NON-PARAMETRIC HYPOTHESIS TESTS

UNIT-III

"If you haven't observed it you don't know what you are talking about."

Learning Objectives

- Distinguish Parametric & Nonparametric Test Procedures
- Explain commonly used Nonparametric Test Procedures
- Perform Hypothesis Tests Using Nonparametric Procedures

Introduction

Statistical tests, such as the *z*, *t*, and *proportion* tests, are called parametric tests. **Parametric tests** are statistical tests for population parameters such as means, variances, and proportions that involve assumptions about the populations from which the samples were selected.

One assumption is that these populations are normally distributed. But what if the population in a particular hypothesis-testing situation is *not* normally distributed? Statisticians have developed a branch of statistics known as **nonparametric statistics** or **distribution-free statistics** to use when the population from which the samples are selected is not normally distributed.

Nonparametric statistics can also be used to test hypotheses that do not involve specific population parameters, such as μ, σ, or p.

Introduction…

Example, a sportswriter may wish to know whether there is a relationship between the rankings of two judges on the diving abilities of 10 Olympic swimmers.

In another situation, a sociologist may wish to determine whether men and women enroll at random for a specific drug rehabilitation program.

The statistical tests used in these situations are nonparametric or distribution-free tests. The term *nonparametric* is used for both situations.

Introduction…

The nonparametric tests explained in this

chapter are

the Runs test,

The Sign test,

the Wilcoxon rank sum test, and

the Wilcoxon signed-rank test.

Advantages

There are five advantages that nonparametric methods have over parametric methods:

1. They can be used to test population parameters when the variable is not normally distributed.
2. They can be used when the data are nominal or ordinal.
3. They can be used to test hypotheses that do not involve population parameters.
4. In some cases, the computations are easier than those for the parametric counterparts.
5. They are easy to understand.

Disadvantages

There are three disadvantages of nonparametric methods:

1. They are less sensitive than their parametric counterparts when the assumptions of the parametric methods are met. Therefore, larger differences are needed before the null hypothesis can be rejected.
2. They tend to use less information than the parametric tests. For example, the sign test requires the researcher to determine only whether the data values are above or below the median, not how much above or below the median each value is.
3. They are less efficient than their parametric counterparts when the assumptions of the parametric methods are met. That is, larger sample sizes are needed to overcome the loss of information. For example, the nonparametric sign test is about 60% as efficient as its parametric counterpart, the z test. Thus, a sample size of 100 is needed for use of the sign test, compared with a sample size of 60 for use of the z test to obtain the same results.

Example-Ranking

Many nonparametric tests involve the ranking of data, that is, the positioning of a data value in a data array according to some rating scale. Ranking is an ordinal variable.

For example, suppose a judge decides to rate five speakers on an ascending scale of 1 to 10, with 1 being the best and 10 being the worst, for categories such as voice, gestures, logical presentation, and platform personality. The ratings are shown in the chart.

Speaker	A	B	C	D	E
Rating	9	6	10	3	1

Example-Ranking

Speaker	A	B	C	D	E
Rating	9	6	10	3	1

The rankings are shown next.

Speaker	E	D	B	A	C
Rating	1	3	6	9	10
Ranking	1	2	3	4	5

Since speaker E received the lowest score, 1 point, he or she is ranked first. Speaker D received the next-lower score, 3 points; he or she is ranked second; and so on.

Example-Ranking

What happens if two or more speakers receive the same number of points? Suppose the judge awards points as follows:

Speaker	A	B	C	D	E
Rating	9	6	10	6	3

When there is a tie for two or more places, the average of the ranks must be used. In this case, each would be ranked as

Speaker	E	B	D	A	C
Rating	3	6	6	9	10
Ranking	1	2.5	2.5	4	5

Many times, the data are already ranked, so no additional computations must be done.

For example, if the judge does not have to award points but can simply select the speakers who are best, second-best, third-best, and so on, then these ranks can be used directly.

The Runs Test

When samples are selected, you assume that they are selected at random. How do you know if the data obtained from a sample are truly random? Before the answer to this question is given, consider the following situations for a researcher interviewing 20 people for a survey.

Let their gender be denoted by M for male and F for female. Suppose the participants were chosen as follows:

Situation I: M M M M M M M M M M F F F F F F F F F F

It does not look as if the people in this sample were selected at random, since 10 males were selected first, followed by 10 females.

The Runs Test…

Consider a different selection:

Situation II: F M F M F M F M F M F M F M F M F M F M

In this case, it seems as if the researcher selected a female, then a male, etc.

This selection is probably not random either.

Finally, consider the following selection:

Situation III: F F F M M F M F M M F F M M F F M M M F

This selection of data looks as if it may be random, since there is a mix of males and females and no apparent pattern to their selection.

The Runs Test…

Rather than try to guess whether the data of a sample have been selected at random, statisticians have devised a nonparametric test to determine randomness. This test is called the **runs test**.

Definition: A **run** is a succession of identical letters preceded or followed by a different letter or no letter at all, such as the beginning or end of the succession.

The Runs Test…

Determine the number of runs in each sequence.

a. M M F F F M F F

b. H T H H H

c. A B AAA B B A B B B

Solution

a. There are four runs, as shown.

MM FFF M FF

1 2 3 4

b. There are three runs, as shown.

H T HHH

1 2 3

c. There are six runs, as shown.

A B AAA BB A BBB

1 2 3 4 5 6

The runs test: Procedure

Step 1 State the hypotheses and identify the claim.

Step 2 Find the number of runs.

Note: When the data are numerical, find the median. Then compare each data value with the median and classify it as above or below the median. Other methods such as odd-even can also be used. (Discard any value that is equal to the median.)

Step 3 Find the critical value. Use Runs Table.

Step 4 Make the decision. Compare the actual number of runs with the critical value.

Step 5 Summarize the results.

Table M Critical Values for the Number of Runs

This table gives the critical values at $\alpha = 0.05$ for a two-tailed test. Reject the null hypothesis if the number of runs is less than or equal to the smaller value or greater than or equal to the larger value.

Value of n_1	Value of n_2: 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2
	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
3	1	1	1	1	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3
	6	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
4	1	1	1	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4
	6	8	9	9	9	10	10	10	10	10	10	10	10	10	10	10	10	10	10
5	1	1	2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5
	6	8	9	10	10	11	11	12	12	12	12	12	12	12	12	12	12	12	12
6	1	2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6
	6	8	9	10	11	12	12	13	13	13	13	14	14	14	14	14	14	14	14
7	1	2	2	3	3	3	4	4	5	5	5	5	5	6	6	6	6	6	6
	6	8	10	11	12	13	13	14	14	14	14	15	15	15	16	16	16	16	16
8	1	2	3	3	3	4	4	5	5	5	6	6	6	6	6	7	7	7	7
	6	8	10	11	12	13	14	14	15	15	16	16	16	16	17	17	17	17	17
9	1	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8
	6	8	10	12	13	14	14	15	16	16	16	17	17	18	18	18	18	18	18
10	1	2	3	3	4	5	5	5	6	6	7	7	7	7	8	8	8	8	9
	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20
11	1	2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9	9
	6	8	10	12	13	14	15	16	17	17	18	19	19	19	20	20	20	21	21
12	2	2	3	4	4	5	6	6	7	7	7	8	8	8	9	9	9	10	10
	6	8	10	12	13	14	16	16	17	18	19	19	20	20	21	21	21	22	22
13	2	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	10
	6	8	10	12	14	15	16	17	18	19	19	20	20	21	21	22	22	23	23
14	2	2	3	4	5	5	6	7	7	8	8	9	9	9	10	10	10	11	11
	6	8	10	12	14	15	16	17	18	19	20	20	21	22	22	23	23	23	24
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11	12
	6	8	10	12	14	15	16	18	18	19	20	21	22	22	23	23	24	24	25
16	2	3	4	4	5	6	6	7	8	8	9	9	10	10	11	11	11	12	12
	6	8	10	12	14	16	17	18	19	20	21	21	22	23	23	24	25	25	25
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	11	12	12	13
	6	8	10	12	14	16	17	18	19	20	21	22	23	23	24	25	25	26	26
18	2	3	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13
	6	8	10	12	14	16	17	18	19	20	21	22	23	24	25	25	26	26	27
19	2	3	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13	13
	6	8	10	12	14	16	17	18	20	21	22	23	23	24	25	26	26	27	27
20	2	3	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14
	6	8	10	12	14	16	17	18	20	21	22	23	24	25	25	26	27	27	28

Application-1

On a commuter train, the conductor wishes to see whether the passengers enter the train at random. He observes the first 25 people, with the following sequence of males (M) and females (F).

F F F M M F F F F M F M M M F F F F M M F F F M M

Test for randomness at a $\alpha=0.05$.

Application-1: Solution

Step 1: State the hypotheses and identify the claim.

H_0: The passengers board the train at random, according to gender (claim).

H_1: The null hypothesis is not true.

Step 2 Find the number of runs. Arrange the letters according to runs of males and females, as shown.

Run Gender

1 F F F; 2 M M; 3 F F F F; 4 M; 5 F ; 6 M M M; 7 F F F F;
8 M M; 9 F F F; 10 M M

There are 15 females (n1) and 10 males (n2).

Application-1: Solution

Step 3: Find the critical value. Find the number of runs in Table M for $n1$ 15, $n2$ 10, and α=0.05. The values are 7 and 18. *Note:* In this situation the critical value is found after the number of runs is determined.

Step 4: Make the decision. Compare these critical values with the number of runs. Since the number of runs is 10 and 10 is between 7 and 18, do not reject the null hypothesis.

Step 5: Summarize the results. There is not enough evidence to reject the hypothesis that the passengers board the train at random according to gender.

Application-2

Twenty people enrolled in a drug abuse program. Test the claim that the ages of the people, according to the order in which they enroll, occur at random, at $\alpha = 0.05$.

The data are 18, 36, 19, 22, 25, 44, 23, 27, 27, 35, 19, 43, 37, 32, 28, 43, 46, 19, 20, 22.

Application-2: Solution

Step 1 State the hypotheses and identify the claim.

H0: The ages of the people, according to the order in which they enroll in a drug program, occur at random (claim).

H1: The null hypothesis is not true.

Step 2 Find the number of runs.

a. Find the median of the data. Arrange the data in ascending order.

18 19 19 19 20 22 22 23 25 27 27 28 32 35 36 37 43 43 44 46

The median is 27.

b. Replace each number in the original sequence with an A if it is above the median and with a B if it is below the median. Eliminate any numbers that are equal to the median.

B AB B B AB AB AAAAAAB B B

Application-2: Solution

c. Arrange the letters according to runs.

Run Letters

1 B; 2 A; 3 B B B; 4 A; 5 B; 6 A; 7 B; 8 AAAAAA

9 B B B

Step 3: Find the critical value. Table shows that with n1=9, n2=9, and α= 0.05, the number of runs should be between 5 and 15.

Step 4: Make the decision. Since there are 9 runs and 9 falls between 5 and 15, the null hypothesis is not rejected.

Step 5 Summarize the results. There is not enough evidence to reject the hypothesis that the ages of the people who enroll occur at random.

Application-3

A teacher records the number of students absent over a 30-day period. Test for randomness, at $\alpha = 0.05$.

The data are 27 6 19 24 18 12 15 17 18 20 0 9 4 12 3 2 7 7 0 5 32 16 38 31 27 15 5 9 4 and 10

Application-4

As students, faculty, friends, and family arrived for the Spring Wind Ensemble Concert at Shafer Auditorium, they were asked whether they were going to sit in the balcony (B) or on the ground floor (G). Use the responses listed below and test for randomness, at $\alpha = 0.05$.

B B G G B B G B B B B B B G B B G G B B B B G G G G B G B B B G G

Application-5

An irate student believes that the answers to his EVS professor's final true/false examination are random or not. Test the claim, at $\alpha = 0.05$.

The data are 18, 36, 19, 22, 25, 44, 23, 27, 27, 35, 19, 43, 37, 32, 28, 43, 46, 19, 20, 22.

Application-6

A institute dentist wanted to test the claim, at $\alpha = 0.05$, that the number of cavities in third-year students is random. Forty students were checked, and the number of cavities each had is shown here. Test for randomness of the values above or below the median.

0 4 6 0 6 2 5 3 1 5 1 2 2 1 3 7 3 6 0 2 6 0 2 3 1 5 2 1 3 0 2 3 7 3 1 5 1 1 2 2

Application-7

Toss a coin 30 times and record the outcomes (H or T). Test the results for randomness at $\alpha = 0.05$. Repeat the experiment a few times and compare your results.

The Sign Test-One Sample

The simplest nonparametric test, the sign test for single samples, is used to test the value of a median for a specific sample.

When using the sign test, the researcher hypothesizes the specific value for the median of a population; then he or she selects a sample of data and compares each value with the conjectured median.

If the data value is above the conjectured median, it is assigned a plus sign.

If it is below the conjectured median, it is assigned a minus sign.

And if it is exactly the same as the conjectured median, it is assigned a 0.

The Sign Test-One Sample…

Then the numbers of plus and minus signs are compared.
If the null hypothesis is true, the number of plus signs should be approximately equal to the number of minus signs.
If the null hypothesis is not true, there will be a disproportionate number of plus or minus signs.

One sample Sign test: Procedure

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s). For the single-sample test, compare each value with the conjectured median. If the value is larger than the conjectured median, replace it with a positive sign. If it is smaller than the conjectured median, replace it with a negative sign.

For the paired-sample sign test, subtract the after values from the before values, and indicate the difference with a positive or negative sign or 0, according to the value.

Check the data to see whether they support the null hypothesis. If they do, do not reject the null hypothesis. If not, continue with step 3.

One sample Sign test: Procedure

Step 3 Compute the test value. Count the numbers of positive and negative signs found in step 2, and use the smaller value as the test value.

Step 4 Make the decision. Compare the test value with the critical value in Table. If the test value is less than or equal to the critical value, reject the null hypothesis.

Step 5 Summarize the results.

Note: If the sample size n is 26 or more, use Z-Table and the following formula for the test value:

$$Z = \frac{(X + 0.05) - (n/2)}{\sqrt{n/2}}$$

where

X =smaller number of + or - signs

n =sample size

Table J Critical Values for the Sign Test

Reject the null hypothesis if the smaller number of positive or negative signs is less than or equal to the value in the table.

n	One-tailed, $\alpha = 0.005$ / Two-tailed, $\alpha = 0.01$	$\alpha = 0.01$ / $\alpha = 0.02$	$\alpha = 0.025$ / $\alpha = 0.05$	$\alpha = 0.05$ / $\alpha = 0.10$
8	0	0	0	1
9	0	0	1	1
10	0	0	1	1
11	0	1	1	2
12	1	1	2	2
13	1	1	2	3
14	1	2	3	3
15	2	2	3	3
16	2	2	3	4
17	2	3	4	4
18	3	3	4	5
19	3	4	4	5
20	3	4	5	5
21	4	4	5	6
22	4	5	5	6
23	4	5	6	7
24	5	5	6	7
25	5	6	6	7

Z-table values

		Z-Value	
		Two-sided test	*One-sided test*
		$Z_{1-\alpha/2}$	$Z_{1-\alpha}$
Significance level	0.01	2.576	2.326
	0.05	1.960	1.645
	0.10	1.645	1.282
		$Z_{1-\beta}$	
Power	0.80	0.842	
	0.90	1.282	
	0.95	1.645	
	0.99	2.326	

Application-1

A convenience store owner hypothesizes that the median number of snow cones she sells per day is 40. A random sample of 20 days yields the following data for the number of snow cones sold each day.

18 43 40 16 22
30 29 32 37 36
39 34 39 45 28
36 40 34 39 52
At α=0.05, test the owner's hypothesis.

Application-1: Solution

Step 1 State the hypotheses and identify the claim.

H_0: median=40 (claim) and H_1: median≠40

Step 2 Find the critical value. Compare each value of the data with the median.

If the value is greater than the median, replace the value with a plus sign.

If it is less than the median, replace it with a minus sign.

And if it is equal to the median, replace it with a 0. The completed table follows.

Application-1: Solution

+	-	o	-	-
-	-	-	-	-
-	-	-	+	-
-	o	-	-	+

using n=18 (the total number of plus and minus signs; omit the zeros) and $\alpha=0.05$ for a two-tailed test; the critical value is 4.

Application-1: Solution

Step 3 Compute the test value. Count the number of plus and minus signs obtained in step 2, and use the smaller value as the test value. Since there are 3 plus signs and 15 minus signs, 3 is the test value.

Step 4 Make the decision. Compare the test value 3 with the critical value 4. If the test value is less than or equal to the critical value, the null hypothesis is rejected. In this case, the null hypothesis is rejected since $3 < 4$.

Step 5 Summarize the results. There is enough evidence to reject the claim that the median number of snow cones sold per day is 40.

Application-2

Based on information from the U.S. Census Bureau, the median age of foreign-born residents is 36.4 years. A researcher selects a sample of 50 foreign-born residents in his area and finds that 21 are older than 36.4 years. At $\alpha = 0.05$, test the claim that the median age of the residents is at least 36.4 years.

Application-2: Solution

Step 1 State the hypotheses and identify the claim.

H_0: MD=36.4 (claim) and H_1: MD <36.4

Step 2 Find the critical value. Since α= 0.05 and n=50, and since this is a left-tailed test, the critical value is -1.65, obtained from Z-Table.

Application-2: Solution

Step 3 Compute the test value.

$$Z = \frac{(X + 0.05) - (n/2)}{\sqrt{n/2}} == \frac{(21 + 0.05) - (50/2)}{\sqrt{50/2}} = -0.79$$

Step 4 Make the decision. Since the test value of -0.79 is greater than -1.65, the
decision is to not reject the null hypothesis.

Step 5 Summarize the results. There is not enough evidence to reject the claim that the median age of the residents is at least 36.4.

Application-3

An athletic director suggests the median number for the paid attendance at 20 local football games is 3000. The data for a sample are shown. At $\alpha = 0.05$, is there enough evidence to reject the claim?

6210	3150	2700	3012	4875
3540	6127	2581	2642	2573
2792	2800	2500	3700	6030
5437	2758	3490	2851	2720

Application-4

A temporary employment agency advertises that its employees are placed in positions where the median income is $500. A random sample of employee records revealed the following weekly earnings. At $\alpha = 0.10$, can the agency's claim be refuted?

510 490 475 495 495 520 500 480 487 498 500 535 500 475 482 480 495 480

Paired Sample- Sign Test

The sign test can also be used to test sample means in a comparison of two dependent samples, such as a before-and-after test. Recall that when dependent samples are taken from normally distributed populations, the t test is used (UNIT-II). When the condition of normality cannot be met, the nonparametric sign test can be used.

Paired samples Sign test: Procedure

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s). For the single-sample test, compare each value with the conjectured median. If the value is larger than the conjectured median, replace it with a positive sign. If it is smaller than the conjectured median, replace it with a negative sign.

For the paired-sample sign test, subtract the after values from the before values, and indicate the difference with a positive or negative sign or 0, according to the value. Use Sign-test Table and n=total number of positive and negative signs.

Check the data to see whether they support the null hypothesis. If they do, do not reject the null hypothesis. If not, continue with step 3.

Paired samples Sign test: Procedure

Step 3 Compute the test value. Count the numbers of positive and negative signs found in step 2, and use the smaller value as the test value.

Step 4 Make the decision. Compare the test value with the critical value in Sign-test Table. If the test value is less than or equal to the critical value, reject the null hypothesis.

Step 5 Summarize the results.

Note: If the sample size n is 26 or more, use Z-Table and the following formula for the test value:

$$Z = \frac{(X + 0.05) - (n/2)}{\sqrt{n/2}}$$

where

X =smaller number of + or - signs

n =sample size

Table J Critical Values for the Sign Test

Reject the null hypothesis if the smaller number of positive or negative signs is less than or equal to the value in the table.

n	One-tailed, $\alpha = 0.005$ Two-tailed, $\alpha = 0.01$	$\alpha = 0.01$ $\alpha = 0.02$	$\alpha = 0.025$ $\alpha = 0.05$	$\alpha = 0.05$ $\alpha = 0.10$
8	0	0	0	1
9	0	0	1	1
10	0	0	1	1
11	0	1	1	2
12	1	1	2	2
13	1	1	2	3
14	1	2	3	3
15	2	2	3	3
16	2	2	3	4
17	2	3	4	4
18	3	3	4	5
19	3	4	4	5
20	3	4	5	5
21	4	4	5	6
22	4	5	5	6
23	4	5	6	7
24	5	5	6	7
25	5	6	6	7

Z-table values

		Z-Value	
		Two-sided test	*One-sided test*
		$Z_{1-\alpha/2}$	$Z_{1-\alpha}$
Significance level	0.01	2.576	2.326
	0.05	1.960	1.645
	0.10	1.645	1.282
		$Z_{1-\beta}$	
Power	0.80	0.842	
	0.90	1.282	
	0.95	1.645	
	0.99	2.326	

Application-1

A medical researcher believed the number of ear infections in swimmers can be reduced if the swimmers use earplugs. A sample of 10 people was selected, and the number of infections for a four-month period was recorded. During the first two months, the swimmers did not use the earplugs; during the second two months, they did. At the beginning of the second two-month period, each swimmer was examined to make sure that no infections were present. The data are shown here. At $\alpha=0.05$, can the researcher conclude that using earplugs reduced the number of ear infections?

Application-1

Number of ear infections

Swimmer	Before, X_B	After, X_A
A	3	2
B	0	1
C	5	4
D	4	0
E	2	1
F	4	3
G	3	1
H	5	3
I	2	2
J	1	3

Application-1: Solution

Step 1 State the hypotheses and identify the claim.

H_0: The number of ear infections will not be reduced.

H_1: The number of ear infections will be reduced (claim).

Step 2 Find the critical value. Subtract the after values XA from the before values XB and indicate the difference by a positive or negative sign or 0, according to the value, as shown in the table.

Application-1: Solution

Swimmer	Before, X_B	After, X_A	Sign of difference
A	3	2	+
B	0	1	−
C	5	4	+
D	4	0	+
E	2	1	+
F	4	3	+
G	3	1	+
H	5	3	+
I	2	2	0
J	1	3	−

From Table, with n=9 (the total number of positive and negative signs; the 0 is not counted) and α=0.05(one-tailed), at most 1 negative sign is needed to reject the null hypothesis because 1 is the smallest entry in the α=0.05 column of Sign-test Table.

Application-1: Solution

Step 3 Compute the test value. Count the number of positive and negative signs found in **step 2**, and use the smaller value as the test value. There are 2 negative signs, so the test value is 2.

Step 4 Make the decision. There are 2 negative signs. The decision is to not reject the null hypothesis. The reason is that with n=9, T.V.=1 and 1<2.

Step 5 Summarize the results. There is not enough evidence to support the claim that the use of earplugs reduced the number of ear infections.

Application-2

A professor wants to investigate the relationship between a student's midterm examination score and the score on the final. Eight students were selected, and their scores on the two examinations are noted below. At $\alpha=0.10$ level of significance, is there sufficient evidence to conclude that there is a difference in scores?

Student	1	2	3	4	5	6	7	8
Midterm	75	92	68	85	65	80	75	80
Final	82	90	79	95	70	83	72	79

Application-3

Is there a difference in weekend movie attendance based on the evening in question? Eight small-town movie theaters were surveyed to see how many movie patrons were in attendance on Saturday evening and on Sunday evening. Is there sufficient evidence to reject the claim that there is no difference in movie attendance for Saturday and Sunday evenings? Use $\alpha=0.10$.

Theater	A	B	C	D	E	F	G	H
Saturday	210	100	150	50	195	125	120	204
Sunday	165	42	92	60	172	100	108	136

Application-4

A study was conducted to see whether a certain diet medication had an effect on the weights (in pounds) of eight women. Their weights were taken before and six weeks after daily administration of the medication. The data are shown here. At $\alpha = 0.05$, can you conclude that the medication had an effect (increase or decrease) on the weights of the women?

Subject	A	B	C	D	E	F	G	H
Weight before	187	163	201	158	139	143	198	154
Weight after	178	162	188	156	133	150	175	150

Application-5

A researcher wishes to determine if the number of viewers for 10 returning television shows has not changed since last year. The data are given in millions of viewers. At $\alpha=0.01$, test the claim that the number of viewers has not changed. Depending on your answer, would a television executive plan to air these programs for another year?

Show	1	2	3	4	5	6
Last year	28.9	26.4	20.8	25.0	21.0	19.2
This year	26.6	20.5	20.2	19.1	18.9	17.8

Show	7	8	9	10
Last year	13.7	18.8	16.8	15.3
This year	16.8	16.7	16.0	15.8

The Wilcoxon Rank Sum Test or Mann-Whitney U test

The sign test does not consider the magnitude of the data. For example, whether a value is 1 point or 100 points below the median, it will receive a negative sign. And when you compare values in the pretest/posttest situation, the magnitude of the differences is not considered. The Wilcoxon tests consider differences in magnitudes by using ranks.

The Wilcoxon Rank Sum Test…

In the Wilcoxon tests, the values of the data for both samples are combined and then ranked. If the null hypothesis is true—meaning that there is no difference in the population distributions—then the values in each sample should be ranked approximately the same. Therefore, when the ranks are summed for each sample, the sums should be approximately equal, and the null hypothesis will not be rejected. If there is a large difference in the sums of the ranks, then the distributions are not identical, and the null hypothesis will be rejected.

The Wilcoxon Rank Sum Test : Procedure

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s). Use Z-Table.

Step 3 Compute the test value.

a. Combine the data from the two samples, arrange the combined data in order, and rank each value.

b. Sum the ranks of the group with the smaller sample size. (Note: If both groups have the same sample size, either one can be used.)

c. Use these formulas to find the test value.

The Wilcoxon Rank Sum Test : Procedure

$$Z = \frac{R - \mu_R}{\sigma_R} \quad \text{where} \quad \mu_R = \frac{n_1(n_1 + n_2 + 1)}{2} \qquad \sigma_R = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

R sum of ranks for smaller sample size (n_1 or n_2)

$n1$ First sample size

$n2$ Second sample size

$n1 \geq 10$ and $n2 \geq 10$

Note that if both samples are the same size, either size can be used as $n1$.

Step 4 Make the decision.

Step 5 Summarize the results.

Z-table values

		Z-Value	
		Two-sided test	*One-sided test*
		$Z_{1-\alpha/2}$	$Z_{1-\alpha}$
Significance level	0.01	2.576	2.326
	0.05	1.960	1.645
	0.10	1.645	1.282
		$Z_{1-\beta}$	
Power	0.80	0.842	
	0.90	1.282	
	0.95	1.645	
	0.99	2.326	

Application-1

Two independent samples of army and marine recruits are selected, and the time in minutes it takes each recruit to complete an obstacle course is recorded, as shown in the table. At $\alpha=0.05$, is there a difference in the times it takes the recruits to complete the course?

Army	**15**	**18**	**16**	**17**	**13**	**22**	**24**	**17**	**19**	**21**	**26**	**28**
Marine	14	09	16	19	10	12	11	08	15	18	25	

Application-1: Solution

Step 1 State the hypotheses and identify the claim.

H_0: There is no difference in the times it takes the recruits to complete the obstacle course.

H_1: There is a difference in the times it takes the recruits to complete the obstacle course (claim).

Step 2 Find the critical value. Since $\alpha=0.05$ and this test is a two-tailed test, use the z values of 1.96 from Z-table.

Step 3 Compute the test value.

a. Combine the data from the two samples, arrange the combined data in order, and rank each value. Be sure to indicate the group.

Application-1: Solution

Time	8	9	10	11	12	13	14	15	15	16	16	17
Group	M	M	M	M	M	A	M	A	M	A	M	A
Rank	1	2	3	4	5	6	7	8.5	8.5	10.5	10.5	12.5

Time	17	18	18	19	19	21	22	24	25	26	28
Group	A	M	A	A	M	A	A	A	M	A	A
Rank	12.5	14.5	14.5	16.5	16.5	18	19	20	21	22	23

b. Sum the ranks of the group with the smaller sample size. (Note: If both groups have the same sample size, either one can be used.) In this case, the sample size for the marines is smaller.

$R = 1+2+3+4+5+7+8.5+10.5+14.5+16.5+21 = 93$

Application-1: Solution

c. Substitute in the formulas to find the test value.

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2} = \frac{(11)(11 + 12 + 1)}{2} = 132$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1 + n_2 + 1)}{12}} = \sqrt{\frac{(11)(12)(11 + 12 + 1)}{12}}$$

$$= \sqrt{264} = 16.2$$

$$z = \frac{R - \mu_R}{\sigma_R} = \frac{93 - 132}{16.2} = -2.41$$

Step 4 Make the decision. The decision is to reject the null hypothesis, since -2.41<-1.96.

Step 5 Summarize the results. There is enough evidence to support the claim that there is a difference in the times it takes the recruits to complete the course.

Application-2

A random sample of men and women in prison was asked to give the length of sentence each received for a certain type of crime. At $\alpha=0.05$, test the claim that there is no difference in the sentence received by each gender. The data (in months) are shown here.

Males	8	12	6	14	22	27	32	24	26
Females	7	5	2	3	21	26	30	9	4

Males	19	15	13		
Females	17	23	12	11	16

Application-3

Are all deli sandwiches created equal? Ten sandwiches were selected from deli A and ten from deli B, and the number of calories was calculated for each sandwich. At α=0.05 level of significance, is there sufficient evidence to conclude that there is a difference in the number of calories contained in sandwiches for the two delis?

Deli A	420	630	790	590	610	480	570	740	620	420
Deli B	680	750	430	760	450	710	430	400	860	690

Application-4

Samples of enrollments from medical schools that specialize in research and in primary care are listed below. At $\alpha=0.05$, can it be concluded that there is a difference?.

Research	474	577	605	663	813	443	565	696	692	217	
Primary care	783	546	442	662	605	474	587	555	427	320	293

Application-5

A study was conducted to see whether there is a difference in the time it takes employees of a factory to assemble the product. Samples of high school graduates and non-graduates were timed. At $\alpha=0.05$, is there a difference in the distributions for the two groups in the times needed to assemble the product? The data (in minutes) are shown here.

Graduates	3.6	3.2	4.4	3.0	5.6	6.3	8.2
Nongraduates	2.7	3.8	5.3	1.6	1.9	2.4	2.9

Graduates	7.1	5.8	7.3	6.4	4.2	4.7
Nongraduates	1.7	2.6	2.0	3.1	3.4	3.9

The Wilcoxon Signed-rank Test

When the samples are dependent, as they would be in a before-and-after test using the same subjects, the Wilcoxon signed-rank test can be used in place of the t test for dependent samples. Again, this test does not require the condition of normality. Wilcoxon Signed-rank Table is used to find the critical values.

The Wilcoxon Signed-rank Test : Procedure

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value from Wilcoxon Signed-rank-Table.

Step 3 Compute the test value.

a. Make a table, as shown.

Before	After	Difference	Absolute value	Rank	Signed Rank
XB	XA	D=XB-XA	IDI		

b. Find the differences (before-after), and place the values in the Difference column.

c. Find the absolute value of each difference, and place the results in the Absolute value column.

d. Rank each absolute value from lowest to highest, and place the rankings in the Rank column.

The Wilcoxon Signed-rank Test : Procedure

e. Give each rank a positive or negative sign, according to the sign in the Difference column.

f. Find the sum of the positive ranks and the sum of the negative ranks separately.

g. Select the smaller of the absolute values of the sums, and use this absolute value as the test value **Ws**.

Step 4 Make the decision. Reject the null hypothesis if the test value is less than or equal to the critical value.

Step 5 Summarize the results.

The Wilcoxon Signed-rank Test : Procedure

Note: When $n \geq 30$, use Z-Table and the test value

$$Z = \frac{Ws - \{\frac{n(n+1)}{4}\}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$

where

$n =$ number of pairs where difference is not 0

$Ws =$ smaller sum in absolute value of signed ranks

Critical Values for the Wilcoxon Signed-Rank Test

Reject the null hypothesis if the test value is less than or equal to the value given in the table.

n	One-tailed, $\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.005$
	Two-tailed, $\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$
5	1			
6	2	1		
7	4	2	0	
8	6	4	2	0
9	8	6	3	2
10	11	8	5	3
11	14	11	7	5
12	17	14	10	7
13	21	17	13	10
14	26	21	16	13
15	30	25	20	16
16	36	30	24	19
17	41	35	28	23
18	47	40	33	28
19	54	46	38	32
20	60	52	43	37
21	68	59	49	43
22	75	66	56	49
23	83	73	62	55
24	92	81	69	61
25	101	90	77	68
26	110	98	85	76
27	120	107	93	84
28	130	117	102	92
29	141	127	111	100
30	152	137	120	109

Z-table values

		Z-Value	
		Two-sided test	One-sided test
		$Z_{1-\alpha/2}$	$Z_{1-\alpha}$
Significance level	0.01	2.576	2.326
	0.05	1.960	1.645
	0.10	1.645	1.282
		$Z_{1-\beta}$	
Power	0.80	0.842	
	0.90	1.282	
	0.95	1.645	
	0.99	2.326	

Application-1

In a large department store, the owner wishes to see whether the number of shoplifting incidents per day will change if the number of uniformed security officers is doubled. A sample of 7 days before security is increased and 7 days after the increase shows the number of shoplifting incidents. Is there enough evidence to support the claim, at $\alpha=0.05$, that there is a difference in the number of shoplifting incidents before and after the increase in security?

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Before	7	2	3	6	5	8	12
After	5	3	4	3	1	6	4

Application-1: Solution

Step 1 State the hypotheses and identify the claim.

H_0: There is no difference in the number of shoplifting incidents before and after the increase in security.

H_1: There is a difference in the number of shoplifting incidents before and after the increase in security (claim).

Step 2 Find the critical value from Table. Since $n=7$ and $\alpha=0.05$ for this two-tailed test, the critical value is 2.

Application-1: Solution

Step 3 Find the test value.

a. Make a table.

b. Find the differences (before minus after), and place the values in the Difference column.

c. Find the absolute value of each difference, and place the results in the Absolute value column. **(Note: The absolute value of any number except 0 is the positive value of the number. Any differences of 0 should be ignored.)**

d. Rank each absolute value from lowest to highest, and place the rankings in the Rank column.

e. e. Give each rank a plus or minus sign, according to the sign in the Difference column. The completed table is shown here.

Application-1: Solution

Day	Before, X_B	After, X_A	Difference $D = X_B - X_A$	Absolute value $\lvert D \rvert$	Rank	Signed rank
Mon.	7	5	2	2	3.5	+3.5
Tues.	2	3	−1	1	1.5	−1.5
Wed.	3	4	−1	1	1.5	−1.5
Thurs.	6	3	3	3	5	+5
Fri.	5	1	4	4	6	+6
Sat.	8	6	2	2	3.5	+3.5
Sun.	12	4	8	8	7	+7

Application-1: Solution

f. Find the sum of the positive ranks and the sum of the negative ranks separately.

Positive rank sum= (3.5)+(5)+(6)+(3.5)+(7)=25

Negative rank sum (-1.5)+(-1.5)=-3

g. Select the smaller of the absolute values of the sums (|-3|), and use this absolute value as the test value **Ws**. In this case, **Ws**=|-3|=3.

Step 4 Make the decision. Reject the null hypothesis if the test value is less than or equal to the critical value. In this case, 3>2; hence, the decision is not to reject the null hypothesis.

Step 5 Summarize the results. There is not enough evidence to support the claim that there is a difference in the number of shoplifting incidents. Hence, the security increase probably made no difference in the number of shoplifting incidents.

Application-2

Nine students were selected to participate in an experiment. At the end of a particular statistics unit, they were given a quiz for which they were asked to memorize the necessary formulas. The results are recorded under test 1. The next day they were given a similar quiz but were allowed to use a formula sheet. The results are recorded as test 2. At $\alpha=0.05$, can a difference in scores be concluded?

Can you think of other factors which may have affected the scores? (Wstab=6)

Test 1:	78	95	72	65	70	70	79	85	75
Test 2:	85	92	70	68	69	76	88	96	80

Application-3

Eight drugs were selected, and the prices for the human doses and the animal doses for the same amounts were compared. At $\alpha=0.05$, can it be concluded that the prices for the animal doses are significantly less than the prices for the human doses? If the null hypothesis is rejected, give one reason why animal doses might cost less than human doses. (Wstab=4)

Human dose: 0.67 0.64 1.20 0.51 0.87 0.74 0.50 1.22

Animal dose: 0.13 0.18 0.42 0.25 0.57 0.57 0.49 1.28

Application-4

A sample of legal costs (in thousands) for school districts for two recent consecutive years is shown. At $\alpha=0.05$, is there a difference in the costs?. (Wstab=4)

Year 1:	108	36	65	108	87	94	10	40
Year 2:	138	28	67	181	97	126	18	67

Application-5

In a corporation, female and male workers were matched according to years of experience working for the company. Their salaries were then compared. The data (in thousands of dollars) are shown in the table. At α=0.10, is there a difference in the salaries of the males and females? (Wstab=6)

Males	18	43	32	27	15	45	21	22
Females	16	38	35	29	15	46	25	28

Summary (Parametric vs. Non-Parametric tests

Nonparametric	Parametric
Sign Rank test	One sample t-test
Wilcoxon Rank – Sum test (Mann-Whitney U test)	Two sample t-test
Wilcoxon Signed-Rank test	Two paired sample t-test

“If you haven't observed it you don't know what you are talking about.”

THANK YOU…