Filtering Techniques For Dynamic System Identification

Report for End-Semester evaluation of CE499 course

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CERTIFICATE

It is certified that the work contained in the project report entitled **" Filtering Techniques For Dynamic System Identification "**, by **M.Karthik (200104056)** has been carried out under my/our supervision and that this work has not been submitted elsewhere for the award of a degree or diploma.

Date:- 29-04-2024

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Date:- 29-04-2024 **Male Karthik**

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Introduction

System identification plays a crucial role in assessing the condition, damage, and deterioration of structures while also predicting their responses to various external loads. This field originated in electrical engineering and subsequently expanded into mechanical, control, and aeronautical engineering. In recent years, its significance has grown within structural engineering due to its applications in predicting structural responses and assessing the condition of buildings and infrastructure. The importance of system identification in structural engineering can be broken down into several key areas **Assessing Existing Condition:** System identification methods are employed to evaluate the current state of a structure. This involves gathering data and analyzing it to understand the structural behavior and detect any existing damage or deterioration. **Predicting Responses:** System identification techniques are used to predict how a structure will respond to different types of external loads, such as earthquakes, wind, or traffic loads. This predictive capability is invaluable for ensuring the safety and performance of structures. **Degree of Damage and Deterioration:** By analyzing data collected from sensors or monitoring systems, engineers can estimate the degree of damage and deterioration in a structure. This information is critical for making informed decisions about maintenance, repair, or retrofitting. A wide range of system identification techniques has been developed for structural engineering. These techniques include traditional methods like modal analysis and finite element modeling, as well as more recent innovations in filtering techniques. In recent years, identification using Filtering techniques, such as the Kalman filter and particle filter, have gained popularity among researchers in structural engineering. These filters are used to estimate the current state of a system based on noisy or incomplete sensor data. They are particularly useful for real-time monitoring and control of structures. Overall, system identification is an interdisciplinary field that bridges various engineering disciplines. It has become increasingly important in recent years as infrastructure ages, and the need for accurate predictions and assessments of structural behavior continues to grow. Researchers and engineers in structural engineering are constantly exploring new methods and technologies to improve the accuracy and efficiency of system identification processes, ensuring the safety and reliability of critical structure.

1.1 The General Problem of system Identification

In **structural engineering**, structures like buildings and bridges are designed to serve specific functions. However, over time, they can experience degradation due to various factors such as environmental stresses or repeated use. **System identification** is a crucial process that allows engineers to assess the current state of a structure by understanding how it responds to certain forces, like earthquakes. This involves determining the structure's current behavior and characteristics, known as the "state of the system." Engineers then expose the structure to known inputs, such as ground acceleration, and measure its response using tools like accelerometers. While these measurements provide valuable data, they can sometimes contain inaccuracies due to device limitations. To make sense of these measurements, engineers use *mathematical models*—theoretical representations of the structures. The primary goal is to refine these models using the collected data, ensuring they accurately predict the structure's future behavior. This refinement often involves pinpointing unknown parameters within the models, a process termed "parameter estimation."

The Filtering Problem: The filtering problem can be described graphically in Figure 1.2. The system's state is signified by X_t and is seen through a distorted measurement, Y_t. Noise introduced into the system is denoted by W_t, while V_t represents the noise in the measurement. Using a filter, we derive an estimated value X_t' from the observed data. The task of identifying a system's state based on distorted measurements is termed filtering.

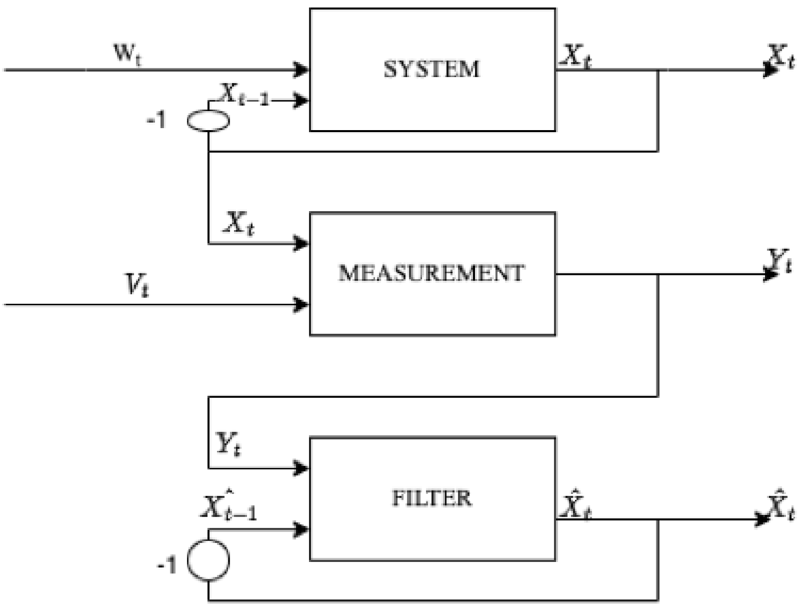

Figure 1.1: The Filtering problem

1.2 Optimal Filters: Kalman Filter

While Bayesian filtering equations are effective for addressing tracking challenges, it is crucial to determine appropriate methods for their application. Various approaches exist for deploying Bayesian filters effectively, especially when specific conditions are met, allowing for the identification of an optimal resolution to the tracking issue. Such methods encompass the Kalman filter and the grid-based filter. The Kalman filter represents the definitive solution to the Bayesian filtering equations when the posterior density remains Gaussian at each time step and the state space model is linear. If the system is linear, the equations are given by:

$$x_k = M_{k-1}x_{k-1} + N_{k-1}u_{k-1} + v_{k-1} \tag{1.1}$$

$$y_k = H_k x_k + n_k \tag{1.2}$$

Where M_{k-1} is the transition Matrix, N_{k-1} is the input matrix, H_{k-1} is the measurement matrix, $x_k \in \mathbb{R}^n$ is the state at time step k, $y_k \in \mathbb{R}^n$ is the measurement, u_{k-1} is the input vector, $v_{k-1} \sim \mathcal{N}(0, Q_{k-1})$ is the process and $n_k \sim \mathcal{N}(0, R_k)$ is the measurement noise. It also assumes that the prior distribution is Gaussian $x_0 \sim \mathcal{N}(m_0, P_0)$.

Considering the property that a gaussian random variable $X \sim \mathcal{N}(\mu, \sigma^2)$ can be written as $X \sim \mu + \sigma\mathcal{N}(0, 1)$, Equations (1.1) and (1.2) can be expressed in probabilistic term as

$$\begin{aligned} p(x_k|x_{k-1}) &= M_{k-1}x_{k-1} + N_{k-1}u_{k-1} + v_{k-1} \\ &= M_{k-1}x_{k-1} + N_{k-1}u_{k-1} + \mathcal{N}(0, Q_{k-1}) \\ &= \mathcal{N}(x_k|M_{k-1}x_{k-1} + N_{k-1}u_{k-1}, Q_{k-1}) \end{aligned} \tag{1.3}$$

$$\begin{aligned} p(y_k|x_k) &= H_k + n_k \\ &= H_k x_k + \mathcal{N}(0, R_k) \\ &= \mathcal{N}(y_k|H_k x_k, R_k) \end{aligned} \tag{1.4}$$

The probability distribution at time step $k-1$ serves as the prior distribution for time step k, in a manner consistent with the iterative estimation process. The expression for this is as follows:

$$p(x_{k-1}|y_{1:k-1}) = \mathcal{N}(x_{k-1}|m_{k-1}, P_{k-1}) \tag{1.5}$$

Where in this context, m and P represent the mean and covariance of the said distribution. By leveraging these three statistical distributions, one can unravel the Bayesian filtering equations. This method is structured into two phases: the *Prediction phase* and the *Update phase*. Initially, the dynamical system is utilized to estimate or predict the state for the upcoming time step. Subsequently, this prediction is refined using the measurement obtained in the prior phase.

Prediction step:

Initially, it is necessary to compute the joint distribution of x_k and x_{k-1} given $y_{1:k-1}$. It is the same that the joint distribution of $p(x_k|x_{k-1})$ and $p(x_{k-1}|y_{1:k-1})$ in Equations (1.3) and (1.5). Considering the equations for joint distribution of Gaussian random variables

$$\begin{aligned} p(x_{k-1}, x_k|y_{1:k-1}) &= p(x_k|x_{k-1})p(x_{k-1}|y_{1:k-1}) \\ &= \mathcal{N}\left(\begin{bmatrix} x_{k-1} \\ x_k \end{bmatrix} |m', P'\right) \end{aligned} \tag{1.6}$$

Where

$$\begin{aligned} m' &= \begin{pmatrix} m_{k-1} \\ M_{k-1}m_{k-1} + N_{k-1}u_{k-1} \end{pmatrix}, \\ P' &= \begin{pmatrix} P_{k-1} & P_{k-1}M_{k-1}^T \\ M_{k-1}P_{k-1} & M_{k-1}P_{k-1}M_{k-1}^T + Q_{k-1} \end{pmatrix}, \end{aligned}$$

Using Equation (1.6) in the Chapman-Kolmogorov equation is computed the marginal distribution of x_{kp} necessary for the next step.

$$p(x_k|y_{1:k-1}) = \mathcal{N}(x_k|m_k^*, P_k^*) \tag{1.7}$$

Where

$$m_k^* = M_{k-1}m_{k-1} + N_{k-1}u_{k-1},$$
$$P_k^* = M_{k-1}P_{k-1}M_{k-1}^T + Q_{k-1}.$$

Update step:

Here, the joint distribution of y_k and x_k is needed. It can be obtained from Equations (1.4) and (1.7) and using joint distribution of Gaussian variables equation.

$$p(x_k, y_k|y_{1:k-1}) = p(y_k|x_k)p(x_k|y_{1:k-1}) = \mathcal{N}\left(\begin{bmatrix} x_k \\ y_k \end{bmatrix} |m'', P''\right) \tag{1.8}$$

Where

$$m'' = \begin{pmatrix} m_k^* \\ H_k m_k^* \end{pmatrix},$$
$$P'' = \begin{pmatrix} P_k^* & P_k^* H_k^T \\ H_k P_k^* & H_k P_k^* H_k^T + R_k \end{pmatrix}.$$

Finally, the conditional distribution of x_k and $y_{1:k}$ is computed from Equation (1.8) and by using conditional distribution of Gaussian variables equations

$$p(x_k|y_{1:k}) = \mathcal{N}(x_k|m_k, P_k) \tag{1.9}$$

Where

$$m_k = m_k^* + K_k r_k, \quad P_k = P_k^* - K_k S_k K_k^T$$

and

$$r_k = y_k - H_k m_k^*, \quad S_k = H_k P_k^* H_k^T + R_k, \quad K_k = P_k^* H_k^T S_k^{-1}$$

All the previous steps are summarized below in Algorithm 1: Kalman filter algorithm

Algorithm 1 Kalman filter algorithm

Initialize m_0 and P_0
for $k = 1$ to T **do**
    Prediction step:
    $m_k^* = M_{k-1}m_{k-1} + N_{k-1}u_{k-1}$
    $P_k^* = M_{k-1}P_{k-1}M_{k-1}^T + Q_{k-1}$
    Update step:
    $r_k = y_k - H_k m_k^*$
    $S_k = H_k P_k^* H_k^T + R_k$
    $K_k = P_k^* H_k^T S_k^{-1}$
    $m_k = m_k^* + K_k r_k$
    $P_k = P_k^* - K_k S_k K_k^T$
end for

1.3 Uncented Kalman filter

In the Uncented Kalman Filter, the state distribution is represented by a Gaussian random variable, like in the Kalman Filter, which is specified by a set of minimal points that are carefully chosen. The task of these sample points is to capture the true mean and covariance of the Gaussian distribution; once these points are propagated through the full nonlinear system, the posterior mean and covariance are represented with a second order (Taylor series expansion) precision. In the UKF, the process and measurement equations are not required to be formally differentiable, however, the UKF requires slightly more computational operations than the EKF.

The UKF has a lot of different applications in non-linear estimation problems, including non-linear system identification, training of neural networks and dual estimation problems. Here we use as reference [25] and [27]. To develop the mathematical formulation of the UKF, is necessary to review the Un- scented Transform.

Unscented Transform:

The Unscented Transform (UT) is a numerical method that tries to approximate the mean and the covariance of a distribution instead of approximate the non-linear function. This is done by choosing a fixed number of the so-called sigma points that capture the mean and covariance of the original distribution exactly. Although the UT resembles Monte Carlo estimation (which is going to be described later in the chapter), the UT sigma points are selected deterministically.

Consider two random vectors defined as

$$\mathbf{X} \sim \mathcal{N}(\mathbf{m}_x, \mathbf{P}_x)$$

$$\mathbf{Y} = \mathbf{f}(\mathbf{X})$$

to calculate the statistics of $\mathbf{Y}$, a matrix $\mathbf{X}$ of $2n+1$ sigma vectors $\mathbf{x}_i$ is formed as follows

$$\mathbf{x}_0 = \mathbf{m}_x$$

$$\mathbf{x}_i = \mathbf{m}_x + \left(\sqrt{(n+\lambda)\mathbf{P}_x}\right)_i \quad i = 1, \ldots, n$$

$$\mathbf{x}_{i+n} = \mathbf{m}_x - \left(\sqrt{(n+\lambda)\mathbf{P}_x}\right)_i \quad i = 1, \ldots, n$$

where n is the length of $\mathbf{Y}$ and $\lambda = \alpha^2(n+\kappa)-n$ is a scaling parameter. The constant α determines the spread of the sigma points around the mean. The constant κ is a secondary scaling parameter.$(\sqrt{\mathbf{P}_x})_i$ denotes the ith column of the matrix square root such that $\sqrt{P}\sqrt{P}^T = P$. This square root can be estimated, for example, by the *Cholesky decomposition*.

These sigma vectors are propagated through the non-linear function $\mathbf{f}(\cdot)$ as them

$$\mathbf{y}_i = \mathbf{f}(\mathbf{x}_i) \quad i = 0, \ldots, 2n$$

which are the transformed sigma points.

Then, the mean and covariance of the transformed variable are approximated using a weighted sample mean and covariance of the posterior sigma points:

$$\mathbb{E}[\mathbf{f}(\mathbf{X})] \approx \mathbf{m}_y = \sum_{i=0}^{2n} W_i^{(m)} \mathbf{y}_i$$

$$\mathrm{Cov}[\mathbf{f}(\mathbf{X})] \approx \mathbf{P}_y = \sum_{i=0}^{2n} W_i^{(c)} (\mathbf{y}_i - \mathbf{m}_y)(\mathbf{y}_i - \mathbf{m}_y)^T$$

Where the constant weights $W_0^{(m)}$ and $W_i^{(c)}$ are given by

$$W_0^{(m)} = \frac{\lambda}{n+\lambda}, \quad W_0^{(c)} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta)$$

$$W_i^{(m)} = W_i^{(c)} = \frac{\lambda}{2(n+\lambda)} \quad \text{for } i = 1, \ldots, 2n$$

And β is used to incorporate prior knowledge of the distribution of $\mathbf{X}$. The unscented transform based Gaussian approximation to the joint distribution of $\mathbf{X}$ and the transformed random variable $\mathbf{Y} = \mathbf{g}(\mathbf{X}) + \mathbf{q}$, where $\mathbf{X} \sim \mathcal{N}(\mathbf{m}, \mathbf{P})$ and the noise $\mathbf{q} \sim \mathcal{N}(0, \mathbf{Q})$, is given as

$$\begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mathbf{m}_x \\ \mathbf{m}_y \end{pmatrix}, \begin{pmatrix} \mathbf{P}_x & \mathbf{P}_{x,y} \\ \mathbf{P}_{x,y}^T & \mathbf{P}_y \end{pmatrix}\right)$$

Unscented Kalman filter algorithm:

With the information of Unscented transform algorithm, it is possible to build the algorithm for the Unscented Kalman filter for the additive-noise case, considering the additive-noise the dynamical model equations are rewritten as

$$\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}) + \mathbf{v}_{k-1}$$

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{n}_k$$

where $\mathbf{f}(\cdot)$ is the dynamic process equation, $\mathbf{h}(\cdot)$ is the measurement equation, $\mathbf{x}_k$ is the state, $\mathbf{y}_k$ is the measurement, $\mathbf{v}_{k-1} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{k-1})$ is the process noise and $\mathbf{n}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k)$ is the measurement noise. It also assumes that the prior distribution of the initial states Gaussian $x_0 \sim \mathcal{N}(m_0, P_0)$, following the dynamical model, the state transition probability $p(x_k|x_{k-1})$ necessary in Bayesian probability equations and the measurement probability or likelihood $p(y_k|x_k)$ are determined by the state-space model. With that, it is possible to say that the process noise density v_{k-1} determines $p(y_k|x_k)$. For example, given that $v_{k-1} \sim \mathcal{N}(0, Q_k)$ and $x_k = f(x_{k-1}) + v_{k-1}$, then, the state transition probability is equal to

$$p(x_k|x_{k-1}) \sim \mathcal{N}(f(x_{k-1}), Q_{k-1}) \tag{1.10}$$

and using $n_k \sim \mathcal{N}(0, R_k)$ and $y_k = h(x_k) + n_k$, then, the likelihood distribution is given as

$$p(y_k|x_k) \sim \mathcal{N}(h(x_k), R_k) \tag{1.11}$$

Knowing the probability distributions from Equations 1.2.1 and 1.2.2 and that the main idea is to assume Gaussian approximations, the posterior distribution is considered

$$p(x_k|y_{1:k}) \sim \mathcal{N}(x_k|m_k, P_k)$$

for every step k, where m_k and P_k are the mean and covariance computed by the algorithm.

This assumption leads to a recursive estimation

$$m_k = \text{prediction of } x_k + K_k[y_k - \text{prediction of } y_k]$$

$$P_k = \text{covariance of } x_k - K_k[\text{covariance of } y_k]K_k^T$$

But this is a linear recursion, and linearity of the model has not been assumed. The optimal terms are given by

$$m_k = \mathbb{E}[f(x_{k-1}) + v_{k-1}]$$
$$K_k = P_{x_k y_k} P_{y_k y_k}^{-1}$$
$$\mu_k = \mathbb{E}[h(x_k) + n_k]$$

These terms are computed exactly by the Kalman filter in the linear case. But, for non-linear models, these terms can be used as an approximation.

Literature Review

System identification is an important step toward evaluating the existing condition, assessing the degree of damage and deterioration, and predicting the response of structures. In recent years, identification using filtering techniques such as Kalman filter and particle filter has attracted many researchers. Some of the studies are presented in this section.

Swarup Mahato and A. Chakraborty [20] in thier research, focused on employing the Extended Kalman Filter (EKF) for parameter identification of Linear Time Invariant (LTI) systems using seismic response measurements. The researchers conduct experiments using a laboratory model consisting of a three-storied steel frame subjected to ground motion. They record the acceleration response at the top of the structure and apply the EKF-based predictor-corrector algorithm to identify the model parameters, particularly stiffness values. The result details the laboratory model setup and the application of the EKF algorithm. The stiffness values at different floor levels and the corresponding natural frequencies are presented and compared with their original values. Despite variations in stiffness values across different tests, the eigen solutions consistently converge to their original values, indicating the algorithm's robustness in capturing the dynamic characteristics of the structure. The conclusion summarizes the key findings of the research. It highlights the convergence behavior of the EKF algorithm to different stiffness values in different tests but emphasizes the consistency in eigen solutions, essential for accurate structural assessment.

The research conducted by Hoshiya and Saito [10] delves into the application of the Extended Kalman Filter (EKF) for addressing system identification challenges in seismic structural systems. They propose a novel approach aimed at enhancing the stability and convergence of solutions by introducing a weighted global iteration procedure with an objective function integrated into the EKF algorithm. Through comprehensive investigations across various structural systems, including multiple degree-of-freedom linear systems, bilinear hysteretic systems, and equivalent linearization of bilinear hysteretic systems, Hoshiya and Saito demonstrate the efficacy of their proposal. The numerical examples provided in the abstract and conclusion sections illustrate the practical utility of the weighted global iteration procedure in achieving stable estimations, thereby offering promising prospects for advancing system identification methodologies in structural engineering applications. In summary, the contributions of Hoshiya and Saito highlight the importance of incorporating innovative techniques, such as the weighted global iteration procedure, into the EKF algorithm to address system identification challenges effectively. Their research findings provide valuable insights into optimizing structural identification methodologies, with broad applications across diverse structural systems. Toki, Sato, and Kiyono [12] present a novel procedure for identifying structural parameters and input ground motion solely from measured responses. They leverage the assumption that the coda segment of the response time history represents the free vibration response of the structural system, unaffected by the input ground motion. By initially identifying structural parameters such as masses, damping coefficients, and spring constants from this segment, they then estimate the input ground motion from the entire record using the identified parameters. The authors employ the Kalman filter for the identification and estimation process. Through simulations on a linear, three-degree-of-freedom system with various earthquake inputs, they verify the efficacy of their procedure. Their results demonstrate that structural parameters can be accurately identified from the coda portion of the response time history, even without knowledge of the input motion. Moreover, they extend the use of the Kalman error filter to multi-degree-of-freedom systems and investigate its efficiency in estimating input motion from structural response and identified structural parameters. The study concludes that their serial procedure enables both the identification of structural parameters and the estimation of input motion solely from the seismic response of a structure, showcasing promising advancements in structural identification methodologies. In summary, the contributions of Toki, Sato, and Kiyono underscore the significance of their innovative procedure in extracting valuable information from response time histories for structural parameter identification and input ground motion estimation. Their findings offer valuable insights into optimizing structural identification methodologies, with broad applications across diverse structural systems. Feng Gao, Yong Lu [9] presents a procedure for the time- sphere analysis of noise- defiled vibration signals for global structural damage opinion. The paper extends from a preliminarily established acceleration response-only time- sphere bus-Accumulative- with- e Xogenous input(ARX) model, where the acceleration response at a given degree of freedom(dof) is regarded as the input, while the accelerations at other dofs are the " state " with which the " measures " are associated. The new idea in the present procedure is to recoup the natural input – affair set from noisy signals by using the Kalman sludge, so that the underpinning physical system is stylish presented to the posterior opinion operation. The theoretical base of representing the system by pairing the raw measured input and the filtered response through the

Kalman sludge is bandied. When similar raw input and filtered response signals are fed into the reference ARX model, the error point becomes reflective of the change of the physical system. By assaying the residual error, the damage status of the structure can be diagnosed. The paper also examines the ARX model concerned for its vulnerability to the noise content in the measured signals. The applicable way of using the Kalman sludge to reuse the noisy signals, given the ultimate ideal as being to save the state information, and the effectiveness of such an approach are demonstrated mathematically. The paper presents numerical and experimental exemplifications to demonstrate the effectiveness of the approach in diving the noises, and both the circumstance and relative extent of damage can be assessed with an applicable damage point. Qijun xia, Ming Rao, Yiqun Ying and Xuemin Shen [28] focuses on the application of the Kalman filter to the head box section of a paper machine, where the filter is used to estimate the dry basis weight and other states of the system. The authors also propose an adaptive fading Kalman filter (AFKF) to address the issue of model errors and divergence in the Kalman filter. The authors consider a linear, discrete-time, stochastic multivariable system with state vector , measurement vector and system matrices. The Kalman filter equations are presented, but the authors also introduce the concept of exponential fading of past data via forgetting factors to address the issue of model errors and divergence. The paper presents three algorithms for choosing optimal forgetting factors to improve the convergence and optimality of the Kalman filter. The authors also introduce a performance criterion based on the residual covariance to judge the optimality of the filter. The steepest descent AFKF algorithm is proposed, which iteratively adjusts the forgetting factor to minimize the performance criterion. The paper presents simulation results with unknown drifts, demonstrating the effectiveness of the AFKF in tackling the noises and both the occurrence and relative extent of damage. The AFKF is shown to be more robust and convergent than the normal Kalman filter in the presence of model errors. The paper applies the AFKF to the head box section of a paper machine, where the filter is used to estimate the dry basis weight and other states of the system. The authors demonstrate that the AFKF can be successfully applied to the head box section of a paper machine for state estimation.

JannN. Yang, Silian Lin, Hongwei Huang, and Li Zhou proposes [29] an adaptive shadowing fashion for the extended Kalman sludge(EKF) to track changes in structural parameters, similar as stiffness and damping, in the presence of damage events. The adaptive factor matrix, denoted is a crucial element of the proposed fashion. The matrix is slant, with the first 2m slant rudiments set to1.0, and the remaining n slant rudiments, corresponding to unknown parameters. The adaptive shadowing fashion is grounded on the current measured data to determine the parameter variations, so that the residual error is contributed only by noise. The adaptive factor matrix is acclimated to minimize the sum of the squared estimation crimes, which are expressed in terms of the unknown parameters. The paper presents simulation results with nonlinear elastic, hysteretic, and direct structures to demonstrate the effectiveness of the proposed fashion in tracking the changes of system parameters when vibration data involve damage events. The paper applies the adaptive shadowing fashion to a two- storey nonlinear elastic- storey hysteretic structure, five- storey direct structure, ASCE Phase 1 standard structure ways able of tracking the abrupt changes of structural parameters due todamages. The paper presents a new approach to address the issue of model crimes and divergence in the Kalman sludge, and demonstrates the effectiveness of the adaptive shadowing fashion in diving the noises and both the circumstance and relative extent of damage. The adaptive factor matrix is a crucial element of the proposed fashion, and its slant rudiments are acclimated to minimize the sum of the squared estimation errors. The paper also applies the adaptive shadowing fashion to colourful structures, including nonlinear elastic, hysteretic, and direct structures, to demonstrate the effectiveness of the proposed fashion in tracking the changes of system parameters when vibration data involve damage events.

R. E. Kalman, R. S. Bucy [14] highlights the fundamental equations and their derivation, as well as the proofs and analytic solutions of the variance equation. The fundamental equations are derived using the state-transition method and linear filtering regarded as orthogonal projection in Hilbert space. The equations include the differential equation governing the optimal filter, error differential equations, time-varying gains, and the nonlinear differential equation governing the covariance matrix of the errors, known as the variance equation. The proofs are outlined based on the Duality Principle, which provides a link between stochastic filtering theory and deterministic control theory. The variance equation is of the Riccati type, and its analytic solutions are discussed, which are of great interest in the theory of adaptive systems. In conclusion highlights the importance of the Duality Principle and the Riccati equation in the field of adaptive systems, as well as the potential applications of these results to a variety of problems. Jeen- Shang Lin and Yigong Zhang [16] presents a study on the performance of a weighted global replication for the extended Kalman sludge in relating the parameters of a hysteretic SDOF system defined The paper investigates the goods of noise on the identification and the proper selection of observation variables. The paper presents several vital sections, including original estimates, observances that contain no noise, observances that contain non-white noises, and change the observation variable. The paper discusses about the weighted global replication procedure was used to modernize the state and error covariance associated with parameters to be linked. The paper set up that the proper selection of observation variables was important and that the presence of non-white noise in the obedience's didn't appear to constitute a problem. still, identification deteriorated as the noise- to- signal rate increased. The paper also discusses the goods of noise on the identification of the parameters of the system. The paper used the approach of Levine to model the noise as compatible with that to digitize a straight line. The paper introduced noises singly to both the base and the responses accelerations, and from them, the relative acceleration, haste, and deportations were attained. Because of integration, non-white noises were introduced into the haste and expatriation obedience's.

Alberto Corigliano and Stefano Mariani [7] investigates the performance of the extended Kalman filter (EKF) for structural systems subject to dynamic loadings, aiming to simultaneously estimate the state and calibrate constitutive parameters. The paper focuses on both single degree-of- freedom (DOF) and multi-DOF dynamic systems. For single DOF systems, the EKF is used to estimate the state (nodal displacements and velocities) and the parameters of the constitutive model. The paper demonstrates that the EKF can accurately estimate the state of the system, but unsatisfactory parameter estimations can be obtained in the case of degrading strength of the structure, i.e., in the case of softening. For multi-DOF systems, the paper investigates the performance of the EKF in estimating the state and calibrating constitutive parameters. The paper demonstrates that the EKF can accurately estimate the state of the system, but unsatisfactory parameter estimations can be obtained in the case of degrading strength of the structure, i.e., in the case of softening. The paper presents a study on the performance of the EKF for nonlinear structural identification using a simulated earthquake input- response pair. The paper investigates the effects of noise on the identification and the proper selection of observation variables. The paper found that the proper selection of observation variables was important and that the presence of non-white noise in the observations did not appear to constitute a problem. However, identification deteriorated as the noise-to-signal ratio increased. The paper provides valuable insights into the use of the extended Kalman filter for nonlinear structural identification. The paper found that the proper selection of observation variables was important and that the presence of non-white noise in the observations did not appear to constitute a problem. However, identification deteriorated as the noise-to-signal ratio increased. Tadanobu SATO and Makoto SA [26] discusses the application of neural network and Kalman filter algorithms in structural identification. The authors, Tadanobu Sato and Makoto Sato, present a learning algorithm using the extended Kalman filter to identify the dynamic characteristics of structural systems. The survey is divided into several sections, including an introduction, discussion of neural network characteristics, the back propagation algorithm, learning algorithm using the extended Kalman filter, application to a linear structure, application to a model building, application to stimulate nonlinear structural response, and a conclusion. The authors discuss the limitations of frequency domain analysis methods and the need for time domain data in identifying dynamic characteristics of structural systems. They highlight the challenges of conventional algorithms, such as instability, non-uniqueness, and excessive calculation time, and propose the use of neural networks and the extended Kalman filter to address these issues. The survey provides a detailed explanation of the neural network model, including the multi-layer perceptron structure and the back propagation algorithm for learning. It also introduces the extended Kalman filter as a method to identify dynamic characteristics of a structural system proof against contaminating noise in teaching signals. The application of the algorithm to a linear structure, a two-story building, is demonstrated. The authors discuss the use of noise-contaminated teaching signals and the stability and robustness of the proposed algorithm in identifying dynamic response characteristics of linear and non-linear structural systems. The results obtained using the back propagation algorithm are shown to be strongly affected by noise, while the Kalman filtering algorithm provides stable learning results. The survey further presents the application of the algorithm to a model building, an eight-story steel frame model of a shear building. The authors discuss the use of real observed data from experiments conducted for seismic response control analyses and demonstrate the identification of the dynamic parameters, natural frequency, and damping constants of the model building. In conclusion, the authors emphasize the robustness of the learning algorithm based on the extended Kalman filtering technique, particularly in the presence of noise in the teaching signals. They also highlight the stability and efficiency of the proposed algorithm in identifying the dynamic characteristics of structural systems. Eleni Chatzi and Andrew Smyth [6] focusing on dual state and parameter estimation, provides a comprehensive comparison of these methods for nonlinear structural system identification with heterogeneous sensing. The UKF is a nonlinear state estimation algorithm that approximates the posterior density by a set of deterministically chosen points. It is more robust and efficient than the Extended Kalman Filter (EKF) for highly nonlinear systems. It is particularly useful for non-Gaussian and nonlinear systems, but it can be computationally expensive. The system is subject to non-collocated heterogeneous sensing, which means that acceleration and displacement measurements are available for different degrees of freedom. The UKF and PF are both effective for dual state and parameter estimation in nonlinear structural systems. The UKF is computationally more efficient and less sensitive to the choice of initial conditions and sample depletion. The PF, however, is more suitable for non-Gaussian and nonlinear systems, but it can be computationally expensive.

Hoshiya and Maruyama [19], along with Yun and Shinozuka [5], have been instrumental in advancing this field. Their research underscores the importance of system identification in assessing structural responses to external stimuli and gauging their overall health. The paper provides an in-depth perspective on the Kalman filter's application in system identification within structural engineering. This technique, which has garnered significant attention in recent years, is adept at filtering out noise and ensuring accurate parameter estimation. The research meticulously outlines the linear discrete Kalman filter algorithm and introduces a weighted global iteration procedure, ensuring convergence in parameter estimation. An illustrative example, focusing on a single degree-of-freedom system, further cements the practicality of the Kalman filter in system identification.In conclusion, the research offers a panoramic view of system identification using the Kalman filter in the structural engineering landscape. It emphasizes the intricate relationship between system identification, parameter estimation, and filtering. The paper also underscores the vast potential for further exploration, especially in the realm of in-situ structures, advocating for continued research and innovation in this pivotal area of structural engineering. Ambient vibration surveys have emerged as a pivotal tool in understanding the structural performance of large civil infrastructures, such as bridges. Over the years, while there have been significant technological advancements in data collection, the processing techniques have often struggled to reliably extract intricate parameters, notably damping.

The research paper by Nicholas P. Jones, Tinghui Shi, J. Hugh Ellis, and Robert H. Scanlan from The Johns Hopkins University [22] delves deep into this challenge, presenting a method rooted in the application of the Kalman filter for the identification of both system and excitation characteristics. The paper underscores the limitations of wind tunnel modeling due to scaling effects, emphasizing the irreplaceable value of full-scale testing and measurement for structures. While there have been short-term ambient vibration studies on several bridges, the primary focus has been on determining the dynamic characteristics of the structure rather than assessing wind effects. The research highlights the challenges associated with estimating parameters such as damping, especially when data is corrupted with significant noise. The paper introduces a promising technique based on the application of a Kalman filter, which has shown potential in extracting meaningful signals from noisy data, enabling simultaneous estimation of both system and input characteristics. Furthermore, the research delves into the intricacies of system identification in both time and frequency domains. For single-degree-of-freedom (SDOF) systems, the paper presents equations of motion and their representation in augmented state-space form. The Kalman filter's application allows for the simultaneous estimation of state variables, system frequency, and mechanical damping components. The frequency domain formulation, on the other hand, offers advantages such as the natural introduction of the power spectral density (PSD) of the input as a function with parameters and a reduction in the number of state variables compared to the time domain formulation. In conclusion, the research by Jones et al. offers a comprehensive insight into the challenges and solutions associated with system identification in ambient vibration surveys. The application of the Kalman filter, as presented in the paper, holds significant promise in advancing the field and ensuring more accurate and reliable system identification Yaohua YANG, Tomonori NAGAYAMA, Di SU [30] Proposed an extended Kalman filter (EKF) based displacement estimation method for nonlinear SDOF systems under seismic excitation.In the proposed approach, time segments are first identified based on whether the system undergoes significant nonlinearity. During elastic phases, the extended Kalman filter (EKF) utilizes observations such as acceleration, displacement derived from twice integrating the acceleration, and residual movement. In periods of pronounced nonlinearity, observations comprise acceleration and a hypothetical displacement measurement. During these periods, displacement and evolving stiffness are simultaneously estimated using an expanded state vector within the EKF. Subsequent refinements are made using an extended Kalman smoother (EKS). This approach has been thoroughly tested on a single-degree-of-freedom (SDOF) system with a bi-linear hysteresis model. It has also been validated against multiple hysteresis models and seismic disturbances, yielding precise displacement estimations. Post-earthquake structural evaluation is crucial for both safety and efficient damage mitigation. Analyzing signals from earthquakes can offer insights into a structure's health. Displacement response, especially, is a vital metric. While displacement measurement is essential for structural health monitoring, practical implementation is challenging due to issues like finding a reliable platform for sensors and limitations of non-contact methods like GPS. Skolnik et al. [23] delve into these measurement techniques. This paper introduces a displacement estimation method for nonlinear SDOF systems under seismic conditions using the Extended Kalman Filter (EKF). The method factors in acceleration and assumes known residual displacement, a parameter highlighted by Kawashima et al.9). Differentiating between periods of significant non-linearity, the approach incorporates techniques by Kuleli et al. [15] and optimizes results with the Extended Kalman Smoother (EKS), influenced by the Robbins-Monro (RM) method used in machine learning.

P. Caravani et al. [4] published a technical document discussing a least- squares formulation for identifying parameters in the dynamics of multi-storey buildings. The focus is on a lumped parameter linear model representing the motion of the building in a one-dimensional manner. The document introduces a matrix equation of motion, where the goal is to determine coefficients related to understory stiffness and damping. The text outlines a least-squares solution to the identification problem, considering discrete measurements of the system's response over time. It presents a recursive formulation of the algorithm and discusses the inclusion of measurement/modelling uncertainty, introducing concepts related to Gaussian white noise and probabilistic interpretations. The algorithm is then applied to numerical tests using a two-degree-of-freedom system, considering different types of excitations and levels of noise. The results are analyzed based on the identified parameters and the convergence of the algorithm under various conditions. As a reader, the content of the document is highly technical and specialized, primarily targeted at individuals with a background in structural dynamics, control systems, or a related field. The document uses mathematical notations, matrix equations, and concepts from control theory, making it challenging for a general reader without this specific background to fully understand.

R. E. Kalman [13] address a wiener challenge having an emphasis on noise-free regulation and optimal estimation in a system that is dynamic. This research proposes a state-space approach to tackling the Wiener challenge that is the prediction of a signal in the presence of noise. The optimal prediction in a model that is dynamically impacted by Gaussian random processes—is addressed in the current research. Determining an estimate that eliminates anticipated loss given observed values is the intent of the dynamic model that comprises equilibrium equations and observation equations. The dynamic system for the ideal estimation proposed in the current research utilises matrices to depict the dynamics of the system. The diagonal projection of the current state tensor onto the simple linear manifold generated by the observed random variables is the solution to the above issue.With the goal to determine the matrices iteratively, the authors establish recursion relations. The Wiener problem could be addressed systematically using the assistance of these relations. In this research, the concept of an optimal filter—a linear dynamic system that produces the most accurate estimate—is proposed. The most recent measured value is the input, the state of the filter is the previous estimate, and the recursive generates the transition matrix. define the matrix of covariance of the determine error and the anticipated quadratic loss. These metrics indicate the way the best estimate operates and how precise it is. Attempting to provide a conceptual connection between the noise-free regulator problem and the Wiener problem for optimal projection, the research offers the Duality Theorem. The swap of constraints and matrices between the two challenges provides the foundation of the duality. The duality between the Wiener problem and the noise-free regulator problem is emphasized by the authors as they discuss regarding the discoveries that acquired through their approach. The Duality Theorem's implications are underscored, offering an opportunity of continuing theoretical investigation. The author admits that several issues need more investigation, such as the constancy of the finest filter and the physical significance of the assumptions. Further studies, stated to the authors, will focus on these aspects.The article offers comprehensive derivatives, calculations, and recursive relations while upholding an exceptionally high level of mathematical discipline. The authors defend their claims with mathematical evidence and logical reasoning. Chin-Hsiung Loh et.al [18] explores the application of two identification techniques—sequential regression analysis and the Kalman filtering algorithm—to analyze strong-motion records obtained from the Milikan Library Building and JPL Building at the California Institute of Technology during the 1971 San Fernando earthquake. Three mathematical models of structural response, including an equivalent linear model, a bilinear hysteretic model, and a modified Clough model, are employed to approximate the fundamental mode response of the multi-degree-offreedom structural system. The Kalman filtering technique is utilized for estimating structural parameters in the equivalent linear model, revealing that the structural behavior is not strongly nonlinear. The bilinear hysteresis model and modified Clough model are also applied, with the extended Kalman filtering technique used for parameter identification. The results indicate that both the equivalent linearization model and the bilinear hysteresis model can be applied to the structure during the earthquake, while the stiffness degradation model does not align well with the real data unless strong nonlinear behavior is present. Overall, the proposed identification process yields reliable estimates of test response time history and structural parameters, providing valuable insights into the seismic behavior of the studied buildings. Graeme H. Mcverry [21] describes a method developed for identifying the parameters of lower modes in linear, time-invariant models of structures based on recorded earthquake responses. The approach utilizes a weighted global iteration procedure, incorporating the extended Kalman filter algorithm, to achieve stable and convergent solutions. By considering the effects of initial and final conditions, the method allows for the identification of parameters using only specific segments of recorded excitation and response acceleration histories, proving advantageous for both generated test data and measured earthquake responses. The study successfully applies this identification technique to analyze the dynamic response of the Millikan Library during two earthquakes, revealing differences in structural behavior under low-level vibrations and Shaking. George A. Bekey [2] explores the application of stochastic approximation techniques, specifically the Kiefer-Wolfowitz algorithm, in the field of system identification. In contrast to deterministic gradient algorithms, stochastic approaches, proposed by Brooks and Rastrigin, offer advantages in handling optimization problems with multiple minima in the criterion function surface. The Kiefer-Wolfowitz algorithm, initially designed for scalar parameter estimation in the presence of noise, has been extended to vector parameter cases by researchers such as Sakrison and Kirvaitis. This methodology, rooted in the principles of random search procedures, proves particularly useful for parameter optimization when dealing with noisy observations. The algorithm's mean-square convergence properties make it applicable to various system identification scenarios, including the estimation of parameters in linear and nonlinear differential equations. Stochastic approximation stands out as a robust iterative optimization technique, demonstrating its efficacy in tackling complex parameter estimation challenges in the presence of measurement noise. The review emphasizes the adaptability and efficiency of stochastic approximation in the context of system identification and optimization. James L. Beck et.al [1] describes a practical method for systematically determining the best estimates of parameters for linear structural models based on records of base motion and response during an earthquake. The method is set within the framework of the output-error approach, where parameters are estimated by minimizing a measure- of-fit between structural output and model output. The focus is on estimating modal periods, damping factors, and effective participation factors at the measurement points, with an emphasis on dominant modes. The document acknowledges the use of the output-error approach, a well-established method in structural dynamics. This approach involves minimizing the difference between observed and predicted structural responses. The text addresses fundamental limitations in obtaining reliable estimates of stiffness and damping matrices from earthquake records. Due to these limitations, the focus is on estimating modal periods, damping factors, and participation factors. The challenge is outlined, and the need for a systematic approach to parameter estimation is emphasized. The literature discusses the importance of identifying and estimating parameters for dominant modes in earthquake records. It suggests a step-wise approach, where modes are successively added to the models until the measure-of-fit no longer significantly decreases. This iterative process is employed for accurate parameter estimation. The document details the application of the method to strong-motion records from a specific 42-storey steel-frame structure during the 1971 San Fernando earthquake. The study provides new insights into the properties of lower modes, especially those with dominant motion in the longitudinal direction. The literature reports on the investigation of temporal changes in equivalent linear parameters during the strong-motion portion of the earthquake records. It notes progressive increases in periods and damping factors for the first and second modes, offering valuable information on structural behavior during seismic events. A key advantage highlighted is that the method determines a model of a prescribed mathematical form that optimally matches the response quantity. This systematic approach is contrasted with ad hoc methods, emphasizing the importance of evaluating the assumed form of the model based on the quality of the optimal match. Hoshiya et.al [11] provided text discusses the application of the extended Kalman filter to system identification problems in seismic structural systems. The focus is on obtaining stable and convergent solutions, and a weighted global iteration procedure with an objective function is proposed to enhance stability in estimation. The proposed procedure is integrated into the extended Kalman filter algorithm. The effectiveness of this approach is then explored through the investigation of identification problems for multiple degree-of-freedom linear systems, bilinear hysteretic systems, and the equivalent linearization of bilinear hysteretic systems. The numerical examples demonstrate the potential usefulness of the weighted global iteration procedure for these identification problems. The text assumes a background understanding of the extended Kalman filter (EKF) and its application to system identification in the context of seismic structural systems. EKF is a widely used recursive estimation algorithm that extends the Kalman filter to nonlinear systems. The primary concern in the literature appears to be stability and convergence in the estimation process. This aligns with a common challenge in system identification, especially for complex and nonlinear systems such as seismic structures. The proposed approach introduces a weighted global iteration procedure with an objective function to address stability issues in the estimation process. The use of a global iteration procedure suggests a holistic strategy to enhance the overall stability of the identification process. The proposed weighted global iteration procedure is not a standalone method but is integrated into the extended Kalman filter algorithm. Chin-Hsuing et.al [17] explores the potential of a new time domain identification procedure to detect changes in structural dynamic characteristics based on measurements. It is verified using mathematical models simulated on the computer and experiments involving two eight-storey steel structures with and without energy devices, and a 47- storey building during the Loma Prieta earthquake.The recursive necessary variable system and extended Kalman sludge algorithm are used as identification algorithms. The study proposes identification schemes on simulation data, including the use of band-pass filters and the sweep modal identification procedure. The identification study is conducted on two structures, a 47-storey building, and an eight-storey steel frame with and without energy devices. The identified modal parameters for the 47-storey building during the Loma Prieta earthquake are also presented. The conclusion emphasizes the importance of the change of storey drift in detecting changes in structural parameters and the applicability of the proposed system identification algorithm with an appropriate inelastic model to simulate the dynamic behavior of real structures undergoing strong ground motion excitations. C.W. Steeg,Jr. [24] focuses on the time- sphere conflation procedure for optimum extrapolators. The paper discusses the design and conflation of optimum predictors, particularly for situations where the vaticination interval is anon- negative function of time, rather than a fixed number of seconds in the future. The time- sphere conflation procedure is introduced to simplify the design of extrapolators, which are used to prognosticate the value of a signal at a specific moment that's a continuously dwindling number of seconds in the future. The paper explains a procedure for avoiding the result of integral equations in the conflation of optimum extrapolators. The paper emphasizes the significance of the time- sphere conflation procedure, which is a means for circumventing the practical difficulties encountered in the factual design of optimum predictors grounded upon the ways deduced byWiener.Steeg's paper provides a comprehensive discussion of the fine proposition of optimum vaticination, fastening on the time- sphere conflation procedure. The paper is significant because it addresses the limitations of frequence- sphere styles and extends the operation of statistical optimization proposition to time- varying systems. The paper also provides a detailed explanation of the time- sphere conflation procedure, which is a precious resource for masterminds and experimenters interested in the design and conflation of optimum predictors. Lingling Fan and Yasser Wehbe [8] provides a case study on the operation of EKF and model divorcing in power systems using Phasor Measurement Units(PMUs). The paper demonstrates the estimation of dynamic countries and parameters related to electromechanical dynamics, similar as rotor angle, rotor speed, and creator parameters, using PMU data. The Extended Kalman Filter(EKF) is a nonlinear estimation fashion that approximates the nonlinear system dynamics and measures using linearization. EKF is suitable for real- time estimation and can handle nonlinear systems perturbed by white noise. The EKF algorithm consists of two main way vaticination and correction. Model decoupling is a fashion that separates the system into lower subsystems, allowing for more effective estimation. In power systems, model decoupling can be achieved by treating measures as inputs and labors in the EKF problem. This approach reduces the computational burden and allows for the estimation of lower subsystems, similar as individual creators, without the need to model the entire power system. The EKF algorithm, combined with model decoupling, provides a important tool for real- time estimation in complex systems like power grids. The fashion allows for the estimation of dynamic countries and parameters, indeed in the presence of nonlinearities and unmodeled dynamics. The case study presented in the literature demonstrates the effectiveness of EKF and model divorcing in power systems, with operations in creator model and parameter estimation, and subsystem identification. G. Bolzon, R. Fedele, and G. Maier [3] presents a stochastic parameter identification technique using the Kalman filter (KF) methodology for the identification of material parameters in a mode I cohesive crack model. The KF methodology consists of a time-stepping sequence of estimations, which starts from an a priori estimation by an expert (Bayesian approach) and exploits a time-stepping flow of experimental data until convergence is empirically ascertained. The performance of the method is assessed by the use of "pseudo-experimental" data, i.e. data which are computer-generated. The paper "Identification of cohesive crack models for concrete on the basis of three-point bending tests" by G. Bolzon and G. Maier presents a mathematical model for quasi-brittle fracture simulations in concrete structures.The model is based on a piecewise-linear cohesive model with four parameters to identify on the crack path and linear elasticity elsewhere. The simulation of the experiments is formulated as a linear complementarity problem (LCP) to obtain the sensitivity matrix, a key ingredient of the Kalman filter (KF) extended to nonlinear inverse problems.

Summary

In the past decade, the problem of system identification had started to be recognized in the field of structural engineering, and its importance has steadily increased in recent years in connection with the prediction of responses of the structure due to various external loads and also with respect to the estimation of the existing condition for assessment of the degree of damage and deterioration of structures. In recent years, identification using filtering techniques such as Kalman filter and particle filter has attracted many researchers. A discussion of the importance of system identification in structural engineering and identification using filtering techniques forms the objective of the present study.

3.1 Objectives of the Work

The main goal of this study is to analyse the performance of some Bayesian filters that are mainly used to estimate the state and parameter estimation of structural dynamical systems. The specific objectives are listed below

- Learning the theory concerning the Kalman Filter and Unscented Kalman filter in order to develop their respective algorithms.
- Performance evaluation of Kalman Filter and Unscented Kalman filter for State estimation of a linear single-degree and two degrees of freedom system under noise and real earthquake excitation.
- State and parameter duel estimation of a linear single degree and two degrees of freedom system under noise and real earthquake excitation using Unscented Kalman filter.

Numerical Simulations

In this chapter we're going to see a Numercial simulation of State and Parameter Estimation in a Single Degree of Freedom System Linear, Two Degree of Freedom System linear, Single Degree of Freedom System Non Linear.

The loads used in the examples are generated by El Centro earthquake as shown in figure 4.1 which have a sampling frequency of 50Hz . Ground accelerations are the inputs of the system and the total accelerations are the measurements; finally, in order to simulate the noise, a 5 % RMS noise-to-signal ratio is superimposed.

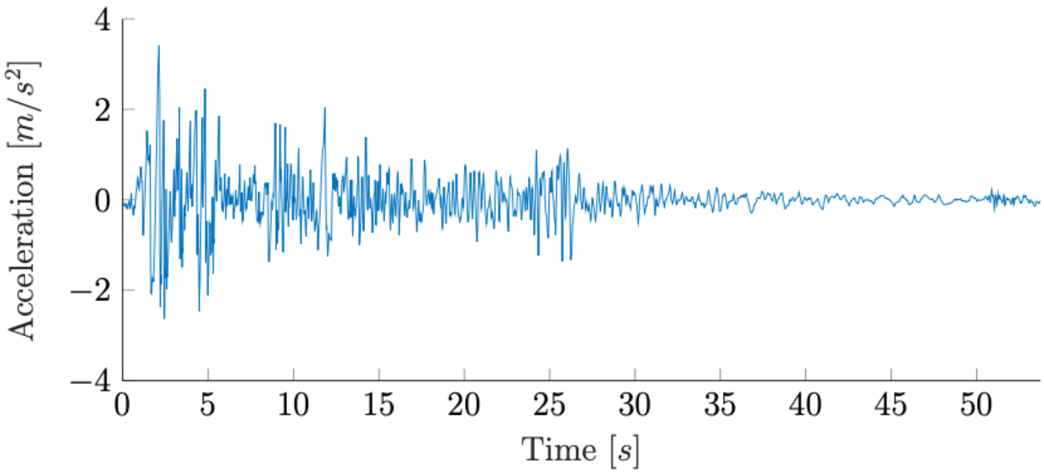

Figure 4.1: El Centro N-S ground acceleration

4.1 SDOF linear system simulation

In this example,the following single degree of freedom linear system is considered

$$m\ddot{x} + c\dot{x} + kx = -m\dot{x}_g \tag{4.1}$$

where $m = 1$, $c = 0.3$ and $k = 9$. The measurements used in the problems are shown in Figure 4.2

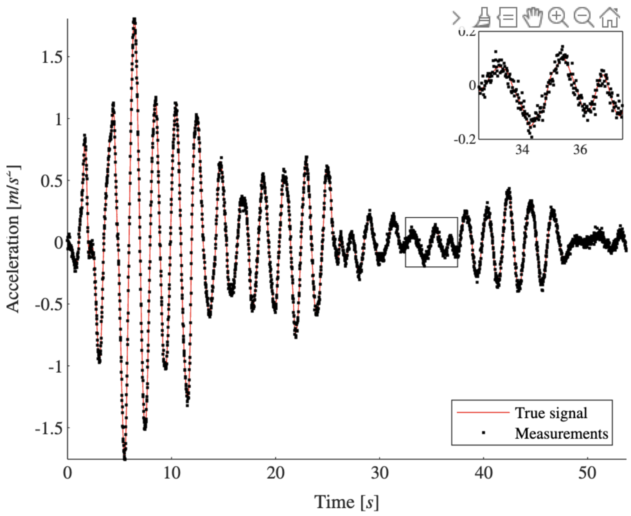

Figure 4.2: Acceleration mesurements SDOF linear system

State estimation:

Due to the system linearity, the KF, UKF can be used to solve the state estimation problem. To write the process equation in a state-space representation, the state vector is defined as

$$\mathbf{x} = \begin{bmatrix} \dot{x} \\ x \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

replacing these terms in Equation (4.1) and considering the derivative, the equations may be written as

$$\dot{x}_1 = x_2$$

$$m\ddot{x}_2 + c\dot{x}_2 + kx_1 = -mg,$$

that leads to the state space representation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{x}_2 \\ -\ddot{x}_g - \frac{(cx_2+kx_1)}{m} \end{bmatrix}$$

Then, the process equation may be written as

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$

where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \ddot{x}_g \end{bmatrix}$$

Now, for the observation equation, as

$$\ddot{x}_T = \dot{x} + \dot{x}_g = -\frac{c\dot{x}_2 + kx_1}{m}$$

it is possible to assert that

$$\mathbf{y} = H\mathbf{x}$$

where

$$\mathbf{y} = \begin{bmatrix} \dot{x}_T \end{bmatrix}, \quad H = \begin{bmatrix} -k/m & -c/m \end{bmatrix}$$

The initial state vector is set as $\mathbf{x}_0 = [0 \quad 0]^T$. The initial, process, and measurement covariance matrices are designated as $P_0 = [0.0001\mathbf{I_2}]$, $Q = [0.01\mathbf{I_2}]$ and $R = [0.001]$. The outcomes of applying the Kalman Filter, the Unscented Kalman Filter are illustrated in Figures 4.3, 4.4 respectively.

In Figures 4.3, 4.4 one can observe the robust performance of both the filters, where the estimated signal closely mirrors the actual signal. The big differences are evident in the deviation band, in both the KF and UKF scenarios. The bandwidth's susceptibility to alterations in the covariance parameters could potentially impact the precision of the estimation. As per Table 4.1, it is evident that the UKF offers enhanced performance over the KF.

Table 4.1: SDOF Linear system state estimation error

		Displacement	**Velocity**
	Mean	0.0578	0.0100
KF	Variance	3.1626	1.0936
	MSE	3.1659	1.0937
	Mean	0.0551	0.0157
UKF	Variance	3.1366	0.0936
	MSE	3.1396	0.9105

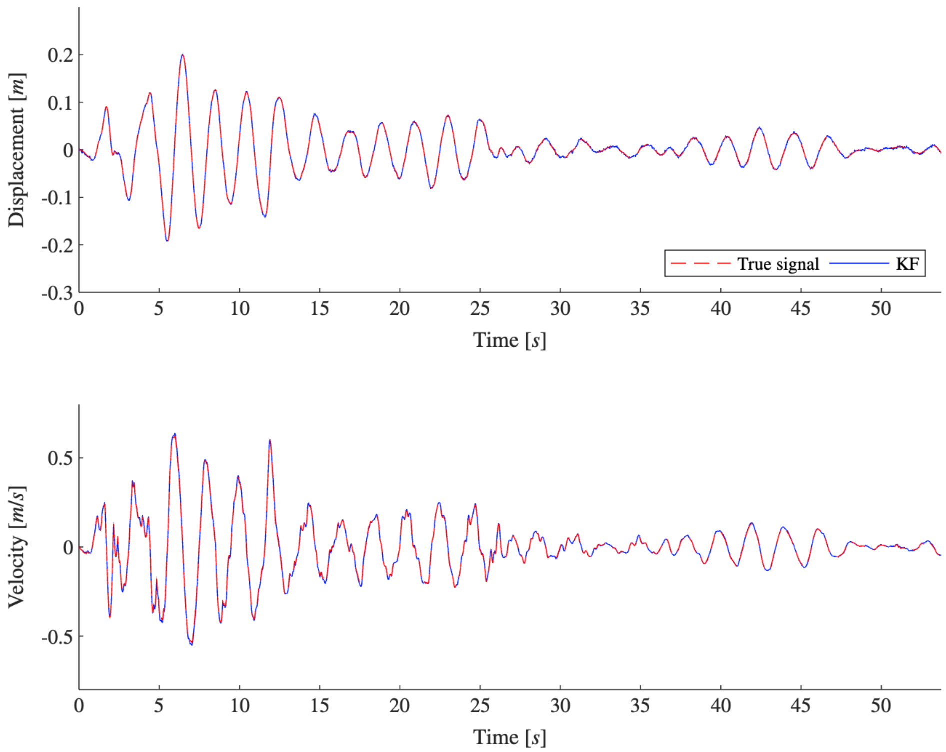

Figure 4.3: KF linear SDOF state estimation

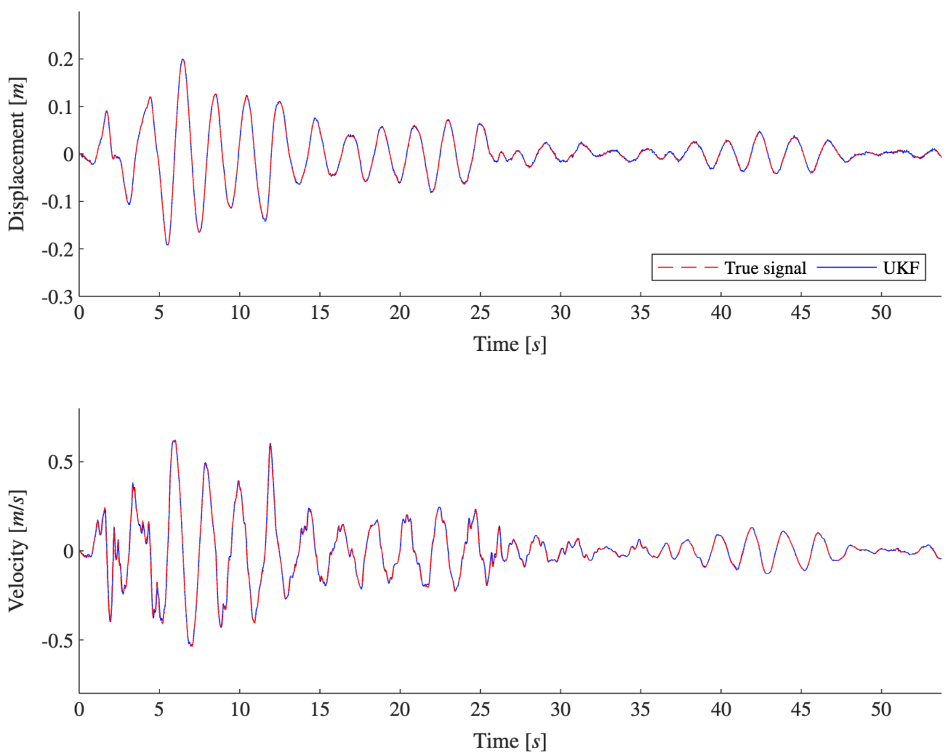

Figure 4.4: UKF linear SDOF state estimation

Dual estimation:

To ascertain the parameters of the system, a process termed dual estimation is warranted. This dual approach encompasses both the system's state and its intrinsic parameters. In pursuit of this objective, the system's state vector is expanded such that

$$\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \\ k \\ c \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Employing this variable substitution, the dynamics can be recast as

$$\dot{x}_1 = x_2$$

$$m\ddot{x}_2 + x_4 x_2 + x_3 x_1 = -m\dot{x}_g$$

culminating in the system representation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{(x_4 x_2 + x_3 x_1)}{m} - \dot{x}_g \\ 0 \\ 0 \end{bmatrix}$$

which diverges from a linear construct. Thus, the process model is formulated as

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$$

In regards to the observation model,

$$\ddot{x}_T = \dot{x} + \dot{x}_g = -\frac{(x_4 x_2 + x_3 x_1)}{m}$$

which, when expressed compactly by invoking State-space representation equation, becomes

$$\mathbf{y} = h(\mathbf{x})$$

where

$$\mathbf{y} = \begin{bmatrix} \dot{x}_T \end{bmatrix}, \quad h(\mathbf{x}) = -\left(\frac{(x_4 x_2 + x_3 x_1)}{m} \right)$$

The initial state vector is defined as $\mathbf{x}_0 = [0 \quad 0 \quad 5 \quad 0.2]^T$. The covariance matrices are prescribed as $P_0 = \mathrm{diag}([0.0001, 10])$ with the initial entry for state estimation and the subsequent for parameter estimation, $Q = \mathrm{diag}([0.01, 0.1])$ and $R = 0.001$. The states of the Unscented Kalman Filter performance is depicted in Figures 4.5. The parameter values are illustrated in Figure 4.6 . Referring to Figure 4.5, UKF demonstrates commendable tracking capabilities, yet the UKF's confidence interval is substantially broader. Observation of parameter estimation in Figure 4.6 reveals that the UKF offers a good estimation of parameters.

Table 4.2: UKF SDOF linear system dual estimation error

State	**Mean**	**Variance**	**MSE**
Displacement	0.0306	4.6257	4.6266
Velocity	0.0223	2.5229	2.5234

Table 4.3: UKF SDOF linear system parameters error

Parameter	**Exact value**	**UKF estimate**	**Error%**
k	9	8.7950	2.2780
c	0.3	0.4240	41.3240

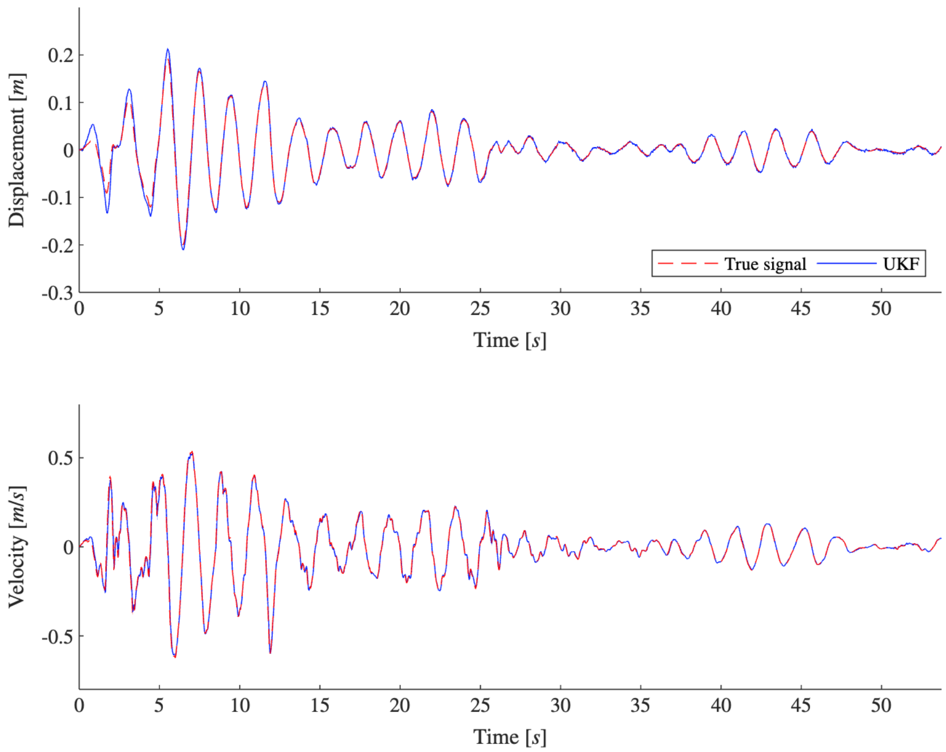

Figure 4.5: UKF linear SDOF dual estimation. States.

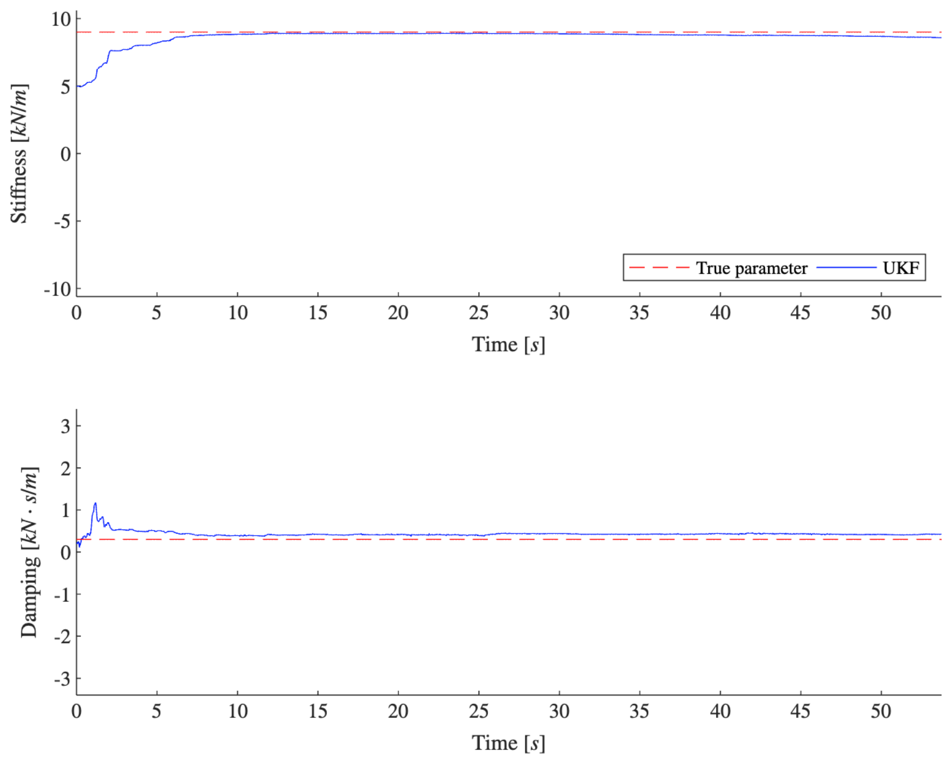

Figure 4.6: UKF linear SDOF dual estimation. Parameters.

4.2 Two-DOF linear structural system simulation

We examine a system with two degrees of freedom, characterized by the following equations:

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = -m_1\ddot{x}_g, \tag{4.2a}$$

$$m_2\ddot{x}_2 - c_2\dot{x}_1 + c_2\dot{x}_2 - k_2x_1 + k_2x_2 = -m_2\ddot{x}_g, \tag{4.2b}$$

where $m_1 = 1$, $m_2 = 1$, damping coefficients $c_1 = 0.6$ and $c_2 = 0.5$, and stiffness values $k_1 = 12$ and $k_2 = 10$.

Acceleration data for the system's two degrees of freedom, which are utilized in estimations, are presented in Figure 4.7.

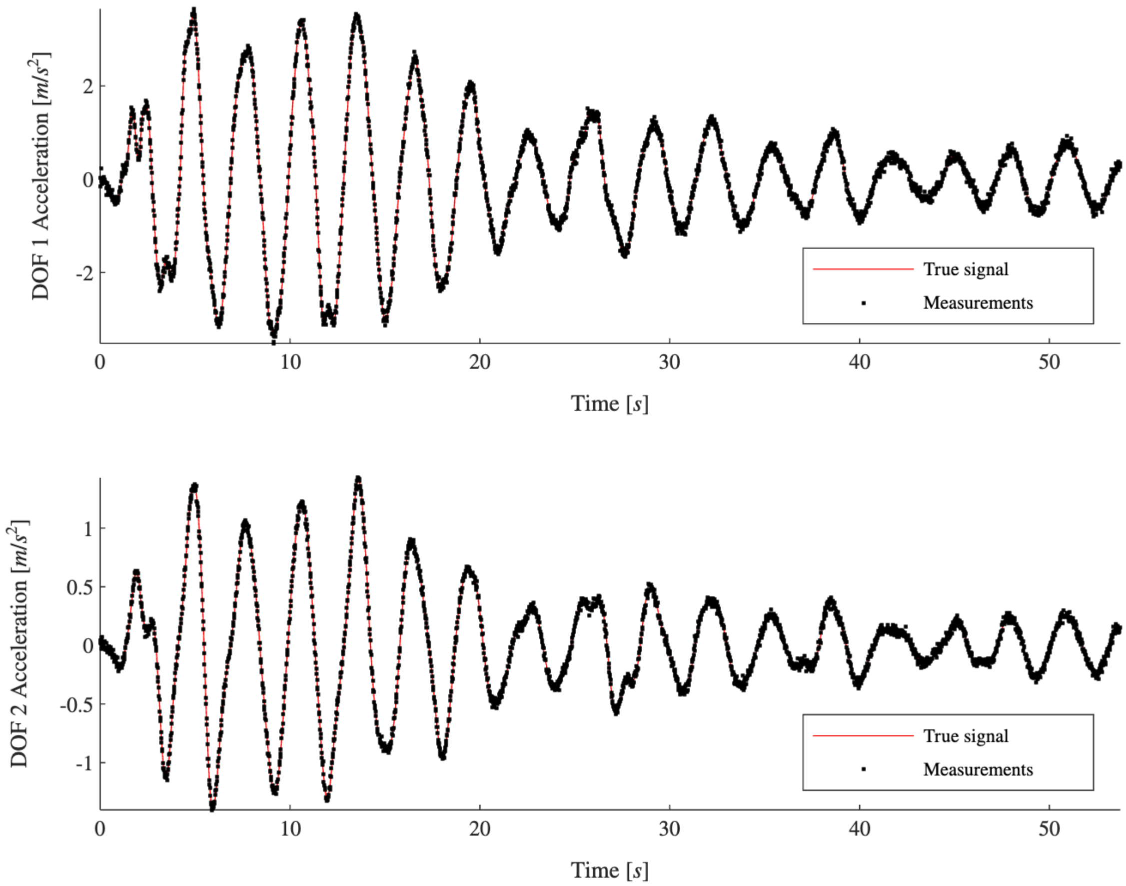

Figure 4.7: 2-DOF linear system Acceleration measurements

State estimation
For state estimation, the Kalman Filter (KF), Unscented Kalman Filter (UKF), and Particle Filter (PF) are all suitable due to the system's linear dynamics. The state vector is articulated as:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

Utilizing Equations (4.2a) and (4.2b), the system's dynamics can be reformulated as:

$$\begin{gathered} \dot{x}_1 = \dot{x}_1 \\ \dot{x}_2 = \dot{x}_2, \\ m_1\dot{x}_3 + (c_1 + c_2)x_3 - c_2x_4 + (k_1 + k_2)x_1 - k_2x_2 = -m_1\ddot{x}_g \\ m_2\dot{x}_4 - c_2x_3 + c_2x_4 - k_2x_1 + k_2x_2 = -m_2\ddot{x}_g. \end{gathered}$$

Alternatively, the state dynamics can be captured by the matrix equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ -\ddot{x}_g - \frac{1}{m_1}((c_1 + c_2)x_3 - c_2x_4 + (k_1 + k_2)x_1 - k_2x_2) \\ -\ddot{x}_g - \frac{1}{m_2}(-c_2x_3 + c_2x_4 - k_2x_1 + k_2x_2) \end{bmatrix}$$

proceeding from which the process equation condenses to:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u},$$

where the matrices $\mathbf{A}$ and $\mathbf{B}$ are defined as:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & -\frac{(c_1+c_2)}{m_1} & \frac{c_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{c_2}{m_2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{m_1} \\ -\frac{1}{m_2} \end{bmatrix}.$$

Turning to the observation equation, we have:

$$\ddot{x}_{T1} = \ddot{x}_1 + \ddot{x}_g = -\frac{1}{m_1}((c_1 + c_2)x_3 - c_2x_4 + (k_1 + k_2)x_1 - k_2x_2)$$

$$\ddot{x}_{T2} = \dot{x}_2 + \ddot{x}_g = \frac{-1}{m_2}(-c_2x_3 + c_2x_4 - k_2x_1 + k_2x_2)$$

thus maintaining the form of the system as:

$$\mathbf{y} = \mathbf{H}\mathbf{x},$$

where $\mathbf{y}$ is the output vector and $\mathbf{H}$ is defined as:

$$\mathbf{y} = \begin{bmatrix} \ddot{x}_{T1} \\ \ddot{x}_{T2} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} -\frac{(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & -\frac{(c_1+c_2)}{m_1} & \frac{c_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{c_2}{m_2} \end{bmatrix}.$$

The initial state is outlined as $x_0 = [0\,0\,0\,0]^T$, with the process and measurement covariance matrices chosen as $\mathbf{P_0} = [0.0001\mathbf{I_4}]$, $\mathbf{Q} = [0.0001\mathbf{I_4}]$ and $\mathbf{R} = [\mathbf{I_2}]$. The Kalman Filter results are presented in Figures 4.8 and 4.9, whereas results for the UKF are shown in 4.10 and 4.11. A comparison of these results reveals that the UKF estimations surpass those made by the KF, especially evident in the initial figures where actual and estimated signals overlay. In Figures 4.10 and 4.11, the UKF's predictions show a constricted confidence band, enhancing trust in the estimation where the confidence band is so narrow it nearly disappears. The data tabulated in Table 4.4 corroborates the conclusions drawn from the preceding analysis.

Table 4.4: 2-DOF linear system state estimation error.

DOF	Response		KF	UKF
First	Displacement	Mean	0.2095	−0.3008
		Variance	708.1153	288.4419
		MSE	708.1592	288.5324
	Velocity	Mean	0.3910	0.0247
		Variance	453.2551	8.4804
		MSE	453.4079	8.4810
Second	Displacement	Mean	0.5912	−0.0055
		Variance	243.1311	0.7928
		MSE	243.4805	0.7928
	Velocity	Mean	−3.2326	0.1261
		Variance	2.5536×10^4	49.6667
		MSE	2.5546×10^4	49.6826

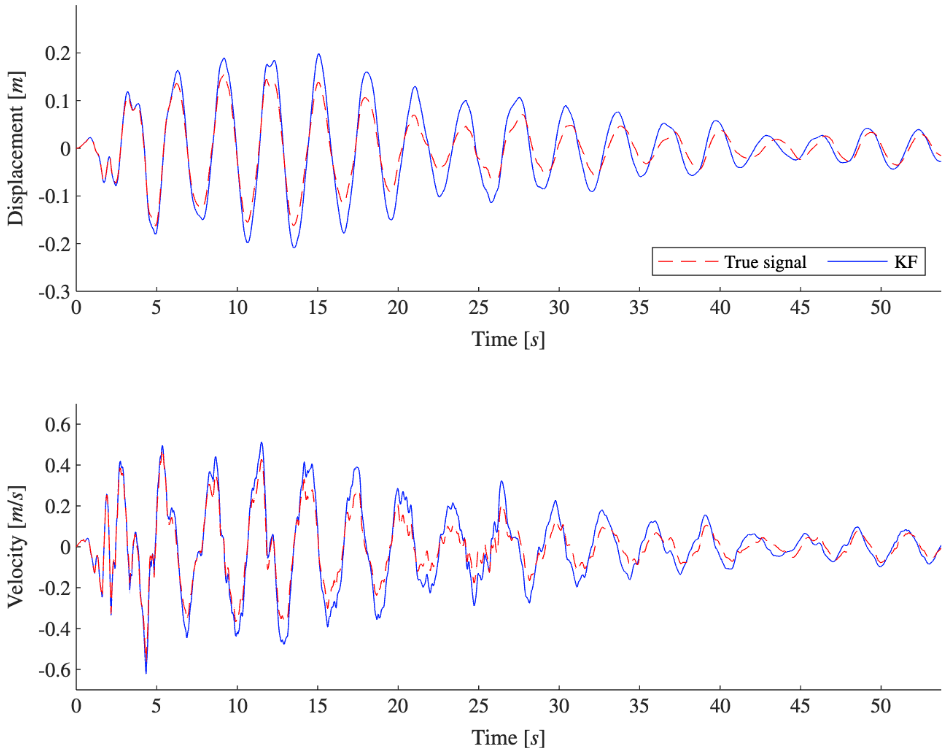

Figure 4.8: KF 2-DOF state estimation. First DOF.

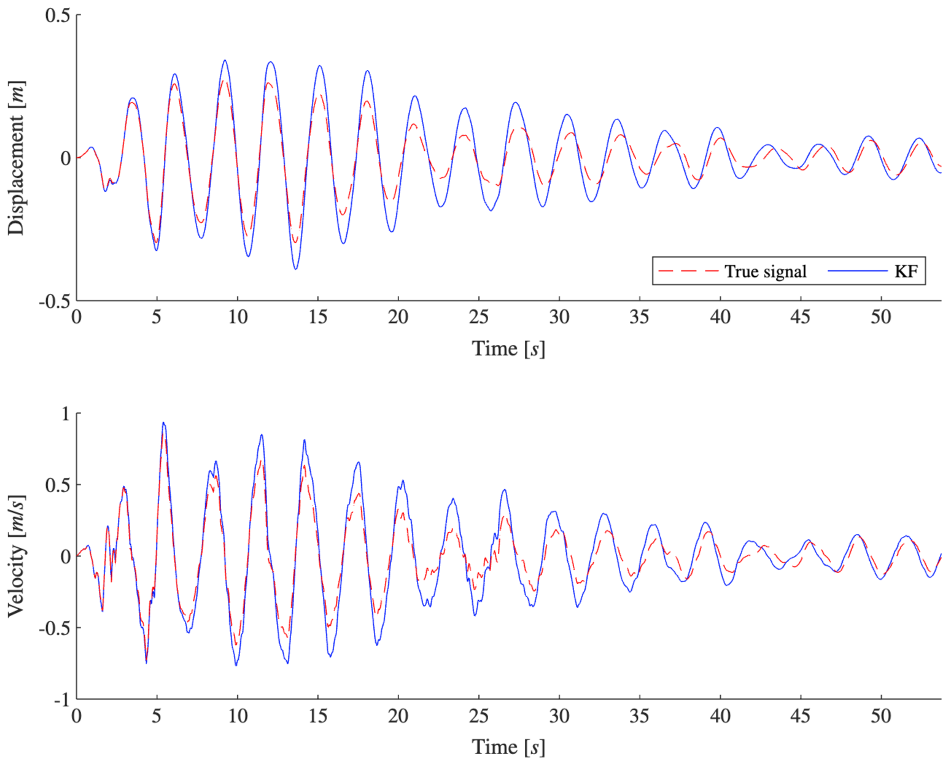

Figure 4.9: KF 2-DOF state estimation. Second DOF.

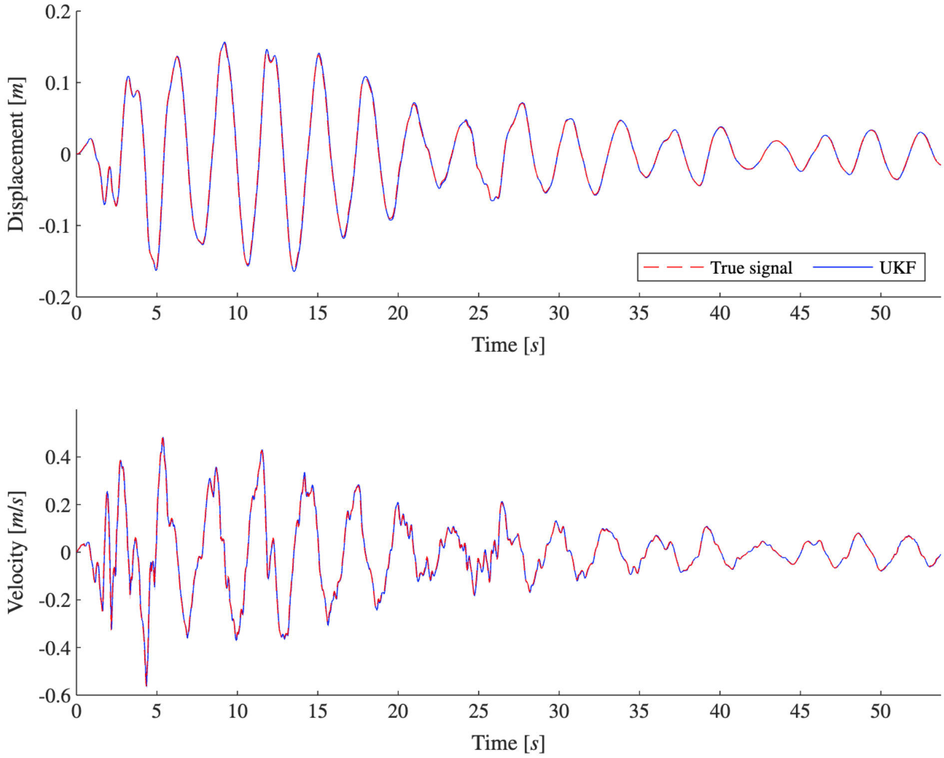

Figure 4.10: UKF 2-DOF state estimation. First DOF.

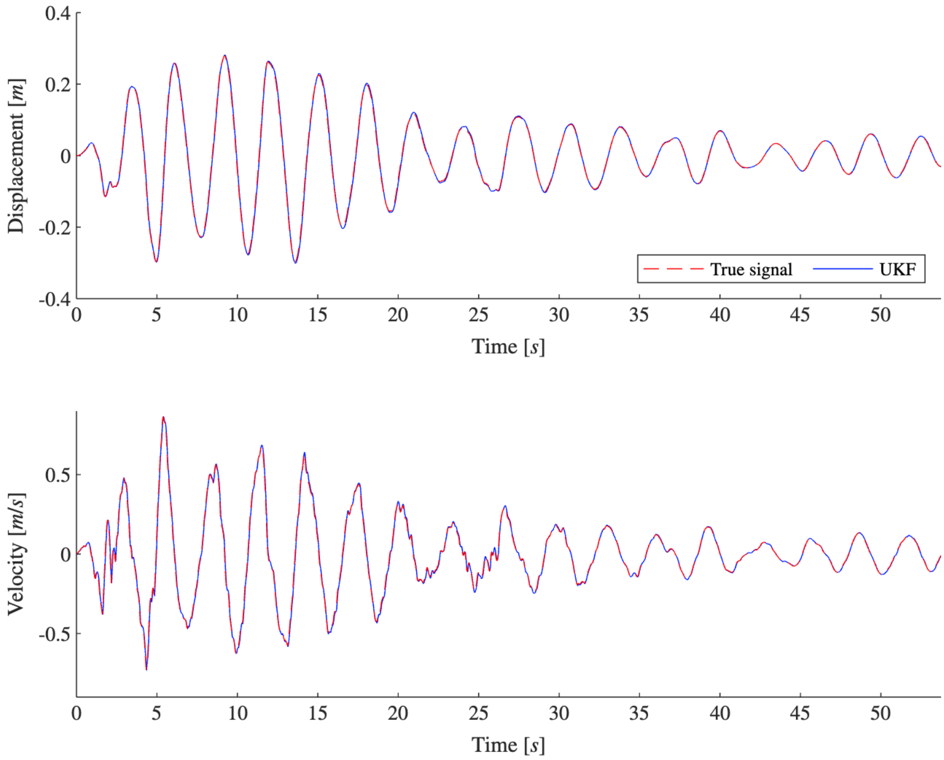

Figure 4.11: UKF 2-DOF state estimation. Second DOF.

Dual estimation

For this section, the system's state vector encapsulates both state variables and parameters, defined as:

$$
\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \\ k_1 \\ k_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}
$$

resulting in the following set of equations:

$$
\begin{gathered}
\dot{x}_1 = x_3 \\
\dot{x}_2 = x_4 \\
m_1\dot{x}_3 + (x_7 + x_8)x_3 - x_8x_4 + (x_5 + x_6)x_1 - x_6x_2 = -m_1\ddot{x}_g \\
m_2\dot{x}_4 - x_8x_3 + x_8x_4 - x_6x_1 + x_6x_2 = -m_2\ddot{x}_g
\end{gathered}
$$

or

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ -\ddot{x}_g - \left((x_7 + x_8)x_3 - x_8x_4 + (x_5 + x_6)x_1 - x_6x_2\right)/m_1 \\ -\ddot{x}_g - \left(-x_8x_3 + x_8x_4 - x_6x_1 + x_6x_2\right)/m_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.
$$

With these relationships, the dynamic model can be compactly expressed as a function of state and input:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}).$$

The observation model is described by:

$$\ddot{x}_{T1} = \dot{x}_1 + \ddot{x}_g = -\frac{1}{m_1}((x_7 + x_8)x_3 - x_8 x_4 + (x_5 + x_6)x_1 - x_6 x_2)$$
$$\ddot{x}_{T2} = \dot{x}_2 + \ddot{x}_g = -\frac{1}{m_2}(-x_8 x_3 + x_8 x_4 - x_6 x_1 + x_6 x_2)$$

allowing the system to be rewritten in terms of the output equation:

$$\mathbf{y} = \mathbf{h}(\mathbf{x}).$$

The initiation of the state vector and corresponding covariance matrix are denoted as $\mathbf{x}_0 = [0 \quad 0 \quad 0 \quad 0 \quad 2 \quad 2 \quad 0.2 \quad 0.2]^T$ with the initiation matrices for process and measurement defined as $P_0 = \text{diag}(0.0001, 0.0001, 100, 100)$, and $Q = \text{diag}(0.001, 0.001)$, $R = \text{diag}(0.01, 0.01)$ respectively.

The state and parameters estimations' for the first and second degree of freedom are illustrated in Figures 4.12, 4.14 and 4.13, 4.15 respectively for the UKF. In Figure 4.12, challenges in fitting the data are evident with the filter overestimating the states in the first degree of freedom. Parameter estimations are less precise as indicated by Figure 4.13, with this assertion supported by the errors presented in Table **??**. Throughout the initial estimations with the UKF in Figure 4.14, the filter struggles with state approximation but eventually adapts, resulting in improved estimations towards the end. The parameter estimations in this case do not appear accurate, occasionally resulting in negative outcomes.

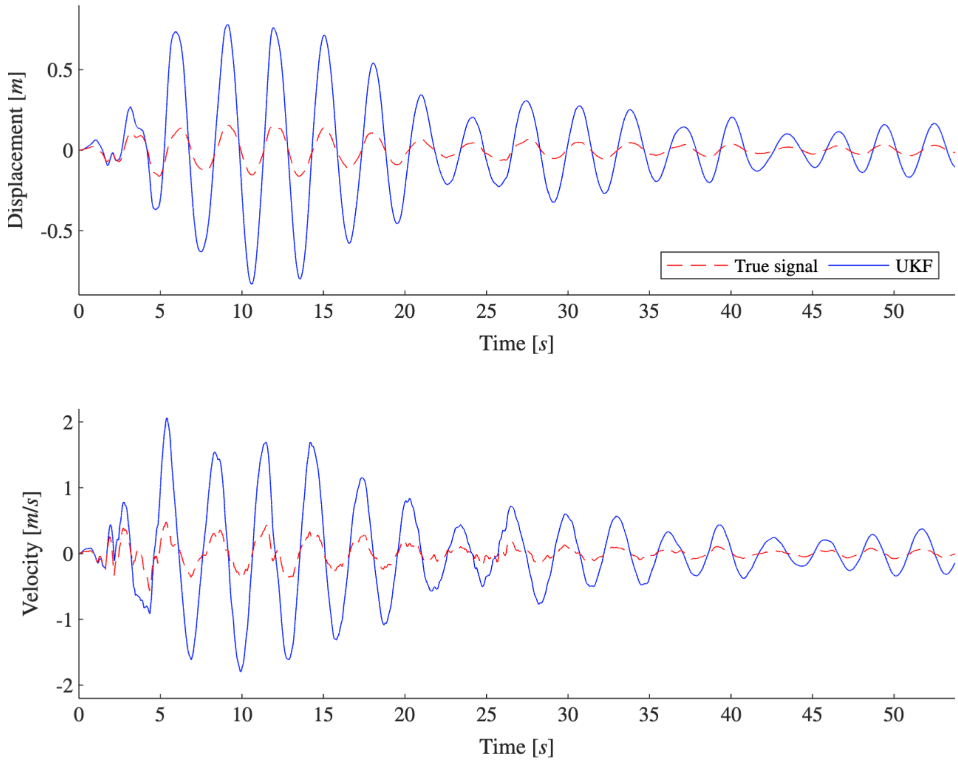

Figure 4.12: UKF 2-DOF dual estimation. First DOF. States

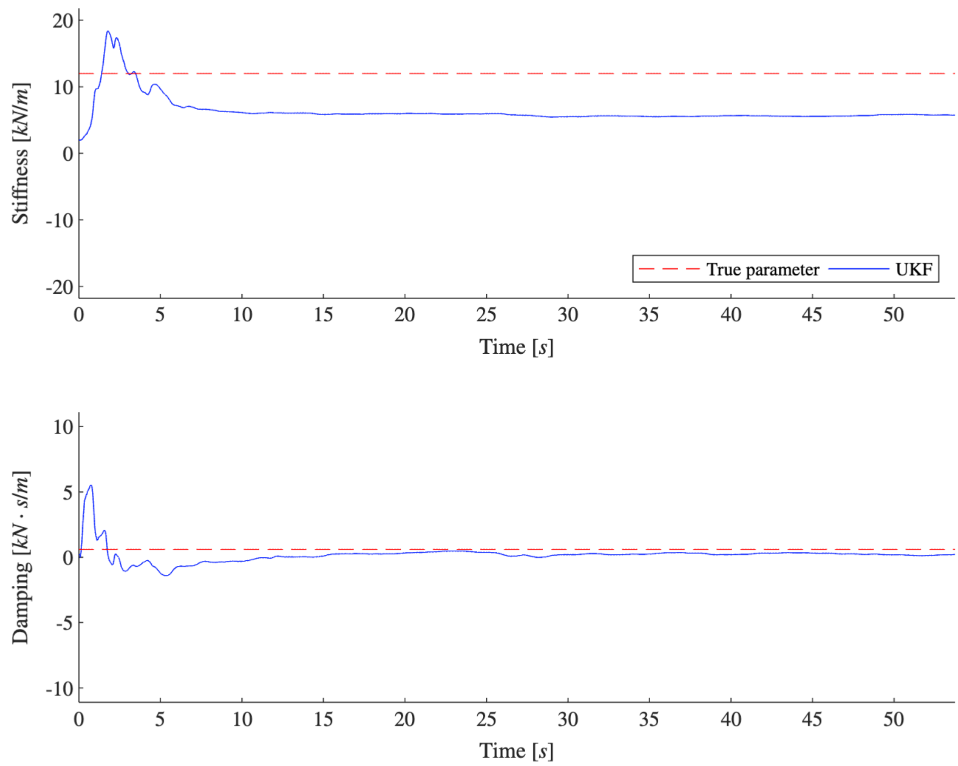

Figure 4.13: UKF 2-DOF dual estimation. First DOF. Parameters

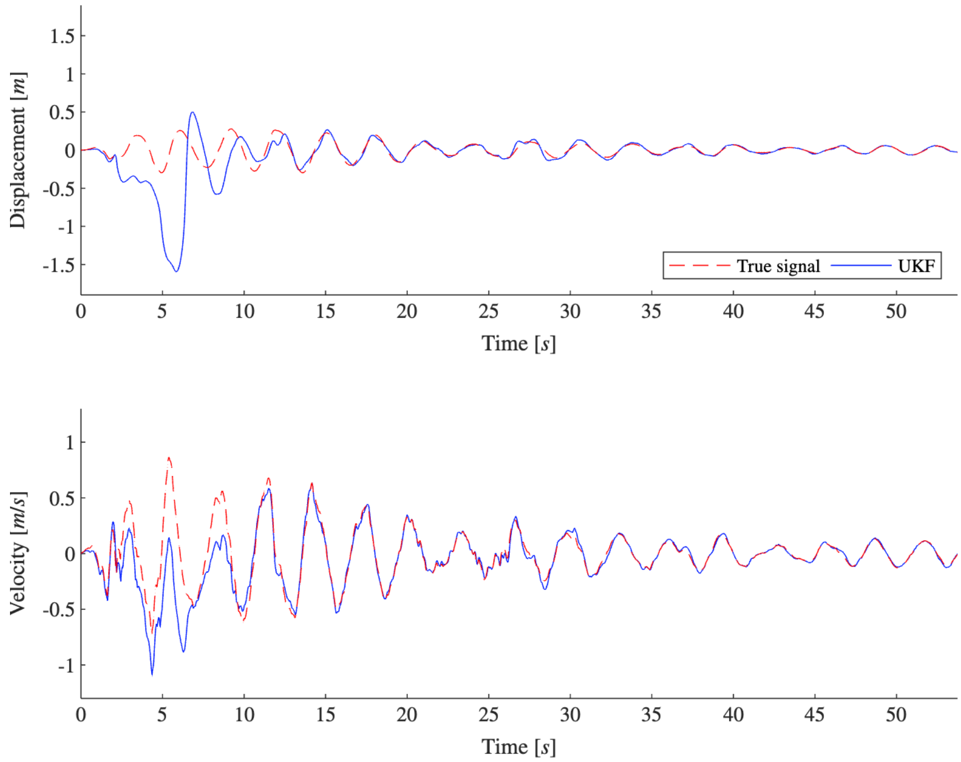

Figure 4.14: UKF 2-DOF dual estimation. Second DOF. States

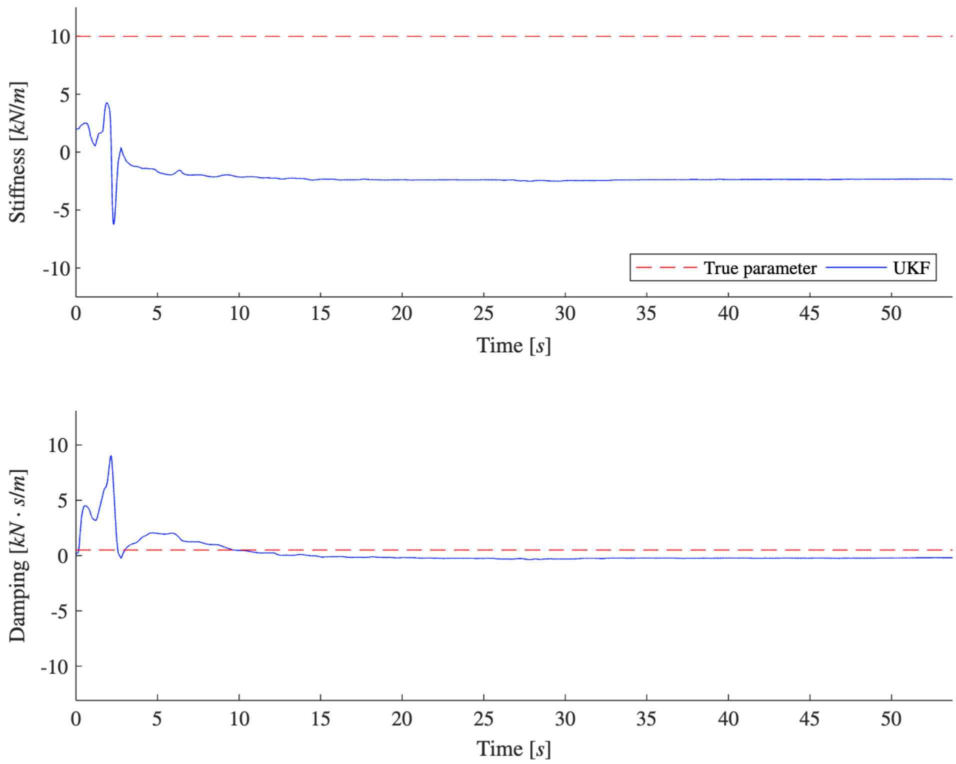

Figure 4.15: UKF 2-DOF dual estimation. Second DOF Parameters

Table 4.5: 2-DOF linear system dual estimation error.

DOF	State		UKF
First	Displacement	Mean	2.1138
		Variance	7.2979×10^3
		MSE	7.3024×10^3
	Velocity	Mean	4.1344
		Variance	3.9692×10^3
		MSE	3.9863×10^3
Second	Displacement	Mean	0.3254
		Variance	452.6163
		MSE	452.7222
	Velocity	Mean	1.9443
		Variance	5.8285×10^3
		MSE	5.8285×10^3

Table 4.6: 2-DOF linear system parameters error.

Parameter	Exact value	UKF estimate	Error%
c_1	0.6	0.2610	56.4982
k_1	12	5.6907	52.5773
c_2	0.5	-0.2438	148.7656
k_2	10	-2.3851	123.8507

Results & Discussion

This study examines the precision and computational efficiency of various Bayesian filters founded on principles of dynamic systems, Bayesian inference, and numerical techniques tailored to meet specific objectives. The selected problems represent a range of dynamic systems prevalent in civil engineering and dynamic analyses where state estimation is relevant.

The *Kalman Filter* (KF) is suitable for linear systems where states are presumed to adhere to a Gaussian distribution. Its application in structural dynamic systems is confined to the state estimation of linear models; it is not viable for parameter estimation in non-linear contexts. This filter serves as a foundational reference for more sophisticated filters.

The *Unscented Kalman Filter* (UKF) is a prominent extension of the KF that accommodates both linear and non-linear systems. It employs the Unscented Transform to better handle probability distributions by converting them into a Gaussian format, thus making the distributions more manageable. Consequently, the Unscented Transform enhances the filter's performance substantially.

While all filters serve the purpose of monitoring states and parameters effectively, each possesses unique strengths and weaknesses. For this investigation, the selection of the initial state vector and covariance matrices was optimized to ensure high accuracy across all filters. When addressing state estimation for a single degree of freedom linear system, both filters demonstrated capable performance. In the context of a Two-degree-of-freedom linear system, the KF's efficacy falls short when juxtaposed with the UKF. Only the UKF is adept at navigating such scenarios. Turning to dual estimation challenges, the UKF emerge as the sole contender equipped to manage these scenarios. Even with linear dynamic systems, the transition to a non-linear representation is inevitable.

The single-degree-of-freedom hysteretic system presents significant challenges in dual estimation due to its pronounced non-linearity. The UKF, although not performing optimally as in simpler systems, can still effectively track the state, providing valuable insights into system behavior. Parameter estimation with the UKF yields varying precision, with less accuracy for stiffness and damping coefficients compared to state estimation. Accuracy takes precedence over computational speed in this instance.

With a two-degree-of-freedom linear system, the increase in complexity presents additional hurdles. As the system evolves, the UKF shows convergence, yet with diminishing errors. The state estimation is more successful for the second degree of freedom than the first; the parameter convergence is also less reliable. Enhancing performance might require alternative initial parameter choices or adjustments to the UKF's approach to non-linearity.

The KF, being the quickest, faces limitations due to its dependence on linearity and Gaussian assumptions. The UKF stands out for its speed and ability to handle non-linear systems effectively, although its success may be contingent on the choice of initial parameters, given the sensitivity of the unscented transform to the covariance matrix's square root.

Conclusion

- The study successfully achieved its primary objective of evaluating the performance of Bayesian filters, specifically the Kalman Filter (KF) and Unscented Kalman Filter (UKF), in estimating the states and parameters of structural dynamical systems. It demonstrated that while both filters are effective in monitoring states, their effectiveness varies with system complexity and non-linearity.

- For linear systems with single-degree of freedom under noise and earthquake excitation, both the KF and UKF displayed capable state estimation performance. However, the KF's application is limited as it cannot perform parameter estimation in non-linear contexts and is thus considered a baseline reference for more advanced filters.

- The UKF proved superior in managing non-linear dynamics and dual estimation challenges, outperforming the KF in scenarios involving two-degree-of-freedom linear systems. This is attributed to its ability to handle non-Gaussian distributions via the Unscented Transform, which makes it a robust tool for more complex system analyses.

- While the KF boasts the fastest computational time due to its simplicity, it is constrained by its assumptions of linearity and Gaussian distributions. The UKF's ability to converge and its adaptability to non-linear systems make it the preferable choice for complex state and parameter estimation tasks, although the success heavily relies on the proper selection of initial parameters and accounting for the non-linearity in the system's representation.

Future work

Based on the comprehensive research conducted thus far in the field of structural dynamics and system estimation, future work can delve into several promising areas to further refine and optimize the application of Bayesian filters. Here are some directions for future work based on the findings of this study

- Sensitivity analysis of parameters and covariance to minimize the error and achieve better results.
- Performance evaluation of parametric and non-parametric filter for State and parameter estimation of Non-linear system under noise and real earthquake excitation
- Performance evaluation of non-parametric filter for State and parameter estimation of Non-linear system under noise and real earthquake excitation

Bibliography

[1] James L. Beck and Paul C. Jennings. Structural identification using linear models and earthquake records. *Earthquake Engineering & Structural Dynamics*, 8(2):145–160, 1980.

[2] George A. Bekey. System identification- an introduction and a survey. *SIMULATION*, 15(4):151–166, 1970.

[3] Gabriella Bolzon, Roberto Fedele, and Giulio Maier. Parameter identification by kalman filter of a cohesive crack model. *Computer Methods in Applied Mechanics and Engineering*, 191:2847–2871, 01 2002.

[4] P. Caravani, M. L. Watson, and W. T. Thomson. Recursive Least-Squares Time Domain Identification of Structural Parameters. *Journal of Applied Mechanics*, 44(1):135–140, 03 1977.

[5] M. Shinozuka C.B.Yun. Identification of nonlinear structural dynamic systems. *Jstruct. Mech*, 8(2):187–203, 1980.

[6] Eleni Chatzi and Andrew Smyth. The unscented kalman filter and particle filter methods for nonlinear structural system identification with non-collocated heterogeneous sensing. *Control*, 00:1–6, 02 2002.

[7] Alberto Corigliano and Stefano Mariani. Parameter identification in explicit structural dynamics: Performance of the extended kalman filter. *Computer Methods in Applied Mechanics and Engineering - COMPUT METHOD APPL MECH ENG*, 193, 09 2004.

[8] Lingling Fan and Yasser Wehbe. Extended kalman filtering based real-time dynamic state and parameter estimation using pmu data. *Electric Power Systems Research*, 103:168–177, 10 2013.

[9] Qingfei Gao, Xiang Wang, and Liu Yang. A kalman filter-based method for diagnosing the structural condition of medium- and small-span beam bridges during brief traffic interruptions. *Sensors*, 20:4130, 07 2020.

[10] Masaru Hoshiya and Etsuro Saito. Structural identification by extended kalman filter. *Journal of Engineering Mechanics*, 110(12):1757–1770, 1984.

[11] Masaru Hoshiya and Etsuro Saito. Structural identification by extended kalman filter. *Journal of Engineering Mechanics*, 110(12):1757–1770, 1984.

[12] J KIYONO K Toki, T SATO. Identification of structural parameters and input ground motion from response time histories. *proc. Of the Japan Society of Civil Engineers*, 6:413–421, 10 1989.

[13] R. E. Kalman. A New Approach to Linear Filtering and Prediction Problems. *Journal of Basic Engineering*, 82(1):35–45, 03 1960.

[14] R. E. Kalman and R. S. Bucy. New Results in Linear Filtering and Prediction Theory. *Journal of Basic Engineering*, 83(1):95–108, 03 1961.

[15] M. Kuleli. Stiffness condition assessment of bridge lateral resisting systems with unscented kalman filter using seis- mic acceleration response measurements. *Ph.D. thesis, Department of Civil Engineering, The University of Tokyo*, 2018.

[16] Jeen-Shang Lin and Yigong Zhang. Nonlinear structural identification using extended Kalman filter. *Computers and Structures*, 52(4):757–764, August 1994.

[17] Chin-Hsiung Loh and Iat-Chun Tou. A system identification approach to the detection of changes in both linear and non-linear structural parameters. *Earthquake Engineering & Structural Dynamics*, 24(1):85–97, 1995.

[18] Chin-Hsiung Loh and Yuan-Huei Tsaur. Time domain estimation of structural parameters. *Engineering Structures*, 10(2):95–105, 1988.

[19] O. Maruyama M Hoshiya. Identification of nonlinear structural systems. *Proc. 5th Int. Conf. on Applications of Statistics and Probability in Soil and Structural Engg. Vancouver, Canada.*

[20] Swarup Mahato and Arunasis Chakraborty. Ekf based parameter identification of lti system from seismic response measurements. *Procedia Engineering*, 144:360–365, 2016. International Conference on Vibration Problems 2015.

[21] Graeme H. McVerry. Structural identification in the frequency domain from earthquake records. *Earthquake Engineering & Structural Dynamics*, 8(2):161–180, 1980.

[22] J. Hugh Ellis Robert H. Scanlan Nicholas P. Jones, Tinghui Shi. System-identification procedure for system and input parameters in ambient vibration surveys.

[23] Derek Skolnik and John Wallace. A critical assessment of interstory drift measurements. *Journal of Structural Engineering-asce - J STRUCT ENG-ASCE*, 136, 12 2010.

[24] C. Steeg. A time domain synthesis for optimum extrapolators. *IRE Transactions on Automatic Control*, 3:32–41, 12 1957.

[25] Simo Särkkä. *Bayesian Filtering and Smoothing*. 10 2013.

[26] Makoto SATO Tadanobu SATO. Structural identification using neural network and kalman filter algorithms.

[27] Eric A. Wan and Rudolph van der Merwe. *The Unscented Kalman Filter*, chapter 7, pages 221–280. John Wiley Sons, Ltd, 2001.

[28] Qijun Xia, Ming Rao, Yiqun Ying, and Xuemin Shen. Adaptive fading kalman filter with an application. *Automatica*, 30(8):1333–1338, 1994.

[29] Jann N. Yang, Silian Lin, Hongwei Huang, and Li Zhou. An adaptive extended kalman filter for structural damage identification. *Structural Control and Health Monitoring*, 13(4):849–867, 2006.

[30] Di SU Yaohua YANG, Tomonori NAGAYAMA. Displacement estimation of nonlinear sdof system under seismic excitation using kalman filter for state-parameter estimation.