**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

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Mean of Sample=45m; SD of Sample=8.0

To finish in one hour, X ≤ 50 and Z = (X-Mean)/SD = (X - 45)/8.0=(50 - 45)/8.0=0.625

Pr(X ≤ 50)=Pr(Z ≤ 0.625)=0.734

Hence, Probability that the service manager cannot meet his commitment =Pr(X > 50)

= 1 - Pr(X ≤ 50)= 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Mean of Sample=38; SD of Sample=6

(a)

Pr(X <38)=Pr(Z<0)=0.5

Pr(X <44)=Pr(Z<1)=0.84134

Pr(38< X <=44)=Pr(0< Z <= 1)=0.34134

Pr(X >44)=1-Pr(X <44)=1-Pr(Z<1)=1-0.84134=0.15866

Since probability of Employees with age greater than 44 is less than the probability of Employees with age between 38-44, the statement “More employees at the processing center are older than 44 than between 38 and 44” is FALSE.

(b)

Pr(X <30)=Pr(Z<-1.33)=0.09175(9.175%)

9.715% of 400= 36.7

Therefore, the statement "A training program for employees under the age of 30 at the center would be expected to attract about 36 employees" of the question is TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

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Let X ~ N(µ1, σ1^2 ) and Y ~ N(µ2, σ2^2 ) are two independent random variables then,

VAR(X+Y) = VAR(X) + VAR(Y)

VAR(X-Y) = VAR(X) + VAR(Y)

and

Mean(X+Y) = Mean(X) + Mean(Y)

Mean(X-Y) = Mean(X) - Mean(Y)

Therefore,

X + Y ~ N(µ1 + µ2, σ1^2 + σ2^2 )

X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 )

Extending the above formula to our problem,

2X1~ N(2 u,4 σ^2)

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

Difference between 2 X1 and X1 + X2:

2X1-[X1+X2] ~ N(2µ - 2µ, 4σ^2 + 2σ^2 ) ~ N(0, 6σ^2 )

Hence, the Resulting Random Variable will have 0 Mean and a variance of 6σ^2 which means the Mean of the resulting Variable will fall at 0 and there is much Variation in the Data compared to the Input Variables [2X1] and [X1 + X2] but resultant variable will also be Normally Distributed cause any sum or difference of independent normal random variables is also normally distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

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The Probability excluding the region between a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = 0.005 (ie. 0.01/2)

The Probability towards right from b = 0.005 (ie. 0.01/2)

By finding the Standard Normal Variable Z (Z Value),calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(0.005+0.99)\*20+100 = (2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, a and b values are [48.5, 151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

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Let 2 Divisions be A and B

Profit made by A ~ N(5, 3^2)

Profit made by B ~ N(7, 4^2)

Profit made by Company(A+B) ~ N(12, 5^2)

Probability for the annual profit of the company centered on the mean=95%=0.95

Probability of excluding the mentioned region=1-0.95=0.05

Probability towards left from Critical value = 0.025

Probability towards right from Critical value = 0.025

Z(0.025+0.95)\*5+12 = (1.96)\*5+12 = 21.8

Z(+0.025)\*5+12 = (-1.96)\*5+12 = 2.2

In Dollars,

Range in which mean is Centered with 95% probability for the annual profit of the company=[2.2,21.8] Million Dollars

In Rupees, 1Million Dollars= 4.5crores Rupees=45Million Rupees

Range in which mean is Centered with 95% probability for the annual profit of the company=[99,981] Million Rupees

1. Specify the 5th percentile of profit (in Rupees) for the company

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Probability for the annual profit of the company corresponding to 5th percentile=0.05

Z(0.05)\*5+12 = (-1.64)\*5+12 = 3.8

Profit made by the Company corresponding to 5th percentile in Rupees= 171 Million Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

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If the Company's Divisions has to under go loss then the mean profit should be LESS THAN 0 since the mean profit is Greater than 0 for both the divisions we would account for the Probability of making Profit less than 0 in this case for both the divisions. Greater the probability greater the chance of loss for that particular division.

Z-Score of A Division with Less than or Equal to 0 profit: 0-5/3= -1.66666

Z-Score of B Division with Less than or Equal to 0 profit: 0-7/4= -1.75

Probability of making Loss for Division A=Pr(Z<-1.66)=0.04846=4.846%

Probability of making Loss for Division B=Pr(Z<-1.75)=0.04006=4.006%

Since Probability of making Loss for Division A is greater than Probability of making Loss for Division B, Hence Division A has larger Probability of making loss than Division B