Q1 Principal Component Analysis:

The explanation and answers to the assignment questions are included with this report

In [32]:

```
#Importing required packages
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

In [33]:

```
df = pd.read_csv("CompPCA.csv", header = None)    #Reading in the data as a Pandas data frame
```

Considering first 15 rows of the data first

In [34]:

```
df.head(15)
```

Out[34]:

	0	1	2	3
0	5.1	3.5	1.4	0.2
1	4.9	3.0	1.4	0.2
2	4.7	3.2	1.3	0.2
3	4.6	3.1	1.5	0.2
4	5.0	3.6	1.4	0.2
5	5.4	3.9	1.7	0.4
6	4.6	3.4	1.4	0.3
7	5.0	3.4	1.5	0.2
8	4.4	2.9	1.4	0.2
9	4.9	3.1	1.5	0.1
10	5.4	3.7	1.5	0.2
11	4.8	3.4	1.6	0.2
12	4.8	3.0	1.4	0.1
13	4.3	3.0	1.1	0.1
14	5.8	4.0	1.2	0.2

De-meaning the data,

In [35]:

```
df = df - df.mean()
```

The data now looks like this

In [36]:

```
df.head(15)
```

Out[36]:

	0	1	2	3
0	-0.743333	0.446	-2.358667	-0.998667
1	-0.943333	-0.054	-2.358667	-0.998667
2	-1.143333	0.146	-2.458667	-0.998667
3	-1.243333	0.046	-2.258667	-0.998667
4	-0.843333	0.546	-2.358667	-0.998667
5	-0.443333	0.846	-2.058667	-0.798667
6	-1.243333	0.346	-2.358667	-0.898667
7	-0.843333	0.346	-2.258667	-0.998667
8	-1.443333	-0.154	-2.358667	-0.998667
9	-0.943333	0.046	-2.258667	-1.098667
10	-0.443333	0.646	-2.258667	-0.998667
11	-1.043333	0.346	-2.158667	-0.998667
12	-1.043333	-0.054	-2.358667	-1.098667
13	-1.543333	-0.054	-2.658667	-1.098667
14	-0.043333	0.946	-2.558667	-0.998667

We shall now proceed to obtaining the dot product of X^T and X; where X represents the de-meaned data (in this case dataframe df)

In [37]:

```
df_cov = np.dot(np.transpose(df.values),df.values)
```

X^TX looks like this

In [38]:

```
df_cov
```

Out[38]:

```
array([[ 102.16833333,   -5.851     ,  189.77866667,   77.01866667],
       [  -5.851     ,   28.0126    ,  -47.9352    ,  -17.5792    ],
       [ 189.77866667,  -47.9352    ,  463.86373333,  193.16173333],
       [  77.01866667,  -17.5792    ,  193.16173333,   86.77973333]])
```

The eigen values and eigen vectors of X^TX are obtained so that we can have the principal components for the data and the respective variances

In [39]:

```
w,v = np.linalg.eig(df_cov)
```

Eigen values of X^TX a.k.a. variances of the components

In [40]:

```
w
```

Out[40]:

```
array([ 629.50127448,   36.09429217,   11.70006231,    3.52877104])
```

Eigen vectors of X^TX a.k.a. principal components

The columns in v represent the principal components

In [41]:

```
v
```

Out[41]:

```
array([[ 0.36158968, -0.65653988, -0.58099728,  0.31725455],
       [-0.08226889, -0.72971237,  0.59641809, -0.32409435],
       [ 0.85657211,  0.1757674 ,  0.07252408, -0.47971899],
       [ 0.35884393,  0.07470647,  0.54906091,  0.75112056]])
```

In [42]:

```
w_percent = (w/np.sum(w))*100                #This provides us with the percent of var
iability contained within each component
print(w, '\033[1m'+" -> eigenvalues" +'\033[0m')
print(w_percent, '\033[1m' + " -> percent of variability explained")
```

```
[ 629.50127448   36.09429217   11.70006231    3.52877104]  -> eigenvalues
[ 92.46162072   5.30155679   1.71851395   0.51830855]  -> percent of varia
bility explained
```

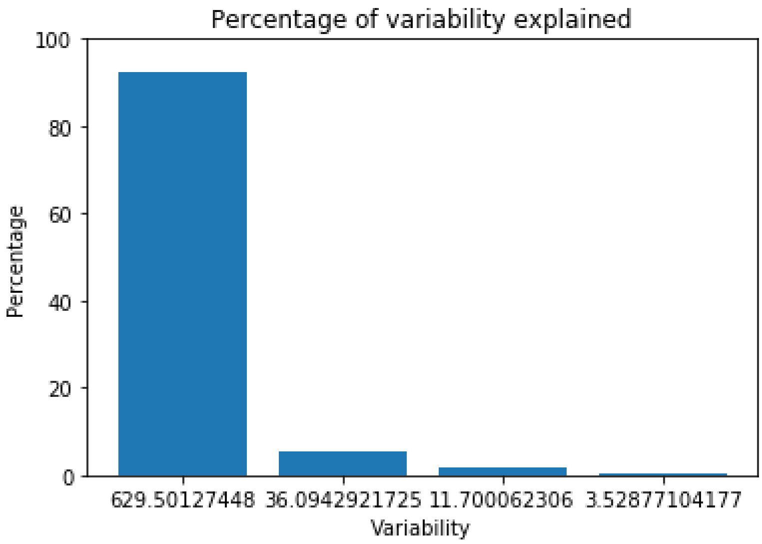
First principal component has the majority of the variance (92.46%) while the others have very small variances.

This implies that the projection of the data on to the first component will provide us with a compressed data having minimum information loss from the original data

Looking at the percentage of variances

In [43]:

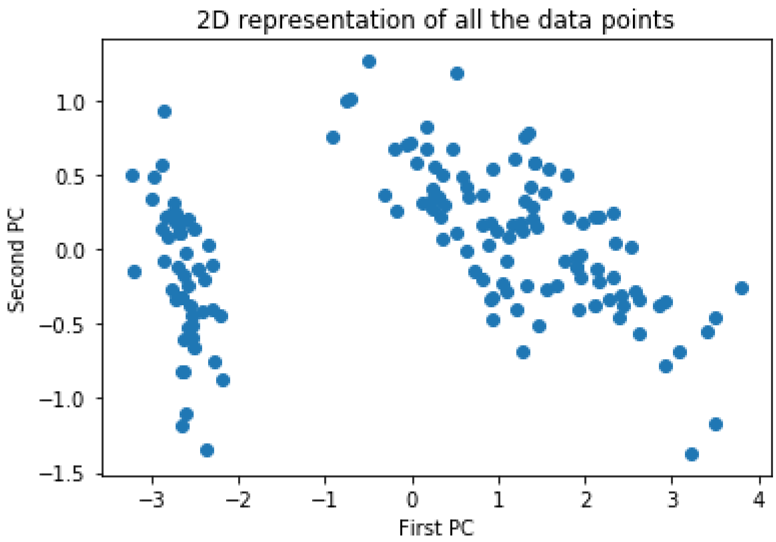
```
plt.bar(np.arange(len(w)),list(w_percent))
plt.xticks(np.arange(len(w)),list(w))
plt.title("Percentage of variability explained")
plt.xlabel("Variability")
plt.ylabel("Percentage")
plt.ylim(ymax = 100)
plt.show()
```



Lets project the data onto the first two principal components to obtain a 2d representation on a scatter plot

In [44]:

```
prin_comp_2d = np.dot(df.values,v[:,:2])
x = prin_comp_2d[:,0]
y = prin_comp_2d[:,1]
plt.scatter(x,y)
plt.title("2D representation of all the data points")
plt.xlabel("First PC")
plt.ylabel("Second PC")
plt.show()
```



From the scatter plot, 2 clusters are evident which could possibly mean that the dataset represents 2 distinct species of flowers

Part 4 PCA with Standardized data

In [45]:

```
df_new = pd.read_csv("CompPCA.csv", header = None)     #Reading in the data as a Pandas
 dataframe
```

De-meaning the data, i.e. subtracting the column mean from each element in the respective column and standardizing the data, i.e. scaling by the inverse of the sample standard deviation

In [46]:

```
df_new = (df_new - df_new.mean())/df_new.std()
```

Viewing the first 15 rows of the de-meaned and standardized data

In [47]:

```
df_new.head(15)
```

Out[47]:

	0	1	2	3
0	-0.897674	1.028611	-1.336794	-1.308593
1	-1.139200	-0.124540	-1.336794	-1.308593
2	-1.380727	0.336720	-1.393470	-1.308593
3	-1.501490	0.106090	-1.280118	-1.308593
4	-1.018437	1.259242	-1.336794	-1.308593
5	-0.535384	1.951133	-1.166767	-1.046525
6	-1.501490	0.797981	-1.336794	-1.177559
7	-1.018437	0.797981	-1.280118	-1.308593
8	-1.743017	-0.355171	-1.336794	-1.308593
9	-1.139200	0.106090	-1.280118	-1.439627
10	-0.535384	1.489872	-1.280118	-1.308593
11	-1.259964	0.797981	-1.223442	-1.308593
12	-1.259964	-0.124540	-1.336794	-1.439627
13	-1.863780	-0.124540	-1.506822	-1.439627
14	-0.052331	2.181763	-1.450146	-1.308593

The dot product of X^T and X are obtained; where X represents the de-meaned and standardized data(in this case dataframe df_new)

In [48]:

```
df_new_cov = np.dot(np.transpose(df_new.values),df_new.values)
```

X^TX looks like this

In [49]:

```
df_new_cov
```

Out[49]:

```
array([[ 149.        ,  -16.29601824,  129.89136944,  121.87509137],
       [ -16.29601824,  149.        ,  -62.65689836,  -53.12506935],
       [ 129.89136944,  -62.65689836,  149.        ,  143.45080746],
       [ 121.87509137,  -53.12506935,  143.45080746,  149.        ]])
```

Obtain the eigen values and eigen vectors of X^TX so that we can have the principal components for the data and their respective variances

In [50]:

```
w,v = np.linalg.eig(df_new_cov)
```

Eigen values of X^TX a.k.a. variances of the components

In [51]:

```
w
```

Out[51]:

```
array([ 433.71189448,  137.26191868,   21.95563847,    3.07054838])
```

Eigen vectors of X^TX a.k.a. principal components

The columns in v represent the principal components

In [52]:

```
v
```

Out[52]:

```
array([[ 0.52237162, -0.37231836, -0.72101681,  0.26199559],
       [-0.26335492, -0.92555649,  0.24203288, -0.12413481],
       [ 0.58125401, -0.02109478,  0.14089226, -0.80115427],
       [ 0.56561105, -0.06541577,  0.6338014 ,  0.52354627]])
```

In [53]:

```
w_percent = (w/np.sum(w))*100                 #This provides us with the percent of var
iability contained within each component
print(w, '\033[1m'+" -> eigenvalues" +'\033[0m')
print(w_percent, '\033[1m' + " -> percent of variability explained")
```

```
[ 433.71189448  137.26191868   21.95563847    3.07054838]  -> eigenvalues
[ 72.77045209  23.03052327   3.68383196   0.51519268]  -> percent of varia
bility explained
```

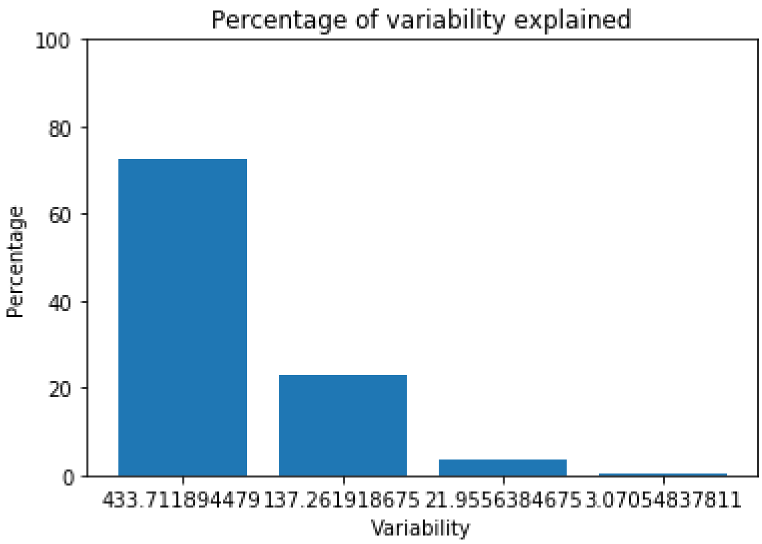
The first principal component has the majority of the variance (72.77%) followed by the second principal component (23%) while the others have very small variances. The first two components provide a cumulative variability percentage of 95.8%

This indicates that the projection of the data on to the first and second component will provide us with a compressed data having minimum information loss from the original data

Looking at the percentage of variance contained by each component in a graphical setting

In [56]:

```
plt.bar(np.arange(len(w)),list(w_percent))
plt.xticks(np.arange(len(w)),list(w))
plt.title("Percentage of variability explained")
plt.xlabel("Variability")
plt.ylabel("Percentage")
plt.ylim(ymax = 100)
plt.show()
```



Projecting the data onto the first two principal components to obtain a 2d representation on a scatter plot