Lecture - 20

07 July 2021 10:05 AM

Discrete Fourier transform (DFT) $x[n] \xleftrightarrow{DFT} X[k]$

$$\left.\begin{aligned} \text{DFT}: \quad & X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn} \\ \text{IDFT}: \quad & x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi}{N}kn} \end{aligned}\right\} \longrightarrow \text{N-length sequences}$$

- periodicity property of $x[n]$ & $X[k]$
- DFT as samples of DTFT spectrum
- DFT as linear combination
- DFT as linear transformation, DFT matrix (F_N)

$\underline{X} = F_N \underline{x}$

* $x[0], x[1]$ length-2 sequence

$$F_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \Rightarrow \begin{matrix} X[0] \\ X[1] \end{matrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x[0] \\ x[1] \end{bmatrix} = \begin{bmatrix} x[0] + x[1] \\ x[0] - x[1] \end{bmatrix}$$

Ex.

$$x[n] = \begin{cases} 1, & n = 0, 1, \cdots L-1 \\ 0, & n = L, \cdots N-1 \end{cases} \Rightarrow X[k] = e^{-j\frac{\pi}{N}k(L-1)} \frac{\sin\left(\frac{\pi k L}{N}\right)}{\sin\left(\frac{\pi k}{N}\right)}$$

Q. what happens as N is increased for fixed L ?

$\longrightarrow$

* special case, $N = L$.

$$|X[k]| = \frac{\left|\sin\left(\frac{\pi k L}{N}\right)\right|}{\left|\sin\left(\frac{\pi k}{N}\right)\right|}$$

$$x[n] = \{\underset{\substack{\uparrow \\ x[0]}}{1}, 1 \cdots 1\}$$

$X[k]$ is sample of $X(e^{j\omega})$

at $\omega = \frac{2\pi k}{N}$, $k = 0, 1, \cdots N-1$

$$\text{DFT} \Rightarrow X[k] = \{\underset{\substack{\uparrow \\ X[0]}}{N}, 0, \cdots 0\}$$

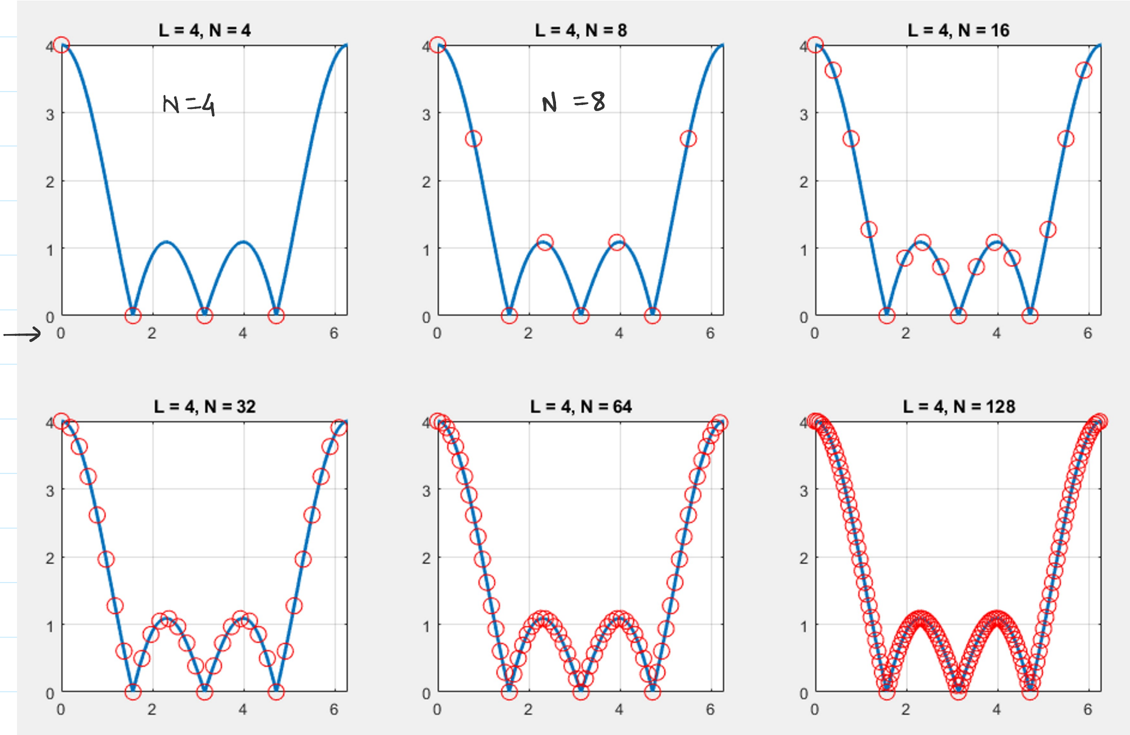
* $N > L$

periodic sinc

Blue - DTFT of $x[n]$ ie. $|X(e^{j\omega})|$

red - DFT of $x[n]$ for $N \uparrow$

$[0, 2\pi] \rightarrow$



* As N increases, we get more samples of DTFT

* zero padding - adding zeros to given sequence & computing is DFT.

as more zeros are added, DFT approaches DTFT.

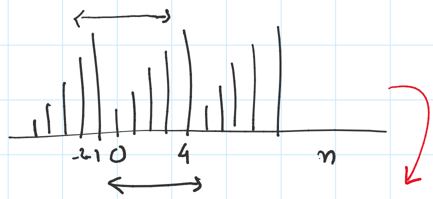
$x[n] \longrightarrow x[n] + c[n]$ } any other padding will not work; it will distort (try in matlab & see!)

* circular shift of $x[n]$

Ex. $x[n] = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$

$y[n] = x[n-1] = \{\underset{\uparrow}{5}, 1, 2, 3, 4\}$

$y[0]$



$x[n-1]$, for $n=0$, $\underbrace{x[-1] = x[-1+5]} = x[4]$

* for circular-shift & time reversal, useful to think of $x[n]$ periodic

$x[n+1] = \{\underset{\uparrow}{2}, 3, 4, 5, 1\}$

$x[n-n_0] \equiv x[\langle n-n_0 \rangle_N]$ modulo operator $\rightarrow$ mod $(n-n_0, N)$

Reversal of time i.e. $x[-n] \equiv x[\langle -n \rangle_N]$

$$y[n] = x[-n] = \{ \underset{\uparrow}{1}, 5, 4, 3, 2 \}$$

$x_1[n]$ & $x_2[n]$ N – length seq.

$n = 0, 1, \cdots N-1$

linear convolution : $x_3[n] = x_1[n] * x_2[n]$ } → length = $2N-1$

Ex. $\{ \underset{\uparrow}{1}\ 2\ 1\ 2 \} * \{ \underset{\uparrow}{1}\ 2\ 3\ 4 \}$

regular shift → $x_2[-n] = \{ 4\ 3\ 2\ \underset{\uparrow}{1}\ 0\ 0\ 0 \}$

Circular convolution $x_3[n] = x_1[n] \circledast x_2[n]$

↑ length – N sequence.

$$x_3[n] = \sum_{m=0}^{N-1} x_1[m]\, \underbrace{x_2[n-m]}_{\text{circular shift}}$$ ← $x_3[n]$ will be length N.

Ex. $\underbrace{\{ \underset{\uparrow}{1}\ 2\ 1\ 2 \}}_{x_1[m]} \circledast \underbrace{\{ \underset{\uparrow}{1}\ 2\ 3\ 4 \}}_{x_2[m]}$

$$x_3[0] = \sum_{m=0}^{3} x_1[m]\, x_2[-m]$$

$\{ 1\ 2\ 1\ 2 \}$ ← $x_1[m]$

×

$\{ 1\ 4\ 3\ 2 \}$ ← $x_2[-m]$

$\{ 1\ 8\ 3\ 4 \} \xrightarrow{\text{sum}} 16$

similarly $x_3[1]$, $x_3[2]$, $x_3[3]$.