Signal Processing Project Report - Analog to Digital Compression

Ananya Sane - 2020102007 L Lakshmanan - 2020112024

Project summary

List of objectives

References

- M. Unser, "Splines: A perfect fit for signal and image processing," IEEE Signal Process. Mag., vol. 16, no. 6, pp. 22–38, Nov. 1999.
- https://www.youtube.com/watch?v=jgi8hbOmUmk
- https://www.youtube.com/watch?v=g-eNeXlZKAQ&t=323s
- https://www.youtube.com/watch?v=7zx3MT9FgT0

Fafer Notes =P

The major criterion here for reconstruction of signals is consistency. This refers to teh reconstruction having the same measurements as the original when measured using the same sampling functions.

The sampling functions and reconstruction fucntions are considered to be linearly independent.

In this paper, the case where we have the same umber of sampling funtions and reconsrruction functions is discussed.

The coefficients of the reconstruction functions are traditionally obtained from the samples by using inner products. This process involves matrix inversion.

The process of inverting a matrix with large amounts of data can be very troublesome and complex. This paper proposes an efficient method to compute the coefficients of consistent reconstruction without involving matrix inversion when we are dealing with shift invariant sampling and shift invariant functions. This method uses the FFT to achieve a lower complexity.

Froblem Formulation

The signals belong to the Hilbert space $H = L^2(0, l)$.

This Hilbert space limits the functions to the Domain (0, l). For $L^2(R)$, we have

$$\int_{-\infty}^{\infty} |f(x)|^2 < \infty$$

In our fafer, $f \rightarrow signal\ l \rightarrow length\ of\ signal$ Inner froduct is defined as

$$< f, g >= \frac{1}{l} \int_0^l f(x) \cdot \overline{g(x)}$$

where $\overline{g(x)}$ is the complex conjugate of $g(x)$. The measurements of the signal f are represented by $\{d_n\}_{n=0}^{N-1} \in H$

They are modeled in the space by the inner product with the sampling functions $\{\psi_n\}_{n=0}^{N-1}$ as,

$$d_n =< f, \psi_n >$$

The subspace spanned by the sampling functions $\{\psi_n\}_{n=0}^{N-1}$ is called the sampling space. The sampling space is denoted by V_s. Sampling is assumed to be shift invariant (**DOUBT**) as,

$$\psi_n(x) = \psi_l(x - x_n)$$

Where

$$\psi_l(x) = \sum_m \psi(x - ml)$$

(Expressing ψ_l as a summation of the kernel \$\psi(x)) and,

$$x_n = \frac{nl}{N} \quad (n = 0, 1, ..., N-1)$$

The kernel $\psi(x)$ is COMPACTLY SUPPORTED on $[\frac{-r}{2}, \frac{r}{2}]$ with $0 < r < l/N$.

$\hat{\psi}(\omega)$ is the fourier transform of $\psi(x)$.

Similarly, $\{\varphi_k\}_{k=0}^{N-1}$ spans the reconstruction space. The reconstruction space is denoted by V_r. The signal f is reconstructed by a linear combination of these functions with coefficients c_k.

$$\overline{f} = \sum_{k=0}^{N-1} c_k \cdot \psi_k$$

where $\overline{f}$ is the reconstruction.

Now if we take the N dimensional coefficient vectors d and c. Then, we need an NxN square matrix to satisfy

$$X \cdot d = c$$

So now, our problem is equivalent to finding a suitable matrix X. By our consistency criterion, whatever matrix X we find must give us outputs that satisfy

$$< \overline{f}, \psi_n >=< f, \psi_n > \quad (n = 0, 1, ..., N - 1)$$