(2)} = \cancel{0.986}\ 0.974}$$

Now 3rd approximations are

$$x_1^{(3)} = \frac{1}{8}\left[8 - 2x_2^{(2)} + 2x_3^{(2)}\right]$$

$$= \frac{1}{8}\left[8 - 2(1.028) + 2(0.974)\right]$$

$$\boxed{x_1^{(3)} = 0.9865}$$

$$\cancel{x_2^{(3)} = \frac{1}{8}\left[-4 - 0.986 - 3(0.97\right.}$$

$$x_2^{(3)} = -\frac{1}{8}\left[-4 - x_1^{(3)} - 3.\ x_3^{(2)}\right]$$

$$= -\frac{1}{8}\left[-4 - 0.9865 - 3(0.974)\right]$$

(8)

$$\boxed{x_2^{(3)} = 0.9886}$$

$$x_3^{(3)} = \frac{1}{9}\left[12 - 2x_1^{(3)} - x_2^{(3)}\right]$$

$$= \frac{1}{9}\left[12 - 2(0.9865) - 0.9886\right]$$

$$= 1.004$$

$$\therefore \boxed{x_3^{(3)} = 1.004}$$

Now $x_1^{(3)} = 0.9865$, $x_2^{(3)} = 0.9886$

$x_3^{(3)} = 1.004$.

Then the 4th approximation are

$$x_1^{(4)} = \frac{1}{8}\left[8 - 2x_2^{(3)} + 2x_3^{(3)}\right]$$

$$= \frac{1}{8}\left[8 - 2(0.9886) + 2(1.004)\right]$$

$$= 1.004$$

$$\boxed{x_1^{(4)} = 1.004}$$

9

$$x_2^{(4)} = -\frac{1}{8}\left[-4 - x_1^{(4)} - 3x_3^{(3)}\right]$$

$$= \frac{1}{8}\left[4 + 1.004 + 3(1.004)\right]$$

$$= 1.002$$

$$\boxed{x_2^{(4)} = 1.002}$$

$$x_3^{(4)} = \frac{1}{9}\left[12 - 2x_1^{(4)} - x_2^{(4)}\right]$$

$$= \frac{1}{9}\left[12 - 2(1.004) - 1.002\right]$$

$$= \frac{1}{9}\left[8.99\right] = 0.999 \approx 1.00$$

$$\therefore \boxed{x_3^{(4)} = 1.00}$$

$$\therefore \boxed{x_1 = 1,\ x_2 = 1,\ x_3 = 1}$$ Ans

verification only for you (not write in exam copy)

eq^n ① $8 - 2 - 2 = 8$

eq^n ② $1 - 8 + 3 = -4$

eq^n ③ $2 + 1 + 9 = \ldots$

(10)

Q. Using Gauss-Seidel method solve the following system of linear eqn.

$$x + y + 3z = 6$$
$$x + 3y + z = 8$$
$$2x + y + z = 5$$

Soln Given that

$$x + y + 3z = 6 \quad \text{--- (1)}$$
$$x + 3y + z = 8 \quad \text{--- (2)}$$
$$2x + y + z = 5 \quad \text{--- (3)}$$

The above eqns are not satisfying diagonally dominant / convergent condition.

So we rearrange the eqns

$$2x + y + z = 5 \quad \text{--- (a)}$$
$$x + 3y + z = 8 \quad \text{--- (b)}$$
$$x + y + 3z = 6 \quad \text{--- (c)}$$

11

In the eqn (a), (b), (c)

$|a_{11}| \geq |a_{12}| + |a_{13}|$

$|2| \geq |1| + |1|$

i.e $2 = 2$

Next $|a_{22}| \geq |a_{21}| + |a_{23}|$

$|3| \geq |1| + |1|$

$\Rightarrow 3 > 2$

and $|a_{33}| \geq |a_{31}| + |a_{32}|$

$|3| \geq |1| + |1|$

$\Rightarrow 3 > 2$

Since condition of convergen is satisfied then apply Gauss-Seidel formula.

12

Here
$$x = \frac{1}{a_{11}}[b_1 - y - z]$$
$$y = \frac{1}{a_{22}}[b_2 - x - z]$$
$$z = \frac{1}{a_{33}}[b_3 - x - y]$$

Here $a_{11} = 2$, $a_{22} = 3$, $a_{33} = 3$

Taking $x^{(0)} = 0$, $y^{(0)} = 0$, $z^{(0)} = 0$.

1st approximation

$x^{(1)} = \frac{1}{2}[5 - 0 - 0]$

$= \frac{5}{2} = 2.5$

$\boxed{x^{(1)} = 2.5}$

$y^{(1)} = \frac{1}{3}[8 - 2.5 - 0]$

$\boxed{y^{(1)} = 1.83}$

$z^{(1)} = \frac{1}{3}[6 - 2.5 - 1.83]$

$\boxed{z^{(1)} = 0.557}$

2nd approximation

$x^{(2)} = \frac{1}{2}[5 - 1.83 - 0.557]$

$\boxed{x^{(2)} = 1.307}$

$y^{(2)} = \frac{1}{3}[8 - 1.307 - 0.557]$

$\boxed{y^{(2)} = 2.045}$

$z^{(2)} = \frac{1}{3}[6 - 1.307 - 2.045]$

$\boxed{z^{(2)} = 0.883}$

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3rd approximation

$x^{(3)} = \frac{1}{2}[5 - 2.045 - 0.883]$

$\boxed{x^{(3)} = 1.036}$

$y^{(3)} = \frac{1}{3}[8 - 1.036 - 0.883]$

$\boxed{y^{(3)} = 2.027}$

$z^{(3)} = \frac{1}{3}[6 - 1.036 - 2.027]$

$\boxed{z^{(3)} = 0.979}$

4th approximation

$x^{(4)} = \frac{1}{2}[5 - 2.027 - 0.979]$

$\boxed{x^{(4)} = 0.997}$

$y^{(4)} = \frac{1}{3}[8 - 0.997 - 0.979]$

$\boxed{y^{(4)} = 2.008}$

$z^{(4)} = \frac{1}{3}[6 - 0.997 - 2.008]$

$\boxed{z^{(4)} = 0.998}$

5th approximation

$x^{(5)} = \frac{1}{2}[5 - 2.008 - 0.998]$

$\boxed{x^{(5)} = 0.997}$

$\simeq 1.00$

$\boxed{x = 1}$

$y^{(5)} = \frac{1}{3}[8 - 0.997 - 0.998]$

$\boxed{y^{(5)} = 2.002}$

$\simeq 2$

$\boxed{y = 2}$

$z^{(5)} = \frac{1}{3}[6 - 0.997 - 2.002]$

$\boxed{z^{(5)} = 1.000}$

$\boxed{z = 1}$

(14)

verification

eqⁿ (1): $2(1)+2+1=5$

eqⁿ (2): $1+3(2)+1=8$

eqⁿ (3): $1+2+3(1)=6$

∴ we conclude that

$\boxed{x=1}$, $\boxed{y=2}$, $\boxed{z=1}$.

Solve the system of linear eqⁿ by LU Decomposition method.

Let us consider the system of linear eqⁿ

$$a_{11}x+a_{12}y+a_{13}z=b_1$$

$$a_{21}x+a_{22}y+a_{23}z=b_2$$

$$a_{31}x+a_{32}y+a_{33}z=b_3$$

(15)

The system of liner eqn can be written in matrix form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times 1}$$

$$\Rightarrow \boxed{A \cdot X = B} \text{ --- } (1)$$

where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1}$ $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times 1}$

Now eqn (1) can be expressed as

$$\boxed{LUX = B} \text{ --- } (2)$$

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where $\boxed{A = LU}$

where $L \rightarrow$ lower triangular matrix

$U \rightarrow$ upper triangular matrix.

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}_{3\times3}$$

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}_{3\times3}$$

LU Decomposition

Dolittle's method

Crout's method

(1)

Crout's Method :-

To solve the system of linear eqⁿ by LU decomposition (crout's method) we use following steps.

Step - (I) :- Express the given system of linear eqⁿ in to matrix form

$$\boxed{AX = B} \quad \text{——} (1)$$

Find A, X, B

Step - (II) :- Again ~~taking~~ eqⁿ (1) can be written as

$$\boxed{LUX = B} \quad \text{——} (2)$$

Where $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$ → lower triangular matrix

(2)

$$U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{upper triangular matrix.}$$

Step-(III) : Taking $\boxed{A = LU}$ — (3)

find all elements of L and U.

Step-(IV) : Taking $\boxed{LY = B}$ — (4)

where $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

Also Here $\boxed{UX = Y}$.

find y_1, y_2, y_3 from (4)

(3)

Step (V) :- Taking $Ux = Y$

find the value of x.

i.e we have to find x_1, x_2, x_3.

Q Solve the system of linear eqn by using LU decomposition method (Crout's method)

$$2x_1 + x_2 + 4x_3 = 12$$

$$8x_1 - 3x_2 + 2x_3 = 20$$

$$4x_1 + 11x_2 - x_3 = 33$$

Soln Given that

$$2x_1 + x_2 + 4x_3 = 12 \quad \text{---} (1)$$

$$8x_1 - 3x_2 + 2x_3 = 20 \quad \text{---} (2)$$

$$4x_1 + 11x_2 - x_3 = 33 \quad \text{---} (3)$$

(4)

The system of linear eqⁿ can be expressed into matrix form.

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\Rightarrow \boxed{A \cdot X = B} \quad \text{— (4)}$$

where $A = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}_{3\times 3}$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} \qquad B = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}_{3\times 1}$$

Now eqⁿ (1) can be expressed as $\boxed{LUX = B}$ — (5)

5

where $\boxed{A = LU}$

$$\therefore \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \cdot \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12}+l_{22} & l_{21}u_{13}+l_{22}u_{23} \\ l_{31} & l_{31}u_{12}+l_{32} & l_{31}u_{13}+l_{32}u_{23}+l_{33} \end{bmatrix}$$

Comparing both sides we get

$\boxed{l_{11} = 2}$

$\boxed{l_{21} = 8}$

$\boxed{l_{31} = 4}$

$l_{11}u_{12} = 1$

$2 \cdot u_{12} = 1$

$\boxed{u_{12} = 1/2}$

$l_{21} \cdot u_{12} + l_{22} = -3$

$\cancel{8}^{4} \cdot \frac{1}{\cancel{2}} + l_{22} = -3$

$\boxed{l_{22} = -7}$

$l_{11}u_{13} = 4$

$2 \cdot u_{13} = 4$

$\boxed{u_{13} = 2}$

$l_{21} \cdot u_{13} + l_{22}u_{23} = 2$

$8 \cdot 2 + (-7) \cdot u_{23} = 2$

$u_{23} = 2\cancel{14}/-7$

$\boxed{u_{23} = 2}$

(b)

$l_{31} \cdot u_{12} + l_{32} = 11$

$\cancel{4}^{2} \cdot \frac{1}{\cancel{2}} + l_{32} = 11$

$l_{32} = 11 - 2$

$\boxed{l_{32} = 9}$

$l_{31} \cdot u_{13} + l_{32} u_{23} + l_{33} = -1$

$\Rightarrow 4 \cdot 2 + 9 \cdot 2 + l_{33} = -1$

$l_{33} = -1 - 26$

$\boxed{l_{33} = -27}$

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Now taking

$\boxed{LY = B}$ where

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

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$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2y_1 \\ 8y_1 - 7y_2 \\ 4y_1 + 9y_2 - 27y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\Rightarrow \quad 2y_1 = 12 \Rightarrow \boxed{y_1 = 6}$$

$$8y_1 - 7y_2 = 20$$

$$\Rightarrow 8 \cdot 6 - 7 \cdot y_2 = 20$$

$$-7y_2 = 20 - 48 = -28$$

$$\boxed{y_2 = 4}$$

and $4y_1 + 9y_2 - 27y_3 = 33$

$$\Rightarrow 4 \cdot 6 + 9 \cdot 4 - 27y_3 = 33$$

$$\Rightarrow -27y_3 = 33 - 60 = -27$$

(8)

$$\boxed{y_3 = 1}$$

$$\therefore Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$$

Now taking $\boxed{UX = Y}$

$$\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$$

$$x_1 + \frac{x_2}{2} + 2x_3 = 6 \quad \text{——— (a)}$$

$$x_2 + 2x_3 = 4 \quad \text{——— (b)}$$

$$\boxed{x_3 = 1} \quad \text{——— (c)}$$

From (b) $\quad x_2 + 2(1) = 4$

$$x_2 = 4 - 2 = 2$$

$$\boxed{x_2 = 2}$$

9 10 11 12 13 14 15 16 17
23 24 25 26 27 28 29 30 31

(a)

From (a) $x_1 + \frac{x_2}{2} + 2x_3 = 6$

$\Rightarrow x_1 + \frac{2}{2} + 2(1) = 6$

$\Rightarrow x_1 + 1 + 2 = 6$

$\Rightarrow x_1 = 6 - 3 = 3$

$\boxed{x_1 = 3}$

$\therefore \boxed{x_1 = 3}, \boxed{x_2 = 2}, \boxed{x_3 = 1}$

$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

H.W

Q:- Using LU decomposition (Crout's method) solve following system of linear eqn.

$2x_1 + x_2 + x_3 = 7$

$x_1 + 2x_2 + x_3 = 8$

$x_1 + x_2 + 2x_3 = 9$

Ans $x_1 = 1$, $x_2 = 2$, $x_3 = 3$

(1)

Cholesky's Method

Let us consider the system of linear eqⁿs.

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

It can be express into matrix form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times1}$$

$$\Rightarrow \boxed{A \cdot X = B} \text{ ——— (1)}$$

Where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$

M T W T F S S M T W T F
1 2 3 4 5 6 7 8 9 10 11 12
15 16 17 18 19 20 21 22 23 24 25 26 27
29 30 31

2

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times 1}$$

The necessary condition for cholesky's method is symmetricity.

i.e. $\boxed{A = A^T}$

Also the sufficient condition for cholesky's method is positive definiteness,

i.e. $\boxed{X^T A X > 0}$

Now eqn 1 can be written as $\boxed{L L^T X = B}$ — 2

3

Let $\boxed{L^T X = Y}$ — (3)

where $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}_{3\times1}$

taking $\boxed{LY = B}$ — (4) [from eqn (2)]

we get the value of Y.

i.e. y_1, y_2, y_3.

Finally taking $\boxed{L^T X = Y}$

we get the value of

x_1, x_2, x_3 i.e. X.

which is the required result of cholesky's method.

4

Working Rule :-

To solve the system of linear eqn by Cholesky's method we use following steps.

Step ①:- Express the given eqn's in to matrix form

if $\boxed{AX = B}$ —— ①

find A, X, B

Step ②

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verify

$\boxed{A = A^T}$

(5)

Step-(III) :- eqn (i) can be written as

$$\boxed{LL^T X = B}$$ where

$$\boxed{A = LL^T}$$

$L \rightarrow$ lower triangular matrix

$$= \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{33} & l_{32} & l_{33} \end{bmatrix}$$

Step-(IV) Taking $\boxed{A = LL^T}$

find all elements of Lower triangular matrix 'L'.

M T W T F S S M T W T F S
1 2 3 4 5 6 7 8 9 10 11 12 13
15 16 17 18 19 20 21 22 23 24 25 26 27
29 30 31

6

Step-V Taking $\boxed{LY = B}$

find the value of Y.
i.e y_1, y_2, y_3 where
$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Step-VI : Finally taking
$$\boxed{L^T X = Y}$$

find the value of x_1, x_2, x_3
i.e $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

which is required result.

(7)

Solve the system of linear eqn by Cholesky's method.

$4x_1 + 2x_2 + 14x_3 = 14$
$2x_1 + 17x_2 - 5x_3 = -101$
$14x_1 - 5x_2 + 83x_3 = 155$

Soln Given that

$4x_1 + 2x_2 + 14x_3 = 14$ —— (1)
$2x_1 + 17x_2 - 5x_3 = -101$ —— (2)
$14x_1 - 5x_2 + 83x_3 = 155$ —— (3)

The above eqns are expressed into matrix form

$$\begin{bmatrix} 4 & 2 & 14 \\ 2 & 17 & -5 \\ 14 & -5 & 83 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 14 \\ -101 \\ 155 \end{bmatrix}$$

$$\Rightarrow \boxed{A \cdot X = B} \text{ —— (4)}$$

(8)

where $A = \begin{bmatrix} 4 & 2 & 14 \\ 2 & 17 & -5 \\ 14 & -5 & 13 \end{bmatrix}_{3\times3}$ $\quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1}$

$$B = \begin{bmatrix} 14 \\ -101 \\ 155 \end{bmatrix}_{3\times1}$$

Now $A^T = \begin{bmatrix} 4 & 2 & 14 \\ 2 & 17 & -5 \\ 14 & -5 & 13 \end{bmatrix}$

$R_1 \to C_1$
$R_2 \to C_2$
$R_3 \to C_3$
or vice versa.

$\therefore \boxed{A = A^T}$

It satisfies symmetric condition.

Now eqn (4) can be written as

$$\boxed{L.L^T X = B} \quad \text{— (5)}$$

where $A = LL^T$

(9)

$\therefore$ Here $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$

$\Rightarrow L^T = \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$

Now $\boxed{A = LL^T}$

$$\begin{bmatrix} 4 & 2 & 14 \\ 2 & 17 & -5 \\ 14 & -5 & 83 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \cdot \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 14 \\ 2 & 17 & -5 \\ 14 & -5 & 83 \end{bmatrix} = \begin{bmatrix} l_{11}^2 & l_{11}l_{21} & l_{11}.l_{31} \\ l_{21}l_{11} & l_{21}^2 + l_{22}^2 & l_{21}l_{31} + l_{22}l_{32} \\ l_{31}.l_{11} & l_{31}.l_{21} + l_{32}l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$$

Comparing both sides we get,

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$$l_{11}^2 = 4$$
$$\boxed{l_{11} = 2}$$
$$l_{21} l_{11} = 2$$
$$l_{21} \cdot 2 = 2$$
$$\boxed{l_{21} = 1}$$
$$l_{31} \cdot l_{11} = 14$$
$$l_{31} \cdot 2 = 14$$
$$\boxed{l_{31} = 7}$$

$$l_{11} \cdot l_{21} = 2$$
$$\boxed{l_{21} = 1}$$
$$l_{21}^2 + l_{22}^2 = 17$$
$$(1)^2 + l_{22}^2 = 17$$
$$l_{22}^2 = 16$$
$$\boxed{l_{22} = 4}$$
$$l_{31} \cdot l_{21} + l_{32} l_{22} = -5$$
$$7 \cdot 1 + l_{32} \cdot 4 = -5$$
$$l_{32} = -3$$

$$l_{11} \cdot l_{31} = 14$$
$$\boxed{l_{31} = 7}$$
$$l_{21} \cdot l_{31} + l_{22} l_{32} = -5$$
$$1 \cdot 7 + 4 \cdot l_{32} = -5$$
$$4 l_{32} = -5 - 7$$
$$\Rightarrow l_{32} = \frac{-12}{4}$$
$$\boxed{l_{32} = -3}$$
$$l_{31}^2 + l_{32}^2 + l_{33}^2 = 83$$
$$(7)^2 + (-3)^2 + l_{33}^2 = 83$$
$$l_{33}^2 = 83 - 58$$
$$l_{33}^2 = 25$$
$$\boxed{l_{33} = 5}$$

$$\therefore L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 4 & 0 \\ 7 & -3 & 5 \end{bmatrix}$$

$$L^T = \begin{bmatrix} 2 & 1 & 7 \\ 0 & 4 & -3 \\ 0 & 0 & 5 \end{bmatrix}$$

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Now taking $\boxed{LY = B}$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 1 & 4 & 0 \\ 7 & -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ -101 \\ 155 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2y_1 \\ y_1 + 4y_2 \\ 7y_1 - 3y_2 + 5y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ -181 \\ 155 \end{bmatrix}$$

$$\Rightarrow 2y_1 = 14 \Rightarrow \boxed{y_1 = 7}$$

$$y_1 + 4y_2 = -101$$

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$$\Rightarrow 7 + 4y_2 = -101$$

$$4y_2 = -108$$

$$y_2 = \frac{-108}{4} = -27$$

$$\boxed{y_2 = -27}$$

Next $7y_1 - 3y_2 + 5y_3 = 155$

$$7(7) - 3\cdot(-27) + 5y_3 = 155$$

$$49 + 81 + 5y_3 = 155$$

$$5y_3 = 155 - 130 = 25$$

$$y_3 = 25/5 = 5 \rightarrow \boxed{y_3 = 5}$$

Now $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -27 \\ 5 \end{bmatrix}$

Now Taking $\boxed{L^T X = Y}$

$$\begin{bmatrix} 2 & 1 & 7 \\ 0 & 4 & -3 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -27 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 2x_1 + x_2 + 7x_3 \\ 4x_2 - 3x_3 \\ 5x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -27 \\ 5 \end{bmatrix}$$

13

Comparing both sides we get

$5x_3 = 5 \Rightarrow \boxed{x_3 = 1}$

$4x_2 - 3x_3 = -27$

$4x_2 - 3(1) = -27$

$4x_2 = -27 + 3 = -24$

$x_2 = -24/4 = -6$

$\boxed{x_2 = -6}$

And $2x_1 + x_2 + 7x_3 = 7$

$2x_1 + (-6) + 7(1) = 7$

$2x_1 - 6 + \cancel{7} = \cancel{7}$

$\Rightarrow 2x_1 - 6 = 0 \Rightarrow 2x_1 = 6$

$\boxed{x_1 = 3}$

$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ 1 \end{bmatrix}$.

H.W

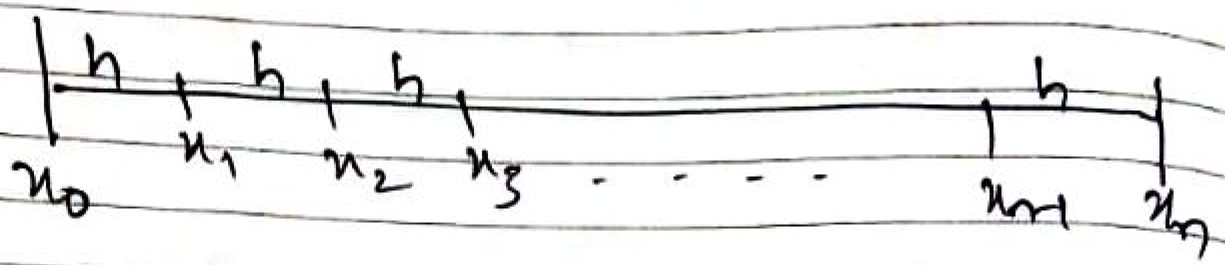
Q Using cholesky's method solve
$9x_1 + 6x_2 + 12x_3 = 87$, $6x_1 + 13x_2 + 11x_3 = 118$
$12x_1 + 11x_2 + 26x_3 = 154$

Ans $[3, 6, 2]^T$

①

Interpolation / Interpolating polynomial

Let $x_0, x_1, x_2, \ldots x_n$ are $(n+1)$ pts or nodes, which are equi spaced.



$h \rightarrow$ step size / step length.

The process of finding intermediate value of a function from a set of its value at specific points is called interpolation.

which is given in the following table.

x	x_0	x_1	x_2	$\ldots$	x_{n-1}	x_n
$f(x)$	f_{x_0}	$f(x_1)$	$f(x_2)$		$f(x_{n-1})$	$f(x_n)$
	y_0	y_1	y_2		y_{n-1}	y_n

In other words we can say the process of finding the values of y corresponding to x within the given range (x_0, x_n) is called interpolation.

Q. Define Extrapolation.

Ans. It is the process of finding the values of y corresponding to x outside the range (x_0, x_n),

i.e. which does not belong to (x_0, x_n).

V.V.Imp

Q. Derive Lagrange's interpolating polynomial.

(3)

Lagrange's linear interpolation

Let $f(x)$ be defined at two distinct points x_0, x_1.

Let $P(x)$ be a linear interpolating polynomial to $f(x)$ in the given interval $[x_0, x_n]$.

Since $P(x)$ is a polynomial of degree 1. Then $P(x)$ can be expressed as

$$\boxed{P(x) = a_0 x + a_1} \quad \text{---} (1)$$

As it is interpolating polynomial to $f(x)$,

$$\therefore \boxed{f(x_0) = P(x_0) = a_0 x_0 + a_1} \quad \text{---} (2)$$

Similarly

$$\boxed{f(x_1) = P(x_1) = a_0 x_1 + a_1} \quad \text{---} (3)$$

Eliminating a_0, a_1 from above three eqn's we get

$$\begin{array}{ccc} (+) & (-) & (+) \end{array}$$
$$\begin{vmatrix} p(x) & x & 1 \\ f(x_0) & x_0 & 1 \\ f(x_1) & x_1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow p(x) \begin{vmatrix} x_0 & 1 \\ x_1 & 1 \end{vmatrix} - x \begin{vmatrix} f(x_0) & 1 \\ f(x_1) & 1 \end{vmatrix} + 1 \begin{vmatrix} f(x_0) & x_0 \\ f(x_1). & x_1 \end{vmatrix} = 0$$

$$\Rightarrow p(x)\left[x_0 - x_1\right] - x\left[f(x_0) - f(x_1)\right] + 1\left[f(x_0)\,x_1 - f(x_1).x_0\right] = 0$$

$$\Rightarrow p(x)\left[x_0 - x_1\right] = x\left[f(x_0) - f(x_1)\right] - \left[f(x_0).x_1 - f(x_1).x_0\right]$$

$$= x f(x_0) - x f(x_1) - f(x_0).x_1 + f(x_1).x_0$$

(5)

$$= [x - x_1] f(x_0) - f(x_1)[x - x_0]$$

$$\therefore P(x) = \frac{(x-x_1) f(x_0) - (x-x_0) \cdot f(x_1)}{x_0 - x_1}$$

$$\Rightarrow P(x) = \frac{(x-x_1) \cdot f(x_0)}{x_0 - x_1} - \frac{(x-x_0) f(x_1)}{x_0 - x_1}$$

$$\Rightarrow P(x) = \frac{x - x_1}{x_0 - x_1} \cdot f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

$$\therefore P(x) = l_0(x) f(x_0) + l_1(x) \cdot f(x_1)$$

where $\boxed{l_0(x) = \frac{x - x_1}{x_0 - x_1}}$

$$\boxed{l_1(x) = \frac{x - x_0}{x_1 - x_0}}$$

It can be expressed as

$$\boxed{f_1(x) = p(x) = l_0(x) f(x_0) + l_1(x) \cdot f(x_1)}$$

which is called Lagrange's linear interpolation

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similarly Lagrange's quadratic interpolation is given by

$$\boxed{f_2(x) = p(x) = l_0(x) \cdot f(x_0) + l_1(x) \cdot f(x_1) + l_2(x) \cdot f(x_2)}$$

where

$$\boxed{l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}}$$

$$\boxed{l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}}$$

$$\boxed{l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}}$$

where $f(x)$ is defined at pts x_0, x_1, x_2

proceeding in this way we get

Lagrange's interpolation of degree 'n' which is

given by

(7)

$$f_n(x) = P(x) = L_0(x) \cdot f(x_0) + L_1(x) \cdot f(x_1) + L_2(x) \cdot f(x_2) + \cdots + L_n(x) f(x_n)$$

where

$$L_0(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}$$

$$\vdots$$

$$L_n(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}$$

Here $f(x)$ is defined at $x_0, x_1, x_2 \cdots x_n$.

Ex Using Lagrange's interpolation find the interpolating polynomial from the datas

$f(0)=1$, $f(1)=3$, $f(3)=55$

(8)

Sol? Given that

$f(0) = 1, \; f(1) = 3, \; f(3) = 55$

i.e $f(x_0) = 1, \; f(x_1) = 3, \; f(x_2) = 55$

Here $x_0 = 0, \; x_1 = 1, \; x_2 = 3$

Now

	x_0	x_1	x_2
x	0	1	3
$f(x)$	1	3	55
	$f(x_0)$	$f(x_1)$	$f(x_2)$

we know Lagrange's interpolating polynomial of ~~.....~~ degree 2

$$\boxed{f_2(x) = P(x) = l_0(x) \cdot f(x_0) + l_1(x) \cdot f(x_1) + l_2(x) \cdot f(x_2)}$$

(A)

where $l_0(x) = \dfrac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$

$l_1(x) = \dfrac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$

$l_2(x) = \dfrac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$

(9)

Now we have $f(x_0), f(x_1), f(x_2)$ we only find $l_0(x), l_1(x), l_2(x)$

Now $l_0(x) = \dfrac{(x-1)(x-3)}{(0-1)(0-3)} = \dfrac{x^2-4x+3}{3}$

$$l_1(x) = \frac{(x-0)(x-3)}{(1-0)(1-3)} = \frac{x^2-3x}{-2}$$

$$l_2(x) = \frac{(x-0)(x-1)}{(3-0)(3-1)} = \frac{x^2-x}{6}$$

Putting all the values in eq^n (A) we get

$$f_2(x) = P(x) = \frac{x^2-4x+3}{3}\cdot(1) + \frac{x^2-3x}{-2}\cdot(3)$$

$$+ \frac{x^2-x}{6}\cdot(55)$$

$$= \frac{x^2-4x+3}{3} - \frac{(3x^2-9x)}{2} + \frac{55x^2-55x}{6}$$

(10)

$$= \frac{2[x^2-4x+3]-3(3x^2-9x)+55x^2-55x}{6}$$

$$= \frac{2x^2-8x+6-9x^2+27x+55x^2-55x}{6}$$

$$= \frac{48x^2-36x+6}{6} = \frac{\cancel{6}(8x^2-6x+1)}{\cancel{6}}$$

$\therefore$ $\boxed{f_2(x) = 8x^2-6x+1}$

which is required interpolating polynomial.

Q Using L.I., find interpolating polynomial from the following table.

x	-1	0	2	5
y	9	5	3	15

(11)

Soln Given that

x	-1	0	2	5
y	9	5	3	15

Here $x_0 = -1$, $x_1 = 0$, $x_2 = 2$, $x_3 = 5$

$f(x_0) = 9$, $f(x_1) = 5$, $f(x_2) = 3$, $f(x_3) = 15$

From the data it is clear that we get a cubic polynomial i.e. a polynomial of degree atmost 3 or less.

$$\boxed{f_3(x) = p(x) = l_0(x).f(x_0) + l_1(x).f(x_1) + l_2(x).f(x_2) + l_3(x).f(x_3)}$$

(A)

SUNDAY

where

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

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$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

Now we have $f(x_0)$, $f(x_1)$, $f(x_2)$, $f(x_3)$ only we find the value of $l_0(x)$, $l_1(x)$, $l_2(x)$, $l_3(x)$.

Now

$$l_0(x) = \frac{(x-0)(x-2)(x-5)}{(-1-0)(-1-2)(-1-5)}$$

$$= \frac{x(x^2-7x+10)}{(-1)(-3)(-6)}$$

$$\therefore \boxed{l_0(x) = -\frac{x^3-7x^2+10x}{18}}$$

$$l_1(x) = \frac{(x-(-1))(x-2)(x-5)}{(0-(-1))(0-2)(0-5)}$$

13

$$= \frac{(x+1)(x-2)(x-5)}{1\cdot(-2)(-5)}$$

$$= \frac{(x+1)(x^2-7x+10)}{10}$$

$$= \frac{x^3-7x^2+10x+x^2-7x+10}{10}$$

$$\boxed{l_1(x) = \frac{x^3-6x^2+3x+10}{10}}$$

$$l_2(x) = \frac{(x-(-1))(x-0)(x-5)}{(2-(-1))(2-0)(2-5)}$$

$$= \frac{(x+1)\,x\,(x-5)}{3\cdot 2(-3)} = \frac{(x^2+x)(x-5)}{-18}$$

$$= \frac{x^3+x^2-5x^2-5x}{-18}$$

$$\boxed{l_2(x) = -\frac{(x^3-4x^2-5x)}{18}}$$

$$l_3(x) = \frac{(x-(-1))(x-0)(x-2)}{(5-(-1))(5-0)(5-2)}$$

(14)

$$= \frac{(x+1) \cdot x \cdot (x-2)}{6 \cdot 5 \cdot 3} = \frac{(x^2+x)(x-2)}{90}$$

$$= \frac{x^3 + x^2 - 2x^2 - 2x}{90}$$

$$\boxed{l_3(x) = \frac{x^3 - x^2 - 2x}{90}}$$

putting all values in eqn (A) we get.

$$f_3(x) = f(x) = -\frac{(x^3 - 7x^2 + 10x)}{\cancel{18}_2} \cdot (\cancel{9})$$

$$+ \frac{x^3 - 6x^2 + 3x + 10}{\cancel{18}_2} \cdot (\cancel{5})$$

$$+ \left\{ -\frac{(x^3 - 4x^2 - 5x)}{\cancel{18}_6} \cdot (\cancel{3}) \right\}$$

$$+ \frac{x^3 - x^2 - 2x}{\cancel{90}_6} \cdot (\cancel{15}).$$

(15)

$$f_3(x) = -\frac{(x^3-7x^2+10x)}{2} + \frac{x^3-6x^2+3x+10}{2}$$

$$-\frac{(x^3-4x^2-5x)}{6} + \frac{x^3-x^2-2x}{6}$$

$$= \frac{-3(x^3-7x^2+10x)+3(x^3-6x^2+3x+10) - x^3+4x^2+5x+x^3-x^2-2x}{6}$$

$$= \frac{\cancel{-3x^3}+21x^2-30x+\cancel{3x^3}-18x^2+9x+30 \cancel{-x^3}+4x^2+5x\cancel{+x^3}-x^2-2x}{6}$$

$$= \frac{6x^2-18x+30}{6} = \frac{\cancel{6}(x^2-3x+5)}{\cancel{6}}$$

$$\boxed{f_3(x) = p(x) = x^2-3x+5}$$

which is required interpolating polynomial.

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HW

Q-1 Using Lagrange's interpolation formula find the polynomial from the following table

x	-1	0	2	3
y	-8	3	1	12

Also find f(1).

Ans 2

Q-2 find the interpolating polynomial from the data

x	0	1	3	4	5
y	0	1	8	256	625

by using Lagrange's interpolation polynomial also find f(2).

Ans 16

1

Newton's Divided difference interpolation

Let $f(x)$ be a function defined at two distinct points x_0 and x_1.

Let $p(x)$ be interpolating polynomial of degree '1'.

$\therefore$ $p(x)$ can be expressed as

$$p(x) = a_0 x + a_1 \quad \text{——} ①$$

As it is interpolating polynomial

$$\therefore f(x_0) = p(x_0) = a_0 x_0 + a_1 \quad \text{——} ②$$

$$f(x_1) = p(x_1) = a_0 x_1 + a_1 \quad \text{——} ③$$

Eliminating a_0 and a_1 from above three eq's we get

2

$$\begin{array}{ccc} (+) & (-) & (+) \\ \end{array}$$
$$\begin{vmatrix} p(x) & x & 1 \\ f(x_0) & x_0 & 1 \\ f(x_1) & x_1 & 1 \end{vmatrix} = 0$$

Expanding with R_1

$$p(x)\begin{vmatrix} x_0 & 1 \\ x_1 & 1 \end{vmatrix} - x\begin{vmatrix} f(x_0) & 1 \\ f(x_1) & 1 \end{vmatrix} + 1\begin{vmatrix} f(x_0) & x_0 \\ f(x_1) & x_1 \end{vmatrix} = 0$$

$$\Rightarrow p(x)[x_0 - x_1] - x[f(x_0) - f(x_1)] + f(x_0).x_1 - x_0.f(x_1) = 0$$

$$\Rightarrow p(x)(x_0 - x_1) = x[f(x_0) - f(x_1)] - f(x_0).x_1 + x_0 f(x_1)$$

③

$$p(x)[x_0 - x_1] = x f(x_0) - x f(x_1) - f(x_0) \cdot x_1 + x_0 \cdot f(x_1)$$

Adding ~~$x_0 f(x_0)$~~ and subtracting $x_0 f(x_0)$ in RHS.

$$p(x)[x_0 - x_1] = x f(x_0) - x f(x_1) - f(x_0) \cdot x_1 + x_0 f(x_1) - x_0 f(x_0) + x_0 f(x_0)$$

$$= x_0 \cdot f(x_0) - x_1 f(x_0) + x_0 [f(x_1) - f(x_0)] - x [f(x_1) - f(x_0)]$$

$$= f(x_0)[x_0 - x_1] + (x_0 - x)[f(x_1) - f(x_0)]$$

$$\Rightarrow P(x) = \frac{f(x_0) \cdot (x_0 - x_1) + (x_0 - x) \cdot [f(x_1) - f(x_0)]}{x_0 - x_1}$$

(4)

$$P(x) = \frac{f(x_0)\cancel{(x_0-x_1)}}{\cancel{(x_0-x_1)}} + \frac{(x_0-x)\cdot[f(x_1)-f(x_0)]}{(x_0-x_1)}$$

$$P(x) = f(x_0) + (x_0-x)\cdot\frac{[f(x_1)-f(x_0)]}{-[x_1-x_0]}$$

$$\Rightarrow P(x) = f(x_0) + \cancel{+}(x-x_0)\cdot\frac{[f(x_1)-f(x_0)]}{\cancel{+}[x_1-x_0]}$$

$$\therefore P(x) = f(x_0) + (x-x_0)\cdot\frac{f(x_1)-f(x_0)}{x_1-x_0}$$

$$\Rightarrow \boxed{P(x) = f(x_0) + (x-x_0)\cdot f[x_0,x_1]}$$

Where $\boxed{f[x_0,x_1] = \dfrac{f(x_1)-f(x_0)}{x_1-x_0}}$

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which is called 1st divided difference of $f(x)$ at x_0, x_1.

It is also expressed as

$$\boxed{f_1(x) = f(x_0) + (x-x_0)\, f[x_0, x_1]}$$

which is also called Newton's first divided difference interpolation formula.

Similarly Newton's 2nd divided difference interpolation formula is

$$\boxed{f_2(x) = f(x_0) + (x-x_0)\, f[x_0, x_1] + (x-x_0)(x-x_1)\, f[x_0, x_1, x_2]}$$

where $f[x_0, x_1, x_2] = \dfrac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$

(2)

Here $f[x_1, x_2] = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Proceeding in this way the nth divided difference interpolation formula is

$$\boxed{\begin{aligned} f_n(x) = p(x) = f(x_0) &+ (x - x_0) \cdot f[x_0, x_1] \\ &+ (x - x_0)(x - x_1) f[x_0, x_1, x_2] \\ &+ (x - x_0)(x - x_1)(x - x_2) f[x_0, x_1, x_2, x_3] \\ &+ \dots \\ &+ (x - x_0)(x - x_1) \dots (x - x_{n-1}) f[x_0, x_1 \dots x_n] \end{aligned}}$$

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where

$$f[x_0, x_1, x_2 \dots x_n] = \frac{f[x_1, x_2 \dots x_n] - f[x_0, x_1 \dots x_{n-1}]}{x_n - x_0}$$

Divided Difference Table :-

x	f(x)	1st D.D	2nd D.D	3rd D.D	4th D.D
x_0	$f(x_0)$				
		$f[x_0, x_1]$			
x_1	$f(x_1)$		$f[x_0, x_1, x_2]$		
		$f[x_1, x_2]$		$f[x_0, x_1, x_2, x_3]$	
x_2	$f(x_2)$		$f[x_1, x_2, x_3]$		$f[x_0, x_1, x_2, x_3, x_4]$
		$f[x_2, x_3]$		$f[x_1, x_2, x_3, x_4]$	
x_3	$f(x_3)$		$f[x_2, x_3, x_4]$		
		$f[x_3, x_4]$			
x_4	$f(x_4)$				

Q Construct a Newton's divided difference table from the following data

x	0	2	4	6	8
y	-18	6	54	174	414

(6)

Sol:

x	$f(x)$	1st D.D Δf	2nd D.D $\Delta^2 f$	3rd D.D $\Delta^3 f$	4th D.D $\Delta^4 f$
0	-18				
		12			
2	6		3		
		24		1	
4	54		9		0
		60		1	
6	174		15		
		120			
8	414				

Explaination

1st D.D

$$\frac{6-(-18)}{2-0}=12$$

$$\frac{54-6}{4-2}=24$$

$$\frac{174-54}{6-4}=60$$

$$\frac{414-174}{8-6}=120$$

2nd D.D

$$\frac{24-12}{4-0}=3$$

$$\frac{60-24}{6-2}=9$$

$$\frac{120-60}{8-4}=15$$

3rd D.D

$$\frac{9-3}{6-0}=1$$

$$\frac{15-9}{8-2}=1$$

4th

$$\frac{1-1}{8-0}=$$

(9)

Find the interpolating polynomial of degree '2' by using Newton's divided ~~...~~ difference formula: Also find f(2). from the following table.

Sol

	x_0	x_1	x_2
x	0	1	3
$f(x)$	1	3	55
	$f(x_0)$	$f(x_1)$	$f(x_2)$

pts IS (0,1), (1,3), (3,55)

Sol Given that

x	0	1	3
y	1	3	55

D.D Table

x	$f(x)$	1st D.D	2nd D.D
x_0 0	1		
		2	
x_1 1	3		8
		26	
x_2 3	55		

(1D)

W.K.T Newton's Divided Difference formula.

$$\boxed{\begin{aligned} f_n(x) &= f(x_0) + (x-x_0) f[x_0, x_1] \\ &+ (x-x_0)(x-x_1) f[x_0, x_1, x_2] + \dots \\ &+ (x-x_0)(x-x_1)\dots(x-x_{n-1}) f[x_0, x_1, \dots x_n] \end{aligned}}$$

Here

$$f_2(x) = f(x_0) + (x-x_0) f[x_0, x_1] + (x-x_0)(x-x_1) f[x_0, x_1, x_2]$$

Putting the value from the table

$$f_2(x) = 1 + (x-0) \cdot 2 + (x-0)(x-1) \cdot 8$$

$$= 1 + 2x + (x^2 - x) 8$$

$$= 1 + 2x + 8x^2 - 8x$$

$$\boxed{f_2(x) = 8x^2 - 6x + 1}$$

(11)

Which is required interpolating polynomial.

We have to find f(2)

putting $x=2$ we get

$$f_2(2) = 8(2)^2 - 6.2+1$$

$$= 32 - 12+1 = 21$$

$\therefore \boxed{f(2) = 21}$

Q Using Newton's divided difference formula find the value of f(4) from the given datas.

$f(0) = 2$, $f(1) = 3$, $f(2) = 12$

$f(5) = 3587$

(12)

Sol Given that

$f(0)=2,\ f(1)=3,\ f(2)=12,\ f(15)=3587$

$f(x_0)=2,\ f(x_1)=3,\ f(x_2)=12$

$f(x_3)=3587$

D.D Table

	x	$f(x)$	1st Δf	2nd $\Delta^2 f$	3rd $\Delta^3 f$
x_0	0	2			
			1		
x_1	1	3		4	
			9		1
x_2	2	12		19	
			275		
x_3	15	3587			

W.K.T Newton's divided difference formula

$$\boxed{f_n(x)=f(x_0)+(x-x_0)f[x_0,x_1]+(x-x_0)(x-x_1)f[x_0,x_1,x_2]+\cdots+(x-x_0)(x-x_1)\cdots(x-x_{n-1})f[x_0,x_1,\ldots,x_n]}$$

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Here

$$f_3(x) = f(x_0) + (x-x_0).f[x_0,x_1] + (x-x_0)(x-x_1)$$
$$.f[x_0,x_1,x_2] + (x-x_0)(x-x_1)(x-x_2)f[x_0,x_1,x_2,x_3]$$

Putting all values from the table we get

$$f_3(x) = 2 + (x-0).1 + (x-0)(x-1).4$$
$$+ (x-0)(x-1)(x-2).1$$

$$= 2 + x + (x^2-x).4$$
$$+ x(x^2-3x+2).$$

$$= 2 + x + 4x^2 - 4x$$
$$+ x^3 - 3x^2 + 2x$$

$$\boxed{f_3(x) = x^3 + x^2 - x + 2}$$

$$f_3(4) = 4^3 + 4^2 - 4 + 2 = 78$$

M	T	W	T	F	S	S	M	T	W	T
					1	2	3	4	5	6
10	11	12	13	14	15	16	17	18	19	20
24	25	26	27	28	29	30				

(14)

HW

Q Using Newton's divided difference interpolation formula find the value of f(2) from the following table.

X	3	4	7	9	10
Y	5	8	11	13	17

Ans $\boxed{f(2) = 1}$

(1)

Newton's forward and Backward Difference

Let $f(x)$ be a function which is defined at $(n+1)$ pts such as $x_0, x_1, x_2 \dots x_n$.

Now $x_1 = x_0 + h$

$$x_2 = x_1 + h = x_0 + h + h = x_0 + 2h$$

$$x_3 = x_0 + 3h$$

$$x_4 = x_0 + 4h$$

$$\vdots$$

$$x_n = x_0 + nh$$

$$\therefore \ nh = x_n - x_0$$

$$\boxed{h = \frac{x_n - x_0}{n}}$$

②

Now we have difference operator.

(I) Forward difference operator (Δ)

(II) Backword difference operator (∇)

<u>Forward difference operator (Δ)</u>

It is defined as

$$\boxed{\Delta f(x_i) = f(x_i+h) - f(x_i)}$$

for $i = 0, 1, 2 \ldots n$

Taking $i=0$

$$\Delta f(x_0) = f(x_0+h) - f(x_0)$$

$$= f(x_1) - f(x_0)$$

$$\therefore \boxed{\Delta f(x_0) = f(x_1) - f(x_0)}$$

3

Taking $i=1$

$$\Delta f(x_1) = f(x_1 + h) - f(x_1)$$

$$= f(x_2) - f(x_1)$$

$$\therefore \boxed{\Delta f(x_1) = f(x_2) - f(x_1)}$$

Taking $i=2$

$$\boxed{\Delta f(x_2) = f(x_3) - f(x_2)}$$

proceeding similarly

for $i=n$ we get

$$\boxed{\Delta f(x_n) = f(x_{n+1}) - f(x_n)}$$

(4)

Forward difference Table :-

Table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
x_0	$f(x_0)$				
		$\Delta f(x_0)$			
x_1	$f(x_1)$		$\Delta^2 f(x_0)$		
		$\Delta f(x_1)$		$\Delta^3 f(x_0)$	
x_2	$f(x_2)$		$\Delta^2 f(x_1)$		$\Delta^4 f(x_0)$
		$\Delta f(x_2)$		$\Delta^3 f(x_1)$	
x_3	$f(x_3)$		$\Delta^2 f(x_2)$		
		$\Delta f(x_3)$			
x_4	$f(x_4)$				

This table is also known as diagonal difference table.

Ex Construct a forward difference table from following data.

x	2	4	6	8	10
y	5	10	17	29	50

Also find $\Delta f(2)$, $\Delta f(6)$, $\Delta^2 f(6)$, $\Delta^2 f(4)$, $\Delta^3 f(2)$, $\Delta^4 f(2)$.

Sol^n Table :-

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
2	5				
		5			
4	10		2		
		7		3	
6	17		5		1
		12		4	
8	29		9		
		21			
10	50				

$\Delta f(2) = 5$, $\Delta f(6) = 12$

$\Delta^2 f(6) = 9$, $\Delta^2 f(4) = 5$

$\Delta^3 f(2) = 3$ $\Delta^4 f(2) = 1$

(6)

Q Construct a forward difference table from the following data table

x	1	2	3	4	5
$y=f(x)$	4	6	9	12	17

Soln find $\Delta f(3)$, $\Delta^2 f(3)$, $\Delta^2 f(4)$, $\Delta^3 f(1)$, $\Delta^3 f(2)$, $\Delta^4 f(1)$.

Table :-

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	4				
		2			
2	6		1		
		3		-1	
3	9		0		3
		3		2	
4	12		2		
		5			
5	17				

(7)

Now $\Delta f_5(3) = 3$, $\Delta^2 f(3) = 2$

$\Delta^2 f(1) = 1$, $\Delta^3 f(1) = -1$

$\Delta^3 f(2) = 2$, $\Delta^4 f(1) = 3$

Backward difference operator (∇)

It is defined as

$$\boxed{\nabla f(x_i) = f(x_i) - f(x_i - h)}$$

for $i = 1, 2, 3 \ldots n$

Taking $i = 1$

$$\nabla f(x_1) = f(x_1) - f(x_1 - h)$$
$$= f(x_1) - f(x_0) \quad (\text{As } x_1 = x_0 + h)$$

$$\therefore \boxed{\nabla f(x_1) = f(x_1) - f(x_0)}$$

8

Taking $i=2$

$$\nabla f(x_2) = f(x_2) - f(x_2 - h)$$

$$= f(x_2) - f(x_1) \quad (\because x_2 = x_1 + h$$

$$\therefore \boxed{\nabla f(x_2) = f(x_2) - f(x_1)}$$

Taking $i=3$

$$\boxed{\nabla f(x_3) = f(x_3) - f(x_2)}$$

proceeding similarly for $i=n$ we get

$$\boxed{\nabla f(x_n) = f(x_n) - f(x_{n-1})}$$

M T W T F S S M T W T F S
1 2 3 4 5 6 7 8
10 11 12 13 14 15 16 17 18 19 20 21 22
24 25 26 27 28 29 30

9

Backward difference Table

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
x_0	$f(x_0)$				
x_1	$f(x_1)$	$\nabla f(x_1)$			
x_2	$f(x_2)$	$\nabla f(x_2)$	$\nabla^2 f(x_2)$		
x_3	$f(x_3)$	$\nabla f(x_3)$	$\nabla^2 f(x_3)$	$\nabla^3 f(x_3)$	
x_4	$f(x_4)$	$\nabla f(x_4)$	$\nabla^2 f(x_4)$	$\nabla^3 f(x_4)$	$\nabla^4 f(x_4)$

This table is also called horizontal difference table.

10

Ex Construct a backward difference table from the following data.

x	1	3	5	7	9
y	8	12	21	36	62

Find the value of $\nabla f(3)$, $\nabla^2 f(7)$, $\nabla^3 f(9)$, $\nabla^4 f(9)$.

Soln
Table :-

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
1	8				
3	12	4			
5	21	9	5		
7	36	15	6	1	
9	62	26	11	5	4

From table it clear that

(11)

$\nabla f(3) = 4$, $\nabla^2 f(7) = 6$

$\nabla^3 f(9) = 5$ $\nabla^4 f(9) = 4$

Q Construct a Backwardword differen table from the following data

x	1	2	3	4	5
y	4	6	9	12	17

Also find $\nabla f(2)$, $\nabla^2 f(3)$, $\nabla^2 f(4)$, $\nabla^3 f(5)$, $\nabla^4 f$

Sol^n

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
1	4				
2	6	2			
3	9	3	1		
4	12	3	0	-1	
5	17	5	2	2	3

From the ~~table~~ it is clear that

$\nabla f(2) = 2$

$\nabla^2 f(3) = 1$

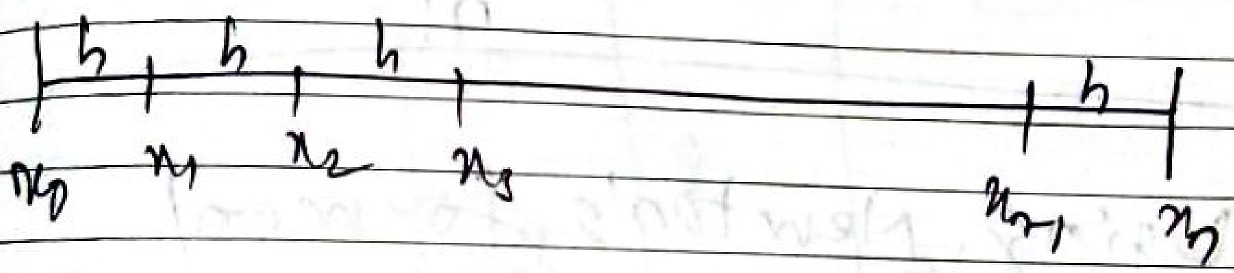
$\nabla^2 f(4) = 0$

$\nabla^3 f(5) = 2$

$\nabla^4 f(5) = 3$

Newton's forward difference interpolation formula

Let us consider $f(x)$ be a function which is defined at $(n+1)$ pts such as $x_0, x_1, x_2, \ldots x_n$ which are equispaced. i.e. step size between two pts is equal to 'h'.



Here $x_n = x_0 + nh$ $\Rightarrow$ $\boxed{h = \frac{x_n - x_0}{n}}$

For Newton's forward difference interpolation formula

taking $x = x_0 + uh$

$$\boxed{h = \frac{x - x_0}{u}}$$

(2)

Then Newton's forward difference interpolation is given by

$$f_n(x) = p(x) = f(x_0) + u\,\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \cdots + \frac{u(u-1)(u-2)\cdots[u-(n-1)]}{n!}\Delta^n f(x_0)$$

Ex Using Newton's forward difference interpolation find $f(x)$ at $x=1$, from the following table.

x	0	2	4	6
y	13	27	49	73

③

06

WK 19 (126-239)

Sol^n. Given that

x	x_0 0	x_1 2	x_2 4	x_3 6
$y=f(x)$	13 $f(x_0)$	27 $f(x_1)$	49 $f(x_2)$	73 $f(x_3)$

Forward difference Table

x	$y=f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$
0	13			
		14		
2	27		8	
		22		-6
4	49		2	
		24		
6	73			

We have to find $f(1)$.

Here $\boxed{x=1}$, $h=2$

$$u = \frac{x-x_0}{h} = \frac{1-0}{2} = \frac{1}{2}$$

(4)

9 $u = \frac{1}{2}$

10 W.K.T Newton's forward

11 difference interpolation

$$\boxed{f_3(x) = f(x_0) + u\,\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(x_0)}$$

$$\therefore f_3(x) = 13 + \frac{1}{2}(14) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\cdot 8 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\cdot(-6)$$

$$= 13 + 7 + \frac{\left(\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)}{2\times 1}(8) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\cdot\left(-\frac{3}{2}\right)}{3\times 2\times 1}\cdot(-6)$$

5

$$= 13 + 7 - \frac{1}{8}\cdot(8) + \left(\frac{3}{8}\right)(-1)$$

$$= 13 + 7 - 1 - \frac{3}{8}$$

$$= 19 - \frac{3}{8} = 18.625$$

$$\therefore \boxed{f_3(x) = 18.625}$$

Q. Find the value of $f(0.5)$ by using Newton's forward difference interpolation from the following table.

x	0	1	2	3	4
y	1	7	23	55	109

Sol^n Given that

	x_0	x_1	x_2	x_3	x_4
x	0	1	2	3	4
$y = f(x)$	1	7	23	55	109
	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$

Forward difference Table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
0	1				
		6			
1	7		10		
		16		6	
2	23		16		0
		32		6	
3	55		22		
		54			
4	109				

Here $x = 0.5$ $h = 1$

$$u = \frac{x - x_0}{h} = \frac{0.5 - 0}{1} = 0.5 = \frac{1}{2}$$

W.K.T Newton's forward interpolation formula

$$f_4(x) = f(x_0) + u\,\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0)$$

$$+\frac{u(u-1)(u-2)}{3!}\Delta^3 f(x_0) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(x_0)$$

$$\therefore f_4(x) = 1 + \frac{1}{2}\cdot 6 + \frac{(\frac{1}{2})(\frac{1}{2}-1)}{2!}\cdot 10$$

$$+\frac{(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)}{3\times2\times1}\cdot 6$$

$$+\frac{(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)(\frac{1}{2}-3)}{4\times3\times2\times1}\cdot 0$$

$$= 1+3 + \frac{\frac{1}{2}(-\frac{1}{2})}{2}\times 10$$

$$+\frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{6}\cdot 6 + 0$$

$$= 1+3+\left(-\frac{5}{4}\right)+\frac{3}{8}$$

$$= 4 - 1.25 + 0.375$$

$$= 3.125$$

$$\therefore \boxed{f_4(0.5) = 3.125}$$

Newton's Backword difference interpolation formula

Let $f(x)$ be a function which is defined at $(n+1)$ pts such as $x_0, x_1, x_2 \cdots x_n$ which are equispaced.

Now taking $x = x_n + uh$

$$\Rightarrow x - x_n = uh$$

$$\Rightarrow \boxed{u = \frac{x - x_n}{h}}$$

Then the Newton's backword difference interpolation formula is given by

9

$$f_n(u) = f(x_n) + u\,\nabla f(x_n) + \frac{u(u+1)}{2!}\nabla^2 f(x_n)$$

$$+ \frac{u(u+1)(u+2)}{3!}\nabla^3 f(x_n) + \cdots +$$

$$\cdots + \frac{u(u+1)(u+2)\cdots[u+(n-1)]}{n!}\nabla^n f(x_n)$$

Q Using Newton's Backword difference interpolation formula, find the value of f(8) from the following data

x	1	3	5	7
$y=f(x)$	12	35	73	128

Soln Given that

	x_0	x_1	x_2	x_3
x	1	3	5	7
$f(x)$	12	35	73	128
	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$

10

Backward difference table

	x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
x_0	1	12			
x_1	3	35	23		
x_2	5	73	38	15	
x_3	7	128	55	17	2

Here $\boxed{h = 2}$

we have to find $f(8)$

W.K.T

$$\boxed{u = \frac{x - x_n}{h}}$$

Here $x_n = x_3 = 7$, $x = 8$

$\therefore$ $u = \frac{8-7}{2} = \frac{1}{2}$

(11)

we know Newton's Backward difference interpolation formula

$$f_3(x) = f(x_3) + u\nabla f(x_3) + \frac{u(u+1)}{2!}\nabla^2 f(x_3) + \frac{u(u+1)(u+2)}{3!}\nabla^3 f(x_3)$$

$$\therefore f_3(8) = 128 + \frac{1}{2}(55) + \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)}{2\times1}\cdot(17) + \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right)}{3\times2\times1}\cdot(2)$$

$$= 128 + \frac{55}{2} + \frac{3}{8}\cdot(17) + \frac{15/8}{3}$$

$$= 128 + \frac{55}{2} + \frac{51}{8} + \frac{5}{8}$$

$$= 162.5$$

$$\therefore \boxed{f(8) = 162.5}$$

(12)

Q Using Newton's Backward difference interpolation formula find the value of $f(3.5)$ from the following data.

x	-2	0	2	4	6
$f(x)$	3	1	6	18	29

Soln Given that

	x_0	x_1	x_2	x_3	x_4
x	-2	0	2	4	6
$f(x)$	3	1	6	18	29
	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$

we have to find

$f(3.5)$

i.e. $\boxed{x = 3.5}$

(13)

Backward difference Table

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
-2	3				
0	1	-2			
2	6	5	7		
4	18	12	7	0	
6	29	11	-4	-8	-8

Here $\boxed{h = 2}$

w.k.T

$$\boxed{u = \frac{x - x_n}{h}}$$

Here $x_n = x_4 = 6$

$\therefore u = \frac{3.5 - 6}{2} = -5/4$

21

WK 21 (141-224)

(14)

$$\boxed{u = -5/4}$$

W.K.T

$$f_4(x) = f(x_4) + u \nabla f(x_4) + \frac{u(u+1)}{2!} \nabla^2 f(x_4)$$

$$+ \frac{u(u+1)(u+2)}{3!} \nabla^3 f(x_4)$$

$$+ \frac{u(u+1)(u+2)(u+3)}{4!} \nabla^4 f(x_4)$$

$$\therefore f_4(3.5) = 29 + \left(-\frac{5}{4}\right)(11)$$

$$+ \frac{\left(-\frac{5}{4}\right)\left(-\frac{5}{4}+1\right)}{2\times1}(-1)$$

$$+ \frac{\left(-\frac{5}{4}\right)\left(-\frac{5}{4}+1\right)\left(-\frac{5}{4}+2\right)}{3\times2\times1}(-8)$$

$$+ \frac{\left(-\frac{5}{4}\right)\left(-\frac{5}{4}+1\right)\left(-\frac{5}{4}+2\right)\left(-\frac{5}{4}+3\right)}{4\times3\times2\times1}\cdot(-8)$$

15

$$= 29 - \frac{55}{4} + \left(-\frac{5}{4}\right)\cdot\left(-\frac{1}{4}\right)\cdot\left(-\frac{1}{2}\right)$$

$$+ \left(-\frac{5}{4}\right)\left(-\frac{1}{4}\right)\left(\frac{3}{4}\right)\cdot\left(-\frac{4}{3}\right)$$

$$+ \left(-\frac{5}{4}\right)\cdot\left(-\frac{1}{4}\right)\left(\frac{3}{4}\right)\cdot\left(\frac{7}{4}\right)\cdot\left(-\frac{1}{3}\right)$$

$$= 28 - \frac{55}{4} - \frac{5}{32} - \frac{5}{16} - \frac{35}{256}$$

$$\boxed{f_4(3.5) = 13.645}$$

$$\therefore \boxed{f(3.5) = 13.645}$$

(1)

Successive over Relaxation Method [SOR Method].

Consider the system of linear eqn

$$a_{11}x + a_{12}y + a_{13}z = b_1$$

$$a_{21}x + a_{22}y + a_{23}z = b_2$$

$$a_{31}x + a_{32}y + a_{33}z = b_3$$

The above system of linear eqn can be written in matrix form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times 1}$$

$$\Rightarrow \boxed{A \cdot X = B} \quad \text{--- (1)}$$

2

Where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

In G. S. method

$$x^{(k+1)} = \frac{1}{a_{11}}\left[b_1 - a_{12}y^{(k)} - a_{13}z^{(k)}\right]$$

$$y^{(k+1)} = \frac{1}{a_{22}}\left[b_2 - a_{21}x^{(k+1)} - a_{23}z^{(k)}\right]$$

$$z^{(k+1)} = \frac{1}{a_{33}}\left[b_3 - a_{31}x^{(k+1)} - a_{32}y^{(k+1)}\right]$$

for $k = 0, 1, 2, \ldots$

3

where $x^{(0)}, y^{(0)}, z^{(0)}$ are initial approximation to chosen by us.

Similarly let 'w' is the relaxation parametre.

Also $x^{(0)}, y^{(0)}, z^{(0)}$ are initial approximations. Then

$$x^{(k+1)} = (1-w)\,x^{(k)} + \frac{w}{a_{11}}\left[b_1 - a_{12}y^{(k)} - a_{13}z^{(k)}\right]$$

$$y^{(k+1)} = (1-w)\,y^{(k)} + \frac{w}{a_{22}}\left[b_2 - a_{21}x^{(k+1)} - a_{23}z^{(k)}\right]$$

$$z^{(k+1)} = (1-w).\,z^{(k)} + \frac{w}{a_{33}}\left[b_3 - a_{31}x^{(k+1)} - a_{32}y^{(k+1)}\right]$$

for $k = 0, 1, 2, \ldots$

M T W T F S S
1 2 3 4 5 6 7
15 16 17 18 19 20 21
29 30

(4)

which is called SOR method.

where the relaxation parametre w lies between 1 and 2.

i.e $\boxed{1 < w < 2}$

Here we put the value of x, y, z successively like G.S. method.

Sometimes initial approximation are given in the questions otherwise we take it.

(5)

Note :- ① w is non-negative.

(II) when $0 < w < 1 \Rightarrow$ under relaxation

(III) when $1 < w < 2 \Rightarrow$ over relaxation

(IV) when $w = 1 \Rightarrow$ G.S. method.

Q. Solve the system of linear eq^ns by SOR method, taking relaxation parametre $w = 1.25$ and initial approximation on $x^{(0)} = y^{(0)} = z^{(0)} = 1$.

$$4x + 3y = 14$$

$$3x + 4y - z = 12$$

$$-y + 4z = 6$$

M T W T F S S M T W T F
1 2 3 4 5 6 7 8 9 10 11 12
15 16 17 18 19 20 21 22 23 24 25 26
29 30

(6)

9 Sol^n Given that

10 $4x + 3y = 14$ ——— (1)

11 $3x + 4y - z = 12$ ——— (2)

12 $-y + 4z = 6$ ——— (3)

1 Also given relaxation

2 parametre $w = 1.25$

3 initial approximation

4 are $x^{(0)} = 1,\ y^{(0)} = 1,\ z^{(0)} = 1.$

5

6 We know by SOR method

7 $$x^{(k+1)} = (1-w)x^{(k)} + \frac{w}{a_{11}}\left[b_1 - a_{12}y^{(k)} - a_{13}z^{(k)}\right]$$

$$y^{(k+1)} = (1-w)y^{(k)} + \frac{w}{a_{22}}\left[b_2 - a_{21}x^{k+1} - a_{23}z^{(k)}\right]$$

$$z^{(k+1)} = (1-w)z^{(k)} + \frac{w}{a_{33}}\left[b_3 - a_{31}x^{k+1} - a_{32}y^{k+1}\right]$$

7

where $k = 0, 1, 2, \ldots$

Here

$$x^{(k+1)} = (1-w)x^{(k)} + \frac{w}{4}\left[14 - 3y^{(k)}\right]$$

$$y^{(k+1)} = (1-w)y^{(k)} + \frac{w}{4}\left[12 - 3x^{(k+1)} + z^{(k)}\right]$$

$$z^{(k+1)} = (1-w)z^{(k)} + \frac{w}{4}\left[6 + y^{(k+1)}\right]$$

for $k = 0, 1, 2, \ldots$

Taking $k=0$, we get 1st approximation

$$x^{(1)} = (1-1.25)x^{(0)} + \frac{1.25}{4}\left[14 - 3y^{(0)}\right]$$

$$= (-0.25)\cdot(1) + 0.3125\left[14 - 3\right]$$

$$\Rightarrow \boxed{x^{(1)} = 3.1875}$$

$$y^{(1)} = (1-1.25)y^{(0)} + \frac{1.25}{4}\left[12 - 3x^{(1)} + z^{(0)}\right]$$

$$= (-0.25)\cdot(1) + 0.3125\left[12 - 3(3.1875) + 1\right]$$

M T W T F S S M T W T F
1 2 3 4 5 6 7 8 9 10 11 12
15 16 17 18 19 20 21 22 23 24 25 26
29 30

8

$= (-0.25) + 0.3125[3.4375]$

$\Rightarrow \boxed{y^{(1)} = -0.824}$

$z^{(1)} = (1-1.25)z^{(0)} + \frac{1.25}{4}[6 + y^{(1)}]$

$= (-0.25)(1) + 0.3125[6 + (-0.824)]$

$\Rightarrow \boxed{z^{(1)} = 1.3675}$

Taking k=1, we get 2nd approx^n

$x^{(2)} = (-0.25).(3.1875) + 0.3125[14 - 3(-0.824)]$

$\Rightarrow \boxed{x^{(2)} = 4.351}$

$y^{(2)} = (-0.25).(-0.824) + 0.3125[12 - 3(4.351) + 1.3675]$

$\Rightarrow \boxed{y^{(2)} = 0.304}$

9

$$z^{(2)} = (-0.25)(1.3675) + 0.3125[6 + 0.304]$$

$$\Rightarrow \boxed{z^{(2)} = 1.628}$$

Taking k = 2, we get 3rd approxm.

$$x^{(3)} = (-0.25)(4.351) + 0.3125[14 - 3(0.304)]$$

$$\Rightarrow \boxed{x^{(3)} = 3}$$

$$y^{(3)} = (-0.25)(0.304) + 0.3125[12 - 3(3) + 1.628]$$

$$\Rightarrow \boxed{y^{(3)} = 1.37}$$

$$z^{(3)} = (-0.25)(1.628) + (0.3125)[6 + 1.37]$$

$$\Rightarrow \boxed{z^{(3)} = 1.9}$$

10

9 Taking $k=3$, we get 4th approx^n.

10 $$x^{(4)} = (-0.25).3 + 0.3125[14 - 3(1.37)]$$

11

$$\Rightarrow \boxed{x^4 = 2.34}$$

1 $$y^{(4)} = (-0.25)(1.37) + 0.3125[12 - 3(2.34) + 1.9]$$

2

3 $$\Rightarrow \boxed{y^{(4)} = 1.81}$$

4

5 $$z^{(4)} = (-0.25)(1.9) + 0.3125[6 + 1.81]$$

6

7 $$\Rightarrow \boxed{z^{(4)} = 1.97}$$

Taking $k=4$, we get 5th approx^n.

$$x^{(5)} = (-0.25).(2.34) + 0.3125[14 - 3(1.81)]$$

(II)

$$\Rightarrow \boxed{x^{(5)} = 2.1}$$

$$y^{(5)} = (-0.25)(1.81) + 0.3125[12 - 3(2.1) + 1.97]$$

$$\Rightarrow \boxed{y^{(5)} = 1.94}$$

$$z^{(5)} = (-0.25)(1.97) + 0.3125[6 + 1.94]$$

$$\Rightarrow \boxed{z^{(5)} = 1.98}$$

Taking $k=5$, we get 6th approx^n

$$x^{(6)} = (-0.25)\cdot(2.1) + 0.3125[14 - 3(1.94)]$$

$$\Rightarrow \boxed{x^{(6)} = 2.03}$$

M T W T F S S M T W T
1 2 3 4 5 6 7 8 9 10 11
15 16 17 18 19 20 21 22 23 24 25

12

$$y^{(6)} = (-0.25)(1.94) + 0.3125[12 - 3(2.03) + 1.98]$$

$$\Rightarrow \boxed{y^{(6)} = 1.98}$$

$$z^{(6)} = (-0.25)(1.98) + 0.3125[6 + 1.98]$$

$$\Rightarrow \boxed{z^{(6)} = 1.99}$$

Taking $K = 6$, we get 7th approx^n.

$$x^{(7)} = (-0.25).(2.03) + (0.3125)[14 - 3(1.98)$$

$$\Rightarrow \boxed{x^{(7)} = 2.01}$$

$$y^{(7)} = (-0.25)(1.98) + 0.3125[12 - 3(2.01) + 1.99]$$

$$\Rightarrow \boxed{y^{(7)} = 1.99}$$

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$$z^{(7)} = (-0.25)(1.99) + 0.3125\left[6 + 1.99\right]$$

$$\boxed{z^{(7)} = 1.99}$$

Taking $k=7$, we get 8th approxm

$$x^{(8)} = (-0.25)(2.01) + 0.3125\left[14 - 3(1.99)\right]$$

$$\boxed{x^{(8)} = 2.01} \simeq 2$$

$$y^{(8)} = (-0.25)\cdot(1.99) + 0.3125\left[12 - 3(2.01) + 1.99\right]$$

$$\boxed{y^{(8)} = 1.99} \simeq 2$$

$$z^{(8)} = (-0.25)\cdot(1.99) + 0.3125\left[6 + 1.99\right]$$

$$\boxed{z^{(8)} = 1.99} \simeq 2$$

$\boxed{x = 2}$, $\boxed{y = 2}$, $\boxed{z = 2}$

M T W T F S S M T W T F S
1 2 3 4 5 6 7 8 9 10 11 12 13
15 16 17 18 19 20 21 22 23 24 25 26 27
29 30

14

Q Solve the system of linear eqn by using method of SOR. Where relaxation parrameter $w = 1.25$ and initial approximations ~~are~~

~~$x_1^{(0)} = 1$, $x_2^{(0)} = 1$, $x_3^{(0)} = 1$~~

$$x_1^{(0)} = 1, \quad x_2^{(0)} = 1, \quad x_3^{(0)} = 1$$

$$4x_1 + 3x_2 = 24$$

$$3x_1 + 4x_2 - x_3 = 30$$

$$-x_2 + 4x_3 = -24$$

Soln Given that

$$4x_1 + 3x_2 = 24 \quad \text{——— (1)}$$

$$3x_1 + 4x_2 - x_3 = 30 \quad \text{——— (2)}$$

$$-x_2 + 4x_3 = -24 \quad \text{——— (3)}$$

15

Also given relaxation paramptre $w = 1.25$.

Initial approximations are

$x_1^{(0)} = 1, \; x_2^{(0)} = 1, \; x_3^{(0)} = 1$.

we know by SOR method

$$x_1^{(k+1)} = (1-w)\,x_1^{(k)} + \frac{w}{a_{11}}\left[b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}\right]$$

$$x_2^{(k+1)} = (1-w)\,x_2^{(k)} + \frac{w}{a_{22}}\left[b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}\right]$$

$$x_3^{(k+1)} = (1-w)\,x_3^{(k)} + \frac{w}{a_{33}}\left[b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}\right]$$

for $k = 0, 1, 2, \ldots$

Here

$$x_1^{(k+1)} = (1-w)\,x_1^{(k)} + \frac{w}{4}\left[24 - 3x_2^{(k)}\right]$$

$$x_2^{(k+1)} = (1-w)\,x_2^{(k)} + \frac{w}{4}\left[30 - 3x_1^{(k+1)} + x_3^{(k)}\right]$$

M T W T F S S M T W T
1 2 3 4 5 6 7 8 9 10 11
15 16 17 18 19 20 21 22 23 24 25
29 30

16

9 $x_3^{(k+1)} = (1-w)x_3^{(k)} + \frac{w}{4}\left[-24 + x_2^{(k+1)}\right]$

10

11 for $k = 0, 1, 2, \ldots$

Taking $K=0$ we get 1st approximation

1 $x_1^{(1)} = (1-1.25).1 + \frac{1.25}{4}\left[24 - 3.(1)\right]$

2

$= -0.25 + 0.3125[21]$

3

$\boxed{x_1^{(1)} = 6.3125}$

4

5 $x_2^{(1)} = (1-1.25).1 + \frac{1.25}{4}\left[30 - 3(6.3125) + 1\right]$

6

7 $= -0.25 + 0.3125\left[31 - 18.9375\right]$

$\boxed{x_2^{(1)} = 3.52}$

$x_3^{(1)} = (1-1.25).1 + \frac{1.25}{4}\left[-24 + 3.52\right]$

$= -0.25 + 0.3125\left[-24 + 3.52\right]$

$\boxed{x_3^{(1)} = -6.65}$

(17)

Taking K=1, we get 2nd approximation

$$x_1^{(2)} = (-0.25)(6.3125) + 0.3125[24 - 3(3.52)]$$

$$= (-0.25)(6.3125) + 4.2$$

$$= -1.578 + 4.2$$

$$\boxed{x_1^{(2)} = 2.62}$$

$$x_2^{(2)} = (-0.25)(3.52) + 0.3125[30 - 3(2.62) + (-6.65)]$$

$$= (-0.25)(3.52) + 4.841$$

$$\boxed{x_2^{(2)} = 3.96}$$

$$x_3^{(2)} = (-0.25)(-6.65) + 0.3125[-24 + 3.96]$$

$$\boxed{x_3^{(2)} = -4.6}$$

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Taking K=2 we get 3rd approx^m

$$x_1^{(3)} = (-0.25)(2.62) + (0.3125)[24 - 3(3.96)]$$

$$\boxed{x_1^{(3)} = 3.1325}$$

$$x_2^{(3)} = (-0.25)(3.96) + (0.3125)[30 - 3(3.1325) + (-4.6)]$$

$$\boxed{x_2^{(3)} = 4.01}$$

$$x_3^{(3)} = (-0.25)(-4.6) + (0.3125)[-24 + 4.01]$$

$$\boxed{x_3^{(3)} = -5.1}$$

Taking K=3 we get 4th approx^m

$$x_1^{(4)} = (-0.25)(3.1325) + (0.3125)[24 - 3(4.01)]$$

$$\boxed{x_1^{(4)} = 2.96}$$

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$$x_2^{(4)} = (-0.25)(4.01) + (0.3125)\left[30 - 3(2.96) + (-5.1)\right]$$

$$\boxed{x_2^{(4)} = 4.004}$$

$$x_3^{(4)} = (-0.25)(-5.1) + (0.3125)\left[-24 + 4.004\right]$$

$$\boxed{x_3^{(4)} = -4.97}$$

Taking K=4 we get 5th approx^m

$$x_1^{(5)} = (-0.25)(2.96) + (0.3125)\left[24 - 3(4.004)\right]$$

$$\boxed{x_1^{(5)} = 3.01}$$

$$x_2^{(5)} = (-0.25)(4.004) + 0.3125\left[30 - 3(3.01) + (-4.97)\right]$$

$$\boxed{x_2^{(5)} = 3.999}$$

20

$$x_3^{(5)} = (-0.25)(-4.97) + 0.3125\left[-24 + 3.999\right]$$

$$\boxed{x_3^{(5)} = -5.01}$$

Taking k=5, we get 6th approximation

$$x_1^{(6)} = (-0.25)(3.01) + (0.3125)\left[24 - 3(3.999)\right]$$

$$\boxed{x_1^{(6)} = 2.99}$$

$$x_2^{(6)} = (-0.25)\cdot(3.999) + (0.3125)\left[30 - 3(2.99) + (-5.01)\right]$$

$$\boxed{x_2^{(6)} = 4.01}$$

$$x_3^{(6)} = (-0.25)(-5.01) + (0.3125)\left[-24 + 4.01\right]$$

$$\boxed{x_3^{(6)} = -4.99}$$

24

Taking $K=6$ we get 7th approx^n.

$$x_1^{(7)} = (-0.25)(2.99) + (0.3125)[24 - 3(4.01)]$$

$$\boxed{x_1^{(7)} = 2.99} \simeq 3$$

$$x_2^{(7)} = (-0.25)(4.01) + (0.3125)[30 - 3(2.99) + (-4.99)]$$

$$\boxed{x_2^{(7)} = 4.01} \simeq 4$$

$$x_3^{(7)} = (-0.25)(-4.99) + (0.3125)[-24 + 4.01]$$

$$\boxed{x_3^{(7)} = -4.99} \simeq -5$$

Now we observe that 6th and 7th approx^{n}s are same.

We stop the procedure.

$\therefore$ The final result is $\boxed{x_1 = 3}$, $\boxed{x_2 = 4}$, $\boxed{x_3 = -5}$

M T W T F S S M T W T F
1 2 3 4 5 6 7 8 9 10 11 12
15 16 17 18 19 20 21 22 23 24 25 26
29 30

22

HW

Q Solve the following system of linear eqⁿ by using SOR method where the relaxation parametre $w = 1.25$ and initial approximations are

$x_1^{(0)} = 0$, $x_2^{(0)} = 0$, $x_3^{(0)} = 0$.

$$3x_1 - x_2 + x_3 = -1$$

$$-x_1 + 3x_2 - x_3 = 7$$

$$x_1 - x_2 + 3x_3 = -7$$

Ans $\boxed{x_1 = 1}$, $\boxed{x_2 = 2}$, $\boxed{x_3 = -2}$