Digital filter design

FIR filter design

- used when linear-phase is a requirement $\rightarrow$ no distortion

- FIR filters are always stable $\left(\sum_{-\infty}^{\infty} |h[n]| < \infty \right)$

- what are conditions on $h[n]$ due to linear-phase requirement?

impulse response $h[n]$ must satisfy : $h[n] = \pm h[N-1-n]$

$n = 0, 1, \cdots N-1$

$\oplus$ - symmetric

$\ominus$ - anti-symmetric

- varias types of FIR structure

Type I : N - odd and $h[n]$ symmetric filter

- $h[n] = h[N-1-n]$; N is odd ; $n = 0, 1, \ldots N-1$

we can show that phase response is linear

define $M = \frac{N-1}{2}$

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h[n] e^{-j\omega n}$$

$$= h[0] + h[1] e^{-j\omega} + h[2] e^{-j2\omega} + \cdots + h[M] e^{-j\omega M} + \cdots h[N-1] e^{-j\omega(N-1)}$$

$$= e^{-j\omega M} \left\{ h[0] e^{j\omega M} + h[1] e^{j\omega(M-1)} + \cdots h[M] + \cdots h[N-1] e^{-j\omega(2M)} e^{j\omega M} \right\}$$

$$= e^{-j\omega M} \left\{ h[M] + \left(h[0] e^{j\omega M} + h[N-1] e^{-j\omega M} \right) + \left(h[1] e^{j\omega(M-1)} + h[N-2] e^{-j\omega(M-1)} \right) + \cdots \right\}$$

$$H(e^{j\omega}) = e^{-j\omega M} \left\{ h[M] + 2h[0] \cos(\omega M) + 2h[1] \cos(\omega(M-1)) + \cdots \right\}$$

real valued

linear phase

$\Rightarrow H(e^{j\omega})$ will have linear-phase. (Type I)

Type II — N - even & $h[n]$ symmetric

Type III — N - odd & $h[n]$ anti - symmetric & $h\left[\frac{N-1}{2}\right] = 0$

Type IV — N - even & $h[n]$ anti - symmetric

Ex.

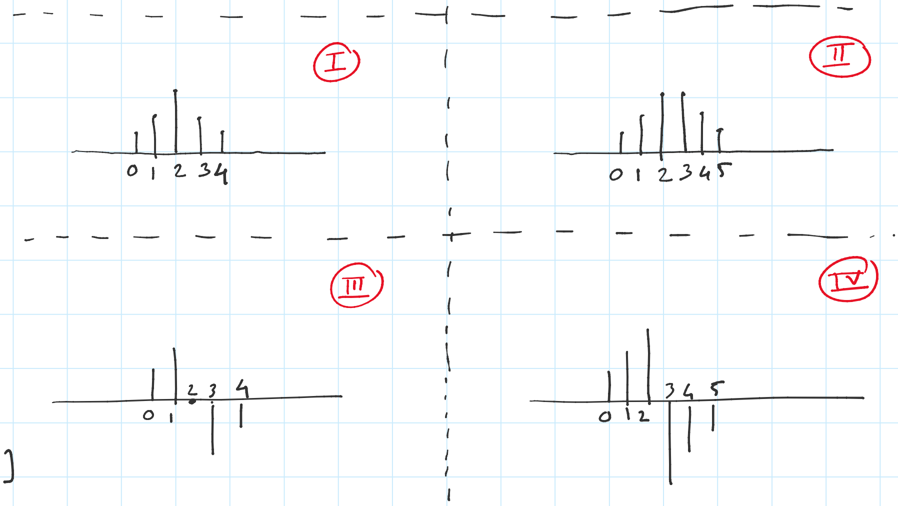
N - odd | N - even

symmetric

$h[n] = h[N-1-n]$

anti - symmetric

$h[n] = -h[N-1-n]$



☆ Type III & IV filters cannot be used to make LPF

proof: $H(e^{j\omega})\Big|_{\omega=0} = \sum_{0}^{N-1} h[n] = 0$ for any type III / IV

☆ Type IV cannot be a HPF .... (HW)

⊛ FIR filter design using windows

Steps:

1. select desired ideal filter $H_{ideal}(e^{j\omega})$

2. shift in time to get desired linear phase $H_d(e^{j\omega}) = H_{ideal}(e^{j\omega})\, e^{-j\omega M}$

3. apply window in time domain to the ideal impulse response

$$h[n] = h_d[n]\,\omega[n]$$

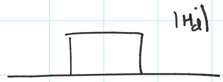
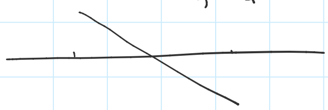
☆ make sure $h[n]$ has desired symmetry / anti-symmetry property

★ by multiplication property : $H(e^{j\omega}) = H_d(e^{j\omega}) \circledast W(e^{j\omega})$

periodic convolution

Ex. ideal LPF and rectangular window

$|H_d|$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $\measuredangle H_d$

$H_d(e^{j\omega}) =$  & 

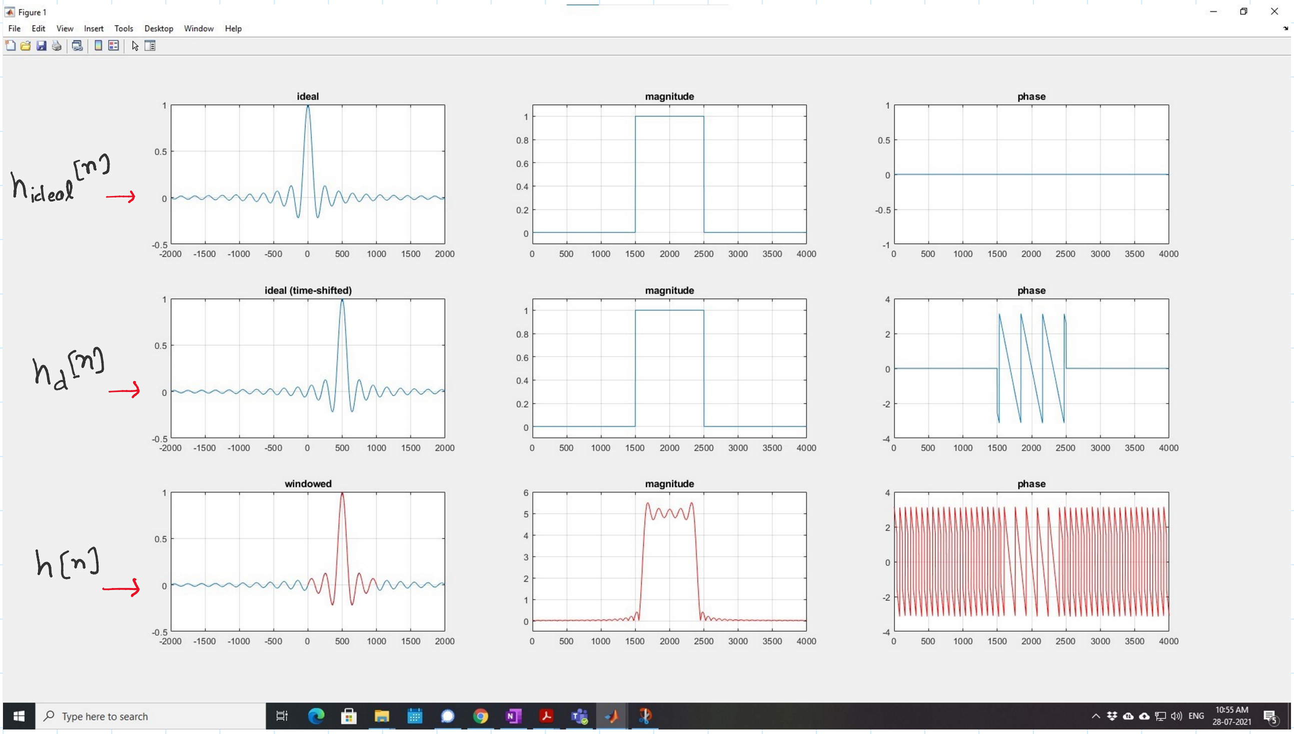
$$W(e^{j\omega}) = \frac{\sin\left(\omega \frac{N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)} e^{-j\omega M}, \quad M = \frac{N-1}{2}, \quad N\text{-odd}$$

Steps

① $h_{ideal}[n] \rightarrow$

② $h_d[n] \rightarrow$

③ $h[n] \rightarrow$



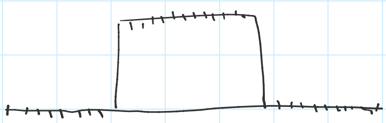
★ different windows will give different amounts of

ⓐ transition band width

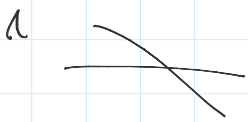
ⓑ side-lobe levels

⊛ FIR filter design using frequency sampling.

$H_d(e^{j\omega})$



$-\pi$ $\quad\quad\quad\quad\quad\quad\quad$ π



$H[k] = H_d(e^{j\omega})\Big|_{\omega=\omega_k}$

| inverse DFT

$\Big|_{\omega=\omega_K}$ $-\pi$ π

$\Big\downarrow$ inverse DFT

$$h[n] \xrightarrow{DTFT} H(e^{j\omega}) \quad \text{with} \quad H[k] = H(e^{j\omega})\Big|_{\omega=\omega_K}$$