PARAMETRIC HYPOTHESIS TESTS

UNIT-II

"If you haven't measured it you don't know what you are talking about."

-William Thompson, Lord Kelvin

Learning Objectives

- Hypothesis Testing
- Parameter & Statistic and its importance
- Large sample tests with applications
- Small sample tests with applications

Data and variables

DATA: the answers to questions or measurements from the experiment

VARIABLE = measurement which varies between subjects e.g. height or gender

One variable per column

	A	B	C	D
1	Subject ID	Gender	Year of study	Height
2	1	Male	1	170
3	2	Female	2	160
4	3	Female	3	165
5	4	Male	PG	175
6	5	Female	3	168

One row per subject

Variable types

Data Variables

- **Scale**
  Measurements/ Numerical/ count data
- **Categorical:**
  appear as categories
  Tick boxes on questionnaires

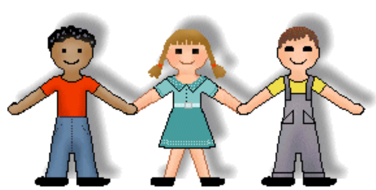
Variable types…

Variables

Scale

Continuous
Measurements
takes any value

Discrete:
Counts/ integers



Categorical

Ordinal:
obvious order

Nominal:
no meaningful
order

What data types relate to following questions?

- Q1: What is your favourite subject?

Stats	English	Science	Art	Social

- Q2: Gender:

Male	Female

- Q3: I consider myself to be good at computers:

Strongly Disagree	Disagree	Not Sure	Agree	Strongly Agree

- Q4: Score in a recent mock grade in CS exam:

Score between 0% and 100%

What data types relate to following questions?

- Q1: What is your favourite subject?

Nominal

Stats	English	Science	Art	Social

- Q2: Gender:

Male	Female

Binary/ Nominal

- Q3: I consider myself to be good at computers:

Strongly Disagree	Disagree	Not Sure	Agree	Strongly Agree

Ordinal

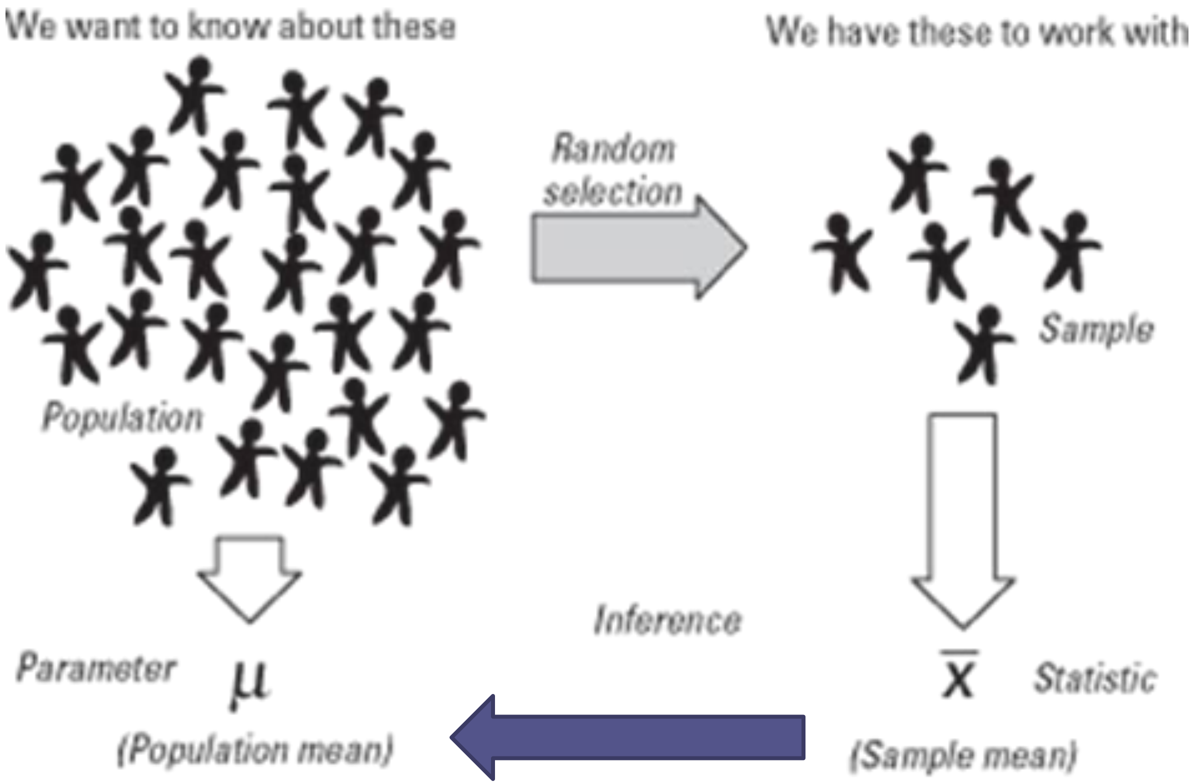
- Q4: Score in a recent mock grade CS exam:

Score between 0% and 100%

Scale

Populations and samples

- Taking a sample from a population



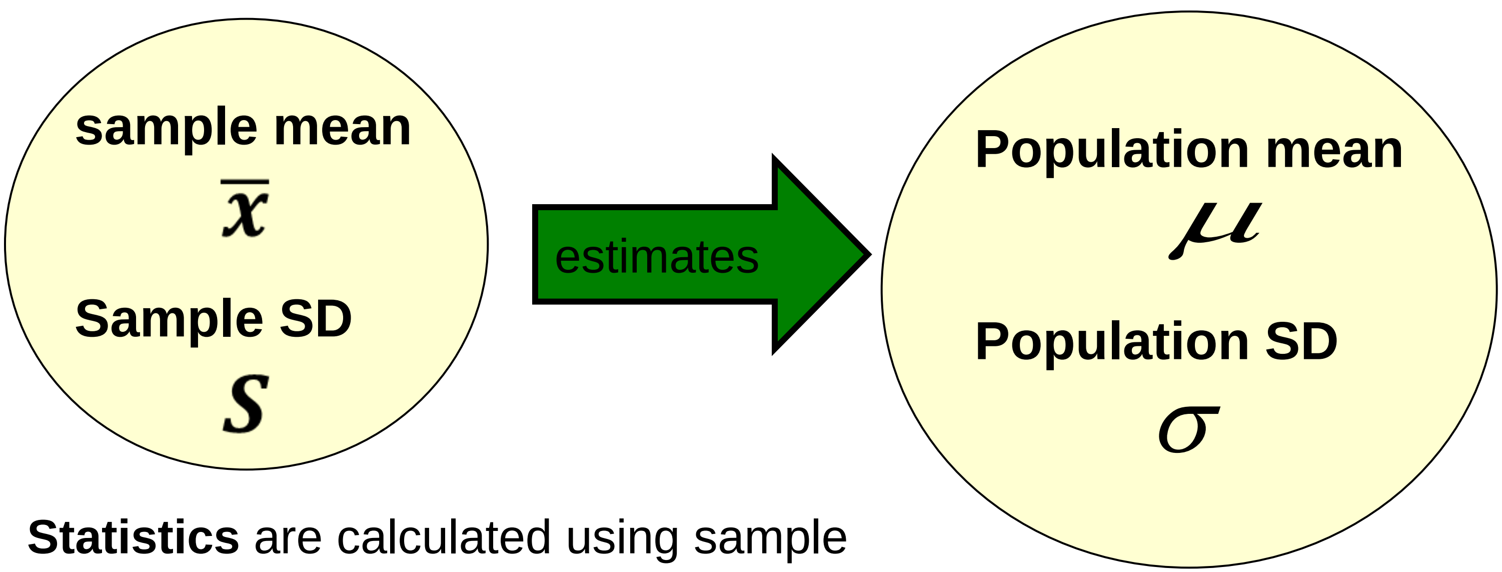
Sample data ‘represents’ the whole population

Point estimation

Sample data is used to estimate parameters of a population

Statistic are the characteristics of sample data.

Parameters are the characteristics of population data



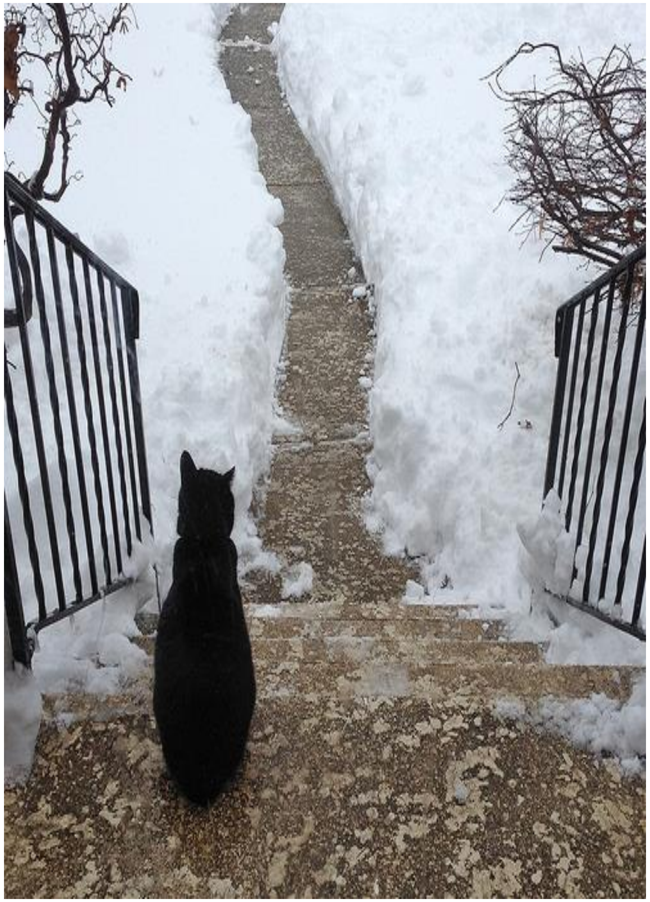
Statistics are calculated using sample data

Hypothesis Testing

- To tell whether our data supports or rejects our ideas, we use statistical hypothesis testing.
- The problem is that we often get data that <u>seem</u> to support our ideas. The literature is full of papers that accept a pet idea uncritically. Statistical testing keeps scientists honest.
- If you read a paper that suggests some alternative hypothesis should be accepted, but there is no statistical test, don't believe it.

Hypothesis testing

- An **objective** method of making decisions or **inferences** from sample data (evidence)
- Sample data used to choose between two choices i.e. **hypotheses** or statements about a population
- We typically do this by comparing what we have observed to what we expected if one of the statements (**Null Hypothesis**) was true



Hypothesis testing Framework
What the text books might say!

Always two hypotheses:

H_o: Null Hypothesis

- What we assume is true to begin with
- Typically that there is **no** difference/effect/relationship etc.

H_A: Research (Alternative) Hypothesis

- What we aim to gather evidence of
- Typically that there **is** a difference/effect/relationship etc.

Could try explaining things in the context of "The Court Case"?

- Members of a jury have to decide whether a person is guilty or innocent based on evidence

Null: The person is innocent
Alternative: The person is not innocent (i.e. guilty)

- The null can only be rejected if there is enough evidence to doubt it
- i.e. the jury can only convict if there is beyond reasonable doubt for the null of innocence
- They do not know whether the person is really guilty or innocent so they may make a mistake

Think of…

Your Decision	The TRUTH	
	God Exists	God Doesn't Exist
Reject God	*BIG MISTAKE*	*Correct*
Accept God	*Correct— Big Pay Off*	*MINOR MISTAKE*

Type I and Type II Error

Your Statistical Decision	True state of null hypothesis	
	H_0 True (example: the drug doesn't work)	H_0 False (example: the drug works)
Reject H_0 (ex: you conclude that the drug works)	*Type I error (α)*	Correct
Do not reject H_0 (ex: you conclude that there is insufficient evidence that the drug works)	Correct	*Type II Error (β)*

Types of Errors

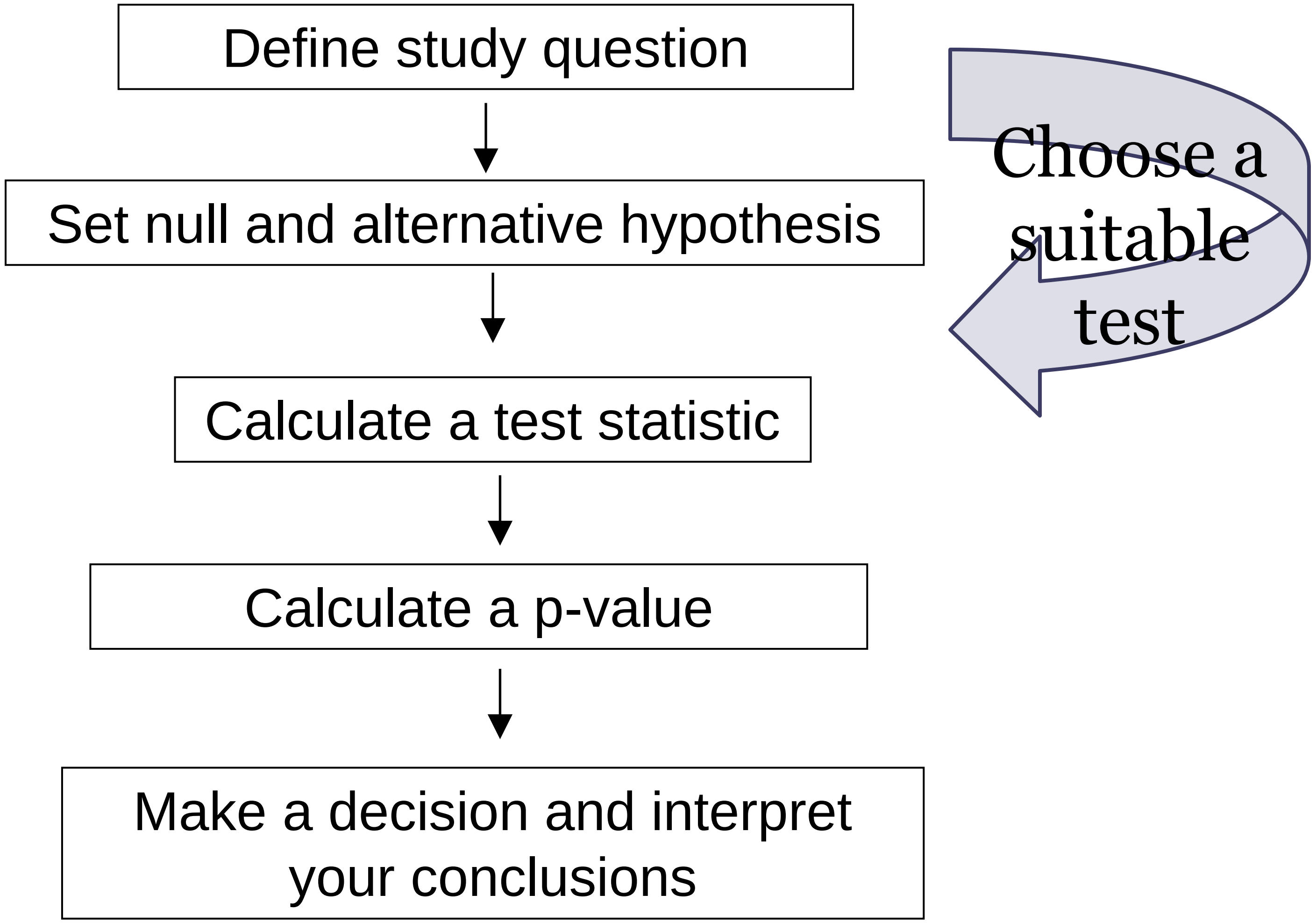
Controlled via sample size (=1-Power of test)

Typically restrict to a 5% Risk = level of significance

	Study reports **NO** difference (Do not reject H_0)	Study reports **IS** a difference (Reject H_0)
H_0 is true Difference Does **NOT** exist in population	✔	X Type I Error
H_A is true Difference **DOES** exist in population	X Type II Error	✔

Prob. of this = Power of test

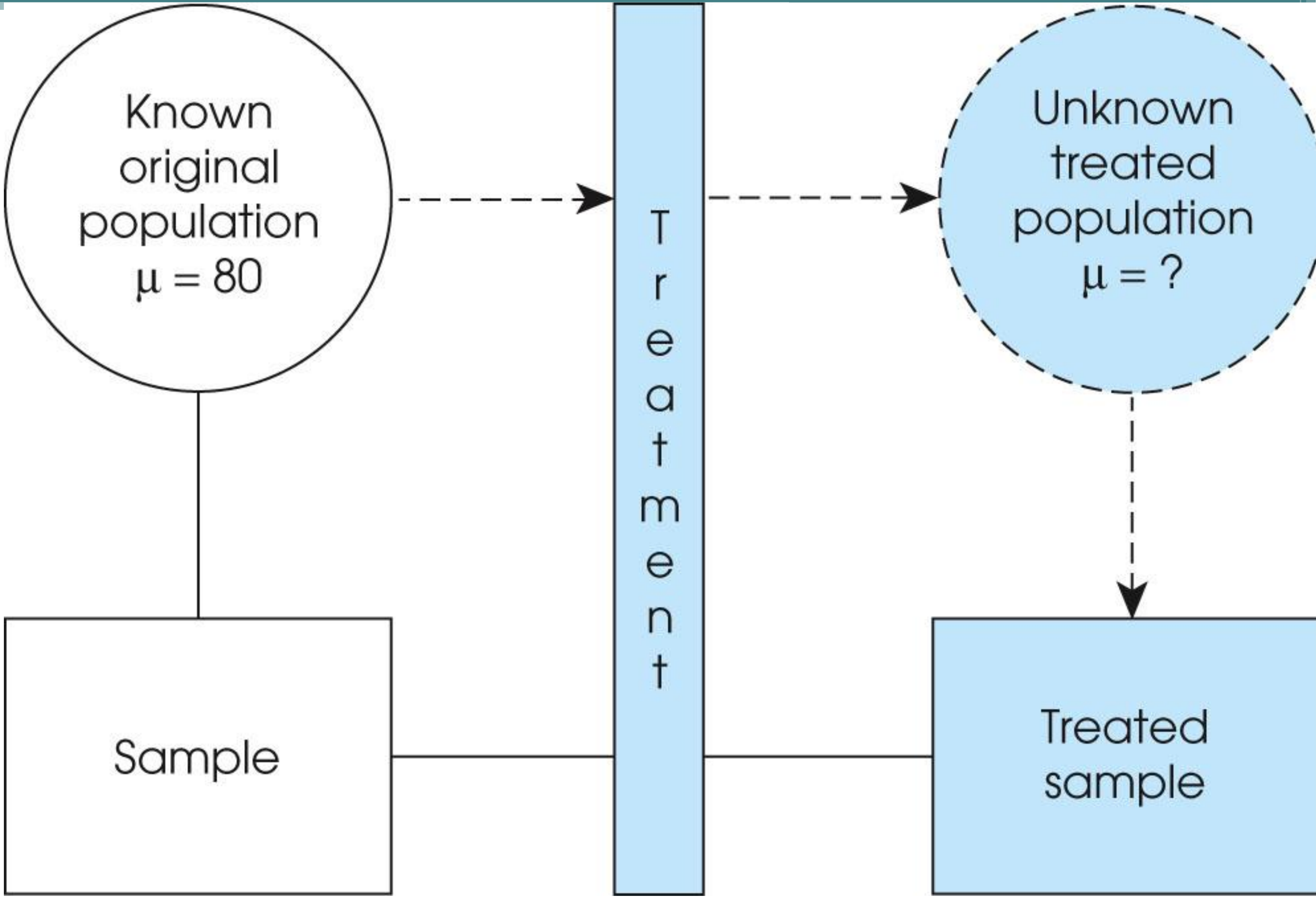
Steps to undertaking a Hypothesis test



Example: Titanic

- The ship Titanic sank in 1912 with the loss of most of its passengers
- 809 of the 1,309 passengers and crew died

  = 61.8%
- **Research question:** Did class (of travel) affect survival?


Known
original
population
μ = 80
Treatment
Unknown
treated
population
μ = ?
Sample
Treated
sample

Hypothesis Tests

A **hypothesis test** is a process that uses sample statistics to test a claim about the value of a population parameter.

If a manufacturer of rechargeable batteries claims that the batteries they produce are good for an average of at least 1,000 charges, a sample would be taken to test this claim.

A verbal statement, or claim, about a population parameter is called a **statistical hypothesis**.

To test the average of 1000 hours, a pair of hypotheses are stated – one that represents the claim and the other, its complement. When one of these hypotheses is false, the other must be true.

Stating a Hypothesis

“H subzero” or “H naught”

A **null hypothesis H_0** is a statistical hypothesis that contains a statement of equality such as $\leq$, $=$, or $\geq$.

“H sub-a”

A **alternative hypothesis H_a** is the complement of the null hypothesis. It is a statement that must be true if H_0 is false and contains a statement of inequality such as $>$, $\neq$, or $<$.

To write the null and alternative hypotheses, translate the claim made about the population parameter from a verbal statement to a mathematical statement.

Stating a Hypothesis

Example:
Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

A manufacturer claims that its rechargeable batteries have an average life of at least 1,000 charges.

$\mu \geq 1000$

Condition of equality

H_0: $\mu \geq 1000$ (Claim)
H_a: $\mu < 1000$

Complement of the null hypothesis

Types of Errors

No matter which hypothesis represents the claim, always begin the hypothesis test **assuming that the null hypothesis is true**.

At the end of the test, one of two decisions will be made:

1. reject the null hypothesis, or
2. fail to reject the null hypothesis.

A **type I error** occurs if the null hypothesis is rejected when it is true.

A **type II error** occurs if the null hypothesis is not rejected when it is false.

Types of Errors

	Actual Truth of H_0	
Decision	**H_0 is true**	**H_0 is false**
Do not reject H_0	Correct Decision	**Type II Error**
Reject H_0	**Type I Error**	Correct Decision

Types of Errors

Example:
Statesville college claims that 94% of their graduates find employment within six months of graduation. What will a type I or type II error be?

H_0: $p = 0.94$ (Claim)
H_a: $p \neq 0.94$

A type I error is rejecting the null when it is true.

The population proportion is actually 0.94, but is rejected. (We believe it is not 0.94.)

A type II error is failing to reject the null when it is false.

The population proportion is not 0.94, but is not rejected. (We believe it is 0.94.)

Level of Significance

In a hypothesis test, the **level of significance** is your maximum allowable probability of making a type I error. It is denoted by α, the lowercase Greek letter alpha.

Hypothesis tests are based on α.

The probability of making a type II error is denoted by β, the lowercase Greek letter beta.

By setting the level of significance at a small value, you are saying that you want the probability of rejecting a true null hypothesis to be small.

Commonly used levels of significance:

$\alpha = 0.10$ $\alpha = 0.05$ $\alpha = 0.01$

Statistical Tests

After stating the null and alternative hypotheses and specifying the level of significance, a random sample is taken from the population and sample statistics are calculated.

The statistic that is compared with the parameter in the null hypothesis is called the **test statistic**.

Population parameter	Test statistic	Standardized test statistic
μ	$\overline{x}$	z $(n \geq 30)$ t $(n < 30)$
p	$\hat{p}$	z
σ^2	s^2	X^2

P-values

If the null hypothesis is true, a ***P*-value** (or **probability value**) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data.

The *P*-value of a hypothesis test depends on the nature of the test.

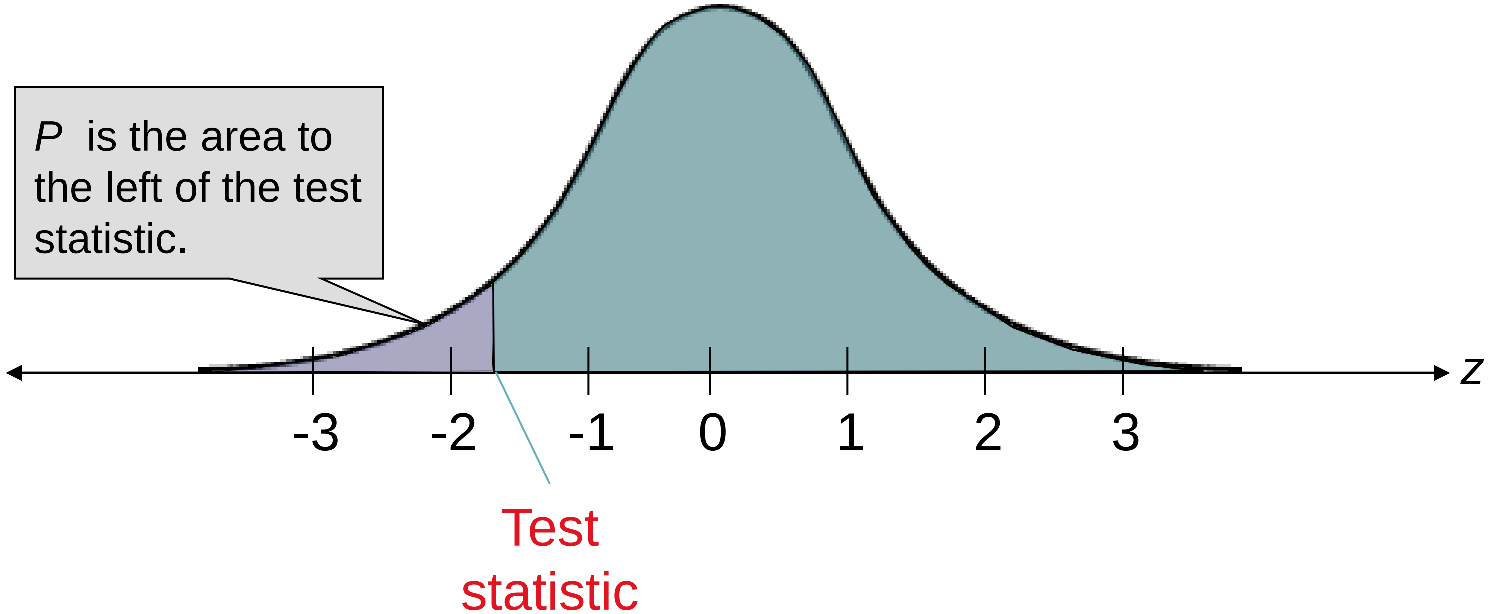
There are three types of hypothesis tests – a left-, right-, or two-tailed test. The type of test depends on the region of the sampling distribution that favors a rejection of H_0. This region is indicated by the alternative hypothesis.

Left-tailed Test

1. If the alternative hypothesis contains the less-than inequality symbol (<), the hypothesis test is a left-tailed test.

H_0: $\mu \geq k$

H_a: $\mu < k$

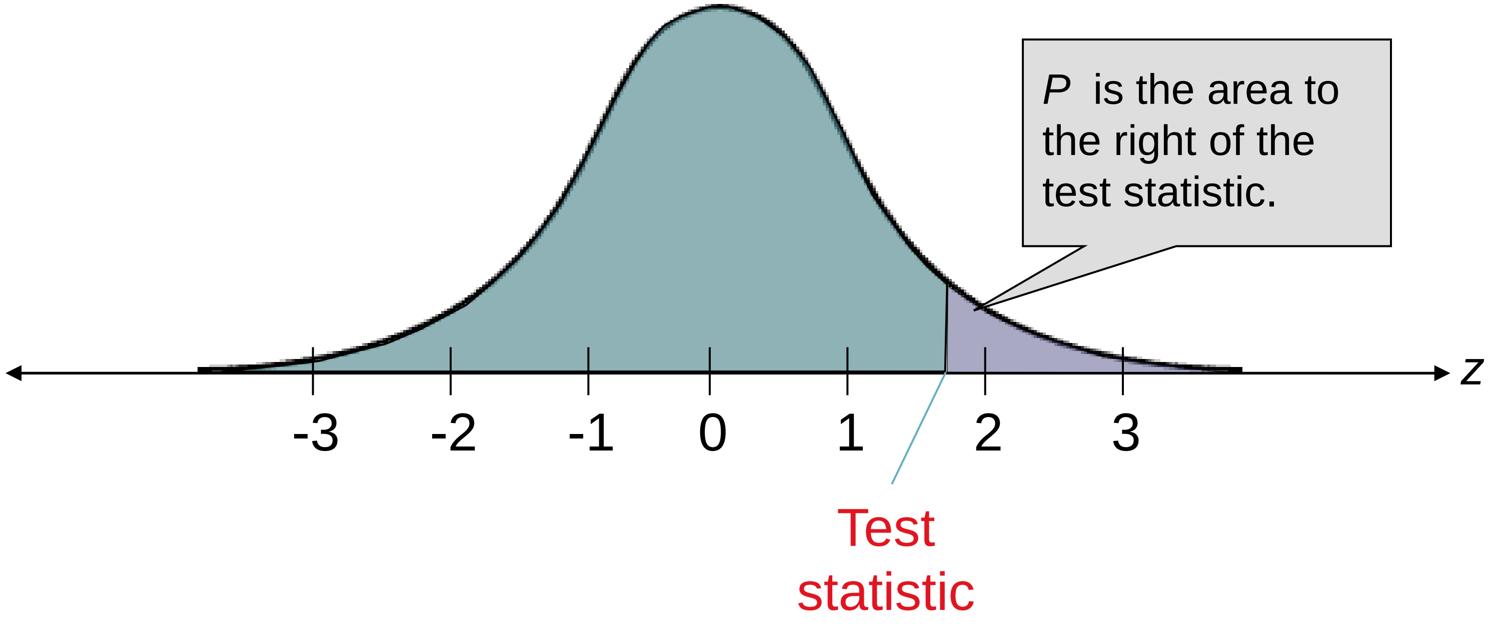


Right-tailed Test

2. If the alternative hypothesis contains the greater-than symbol (>), the hypothesis test is a right-tailed test.

H_0: $\mu \leq k$

H_a: $\mu > k$

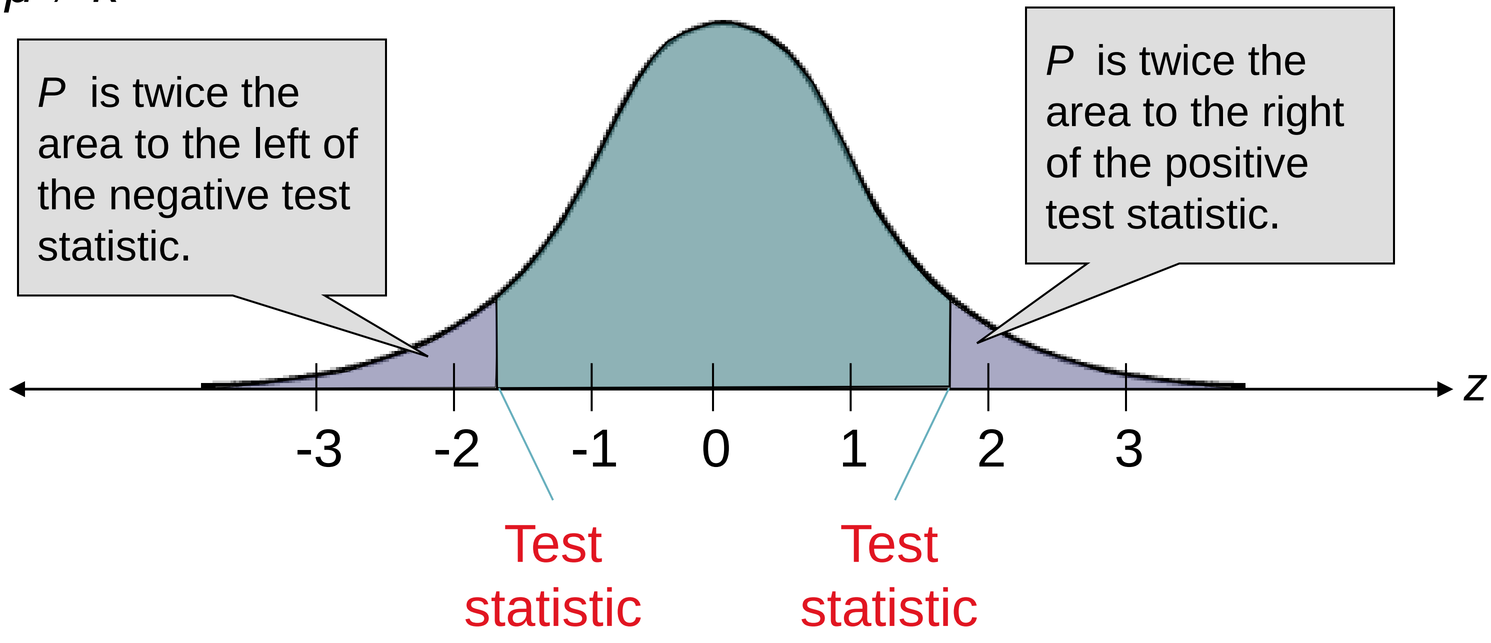


Two-tailed Test

3. If the alternative hypothesis contains the not-equal-to symbol ($\neq$), the hypothesis test is a **two-tailed test**. In a two-tailed test, each tail has an area of $\frac{1}{2}P$.

H_0: $\mu = k$

H_a: $\mu \neq k$



Identifying Types of Tests

Example:
For each claim, state H_0 and H_a. Then determine whether the hypothesis test is a left-tailed, right-tailed, or two-tailed test.

a.) A cigarette manufacturer claims that less than one-eighth of the US adult population smokes cigarettes.

H_0: $p \geq 0.125$

H_a: $p < 0.125$ (Claim)

Left-tailed test

b.) A local telephone company claims that the average length of a phone call is 8 minutes.

H_0: $\mu = 8$ (Claim)

H_a: $\mu \neq 8$

Two-tailed test

Making a Decision

Decision Rule Based on *P*-value

To use a *P*-value to make a conclusion in a hypothesis test, compare the *P*-value with α.

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

	Claim	
Decision	**Claim is H_0**	**Claim is H_a**
Reject H_0	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Do not reject H_0	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

Interpreting a Decision

Example:
You perform a hypothesis test for the following claim. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

H_0: (Claim) A cigarette manufacturer claims that less than one-eighth of the adult population smokes cigarettes.

If H_0 is rejected, you should conclude "there is sufficient evidence to indicate that the manufacturer's claim is false."

If you fail to reject H_0, you should conclude "there is *not* sufficient evidence to indicate that the manufacturer's claim is false."

Steps for Hypothesis Testing

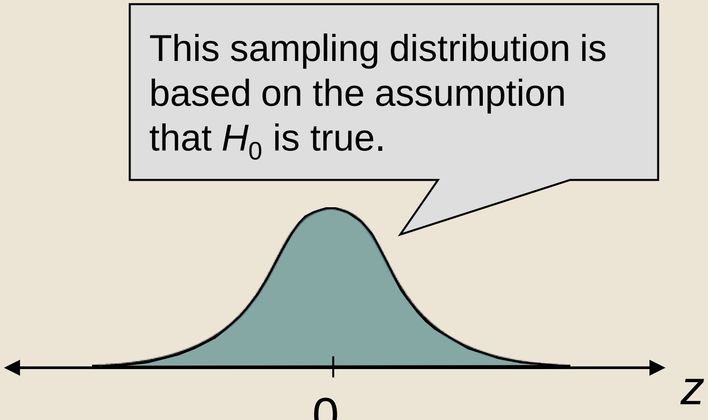
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.

   H_0: ? H_a: ?

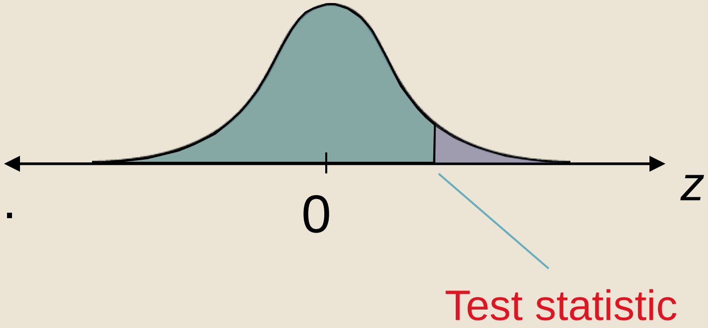
2. Specify the level of significance.

   α = ?

3. Determine the standardized sampling distribution and draw its graph.



4. Calculate the test statistic and its standardized value. Add it to your sketch.



Continued.

Steps for Hypothesis Testing

5. Find the P-value.

6. Use the following decision rule.

Is the P-value less than or equal to the level of significance?

No → Fail to reject H_0.

Yes ↓ Reject H_0.

7. Write a statement to interpret the decision in the context of the original claim.

These steps apply to left-tailed, right-tailed, and two-tailed tests.

Using *P*-values to Make a Decision

Decision Rule Based on *P*-value
To use a *P*-value to make a conclusion in a hypothesis test, compare the *P*-value with α.

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

Recall that when the sample size is at least 30, the sampling distribution for the sample mean is normal.

Using *P*-values to Make a Decision

Example:

The *P*-value for a hypothesis test is $P = 0.0256$. What is your decision if the level of significance is

a.) 0.05,

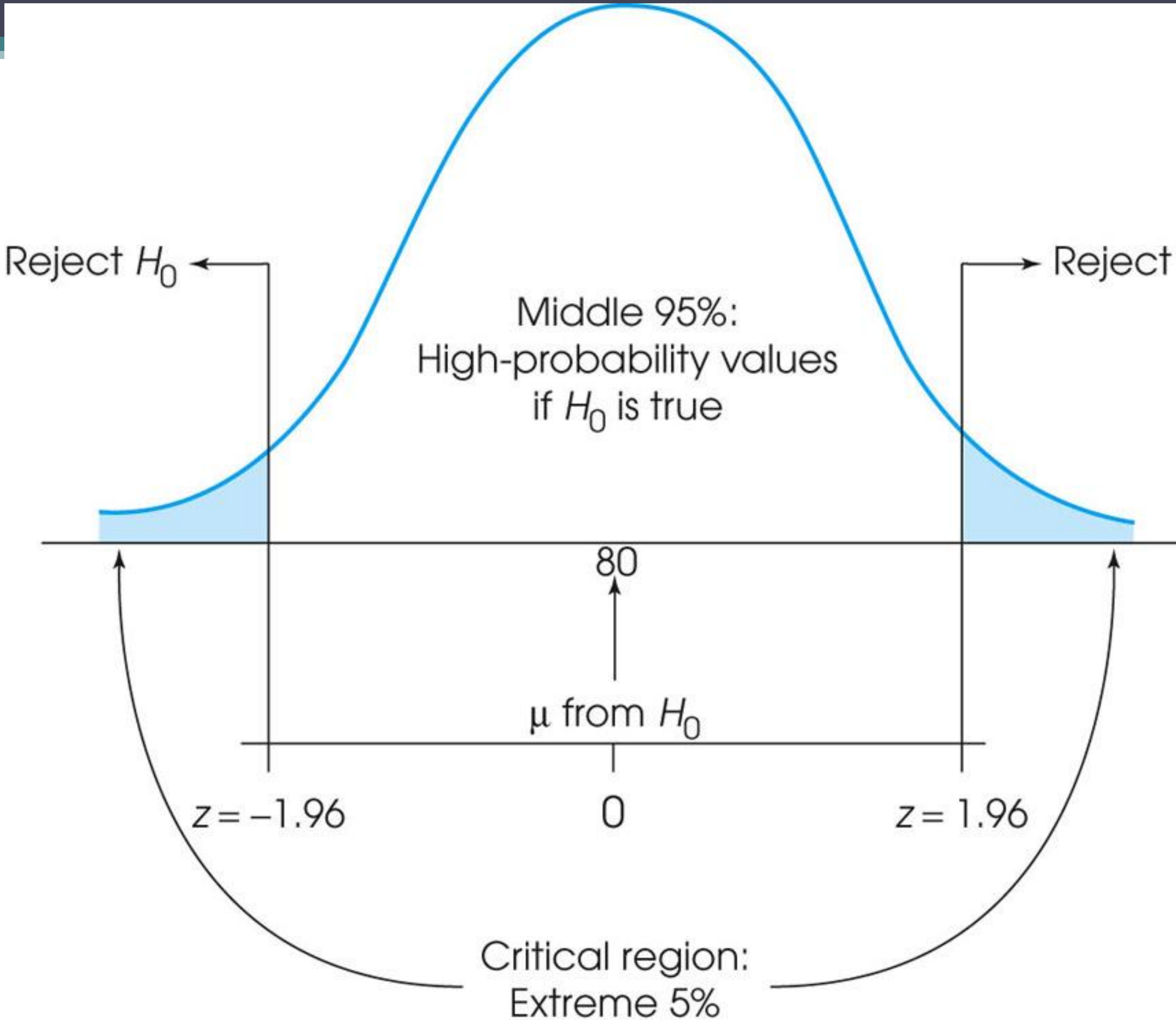
b.) 0.01?

a.) Because 0.0256 is < 0.05, you should reject the null hypothesis.

b.) Because 0.0256 is > 0.01, you should fail to reject the null hypothesis.

Z-table values

		Z-Value	
		Two-sided test	One-sided test
		$Z_{1-\alpha/2}$	$Z_{1-\alpha}$
Significance level	0.01	2.576	2.326
	0.05	1.960	1.645
	0.10	1.645	1.282
		$Z_{1-\beta}$	
Power	0.80	0.842	
	0.90	1.282	
	0.95	1.645	
	0.99	2.326	


Reject H_0
Reject H_0
Middle 95%:
High-probability values
if H_0 is true
80
μ from H_0
$z = -1.96$
0
$z = 1.96$
Critical region:
Extreme 5%

Using *Z-table* for a z-Test

The **z-test for the mean** is a statistical test for a population mean. The *z*-test can be used when the population is normal and σ is known, or for any population when the sample size *n* is at least 30.

The **test statistic** is the sample mean $\bar{x}$ and the **standardized test statistic** is *z*.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \qquad \frac{\sigma}{\sqrt{n}} = \text{standard error} = \sigma_{\bar{x}}$$

When $n \geq 30$, the sample standard deviation *s* can be substituted for σ.

Using *P*-values for a *z*-Test for a Mean *μ*

Using *P*-values for a z-Test

In Words	*In Symbols*
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Determine the standardized test statistic.	$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$
4. Find the area that corresponds to *z*.	

Continued.

Large sample tests (Z-tests)

Z-test for one sample

Z-test for two samples

Z-test: proportion test for one sample

Z-test: proportion test for two samples

One sample Z-test

Example:

A manufacturer claims that its rechargeable batteries are good for an average of more than 1,000 charges. A random sample of 100 batteries has a mean life of 1002 charges and a standard deviation of 14. Is there enough evidence to support this claim at $\alpha = 0.01$?

H_0: $\mu \leq 1000$

H_a: $\mu > 1000$ (Claim)

The level of significance is $\alpha = 0.01$.

The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{1002 - 1000}{14 / \sqrt{100}}$$

$$\approx 1.43$$

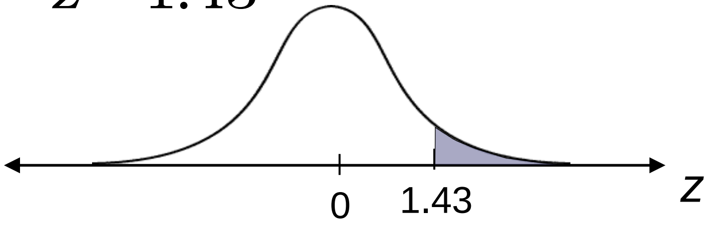
Continued.

Hypothesis Testing (Z-test)

Example continued:
A manufacturer claims that its rechargeable batteries are good for an average of more than 1,000 charges. A random sample of 100 batteries has a mean life of 1002 charges and a standard deviation of 14. Is there enough evidence to support this claim at $\alpha = 0.01$?

H_0: $\mu \leq 1000$ Ha: $\mu > 1000$ (Claim)

$z = 1.43$



Z-value is less than Z-table value at $\alpha = 0.01$, accept H_0.

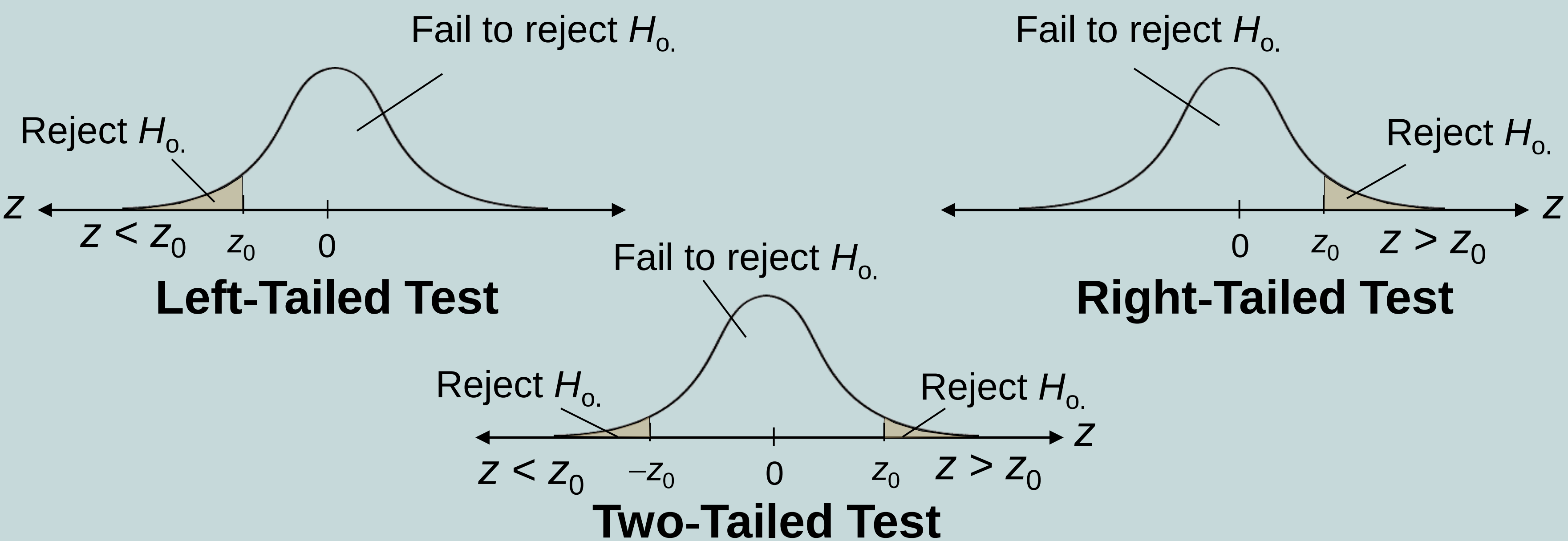
At the 1% level of significance, there is not enough evidence to support the claim that the rechargeable battery has an average life of at least 1000 charges.

Rejection Regions for a z-Test

Decision Rule Based on Rejection Region

To use a rejection region to conduct a hypothesis test, calculate the standardized test statistic, *z*. If the standardized test statistic

1. is in the rejection region, then reject H_0.
2. is *not* in the rejection region, then fail to reject H_0.



Confidence Intervals

Suppose a college president wishes to estimate the average age of students attending classes this semester. The president could select a random sample of 100 students and find the average age of these students, say, 22.3 years. From the sample mean, the president could infer that the average age of all the students is 22.3 years. This type of estimate is called a *point estimate*.

A point estimate is a specific numerical value estimate of a parameter. The best **point estimate** of the population mean μ is the sample mean $\overline{x}$

Confidence Intervals

The sample mean will be, for the most part, somewhat different from the population mean due to sampling error. Therefore, you might ask a second question: How good is a point estimate? The answer is that there is no way of knowing how close a particular point estimate is to the population mean.

This answer places some doubt on the accuracy of point estimates. For this reason, statisticians prefer another type of estimate, called an *interval estimate*.

An interval estimate of a parameter is an interval or a range of values used to estimate the parameter. This estimate may or may not contain the value of the parameter being estimated.

Confidence Intervals

In an interval estimate, the parameter is specified as being between two values. For example, an interval estimate for the average age of all students might be $26.9 < \mu < 27.7$, or 27.3 ± 0.4 years.

Either the interval contains the parameter or it does not. A degree of confidence (usually a percent) can be assigned before an interval estimate is made. For instance, you may wish to be 95% confident that the interval contains the true population mean. Another question then arises. **Why 95%? Why not 99 or 90%?**

A **point estimate** of a parameter is the value of a statistic that estimates the value of the parameter.

The sample mean, $\bar{x}$, is the **best point estimate** of the population mean, μ.

A **confidence interval estimate** of a parameter consists of an interval of numbers along with a probability that the interval contains the unknown parameter.

The **level of confidence** in a confidence interval is a probability that represents the percentage of intervals that will contain if a large number of repeated samples are obtained. The level of confidence is denoted

For example, a 95% level of confidence would mean that if 100 confidence intervals were constructed, each based on a different sample from the same population, we would expect 95 of the intervals to contain the population mean.

The construction of a confidence interval for the population mean depends upon three factors

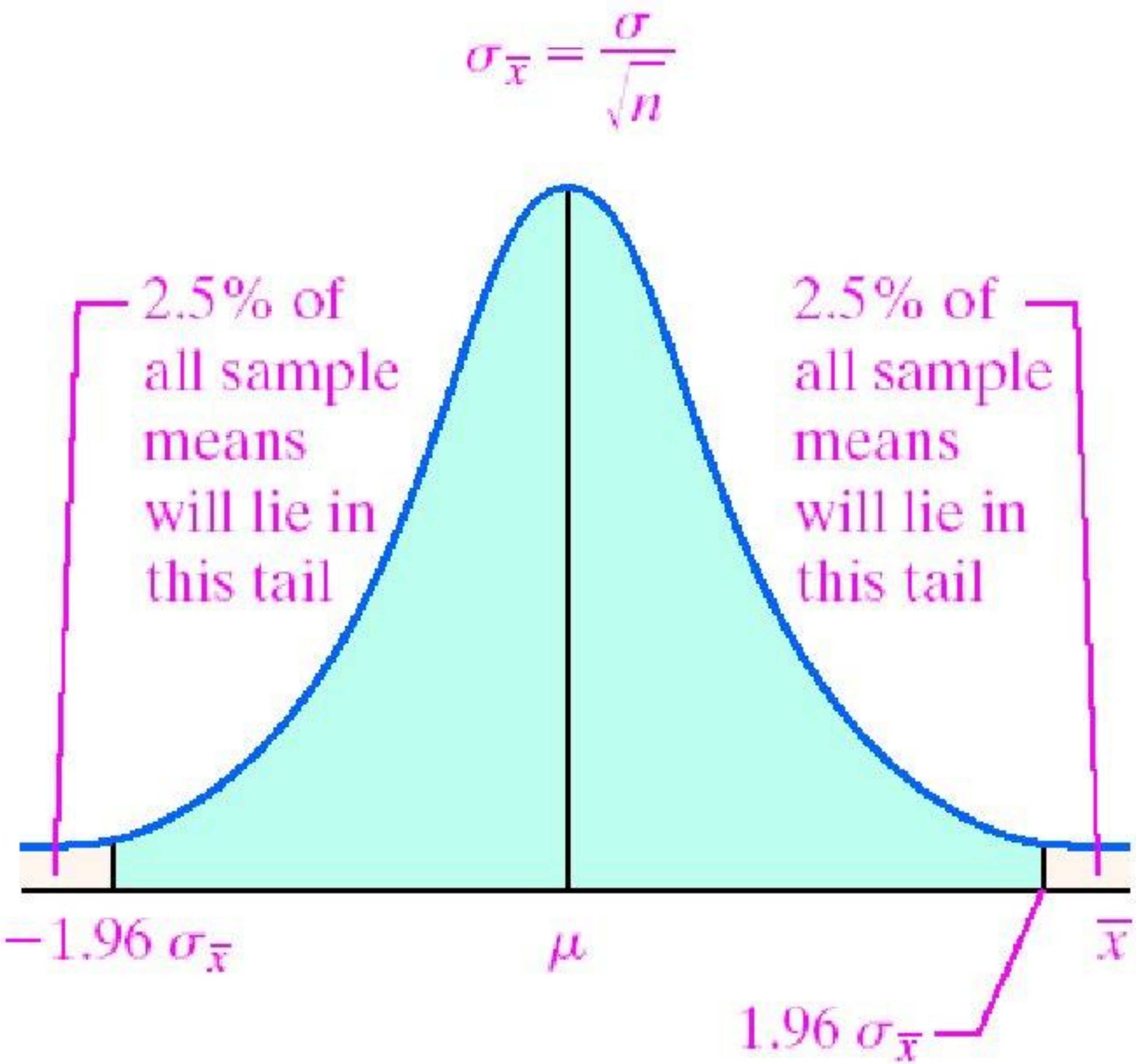
- The point estimate of the population
- The level of confidence
- The standard deviation of the sample mean $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

Suppose we obtain a simple random sample from a population. Provided that the population is normally distributed or the sample size is large, the distribution of the sample mean will be normal with

$$\text{mean } = \mu$$

$$\text{and standard deviation } = \frac{\sigma}{\sqrt{n}}$$

Because $\overline{x}$ is normally distributed, we know that 95% of all sample means should lie within 1.96 standard deviations of the population mean, μ , and 2.5% will lie in each tail.


$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$
2.5% of all sample means will lie in this tail
2.5% of all sample means will lie in this tail
$-1.96\,\sigma_{\overline{x}}$
μ
$\overline{x}$
$1.96\,\sigma_{\overline{x}}$

Now let's look at how we can derive the confidence interval:

$$z = \pm\left(\bar{X} - \mu\right) / \quad \frac{\sigma}{\sqrt{n}} \quad rewriting$$

$$z\frac{\sigma}{\sqrt{n}} = \pm\left(\bar{X} - \mu\right)$$

$$For\ + \quad then \quad z\frac{\sigma}{\sqrt{n}} = \left(\bar{X} - \mu\right)$$

$$or \quad \mu = \bar{X} - z\frac{\sigma}{\sqrt{n}}$$

$$For\ - \quad then \quad z\frac{\sigma}{\sqrt{n}} = \left(-\bar{X} + \mu\right)$$

$$or \quad \mu = \bar{X} + z\frac{\sigma}{\sqrt{n}}$$

$$\bar{X} - z\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + z\frac{\sigma}{\sqrt{n}}$$

$$\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}$$

$$P(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}) = 0.95$$

Interpretation of a Confidence Interval

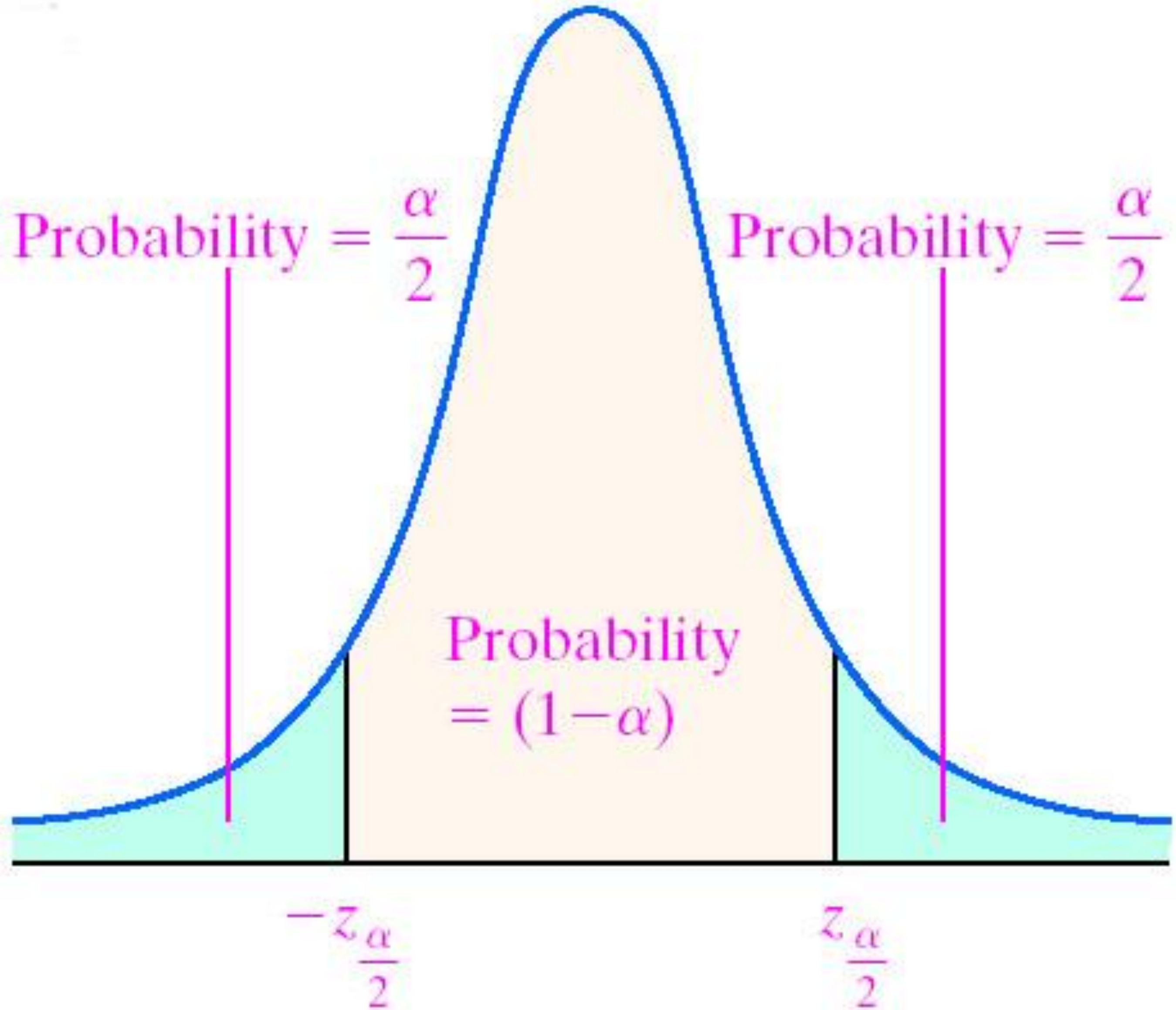
A $(1 - \alpha)100\%$ confidence interval means that if we obtained many simple random samples of size n from the population whose mean, μ, is unknown, then approximately $(1 - \alpha)100\%$ of the intervals will contain μ.

Constructing a (1 - α)100% Confidence Interval about μ, σ Known

Suppose a simple random sample of size *n* is taken from a population with unknown mean μ and known standard deviation σ. A (1 - α)100% confidence interval for is given by

Lower Bound:	$\bar{x} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$	Upper Bound:	$\bar{x} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$

NOTE: The size, *n*, of the population must be greater than or equal to 30 or the population must be normally distributed.


Probability = $\frac{\alpha}{2}$
Probability = $\frac{\alpha}{2}$
Probability
= $(1-\alpha)$
$-z_{\frac{\alpha}{2}}$
$z_{\frac{\alpha}{2}}$

The Margin of Error

The margin of error, E, in a $(1 - \alpha)100\%$ confidence interval in which σ is known is given by

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

where n is the sample size.

NOTE: We require the population from which the sample was drawn be normally distributed or the sample size n be greater than or equal to 30.

Application-1 of one sample Z-test

A researcher wishes to estimate the number of days it takes an automobile dealer to sell a Chevrolet. A sample of 50 cars had a mean time on the dealer's lot of 54 days. Assume the population standard deviation to be 6.0 days. Find the best point estimate of the population mean and the 95% confidence interval of the population mean.

Formula for CI (One sample z-test)

Formula for the Confidence Interval of the Mean for a Specific α

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

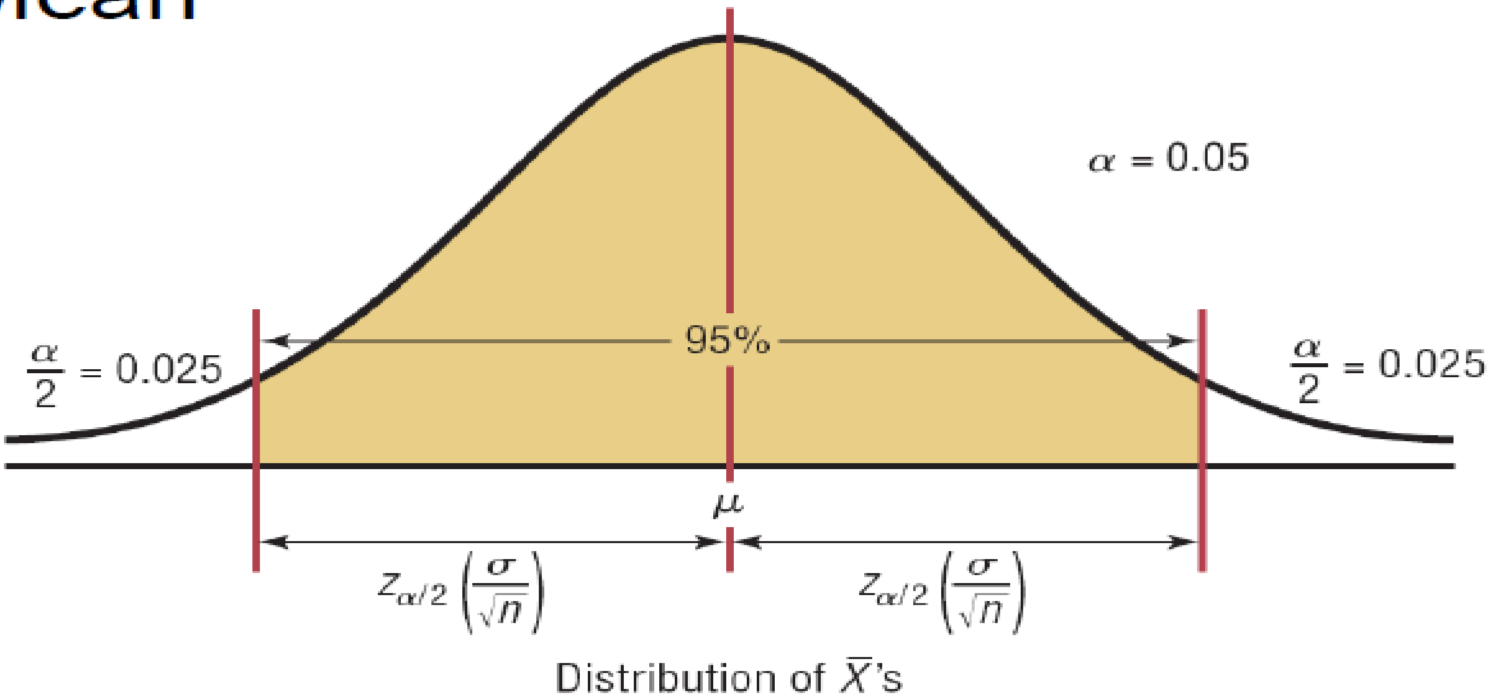
For a 90% confidence interval: $z_{\alpha/2} = 1.65$

For a 95% confidence interval: $z_{\alpha/2} = 1.96$

For a 99% confidence interval: $z_{\alpha/2} = 2.58$

95% CI for Mean

95% Confidence Interval of the Mean



Confidence Intervals-Application-1

$$\bar{X} = 54, s = 6.0, n = 50, 95\% \rightarrow z = 1.96$$

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$54 - 1.96\left(\frac{6.0}{\sqrt{50}}\right) < \mu < 54 + 1.96\left(\frac{6.0}{\sqrt{50}}\right)$$

$$54 - 1.7 < \mu < 54 + 1.7$$

$$52.3 < \mu < 55.7$$

$$52 < \mu < 56$$

One can say with 95% confidence that the interval between 52 and 56 days contains the population mean, based on a sample of 50 automobiles.

Confidence Intervals-Application-2

A survey of 30 adults found that the mean age of a person's primary vehicle is 5.6 years. Assuming the standard deviation of the population is 0.8 year, find the best point estimate of the population mean and the 99% confidence interval of the population mean.

The best point estimate of the mean is 5.6 years.

$$5.6-2.58\left(\frac{0.8}{\sqrt{50}}\right)<\mu<5.6+2.58\left(\frac{0.8}{\sqrt{50}}\right)$$

$$5.2<\mu<6.0$$

One can be 99% confident that the mean age of all primary vehicles is between 5.2 and 6.0 years, based on a sample of 30 vehicles.

Application-3: Credit Union Assets

The following data represent a sample of the assets (in millions of dollars) of 30 credit unions in southwestern Pennsylvania. Find the 90% confidence interval of the mean.

12.23	16.56	4.39
2.89	1.24	2.17
13.19	9.16	1.42
73.25	1.91	14.64
11.59	6.69	1.06
8.74	3.17	18.13
7.92	4.78	16.85
40.22	2.42	21.58
5.01	1.47	12.24
2.27	12.77	2.76

Application-3 Conti...

Step 1: Find the mean and standard deviation. Using technology, we find $\overline{X}$ = 11.091 and s = 14.405.

Step 2: Find $\alpha/2$. 90% CL → $\alpha/2 = 0.05$.

Step 3: Find $z_{\alpha/2}$. 90% CL → $\alpha/2 = 0.05$ → $z_{.05} = 1.65$

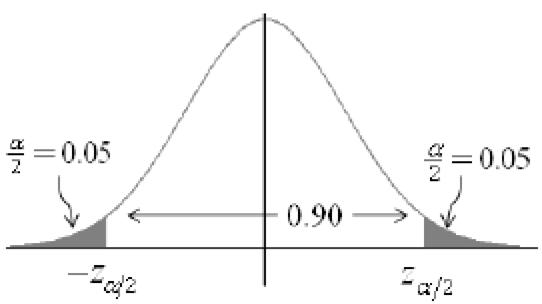


Table E

The Standard Normal Distribution

z	.00	...	.04	.05	...	.09
0.0						
0.1						
⋮						
1.6			0.9495	$\boxed{0.9505}$		

Application-3 Conti…

Step 4: Substitute in the formula.

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$11.091 - 1.65\left(\frac{14.405}{\sqrt{30}}\right) < \mu < 11.091 + 1.65\left(\frac{14.405}{\sqrt{30}}\right)$$

$$11.091 - 4.339 < \mu < 11.091 + 4.339$$

$$6.752 < \mu < 15.430$$

One can be 90% confident that the population mean of the assets of all credit unions is between \$6.752 million and \$15.430 million, based on a sample of 30 credit unions.

Z-Test for one sample

Using Rejection Regions for a *z*-Test for a Mean *μ*

In Words	*In Symbols*
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Sketch the sampling distribution.	
4. Determine the critical value(s).	
5. Determine the rejection regions(s).	

Continued.

Z-Test for one sample

Using Rejection Regions for a *z*-Test for a Mean *μ*

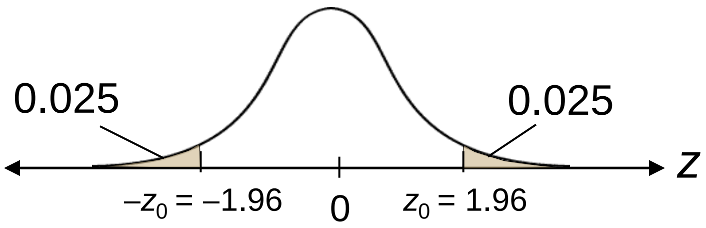
In Words	*In Symbols*
6. Find the standardized test statistic.	$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ or if $n \geq 30$ use $\sigma \approx s$.
7. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Testing with Rejection Regions

Example:
A local telephone company claims that the average length of a phone call is 8 minutes. In a random sample of 58 phone calls, the sample mean was 7.8 minutes and the standard deviation was 0.5 minutes. Is there enough evidence to support this claim at $\alpha = 0.05$?

H_0: $\mu = 8$ (Claim) $\qquad$ H_a: $\mu \neq 8$

The level of significance is $\alpha = 0.05$.



Continued.

Testing with Rejection Regions

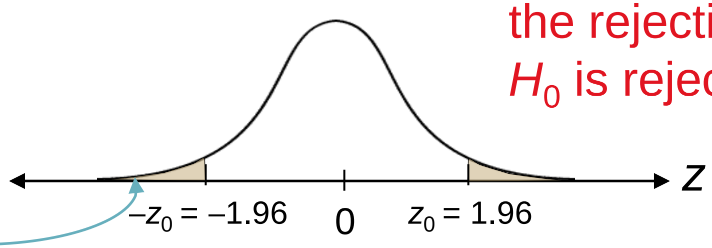
Example continued:
A local telephone company claims that the average length of a phone call is 8 minutes. In a random sample of 58 phone calls, the sample mean was 7.8 minutes and the standard deviation was 0.5 minutes. Is there enough evidence to support this claim at $\alpha = 0.05$?

H_0: $\mu = 8$ (Claim) H_a: $\mu \neq 8$

The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{7.8 - 8}{0.5 / \sqrt{58}}$$

$$\approx -3.05.$$



The test statistic falls in the rejection region, so H_0 is rejected.

At the 5% level of significance, there is enough evidence to reject the claim that the average length of a phone call is 8 minutes.

Application-1

A researcher reports that the average salary of assistant professors is more than 42,000. A sample of 30 assistant professors has a mean salary of 43,260. At a 0.05, test the claim that assistant professors earn more than 42,000 per year. The standard deviation of the population is 5230. Calculate the 95% CIs.

Application-2

A researcher wishes to test the claim that the average cost of tuition and fees at a four year public college is greater than 5700. She selects a random sample of 36 four-year public colleges and finds the mean to be 5950. The population standard deviation is 659. Is there evidence to support the claim at a 5% los and 95% CIs?

Application-3

A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At a 1% los, is there enough evidence to reject the claim and find the 99% CIs?

Application-4

A researcher claims that the average cost of men's athletic shoes is less than 80. He selects a random sample of 36 pairs of shoes from a catalog and finds the following costs (in dollars). (The costs have been rounded to the nearest dollar.) Is there enough evidence to support the researcher's claim at a 0.10? Assume σ= 19.2. Find the 90% CIs.

60	70	75	55	80	55
50	40	80	70	50	95
120	90	75	85	80	60
110	65	80	85	85	45
75	60	90	90	60	95
110	85	45	90	70	70

Two Sample Hypothesis Testing

In a two-sample hypothesis test, two parameters from two populations are compared.

- For a two-sample hypothesis test,

1. the **null hypothesis** H_o is a statistical hypothesis that usually states there is no difference between the parameters of two populations. The null hypothesis always contains the symbol $\leq$, $=$, or $\geq$.
2. the **alternative hypothesis** H_a is a statistical hypothesis that is true when H_o is false. The alternative hypothesis always contains the symbol $>$, $\neq$, or $<$.

Hypothesis Testing

1. State the research question.
2. State the statistical hypothesis.
3. Set decision rule.
4. Calculate the test statistic.
5. Decide if result is significant.
6. Interpret result as it relates to your research question.

Two Sample Hypothesis Testing

To write a null and alternative hypothesis for a two-sample hypothesis test, translate the claim made about the population parameters from a verbal statement to a mathematical statement.

$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_a: \mu_1 \neq \mu_2 \end{cases}$$

$$\begin{cases} H_0: \mu_1 \leq \mu_2 \\ H_a: \mu_1 > \mu_2 \end{cases}$$

$$\begin{cases} H_0: \mu_1 \geq \mu_2 \\ H_a: \mu_1 < \mu_2 \end{cases}$$

Regardless of which hypotheses used, $\mu_1 = \mu_2$ is always assumed to be true.

Two Sample *z*-Test

Three conditions are necessary to perform a *z*-test for the difference between two population means μ_1 and μ_2.

1. The samples must be randomly selected.
2. The samples must be independent. Two samples are **independent** if the sample selected from one population is not related to the sample selected from the second population.
3. Each sample size must be at least 30, or, if not, each population must have a normal distribution with a known standard deviation.

Two Sample z-Test

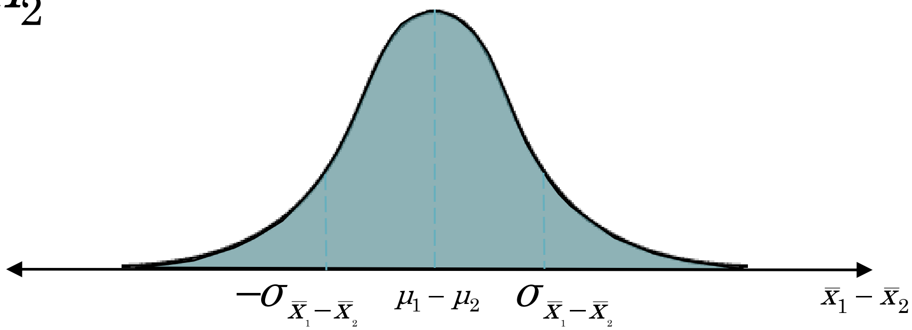
If these requirements are met, the sampling distribution for $\bar{x}_1 - \bar{x}_2$ (the difference of the sample means) is a normal distribution with mean and standard error of

$$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_{\bar{x}_1} - \mu_{\bar{x}_2} = \mu_1 - \mu_2$$

and

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

Sampling distribution for $\bar{x}_1 - \bar{x}_2$



Two Sample z-Test

Two-Sample *z*-Test for the Difference Between Means

A two-sample *z*-test can be used to test the difference between two population means μ_1 and μ_2 when a large sample (at least 30) is randomly selected from each population and the samples are independent. The test statistic is and the standardized test statistic is

When the samples are large, you can use s_1 and s_2 in place of σ_1 and σ_2. If the samples are not large, you can still use a two-sample z-test, provided the populations are normally distributed and the population standard deviations are known.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}} \quad \text{where} \quad \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

Using a Two-Sample *z*-Test for the Difference Between Means (Large Independent Samples)

Two Sample z-Test for the Means

In Words	*In Symbols*
1. State the claim mathematically. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Sketch the sampling distribution.	
4. Determine the critical value(s).	
5. Determine the rejection regions(s).	

Continued.

Two Sample z-Test for the Means

Using a Two-Sample *z*-Test for the Difference Between Means (Large Independent Samples)

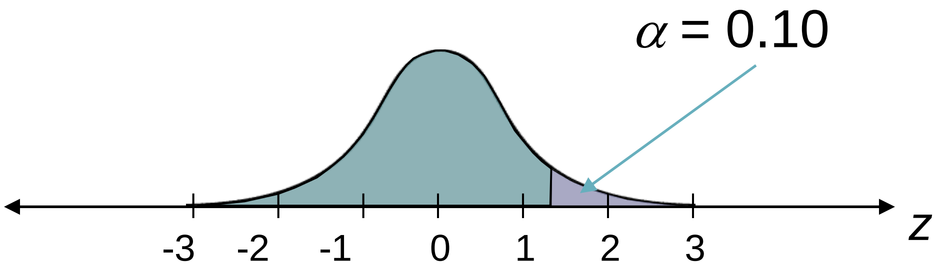
In Words	*In Symbols*
6. Find the standardized test statistic.	$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}}$
7. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Two Sample z-Test for the Means

Example:
A high school math teacher claims that students in her class will score higher on the math portion of the ACT then students in a colleague's math class. The mean ACT math score for 49 students in her class is 22.1 and the standard deviation is 4.8. The mean ACT math score for 44 of the colleague's students is 19.8 and the standard deviation is 5.4. At $\alpha = 0.10$, can the teacher's claim be supported?

H_0: $\mu_1 \le \mu_2$

H_a: $\mu_1 > \mu_2$ (Claim)



$z_0 = 1.28$

Continued.

Two Sample z-Test for the Means

Example continued:

H_0: $\mu_1 \leq \mu_2$

H_a: $\mu_1 > \mu_2$ (Claim)

The standardized error is

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{4.8^2}{49} + \frac{5.4^2}{44}} \approx 1.0644.$$

The standardized test statistic is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{(22.1 - 19.8) - 0}{1.0644} \approx 2.161$$

There is enough evidence at the 10% level to support the teacher's claim that her students score better on the ACT.

Conti….

If population distribution of gain in weight in both the groups is **normal** (either known from earlier studies or established from the random samples), or both the sample sizes are large (n_1 and $n_2 > 30$), the ***test statistic*** is Z and the test is called **normal (Z) Test.**

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

Critical ratio (C.R.)

If $\alpha = 0.05$, C.R. = 1.96; if $\alpha = 0.01$, C.R. = 2.58; and if $\alpha = 0.001$, C.R. = 3.29

Practice Problem

Population	Sample size	Mean	S.D.
A	100	17.4	3.0
B	100	13.2	2.5

Using formula, $Z = 10.77 > 3.29$ ($p < 0.001$), i.e., $\mu_A \neq \mu_B$

i.e., mean weight of school going children is significantly higher than that of children not going to school

Application-1

The Average panicle length of 60 paddy plants in field No. 1 is 18.5 cms and that of 70 paddy plants in field No. 2 is 20.3 cms. With common S.D. 1.15 cms. Test whether there is significant difference between two paddy fields w.r.t panicle length. Find the 95% CIs. (Ztab=1.96 @ 5% los)

Application-2

A survey found that the average hotel room rate in a city is 88.42 and the average room rate in city-A is 80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are 5.62 and 4.83, respectively. At a 0.05, can it be concluded that there is a significant difference in the rates? Find the 95% CIs.

Application-3

- A researcher hypothesizes that the average number of sports that colleges offer for males is greater than the average number of sports that colleges offer for females. A sample of the number of sports offered by colleges is shown. At a α=0.10, is there enough evidence to support the claim? Assume $\sigma 1$ and $\sigma 2$= 3.3. and find the 90% Cis.

Males					Females				
6	11	11	8	15	6	8	11	13	8
6	14	8	12	18	7	5	13	14	6
6	9	5	6	9	6	5	5	7	6
6	9	18	7	6	10	7	6	5	5
15	6	11	5	5	16	10	7	8	5
9	9	5	5	8	7	5	5	6	5
8	9	6	11	6	9	18	13	7	10
9	5	11	5	8	7	8	5	7	6
7	7	5	10	7	11	4	6	8	7
10	7	10	8	11	14	12	5	8	5

Applications-4

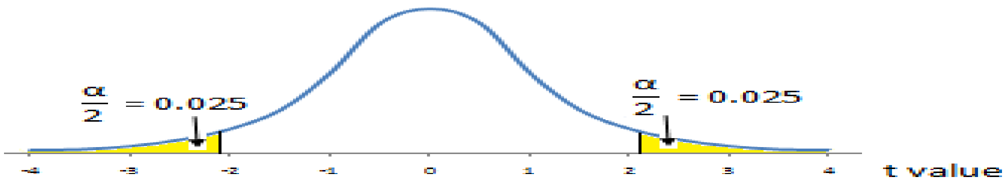
Two groups of students are given a problem-solving test, and the results are compared. Find the 90% CIs of the true different in means.

Mathematics: mean=83.6, SD=4.3, sample size=36

Computers: mean =79.2, SD=3.8, sample size=36

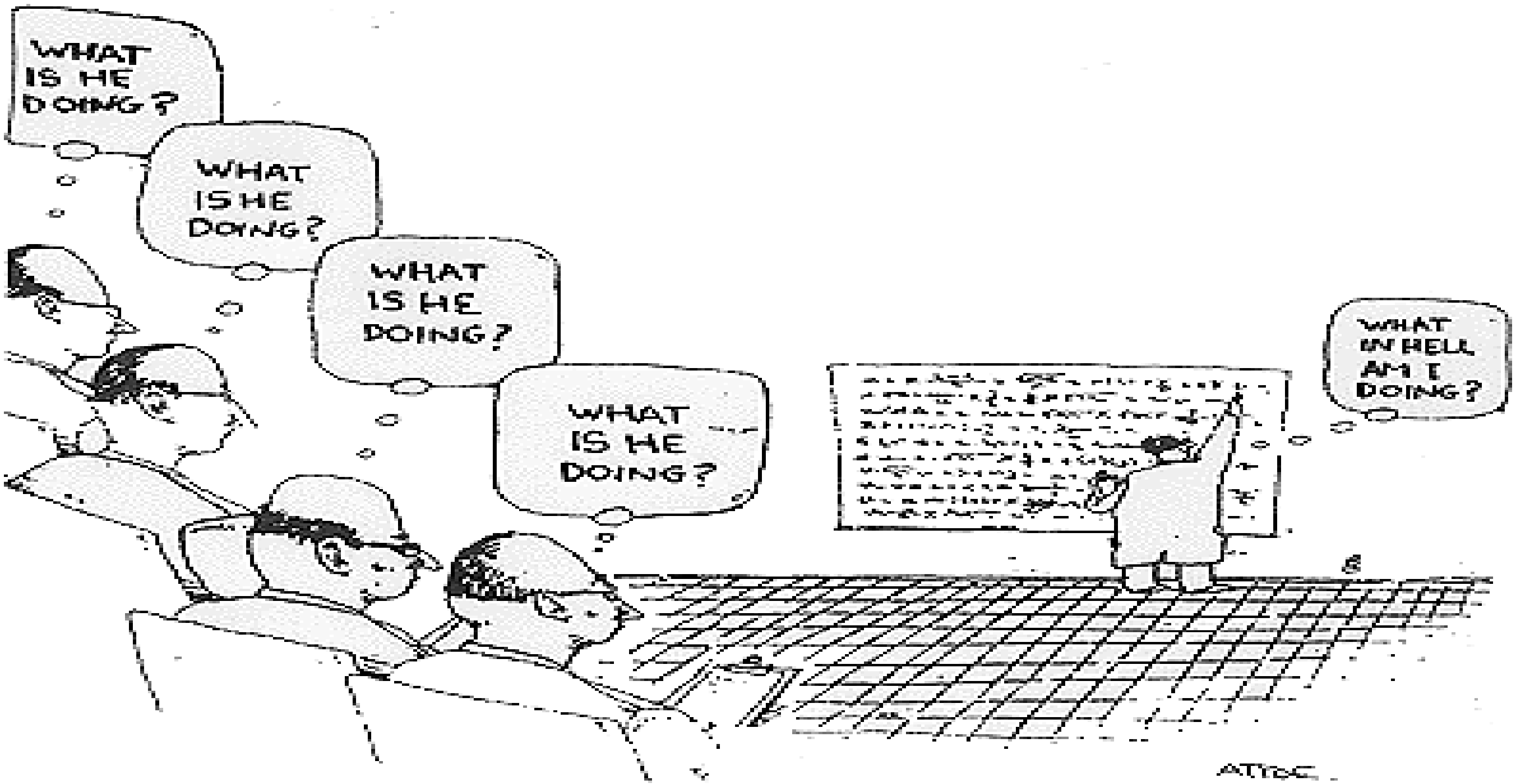
Student's t Distribution Table

For example, the t value for 18 degrees of freedom is 2.101 for 95% confidence interval (2-Tail $\alpha = 0.05$).



	90%	95%	97.5%	99%	99.5%	99.95%	1-Tail Confidence Level
	80%	90%	95%	98%	99%	99.9%	2-Tail Confidence Level
	0.100	0.050	0.025	0.010	0.005	0.0005	1-Tail Alpha
df	0.20	0.10	0.05	0.02	0.01	0.001	2-Tail Alpha
1	3.0777	6.3138	12.7062	31.8205	63.6567	636.6192	
2	1.8856	2.9200	4.3027	6.9646	9.9248	31.5991	
3	1.6377	2.3534	3.1824	4.5407	5.8409	12.9240	
4	1.5332	2.1318	2.7764	3.7469	4.6041	8.6103	
5	1.4759	2.0150	2.5706	3.3649	4.0321	6.8688	
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.9588	
7	1.4149	1.8946	2.3646	2.9980	3.4995	5.4079	
8	1.3968	1.8595	2.3060	2.8965	3.3554	5.0413	
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.7809	
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.5869	
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.4370	
12	1.3562	1.7823	2.1788	2.6810	3.0545	4.3178	
13	1.3502	1.7709	2.1604	2.6503	3.0123	4.2208	
14	1.3450	1.7613	2.1448	2.6245	2.9768	4.1405	
15	1.3406	1.7531	2.1314	2.6025	2.9467	4.0728	
16	1.3368	1.7459	2.1199	2.5835	2.9208	4.0150	
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.9651	
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.9216	
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.8834	
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.8495	
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.8193	
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.7921	
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.7676	
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.7454	
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.7251	
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.7066	
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.6896	
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.6739	
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.6594	
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.6460	

Hypothesis Testing with *t* Tests



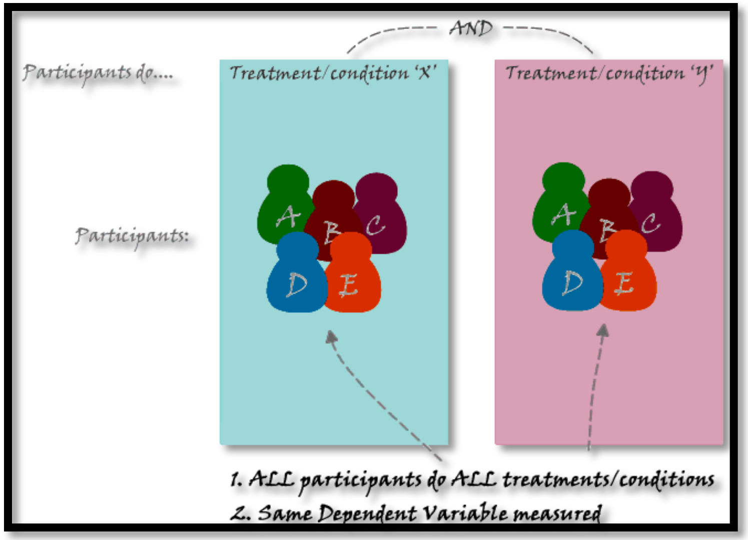
The Story of Student's *t*

"Guinness is the best beer available, it does not need advertising as its quality will sell it, and those who do not drink it are to be sympathized with rather than advertised to." --W.S. Gosset (aka "Student")

- *The pros and cons of ~~beer~~ sampling…*

The many flavors of *t*ea

- Single sample *t*
  - *One sample, compared with <u>known</u> population mean*
  - *Goal: Is our sample different from population?*
- Independent Samples *t*
  - *Different (independent) samples of participants experience each level of IV*
  - *Are our samples from different populations?*
- Paired/Dependent Samples
  - *Same or related (dependent) samples of participants experience each level of IV*
  - *Are our samples from different populations?*



Small Sample Tests
Student ‘t’ test- One Sample

When the population standard deviation is unknown, the z test is not normally used for testing hypotheses involving means. A different test, called the *t test,* is used. The distribution of the variable should be approximately normal.

The t distribution is similar to the standard normal distribution in the following ways.

1. It is bell-shaped.

2. It is symmetric about the mean.

3. The mean, median, and mode are equal to 0 and are located at the center of the
distribution.

4. The curve never touches the x axis.

Student ‘t’ test- One Sample

The *t* distribution differs from the standard normal distribution in the following ways.

1. The variance is greater than 1.

2. The *t* distribution is a family of curves based on the *degrees of freedom,* which is a number related to sample size.

3. As the sample size increases, the *t* distribution approaches the normal distribution.

The *t* test is defined next.

Hypothesis Testing

1. State the research question.
2. State the statistical hypothesis.
3. Set decision rule.
4. Calculate the test statistic.
5. Decide if result is significant.
6. Interpret result as it relates to your research question.

Critical Values in a *t*-Distribution

Finding Critical Values in a *t*-Distribution

1. Identify the level of significance α.
2. Identify the degrees of freedom d.f. = $n - 1$.
3. Find the critical value(s) using Table 5 in Appendix B in the row with $n - 1$ degrees of freedom. If the hypothesis test is
   a. left-tailed, use "One Tail, α" column with a negative sign,
   b. right-tailed, use "One Tail, α" column with a positive sign,
   c. two-tailed, use "Two Tails, α" column with a negative and a positive sign.

t-Test for a Mean μ (n < 30, σ Unknown)

The ***t*-test for the mean** is a statistical test for a population mean. The t-test can be used when the population is normal or nearly normal, σ is unknown, and $n < 30$.

The **test statistic** is the sample mean $\bar{x}$ and the **standardized test statistic** is t.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

The degrees of freedom are d.f. = $n - 1$.

t-Test for a Mean μ (n < 30, σ Unknown)

Using the *t*-Test for a Mean *μ* (Small Sample)

In Words	*In Symbols*
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Identify the degrees of freedom and sketch the sampling distribution.	d.f. = $n - 1$.
4. Determine any critical values.	
5. Determine any rejection region(s).	

Continued.

Testing μ Using Critical Values

Example:
A local telephone company claims that the average length of a phone call is 8 minutes. In a random sample of 18 phone calls, the sample mean was 7.8 minutes and the standard deviation was 0.5 minutes. Is there enough evidence to support this claim at $\alpha = 0.05$?

H_0: $\mu = 8$ (Claim) H_a: $\mu \neq 8$

The level of significance is $\alpha = 0.05$.

The test is a two-tailed test.

Degrees of freedom are d.f. = 18 – 1 = 17.

The critical values are $-t_0 = -2.110$ and $t_0 = 2.110$

Continued.

Testing μ Using Critical Values

Example continued:

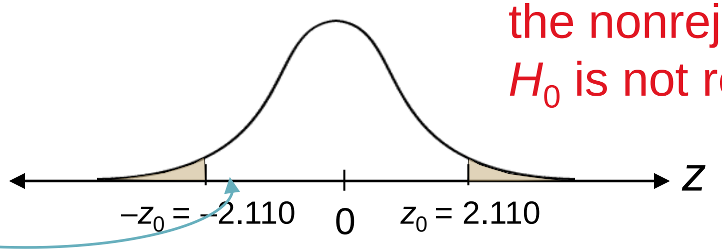
A local telephone company claims that the average length of a phone call is 8 minutes. In a random sample of 18 phone calls, the sample mean was 7.8 minutes and the standard deviation was 0.5 minutes. Is there enough evidence to support this claim at $\alpha = 0.05$?

H_0: $\mu = 8$ (Claim) H_a: $\mu \neq 8$

The standardized test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{7.8 - 8}{0.5/\sqrt{18}}$$

$$\approx -1.70.$$



The test statistic falls in the nonrejection region, so H_0 is not rejected.

At the 5% level of significance, there is not enough evidence to reject the claim that the average length of a phone call is 8 minutes.

Application-1

A medical investigation claims that the average number of infections per week at a hospital in Bengaluru is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at a □ $\alpha = 0.05$? Find the 95% CIs.

Application-2

An educator claims that the average salary of substitute teachers in school districts in a State, is less than 60 per day. A random sample of eight school districts is selected, and the daily salaries (in dollars) are shown. Is there enough evidence to support the educator's claim at a $\alpha=0.10$? Find the 90% CIs.

60, 56, 60, 55, 70, 55, 60, 55

Application-3

The average family size was reported. A random sample of families in a particular school district resulted in the following family sizes:

5, 4, 5, 4, 4, 3, 6, 4, 3, 3, 5, 6, 3, 3, 2, 7, 4, 5, 2, 2, 2, 3, 5, & 2

At a $\alpha=0.05$, does the average family size differ from the national average? Find the 95% CIs.

Applications-4

A group of students are given a problem-solving test, and the results are compared. Find the 90% CIs of the true different in mean.

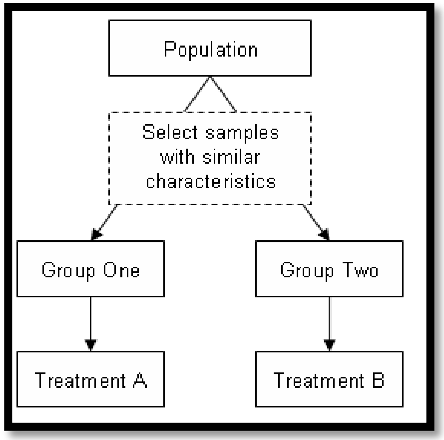
Mathematics: mean=83.6, SD=4.3, sample size=27

Two means t-test or two samples t-test

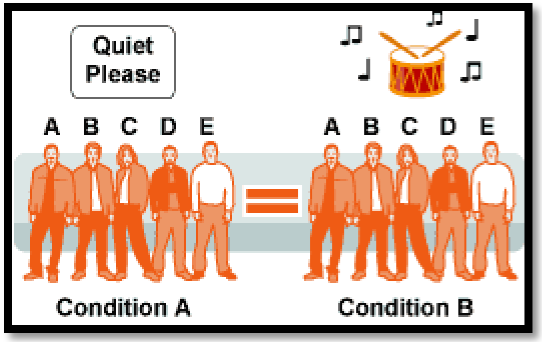
- Imagine two sampling distributions of the mean...
- And then subtracting one from the other…
- If you create a sampling distribution of the difference between the means…
  - Given the null hypothesis, we expect the mean of the sampling distribution of differences, $\mu 1 - \mu 2$, to be 0.
  - We must estimate the standard deviation of the sampling distribution of the difference between means.

Studies with Two Samples

- Independent
  - What is it?
  - Pros
  - Cons



- Paired (Dependent)
  - What is it?
  - Pros
  - Cons



- Hypothetical Beer Tasting Experiment
  - What is the ideal design for this study?

Pooled Estimate of the Population Variance

- Using the assumption of homogeneity of variance, both s1 and s2 are estimates of the *same* population variance.
- If this is so, rather than make two separate estimates, each based on some small sample, it is preferable to combine the information from both samples and make a single pooled estimate of the population variance.

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

Pooled Estimate of the Population Variance

- The pooled estimate of the population variance becomes the average of both sample variances, once adjusted for their degrees of freedom.
  - Multiplying each sample variance by its degrees of freedom ensures that the contribution of each sample variance is proportionate to its degrees of freedom.
  - You know you have made a mistake in calculating the pooled estimate of the variance if it does not come out between the two estimates.
  - You have also made a mistake if it does not come out closer to the estimate from the larger sample.
- The degrees of freedom for the pooled estimate of the variance equals the sum of the two sample sizes minus two, or $(n_1-1)+(n_2-1)$.

Estimating Standard Error of the Difference Between Means

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

$$t = \frac{(\overline{X}_1 - \overline{X}_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

An Example

- Stereotype Threat

Control Subjects				Threat Subjects			
4	9	12	8	7	8	7	2
9	13	12	13	6	9	7	10
13	7	6		5	0	10	8

"Trying to develop the test itself."

"This test is a measure of your academic ability."

The *t* Test for Independent Samples

- State the research question.
  - Does stereotype threat hinder the performance of those individuals to which it is applied?
- State the statistical hypotheses.

$$H_o : \mu_1 - \mu_2 \geq 0$$
$$H_1 : \mu_1 - \mu_2 < 0$$
$$or$$
$$H_o : \mu_1 \geq \mu_2$$
$$H_1 : \mu_1 < \mu_2$$

An Example

- Set the decision rule.

$$\alpha = .05$$
$$df = (n_1 - 1) + (n_2 - 1) = (11 - 1) + (12 - 1) = 21$$
$$t_{crit} = -1.721$$

An Example

- Calculate the test statistic.

Control	Control Sq	Threat	Threat Sq
4	16	7	49
9	81	8	64
12	144	7	49
8	64	2	4
9	81	6	36
13	169	9	81
12	144	7	49
13	169	10	100
13	169	5	25
7	49	0	0
6	36	10	100
		8	64
106	1122	79	621

$$t = \frac{(\overline{X}_1 - \overline{X}_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\overline{X}_1 = \frac{79}{12} = 6.58$$

$$\overline{X}_2 = \frac{106}{11} = 9.64$$

An Example

- Calculate the test statistic.

$$s_1^2 = \frac{12(621) - (79)^2}{12(11)} = 9.18$$

$$s_2^2 = \frac{11(1122) - (106)^2}{11(10)} = 10.05$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

$$s_p^2 = \frac{(12-1)9.18 + (11-1)10.05}{(12-1) + (11-1)} = 9.59$$

$$s_p = 9.59\sqrt{\frac{1}{12} + \frac{1}{11}} = 1.29$$

An Example

- Calculate the test statistic.

$$\overline{X}_1 = 6.58 \qquad \overline{X}_2 = 9.64$$

$$t = \frac{(\overline{X}_1 - \overline{X}_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{6.58 - 9.64}{1.29} = -2.37$$

An Example

- Decide if your result is significant.
  - Reject H_0, $-2.37 < -1.721$
- Interpret your results.
  - Stereotype threat significantly reduced performance of those to whom it was applied.

Assumptions

1) The observations within each sample must be independent.
2) The two populations from which the samples are selected must be normal.
3) The two populations from which the samples are selected must have equal variances.
   - This is also known as **homogeneity of variance,** and there are two methods for testing that we have equal variances:
     - a) informal method – simply compare sample variances
     - b) Levene's test –
4) Random Assignment
   To make causal claims
5) Random Sampling
   To make generalizations to the target population

The *t* Test for Dependent Samples

- You do a *t* test for dependent samples the same way you do a *t* test for a single sample, except that:
  - You use difference scores.
  - You assume the population mean is 0.

$$t = \frac{\overline{X} - \mu_{hyp}}{s_{\overline{X}}} \qquad t = \frac{\overline{D} - \mu_{D_{hyp}}}{s_{\overline{D}}}$$

The *t* Test for Dependent Samples

$$t = \frac{\overline{D} - \mu_{D_{hyp}}}{s_{\overline{D}}}$$

$$s_{\overline{D}} = \frac{s_D}{\sqrt{n}}$$

$$s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

Hypothesis Testing

1. State the research question.
2. State the statistical hypothesis.
3. Set decision rule.
4. Calculate the test statistic.
5. Decide if result is significant.
6. Interpret result as it relates to your research question.

An Example

Eight individuals indicated their attitudes toward socialized medicine before and after listening to a pro–socialized medicine lecture. Attitudes were assessed on a scale from 1 to 7, with higher scores indicating more positive attitudes. The attitudes before and after listening to the lecture were as indicated in the second and third columns of the table. Test for a relationship between the time of assessment and attitudes toward socialized medicine using a correlated groups *t* test.

Individual	Before speech	After speech
1	3	6
2	4	6
3	3	3
4	5	7
5	2	4
6	5	6
7	3	7
8	4	6

An Example

- State the research hypothesis.
  - Does listening to a pro-socialized medicine lecture change an individual's attitude toward socialized medicine?
- State the statistical hypotheses.

$$H_O : \mu_D = 0$$

$$H_A : \mu_D \neq 0$$

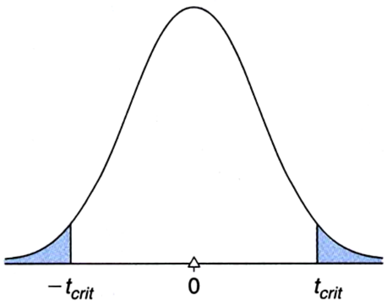
An Example

- Set the decision rule.

$$\alpha = .05$$

$$df = \text{number of differencescores} - 1 = 8 - 1 = 7$$

$$t_{crit} = \pm 2.365$$



Two-tailed or Nondirectional Test
LEVEL OF SIGNIFICANCE
(*p-value in color*)

$p > .05$ $p < .05$ $p < .01$ $p < .($

df	.05*	.01**	.001
1	12.706	63.657	636.62
2	4.303	9.925	31.598
3	3.182	5.841	12.924
4	2.776	4.604	8.610
5	2.571	4.032	6.869
6	2.447	3.707	5.959
7	2.365	3.499	5.408
8	2.306	3.355	5.041
9	2.262	3.250	4.781
10	2.228	3.169	4.587
11	2.201	3.106	4.437
12	2.179	3.055	4.318
13	2.160	3.012	4.221
14	2.145	2.977	4.140
15	2.131	2.947	4.073
16	2.120	2.921	4.015
17	2.110	2.898	3.965
18	2.101	2.878	3.922
19	2.093	2.861	3.883
20	2.086	2.845	3.850
21	2.080	2.831	3.819
22	2.074	2.819	3.792
23	2.069	2.807	3.767
24	2.064	2.797	3.745
25	2.060	2.787	3.725
26	2.056	2.779	3.707
27	2.052	2.771	3.690
28	2.048	2.763	3.674
29	2.045	2.756	3.659
30	2.042	2.750	3.646
40	2.021	2.704	3.551
60	2.000	2.660	3.460
120	1.980	2.617	3.373
∞	1.960	2.576	3.291

An Example

- Calculate the test statistic.

Individual	Before speech	After speech	Difference (D)	D^2
1	3	6	-3	9
2	4	6	-2	4
3	3	3	0	0
4	5	7	-2	4
5	2	4	-2	4
6	5	6	-1	1
7	3	7	-4	16
8	4	6	-2	4
	$\Sigma X_B = 29$	$\Sigma X_A = 45$	$\Sigma D = -16$	$\Sigma D^2 = 42$

$$\overline{D} = \frac{-16}{8} = -2$$

$$s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

$$s = \sqrt{\frac{8(42) - (-16)^2}{8(7)}} = 1.2$$

$$s_{\overline{D}} = \frac{s_D}{\sqrt{n}} \qquad s_{\overline{D}} = \frac{1.2}{\sqrt{8}} = .42$$

$$t = \frac{-2-0}{.42} = -4.76$$

An Example

- Decide if your results are significant.
  - Reject H0, -4.76<-2.365
- Interpret your results.
  - After the pro-socialized medicine lecture, individuals' attitudes toward socialized medicine were significantly more positive than before the lecture.

Issues with Repeated Measures Designs

- Order effects.
  - Use counterbalancing in order to eliminate any potential bias in favor of one condition because most subjects happen to experience it first (order effects).
  - Randomly assign half of the subjects to experience the two conditions in a particular order.
- Practice effects.
  - Do not repeat measurement if effects linger.

Application-1

The average size of a farm in County, is 191 acres. The average size of a farm is 199 acres. Assume the data were obtained from two samples with standard deviations of 38 and 12 acres, respectively, and sample sizes of 8 and 10, respectively. Can it be concluded at $\alpha=0.05$ that the average size of the farms in the two counties is different? Assume the populations are normally distributed. Find the 95% CIs.

Application-2

Health Care Knowledge Systems reported that an insured woman spends on average 2.3 days in the hospital for a routine childbirth, while an uninsured woman spends on average 1.9 days. Assume two samples of 16 women each were used in both samples. The standard deviation of the first sample is equal to 0.6 day, and the standard deviation of the second sample is 0.3 day. At $\alpha=0.01$, test the claim that the means are equal. Find the 99% confidence interval for the differences of the means.

Application-3

Listed below is the moisture content (by percent) for random samples of different fruits and vegetables. At $\alpha=0.05$ level of significance, can it be concluded that fruits differ from vegetables in average moisture content? Find the 99% confidence interval for the differences of the means.

Fruits		Vegetables	
Apricot	86	Artichoke	85
Banana	75	Bamboo shoots	91
Avocado	72	Beets	88
Blackberry	88	Broccoli	89
Clementine	87	Cucumber	95
Fig	79	Iceberg lettuce	96
Pink grapefruit	92	Mushroom	92
Mango	84	Radish	95
		Tomato	94

Applications-4

A physical education director claims by taking a special vitamin, a weight lifter can increase his strength. Eight athletes are selected and given a test of strength, using the standard bench press. After 2 weeks of regular training, supplemented with the vitamin, they are tested again. Test the effectiveness of the vitamin regimen at $\alpha=0.05$. Each value in these data represents the maximum number of pounds the athlete can bench-press. Assume that the variable is approximately normally distributed. Find the 95% CIs.

Athlete	1	2	3	4	5	6	7	8
Before (X_1)	210	230	182	205	262	253	219	216
After (X_2)	219	236	179	204	270	250	222	216

Applications-5

A researcher wishes to test the claim that, on average, more juveniles than adults are classified as missing persons. Records for the last 5 years are shown. At $\alpha=0.10$, is there enough evidence to support the claim? Find the 90% CIs.

Juveniles : 65,513 65,934 64,213 61,954 59,167
Adults : 31,364 34,478 36,937 35,946 38,209

z-Test for a Population Proportion

The ***z*-test for a population** is a statistical test for a population proportion. The *z*-test can be used when a binomial distribution is given such that $np \geq 5$ and $nq \geq 5$.

The **test statistic** is the sample proportion $\hat{p}$ and the **standardized test statistic** is *z*.

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

Using a z-Test for a Proportion p

Verify that $np \geq 5$ and $nq \geq 5$.

Hypothesis Test for Proportions

In Words	*In Symbols*
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Sketch the sampling distribution.	
4. Determine any critical values.	Use Table 4 in Appendix B.

Continued.

Using a z-Test for a Proportion p

Verify that $np \geq 5$ and $nq \geq 5$.

Hypothesis Test for Proportions

In Words	*In Symbols*
5. Determine any rejection regions.	
6. Find the standardized test statistic.	$z = \dfrac{\hat{p} - p}{\sqrt{pq/n}}$
7. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Hypothesis Test for Proportions

Example:
Statesville college claims that more than 94% of their graduates find employment within six months of graduation. In a sample of 500 randomly selected graduates, 475 of them were employed. Is there enough evidence to support the college's claim at a 1% level of significance?

Verify that the products np and nq are at least 5.

$np = (500)(0.94) = 470$ and $nq = (500)(0.06) = 30$

H_0: $p \leq 0.94$

H_a: $p > 0.94$ (Claim)

Continued.

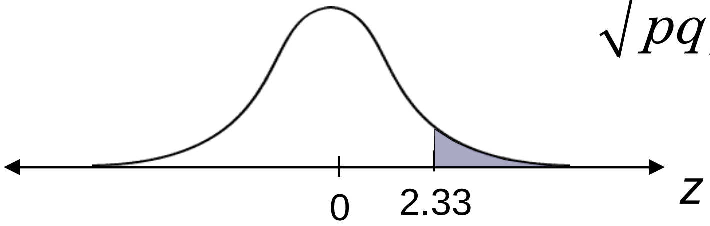
Hypothesis Test for Proportions

Example continued:

Statesville college claims that more than 94% of their graduates find employment within six months of graduation. In a sample of 500 randomly selected graduates, 475 of them were employed. Is there enough evidence to support the college's claim at a 1% level of significance?

H_0: $p \leq 0.94$ H_a: $p > 0.94$ (Claim)

Because the test is a right-tailed test and $\alpha = 0.01$, the critical value is 2.33.



$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.95 - 0.94}{\sqrt{(0.94)(0.06)/500}}$$

$$\approx 0.94$$

Test statistic

Continued.

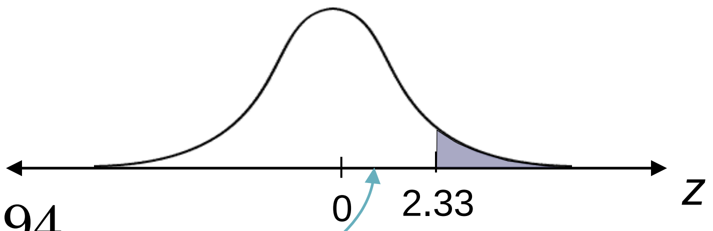
Hypothesis Test for Proportions

Example continued:

Statesville college claims that more than 94% of their graduates find employment within six months of graduation. In a sample of 500 randomly selected graduates, 475 of them were employed. Is there enough evidence to support the college's claim at a 1% level of significance?

H_0: $p \leq 0.94$

H_a: $p > 0.94$ (Claim)



$z = 0.94$

The test statistic falls in the nonrejection region, so H_0 is not rejected.

At the 1% level of significance, there is not enough evidence to support the college's claim.

Hypothesis Test for Proportions

Example:
A cigarette manufacturer claims that one-eighth of the US adult population smokes cigarettes. In a random sample of 100 adults, 5 are cigarette smokers. Test the manufacturer's claim at $\alpha = 0.05$.

Verify that the products *np* and *nq* are at least 5.

$np = (100)(0.125) = 12.5$ and $nq = (100)(0.875) = 87.5$

$H_0: p = 0.125$ (Claim) $H_a: p \neq 0.125$

Because the test is a two-tailed test and $\alpha = 0.05$, the critical values are ± 1.96.

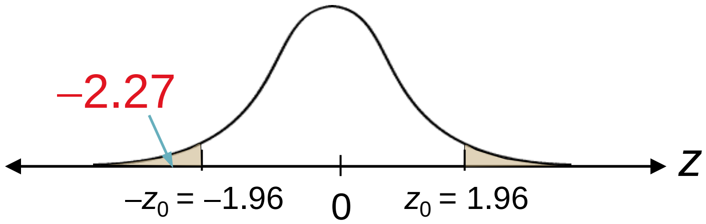
Continued.

Hypothesis Test for Proportions

Example continued:
A cigarette manufacturer claims that one-eighth of the US adult population smokes cigarettes. In a random sample of 100 adults, 5 are cigarettes smokers. Test the manufacturer's claim at $\alpha = 0.05$.

H_0: $p = 0.125$ (Claim)

H_a: $p \neq 0.125$



The test statistic is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.05 - 0.125}{\sqrt{(0.125)(0.875)/100}}$$

$$\approx -2.27$$

Reject H_0.

At the 5% level of significance, there is enough evidence to reject the claim that one-eighth of the population smokes.

Application-1

A dietitian claims that 60% of people are trying to avoid trans fats in their diets. She randomly selected 200 people and found that 128 people stated that they were trying to avoid trans fats in their diets. At α=0.05, is there enough evidence to reject the dietitian's claim? Find the 95% CIs.

Application-2

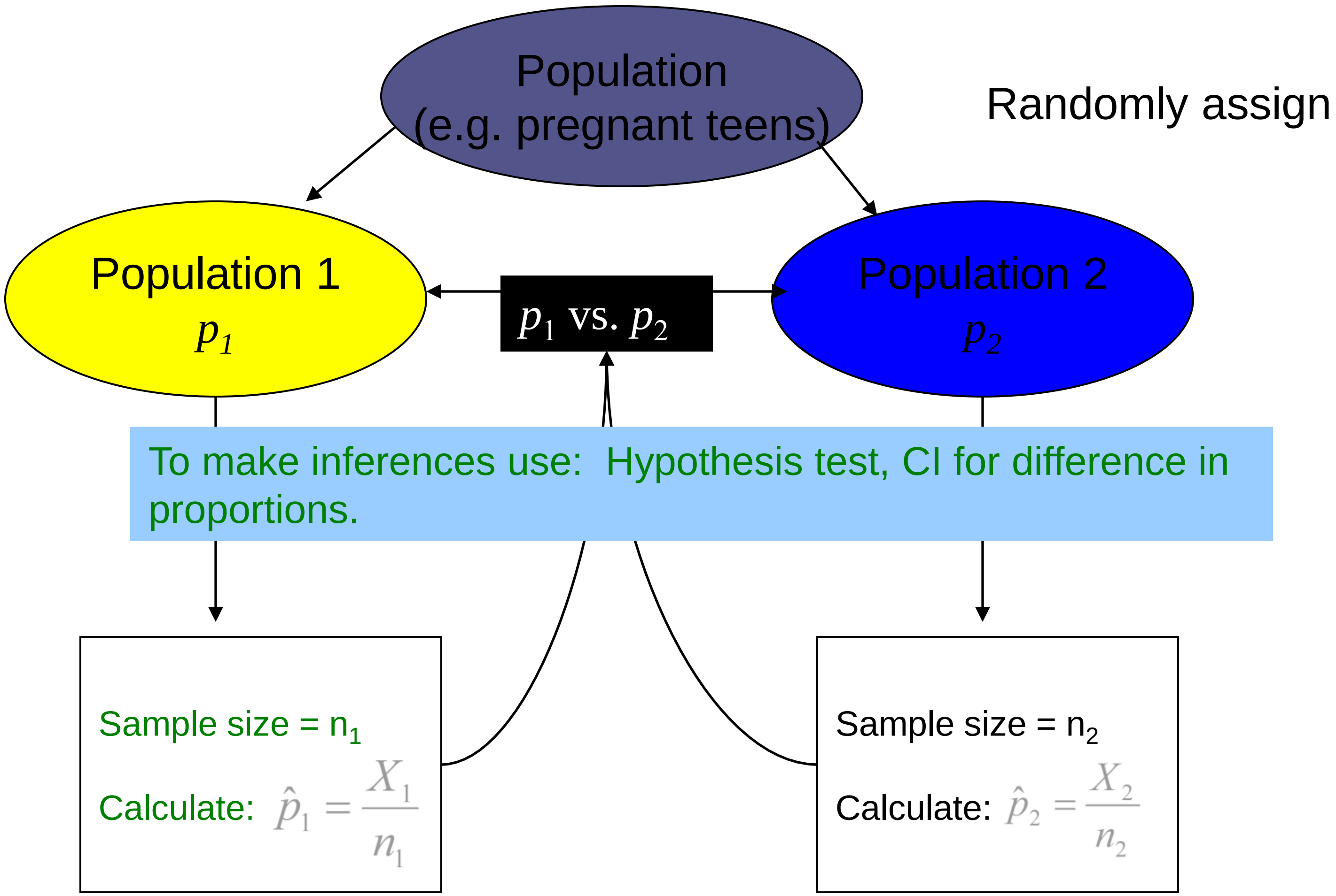
A telephone company representative estimates that 40% of its customers have call-waiting service. To test this hypothesis, she selected a sample of 100 customers and found that 37% had call waiting. At $\alpha=0.01$, is there enough evidence to reject the claim? Find the 99% confidence intervals.

Application-3

A statistician read that at least 77% of the population oppose replacing \$1 bills with \$1 coins. To see if this claim is valid, the statistician selected a sample of 80 people and found that 55 were opposed to replacing the \$1 bills. At $\alpha=0.01$, test the claim that at least 77% of the population are opposed to the change. Find the 99% confidence intervals.

Applications-4

An attorney claims that more than 25% of all lawyers advertise. A sample of 200 lawyers in a certain city showed that 63 had used some form of advertising. At α=0.05, is there enough evidence to support the attorney's claim? Find the 95% CIs.



Experimental Comparative Study

Two Sample z-Test for Proportions

A z-test is used to test the difference between two population proportions, p_1 and p_2.

Three conditions are required to conduct the test.

1. The samples must be randomly selected.
2. The samples must be independent.
3. The samples must be large enough to use a normal sampling distribution. That is,

$$n_1p_1 \geq 5,\ \ n_1q_1 \geq 5,$$

$$n_2p_2 \geq 5,\ \text{ and }\ n_2q_2 \geq 5.$$

Two Sample z-Test for Proportions

Two Sample *z*-Test for the Difference Between Proportions

A two sample *z*-test is used to test the difference between two population proportions p_1 and p_2 when a sample is randomly selected from each population.

The **test statistic** is

$$\hat{p}_1 - \hat{p}_1$$

and the **standardized test statistic** is

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} \text{ and } \bar{q} = 1 - \bar{p}.$$

Note:

$n_1\bar{p}$, $n_1\bar{q}$, $n_2\bar{p}$, and $n_2\bar{q}$ must be at least 5.

Two Sample z-Test for Proportions

Using a Two-Sample *z*-Test for the Difference Between Proportions

In Words	*In Symbols*
1. State the claim. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Determine the critical value(s).	
4. Determine the rejection region(s).	
5. Find the weighted estimate of p_1 and p_2.	$\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

Continued.

Using a Two-Sample *z*-Test for the Difference Between Proportions

Two Sample z-Test for Proportions

In Words	*In Symbols*
6. Find the standardized test statistic.	$z = \dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\bar{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$
7. Make a decision to reject or fail to reject the null hypothesis. 8. Interpret the decision in the context of the original claim.	If z is in the rejection region, reject H_0. Otherwise, fail to reject H_0.

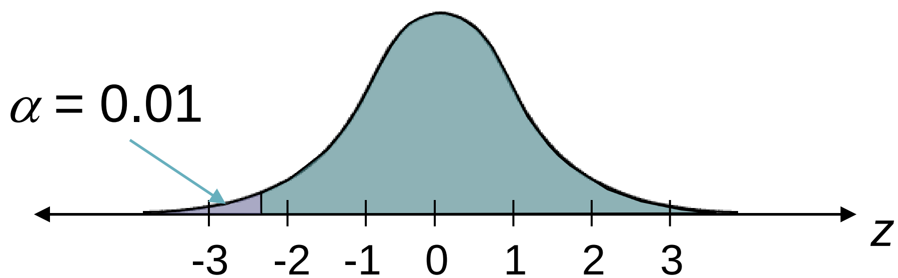
Two Sample z-Test for Proportions

Example:

A recent survey stated that male college students smoke less than female college students. In a survey of 1245 male students, 361 said they smoke at least one pack of cigarettes a day. In a survey of 1065 female students, 341 said they smoke at least one pack a day. At $\alpha = 0.01$, can you support the claim that the proportion of male college students who smoke at least one pack of cigarettes a day is lower then the proportion of female college students who smoke at least one pack a day?

H_0: $p_1 \geq p_2$

H_a: $p_1 < p_2$ (Claim)



$-z_0 = -2.33$

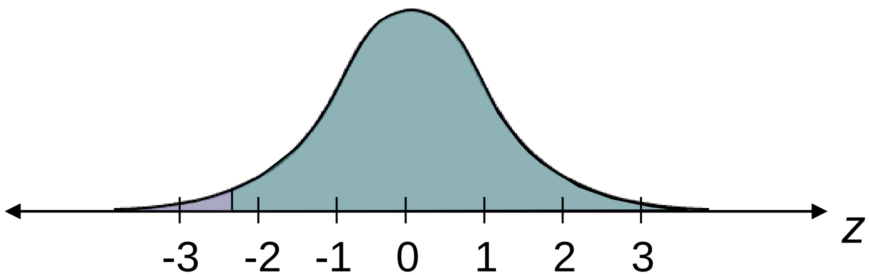
Continued.

Two Sample z-Test for Proportions

Example continued:

H_0: $p_1 \geq p_2$

H_a: $p_1 < p_2$ (Claim)



$-z_0 = -2.33$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{361 + 341}{1245 + 1065} = \frac{702}{2310} \approx 0.304$$

$$\bar{q} = 1 - \bar{p} = 1 - 0.304 = 0.696$$

Because 1245(0.304), 1245(0.696), 1065(0.304), and 1065(0.696) are all at least 5, we can use a two-sample *z*-test.

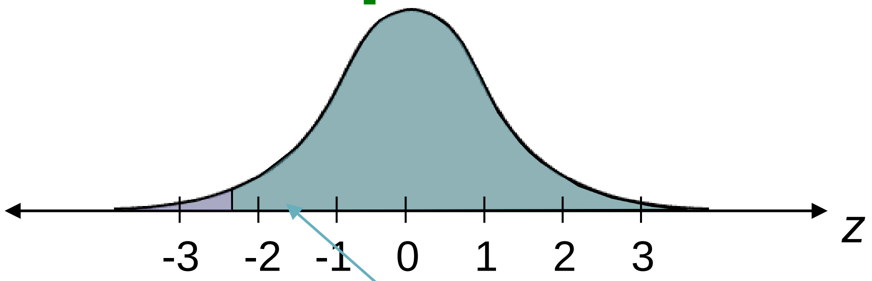
Continued.

Two Sample z-Test for Proportions

Example continued:

H_0: $p_1 \geq p_2$

H_a: $p_1 < p_2$ (Claim)



$-z_0 = -2.33$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.29 - 0.32) - 0}{\sqrt{(0.304)(0.696)\left(\frac{1}{1245} + \frac{1}{1065}\right)}} \approx -1.56$$

Fail to reject H_0.

There is not enough evidence at the 1% level to support the claim that the proportion of male college students who smoke is lower then the proportion of female college students who smoke.

Application-1

In the nursing home study mentioned in the chapter-opening Statistics Today, the researchers found that 12 out of 34 small nursing homes had a resident vaccination rate of less than 80%, while 17 out of 24 large nursing homes had a vaccination rate of less than 80%. At $\alpha=0.05$, test the claim that there is no difference in the proportions of the small and large nursing homes with a resident vaccination rate of less than 80%. Find the 95% CIs.

Application-2

In a sample of 200 workers, 45% said that they missed work because of personal illness. Ten years ago in a sample of 200 workers, 35% said that they missed work because of personal illness. At $\alpha=0.01$, is there a difference in the proportion? Find the 99% confidence intervals.

"If you haven't measured it you don't know what you are talking about."

-William Thompson, Lord Kelvin

THANK YOU…