Laplace Transform for Network Analysis - Detailed Report

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Introduction

This report will carry out computationally finding the Laplace transform of a signal, this task is performed in R with appropriate programming on plotting the curves using `ggplot2`, manually calculating the Laplace transform for visualisation purposes.

Theory

The Laplace transform is a powerful mathematical tool extensively used in network analysis to analyze and solve linear time-invariant (LTI) systems. In network analysis, it offers a way to simplify differential equations describing circuit behavior into algebraic equations, facilitating their analysis.

- **Definition**: The Laplace transform of a function $f(t)$ is defined as the integral of the function multiplied by e^{-st}, where s is a complex variable:

$$F(s) = \int_0^\infty e^{-st} f(t) dt$$

- **Impulse Response**: The Laplace transform provides a way to compute the impulse response of linear systems, which is crucial for understanding their dynamic behavior.
- **Circuit Analysis**: In network analysis, Laplace transforms are extensively used to analyze electrical circuits. They help in determining voltage and current responses, transient and steady-state behaviors, and frequency-dependent characteristics of circuits.
- **Transform Pairs**: Several common transform pairs exist, which are widely used in network analysis, such as the step function, impulse function, exponential function, and trigonometric functions.
- **Applications**: Laplace transform finds applications in various fields, including control theory, signal processing, communication systems, and electrical engineering, making it a fundamental tool in network analysis and system theory.

Question

Find the Laplace transform of $f(t)$ shown in Fig. R.P. 5.8.

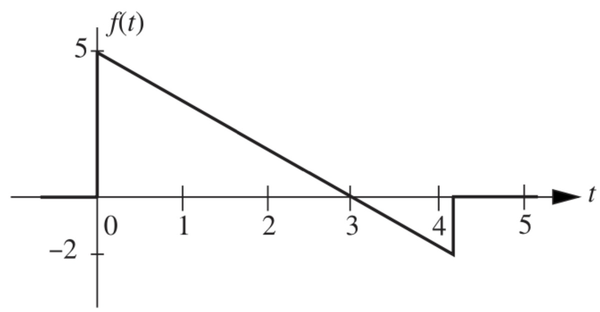


Figure R.P. 5.8

Solution

The equation of a straight line is $y = mx + c$, where $m =$ slope of the line and $c =$ intercept on y-axis. Hence $f(t) = \left(-\frac{5}{3}\right)t + 5$. When $f(t) = -2$, finding the value of t gives us,

$$-2 = (-\frac{5}{3})t + 5$$

Mathematically,

$$t = 4.2 \text{ seconds}$$

Load Packages

```r
library(ggplot2)
```

Define the Function

Let's define the function $f(t)$ in R:

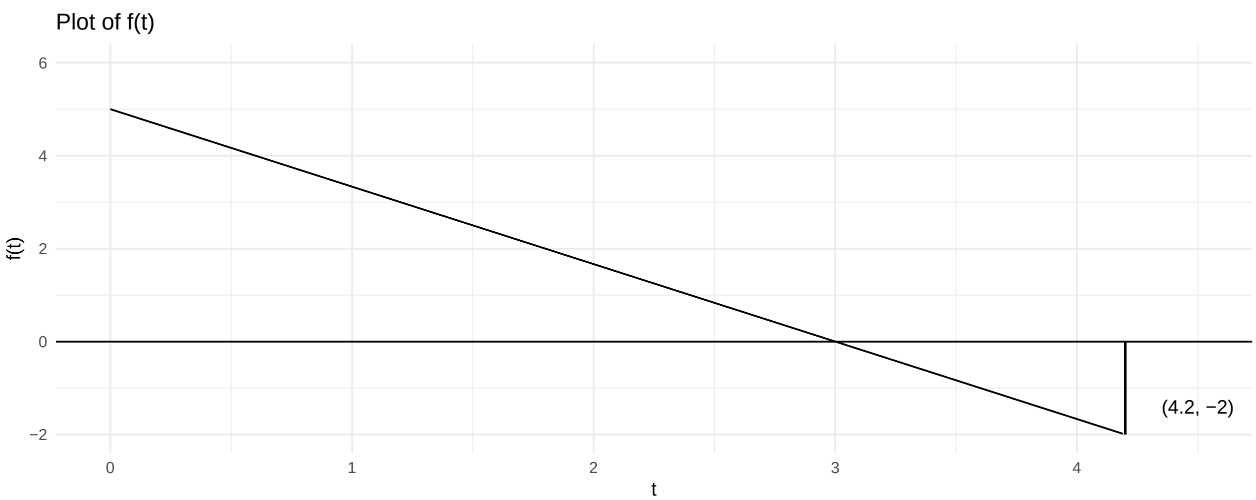
```r
f <- function(t) {
  return((-5/3)*t + 5)
}
```

Plot the Function

Now, let's plot $f(t)$ over a suitable range. We'll use the range from 0 to 4.2 as given in the question:

```r
t_values <- seq(0, 4.2, by = 0.01)
f_values <- f(t_values)

df <- data.frame(t = t_values, f = f_values)

ggplot(df, aes(x = t, y = f)) +
  geom_line() +
  geom_segment(aes(x = 4.2, y = -2, xend = 4.2, yend = 0)) +
```

```r
  geom_hline(yintercept = 0) +
  annotate("text", x = 4.2, y = -2, label = "(4.2, -2)", vjust = -1.5, hjust = -0.5) +
  xlim(0, 4.5) +
  ylim(-2, 6) +
  labs(title = "Plot of f(t)", x = "t", y = "f(t)") +
  theme_minimal()
```



The plot above illustrates the behavior of the function $f(t)$.

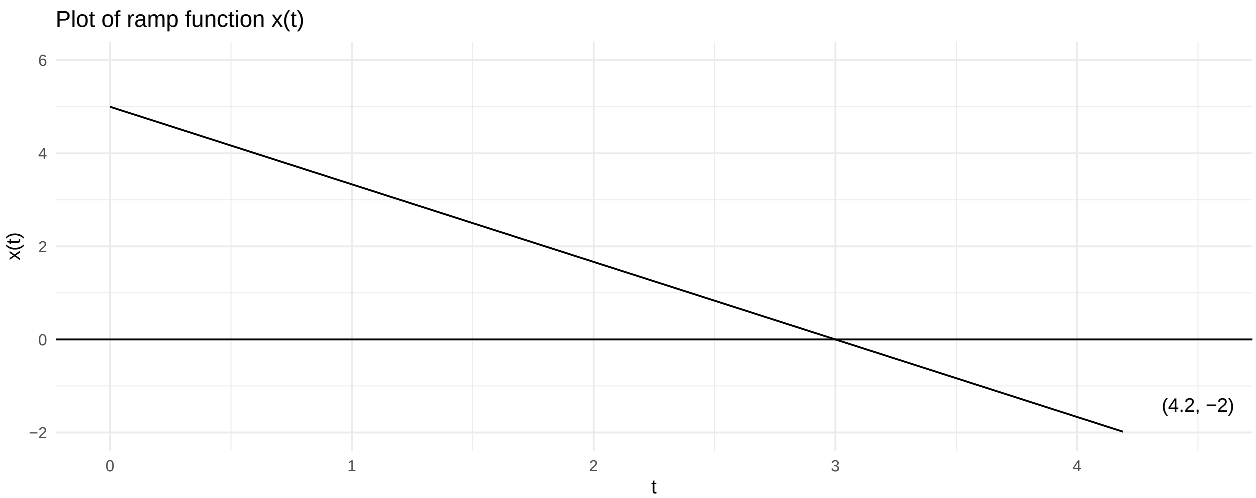
Representing sub signals from the given signal

The above signal can be expressed as,

$$f(t) = x(t)g(t)$$

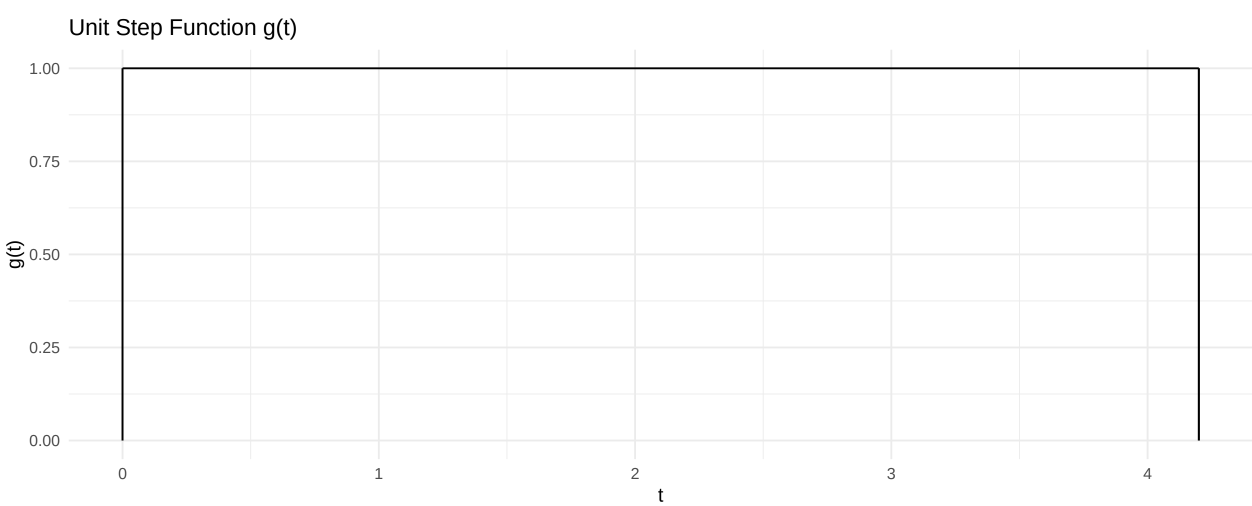
$x(t)$ can be represented graphically,

```r
t_values <- seq(0, 5, by = 0.01)
f_values <- f(t_values)

df <- data.frame(t = t_values, f = f_values)
ggplot(df, aes(x = t, y = f)) +
  geom_line() +
  geom_hline(yintercept = 0) +
  annotate("text", x = 4.2, y = -2, label = "(4.2, -2)", vjust = -1.5, hjust = -0.5) +
  xlim(0, 4.5) +
  ylim(-2, 6) +
  labs(title = "Plot of ramp function x(t)", x = "t", y = "x(t)") +
  theme_minimal()
```



Similarly, $g(t)$ can be represented graphically,

```
step_data <- data.frame(t = c(0, 4.2),
                        g = c(1, 1))

# Create the plot
ggplot(step_data, aes(x = t, y = g)) +
  geom_step(direction = "hv") +
  geom_segment(aes(x = 0, y = 0, xend = 0, yend = 1)) +
  geom_segment(aes(x = 4.2, y = 0, xend = 4.2, yend = 1)) +
  xlim(0, 4.2) +
  ylim(0, 1.0) +
  labs(title = "Unit Step Function g(t)", x = "t", y = "g(t)") +
  theme_minimal()
```



Calculate the Laplace Transform

The Laplace transform of $f(t)$ is given by the formula:

$$F(s) = \int_0^\infty e^{-st} f(t) dt$$

Substituting appropriate values for $x(t)$ and $g(t)$ in. $f(t)$, we get,

$f(t) = [\frac{-5}{3}t + 5][u(t) - u(t - 4.2)]$
$= \frac{-5}{3}tu(t) + \frac{5}{3}tu(t - 4.2) + 5u(t) - 5u(t - 4.2)$
$= \frac{-5}{3}tu(t) + \frac{5}{3}(t - 4.2 + 4.2)u(t - 4.2) + 5u(t) - 5u(t - 4.2)$
$= \frac{-5}{3}tu(t) + \frac{5}{3}(t - 4.2)u(t - 4.2) + 7u(t - 4.2) + 5u(t) - 5u(t - 4.2)$
$= \frac{-5}{3}tu(t) + \frac{5}{3}(t - 4.2)u(t - 4.2) + 2u(t - 4.2) + 5u(t)$

Hence,

$F(s) = L[f(t)]$

$$= \frac{-5}{3s^2} + \frac{5}{3s^2}e^{-4.2s} + \frac{5}{s}$$

Which simplfies to,

$$= \frac{-5 + 5e^{-4.2s} + 6se^{-4.2s} + 15s}{3s^2}$$

Therefore,

$$L[f(t)] = \frac{-5 + 5e^{-4.2s} + 6se^{-4.2s} + 15s}{3s^2}$$

Conclusion

In this report, we've discussed the process of finding the Laplace transform of the function $f(t) = \left(-\frac{5}{3}\right)t + 5$. We've plotted the function and outlined the manual calculation process for finding its Laplace transform. Further numerical or symbolic computations can be performed to obtain $F(s)$ for specific values of s if needed.

Acknowledgement

I would like to express my sincere gratitude to Professor Dr. Niranjan K R sir for granting me permission to carry out this report on Laplace transform for network analysis. His guidance and support have been invaluable throughout the process, and I am deeply thankful for the opportunity to work on this AAT under his supervision.