PRP Mid Sem Exam (2:30pm to 4:00pm 4th July.)

- There are 6 questions
- Please make suitable assumptions, wherever necessary.
- Please show all the steps in your answers.
- Maximum Marks: 30

(1) If X is a non-negative random variable satisfying

$$P(X > m+n \mid X > m) = P(X > n)$$

for all possible integers m and n, then show that X is a geometric random variable. (4 marks)

(2) Consider a probability space (S, F, P). For an infinite sequence of events A_i, $i \geq 1$, show that

$$P\left(\bigcap_{i=1}^{\infty} A_i\right) = \prod_{i=1}^{\infty} P\left(A_i \mid A_1, \ldots, A_{i-1}\right).$$

(4 marks)

(3) A random variable X is defined on the sample space $\Omega = \{\omega : -1 \le \omega \le 6\}$.

$$X(\omega) = \begin{cases} 2 & \text{if } -1 \le \omega < 0 \\ 0 & \text{if } \omega = 0 \\ 1 & \text{if } 0 < \omega < 1 \\ 3 & \text{if } 1 \le \omega \le 3 \\ 5 & \text{if } 3 < \omega < 5 \\ 4 & \text{if } \omega = 5 \\ 7 & \text{if } 5 < \omega \le 6. \end{cases}$$

(a) Specify a suitable event space F such that X is a random variable (4 marks)

(b) Let P be probability measure corresponding to the above event space F. Calculate the following in terms of P: (3 marks)

$$F_X(0), \quad F_X(-2), \quad F_X(3.5).$$

④ A noise voltage X is uniformly distributed between $[-5, 5]$. If it is applied as the input to a circuit whose output is $Y = |X|$, obtain $F_Y(y)$. Given the event $A = \{Y \leq 2.5\}$, determine $f_{Y|A}(y|A)$. (5 marks)

⑤ Consider the joint pdf.

$$f_{X,Y}(x,y) = \begin{cases} 24xy & 0 \leq x \leq 1,\ 0 \leq y \leq 1 \\ & 0 \leq x+y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $f_{X,Y}$ is a valid joint pdf.

(b) Find the marginal pdf $f_X(\cdot)$. (4 marks)

⑥ A total of r keys are to be put, one at a time in k boxes, with each key independently put in box i with probability p_i where $\sum_{i=1}^{k} p_i = 1$. Each time a key is

put in a non-empty box, we say a collision occurs. Find the expected number of collisions. (6 marks)