DELAUNAY TRIANGULATION

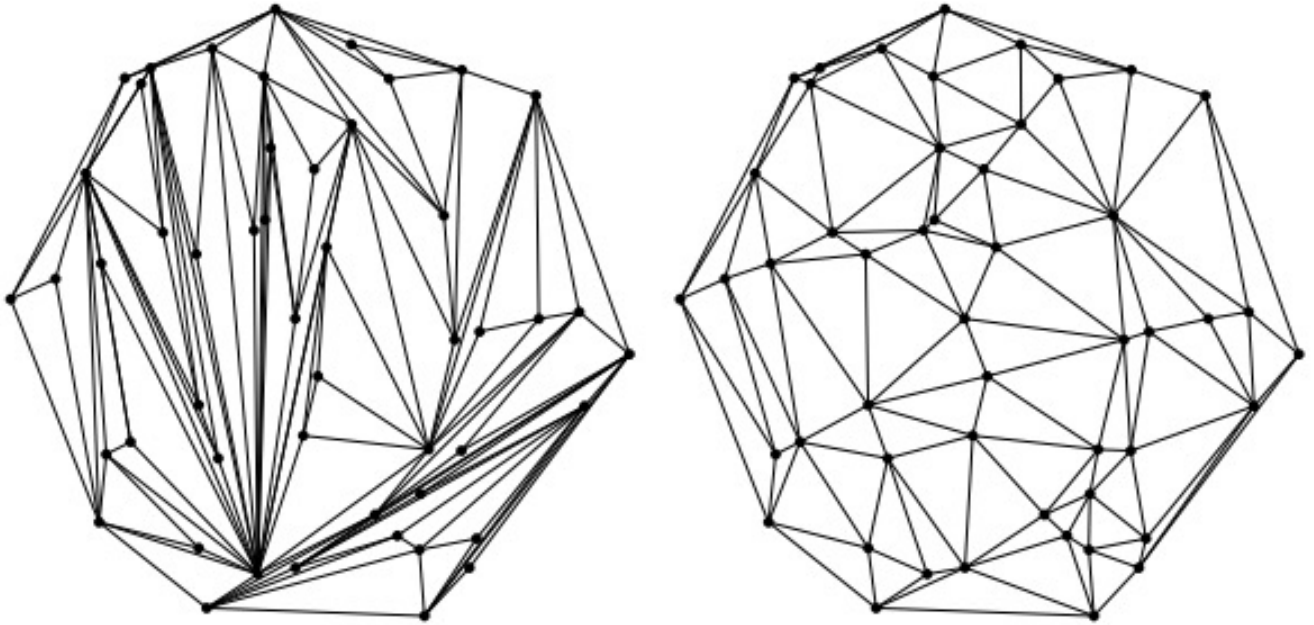
Karthik Iyer

CSCE 645 Geometric Modelling

Delaunay Triangulation

TEXAS A&M UNIVERSITY

A triangulation of a set of discrete points $\{P_i\}$ such that no point P_i is inside the circumcircle of any triangle in the triangulation

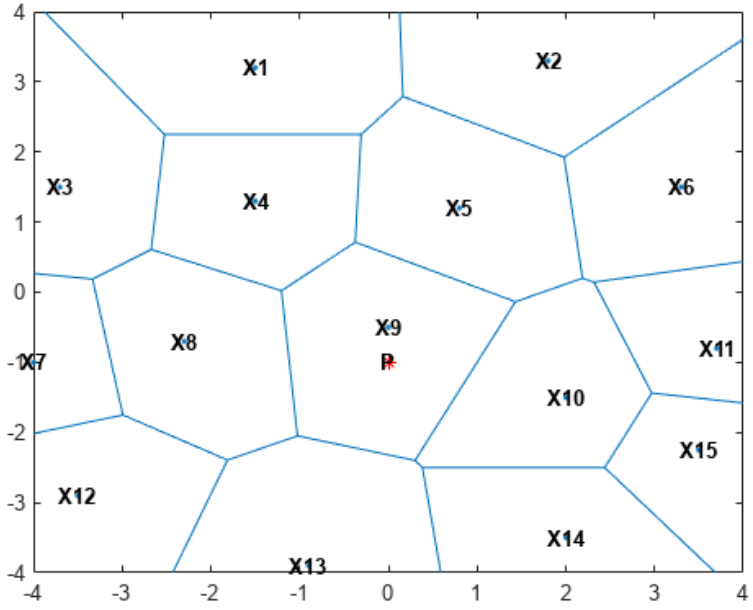


(a) Scan triangulation. (b) Delaunay triangulation.

https://ti.inf.ethz.ch/ew/Lehre/CG13/lecture/Chapter%206.pdf

Voronoi Diagram

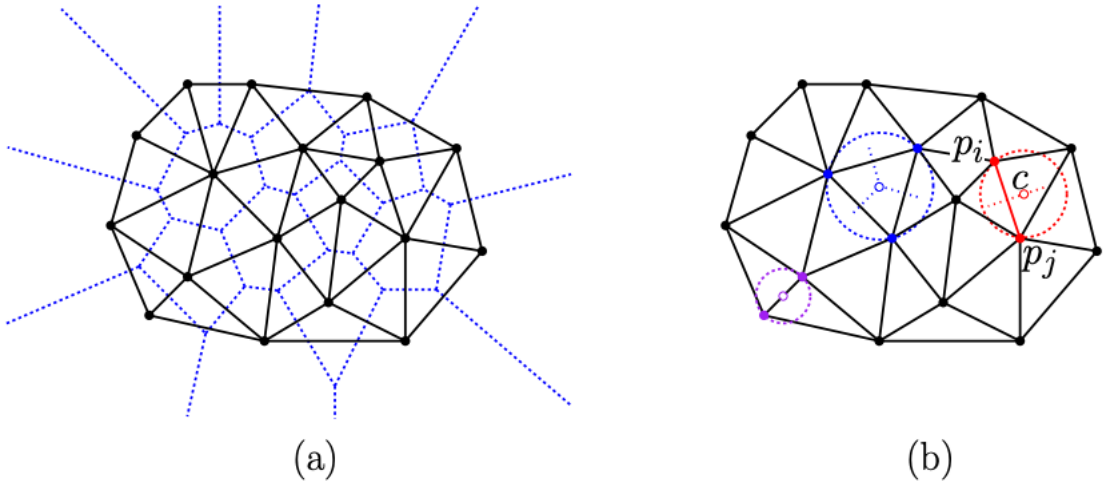
A dual of Delaunay Triangulation where any point is the cell is closest to the site of the cell.



https://www.mathworks.com/help/matlab/math/voronoi-diagrams.html

Properties

- Convex Hull: The boundary of the triangulation is the convex hull of the points Pi
- Empty Circle: Circle with edge as diameter is empty

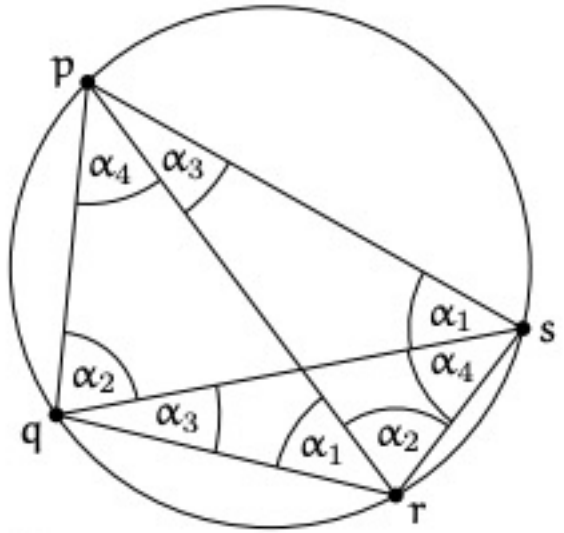


(a) (b)

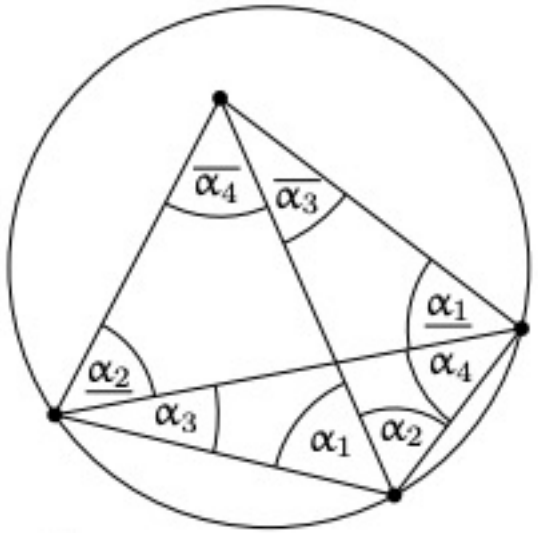
https://www.cs.umd.edu/class/spring2020/cmsc754/Lects/lect12-delaun-prop.pdf

Properties

- Maximizing smallest angle: DT avoids slivers



(a) Four cocircular points and the induced eight angles.

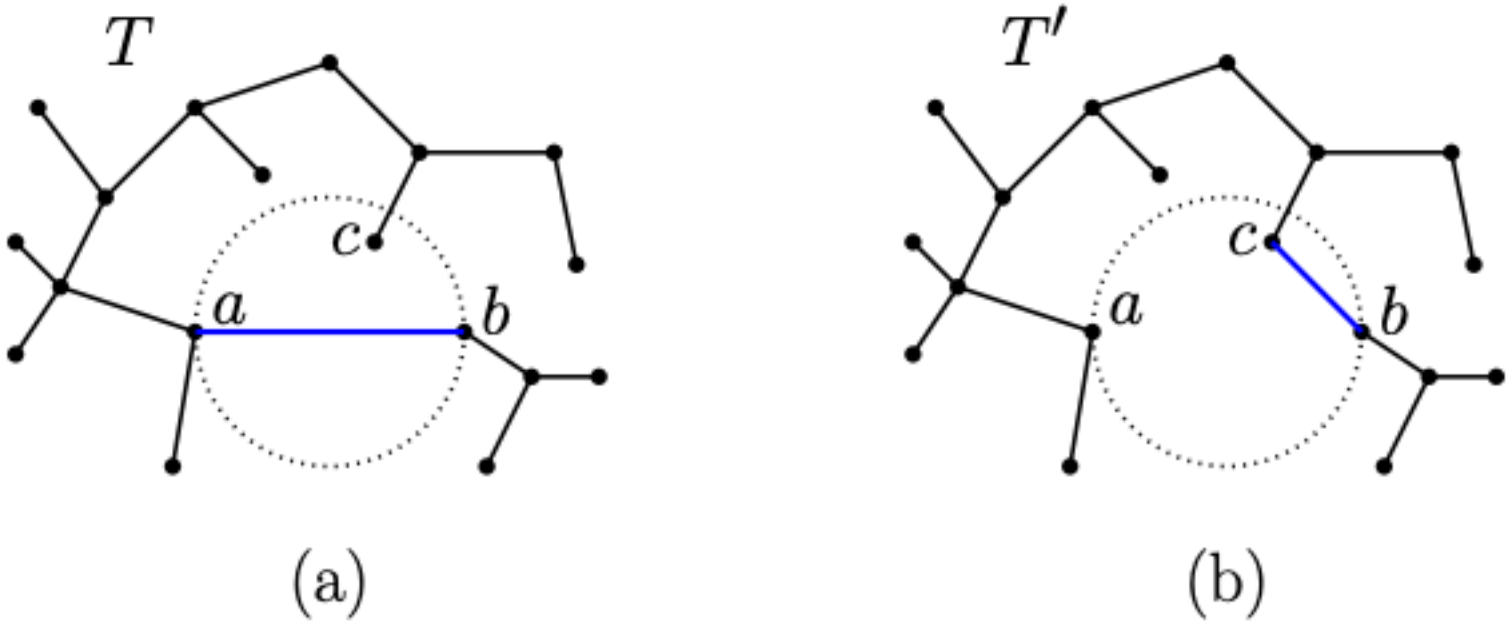


(b) The situation before a flip.

https://ti.inf.ethz.ch/ew/Lehre/CG13/lecture/Chapter%206.pdf

Properties

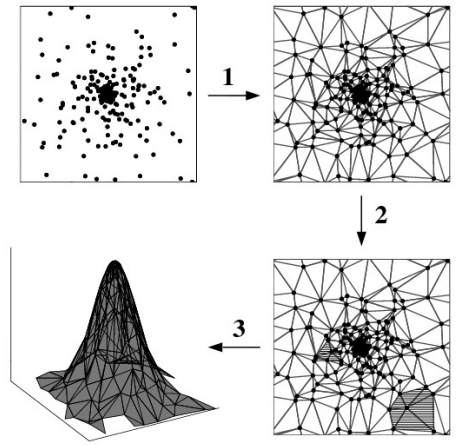
- Minimum spanning tree is part of the DT



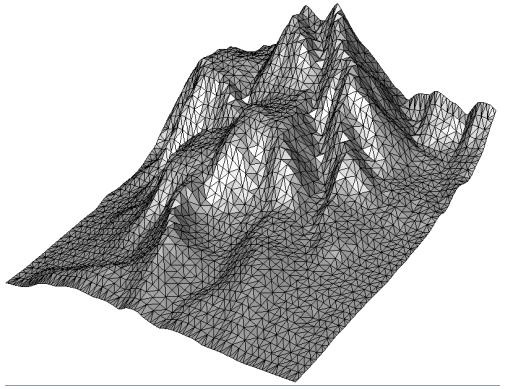
https://www.cs.umd.edu/class/spring2020/cmsc754/Lects/lect12-delaun-prop.pdf

Applications

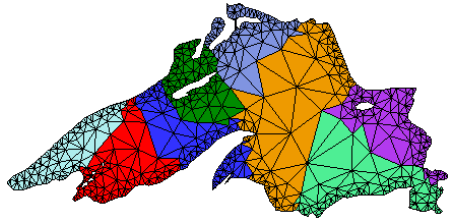
- Terrain Modelling
- Generate meshes for FEM/FVM
- Path planning
- Delaunay Tessellation Field Estimator



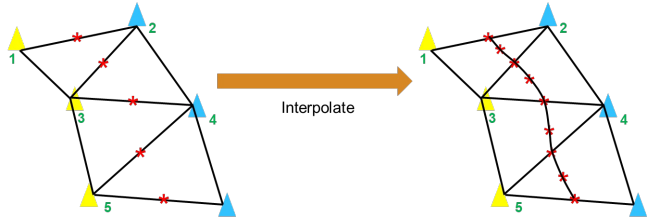
https://en.wikipedia.org/wiki/File:Delaunay_tessellation_field_estimator_(overview).jpg



https://www.geom.at/terrain-triangulation/



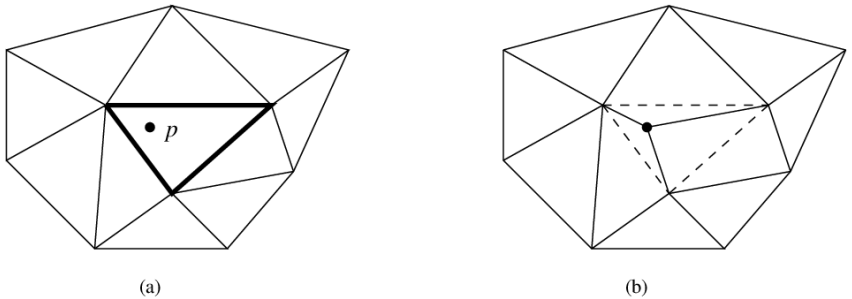
https://www.cs.cmu.edu/~quake/triangle.html



https://blogs.mathworks.com/student-lounge/2022/10/03/path-planning-for-formula-student-driverless-cars-using-delaunay-triangulation/

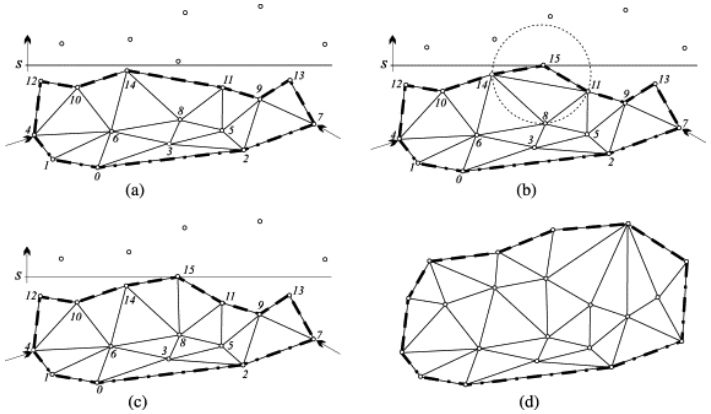
Construction

Incremental



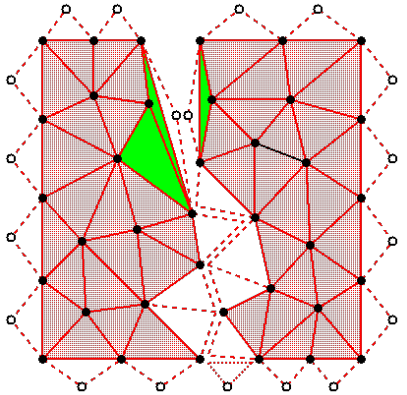
https://doi.org/10.1016/B978-0-12-336156-1.50014-8

Sweepline



https://doi.org/10.1016/j.cad.2004.10.004

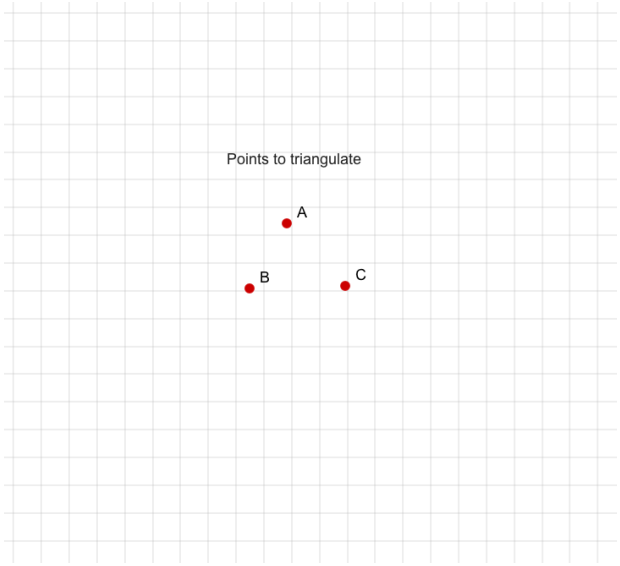
Divide and conquer

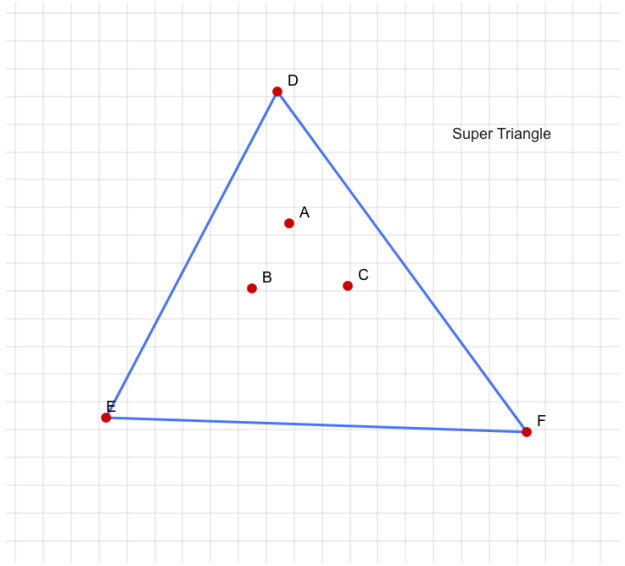


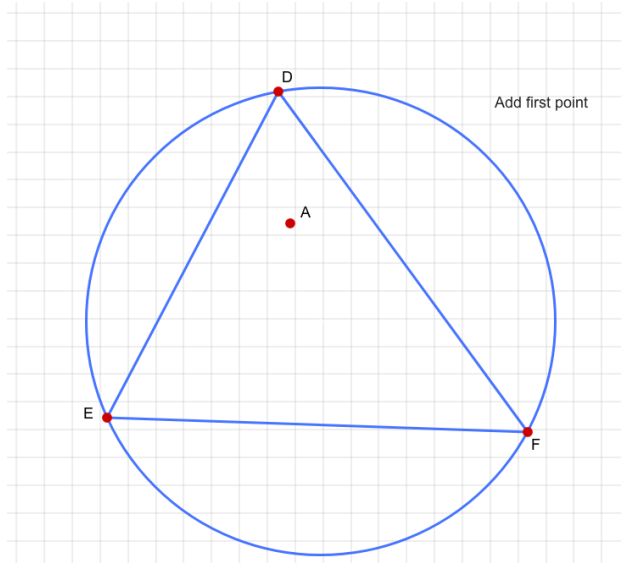
https://www.cs.cmu.edu/~quake/tripaper/triangle2.html

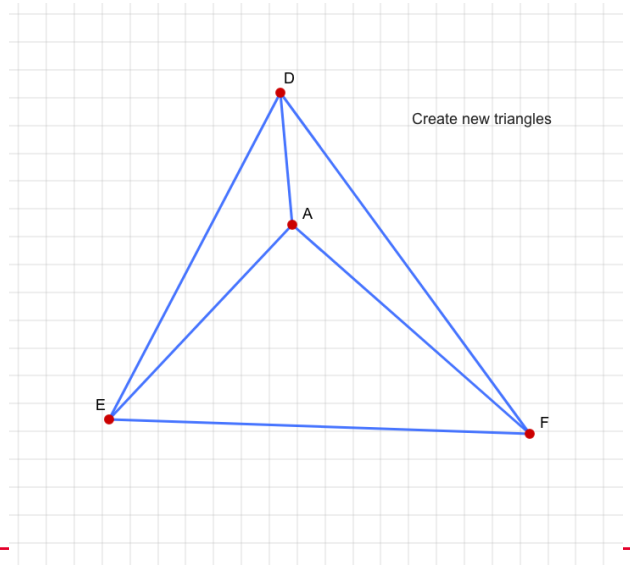
BOWYER-WATSON ALGORITHM

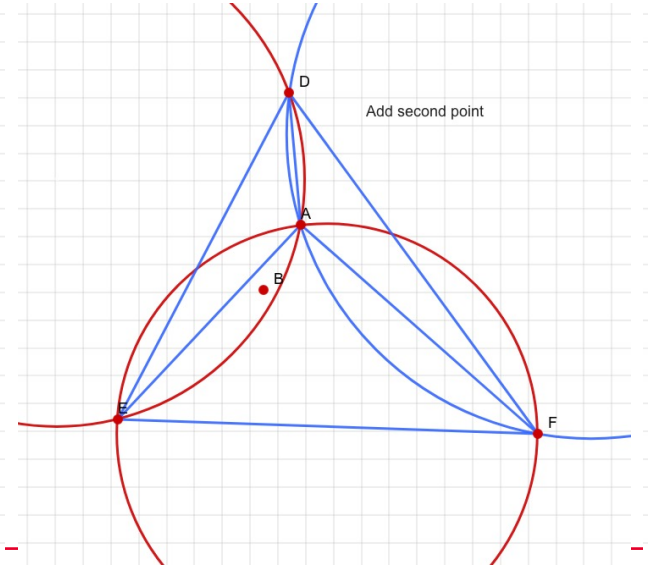
Bowyer-Watson Algorithm

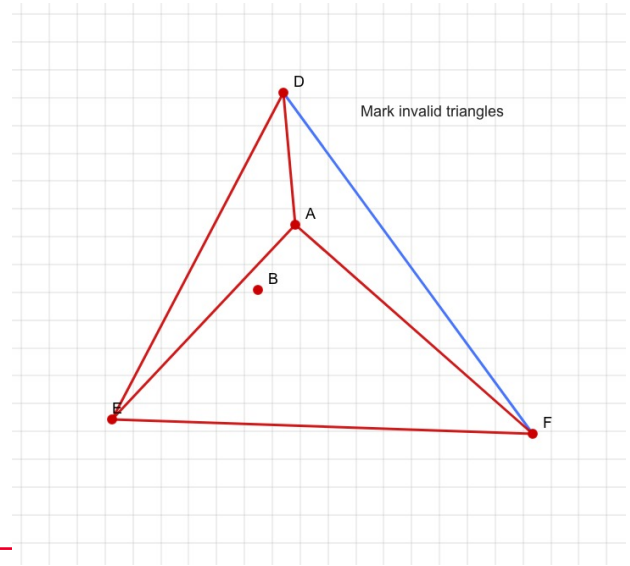




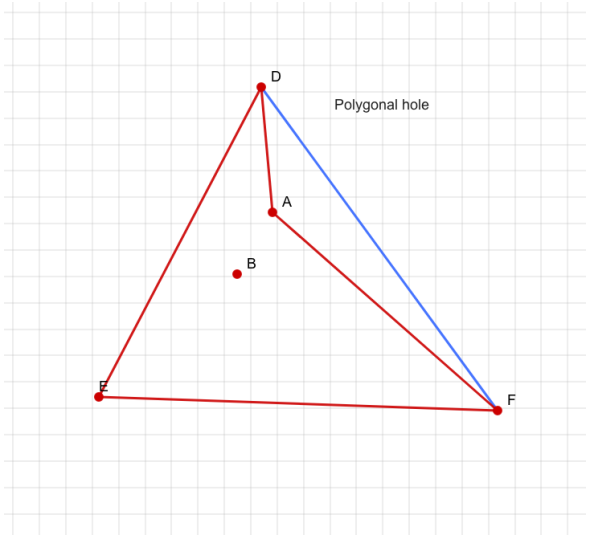


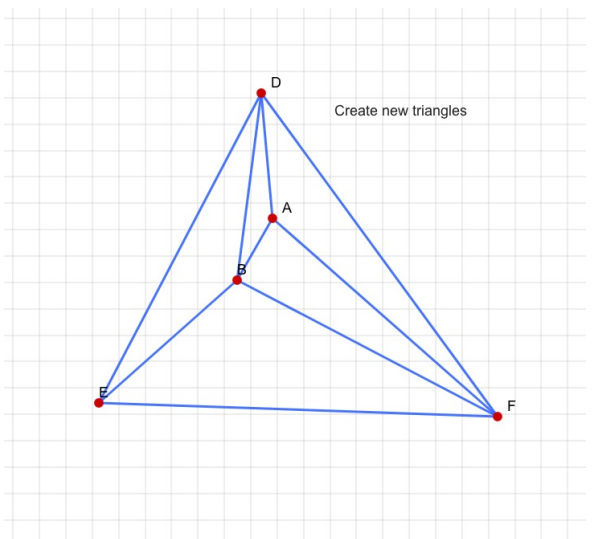


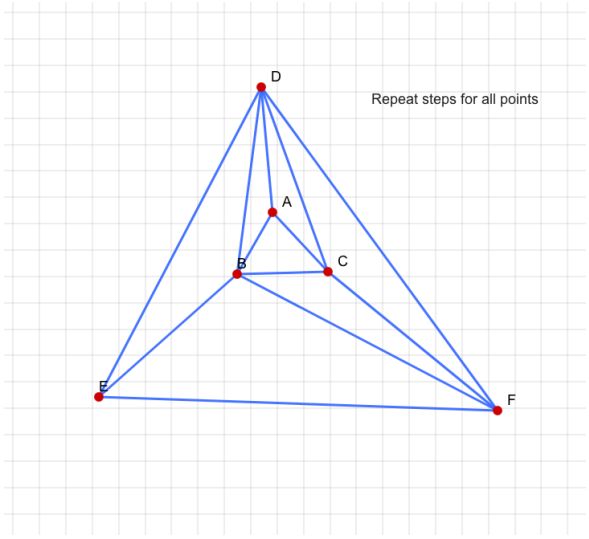


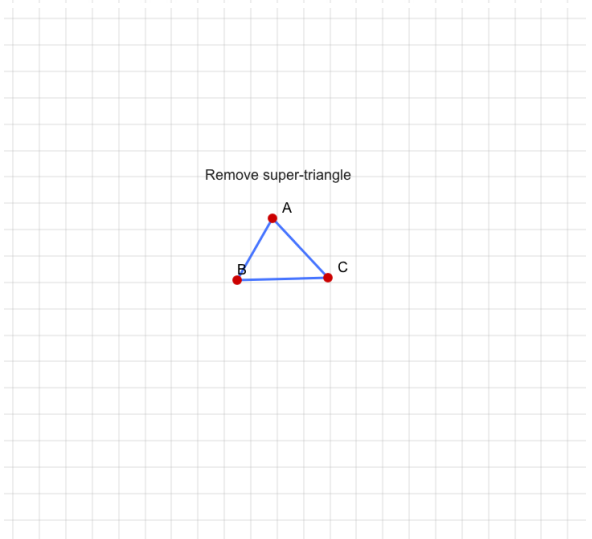


Bowyer-Watson Algorithm

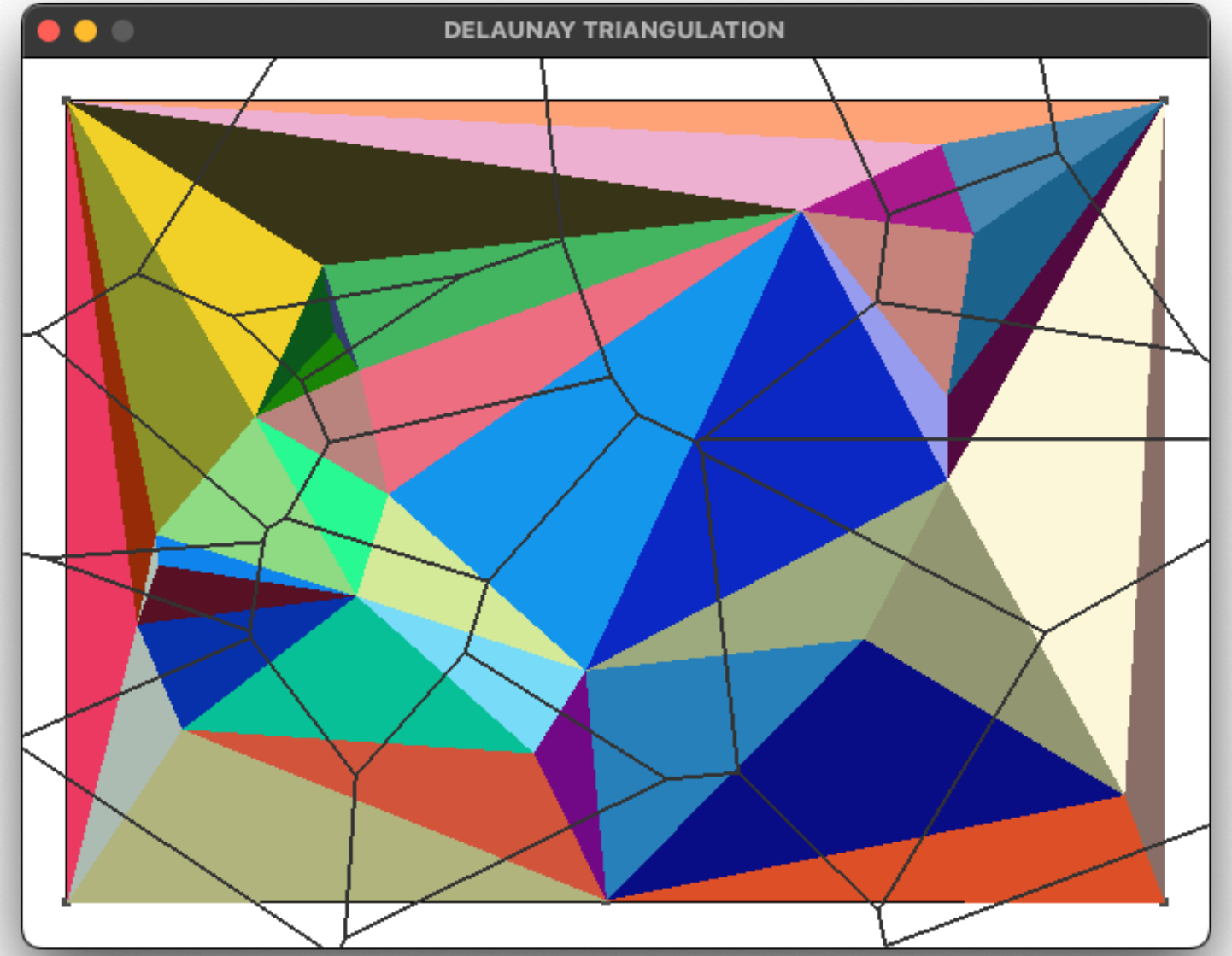








Implementation



Thank You