Delaunay Triangulation and Voronoi Diagram

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1 Overview

A Delaunay Triangulation of a set of points is a triangulation such that no point is inside the circumcircle of any triangle in the triangulation. There can be many different triangulations for a set of points, but Delaunay triangulations maximize the minimum of all angles of the triangles. In other words, the triangulation tends to avoid thin triangles. The geometric properties of Delaunay Triangulations have many applications across various domains such as modeling terrain from a point cloud, generating meshes for finite element method, and can also be used to create algorithmic art.

Delaunay Triangulation can also be used to derive a dual Voronoi diagram, which is a set of polygons with the circumcenters of the triangles as vertices. Voronoi diagrams also have a plethora of applications, such as calculating average precipitation at each weather station, modeling biological structures such as cells, algorithmic art, and architecture.

Through this project, I would like to explore the Delaunay Triangulation and the dual Voronoi diagram for a given set of points. I would like to write a GUI program that allows the user to either specify a set of points, or randomly generate points, and then triangulate them. A lot of research has gone into finding efficient ways to generate triangulations conforming to the Delaunay criterion, but they can be broadly classified into the following categories: Incremental, Divide and Conquer, Sweepline, Gift wrapping, and Convex hull [SS97]. I would like to implement the Bowyer-Watson algorithm, which is one of the most popular incremental triangulation algorithms. It works by adding one point at a time and deleting any existing triangles whose circumcircle contains the newly added point. The hole left behind by the deleted triangles is filled with new triangles with the new point as a vertex. A relatively simple variant of this algorithm was given by Paul Bourke [Bur89]. In addition to the triangulation, I would also like to implement the dual Voronoi diagram, which can be constructed by connecting the circumcenters of the triangles. A special type of Voronoi tesselation called Centroidal Voronoi Tessealtion can be generated, where each generating point of the Voronoi cell is at its centroid or center of mass. A general Voronoi tesselation can be converted to a Centroidal Voronoi Tesselation using Lloyd's algorithm, which involves computing the centroid of each Voronoi cell and moving the points to these centroids iteratively until convergence [QD06].

2 Project Goals

- Implement Delaunay Triangulation based on the Bowyer-Watson algorithm.
- Extend the triangulation implementation to find the dual Voronoi digram by connecting the centroid of the triangles.
- Compute the Centroidal Voronoi Tesselation (CVT) based on Lloyd's relaxation algorithm.
- Stretch Goals: If the above mentioned goals are achieved in advance, explore and implement other CVT algorithms such as Newton's method and BFGS.

References

[Bur89] Paul Burke. Efficient triangulation algorithm suitable for terrain modelling. *Pan Pacific Computer Conference*, 1989.

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