Matrices in Coordinate Geometry

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EE1390 Matrix Project

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Overview

The Original Problem

JEE (Advanced)2018 - Paper 1 - Q.3

Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is(are) TRUE?

(A) The line of intersection of P_1 and P_2 has direction ratios $1, 2, -1$

(B) The line

$$\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$$

is perpendicular to the line of intersection of P_1 and P_2

(C) The acute angle between P_1 and P_2 is 60°

(D) If P_3 is the plane passing through the point $(4, 2, -2)$ and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point $(2, 1, 1)$ from the plane P_3 is $\frac{2}{\sqrt{3}}$

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We can transform the problem into matrices in the following way

- The plane can be written in the form

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- The direction vector of a line in the form:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

is $\vec{L}(a, b, c)$

Solution

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- $$n_1 \times n_2 = \begin{bmatrix} n_{12}n_{23} - n_{13}n_{22} & n_{13}n_{21} - n_{11}n_{23} & n_{11}n_{22} - n_{12}n_{21} \end{bmatrix}^T$$

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- $$n_1 \times n_2 = \begin{bmatrix} 3 & -3 & 3 \end{bmatrix}^T$$
- The resultant ratio can be compared to the given direction ratios

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- The two planes have normal vectors n_1, n_2 and let line of intersection have direction vector $L_1(3, -3, 3)$

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- If L_1 is $\perp$ to L_2, it is implied to be co-planar with P_1, P_2.
- Use Scalar Dot Product to check the above.

$$d \cdot (n_1 \times n_2) = \begin{bmatrix} 2 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 5 \neq 0$$

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- The angles between the planes is the same as the angle between normal vectors $\vec{n_1}, \vec{n_2}$
- The angle is given by

$$\vec{n_1} \cdot \vec{n_2} = \|\vec{n_1}\|\|\vec{n_2}\| \cos\theta$$

$$\theta = \arccos \frac{\begin{bmatrix} 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}}{\sqrt{6} \cdot \sqrt{6}} = 60^\circ$$

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(D) Distance Between Point and Plane

- The component of the vector joining $A(4, 2, -2)$ and $P(2, 1, 1)$ along the normal n_3 is the distance between the point and plane.

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- $$dist = \left| \frac{n^T \cdot (A - P)}{\|n\|} \right|$$
- $$dist = \left| \frac{\begin{bmatrix} 3 & -3 & 3 \end{bmatrix} \cdot \left(\begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right)}{\sqrt{27}} \right| = \frac{2}{\sqrt{3}}$$

Figures

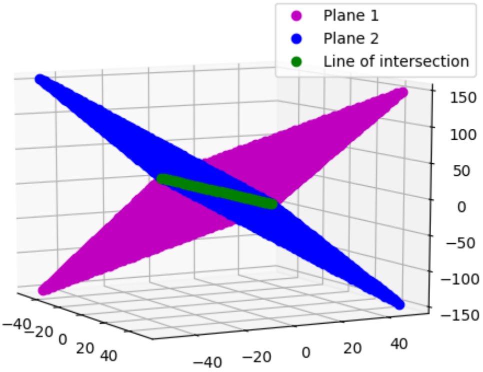


Figure: A - Planes & Line Of Intersection

Figures

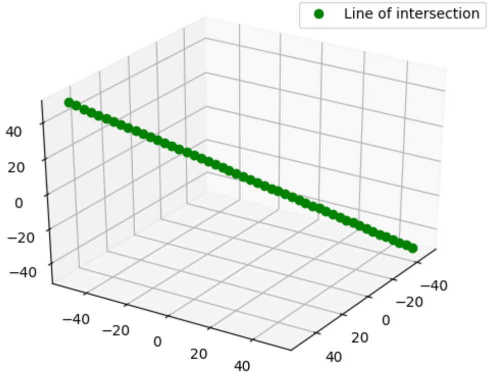


Figure: A - Line Of Intersection

Figures

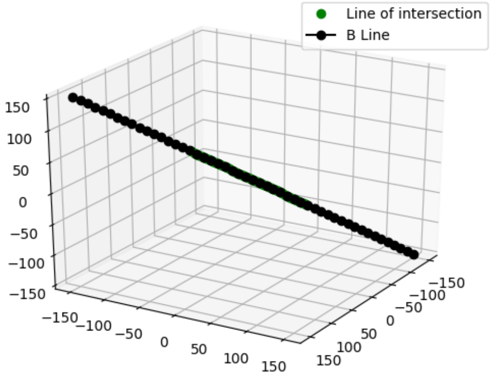


Figure: B- Line In Question

Figures

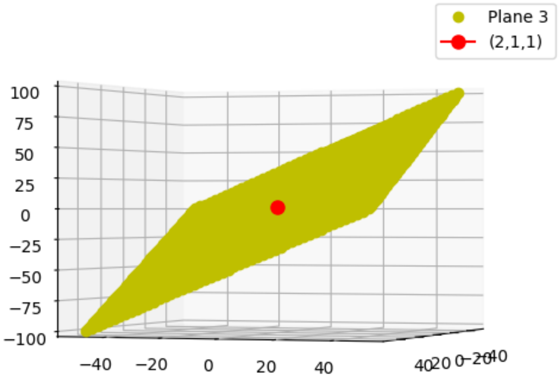
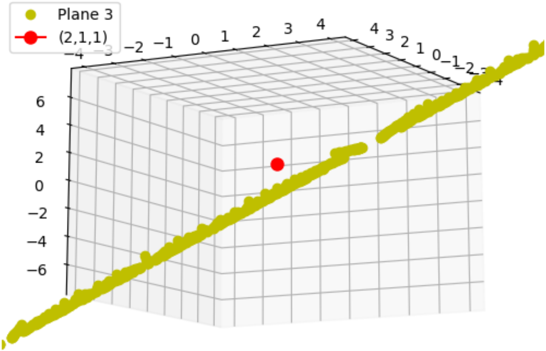


Figure: D- Point and Plane

Figures



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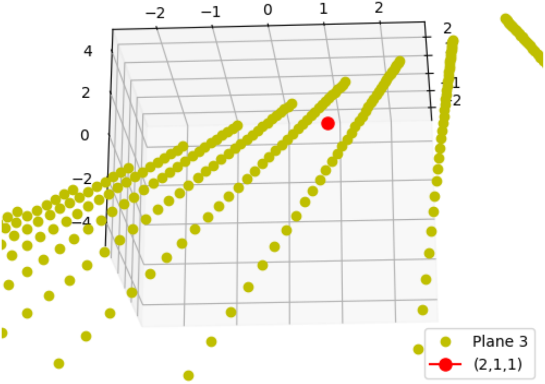

Figure: D- Point and Plane

The End